

Detecting single gravitons with quantum sensing

Germain Tobar^{*}, *Sreenath K Manikandan*^{*}, *Thomas Beitel*,
and Igor Pikovski

Arxiv:2308.15440 (2023)
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Stockholm
University



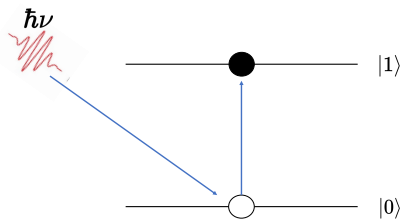
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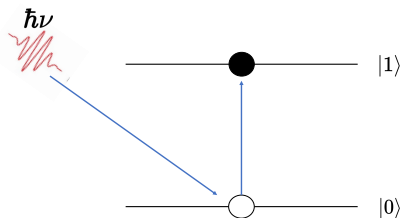
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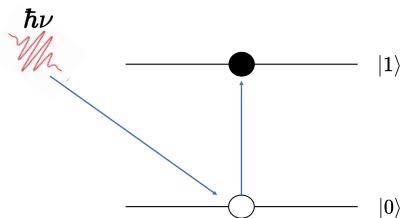


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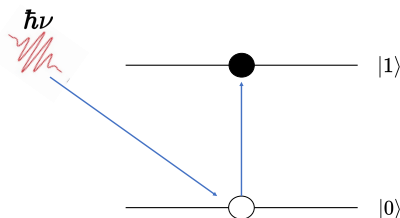
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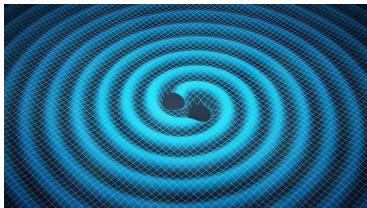
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- 2 The photo-electric effect works on exactly the same principle, but $|0\rangle \rightarrow |k\rangle$, where $|k\rangle$ is a state in the continuum of excited states.
- 3 Original studies of photon detections - stimulated processes (photo-electric effect). Modern view: 'detection' is only when there is a single-photon-input'.

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Question: Can we observe such a detection process for the interaction between gravitational waves and quantum matter?

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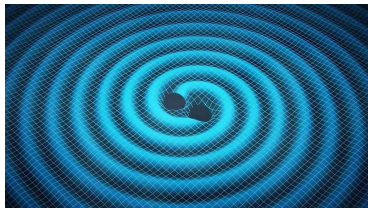
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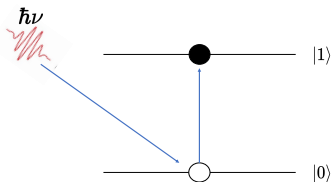
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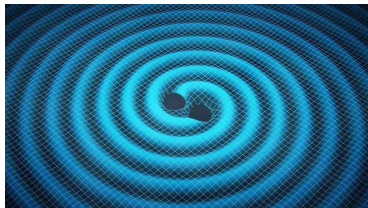
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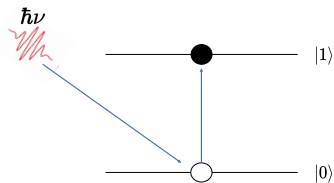
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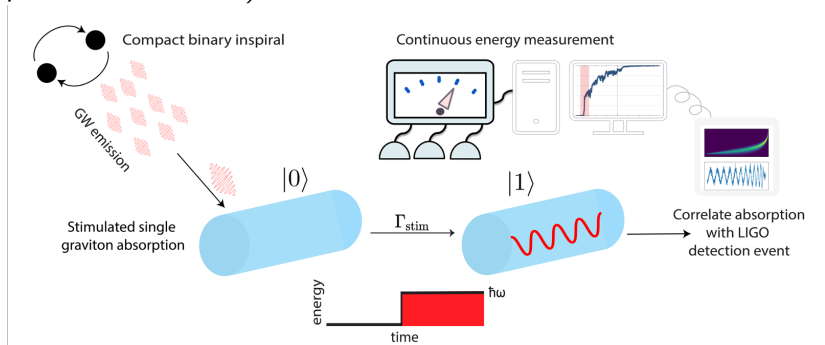
Our answer: Yes!

Tobar, Manikandan, Beitel, Pikovski Arxiv:2308.15440 (2023)

Quantum-jumps between energy levels of a massive quantum acoustic resonator, induced by a gravitational wave.

Particle detection for gravitational waves

You do not need a single graviton input, to infer the exchange of single energy quanta between matter and gravitational waves (as occurs in the photo-electric effect)



Single graviton processes

- Linearized quantum gravity, low energy regime: Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg 1972, Lightman 1973, Boughn and Rothman 2006.

First quantize:

$$\hat{h}^{ij} = \sum_{\mathbf{k}, \lambda} e_{\mathbf{k}, \lambda}^{ij} h_{q\mathbf{k}, \lambda} \hat{a} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} + cc \quad (1)$$

$$h_{q\mathbf{k}, \lambda} = \sqrt{\frac{16\pi G \hbar}{c^2 v_k V}} \quad (2)$$

Then compute the graviton transition rate:

$$\Gamma_{\text{atom}} (3d2 \rightarrow 1s) = \frac{2\pi}{\hbar} \left| \langle 1s | \langle 1 | \hat{H}_{\text{int}} | 0 \rangle | 3d2 \rangle \right|^2 \rho \quad (3)$$
$$\approx 10^{-40} \text{ s}^{-1}.$$

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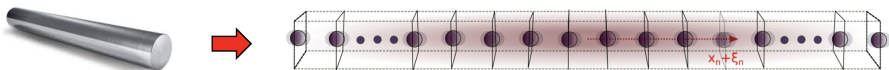
Massive acoustic resonators (Weber Bars)



Cho Adrian. 'Remembering Joseph Weber, the controversial pioneer of gravitational waves'. Science 12 (2016).

An enhancement to the graviton-matter interaction

Weber-BARs provide a macroscopic enhancement for the graviton-matter interaction as compared to the case where the matter is an atom:



$$H_{\text{int}} \approx -m \sum_n \frac{1}{4} \ddot{h}_{xx}(t) (x_n + \xi_n)^2 \approx -\frac{ML\ddot{h}_{xx}(t)}{\pi^2} \sum_{l=1,3,5..} \frac{(-1)^{\frac{l-1}{2}}}{l^2} \chi_l. \quad (4)$$

Now, take the example of a Niobium-cylinder:



$$\rho_m = 8570 \frac{\text{kg}}{\text{m}^3} \quad v_s = 5 \frac{\text{km}}{\text{s}} \quad 2R = L = 1\text{m}$$

$$\Gamma_{\text{spn}} = 10^{-33} \text{s}^{-1}$$

Orders of magnitude larger than the atom, but still vanishingly small!

We now consider stimulated emission and absorption

$$\Gamma_{\text{stim}} (1 \rightarrow 0) = \frac{2\pi}{\hbar} \left| \langle 1 | \langle \alpha | \hat{H}_{\text{int}} | \alpha \rangle | 0 \rangle \right|^2 \rho = \frac{|\alpha|^2 8GML^2 \omega_l^4}{l^4 \pi^4 c^5} \quad (5)$$

with the number of gravitons in the gravitational wave as:

$$|\alpha|^2 \approx N = \frac{h_0^2 c^5}{32\pi G \hbar \omega_l^2} \quad (6)$$

, the stimulated emission rate is

$$\Gamma_{\text{stim}} = \frac{ML^2 \omega_l^2}{4l^4 \pi^5 \hbar} h_0^2 = \frac{Mv_s^2}{4l^4 \pi^3 \hbar} h_0^2. \quad (7)$$

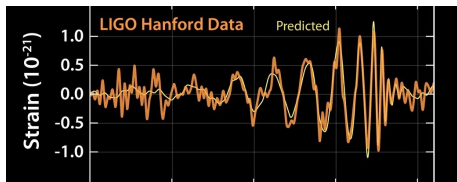
For an Aluminum BAR of Mass 1800 kg, and strain amplitude $h_0 = 5 \times 10^{-22}$ (GW150914), we obtain:

$$\Gamma_{\text{stim}} \approx 1 \text{ Hz}. \quad (8)$$

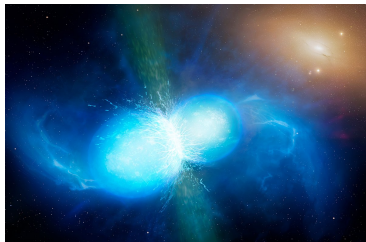
Chirping Gravitational Waves

However, detected gravitational waves chirp, in which case need to solve by accounting for the time-dependent interaction:

$$\hat{H} = \hbar\omega\hat{b}^\dagger\hat{b} + \frac{L}{\pi^2}\sqrt{\frac{M\hbar}{\omega}}\ddot{h}(t)(\hat{b} + \hat{b}^\dagger).$$



Chirping gravitational waves



The dynamics can be solved analytically

$$|0\rangle \rightarrow |\beta(t)e^{-i\omega t}\rangle \quad |\beta| = \frac{L}{\pi^2} \sqrt{\frac{M}{\omega\hbar}} \chi(h, \omega, t) \quad \chi(h, \omega, t) = \left| \int_0^t ds \ddot{h}(s) e^{i\omega s} \right| \quad (9)$$

$$P_{0 \rightarrow 1} = |\langle 1 | \beta e^{-i\omega t} \rangle|^2 = e^{-|\beta|^2} |\beta|^2 \quad (10)$$

$$P_{\max} = \frac{1}{e} \rightarrow \sim 36\% \quad |\beta|_{\max} = 1$$

Chirping gravitational waves

Optimise the mass for a single graviton exchange:

$$|\beta|_{\max} = 1 \quad M = \frac{\pi^2 \hbar \omega^3}{v_s^2 \chi(h, \omega, t)}$$

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GW Source	GW170817 (NS-NS merger)	GW170817 (NS-NS merger)	GW170608 (BH-BH merger)	GW150914 (BH-BH merger)	J1301+0833 (black-widow pulsar)	J1748-2446ad (fast-spinning pulsar)	A0620-00 (BH Super-radiance)	Primordial (rare BH-BH merger)
$f = \frac{\omega}{2\pi}$	100 Hz	150 Hz	175 Hz	200 Hz	1085 Hz	1433 Hz	33 kHz	5.5 MHz
$h_0(f)$	10^{-22}	2×10^{-22}	2×10^{-22}	10^{-21}	$< 10^{-25}$	$< 10^{-25}$	3×10^{-21}	10^{-16}
M_c	$1.19 M_\odot$	$1.19 M_\odot$	$7.9 M_\odot$	$28.6 M_\odot$	Continuous	Continuous	Continuous	$5 \times 10^{-4} M_\odot$
Material	Sapphire	Aluminum	Niobium	CuAl6%	Niobium	Superfluid He-4	Sapphire	Quartz
v_0	10 km/s	5.4 km/s	5 km/s	4.1 km/s	5 km/s	238 m/s	10 km/s	6.3 km/s
T	1 mK	1 mK	1 mK	1 mK	0.1 μ K	0.1 μ K	0.6 K	0.6 mK
Q-factor	10^{10}	10^{10}	10^{10}	10^{10}	10^{10}	10^{13}	10^{10}	10^{10}
M	~ 100 kg	~ 250 kg	~ 9 t	~ 6 t	> 52 t	> 20 t	~ 100 kg	~ 10 g

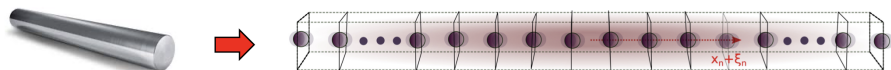
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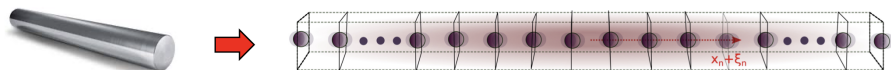
$$\begin{aligned} \text{BAR detector resonance frequency} &: f = 150 \text{ Hz} \\ \text{Strain Amplitude} &: h_0 = 10^{-22} \\ \text{Required environmental temperature} &: 1 \text{ mK} \\ \text{Required Q - factor} &: 10^{10} \\ \text{Optimal detector mass} &: 250 \text{ kg} \end{aligned} \quad (12)$$



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What has been achieved?

Chirping gravitational waves

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Required environmental temperature : 1 mK

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(13)

What parameters have been achieved?

High Sensitivity Gravitational Wave Antenna with Parametric Transducer Readout

D. G. Blair, E. N. Ivanov, M. E. Tobar, P. J. Turner, F. van Kann, and I. S. Heng
Phys. Rev. Lett. **74**, 1908 – Published 13 March 1995

Gravitational wave detection with high frequency phonon trapping acoustic cavities

Maxim Goryachev and Michael E. Tobar
Phys. Rev. D **90**, 102005 – Published 24 November 2014

Progress towards ground state cooling of a 1.5 tonne Niobium BAR, with $Q \sim 10^8$ and $f = 700$ Hz

More recently, near ground state cooling for lower masses (gram scale) and higher frequencies (MHz), with $Q \sim 10^{10}$.

Photo-electric analogue

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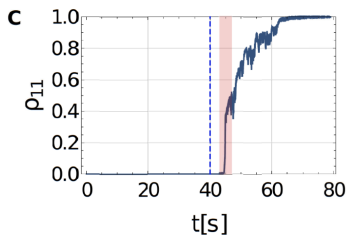
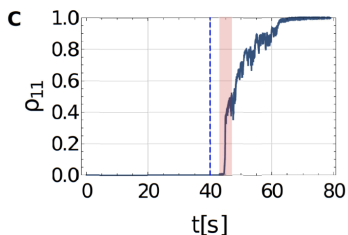


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Gives a direct gravito-phononic analogue of the photo-electric case:

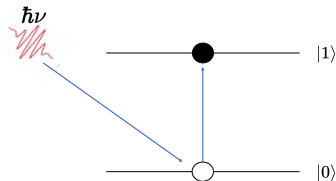
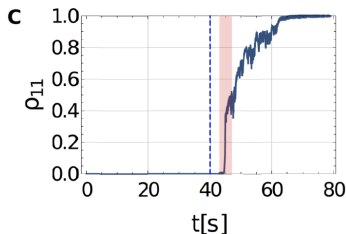


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What has been achieved?

Parity measurement in the strong dispersive regime of circuit quantum acoustodynamics

[Uwe von Lüpke](#) , [Yu Yang](#), [Marius Bild](#), [Laurent Michaud](#), [Matteo Fadel](#) & [Yiwen Chu](#) 

[Nature Physics](#) **18**, 794–799 (2022)

Direct measurement of individual energy levels of microgram mass acoustic resonators

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In our gravito-phononic set-up, we have:

- Threshold frequency: $P_{0 \rightarrow 1} \approx \frac{\hbar_0^2 \omega^3 M L^2}{\hbar \pi^4 (\nu - \omega)^2} \sin^2 \frac{(\nu - \omega)t}{2}$.
- Independence of ejected gravito-phonon energy ($\hbar\omega$) from the GW amplitude h .
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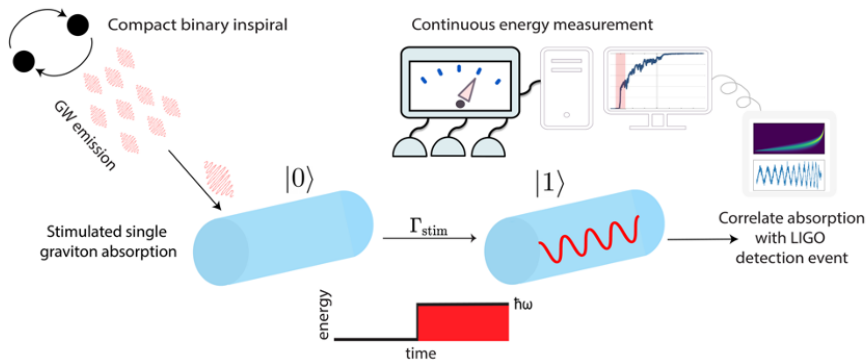
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If energy is conserved, the experiment is inconsistent with the gravitational field treated as a classical-continuous wave that solves the linearised Einstein equations.

Protocol Summary



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