Out-of-Time-Order Correlators & Chaotic Volume

Introduction

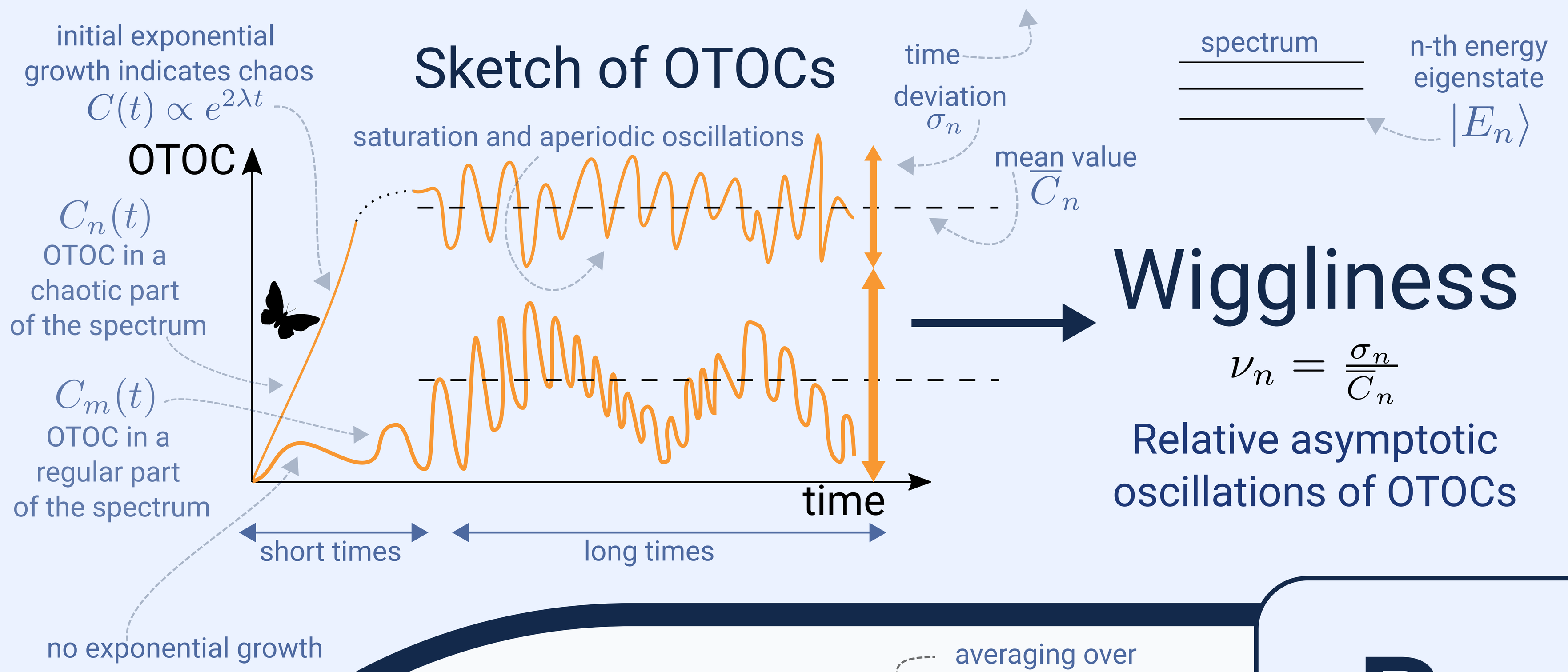
The Out-of-Time-Order Correlators (OTOCs) gained popularity as an indicator of quantum chaos. The connection between their short-time behavior and the classical instability of trajectories brought butterflies into the quantum world. In the last years, studies proposed that the long-time behavior of OTOCs can reveal a different measure of chaoticity. Here we present a numerical study of OTOCs in an algebraic model of a Bose-Einstein condensate. We show that the ratio of the asymptotic standard deviation to the asymptotic mean value (called wiggliness for brevity) correlates with the volume of the regular part of the phase space in the classical limit.

Out-of-Time-Order Correlators

In general, OTOCs are products of operators taken in different times. Here we employ four-point correlation function in a form of a square commutator of operators $[\hat{V}(t),\hat{W}(0)]$, where $\hat{V}(t)=e^{\frac{i}{\hbar}\hat{H}t}\hat{V}e^{-\frac{i}{\hbar}\hat{H}t}$

OTOC as an operator $\quad [\hat{V}(t),\hat{W}(0)]^{\dagger}[\hat{V}(t),\hat{W}(0)]$

Energy expectation value $\quad C_n(t)=\left\langle E_n\middle|[\hat{V}(t),\hat{W}(0)]^{\dagger}[\hat{V}(t),\hat{W}(0)]\middle|E_n\right\rangle$



Wiggliness

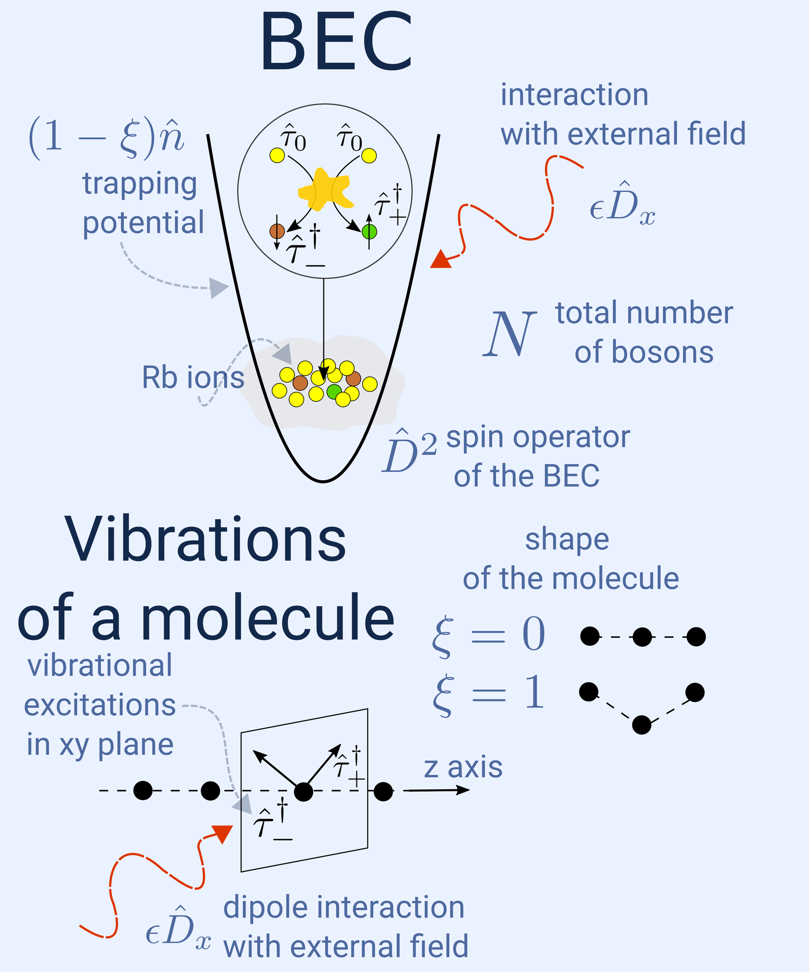
$$\nu_n=\frac{\sigma_n}{C_n}$$

Relative asymptotic oscillations of OTOCs

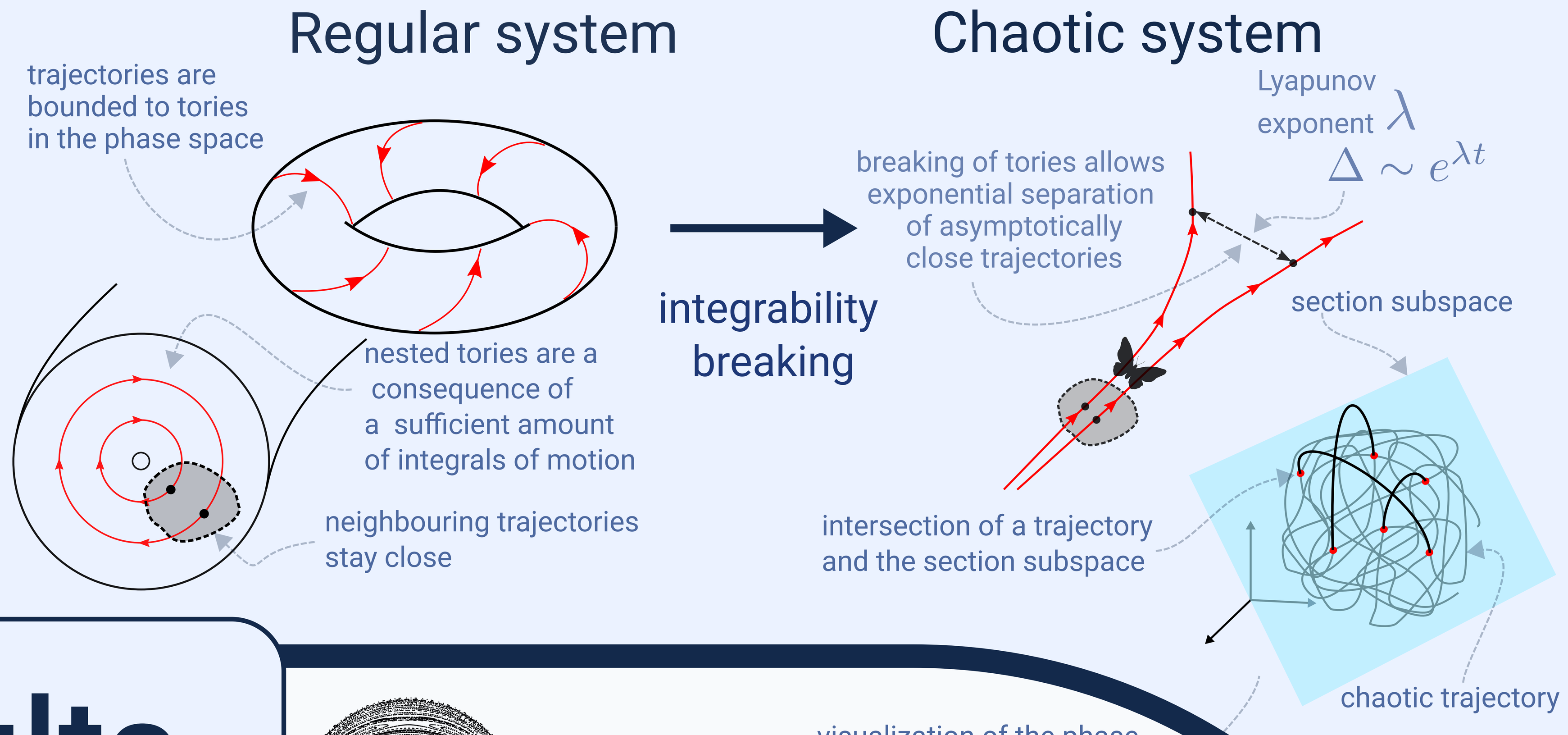
Model

$$\hat{H}=(1-\xi)\hat{n}-\frac{\xi}{N-1}\hat{D}^2+\epsilon\hat{D}_x$$

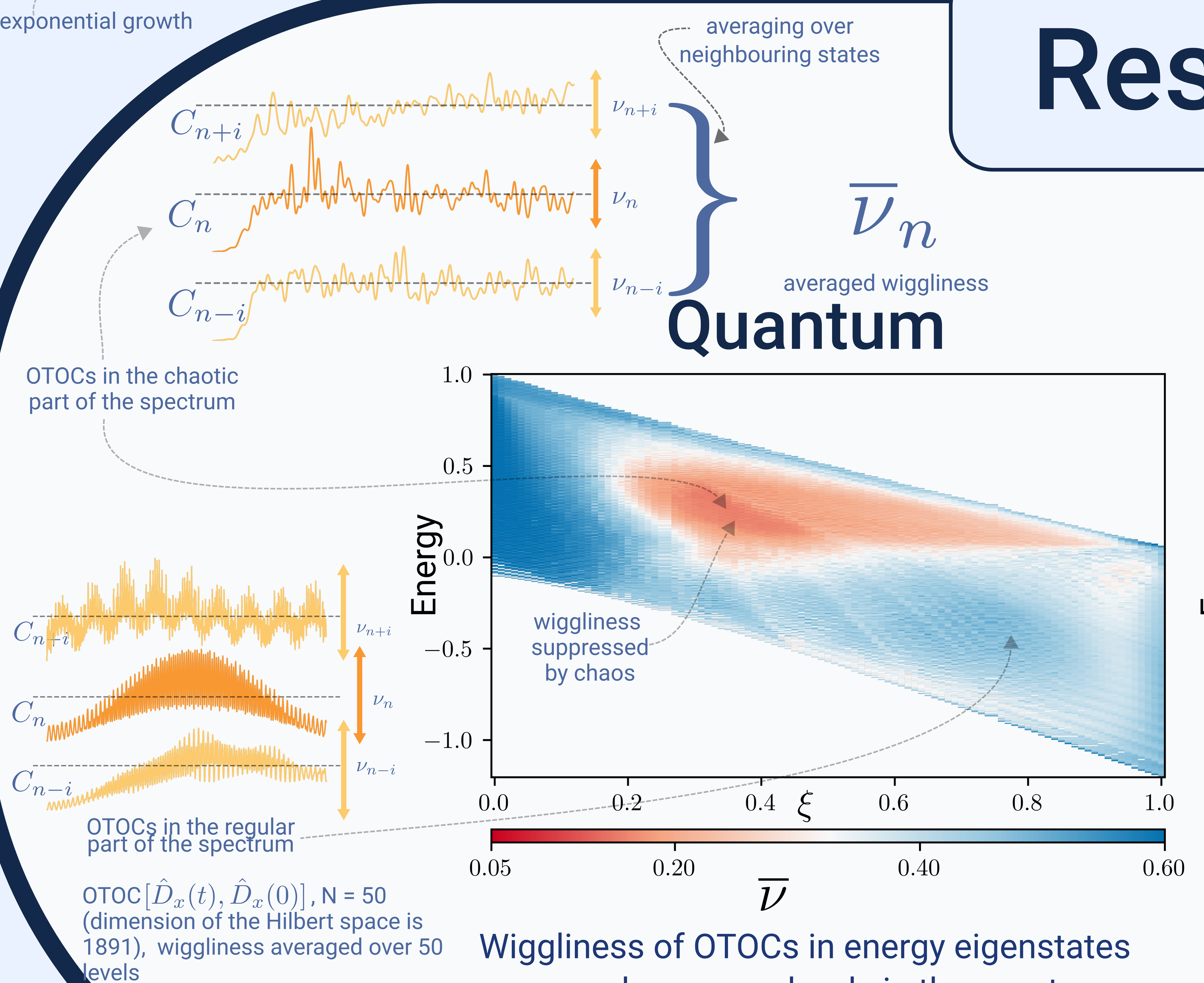
The model Hamiltonian belongs to a class of boson-interacting systems. It is constructed from generators of u(3) Lie Algebra and can be represented by the bilinear products of creation and annihilation operators $\hat{\tau}_+,\hat{\tau}_-,\hat{\tau}_0$. This model has been originally introduced [3] to describe the bending modes of linear polyatomic molecules. Recently, it has also been applied to study the properties of spin-1 Bose-Einstein condensate (condensate of cold Rb atoms) and chaos [4].Theclassical limit corresponds totheinfinitesizelimit. As $N\to\infty$ the Hamitonian becomes a function on a four-dimensional phase space



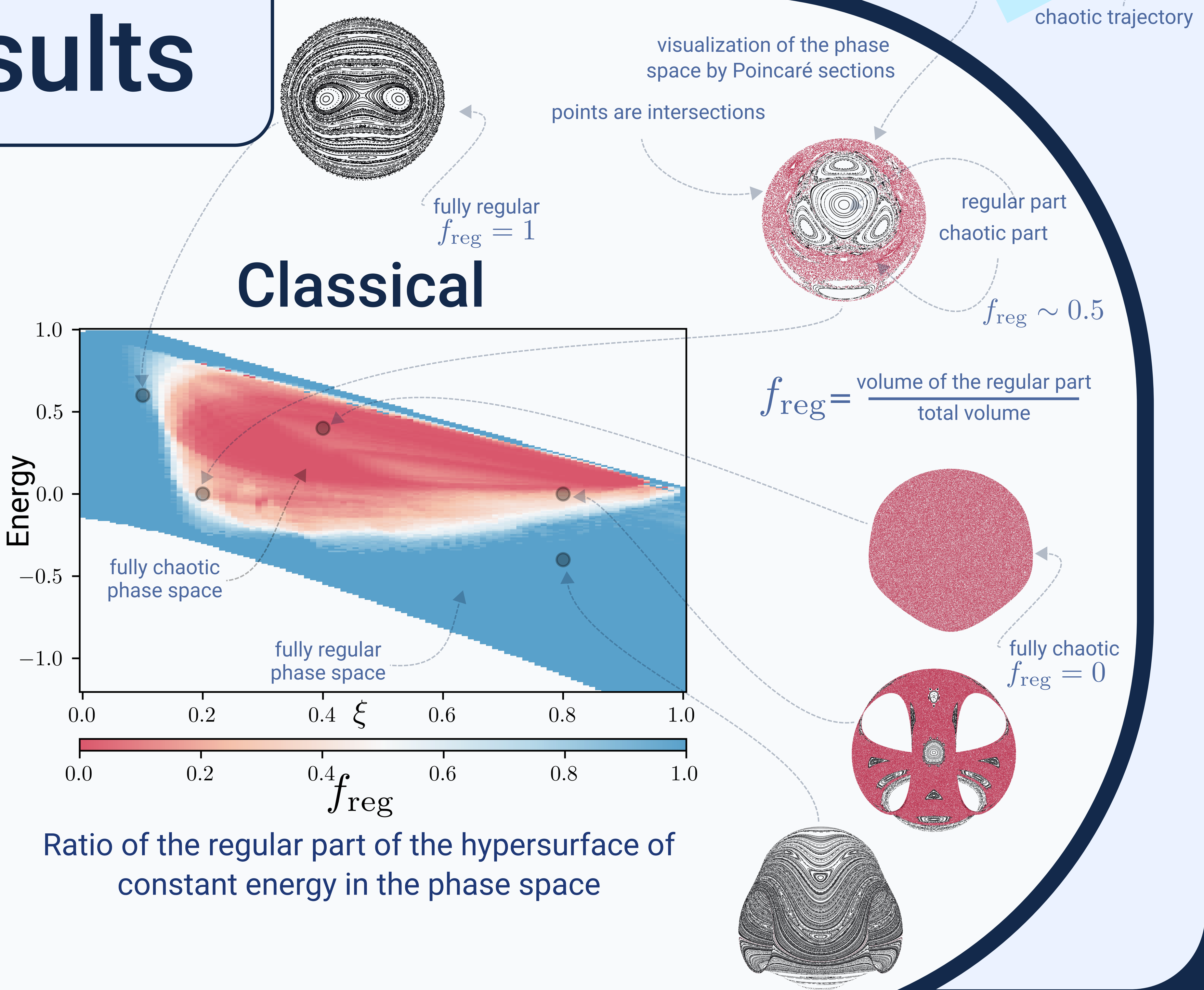
Classical Chaos & Chaotic volume



Results

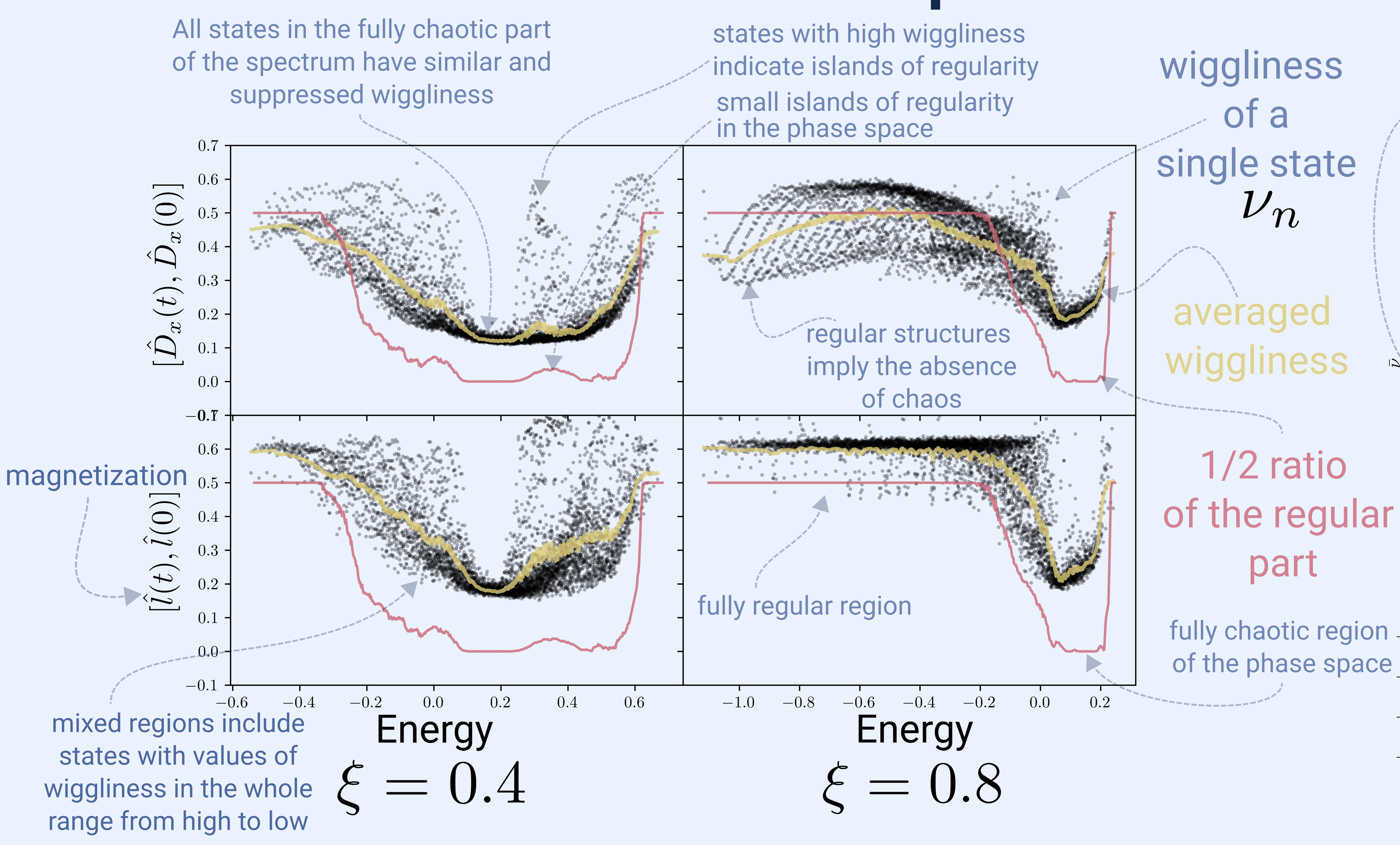


Wiggliness of OTOCs in energy eigenstates averaged over near levels in the spectrum

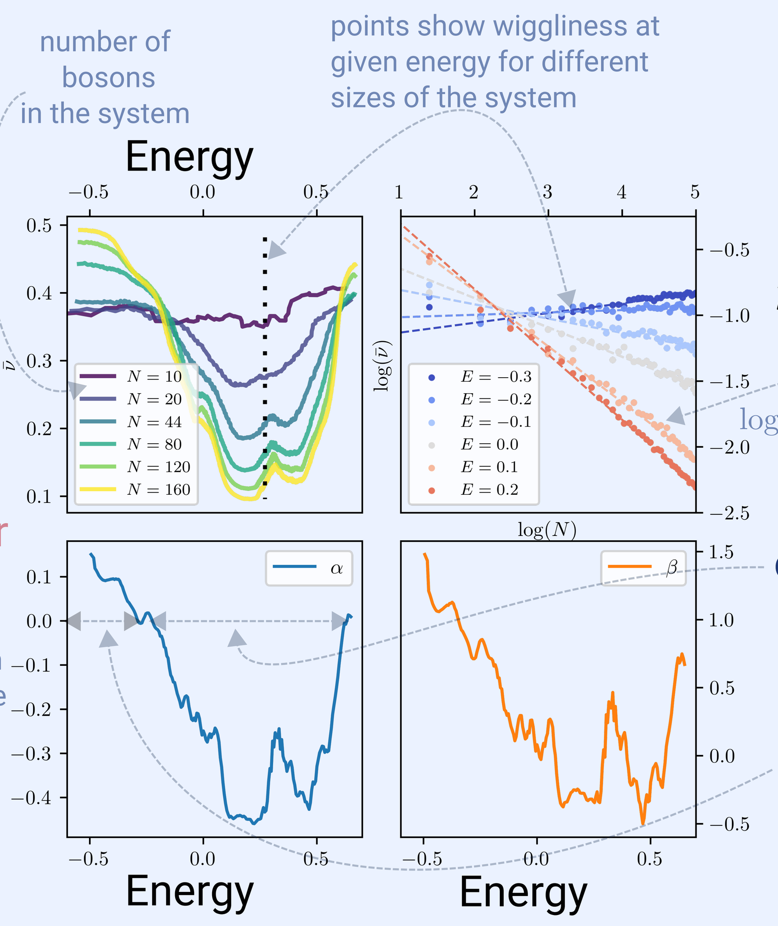


Ratio of the regular part of the hypersurface of constant energy in the phase space

It works for other OTOC operators too



Bigger system = Higher suppression of wiggliness



Wiggliness scales algebraicaly

$$\bar{\nu}(E,N)=N^{\alpha(E)}e^{-\beta(E)}$$

chaotic regions of spectra

$$\alpha<0$$

regular regions of spectra

$$\alpha\geq 0$$

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[2] E. M. Fortes, I. García-Mata, R. A. Jalabert, D. A. Wisniacki, Gauging classical and quantum integrability through out-of-time-ordered correlators, Phys. Rev. E 100, 042201(2019).

[3] F. Iachello and S. Oss, Algebraic approach to molecular spectra: Two-dimensional problems, J. Chem. Phys. 104, 6956 (1996).

[4] M. Rautenberg and M. Gärttner, Classical and quantum chaos in a three-mode bosonic system, Phys. Rev. A 101, 053604 (2020).

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