Liouvillian Tomography

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Abstract

NISQ computers are characterized by a significant noise affecting qubit operations and a number of qubits that, despite large, is still insufficient for error-correcting protocols. Therefore, a large toolbox of methods has been developed to verify the performance of NISQ devices. Here we introduce a new Liouvillian Tomography (LT) algorithm. This algorithm builds on existing protocols, generalizing them for time-dependent, non-Markovian dynamics. A benchmark for the LT algorithm presented is obtained by applying it to synthetic data of a general system of 3 qubits connected to a non-Markovian reservoir. The benchmark shows that LT is capable of retrieving a generator with reasonable precision.

Non-unitary dynamics

Consider a system with some unknown operation, Λ_t , LT will attempt to extract a non-unitary generator for this operation, i.e., estimate a Liouvillian satisfying:

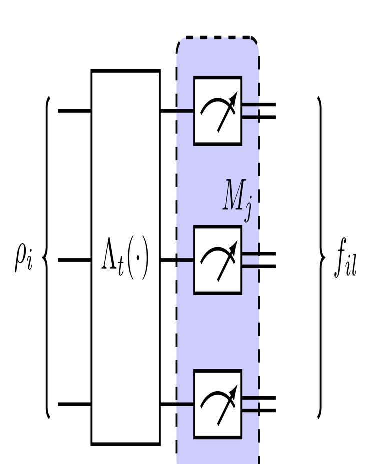
$$\frac{d}{dt}\Lambda_t = \mathcal{L}_t\Lambda_t$$

Such time-local generator's can always be cast in Lindblad-like form: $\mathcal{L}_t(\cdot) = -i\,[H,\cdot] + \sum_\mu \gamma_\mu \left(J_\mu \cdot J_\mu^\dagger - \frac{1}{2}\left\{J_\mu^\dagger J_\mu, \cdot\right\}\right)$

With Lindblad's form, evolutions can be classified as Markovian (CP-divisible) if $\gamma_\mu(t) > 0$ or non-Markovian otherwise.

LT algorithm

The LT algorithm is based on a regression problem for derivatives of experimental probability distributions over outcomes of a Positive Operator-Valued measurement (M_l), for different initial states (ρ_i) .



1) The experimental probabilities are estimated as the relative frequencies, f_{il}, of the outcomes of many ($\approx 10^4$) circuit executions.
2) The derivatives are estimated by a finite difference approximation on f_{il}.

The estimated derivatives are modeled by Born's rule (+ definition of a generator),

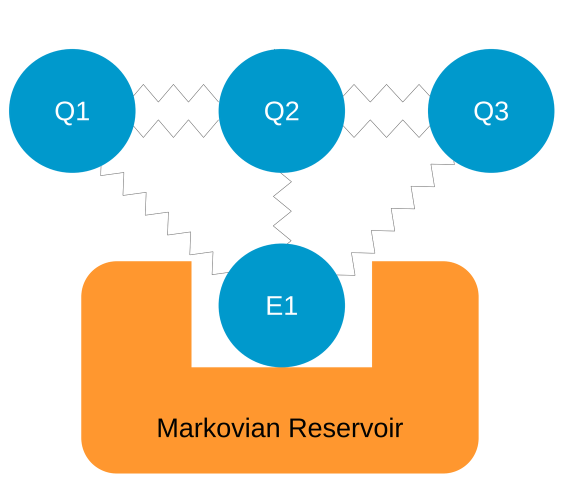
$$\frac{d}{dt}p_{il}(t) = \mathcal{L}_t\Lambda_t(\rho_i)M_l$$

With this model, along with estimates of the map, states and measurements, the Liouvillian is estimated by minimizing the least-squares loss between the models predictions and experimental derivatives.

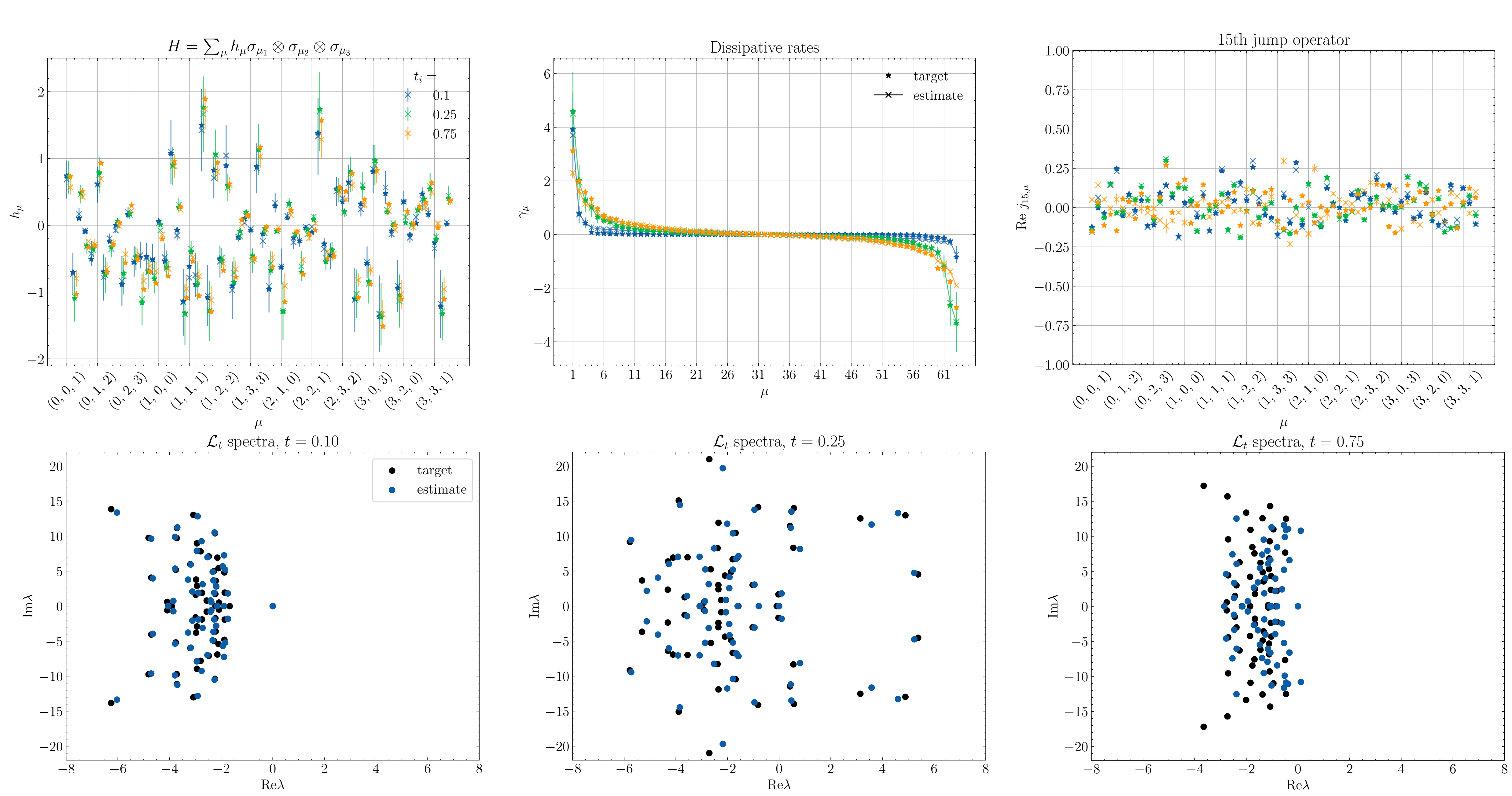
$$\tilde{\mathcal{L}} = \arg\min_{\mathcal{L}} \sum_{ij}\left(\frac{d}{dt}\tilde{p}_{ij}(t) - \mathcal{L}(\theta)\Lambda_t(\rho_i)\,M_j\right)^2$$

Benchmarking on 3-qubit systems

- To benchmark the algorithm a general system of 3 qubits coupled to a non-Markovian reservoir was considered.



- The Liouvillian was retrieved at 3 times during the evolution and compared with the analytical predictions for this system.
- The estimates agree well with the target values with more significant deviations for the t = 0.75, when the system is near the steady state.



Conclusions

- Liouvillian Tomography → estimate a generator of non-unitary dynamics → used to verify and predict the performance of NISQ devices.
- New Liouvillian Tomography algorithm capable of dealing with time-dependant non-Markovian dynamics, based on a regression problem.
- Benchmark results: 3 qubits + Non-Markovian enviroment.
  - Good benchmark performance demonstrating ability to predict Hamiltonian, rates and jump operators with reasonable accuracy.
  - Exhibits more error when the system is close to steady state.
- Next step: characterzie a pulse on a real NISQ device.

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