Measured-induced phase transitions by matrix product states scaling

arXiv:2402.13160

Guillaume Cecile[1], Hugo Lóio[1] and Jacopo De Nardis[1]

[1]Laboratoire de Physique Théorique et Modélisation, CNRS UMR 8089,CY Cergy Paris Université, 95302 Cergy-Pontoise Cedex, France

Abstract

We investigate long quantum spin chains under continuous monitoring, employing matrix product states (MPS) and the time-dependent variational principle (TDVP) algorithm. Our findings reveal a notable phase transition in the error rate during monitoring, detectable even with low bond dimensions. This approach efficiently pinpoints critical parameters for measurement-induced phase transitions (MIPTs) in complex many-body quantum systems. Furthermore, we unveil a distinct charge-sharpening (CS) transition in the context of U(1) global spin charge, validating our TDVP approach for identifying phase transitions across diverse system dimensions and sizes.

Models

We focus on two generic interacting systems in one dimension (whose MIPT has been also studied by different means in [1, 2, 3]) namely a chain of L spin-$1/2$ particles, unitarily evolving with $U(1)$ symmetric (magnetisation conserving) Hamiltonians, defined as

$$\hat{H}_{\text{J-XXX}} = \sum_{i=1}^{L}\left(\hat{S}_i^x\hat{S}_{i+1}^x + \hat{S}_i^y\hat{S}_{i+1}^y + \hat{S}_i^z\hat{S}_{i+1}^z\right) + \sum_{i=1}^{L} J\left(\hat{S}_i^x\hat{S}_{i+2}^x + \hat{S}_i^y\hat{S}_{i+2}^y\right), \qquad (1)$$

with spin operators $\hat{S}_i^\alpha$ and with $J = 0$ (XXX chain) or $J = 1/2$ (J-XXX chain).

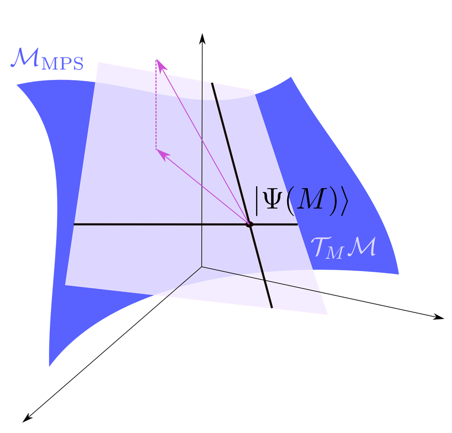
Basic Concepts

Unitary time evolution tends to entangle the system, whereas measurements serve to disentangle it. In measured-induced phase transitions, measurements disrupt the system's coherent evolution and can drive it across phase boundaries.

What is charge sharpening?

This transition separates two distinct entangling phases based on the ease or difficulty of determining the charge of the system through measurements.

Time Dependent Variational Principle



The time-evolved quantum state can be brought back to the initial manifold $\mathcal{M}$ by a projection $P_{\mathcal{T}_M\mathcal{M}}$,

$$i\partial_t|\Psi(M)\rangle = P_{\mathcal{T}_M\mathcal{M}}\hat{H}|\Psi(M)\rangle \ .$$

The projector $P_{\mathcal{T}_M\mathcal{M}}$ which projects onto this tangent space is given by

$$P_{\mathcal{T}_M\mathcal{M}} : \mathcal{H} \to \mathcal{T}_M\mathcal{M}$$

References

[1] Y. Fuji and Y. Ashida. In: *Phys. Rev. B* 102 (2020).

[2] L. Lumia, E. Tirrito, R. Fazio, and M. Collura. In: (2023). arXiv: 2311.09043.

[3] B. Xing, X. Turkeshi, M. Schiró, R. Fazio, and D. Poletti. In: (2023). eprint: arXiv:2308.09133.

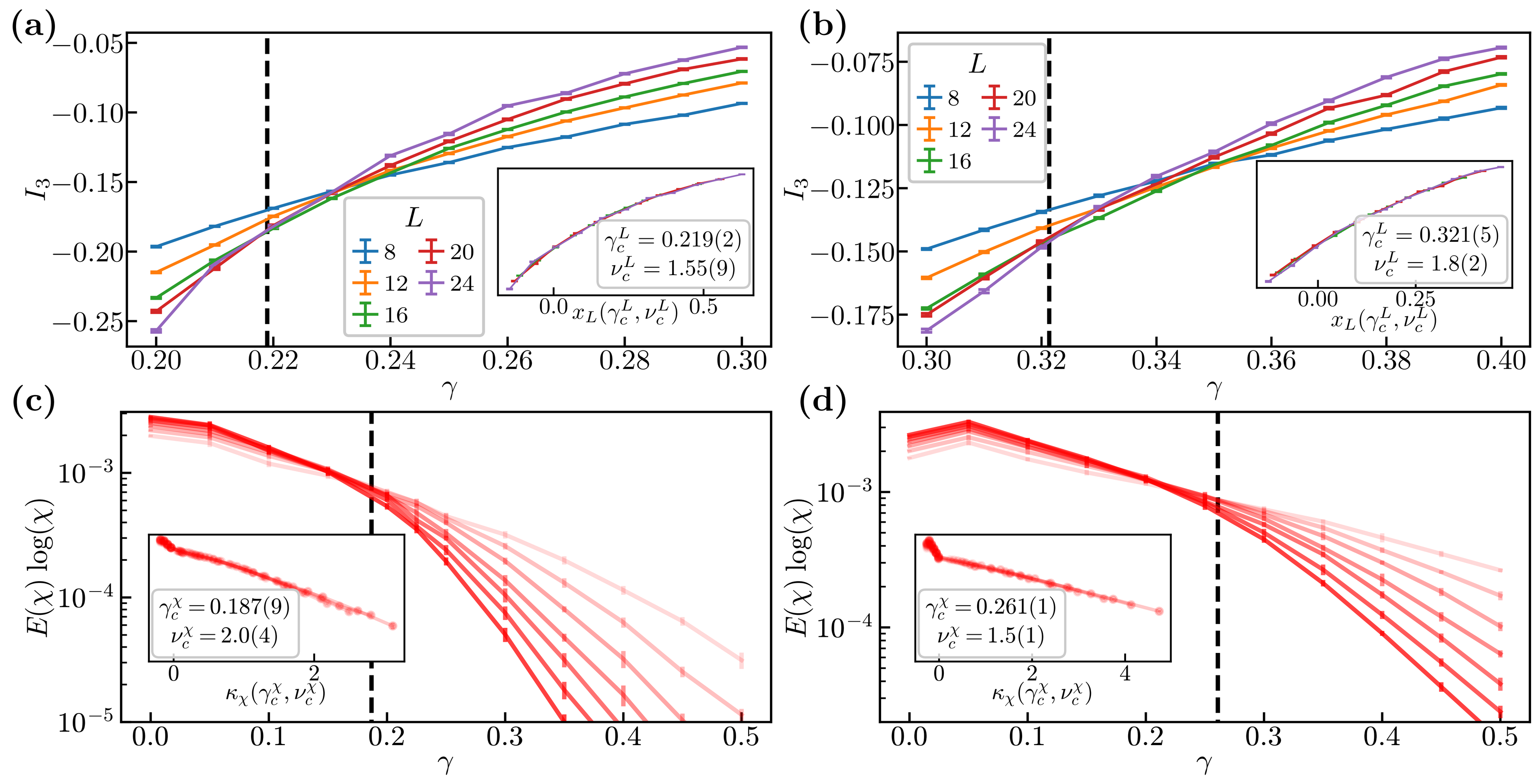
[4] K. Jacobs and D. A. Steck. In: *Contemp. Phys.* 47.4 (2006).

[5] H. M. Wiseman and G. J. Milburn. Cambridge University Press, 2009.

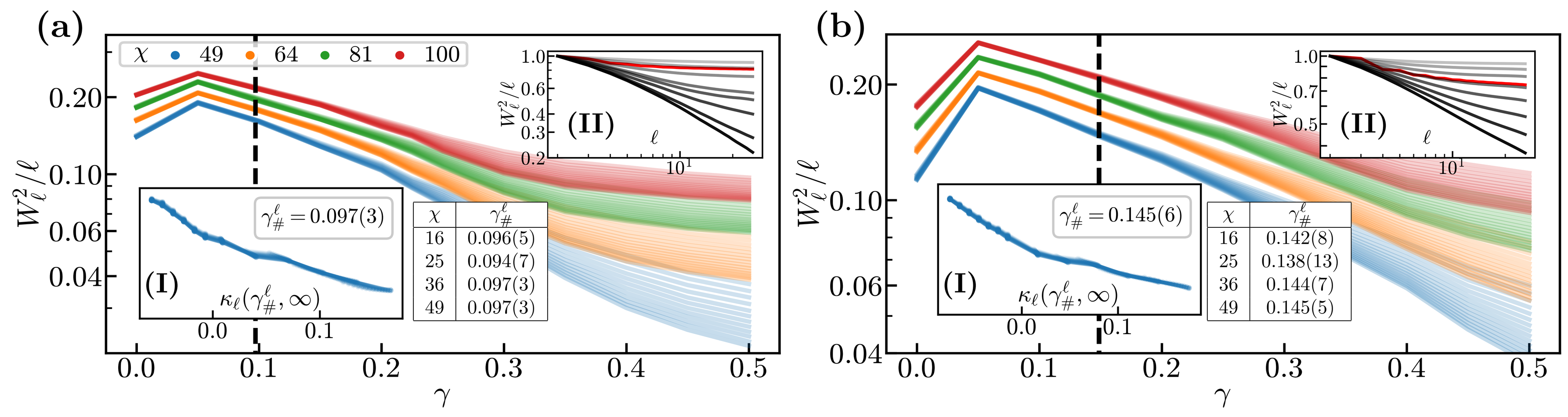
[6] A. Barchielli and M. Gregoratti. Springer Berlin Heidelberg, 2009.

Numerical Results

Entanglement/Error phase transitions in (a),(c) XXX model, and (b),(d) in J-XXX model.



Charge-sharpening phase transitions for (a) the XXX model and (b) the J-XXX model.



Methods

The associated monitored dynamics of the quantum state are described by the following stochastic Schrödinger equation (SSE) [4, 5, 6] for the many-body state $|\psi\rangle$,

$$d|\psi_t\rangle = -iHdt|\psi_t\rangle + \sum_{i=1}^{L}\left[\sqrt{\gamma}(\hat{S}_i^z - \langle\hat{S}_i^z\rangle_t)dW_t^i - \frac{\gamma}{2}(\hat{S}_i^z - \langle\hat{S}_i^z\rangle_t)^2 dt\right]|\psi_t\rangle\,, \qquad (2)$$

with the expectation value of the local magnetisation given by the state at a given time $\langle\hat{S}_i^z\rangle_t = \langle\psi_t|\hat{S}_i^z|\psi_t\rangle$. Eq. (2) can be easily simulated by alternating its unitary and measurement terms via a Trotter splitting,

$$|\psi_{t+\delta t}\rangle \approx C e^{\sum_{j=1}^{L}\left[\delta W_t^j + 2\langle\hat{S}_j^z\rangle_t\gamma\delta t\right]\hat{S}_j^z} e^{-i\hat{H}\delta t}|\psi_t\rangle \ , \qquad (3)$$

where the set of δW^i are generated each time step from a normal distribution with variance $\sqrt{\gamma\delta t}$ and zero mean and C is a normalizing constant. In order to extend the CS protocol to large system sizes, we here consider the variance of the fluctuations of the local magnetization on a sub-system of size ℓ, e.g. by defining $Q_\ell = \sum_{j\in\ell} S_j^z$, we introduce its variance as

$$W_\ell^2 = \langle Q_\ell^2\rangle - \langle Q_\ell\rangle^2 = \sum_{i,j\in\ell}\langle S_i^z S_j^z\rangle^c\,, \qquad (4)$$

In order to fit the MPS transition, given the much quicker convergence in χ on the left of the critical point, we need to introduce a *piece-wise ansatz*, namely

$$x = \kappa_A(\gamma_O, \nu_O) = \begin{cases} (\gamma - \gamma_O) & \gamma < \gamma_O \\ (\gamma - \gamma_O)A^{1/\nu_O} & \gamma > \gamma_O \end{cases}\,, \qquad (5)$$

and we cross-checked this scaling with the one where only the right side of the transition is fitted, giving analogous results.

Discussion

Starting with the initial Néel state $|\Psi(0)\rangle = |\downarrow\uparrow\downarrow\uparrow\ldots\downarrow\uparrow\rangle$, monitored evolution at different χ and L values is performed. Analysis of error rate evolution reveals saturation after a timescale of order L, indicating a transition between volume-law and area-law phases. Precisely determined critical parameters suggest a smaller critical measurement rate γ_c^χ than that of ED simulations. Unitary terms dominate for small L, shifting critical measurement rates leftward for larger L. Although critical exponents differ slightly between ED and MPS, well-converged results are obtained for small bond dimensions. Transition to the CS phase is observed through magnetization variance values, showing a shift from extensive to sub-linear scaling with increasing γ. Cross-checking with ED simulations confirms the observed transition, highlighting the method's importance in studying large systems with continuous time.