QUANTUM RESERVOIR COMPUTING ON RANDOM REGULAR GRAPHS

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INTRODUCTION

Quantum reservoir computing (QRC) is a low-complexity learning paradigm that combines the inherent dynamics of input-driven many-body quantum systems with classical learning techniques for nonlinear temporal data processing. We introduce a strongly interacting spin model on random regular graphs as the quantum component and investigate the interplay between static disorder, interactions, and graph connectivity, revealing their critical impact on quantum memory capacity and learnability accuracy. We tackle quantum and classical tasks, and identify optimal learning and memory regimes through studying information localization, dynamical quantum correlations, and the many-body structure of the disordered Hamiltonian.

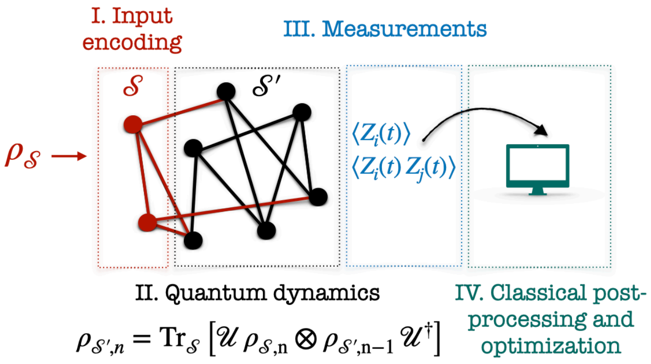
MODEL AND DYNAMICS

Hamiltonian: $$\mathscr{H} = \sum_{ij} J^z_{ij} Z_i Z_j + \sum_{ij} J^x_{ij} X_i X_j + \sum_i h^z_i Z_i + \sum_i h^x_i X_i$$

$J^{x/z}_{ij} = J^{x/z} A_{ij}$ — A_{ij} is a component of the graph adjacency matrix

$h^{x/z}_i = h^{x/z} + \delta^{x/z}_i$ — Disorder parameter: $\delta^{x/z}_i \in [-\Delta^{x/z}, \Delta^{x/z}]$

We fix: $J^z = 1, h^z = 0, h^x = 1, \Delta^z = 0.2$



X_i, Z_i are Pauli spin 1/2

$$\rho_{\mathcal{S}',n} = \mathrm{Tr}_{\mathcal{S}}\left[\mathscr{U}\, \rho_{\mathcal{S},n} \otimes \rho_{\mathcal{S}',n-1}\, \mathscr{U}^\dagger\right]$$

An auxiliary system $\mathcal{S}$ is initialized in density matrix $\rho_{\mathcal{S},n}$, encoding the input data stream at step n. $\mathcal{S}$ interacts with the reservoir $\mathcal{S}'$ for the time-scale $J^z \Delta t$ through $\mathscr{U}(\Delta t) = e^{-i\mathscr{H}\Delta t}$, before the next input is injected. The Hamiltonian $\mathscr{H}$ is defined on a graph with N spins each connected to exactly k random neighbours. Measurement results of the reservoir's spins are then recorded for optimization. Repeating this for a sequence of temporal data, one can construct a quantum recurrent neural channel $\rho_{\mathcal{S}',n} = \mathscr{L}_{\mathcal{S}}\left(\rho_{\mathcal{S}',n-1}\right)$, capable of learning and performing various real-world and neurological tasks.

METHODOLOGY

The input-output relation of a quantum reservoir supplemented with a classical learning layer can be summarized as $\{\bar{y}_n\} = \mathscr{F}\left(\{\rho_{\mathcal{S},n}\}, \rho_{\mathcal{S}\mathcal{S}',n}, \mathscr{W}\right)$. Here $\{\bar{y}_n\}$ indicates the predictions, which is found by classical post-processing and minimizing an error measure with respect to a sequence of desired targets $\{y_n\}$. $\mathscr{W}$ represents the optimal weights obtained after the training stage. Linear memory capacity of the reservoir can be assessed by calculating the correlation coefficient $C_n = \mathrm{cov}^2(y_n, \bar{y}_n)/\sigma^2 y_n \sigma^2 \bar{y}_n$, which is bounded between zero and unity. The error measure is simply defined as $\mathrm{MSE} = \sum_n^L (y_n - \bar{y}_n)^2/L$.

LEARNING TASKS

- **Tomography and quantum memory**

$$\rho_W(\eta, t) = \frac{1-\eta(t)}{4}\mathbb{I} - \eta(t)\,\rho_B,$$

Two-qubit Werner states as inputs:

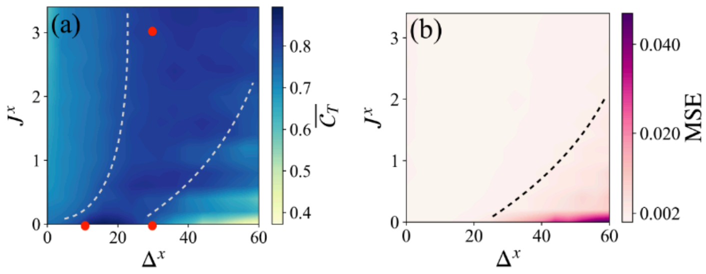
$$\rho_B = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad 0 \le \eta(t) \le 1$$

which is a mixture of a singlet Bell state ρ_B and a maximally mixed state $\mathbb{I}$. Utilizing a simple linear regression, the target y_n is to reconstruct the past mixing parameters by probing the current state of the system:

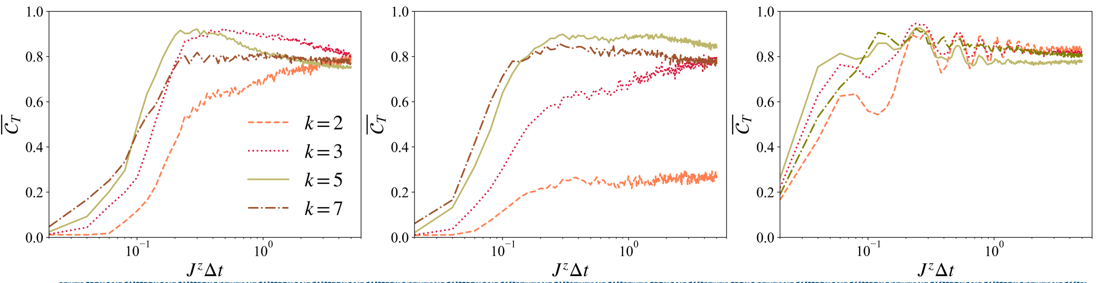
$$y_{n,\tau} = \eta\,(n\Delta t - \tau\Delta t)\,.$$

- **Logical multitasking**

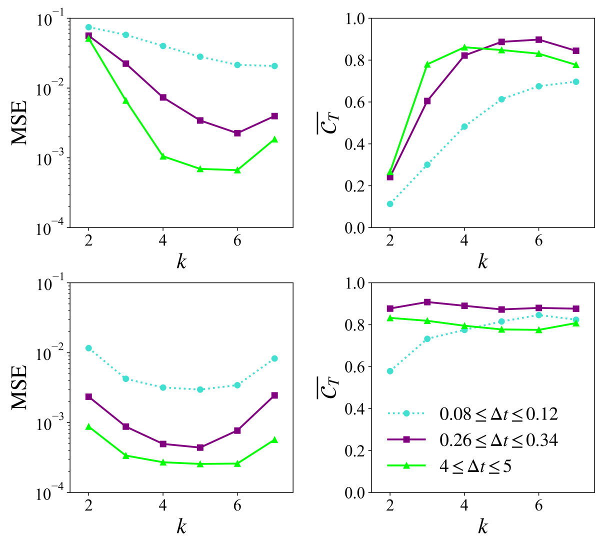
Given two independent sequence of binary inputs, the task is to simultaneously learn their classical logical manipulations **AND**, **OR**, **XOR**. For classification, we use support vector machines with a nonlinear kernel. Input in this case is given as the product of pure states, e.g. $|\uparrow\rangle \otimes |\downarrow\rangle$.

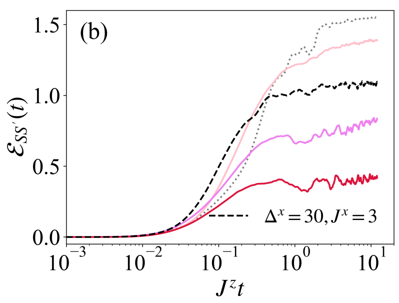


Learning diagrams.
Normalized total memory capacity for $1 \le \tau \le 6$ and MSE for $\tau = 1$. Dashed lines approximately mark regions where $\overline{C_T} \ge 0.75$ and MSE $\le 2.5 \times 10^{-3}$. The optimal learning regime for our model occurs at around the boundary of chaotic-localized phase transitions. Calculated for $(N, k) = (8,3)$ and $J^z\Delta t = 3$, with N indicating the graph size and k the graph degree. Total memory capacity is defined as $C_T = \sum_{n,\tau} C_{n,\tau}$.



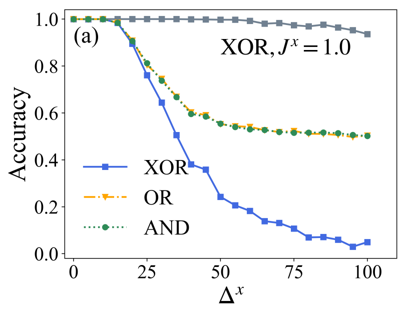
Normalized total memory capacity as a function of $J^z\Delta t$.
Left: $\Delta^x = 10, J^x = 0$. **Middle**: $\Delta^x = 30, J^x = 0$. **Right:** $\Delta^x = 30, J^x = 3$.
There can be an optimal window for the interaction time $J^z\Delta t$ between $\mathcal{S}$ and $\mathcal{S}'$ to achieve the largest memory. Low degree graphs exhibit slow growth of the quantum memory, possibly due to poor fading memory. However, by adding the delocalizing interactions X_iX_j, the memory exhibits a fast growth to optimal values for all degrees!





Dynamical logarithmic negativity;
Entangling quantum correlations are crucial for avoiding localization and improving learning performance.

Time-averaged MSE and normalized total memory capacity.
For $(\Delta^x, J^x) = (30,0)$ and $(\Delta^x, J^x) = (30,3)$. Models with intermediate Δt and k display the best performance. In the limit $k \to N-1$, it becomes exceedingly difficult to extract the non-locally hidden inputs information through only (quasi-) local measurements.



Accuracy of the logical multitasking as a function of disorder.
Remarkably, the presence of entangling interactions is greatly beneficial for the classification accuracy.

CONCLUSION

We have shown that, in a favourable dynamical regime, our quantum spin model acts as a high-performing learning reservoir for temporal and non-linear data processing. Our work paves the way toward optimal design of such quantum learning platforms by relating their performance to their fundamental physical and geometrical properties.

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