The fastest generation of multipartite entanglement with natural interactions

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Abstract

We study the possibility and time-energy relations for creating strongly nonclassical multipartite correlations with a single unitary generated by natural (two-body) interactions. Our main focus is on the fastest generation of the GHZ, W, Dicke and absolutely maximally entangled (AME) states for up to seven qubits. These results are obtained by constraining the total energy in the system and accordingly can be seen as state-dependent quantum speed limits for natural interactions.

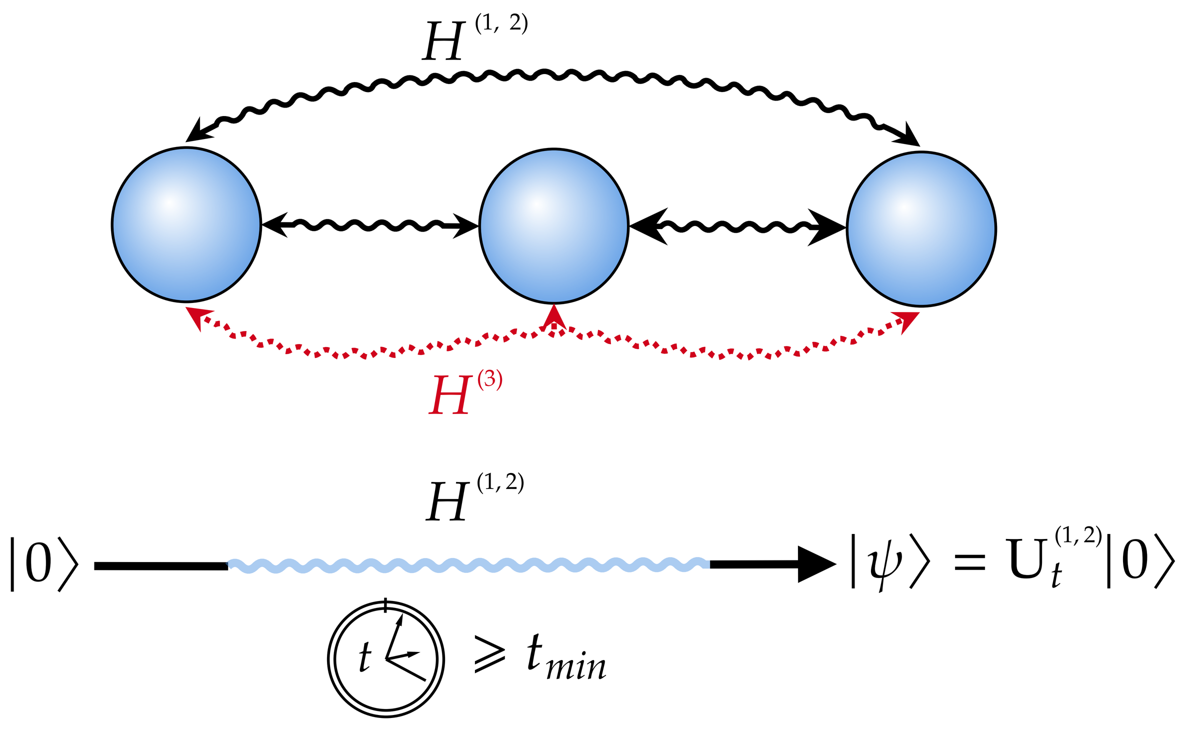

Figure 1. ***Motivation*** Is it possible to create strongly nonclassical multipartite correlations with evolution generated by two-body Hamiltonians? How does the interaction constraint affect the time and energy resources needed for that?

Physical setting

General two-body interaction (2-local) Hamiltonian for qubits

$$H = \sum_{i,j} \sum_{\mu_i,\mu_j} h_{\mu_i\mu_j} \sigma_{\mu_i}^{(i)} \sigma_{\mu_j}^{(j)} + b_{\mu_i} \sigma_{\mu_i}^{(i)}, \tag{1}$$

where the first sum is defied by the chosen interaction graph, see Fig. 2.
Energy constraint

$$H \to H' = \frac{H - \mathbb{I}E_{\min}}{\tilde{E}_{\max}} \Rightarrow E'_k \in [0,1]. \tag{2}$$

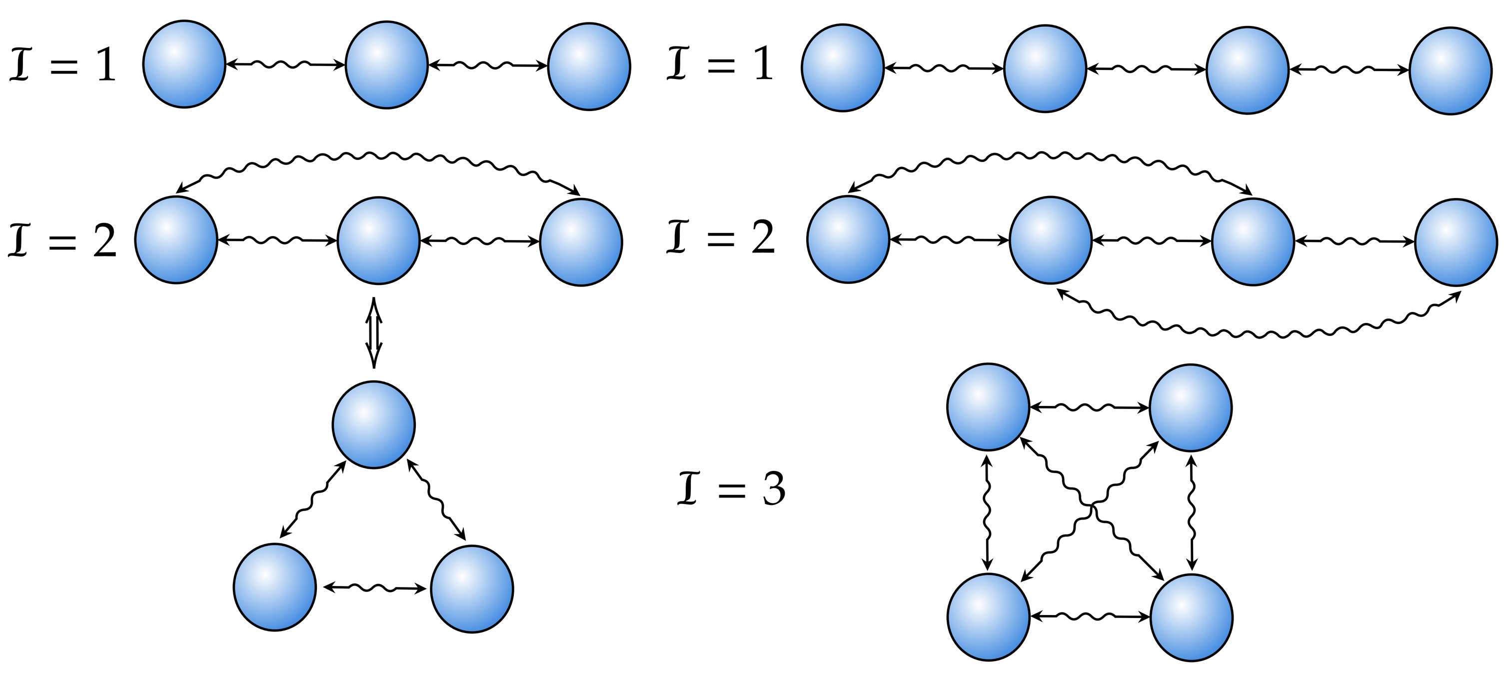

Figure 2. Various scenarios for possible interaction with different ranges in the case of 3 and 4 qubits.

Natural interaction quantum speed limits

Based on the minimal no. of distinct E_k needed for $U_t|0\rangle^{\otimes N} = |\psi\rangle$ we estimate the standard deviation of H in the three-qubit case. By the the Mandelstam-Tamm relation [1] we get

$$t \geq t_{min} = \arccos(|\langle\psi(0)|\psi(t)\rangle|)\frac{1}{\Delta H_0}, \tag{3}$$

$$\Rightarrow t_{\min}^{\mathrm{W},2-b} = \pi \text{ and } t_{\min}^{\mathrm{GHZ},2-b} = 2\pi. \tag{4}$$

Numerical results

For larger systems, we define the following optimisation problem

$$\mathcal{F}(t) = \max_{h_{\mu\nu}, b_\mu} |\langle\psi| \exp(-iHt)|0\cdots 0\rangle|^2. \tag{5}$$

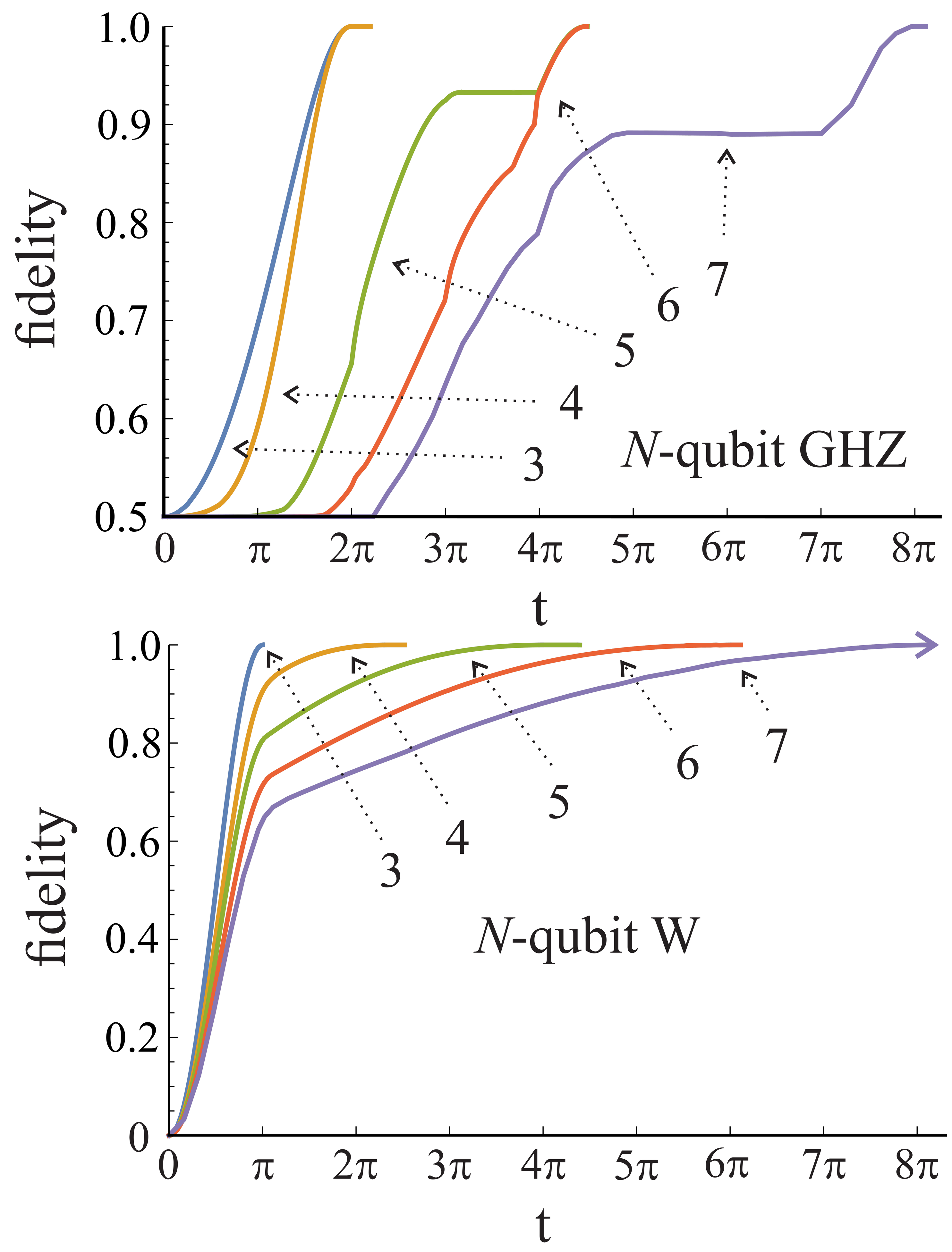

Figure 3. The fastest generation of N-qubit GHZ and W state with natural interactions.

Results summary

1. Strongly entangled GHZ, W, Dicke and even approximate AME(5,2) states **can be obtained** by the two-body evolution
2. The amount of **time (energy)** required for the creation of correlations **does not** have to **rise** strictly with the **number of qubits**
3. The analytical estimations agree with numerics
4. **Explicit formulas** for the **fastest generating Hamiltonians** have been found

References

[1] L. Mandelstam and Ig. Tamm, The Uncertainty Relation Between Energy and Time in Non-relativistic Quantum Mechanics, Springer Berlin Heidelberg, 1991.

Acknowledgements

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