A simple model of a black hole ~~and its radiation~~

AdS/CFT for Relativistic Quantum Information

Bo Sundborg, Stockholm University, Nov 17 2023

Take-aways

for today

- *Standard quantum mechanics* in the form of quantum field theory *can model quantum effects of gravity*, eg around black holes.

- *Composite objects* probe tidal effects of gravity through *scattering-like processes*.

- Black hole-like objects can be modeled by thermal equilibrium in a *gravitational cavity - AdS*, or by energetic pure states.

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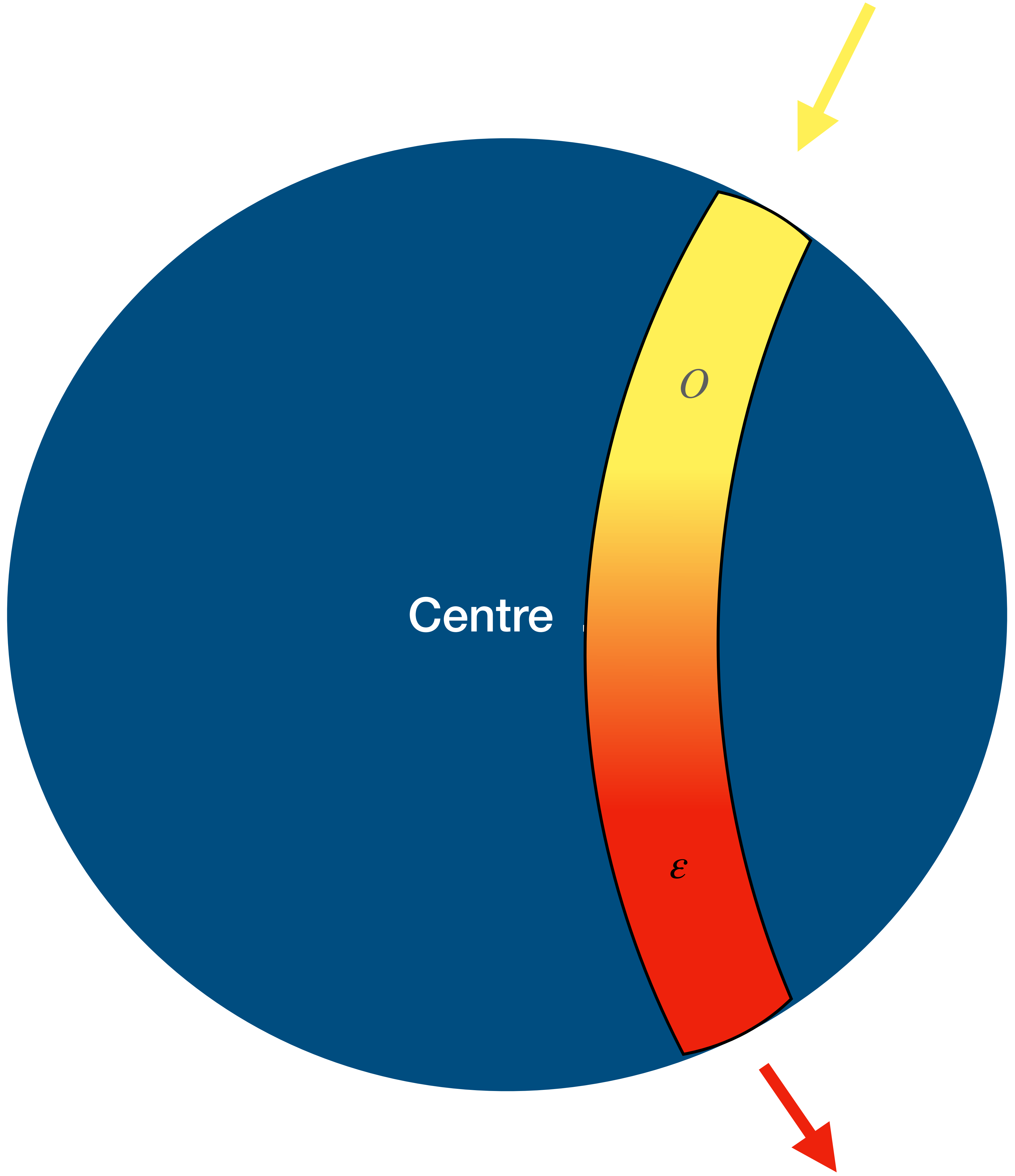

Design goals

For the talk and the research

- *Operational approach*: Define gravity by *measurable quantities* - Green functions of quantum field theory (QFT).

- Background independence: Avoid coordinate choice ambiguities by *measurements in weak fields* - at infinity

- ~~Unruh-DeWitt~~: Probe gravity by *composite objects* represented by composite QFT operators.

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- *Operational approach*: Define gravity by *measurable quantities* - Green functions of quantum field theory (QFT).
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- ~~Unruh-DeWitt~~: Probe gravity by *composite objects* represented by composite QFT operators.

Unruh-DeWitt detectors replaced:

- ☑ Reliable detection only in asymptotic regions with vanishing fields
- ☑ Detectors are composite objects
- ☑ Detectors propagate just as any quantum object - no fixed trajectory
- ☑ Detectors are made of the same stuff as the fields they detect

Gravity represented by Quantum Field Theory

AdS/CFT

For RQI

AdS/CFT is an observation from 1997/1998 which connects results in *gravity and quantum field theory*.

It is still a conjecture.

- Evidence for it has been accumulating.
- There is no counterexample yet.
- The exact scope may be debated.

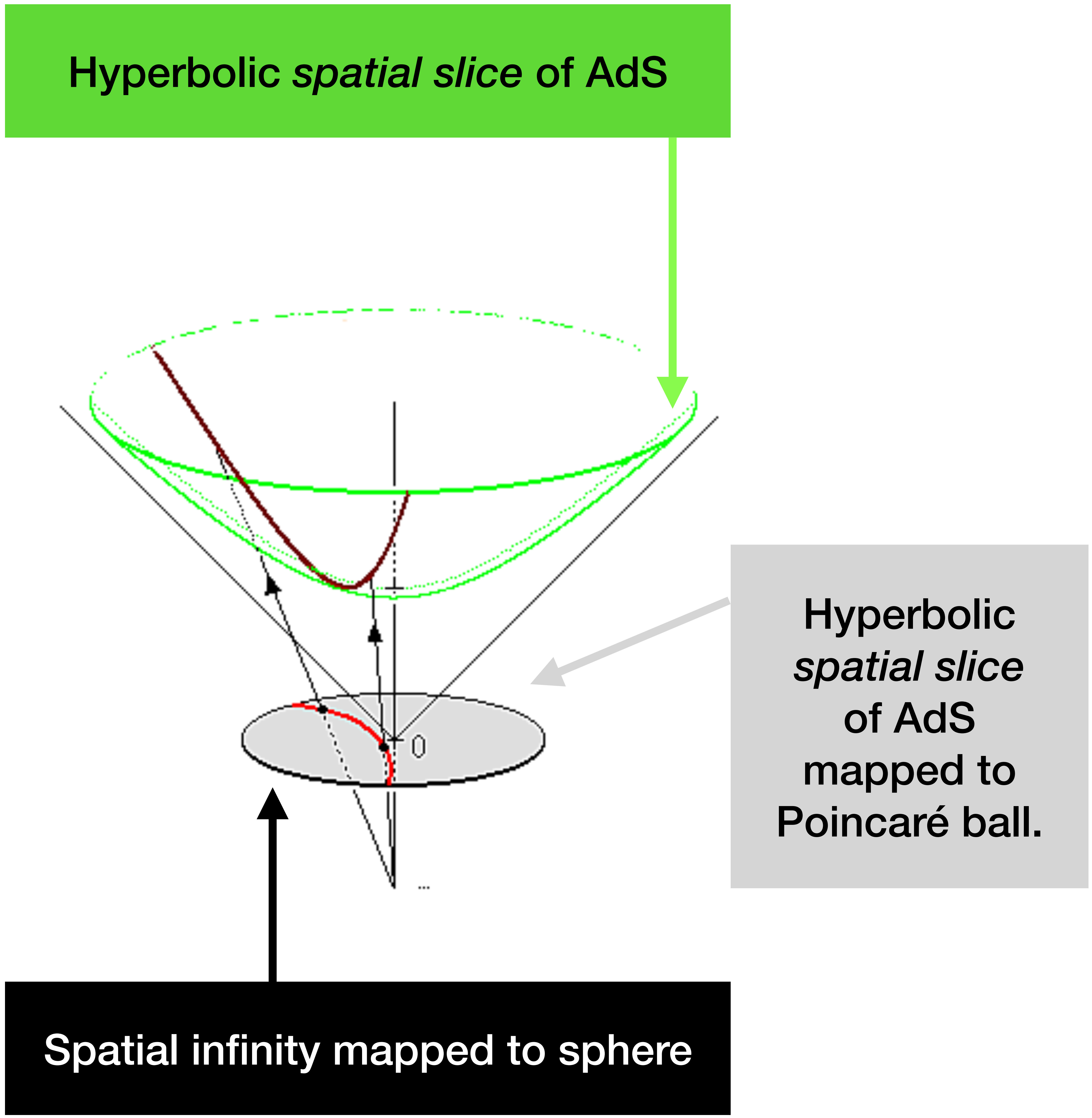
Simplest facts about AdS - Anti-De Sitter space, and CFT - conformal field theory

AdS

For RQI

Anti-De Sitter (*AdS*) *space* is the gravitational equivalent of a harmonic oscillator.

- It confines the motion of all waves and all massive particles.
- It is effectively a cavity.
- It has a natural *conformal structure* at infinity - simple light propagation.



CFT

For RQI

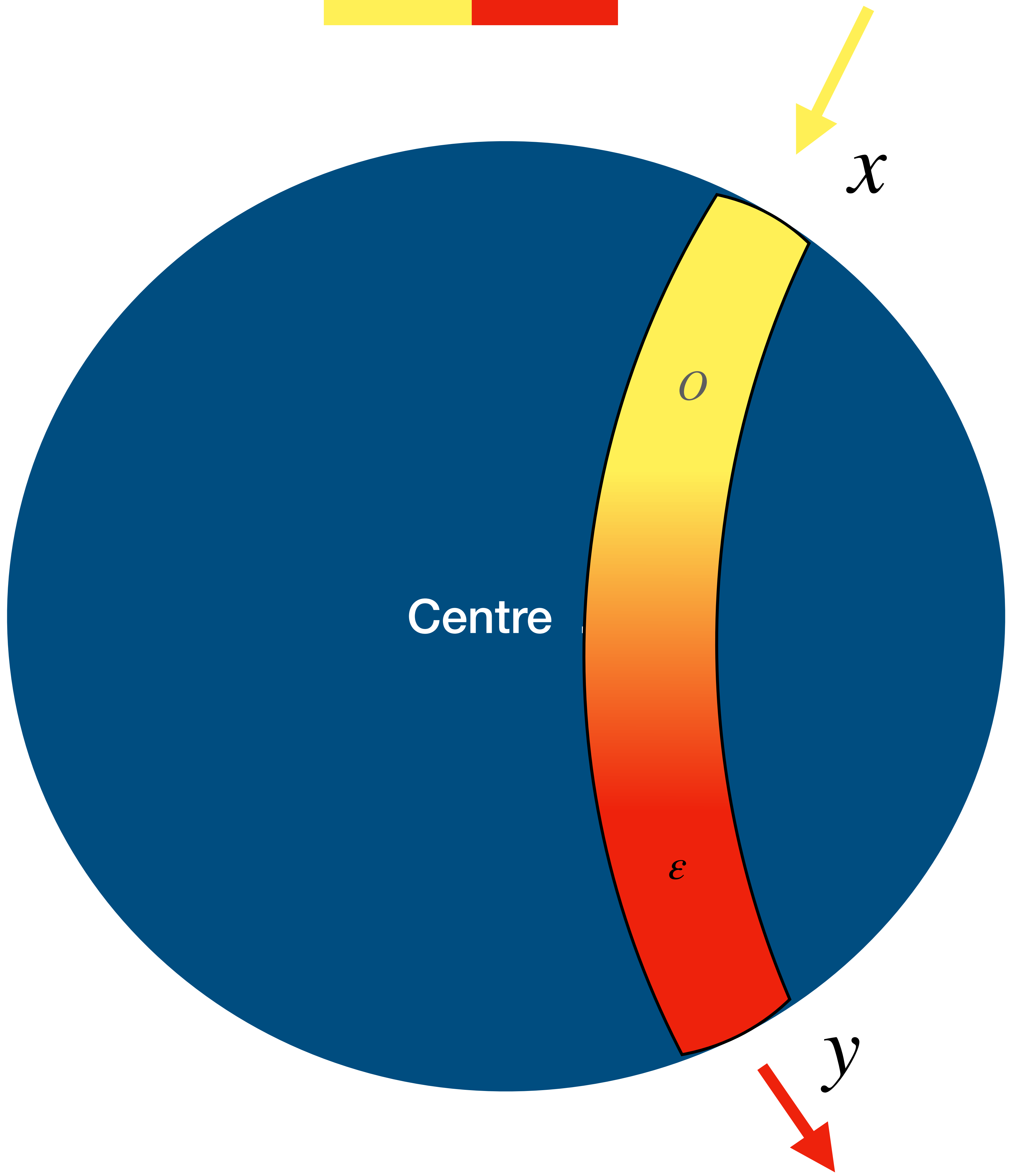
Conformal Field Theory (*CFT*) are the simplest (interacting) quantum field theories. CFTs require a *conformal structure* and are characterised by

- their spectrum of local operators $O_{\Delta_i, J_i}(x)$ of spin J_i and scaling dimension Δ_i and
- their "operator product coefficients" C_{ijk} coupling three operators O_i.

AdS/CFT

For RQI

$$G_{O\varepsilon}(x,y) = \langle O(x)\varepsilon(y)\rangle$$



The structure at infinity in AdS (conformal compactification) matches the conformal structure of CFTs.

The claim of AdS/CFT is that

- Fields in an AdS gravity theory matches the CFT spectrum O_i
- Green functions in (asymptotic) AdS and the CFT are directly related.

Calculate Green functions of O_i to obtain “scattering theory” in AdS

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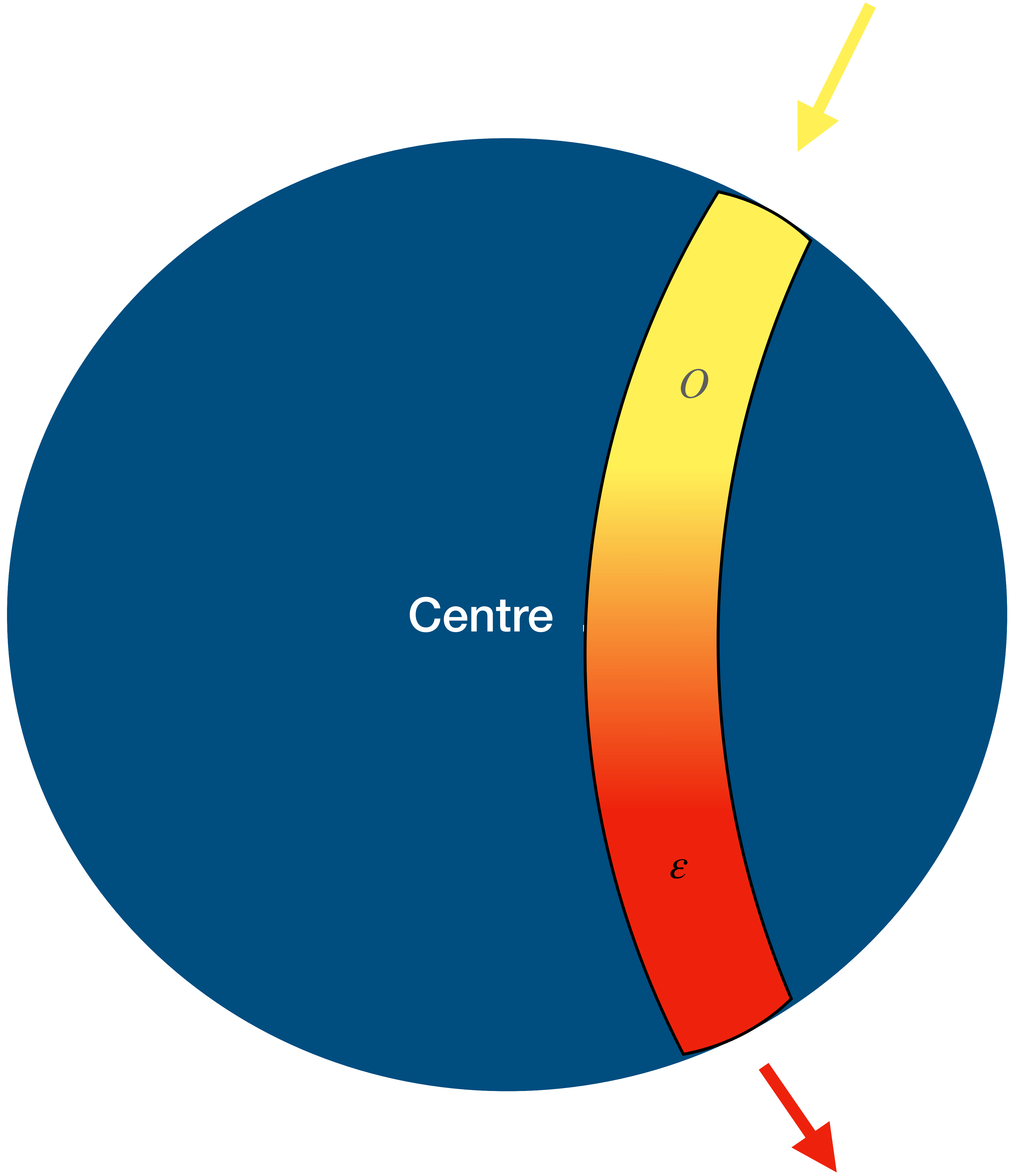
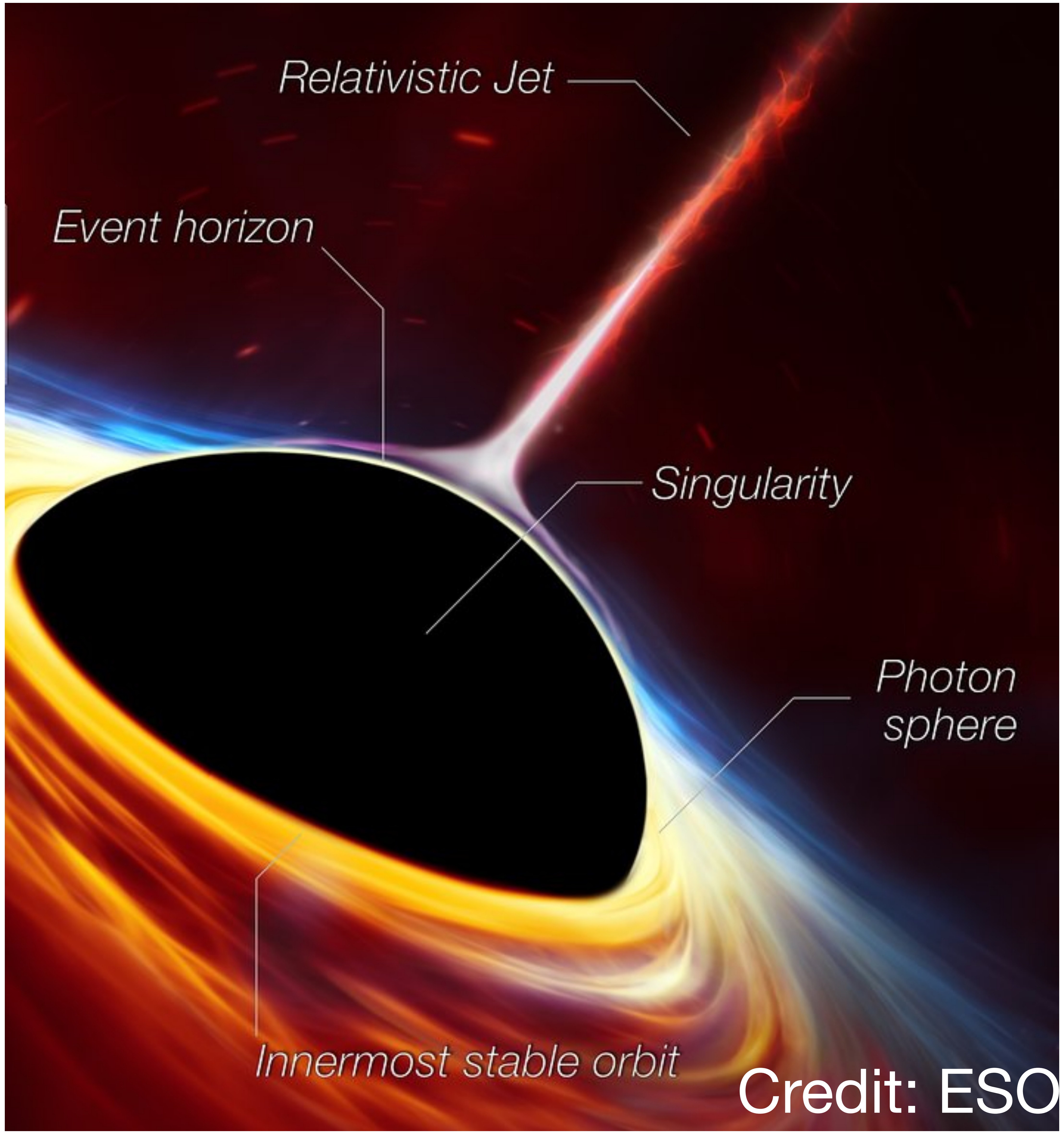


Black hole like objects

- Black holes have an event horizon from which no light can escape.
- Unclear whether there are actual black holes in quantum gravity.
- There are interesting strong gravity effects as soon as there is a photon sphere - light rays bent to circles.
- Look for objects with photon spheres!



Thermal equilibrium

And AdS black holes

- Black holes of mass M emit thermal Hawking radiation of temperature $T(M)$.
- For black holes in flat space thermal equilibrium is unstable.
- In AdS there are large thermally stable black holes for $T > T_{HP}$.
- There is a Hawking-Page phase transition at $T = T_{HP}$.

Flat space instability: $$\frac{\partial T(M)}{\partial M} < 0$$

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Flat space instability: $\frac{\partial T(M)}{\partial M} < 0$

We may also count states at fixed energy in AdS (without divergences) since AdS is effectively a cavity.

For large mass in AdS, $M(T)$ is increasing

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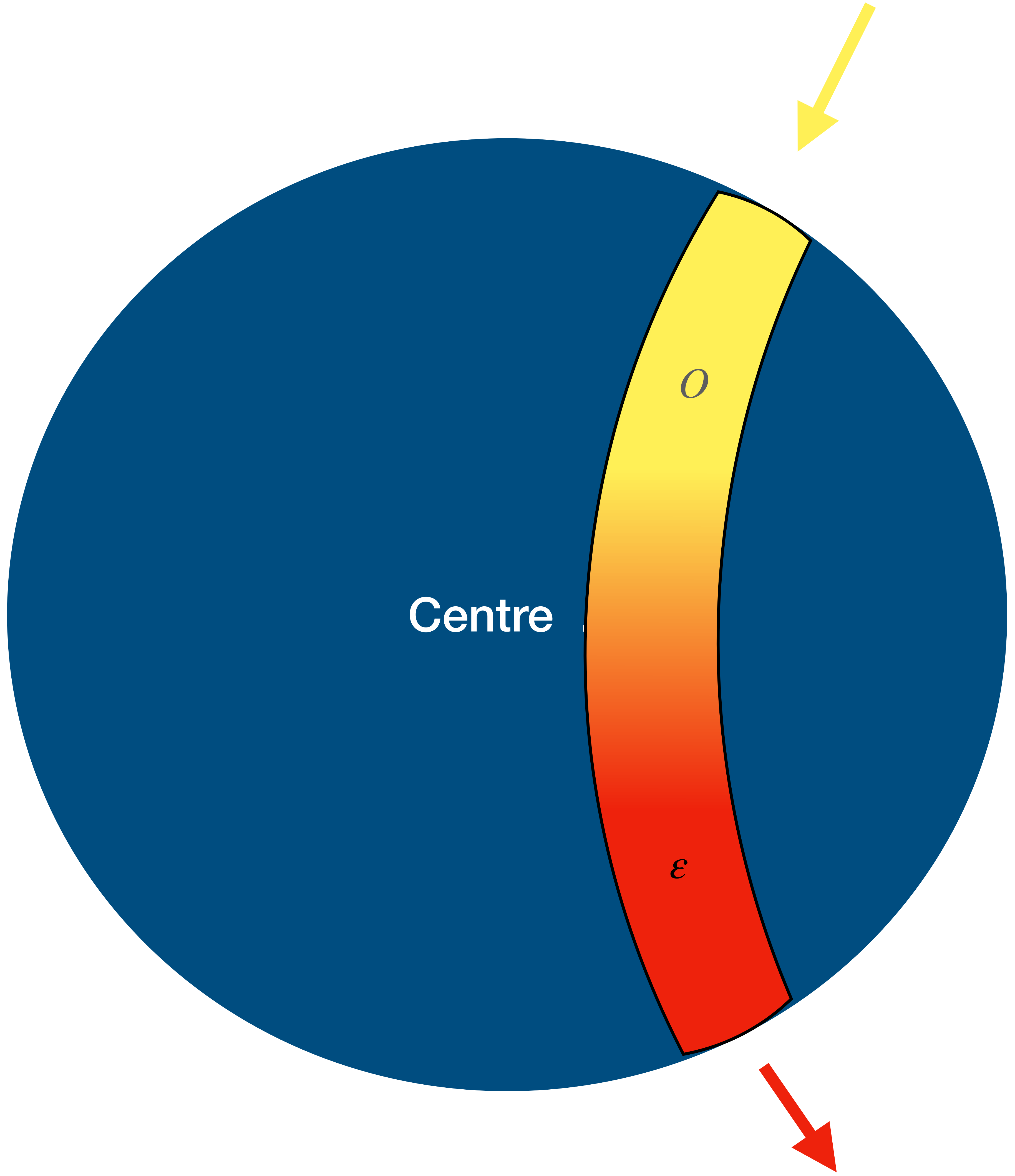
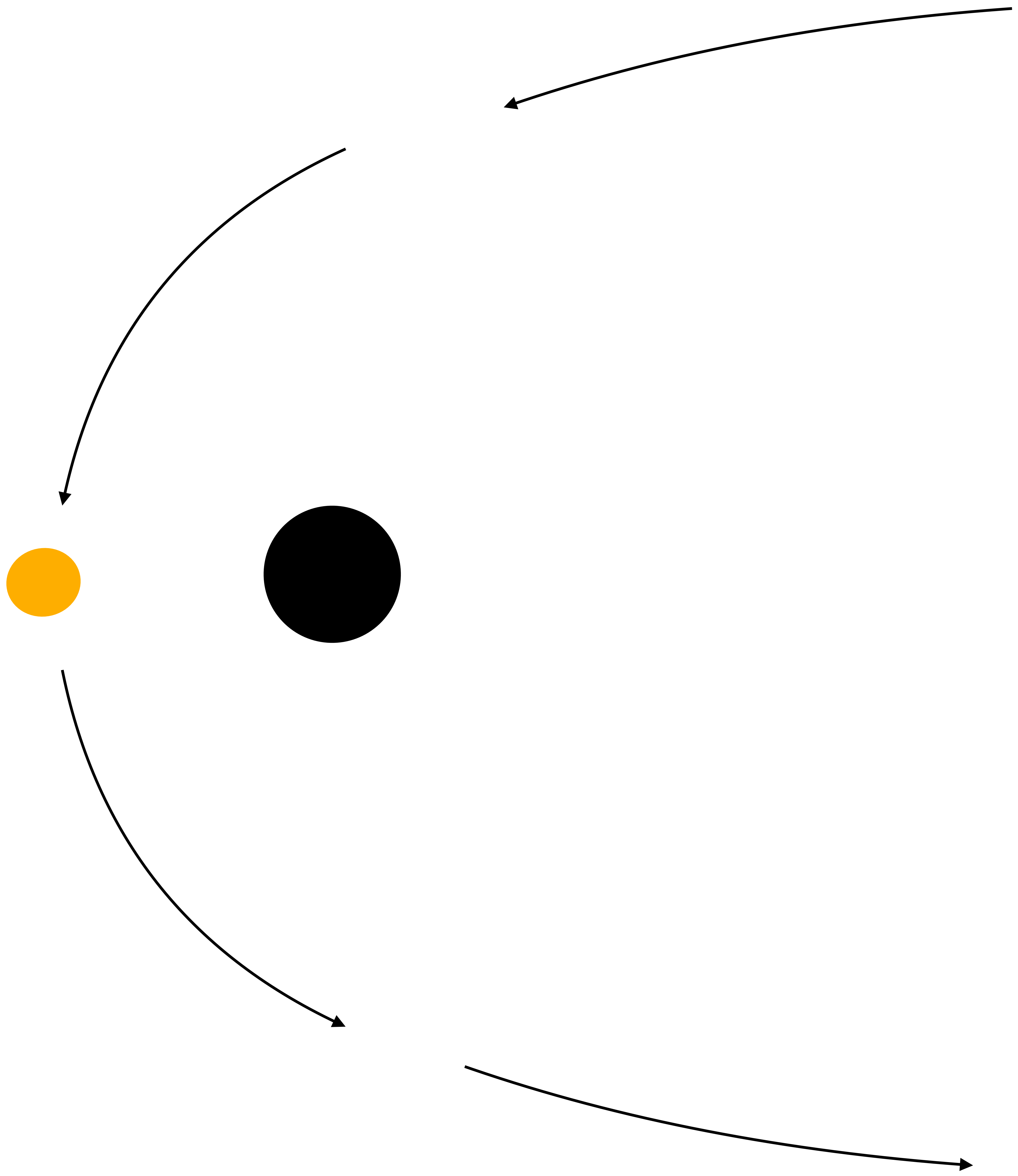


Composite objects

- probes of gravity

Test particles do not test gravity well - left unchanged by travel through a gravitational field

- Composite quantum objects are clocks - different energy levels
- Clocks measure gravitational time dilation



Composite objects

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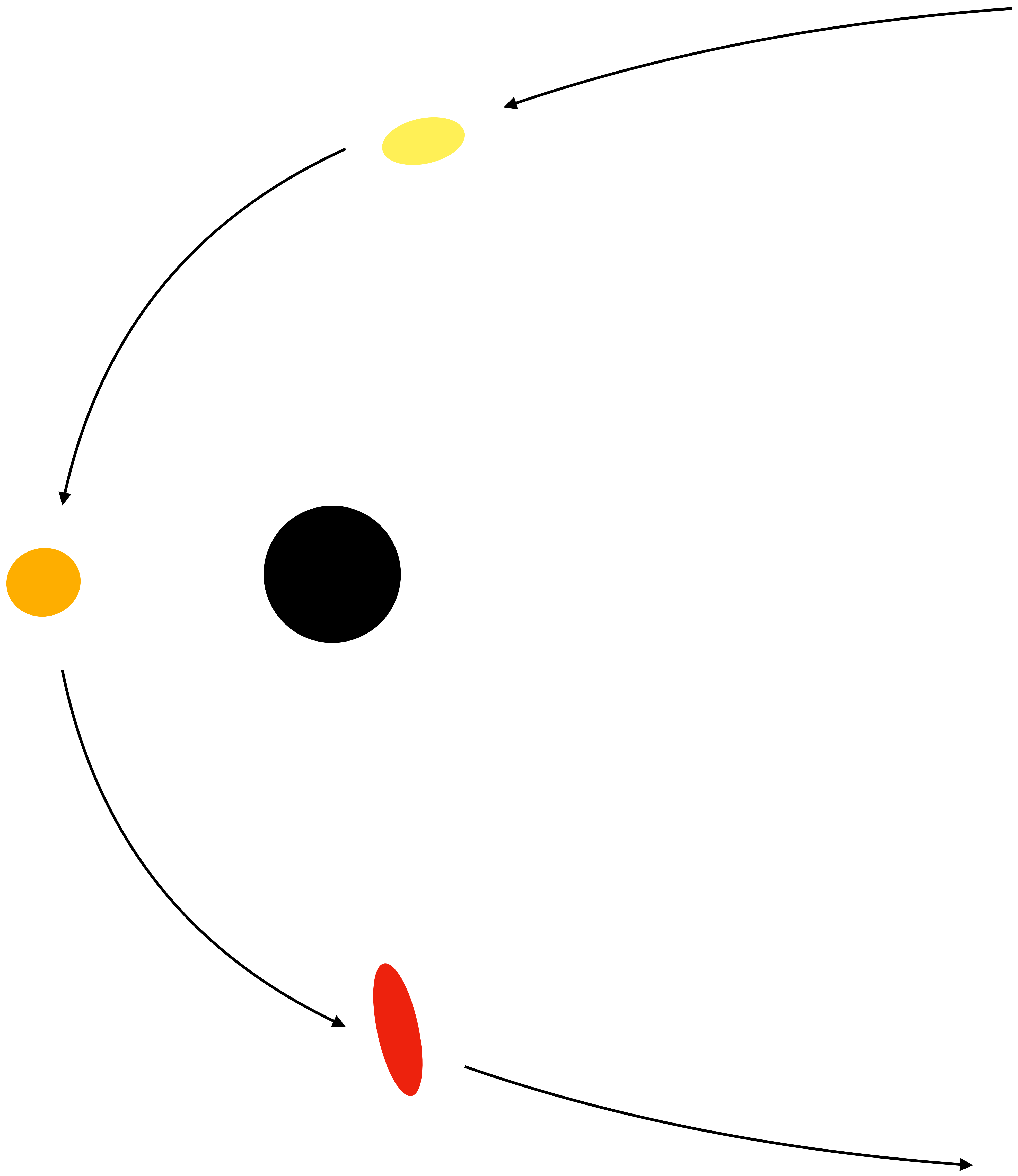
Test particles do not test gravity well - left unchanged by travel through a gravitational field

- Composite quantum objects are clocks - different energy levels
- Clocks measure gravitational time dilation
- Composite objects also experience *tidal forces.*



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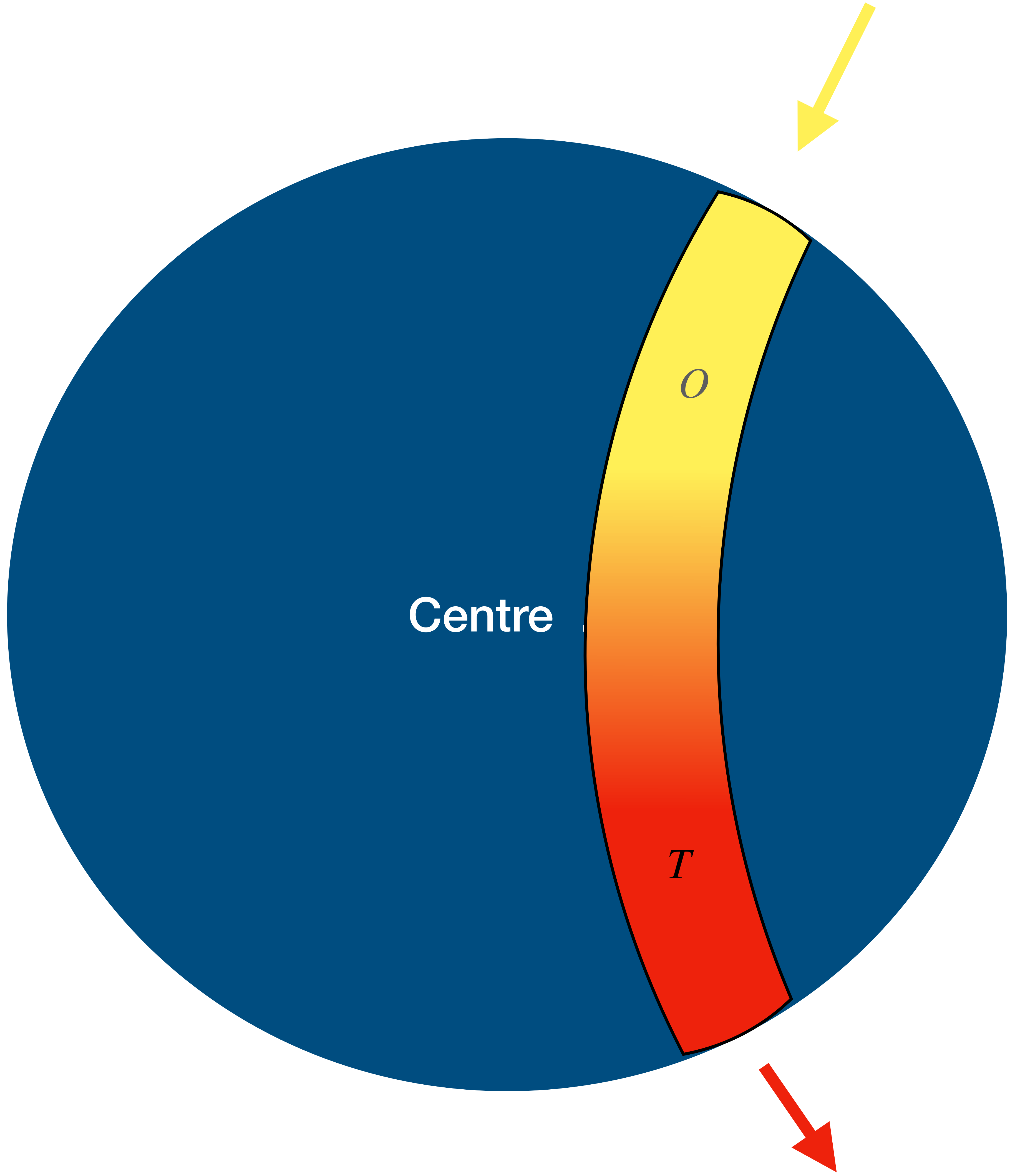

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Tie this together!

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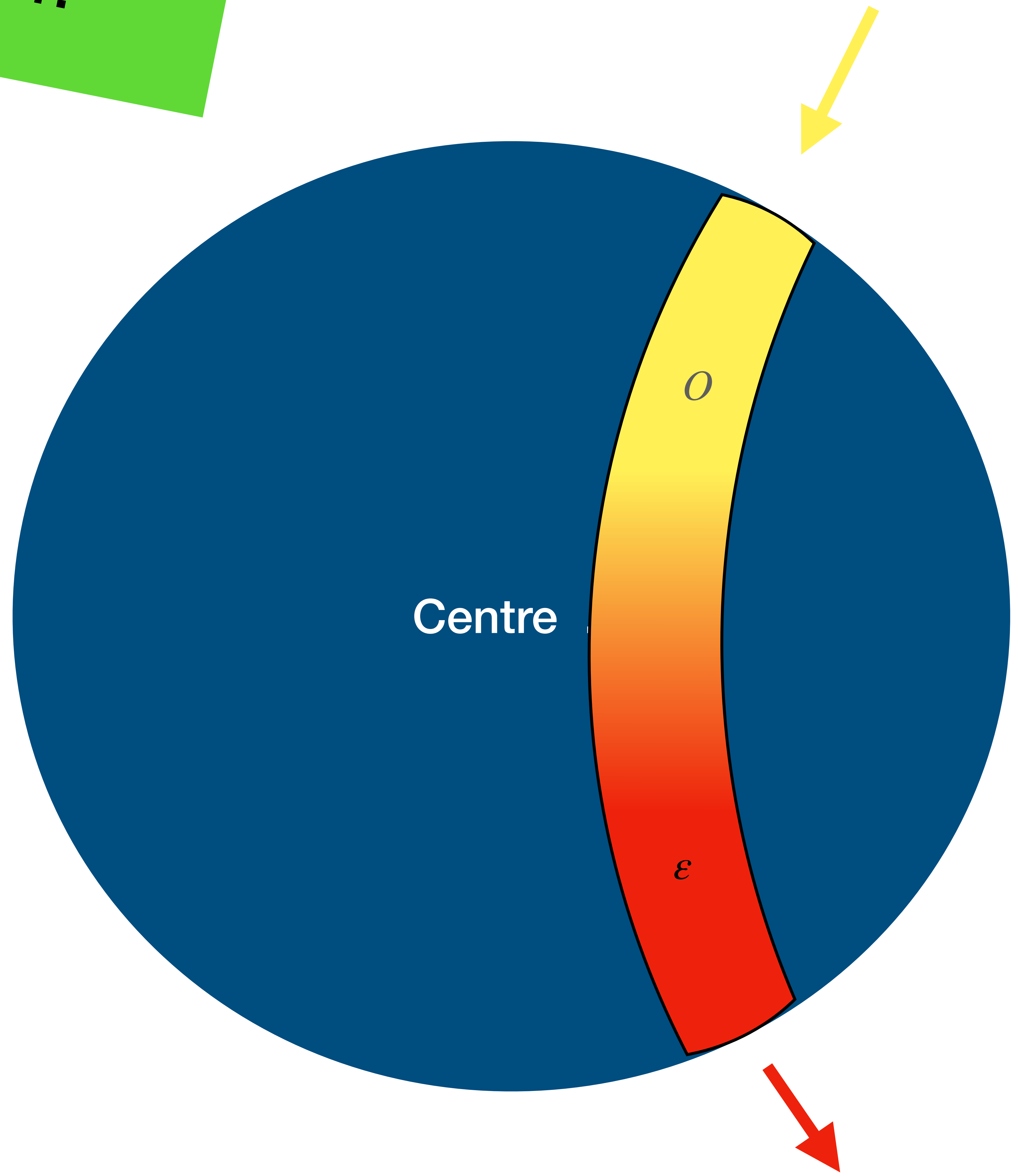


Composite operators in QFT can probe tidal effects in quantum gravity

Composite operators

in CFT - technical remarks

Dipoles are simple composite objects.

Technically, we can get a simple spectrum of dipole operators

$$O(x) = \phi_n^\dagger \phi_n(x),$$

by taking

- charge $g \to 0$
- number of charges, $N \to \infty$

For weakly coupled gravity:

$$G_N \sim \frac{1}{N}$$

Green functions

of dipoles - technical remarks

$$G_{O\varepsilon}(x,y) = \langle O(x)\varepsilon(y)\rangle$$

Dipole operators

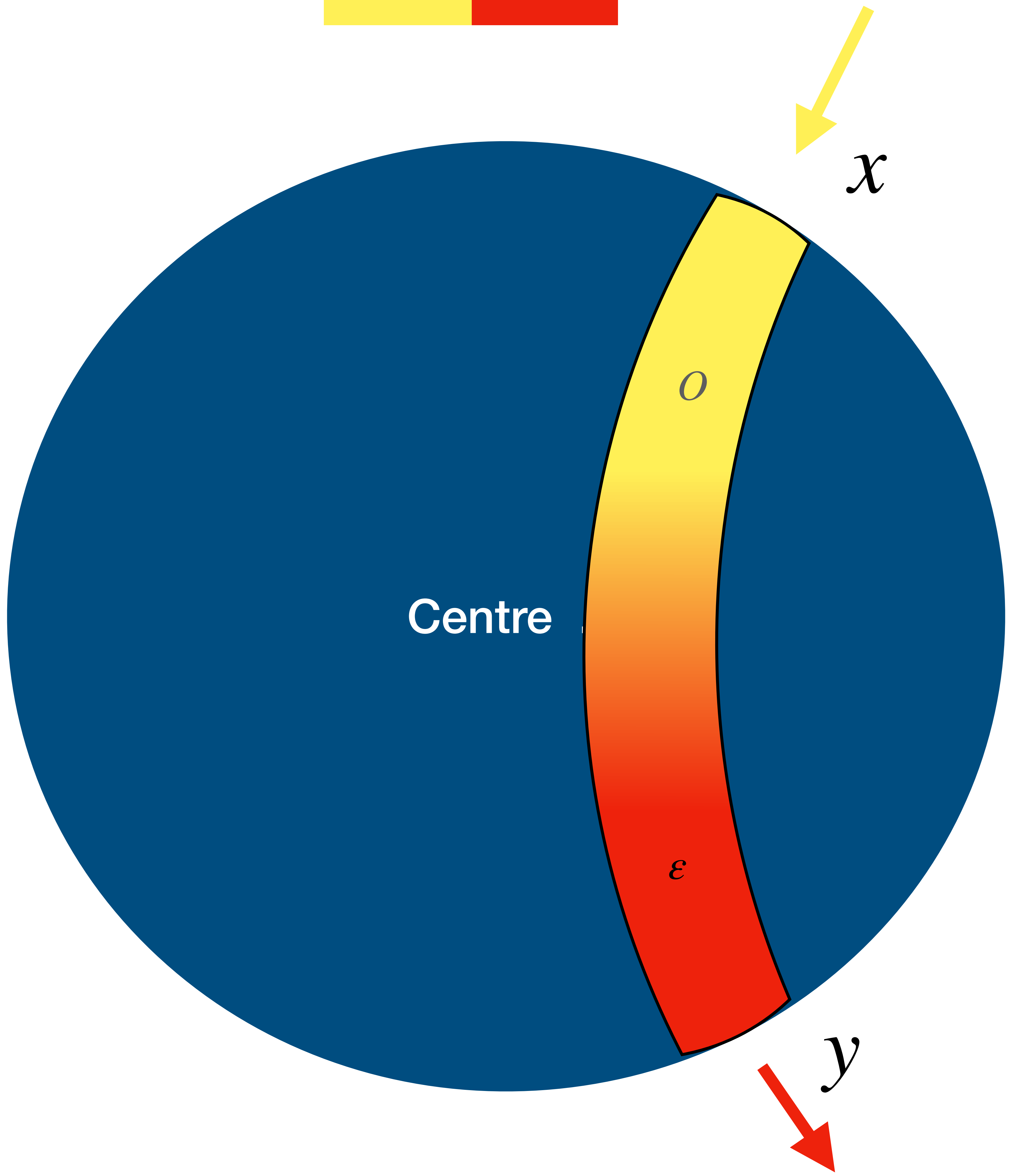
$$O(x) = \phi_n^\dagger \phi_n(x),$$

Are related to the energy density

$$\varepsilon(x)$$

of the same field, which is also quadratic.

Green functions yield propagation between different asymptotic regions.

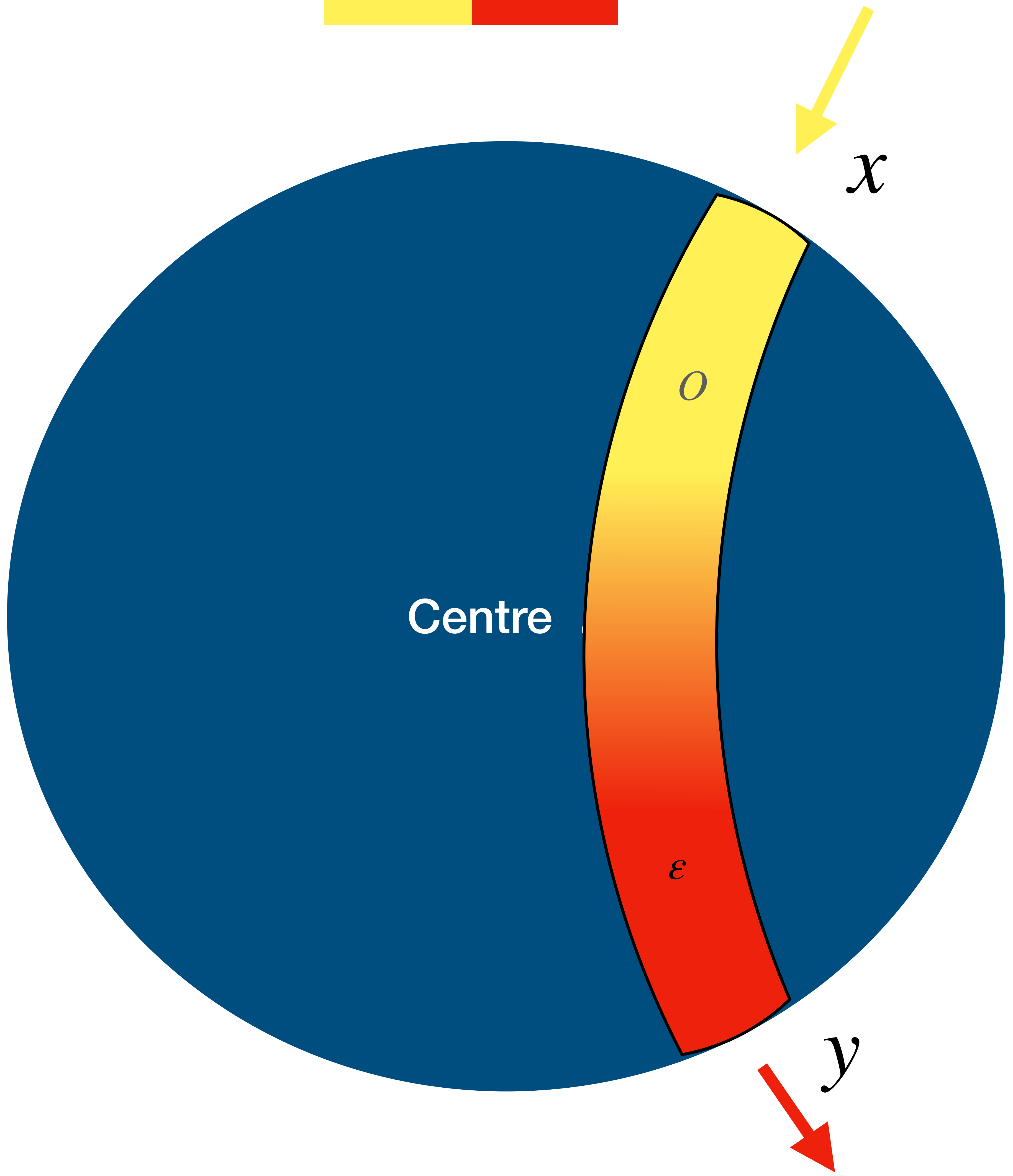


Tidal conversion
between gravity theory fields

$$G^{(T)}_{O\varepsilon}(x,y) = \langle O(x)\varepsilon(y)\rangle_T$$

Mixed Green functions between $O(x)$ and $\varepsilon(y)$ in different asymptotic regions represent

- tidal conversion between a scalar field $\Phi_O(X)$ and a graviton $h_\varepsilon(Y)$.
- The conversion depends on temperature T.
- The total mass of the central object, $M(T)$, increases with T.

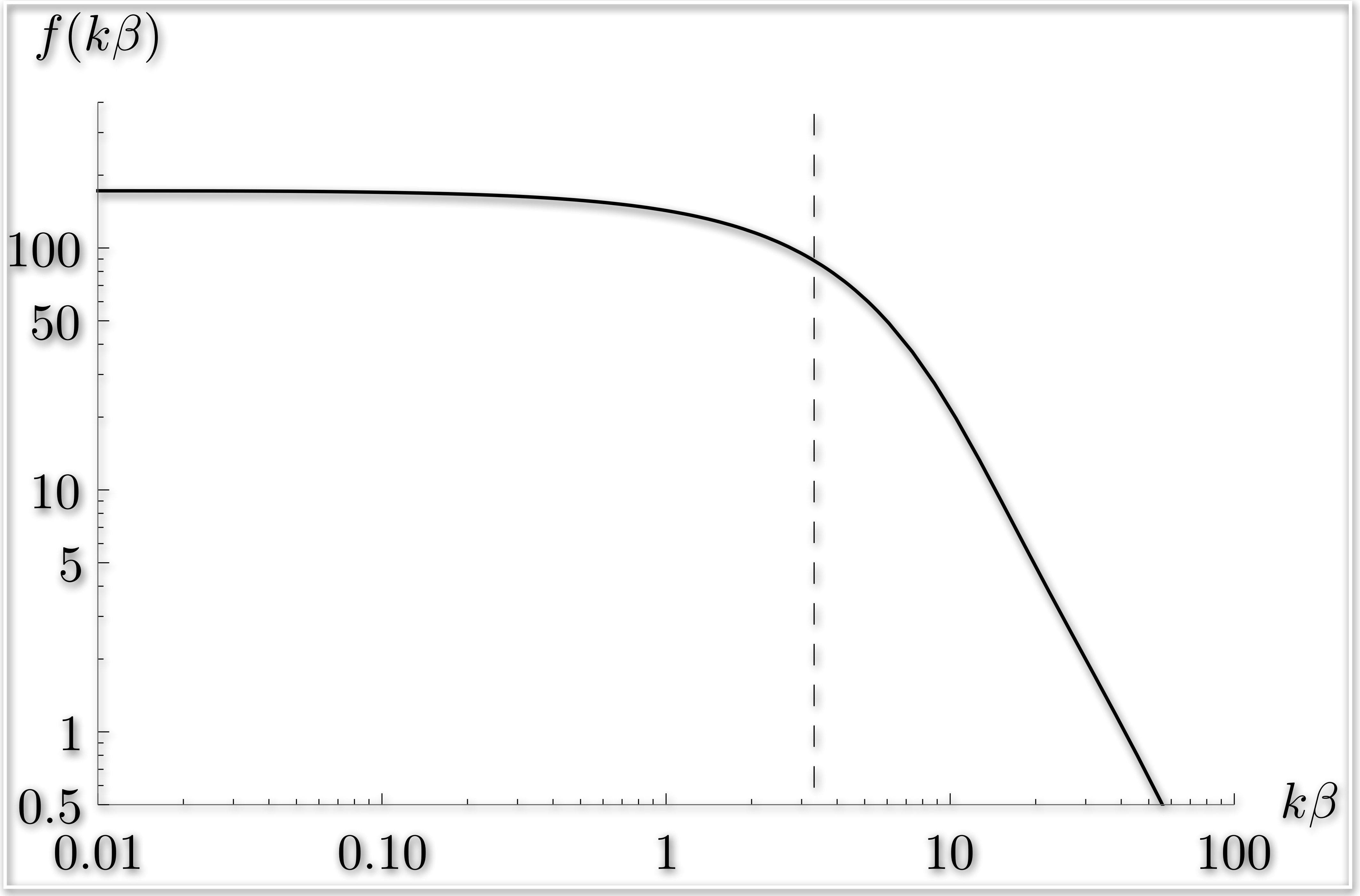


Tidal conversion

between gravity theory fields

$$G^{(T)}_{O\varepsilon}(x,y) = \langle O(x)\varepsilon(y)\rangle_T$$

- Fourier transform to ω, k space.
- $G^{(T)}_{\varepsilon O}(\omega = 0, k) = -T^2 f(k/T)$
  - The mixed correlator is constant for $k/T \ll 1$.
  - It falls as an inverse power law for $k/T \gg 1$.
  - Only appreciable conversion for regions of order $\Delta x \gg 1/T$.



Higher temperature/mass yields conversion at smaller angles ⇒ larger object

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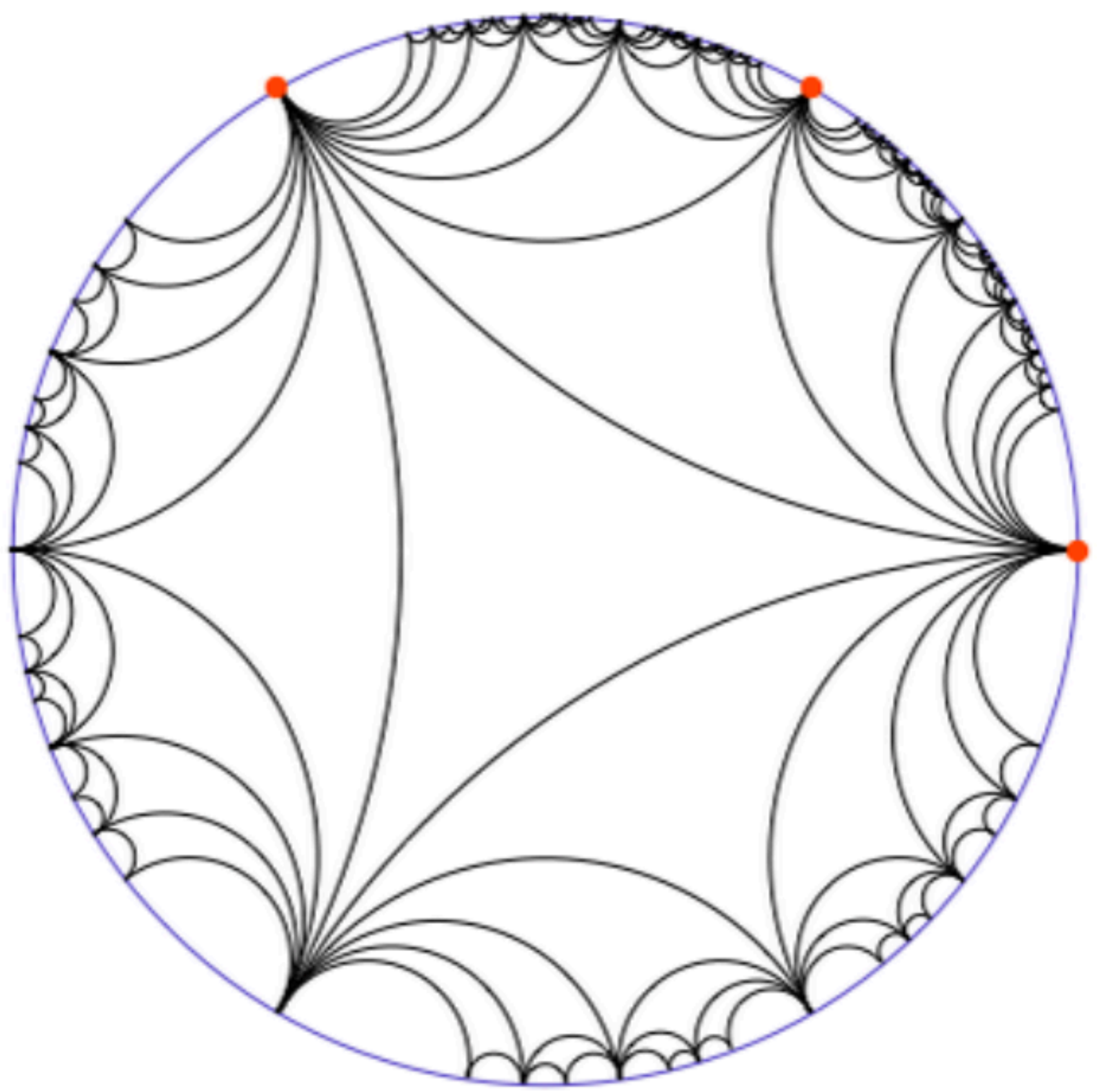

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Technical calculation of partition function

There is a crucial difference from the naive free field theory:
Gauss law is kept for a smooth free limit of gauge theory.

$$Z[\beta] = \int_{\varphi \text{ on } S^{d-1}\times S^1} \mathcal{D}A_\mu \mathcal{D}\varphi \mathcal{D}\varphi^\dagger e^{-S\left[A_\mu, \varphi, \varphi^\dagger; \beta\right]}$$

$$S = \int_{S^{d-1}\times S^1} \mathrm{d}^d x \sqrt{g} \left(g^{\mu\nu}(D_\mu\varphi)^\dagger D_\nu \varphi - \frac{(d-2)^2}{4R^2}\varphi^\dagger\varphi \right)$$

Gauge covariant derivative

Conformal coupling of scalars

Conclusions

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