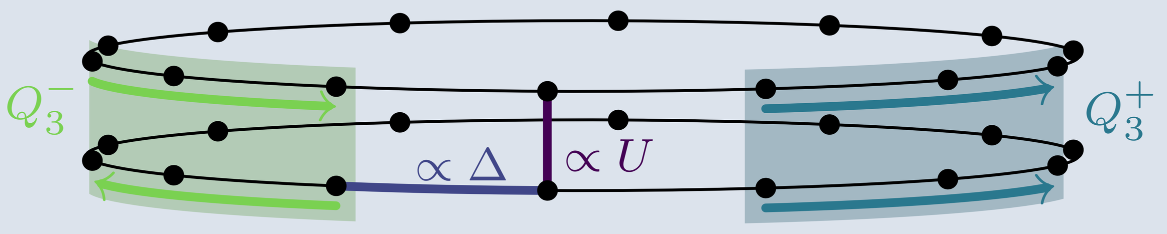
# Long-living prethermalization in nearly integrable spin ladders

**J.Pawłowski**[a], M.Panfil[b], J. Herbrych[a], M. Mierzejewski[a]

[a] Institute of Theoretical Physics, Faculty of Fundamental Problems of Technology, WUST
[b] Faculty of Physics, University of Warsaw

Wrocław University of Science and Technology

## Introduction




We investigate dynamics of observables in a **spin-$\frac{1}{2}$ ladder**, consisting of **two XXZ chains coupled via anisotropic spin-spin interaction of strength** $U$

$$H = \sum_{\ell=1}^{2} H_\ell + U \sum_{j=1}^{L} S^z_{j,1} S^z_{j,2} \,,$$

$$H_\ell = \frac{J}{2} \sum_{j=1}^{L} \left( S^+_{j,\ell} S^-_{j+1,\ell} + \text{H.c.} \right) + \Delta \sum_{j=1}^{L} S^z_{j,\ell} S^z_{j+1,\ell} \,.$$

For $U = 0$ the model is integrable and inherits a complete set of **local integrals of motion** (LIOMs) $\{Q_n^\pm\} \equiv \{Q_{n,1} \pm Q_{n,2}\}$ from both XXZ-chains:

$$[H(U=0), Q_{n,\ell}] = 0 \,, \quad \ell = 1,2 \,, \quad n = 1,2,\ldots \,.$$

In particular:

- $Q_{1,\ell} \propto \sum_{j=1}^{L} S^z_{j,\ell}$ - total magnetization,
- $Q_{2,\ell} \propto H_\ell(U=0)$ - Hamiltonian of the $\ell$-th chain,
- $Q_{3,\ell} \propto i \sum_{j=1}^{L} \left( J S^-_{j,\ell} S^z_{j+1,\ell} S^+_{j+2,\ell} + \Delta S^z_{j,\ell} S^+_{j+1,\ell} S^-_{j+2,\ell} + \Delta S^+_{j,\ell} S^-_{j+1,\ell} S^z_{j+2,\ell} \right) + \text{H.c.}$ - XXZ energy current,
- $Q_{4,\ell} \propto$ complicated, 4-local sum of operators.

We focus on the first two nontrivial LIOMs, $Q_3^\pm$ and $Q_4^\pm$, and study relaxation dynamics of **autocorrelation functions** in Hilbert space sector with zero total magnetization

$$C_n^\pm(t) = \langle e^{iHt} Q_n^\pm e^{-iHt} Q_n^\pm \rangle = \frac{1}{\mathcal{Z}} \text{Tr} \left( e^{iHt} Q_n^\pm e^{-iHt} Q_n^\pm \right)$$

under presence of nonzero, integrability-breaking perturbation $U \neq 0$. Here, $\mathcal{Z}$ is the dimension of the Hilbert space, so the averaging corresponds to the infinite-temperature canonical ensemble.

## Numerical methods

We evaluate the correlation functions using **Quantum Typicality**. The idea is to approximate the trace over the full Hilbert space with an expectation value in a random pure state drawn from a suitable ensemble

$$C_n^\pm(t) \simeq \langle \psi | e^{iHt} Q_n^\pm e^{-iHt} Q_n^\pm | \psi \rangle \,.$$

Imposing unitary invariance on the distribution of the random states $|\psi\rangle = \sum_{j=1}^{\mathcal{Z}} c_j |j\rangle$ and assuming independence of all $\text{Re}\, c_j$ and $\text{Im}\, c_j$, yields a Gaussian distribution for the coefficients, $\text{Re}\, c_j$ and $\text{Im}\, c_j$, in arbitrary orthonormal basis $\{|j\rangle\}$. Then, it can be shown that:

$$\mathbb{E}_{|\psi\rangle} \left[ \langle \psi | Q_n^\pm(t) Q_n^\pm | \psi \rangle \right] = C_n^\pm(t)$$

$$\sigma \left[ \langle \psi | Q_n^\pm(t) Q_n^\pm | \psi \rangle \right] \leq \mathcal{O}\left( \frac{1}{\sqrt{\mathcal{Z}}} \right)$$

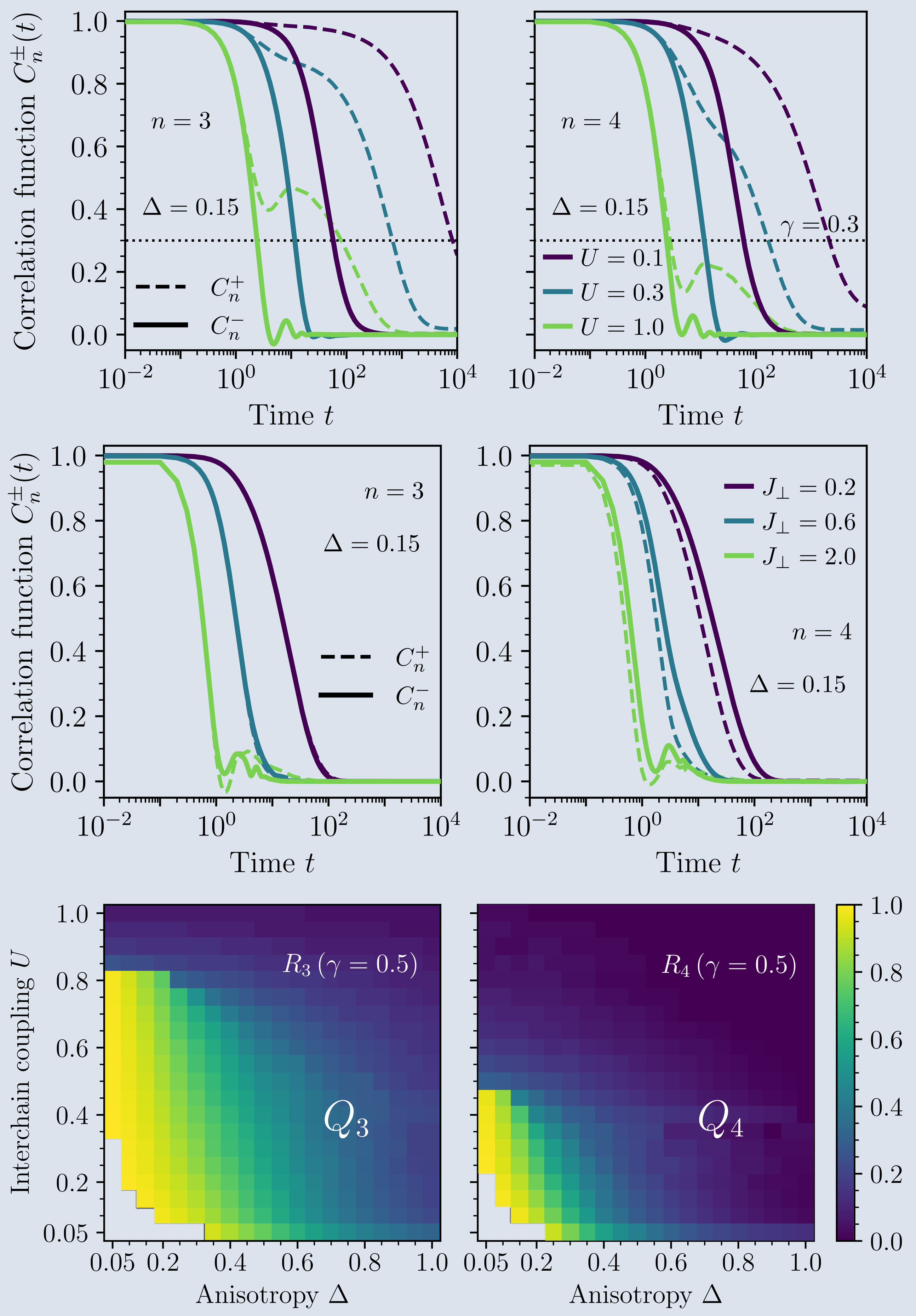
Contribution of a single pure state can already be an exponentially good approximation of $C_n^\pm(t)$, which for small systems can be further improved by additional sampling. We can then shift the time evolution to two auxiliary pure states $|\psi\rangle$, $|\phi_n^\pm\rangle = Q_n^\pm |\psi\rangle$ and calculate it using the **Lanczos time evolution** method.

## References & Acknowledgements

[1] J. Pawłowski, M. Panfil, J. Herbrych, and M. Mierzejewski, "Long-living prethermalization in nearly integrable spin ladders," *Physical Review B*, vol. 109, no. 16, p. L161109.

This project was supported by National Science Centre (NCN), Poland via project 2020/37/B/ST3/00020. The calculations were carried out using resources provided by the Wrocław Centre for Networking and Supercomputing.

## Relaxation of XXZ local integrals of motion




- **relaxation times** for $Q_n^+$ and $Q_n^-$ differ by two orders of magnitude
- prethermalization with twice smaller number of nearly conserved operators than expected
- relaxation times for $Q_n^+$ are very long compared to $1/U^2$
- no difference in relaxation times for $Q_n^+$ and $Q_n^-$ if coupling between chains is different:

$$\sum_{j=1}^{L} \frac{J_\perp}{2} \left( S^+_{j,1} S^-_{j,2} + \text{H.c.} \right)$$

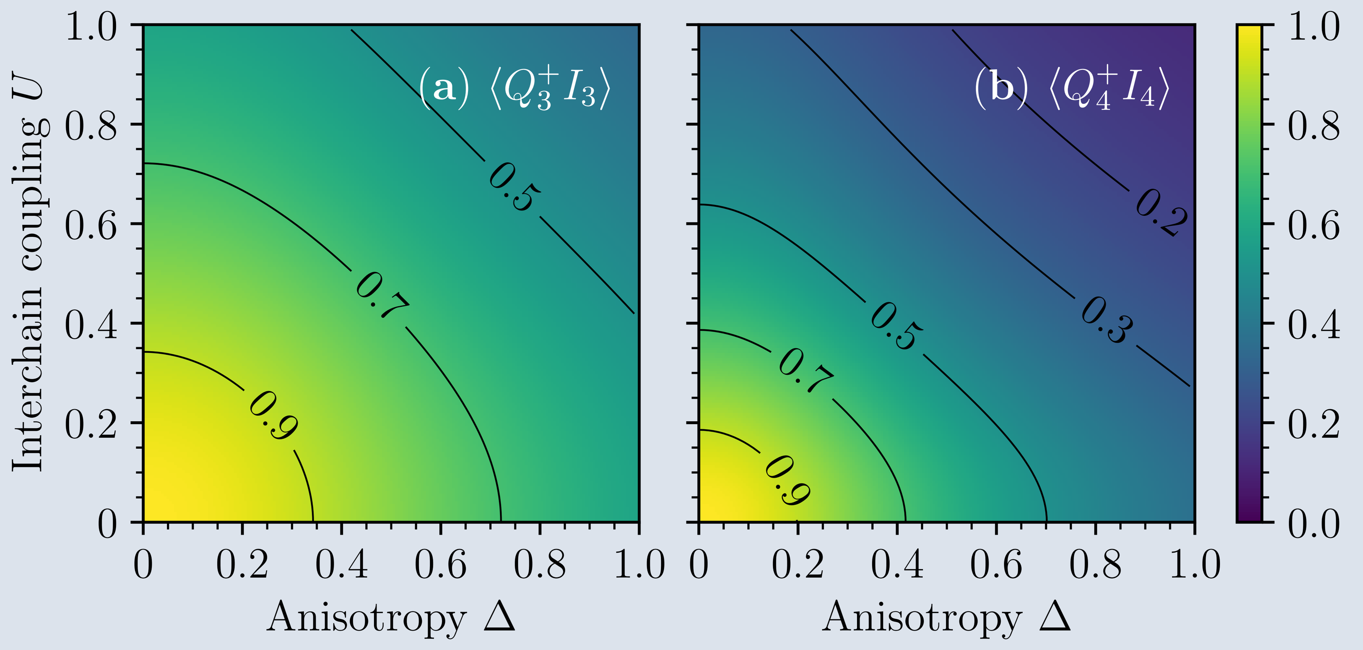
- not a finite size effect (see supplementary material in [1])
- quantitative difference in relaxation times

$$R_n(\gamma) = \frac{t^+_{n,\gamma} - t^-_{n,\gamma}}{t^+_{n,\gamma} + t^-_{n,\gamma}}$$

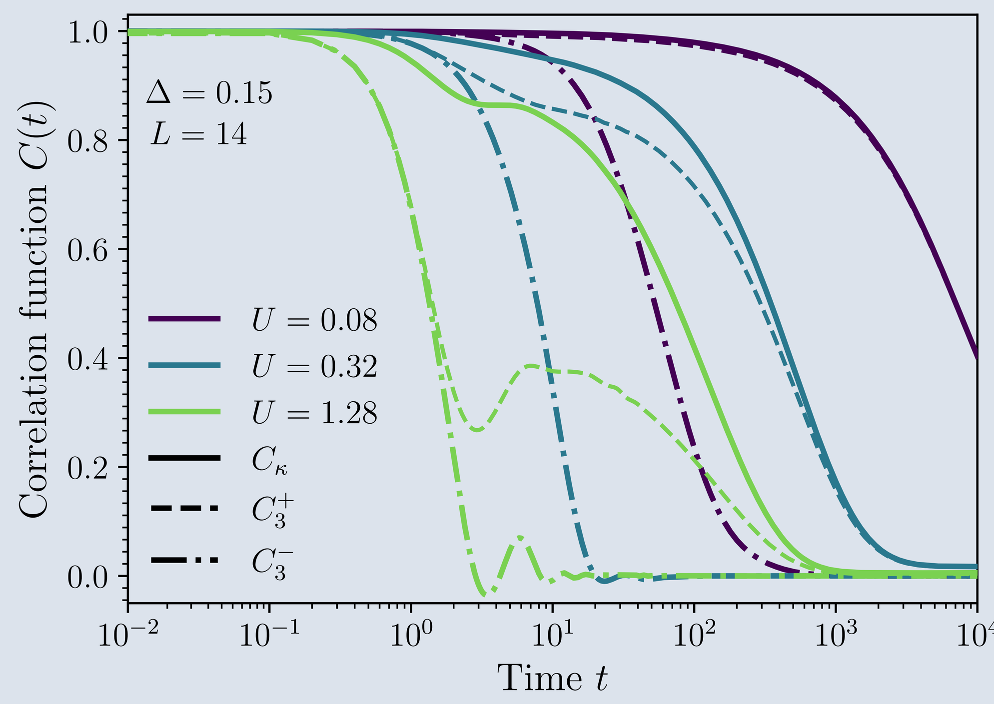
- most pronounced for small $\Delta$ and $U$

## Relation to spin-Hubbard model and impact on measurable quantities

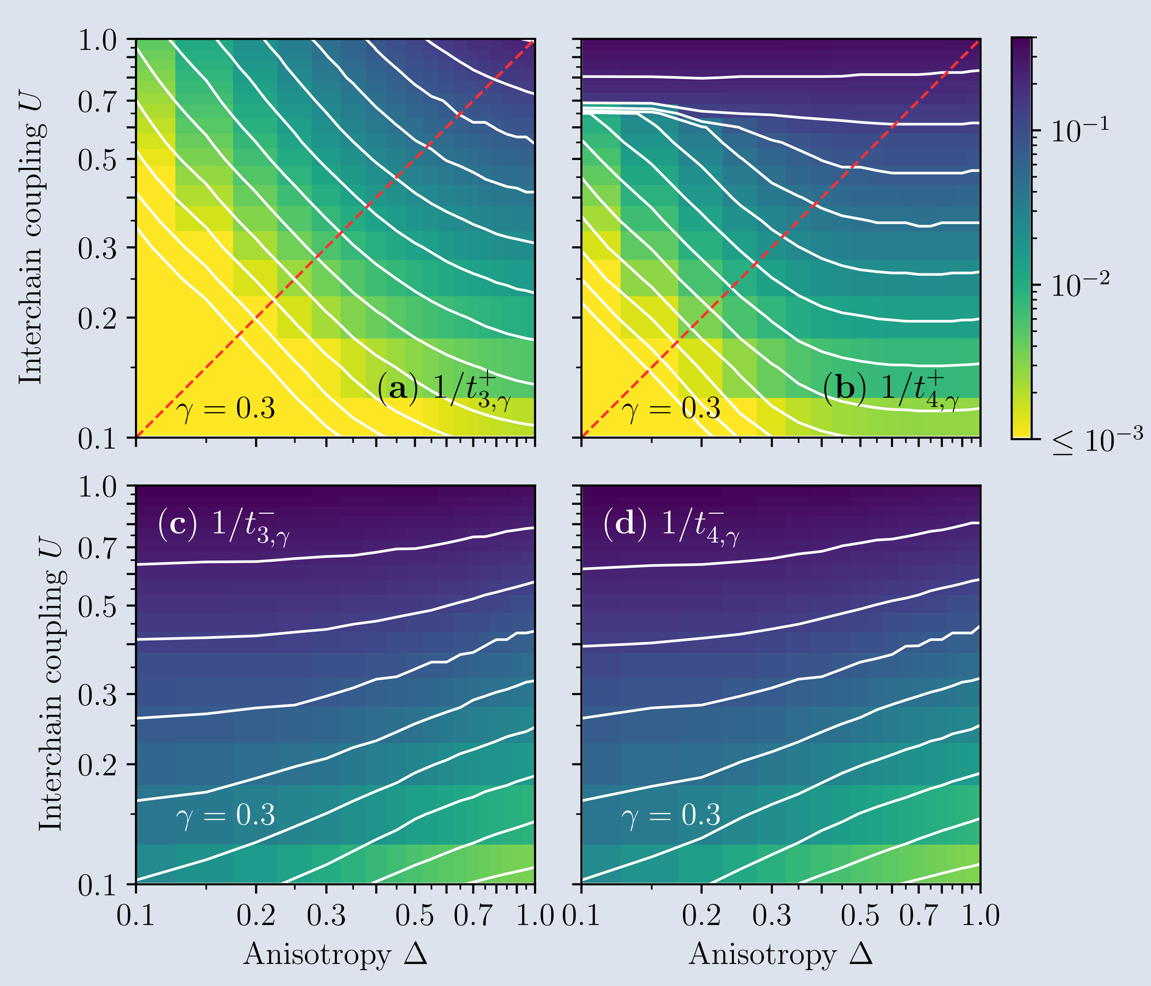
- $H(\Delta = 0)$ reduces to the Hubbard model $\longrightarrow$ another family $\{I_n\}$ of LIOMs!
- $\langle Q_n^- I_n \rangle$ vanishes identically due to parity under ladder leg exchange $\ell \to 3 - \ell$
- $\langle Q_n^+ I_n \rangle$ is large in the same area of parameter space as $R_n(\gamma)$

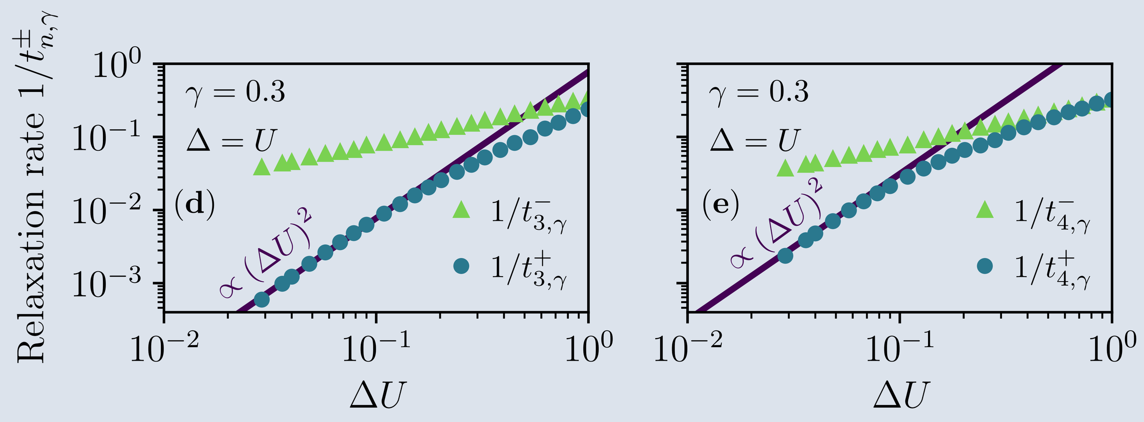



- both $Q_3^+$ and $I_3$ are closely related to the **energy current** of the spin ladder
- energy current relaxes very slowly for all points $(\Delta, U)$ in the parameter space, close to $(0,0)$
- should be measurable in experiment as **high thermal conductivity**




## Lowest-order contributions to relaxation rates




- $1/t^-_{n,\gamma} \propto U^2$, consistent with the Fermi golden rule
- $1/t^+_{n,\gamma} \propto$ ~~$U$~~, ~~$\Delta$~~, ~~$U^2$~~, ~~$\Delta^2$~~, ~~$U\Delta$~~, ~~$U^4$~~, ~~$\Delta^4$~~, $(U\Delta)^2$
- relaxation rates increase with **fourth power of the distance** in the parameter space from the intersection point

This reasoning is general, and is expected to hold true for other systems which are integrable along two intersecting lines in the parameter space, provided that both families of LIOMs have large overlaps close to the crossing point.