Error accumulation in random quantum circuits

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Abstract

Fidelity decay captures the inevitable state degradation in any practical implementation of a quantum process. We devise bounds for the decay of fidelity for a generic evolution given by a random quantum circuit model that encompasses errors arising from the implementation of two-qubit gates and qubit permutations. We show that fidelity decays exponentially with both circuit depth and the number of qubits raised to an architecture-dependent power and we determine the decay rates as a function of the amplitude of the aforementioned errors.

Introduction and motivation

In this work, we aim to characterize the accumulation of errors in generic random quantum circuits, which serve as models for complex quantum dynamics and are of paramount importance in various areas of physics.

Given the states

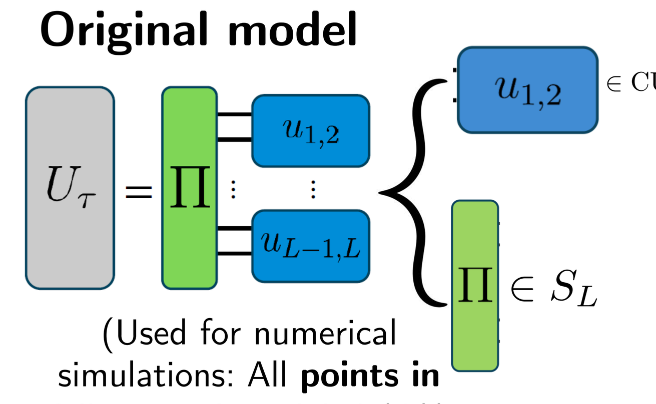
$$\begin{cases} |\Psi\rangle = \prod_{\tau=1}^{T} U_\tau |\psi_0\rangle & \text{Ideal} \\ |\tilde\Psi\rangle = \prod_{\tau=1}^{T} \tilde U_\tau |\psi_0\rangle & \text{Faulty} \end{cases}$$

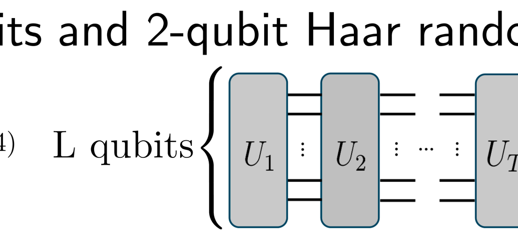
We are interested in the average fidelity

$$\mathcal{F} = |\langle \Psi | \tilde\Psi \rangle|^2 \qquad \overline{\mathcal{F}} \in [\tfrac{1}{2^L}, 1]$$

In particular, we consider the following layer configuration made of a random permutation Π of L qubits and 2-qubit Haar random gates

Original model





(Used for numerical simulations: All **points in following plots**, label **(O)**)

Each Π is decomposed in swaps S. The optimal decomposition depends on the underlaying qubit connectivity (see example).

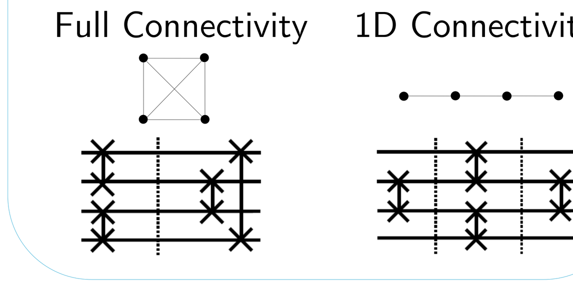
Example

Given the permutation of 4 qubits

0123 → 2301

The optimal decomposition is

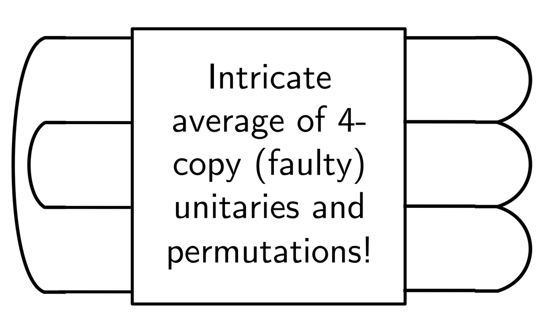
Full Connectivity 1D Connectivity



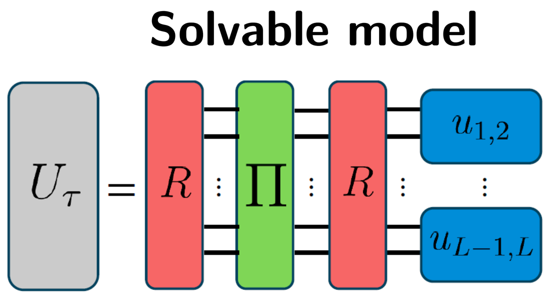
Solvable model: closed formula

$$\overline{\mathcal{F}} = \sum_{nm} \langle \boldsymbol{mnnm} \| \overleftarrow{\prod_\tau} \left[\tilde U_\tau \otimes \tilde U_\tau^* \otimes U_\tau \otimes U_\tau^*\right] \| \psi_0\psi_0\psi_0\psi_0\rangle$$

$$= \frac{1}{2^L} \sum_{nml} \langle \boldsymbol{mnnm} \| \overleftarrow{\prod_\tau} \left[\tilde U_\tau \otimes \tilde U_\tau^* \otimes U_\tau \otimes U_\tau^*\right] \| \boldsymbol{llll}\rangle = \frac{1}{2^L}$$



Solvable model



(Used for analytical computations: All **lines in following plots**, label **(S)**)

To compute the average fidelity, we introduce the Solvable Model. The 2-qubit gates and permutations are uncorrelated due to the insertion of large faultless Haar random unitaries R.

$$\overline{\mathcal{F}}(\alpha, p) = \left(1 - \frac{1}{2^L}\right)\left(\frac{(\delta(p)-1)(\Delta(\alpha)-1)}{(4^L-1)^2}\right)^T + \frac{1}{2^L}.$$

Unitary Deviations

2-qubit gates noise

Each gate is independently poisoned with a new unitary generated from a unstructured Hamiltonian

$$\tilde u_{1,2} = e^{-i\alpha h_{12}} u_{1,2}$$

$$h \in \mathrm{GUE}(4) \qquad \alpha >= 0$$

The effect of the faulty gates is encoded in

$$\Delta(\alpha) = 2^L\,(3f(\alpha)+1)^{L/2}$$

f(α) is closely related to GUE(4) **spectral form factor**: $f(\alpha) \sim e^{-5\alpha^2}$

Faulty Permutation

Each swap is implemented with an imprecise pulse $\tilde S(\beta) = S^\beta$ where β is drawn from Gaussian with variance σ². This is equivalent to model that each S can be independently omitted with probability p=(1−exp(−π²σ²/2))/2

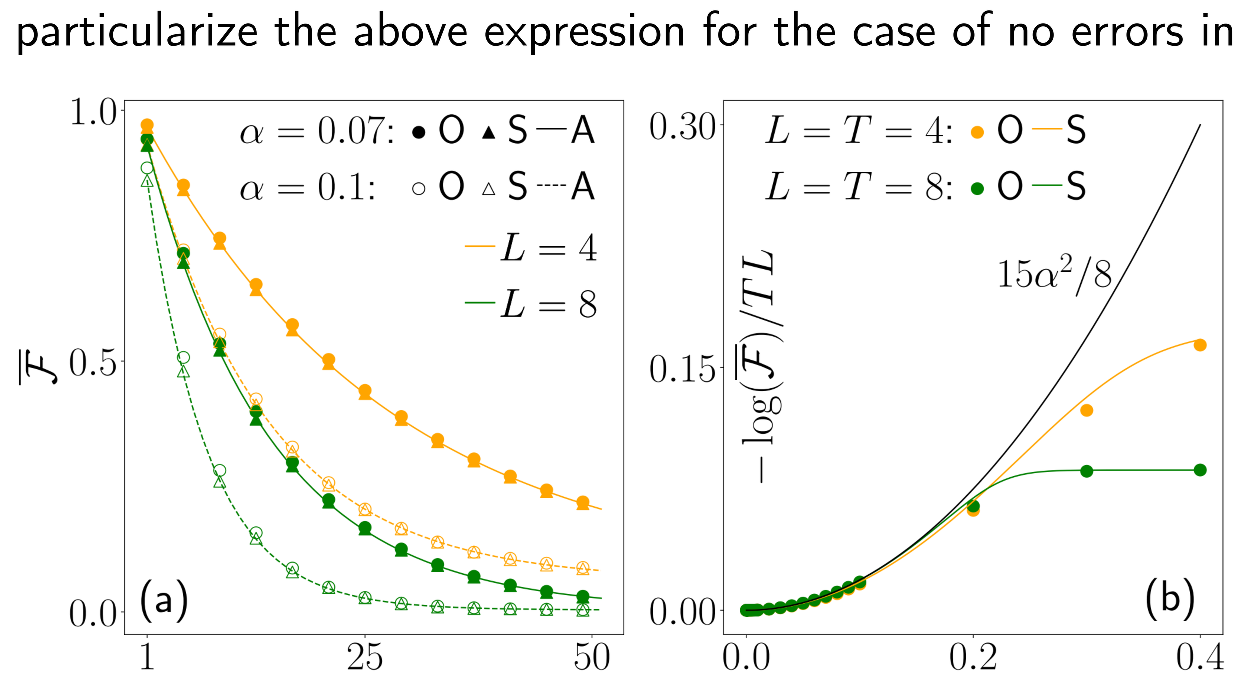
The effect of faulty permutations is encoded in the **error factor**

$$\delta(p) = \overline{4^{m(\Pi,p)}} \qquad m(\Pi,p)\text{: nº of cycles in permutation } \tilde\Pi(p)\Pi^T$$

$\delta(p)$ depends on architecture. We compute it in sparse error regime $p \ll 1$

Only faulty 2-qubit gates

First, we particularize the above expression for the case of no errors in permutations.



The expression is simpler in the small error regime:

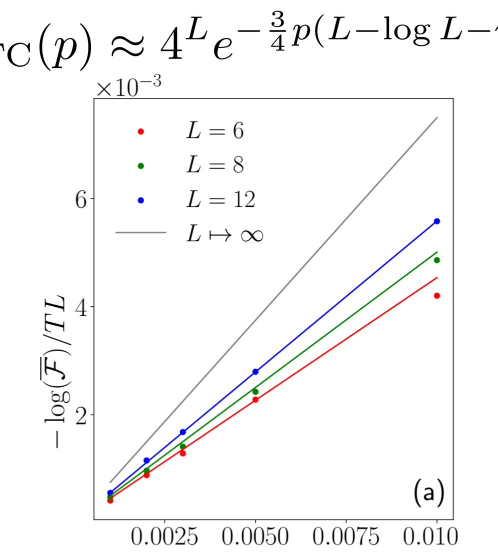
$$\mathcal{F}(\alpha) \sim e^{-\frac{15}{8}\alpha^2 LT}, \quad \alpha \ll 1$$

- The agreement is remarkable even for shallow circuits.
- The exponent is **universal:** we also found it in the brick-wall circuit.

Only faulty permutations

Full connectivity

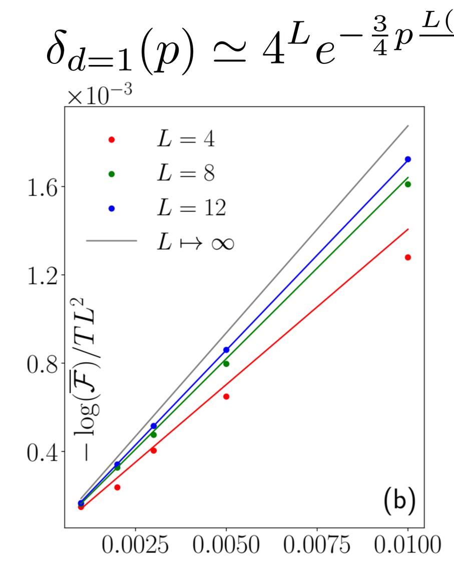
$$\delta_{\mathrm{FC}}(p) \approx 4^L e^{-\frac{3}{4}p(L - \log L - \gamma)}$$



$$\overline{\mathcal{F}}_{\mathrm{FC}} \approx e^{-\frac{3}{4}pTL}$$

1D connectivity

$$\delta_{d=1}(p) \simeq 4^L e^{-\frac{3}{4}p\frac{L(L-1)}{4}}$$



$$\overline{\mathcal{F}}_{d=1} \approx e^{-\frac{3}{16}pTL^2}$$

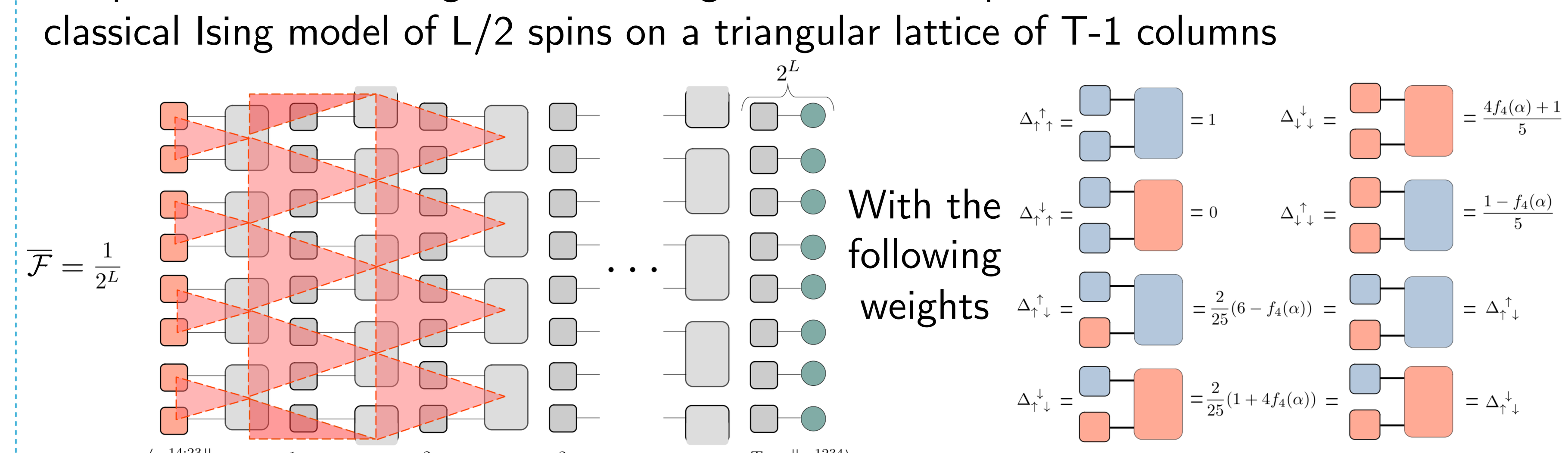
In addition, we devised an algorithm for finding optimal decomposition in higher dimensional d cubic lattices:

$$\delta_d \gtrsim 4^L e^{-\frac{3}{4}(d-1/2)pL^{1+\frac{1}{d}}}$$

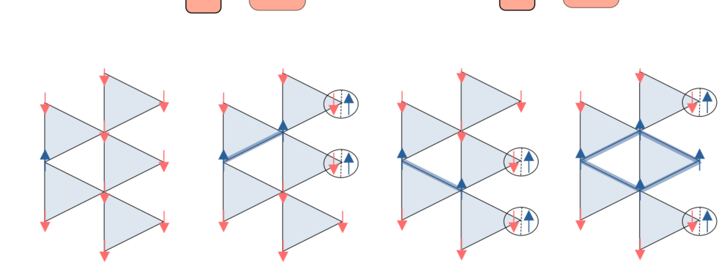
See details at: https://github.com/RafalBistron/Hypercube_sorting

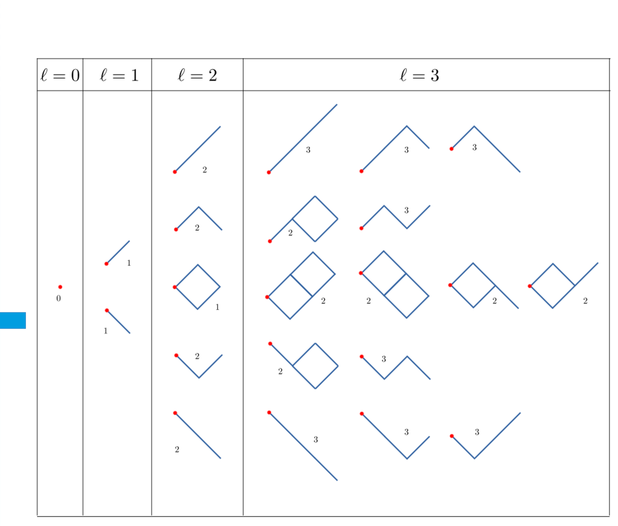
The brick-wall circuit

We have also considered the brick-wall circuit, which consists of a sequential alternation of leftward and rightward single-qubit shifts. The average fidelity is found by mapping the problem to counting allowed configurations in the partition function of a 2D classical Ising model of L/2 spins on a triangular lattice of T-1 columns



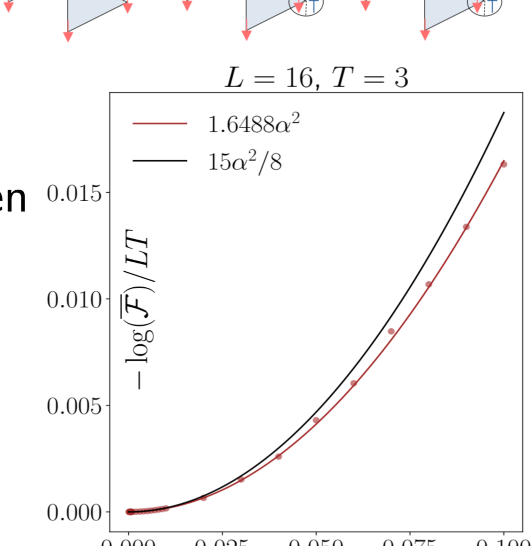
A defect can propagate towards the left (never turn around). For instance, for a defect that appears at τ-1, at leading order we have:





The number of configurations with defects of length m is given by the Catalan numbers

$$\chi_m = \frac{(2m)!}{(m+1)!m!}$$



Conclusions

- We quantify error accumulation for generic quantum dynamics implemented in a wide class of random quantum circuits.
- We find universal decay of the average fidelity

$$\overline{\mathcal{F}}_d \approx \exp\left(-\frac{15}{8}\alpha^2 LT\right)\exp\left(-\frac{3}{4}\left(d-\frac{1}{2}\right)pL^{1+\frac{1}{d}}T\right)$$

Scan me...

To uncover a connection between Fidelity and Quantum Volume figure of merit of Quantum Computing!

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