The quantum triangular plaquette model in a longitudinal field

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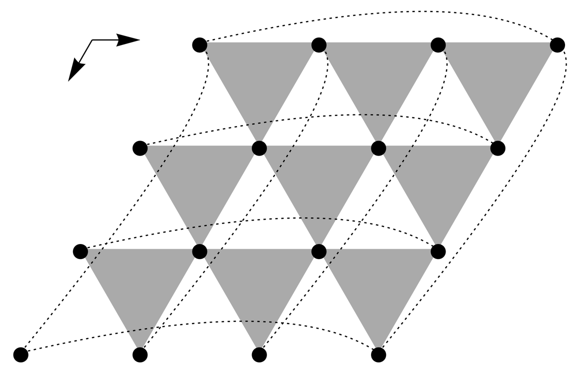
The QTPMz



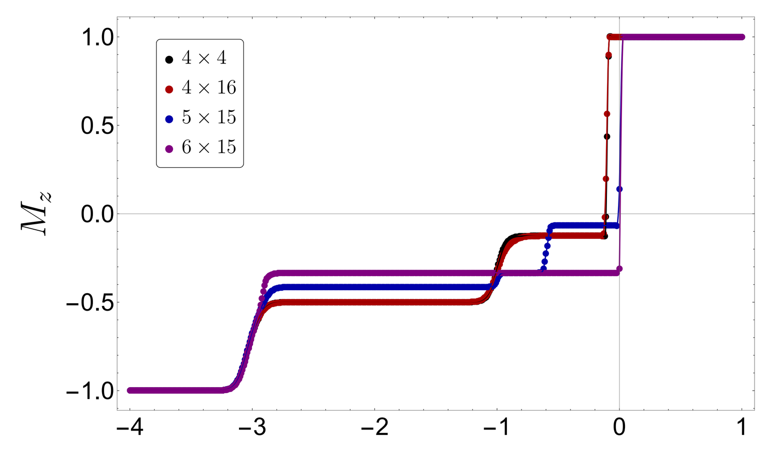
- Ising spins on the sites $i = 1, \dots, N$ of a square lattice
- Fully periodic boundaries
- Cubic interactions between the spins on the corners of downward-pointing triangles, minimally coupled with a transverse and a longitudinal field
- $H_{\mathrm{QTPMz}} = -J \sum_{i,j,k \in \nabla} Z_i Z_j Z_k - h \sum_i X_i - k \sum_i Z_i$

Classical TPM $(k = 0, h = 0)$

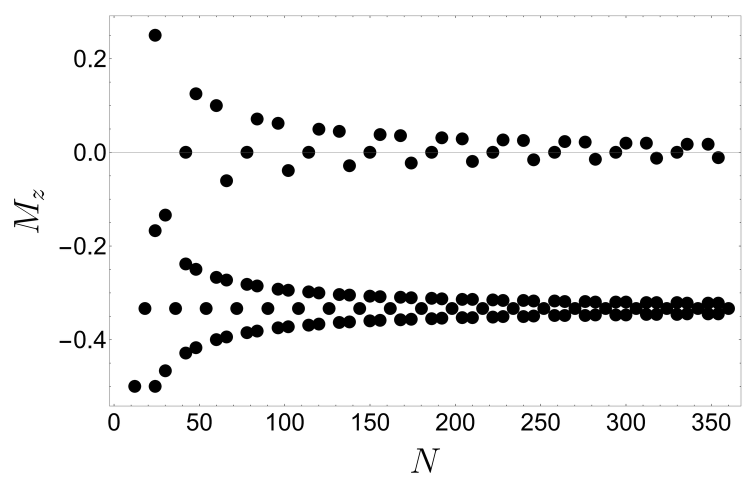
- $E_{\mathrm{TPM}} = -J \sum_{i,j,k \in \nabla} s_i s_j s_k$
- For power of 2 system sizes there is an exact spin-defect duality, $E_{\mathrm{TPM}} = -J \sum_{\nabla} d_{\nabla}$
- Subextensive number of ground states at most, a single for powers of 2.
- Zero-temperature ground states and symmetries found systematically from an 1+1D cellular automaton, rule 60.

Classical TPMz $(k \neq 0, h = 0)$ - Frustration

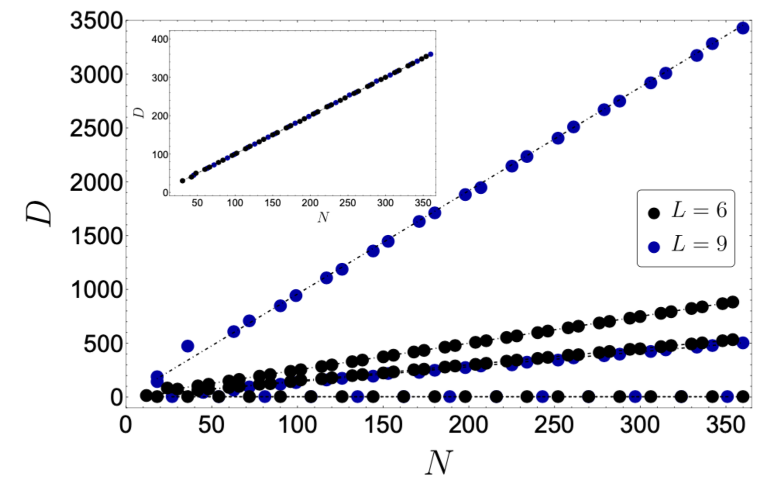
Emergence of plateaus for $-3 < k < 0$



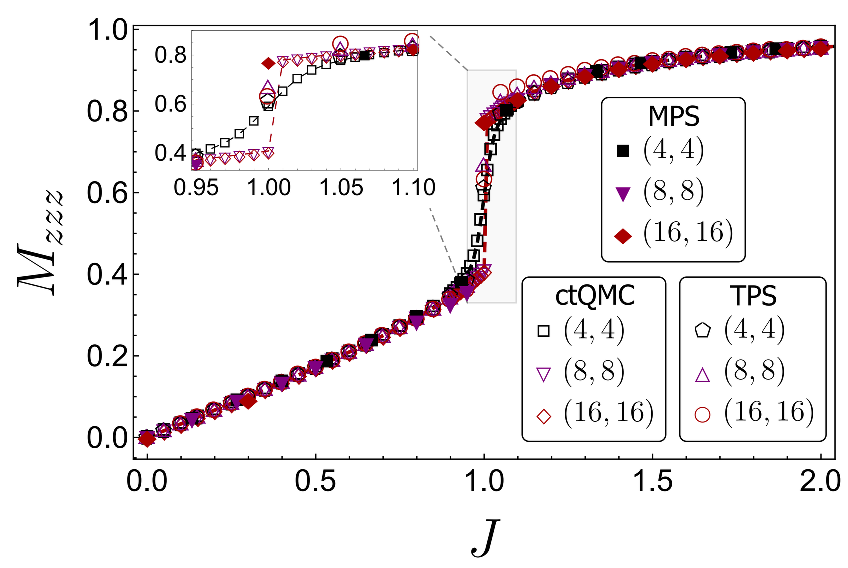
Plateaus, $L = 6$

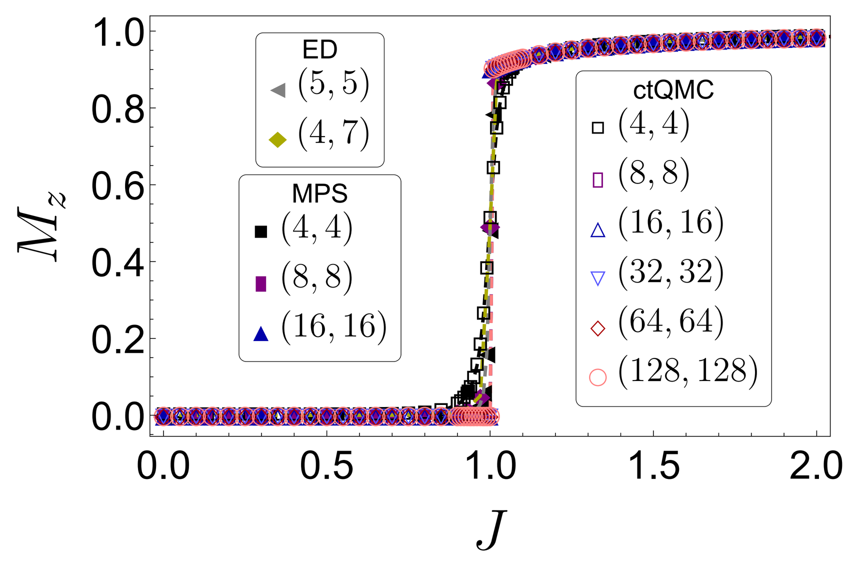


Degeneracy, $L = 6$ and $L = 9$

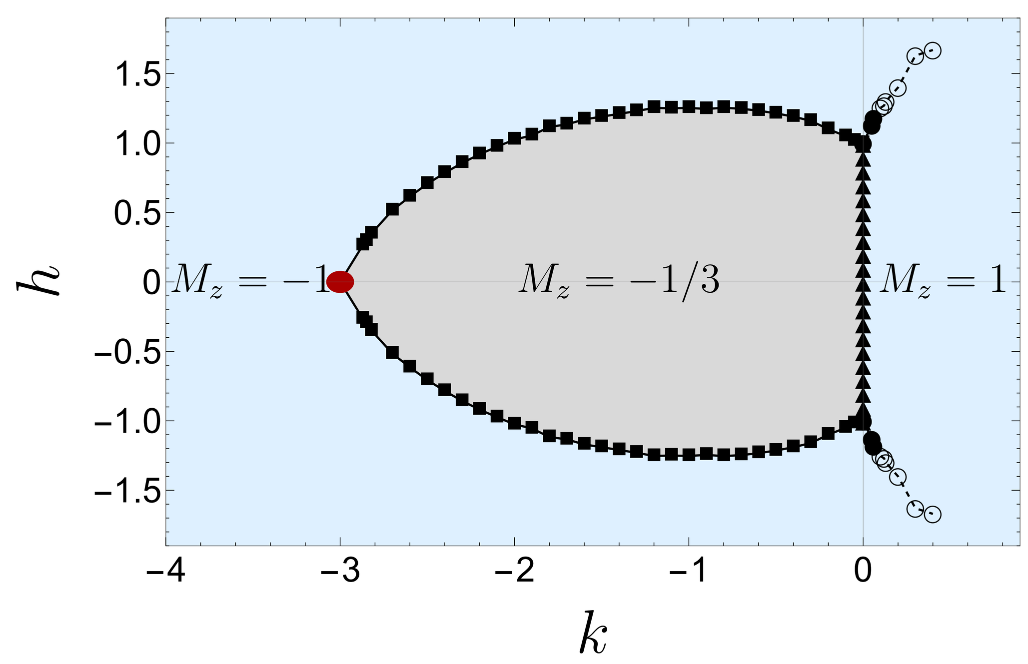


The QTPM QPT





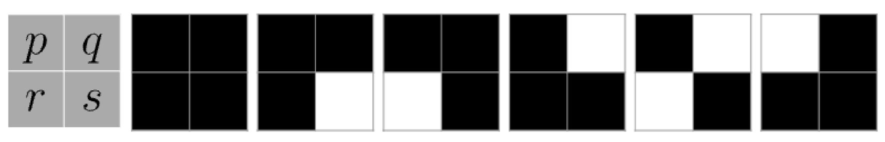
QTPMz: Phase Diagram



- $k \ll -3$: no ph.tr.
- $-3 < k < 0$: VBS - PM, (1-QPT), TSSB
- $0 < k \lesssim 0.1$: FM - PM (1-QPT)
- $0.1 \gtrsim k$: FM - PM (crossover)
- $k = 0, h < 1$: FM - Frustr. (1-PT)
- ▲ $h = k = 1$, triple point
- ▲ $k \sim 0.1, h \sim 1.25$, critical point

Effective Rydberg Model and Lattice Gauge Theory

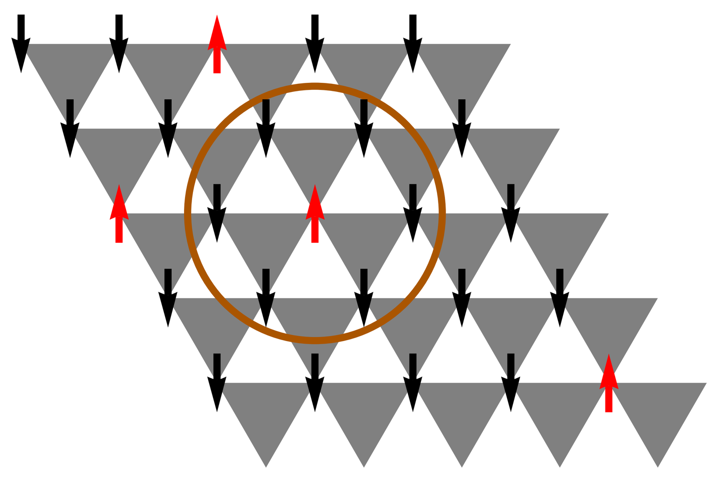
For $h = 0, k = -3$, CSP with $pq + ps + qs = 1$, mod 2



Now, deform to a triangular lattice. We get an effective Rydberg model,

$$H_{\mathrm{P_6X}} = -h \sum_i P_6^{(i)} X_i - \delta k \sum_i Z_i,$$

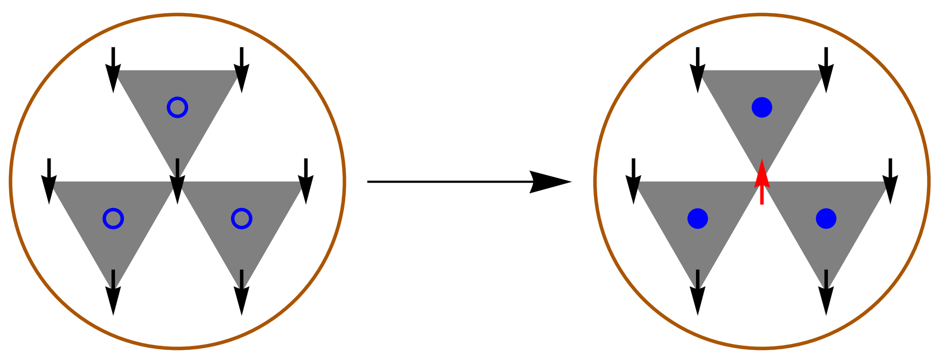
with $P_6^{(i)} = \bigotimes_{\langle i,j \rangle} \frac{1}{2}(1 - Z_j)$.



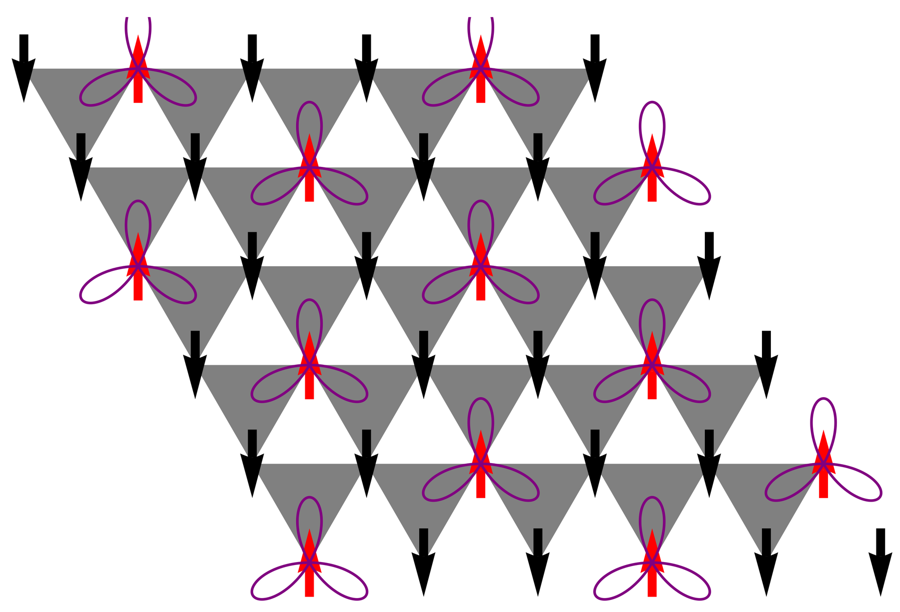
Rephrase the problem as an LGT:

$$H_{\mathrm{LGT}} = -h \sum_i \left(S_i^+ f^\dagger_{\nabla_{i,1}} f^\dagger_{\nabla_{i,2}} f^\dagger_{\nabla_{i,3}} + \mathrm{h.c.} \right) - \delta k \sum_i Z_i,$$

with $G_\nabla \equiv f^\dagger_\nabla f_\nabla - \frac{1}{2} \sum_{i \in \nabla} (Z_i + 1)$. Fixing the gauge condition, $G_\nabla = 0$, for all plaquettes provides the equivalence with the "Rydberg" description, schematically:



Equivalent description in terms of trimers ($k < -3$ monomers, $k \sim -3$ monomer-trimer mixture, $k > -3$ fully packed trimer coverings).



Conclusions

- Classical phase of the TPM viewed as the coexistence of a frustrated and a FM phase.
- Frustration:(i) strong finite size effects for all $N = L \times M$ sizes, (ii) exact results in the thermodynamic limit only for $3P \times 3Q$.
  - For $L \neq 2^k$, $h = 0$: "ph.tr." at $k = 0$.
  - For $L = 2^k$, $h = 0$: "ph.tr." at $k = 0$ only asymptotically.
- Effective Rydberg description
  - The neighborhood of the $k = -3$ point similar to the effective low-energy space of the Ising model on the triangular lattice.
  - LGTs + monomer-trimer description.
- Is the $k = -3$ region confined or deconfined?
- Parallelism to classical stochastic dynamics and large deviations of kinetically constrained model studies.
- Slow dynamics and nonthermal dynamics.
- Tetramers, pentamers, etc for other plaquette models (PIM, SPyM, rule 150, etc).

BCS ansatz - topological order?

Composite spins, τ_i, $|\Downarrow\rangle \equiv |\downarrow\rangle_s \otimes |000\rangle_f$, $|\Uparrow\rangle \equiv |\uparrow\rangle_s \otimes |111\rangle_f$. The Hamiltonian transforms to

$$H_\tau = -h \sum_i X_i^\tau - \delta k Z_i^\tau.$$

Ansatz wavefunction: $|\Psi\rangle = \sum_\Lambda u^{N_\Downarrow(\Lambda)} v^{N_\Uparrow(\Lambda)} |\Lambda\rangle$