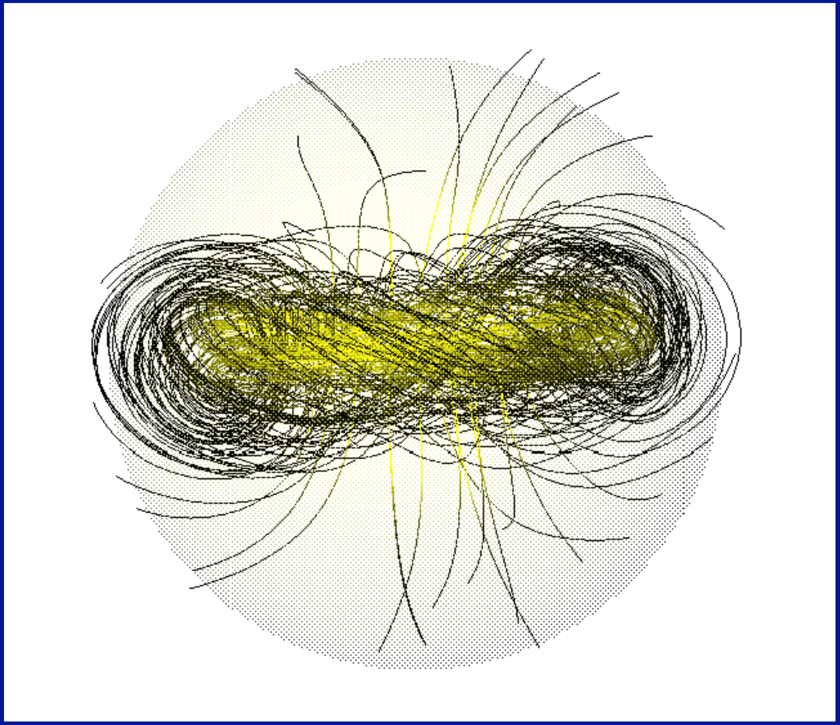
Formation and evolution of magnetic fields in non-convective stars



NORDITA, Stockholm
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Contents

- Summary of formation of and observations of B-fields in non-convective stars
- Model of formation of magnetic equilibria
  - nature of equilibria: axisymmetric & non-axisymmetric
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- Secular evolution of magnetic equilibria
- Conclusions

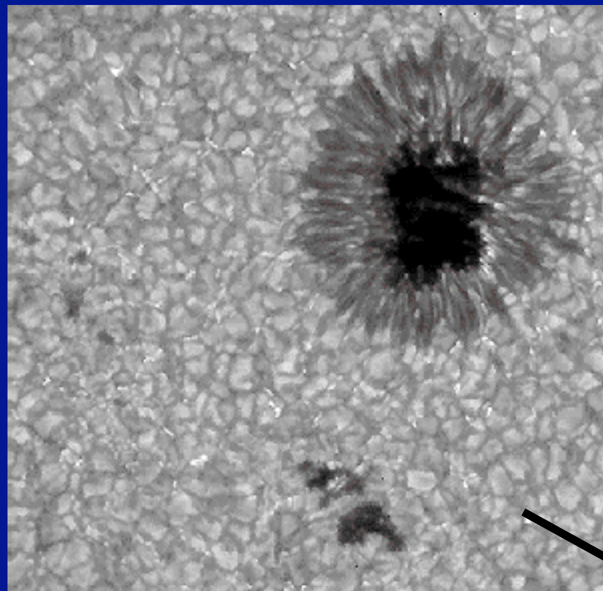
Observations of B-fields on non-convective stars

	Strength	Geometry	Evolution during lifetime
Main-sequence (>1.5Msun)	200 G - 30 kG	Some dipolar, some with strong quadrupolar or octopolar fields	No evidence
White dwarfs	10^4 - 10^9 G	As above	?
Neutron stars	10^9 - 10^{15} G	Uncertain - only dipole easily measurable	10^4 - 10^7 years?

– *see e.g. Kochukhov et al. (2004), Wickramasinghe & Ferrario (2000), Becker et al. (2003)*

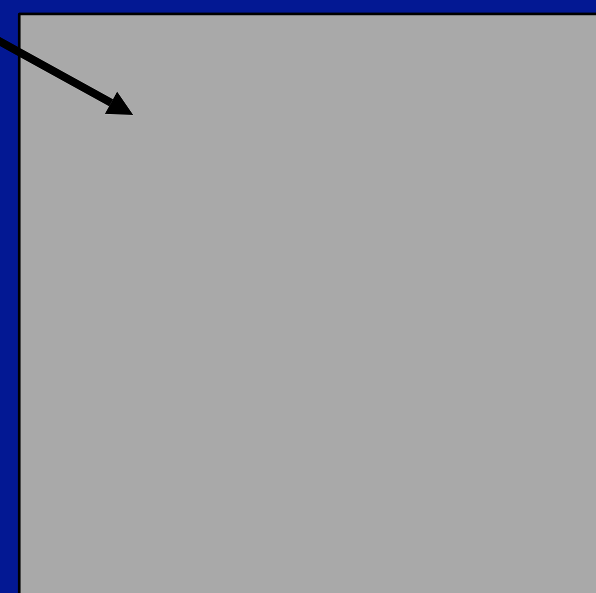
Formation of non-convective stars

- Formation:
  - Main-sequence > 1.5 Msun: convective protostellar phase
  - WDs: Born out of convective part of M-S star?
  - NSs: Neutrino-driven convection in proto-NS lasting ~100s
- All have in common:
  - convective --> non-convective
  - magnetic field:
    small-scale, chaotic --> large-scale, ordered



Convection:
Small-scale chaotic velocity field, small-scale chaotic magnetic field

No convection:
Settled (~zero?) velocity field, large-scale ordered magnetic field



Question:

- Is it possible to predict from first principles what should happen to the magnetic field when convection ends?
- Run numerical simulations to find out!

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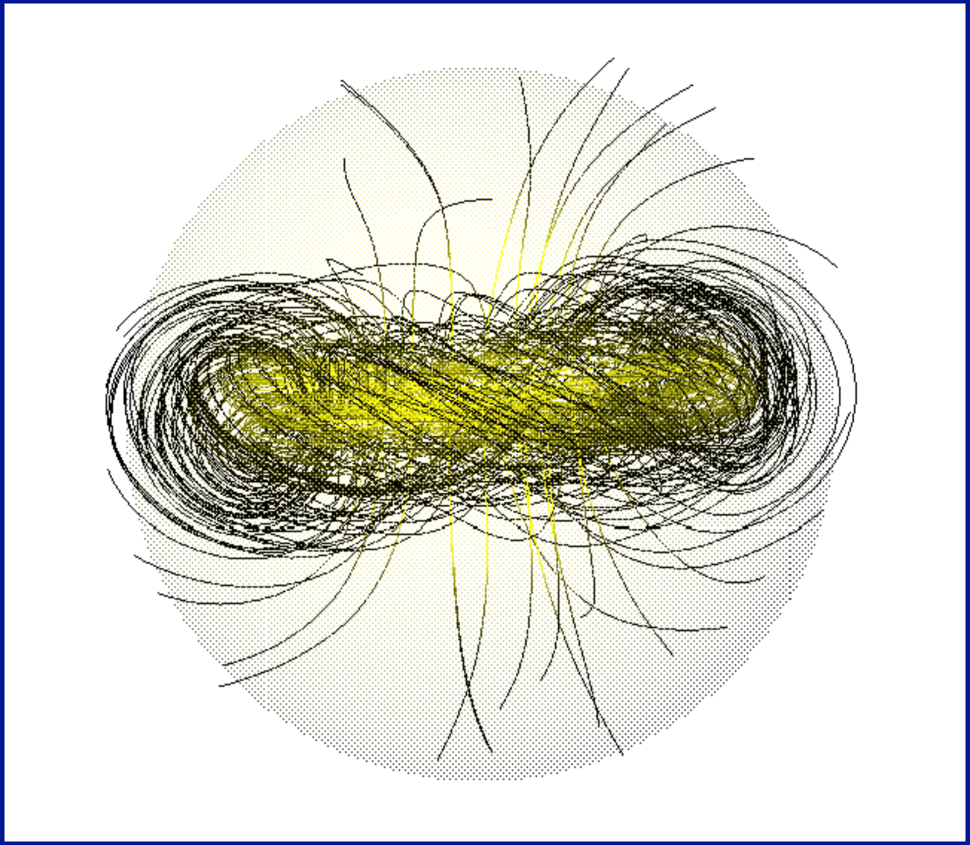
Model of post-convective evolution

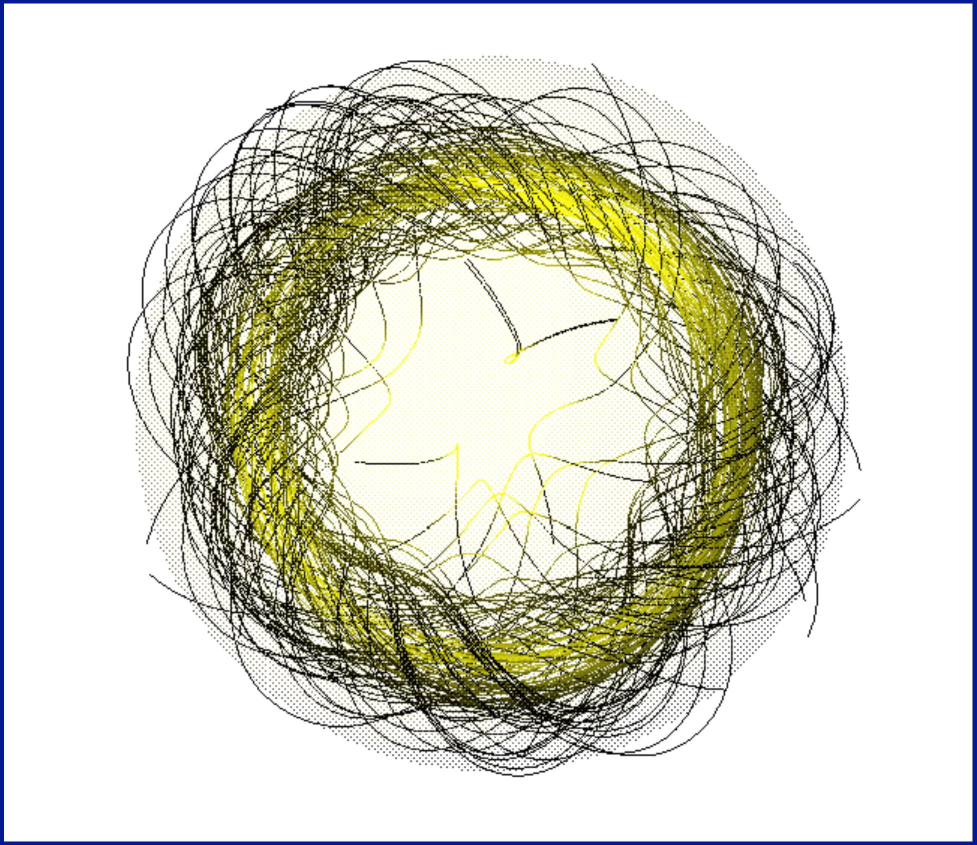
- Run "star in box" simulation with an initially turbulent field
- Star has two important properties:
  - stable stratification (*not* barotropic EOS)
  - high conductivity inside star & approx. potential field outside
- Initial magnetic field: small-scale "turbulent"
- Use Aake Nordlund's "stagger code", high-order finite-difference MHD code, Cartesian

Result

- Field relaxes on Alfven timescale into a stable equilibrium consisting of twisted flux tube(s) lying horizontally at roughly uniform radius
- Shape of flux tube(s) depends on initial conditions: in particular, on radial energy distribution. If $B \sim \rho^p$, then at
  - $p > 1/2$: roughly axisymmetric equilibrium forms
  - $p < 1/2$: non-axisymmetric equilibrium forms

Shape of axisymmetric field

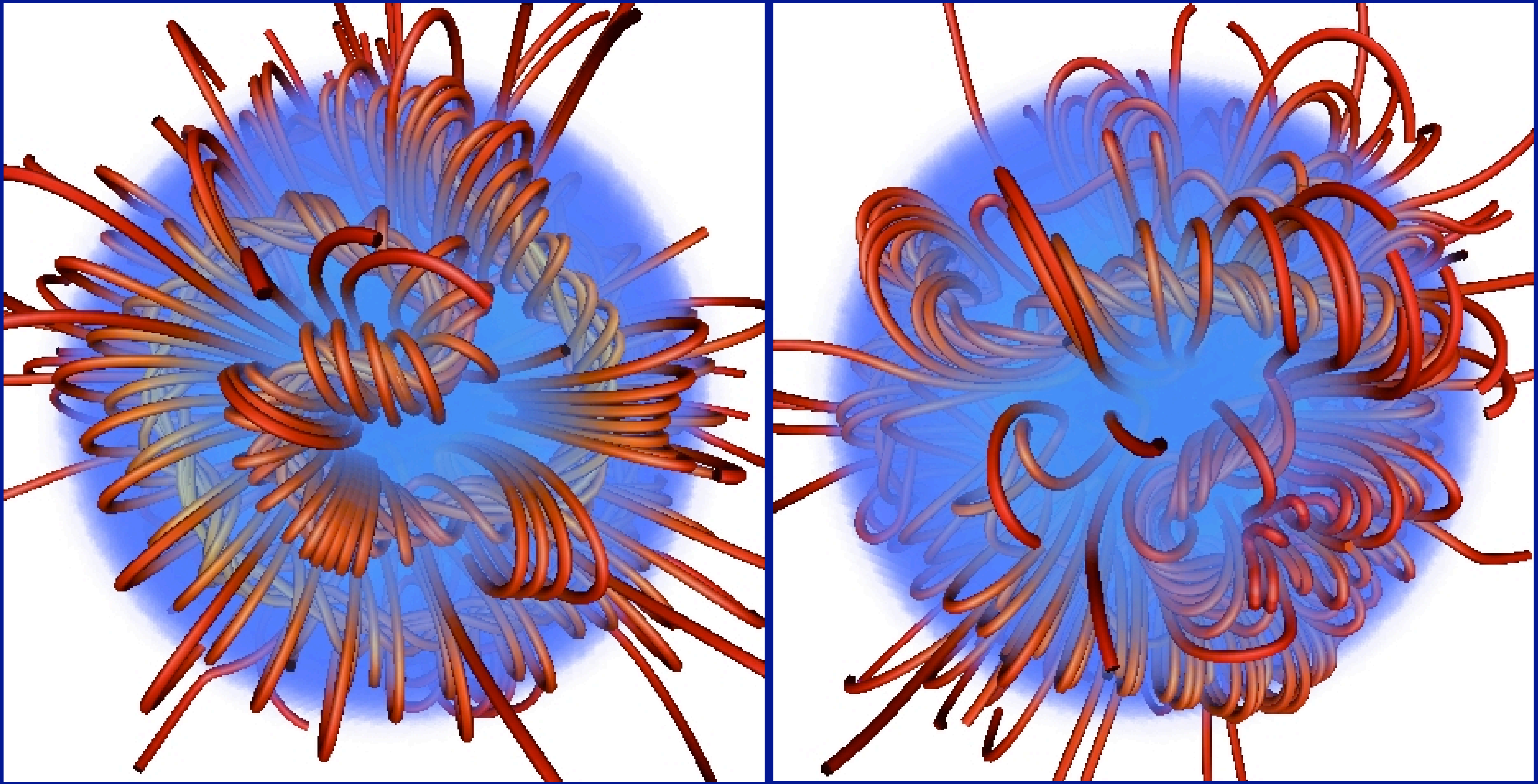




Formed when p>1/2, i.e.
initial field more concentrated
towards centre of star

Braithwaite & Nordlund 2006

Shape of non-axisymmetric field

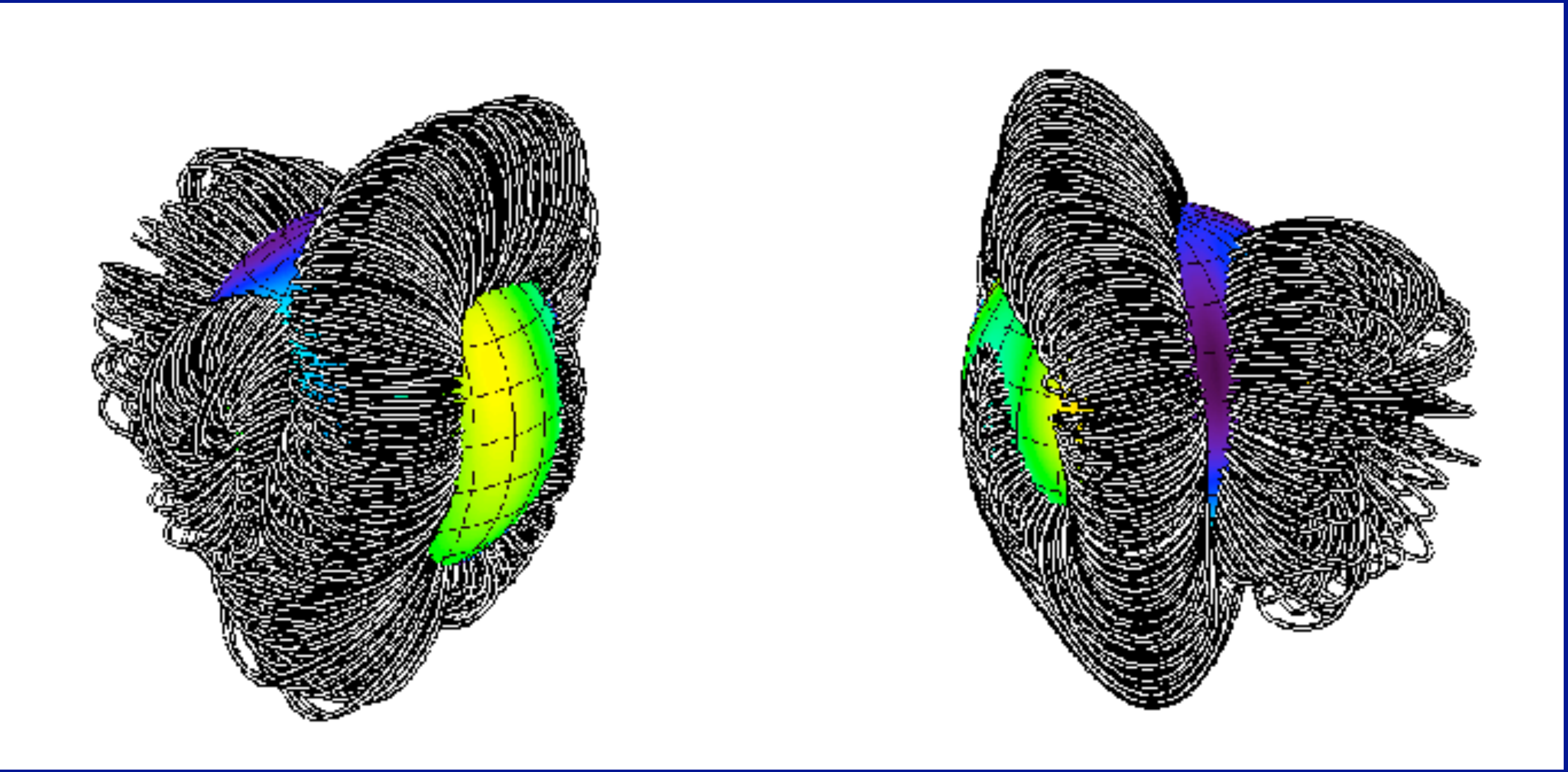


Formed when p<1/2, i.e. initial field less concentrated towards centre of star

Braithwaite 2008

Comparison with observations: analogy with upper-main-sequence star

(radiative apart from small convective core)



Field topology of τ Sco, a B0 main-sequence star (M_V=2.8)

(Donati et al. 2006, using Zeeman-Doppler imaging)

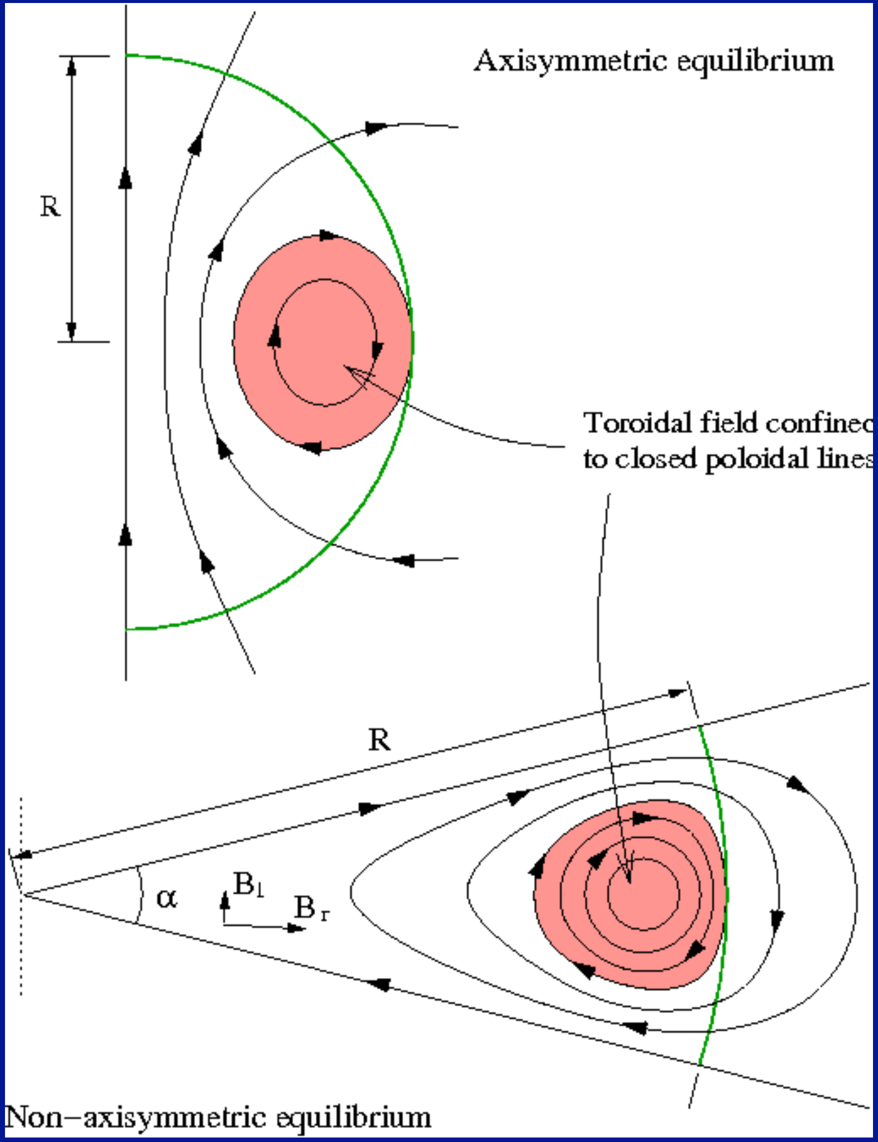
Structure of equilibria

- Consist of twisted flux tube(s) below the stellar surface
- Toroidal flux confined to largest closed poloidal loop
- Energy of tube, $E \approx \frac{AR}{24\pi}[B_t^2 + B_l^2 + B_r^2]$
- If we allow length & width of tube to change adiabatically, then

$$\frac{\partial \ln B_t}{\partial \ln \alpha} = -1 \qquad \frac{\partial \ln B_l}{\partial \ln \alpha} = 1 \qquad \frac{\partial \ln B_r}{\partial \ln \alpha} = 0$$

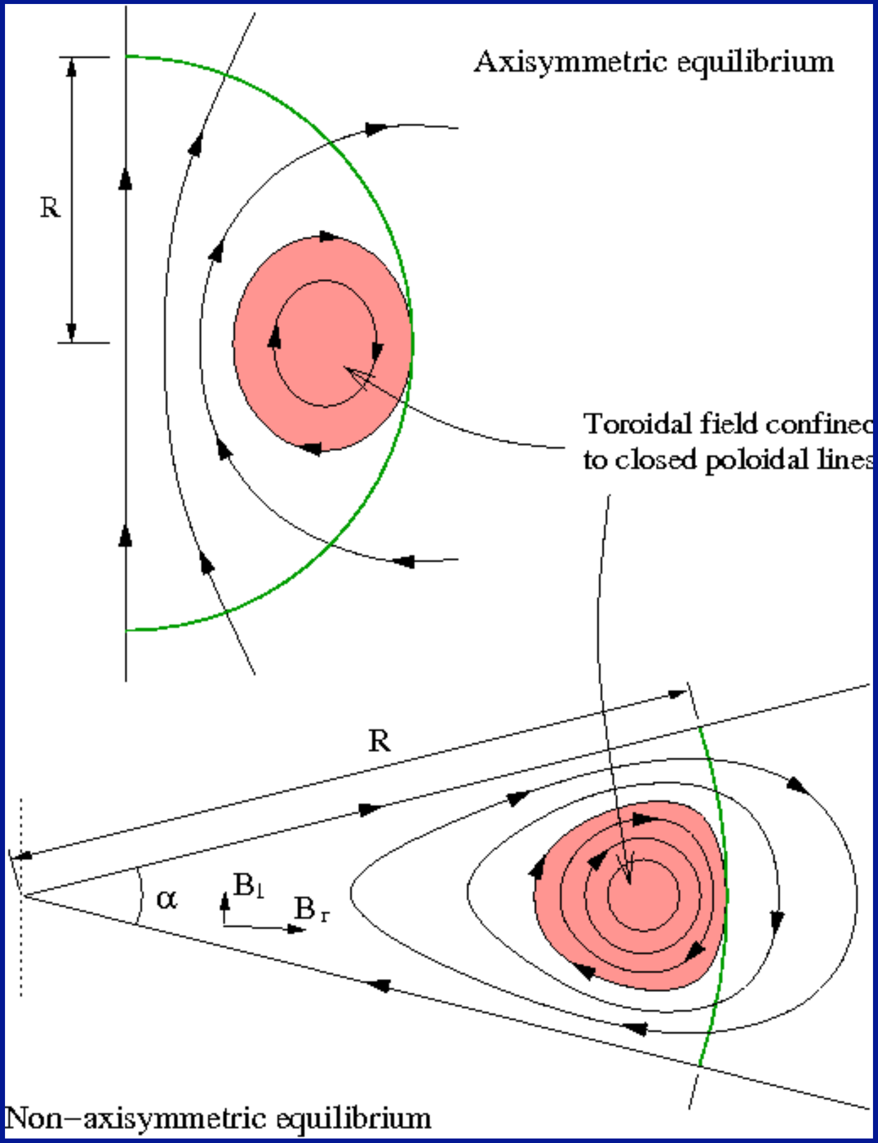
- giving $\frac{\partial E}{\partial \alpha} \approx \frac{AR}{12\pi\alpha}[B_l^2 - B_t^2]$
- Therefore: $B_t \approx B_l$
- And using the geometry to relate B_r & B_l,

$$\alpha B_r \approx B_t \approx B_l$$

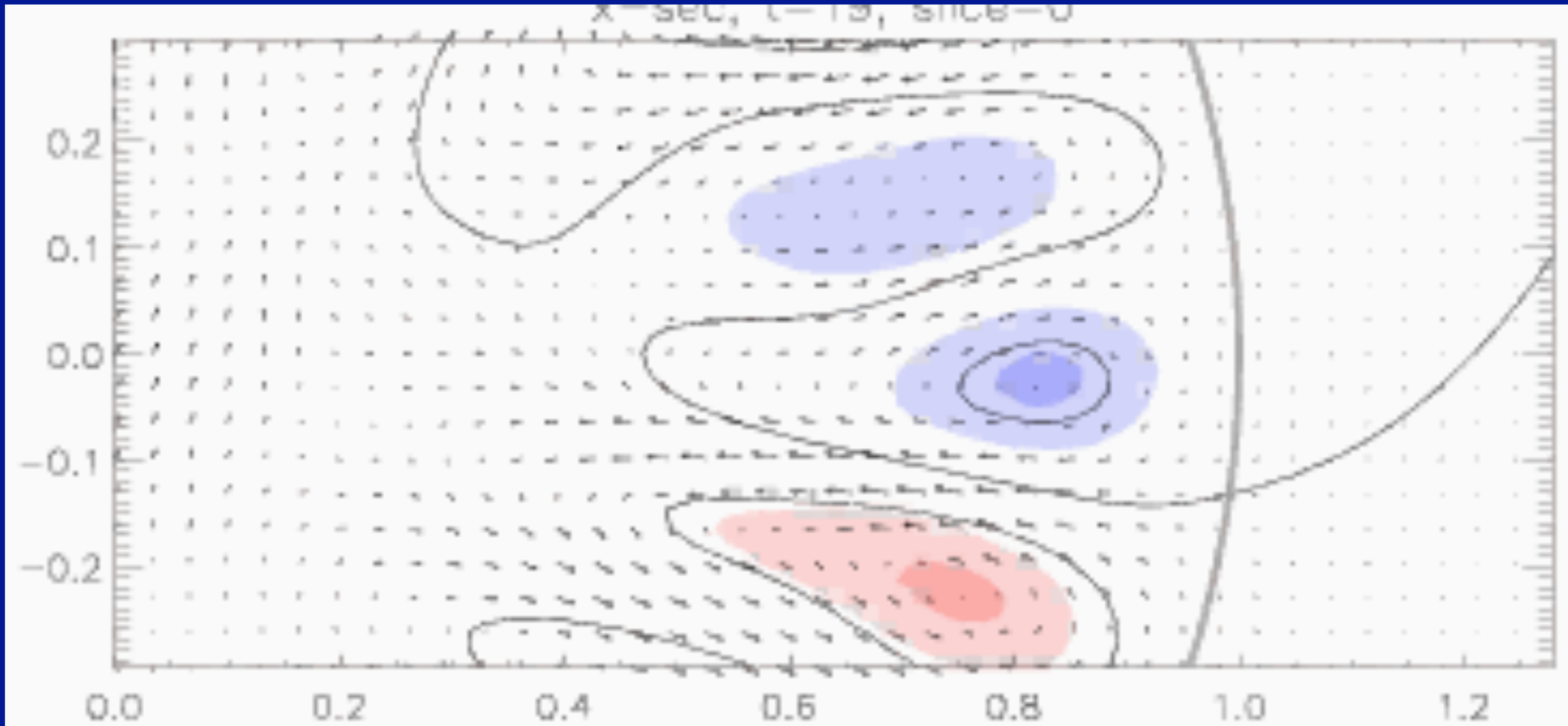


- Roughly equal toroidal & latitudinal components:
  - Too strong toroidal field $\Rightarrow$ flux tube contracts and widens $\Rightarrow$ toroidal field weakens
  - Too strong poloidal field $\Rightarrow$ flux tube lengthens & becomes narrower $\Rightarrow$ poloidal field becomes weaker
- At equilibrium:

$$\alpha^2 \sim 4\pi \frac{\Phi_t}{\Phi_p}$$



Cross-section of flux tube from simulation



Animation follows a flux tube on its path around the star
Lines=poloidal field; red/blue=toroidal field

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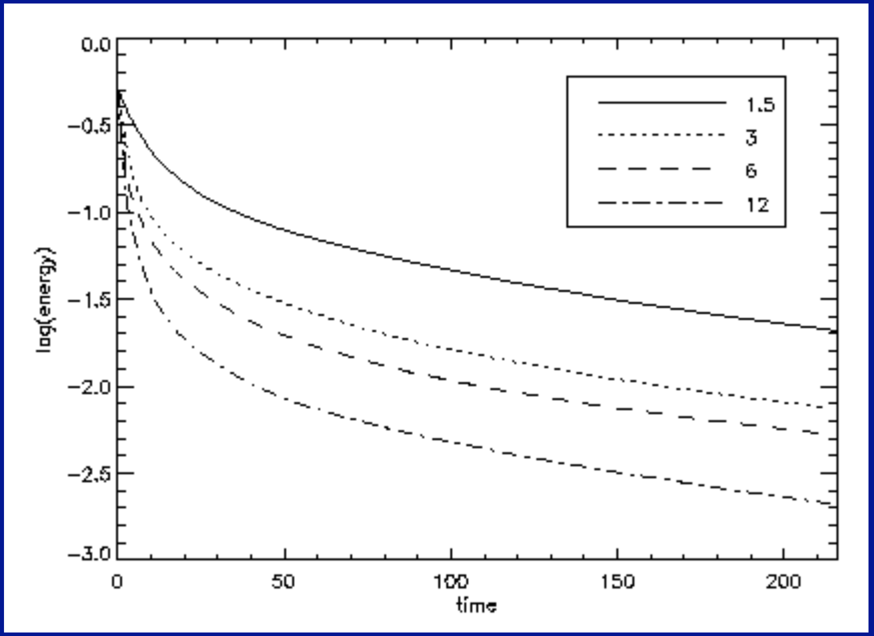
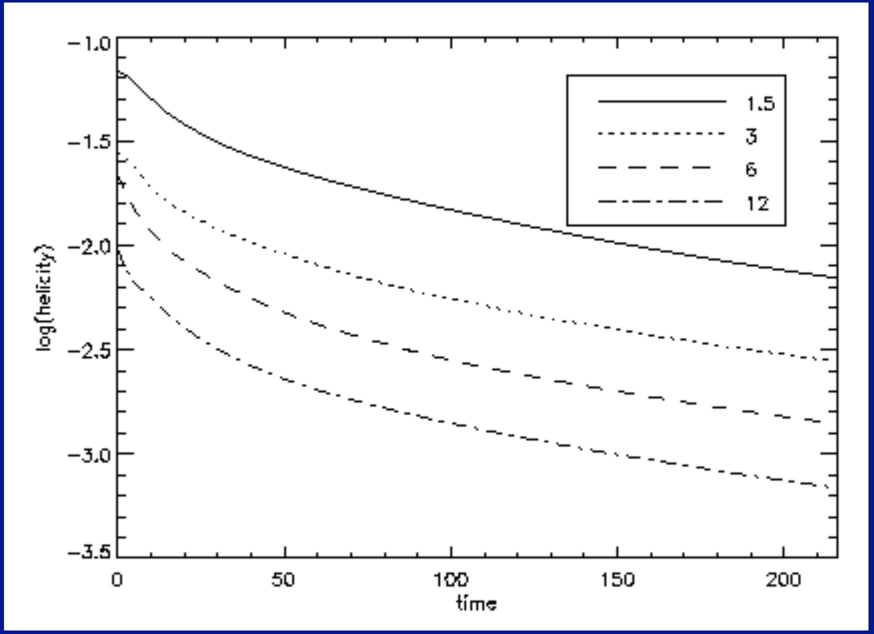
Magnetic helicity of initial field

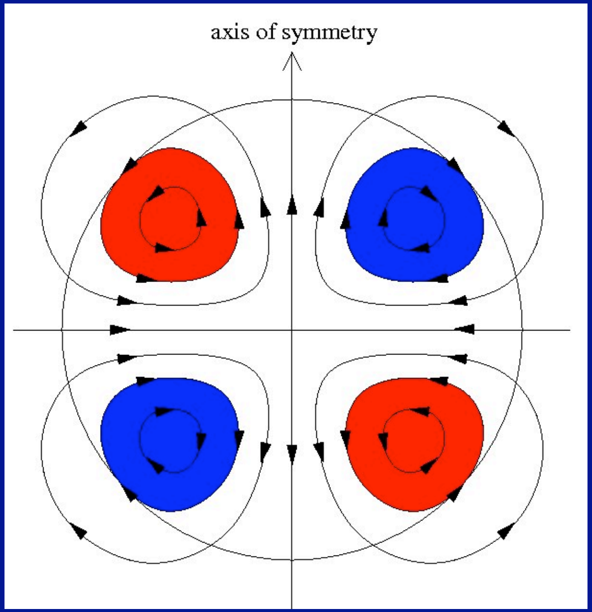
- Initial random field contains wavenumbers up to k_{max}
- Simulations run with $R_* k_{max}/2\pi$ = 1.5, 3, 6 and 12.
- Helicity defined as $H \equiv \int \mathbf{A}.\mathbf{B}\, dV$, where **B** = curl **A**. It is conserved in the limit of infinite conductivity
- Higher k_{max} means lower helicity, because different regions cancel each other out
- *Conclusion*: Lower initial helicity --> lower-energy equilibrium field, since the equilibrium is an energy minimum at given helicity



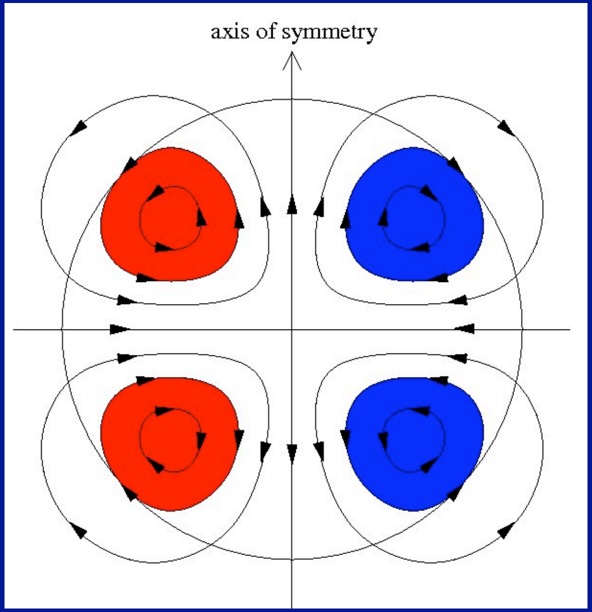


Above right: magnetic energy against time
Below right: magnetic helicity against time

BUT:
A zero-helicity equilibrium?



- Helicity is essentially the product of toroidal and poloidal fluxes
- Equilibrium can contain flux tubes with helicity of opposite signs, since sign of toroidal field has no effect on stability
- Contradicts hypothesis that equilibria are minimum energy states at given helicity
- *Local* energy minima also stable



Cross-sections of simple equlilbria consisting of two flux tubes
Top: finite helicity
Bottom: zero helicity

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Axisymmetric fields: possible toroidal/poloidal strength ratios

- Both poloidal and toroidal fields unstable on their own
- Mixture of the two is required

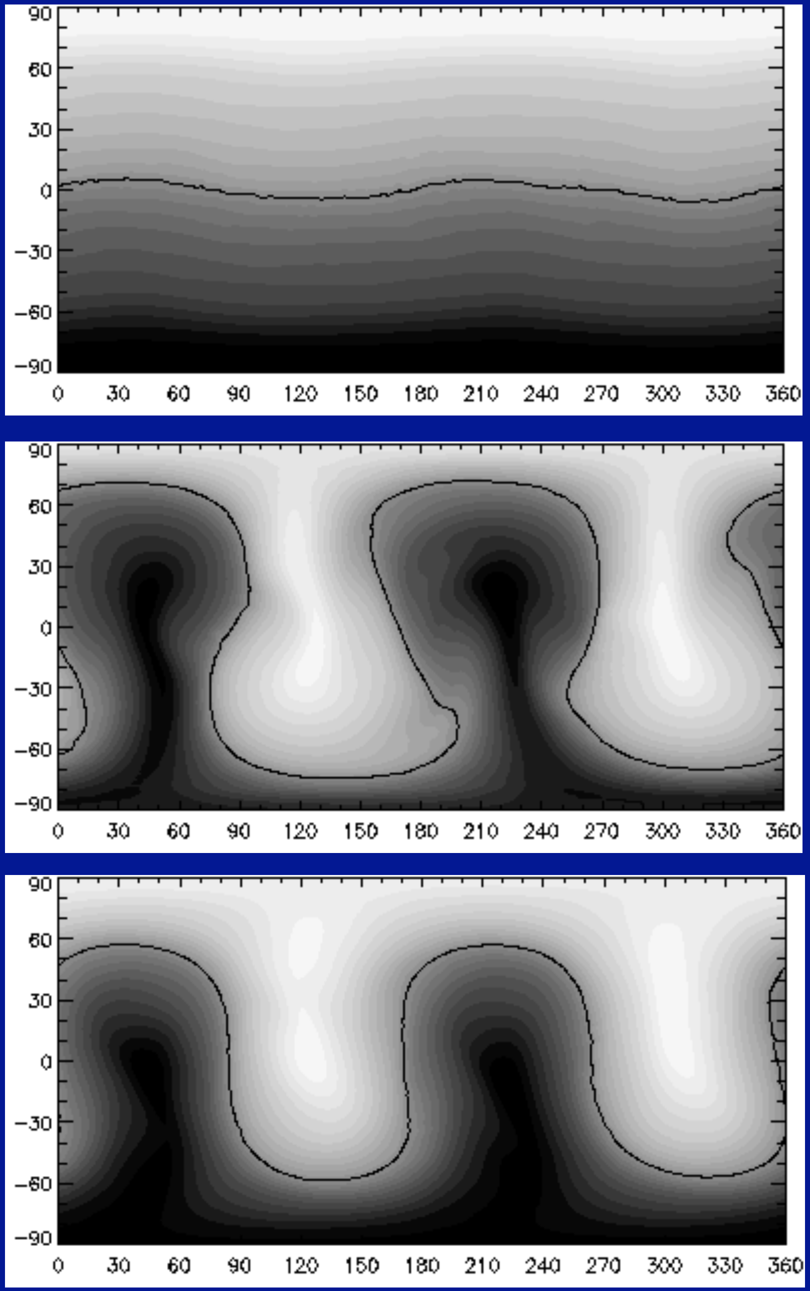
Axisymmetric fields: possible toroidal/poloidal strength ratios

- Very strong poloidal component not possible, as transition to non-axisymmetric equilibrium will occur.

$$\alpha^2 \sim 4\pi \frac{\Phi_{\rm t}}{\Phi_{\rm p}}$$

- However, very strong toroidal component not ruled out. Simulations produce field configurations with $E_p/E \sim 0.05 - 0.1$
- Stability examined with help of simulations
  - Use output of previous simulations where axisymmetric equilibria form
  - change E_p/E by hand, use local stability analysis *and check* by setting simulation running again

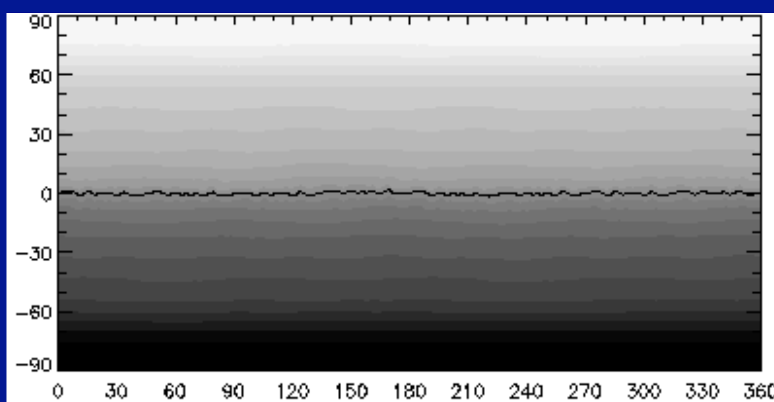
Simulations with high E_p/E



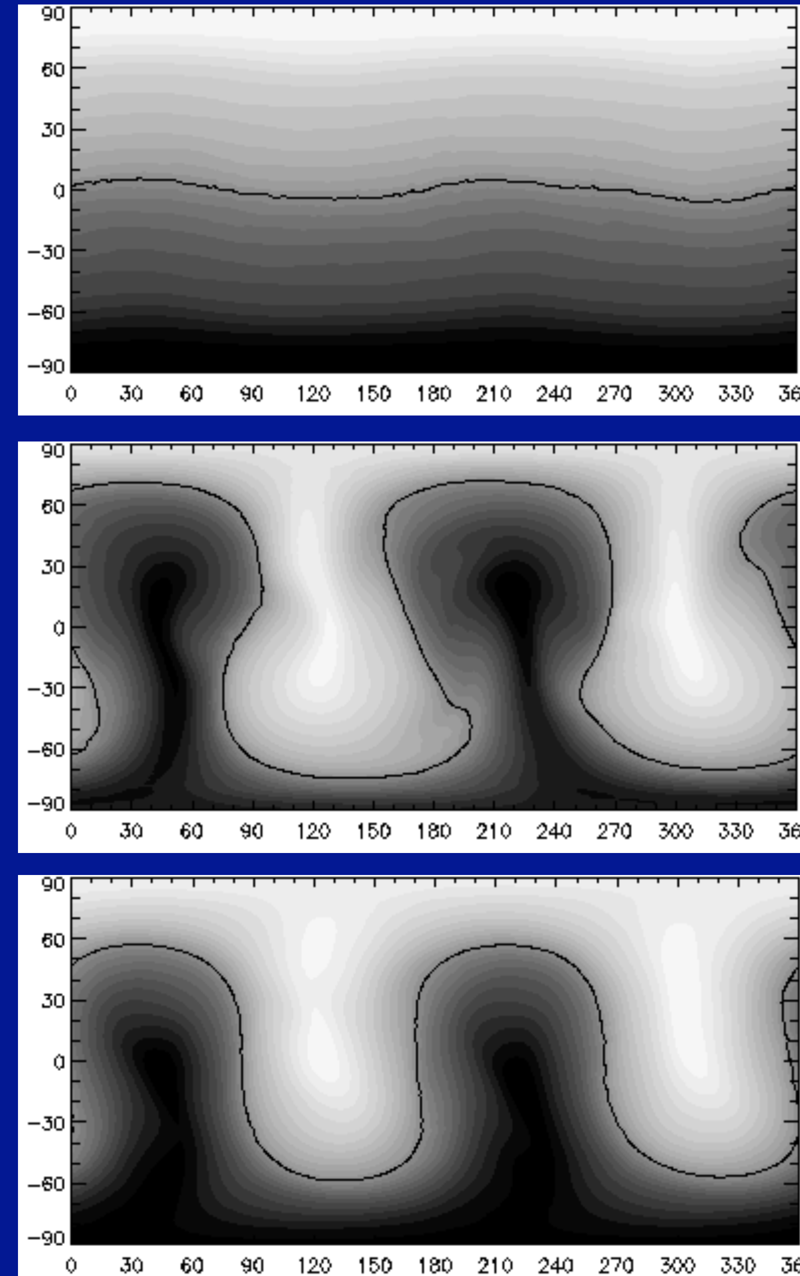
$E_p/E=0.9$, t=3,8 and 15

Simulations with high E_p/E

result: stability threshold at $E_p/E \sim 0.8$

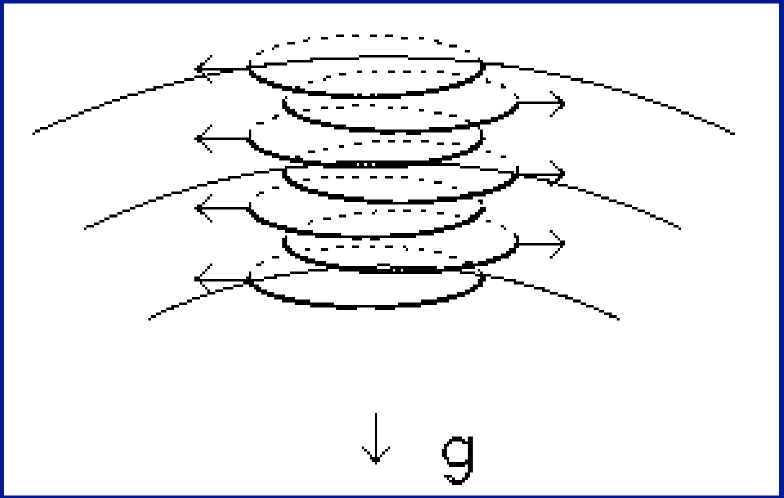


$E_p/E=0.7$, $t=15$



$E_p/E=0.9$, $t=3,8$ and 15

Low E_p/E fields: using Tayler's stability conditions

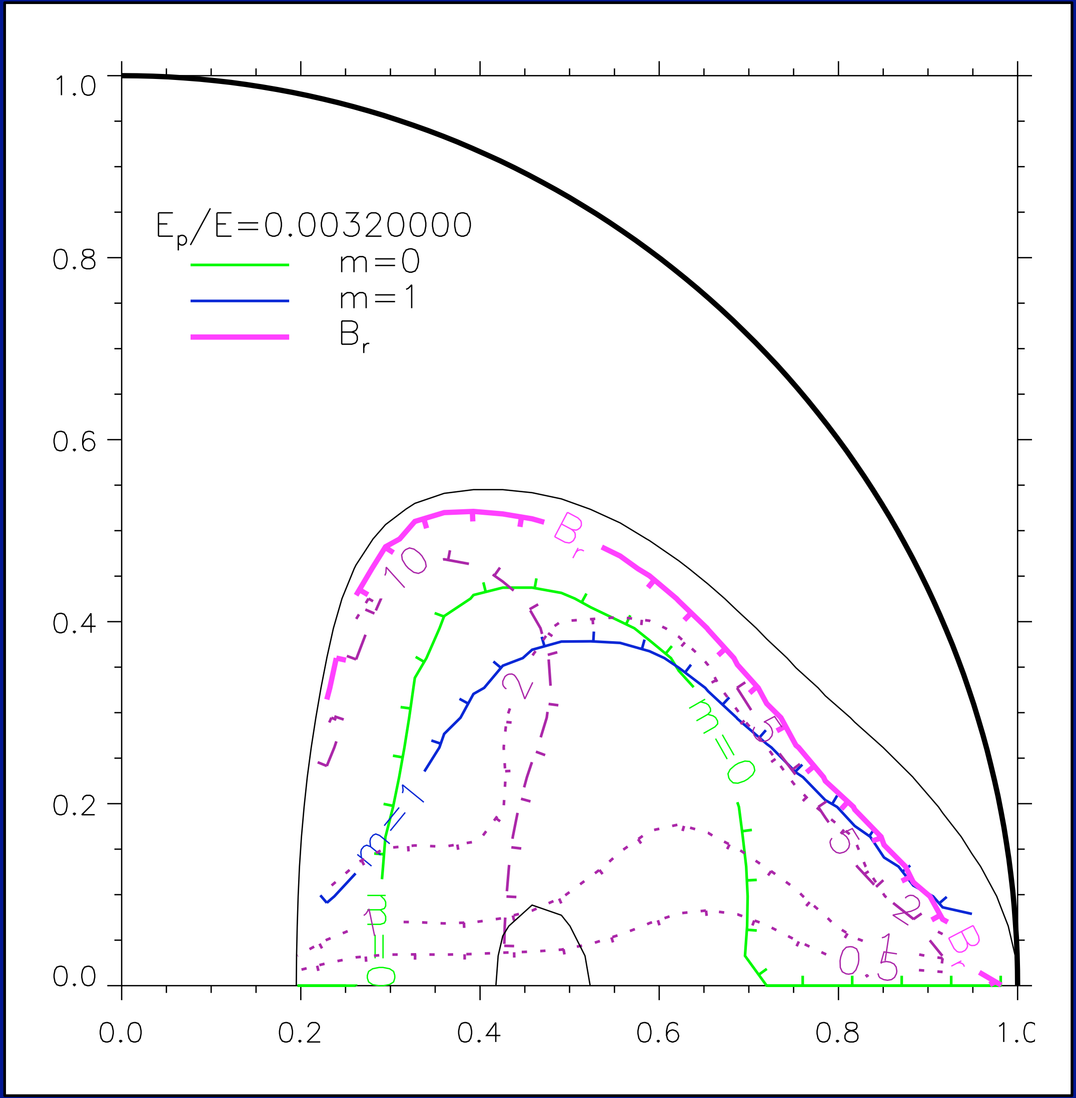


- Tayler (1973) derived necessary and sufficient stability conditions for m=0 and m=1 modes, using energy method of Bernstein et al. (1958)
- Conditions are local in meridional plane

Tayler's conditions applied to configuration found in simulations

- m=0 and m=1 conditions satisfied in only part of the star (hence any purely toroidal field is unstable)
- Weak poloidal component stabilises field if

$$B_r > B_\phi \frac{L_r}{r}$$

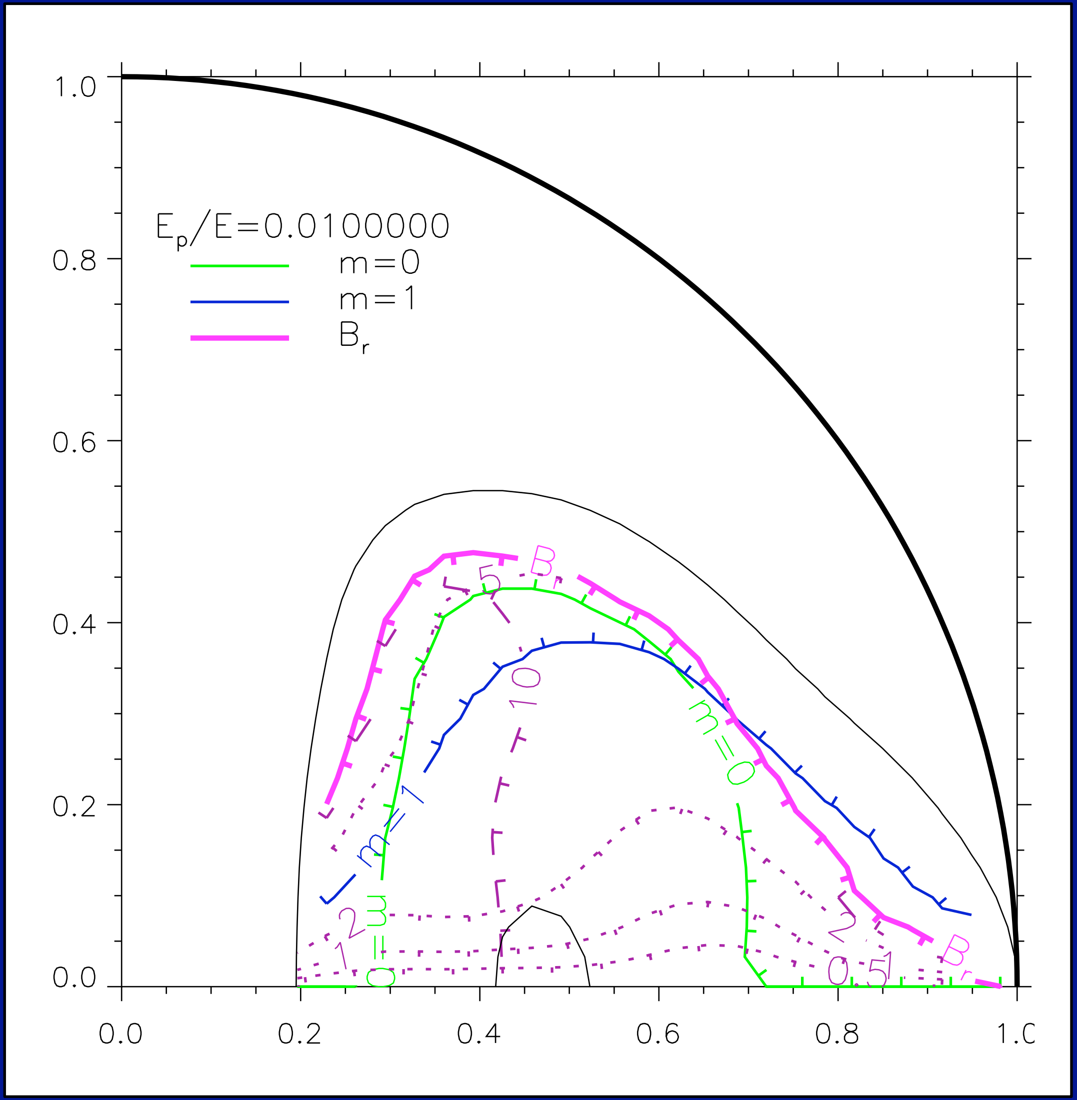


E_p/E=0.0032, unstable

Tayler's conditions applied to configuration found in simulations

- m=0 and m=1 conditions satisfied in only part of the star (hence any purely toroidal field is unstable)
- Weak poloidal component stabilises field if

$$B_r > B_\phi \frac{L_r}{r}$$

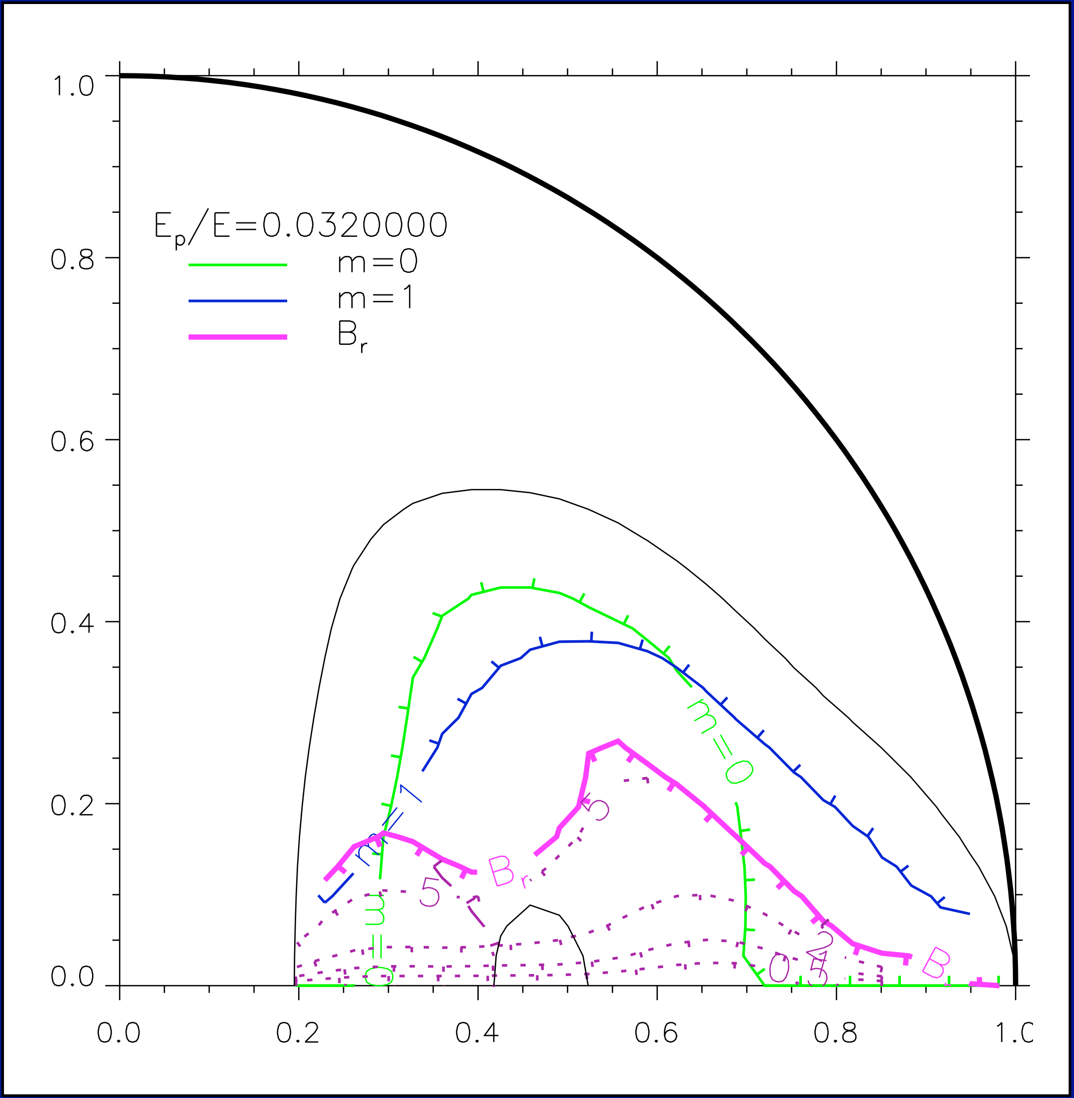


$E_p/E=0.01$, marginally stable

Tayler's conditions applied to configuration found in simulations

- m=0 and m=1 conditions satisfied in only part of the star (hence any purely toroidal field is unstable)
- Weak poloidal component stabilises field if

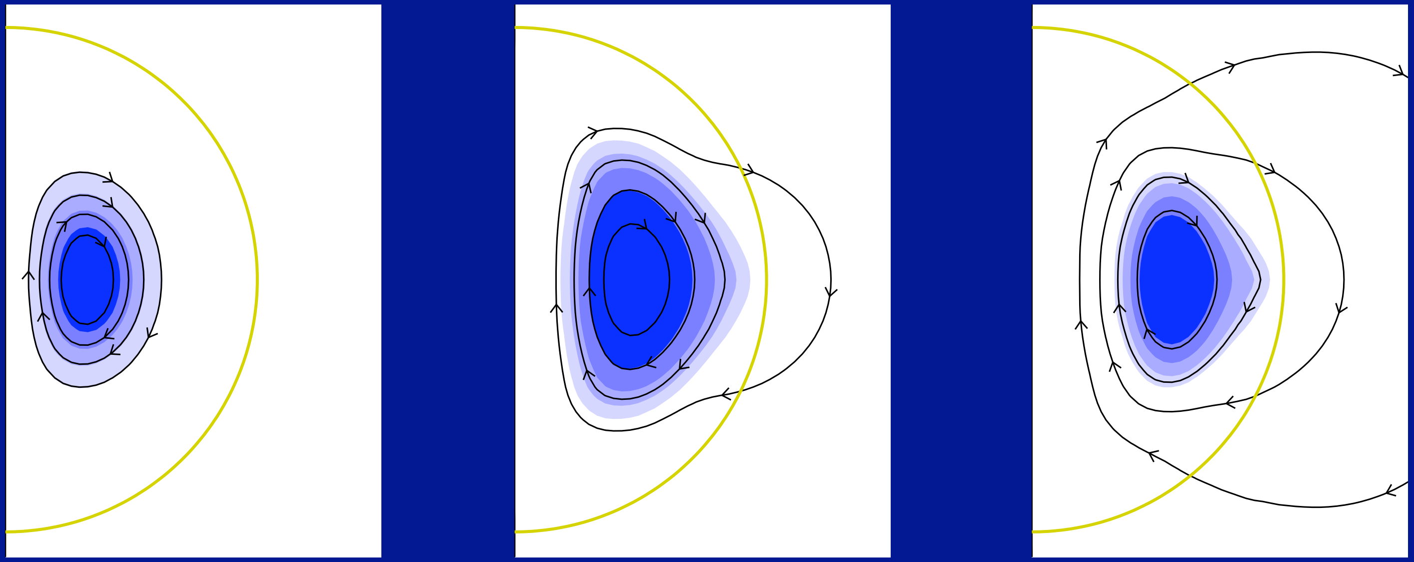
$$B_r > B_\phi \frac{L_r}{r}$$



E_p/E=0.032, stable

Simulations with low E_p/E

- Simulations with a different E_p/E ratios performed for various axisymmetric fields
- Lower limit on E_p/E found to be 1-3%



lower limit	~3%	~1%	~3%
upper limit	~80%	~80%	~80%

Magnetic deformation

- Magnetic field deforms star
  - Predominantly poloidal field (high E_p/E) -> oblate star
  - Predominantly toroidal field (low E_p/E) -> prolate star
- This leads to torque-free precession
- Damping of precession -> minimisation of energy while conserving angular momentum
  - Predominantly poloidal field -> aligned rotator
  - Predominantly toroidal field -> perpendicular rotator
- Non-aligned rotator emits gravitational waves
- But: external torques...

Magnetic energy as inferred from dipole strength: possibility for underestimation

- Energy in higher multipoles
- Dipolar fields can
  - be concentrated towards centre of star, little flux emerging on surface
  - have (and probably do have) large toroidal flux - which we cannot observe directly
- Explanation for differing observational properties of NSs in same place on P-Pdot diagram?

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Secular evolution of magnetic field

- Ohmic diffusion (finite conductivity)
  - Timescale independent of B
- Buoyancy effects (diffusion of elements and N <-> P+e)
  - $t \sim B^{-2}$
- Hall drift: field is “frozen into” charge carriers instead of with the fluid
  - $t \sim B^{-1}$. See e.g. Reisenegger 2007

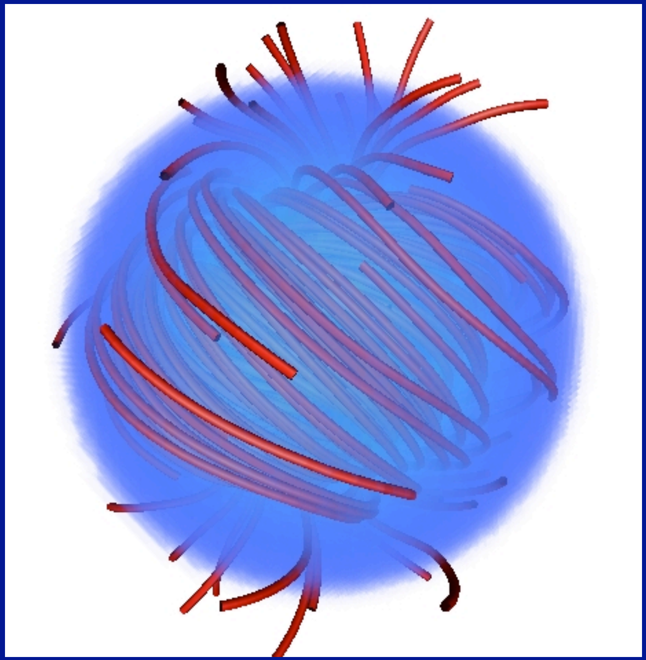
Buoyant rise of flux tubes

- Flux tube in pressure equilibrium with surroundings, so $P_{in} + B^2/8\pi = P_{out}$
- To avoid rapid buoyant rise, we need $\rho_{in} = \rho_{out}$
- Lower P & same ρ: tube must be colder (main-sequence star) or have heavier composition, i.e. lower election fraction Y_e (neutron star)
- Heat/particles diffuses into/out of tube, causing slow rise
- Toroidal flux lost into atmosphere faster than poloidal flux is lost

Relevance of diffusive processes in context

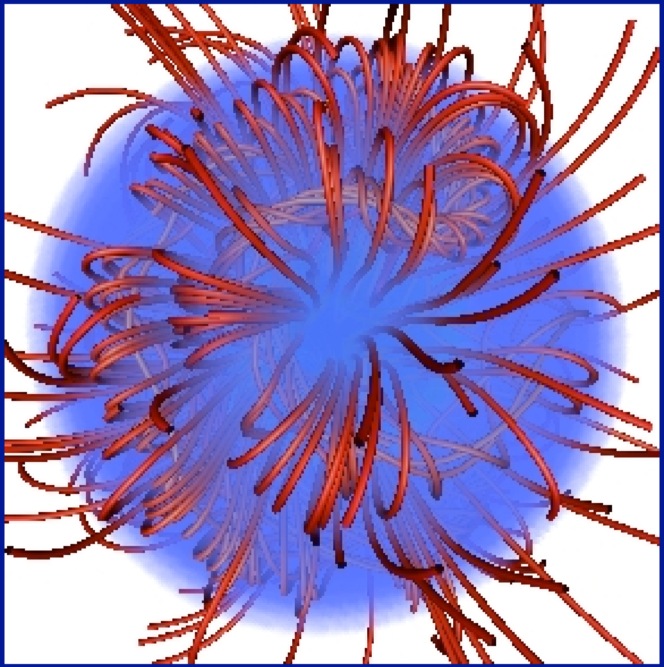
Important?	Finite conductivity	Buoyancy: thermal or species diffusion	Hall drift/ ambipolar diffusion
Main-sequence (>1.5Msun)	Probably too slow	Probably too slow	No
WD	Ohmic timescale ~10^{10} yrs?	Stable stratification from composition gradient required for stability of field! Crystalisation?	Maybe. See Muslimov et al. 1995
NS	Less important than other effects?	Yes, plus weak processes which do not depend on length scale	Yes. See e.g. Reisenegger et al. (various)

Sequence of equilibria



1

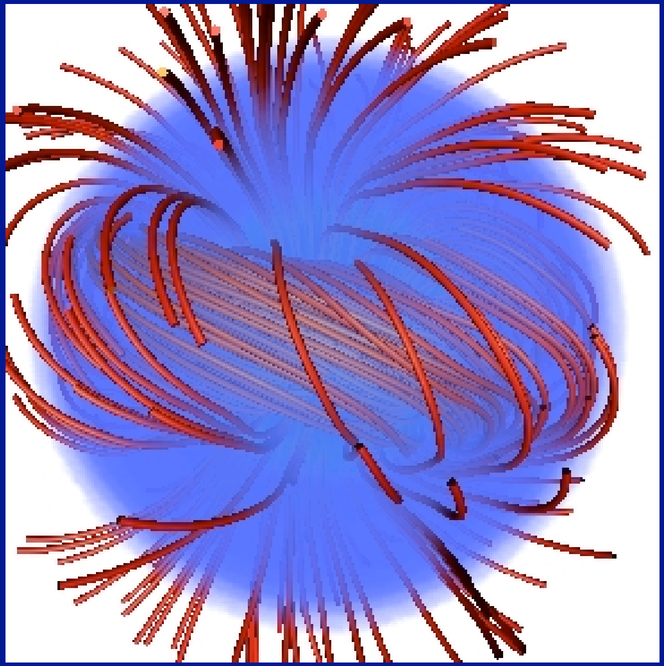
Initial conditions determine starting point



4

Field moves outwards, greater fraction of flux passes through surface

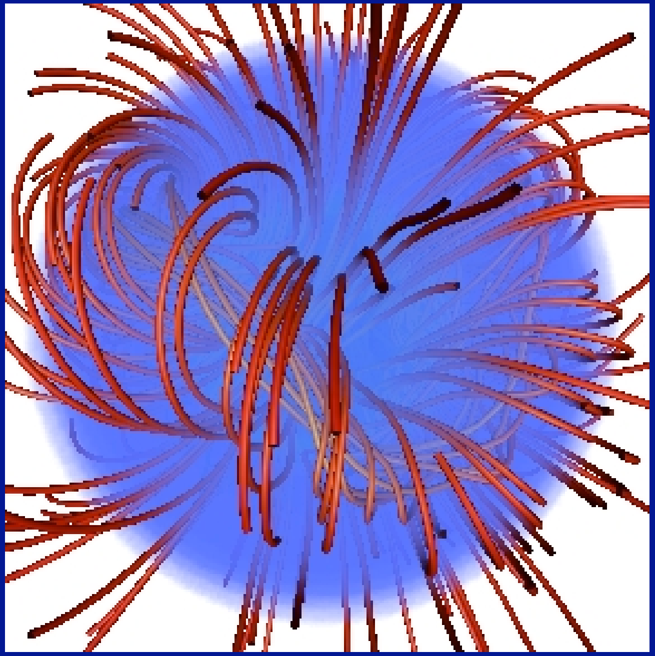
Toroidal flux continually lost through surface, tubes lengthen and narrow



2

Toroidal component becomes weaker, transition to non-axisymmetric equilibrium

3



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Conclusions

- Continuous sequence of stable equilibria consisting of twisted flux tubes
- Field moves quasi-statically along this sequence as a result of diffusive processes
- Initially turbulent field evolves on an Alfven timescale onto some point on this sequence, depending chiefly on the radial distribution of energy
- In axisymmetric equilibria, $0.01 < E_p/E < 0.8$; low ratios likely in practice

Open questions, etc.

- The wide range of field strengths and geometries, even in new-born stars: a generic problem common to all non-convective stars? What about the original (saturated dynamo?) field?
- Upper main-sequence stars: why the low-B cutoff at 200 gauss?
- Initial conditions from convective phase: effect of length scales, helicity, differential rotation, etc.
- Gravitational radiation from magnetically-deformed NSs.. test the ms-magnetar theory?
- Hotspots on surface of NSs from non-isotropic thermal conduction
- Low-frequency QPOs in SGR flares: Alfven modes?