

# Redshifted 21-cm bispectrum from CD-EoR: Impact of Source Model and IGM Physics

**Suman Majumdar**

Department of Astronomy, Astrophysics and Space Engineering  
Indian Institute of Technology Indore

[arXiv: 2406.03118](#) and [2207.09128](#)

**Collaborators:** Leon Noble (IIT Indore), Mohammad Kamran (Uppsala University), Chandra Shekhar Murmu (IIT Indore), Raghunath Ghara (University of Pennsylvania), Garret Mellema (Stockholm University), Ilian T. Iliev (University of Sussex) and Jonathan R. Pritchard (Imperial College London)





## **Mohammad Kamran**

Postdoc at Uppsala University  
PhD from IIT Indore (2022)

arXiv: 2207.09128



## **Leon Noble**

PhD candidate at IIT Indore

arXiv: 2406.03118

# How we can see the IGM during EoR

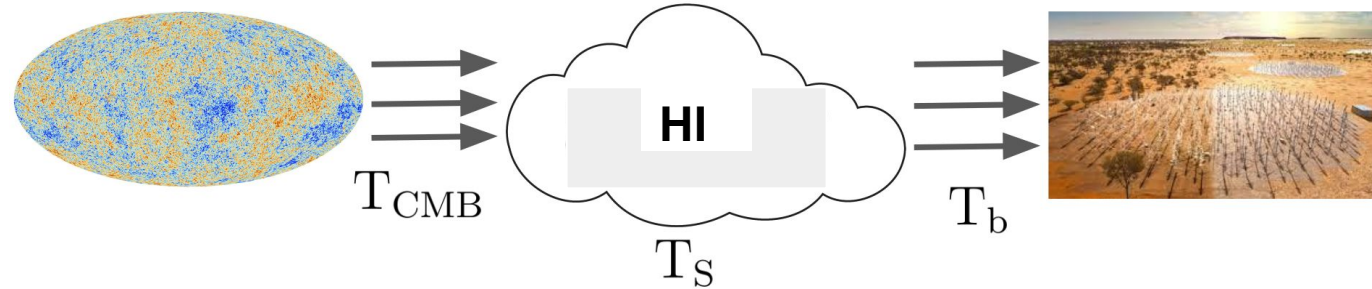
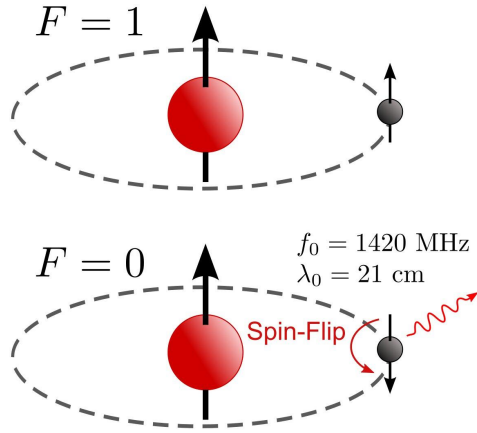


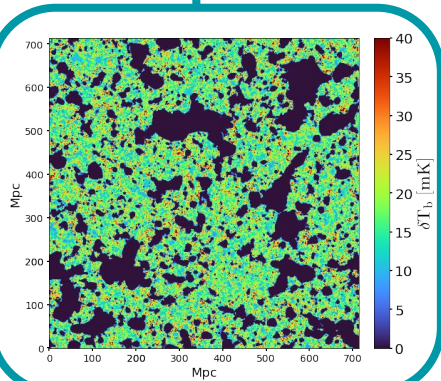
Image credit: Tiltec of Wikipedia

$$\delta T_{\text{b}} = T_{\text{b}}(\mathbf{r}, z) - T_{\text{CMB}}(z)$$

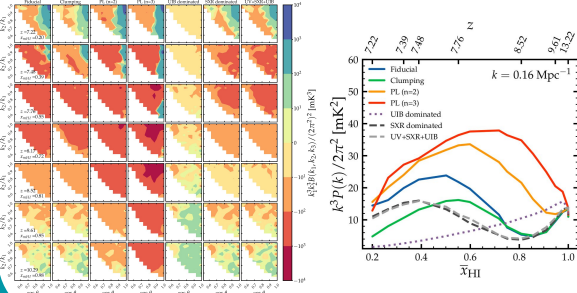
**21-cm differential brightness temperature**

# How to interpret the observations?

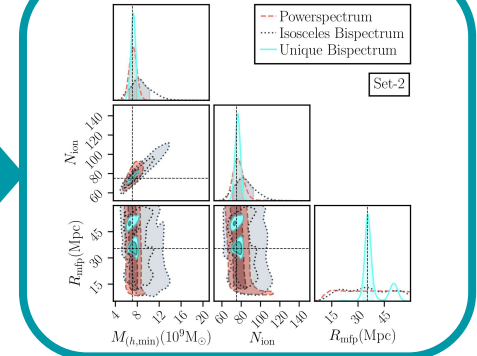
## Simulations



## Summary statistics



## Parameter estimation



Observations

Summary statistics

Bayesian



# Observable summary statistics

## 21-cm Power spectrum

$$\langle \Delta(\vec{k}_1) \Delta^*(\vec{k}_2) \rangle = V \delta_D^3(\vec{k}_1 - \vec{k}_2) P(k)$$

$$\Delta(\vec{k}) \xrightarrow{\mathcal{FT}} \delta T_b$$

Power spectrum can completely characterize the statistical properties of a signal  
i.e. Gaussian in nature.

# IGM 21-cm signal from EoR is highly non-Gaussian

**Intrinsic**

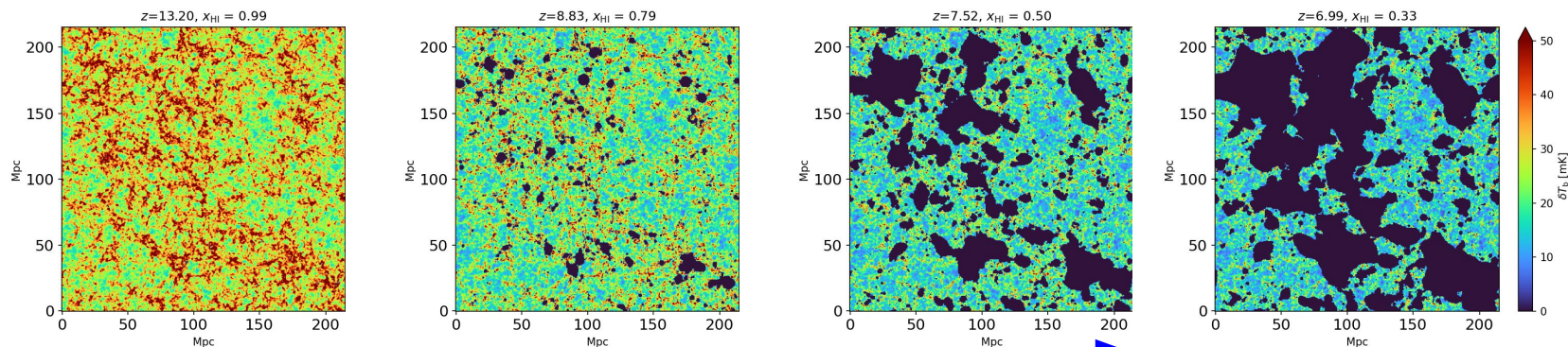


The sources of radiation are distributed at non-random (biased) locations

**Time-evolving**



The physical processes in the IGM, controlled by these sources, also progress with time



Progress of reionization



# Various ways of quantifying non-Gaussianity

- One point statistics → Skewness, Kurtosis → [Watkinson et al 2014](#) etc
- Position dependent power spectrum → [Giri et al 2019](#) etc
- Wavelet Scattering Transforms → [Greig et al 2022](#), [Hoti et al 2024](#) etc
- Different image based statistics →  
Bubble Size Distribution, Minkowski functionals,  
Largest Cluster Statistics →  
[Iliev et al 2005, 2007](#), [Friedrich et al. 2010](#), [Kakiichi et al. 2017](#),  
[Giri et al. 2017, 2018](#), [Pathak et al 2022](#), [Dasgupta et al 2023](#) etc

# 21-cm bispectrum

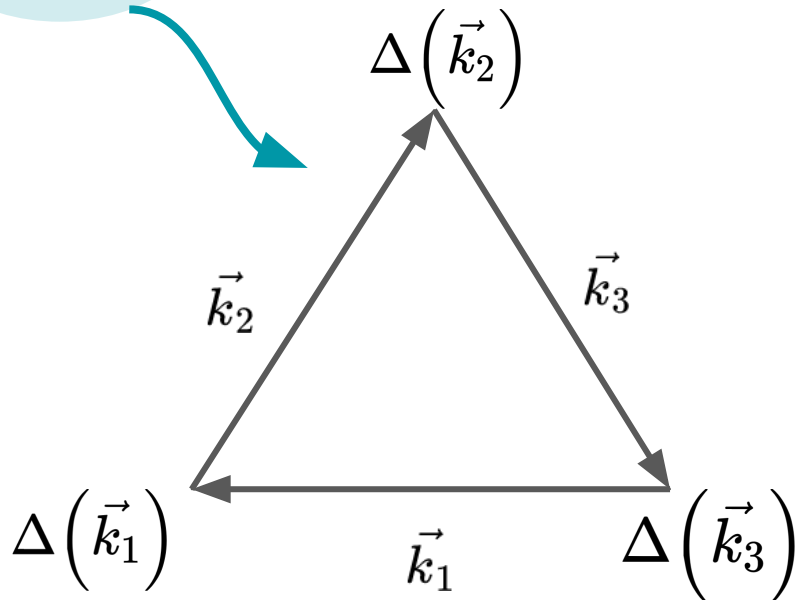
$$\langle \Delta(\vec{k}_1) \Delta(\vec{k}_2) \Delta(\vec{k}_3) \rangle = V \delta_{\vec{k}_1 + \vec{k}_2 + \vec{k}_3, 0}^K B(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

$$\Delta(\vec{k}) \xrightarrow{\mathcal{FT}} \delta T_b$$

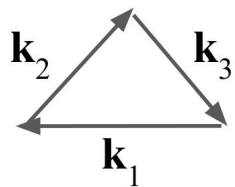
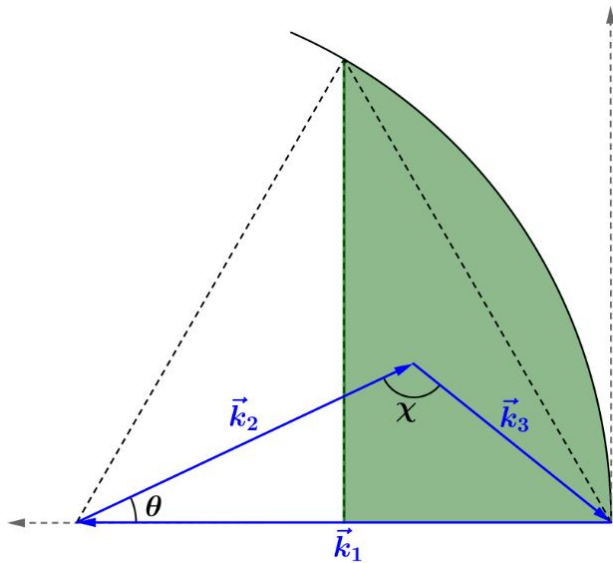
# 21-cm bispectrum

$$\langle \Delta(\vec{k}_1) \Delta(\vec{k}_2) \Delta(\vec{k}_3) \rangle = V \delta_{\vec{k}_1 + \vec{k}_2 + \vec{k}_3, 0}^K B(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

$$\Delta(\vec{k}) \xrightarrow{\mathcal{FT}} \delta T_b$$

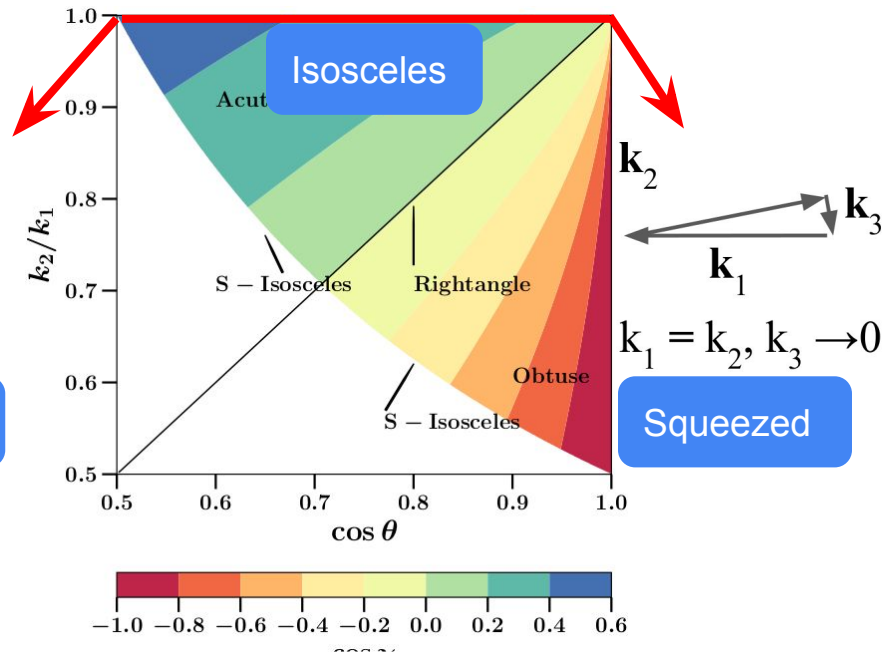


# Unique triangles in the Fourier space

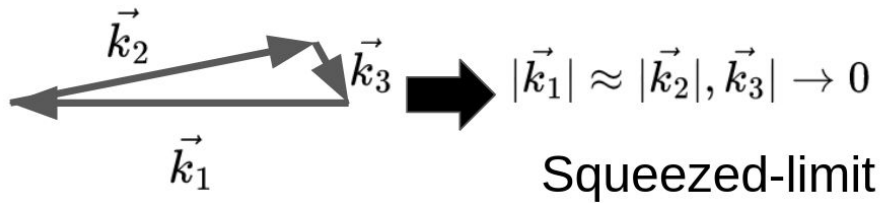


$$k_1 = k_2 = k_3$$

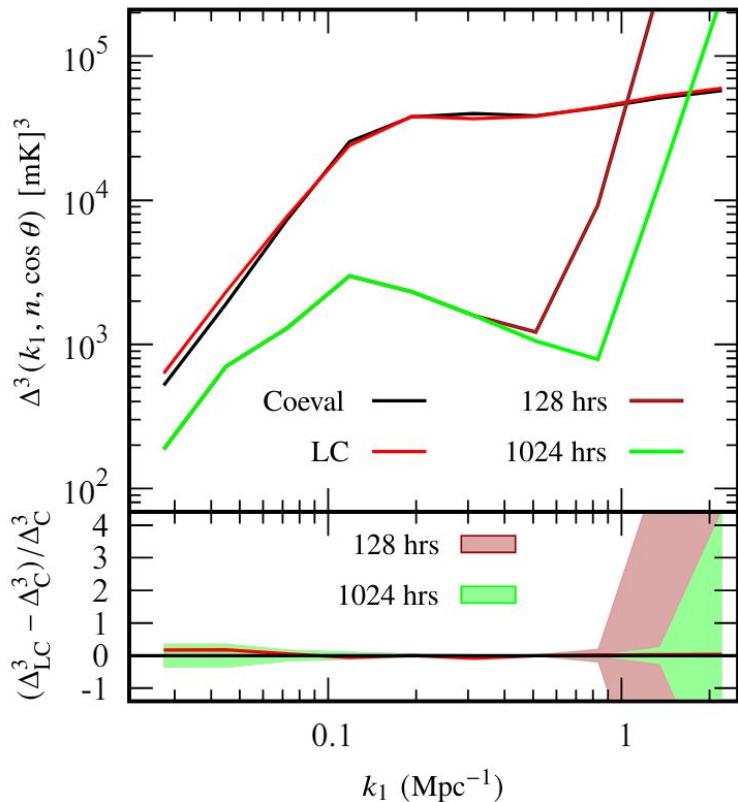
Equilateral



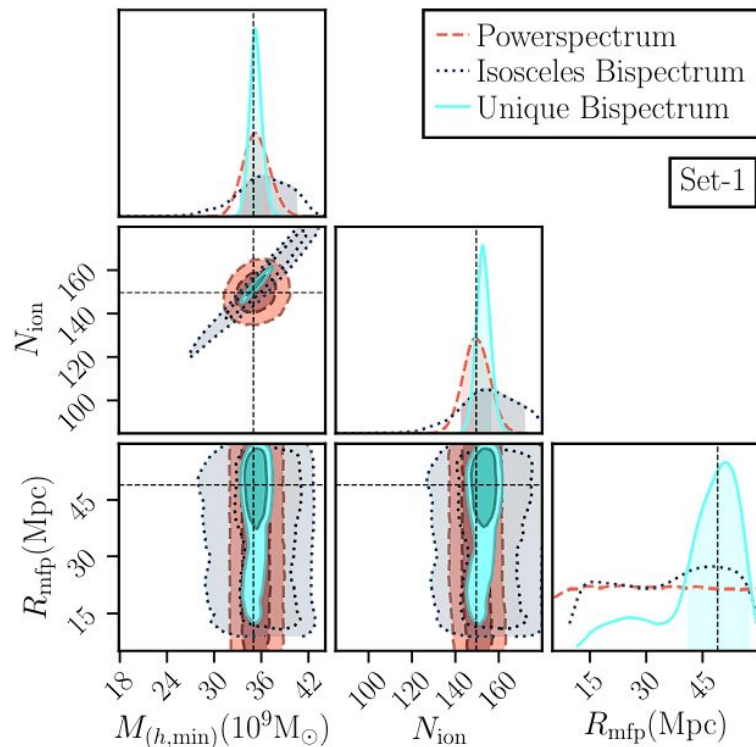
# Squeezed-limit bispectrum



# Detectability of the squeezed-limit bispectrum with SKA



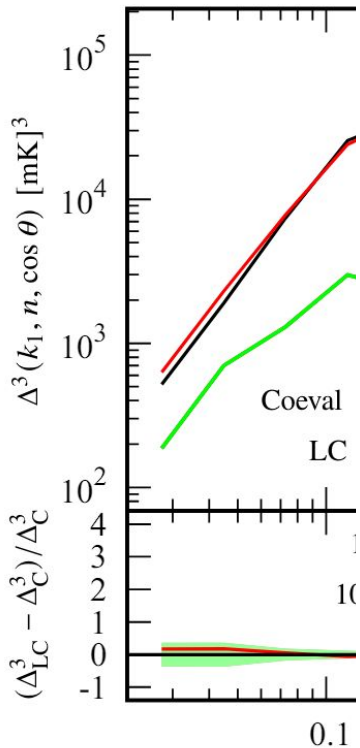
Mondal, ...SM et al 2021



Tiwari, SM et al 2022



# Detectability



Mondal, ...SM

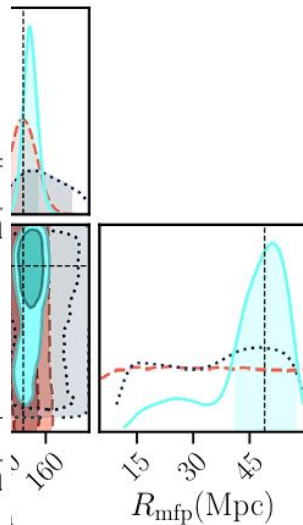
Triangles	Direct	Gridded	Type
<b>EoR0</b>			<b>14m</b>
$k_1 = k_2 = k_3 = 0.007$	$0.166 \pm 2.5e - 7$	$10.4 \pm 4.1e - 8$	Equilateral
$k_1 = 0.2, k_2 = k_3 = 0.1$	$-0.266 \pm 0.0004$	$921.2 \pm 0.3$	Isosceles
$k_1 = 0.4, k_2 = k_3 = 0.2$	$2.84 \pm 0.0044$	$-1766.2 \pm 0.6$	Isosceles
$k_1 = 0.6, k_2 = k_3 = 0.3$	$-4.87 \pm 0.063$	$-427.8 \pm 5.8$	Isosceles
$k_1 = 1.0, k_2 = k_3 = 0.5$	$3.45 \pm 0.60$	<b><math>129.4 \pm 108.9</math></b>	Isosceles
<b>EoR0</b>			<b>28m</b>
$k_1 = k_2 = k_3 = 0.014$	$-0.019 \pm 1.4e - 7$	$-29.3 \pm 8.1e - 6$	Equilateral
$k_1 = 0.2, k_2 = k_3 = 0.1$	$-0.14 \pm 0.002$	$-594.5 \pm 4.0$	Isosceles
$k_1 = 0.4, k_2 = k_3 = 0.2$	$0.360 \pm 0.009$	$948.9 \pm 7.2$	Isosceles
$k_1 = 0.6, k_2 = k_3 = 0.3$	<b><math>0.98 \pm 0.18</math></b>	$-793.1 \pm 37.6$	Isosceles
$k_1 = 1.0, k_2 = k_3 = 0.5$	<b><math>1.08 \pm 1.78</math></b>	$19450 \pm 752$	Isosceles
<b>EoR1</b>			<b>14m</b>
$k_1 = k_2 = k_3 = 0.007$	$-0.004 \pm 1.2e - 8$	$0.61 \pm 3.2e - 9$	Equilateral
$k_1 = 0.2, k_2 = k_3 = 0.1$	$0.044 \pm 0.0001$	$-666.5 \pm 0.03$	Isosceles
$k_1 = 0.4, k_2 = k_3 = 0.2$	$0.19 \pm 0.0004$	$3157.0 \pm 0.82$	Isosceles
$k_1 = 0.6, k_2 = k_3 = 0.3$	$-0.064 \pm 0.007$	$-1861.9 \pm 0.54$	Isosceles
$k_1 = 1.0, k_2 = k_3 = 0.5$	<b><math>-0.12 \pm 0.13</math></b>	$5907.5 \pm 56.1$	Isosceles
<b>EoR1</b>			<b>28m</b>
$k_1 = k_2 = k_3 = 0.014$	$0.0006 \pm 7.0e - 9$	$17.1 \pm 8.4e - 7$	Equilateral
$k_1 = 0.2, k_2 = k_3 = 0.1$	<b><math>0.0001 \pm 0.0005</math></b>	$927.7 \pm 0.43$	Isosceles
$k_1 = 0.4, k_2 = k_3 = 0.2$	$-0.082 \pm 0.002$	$-245.3 \pm 0.15$	Isosceles
$k_1 = 0.6, k_2 = k_3 = 0.3$	<b><math>0.012 \pm 0.030</math></b>	$5881.8 \pm 10.4$	Isosceles
$k_1 = 1.0, k_2 = k_3 = 0.5$	<b><math>6.2 \pm 1.2</math></b>	$4257.6 \pm 15.6$	Isosceles

# with SKA

Trott, ...SM et al 2019

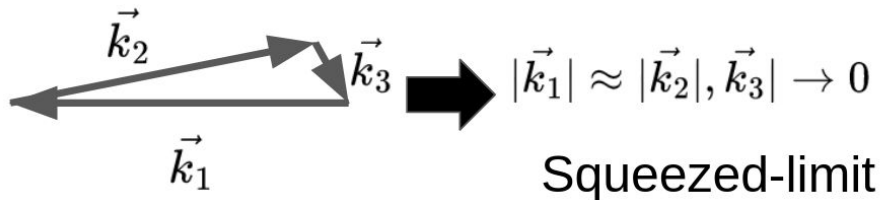
- Powerspectrum
- Isosceles Bispectrum
- Unique Bispectrum

Set-1



al 2022

# Squeezed-limit bispectrum

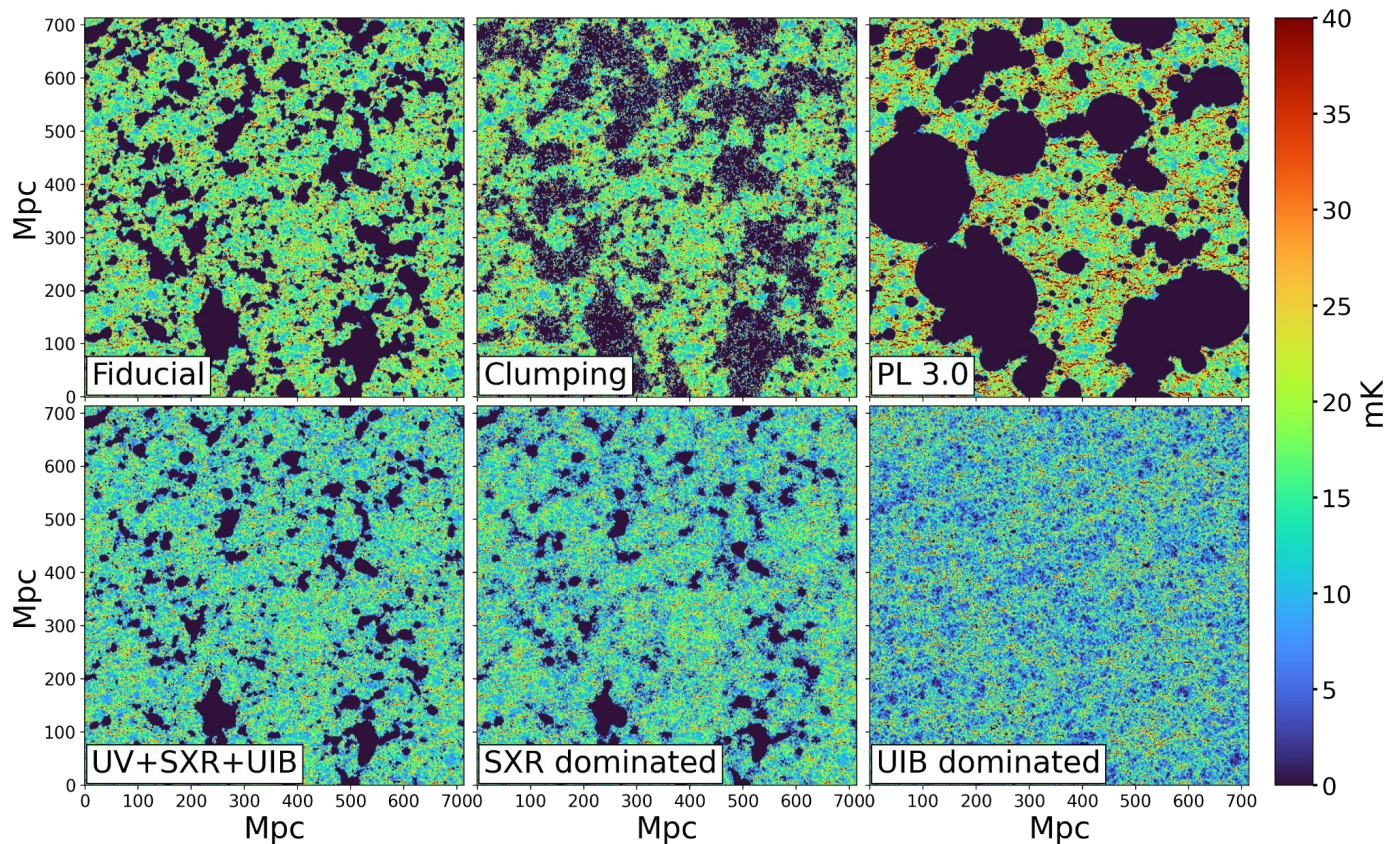


- Among all possible unique k-triangles, **squeezed-limit** triangle bispectrum has **maximum magnitude** (Majumdar et al. 2018, 2020; Hutter et al. 2019, Watkinson et al 2021, Kamran, SM et al. 2021, 2022, Tiwari, SM et al. 2022, Gill, SM et al. 2023, Raste et al. 2024).
- Highest **detection probability** by **SKA** (Mondal, SM et al. 2021, Tiwari, SM, et al. 2022).

**We will focus on large scale ( $k_1 \sim 0.16 \text{ Mpc}^{-1}$ ) squeezed limit bispectrum.**

# Epoch of Reionization 21-cm bispectrum

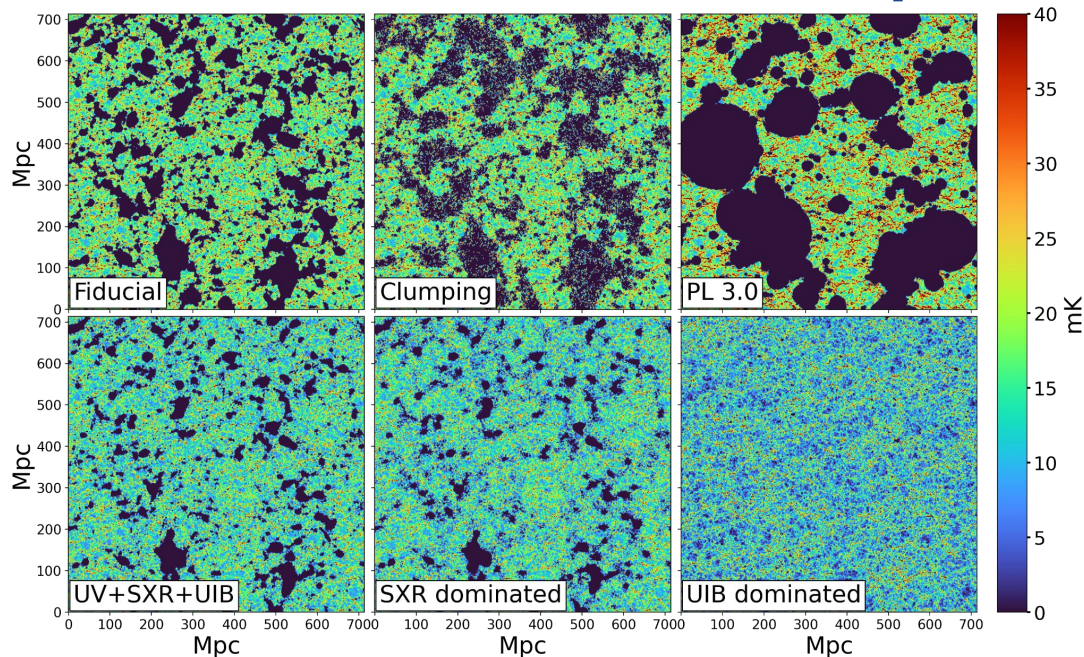
# Impact of the sources of reionization on 21-cm signal



Choudhury et al 2009;  
Watkinson et al. 2014;  
Majumdar et al. 2016;  
Eide et al. 2018;  
Hutter et al. 2019;  
Watkinson et al. 2021  
Pathak et al. 2022;  
Raste et al 2024;  
Schaeffer et al 2024,.....  
and many more

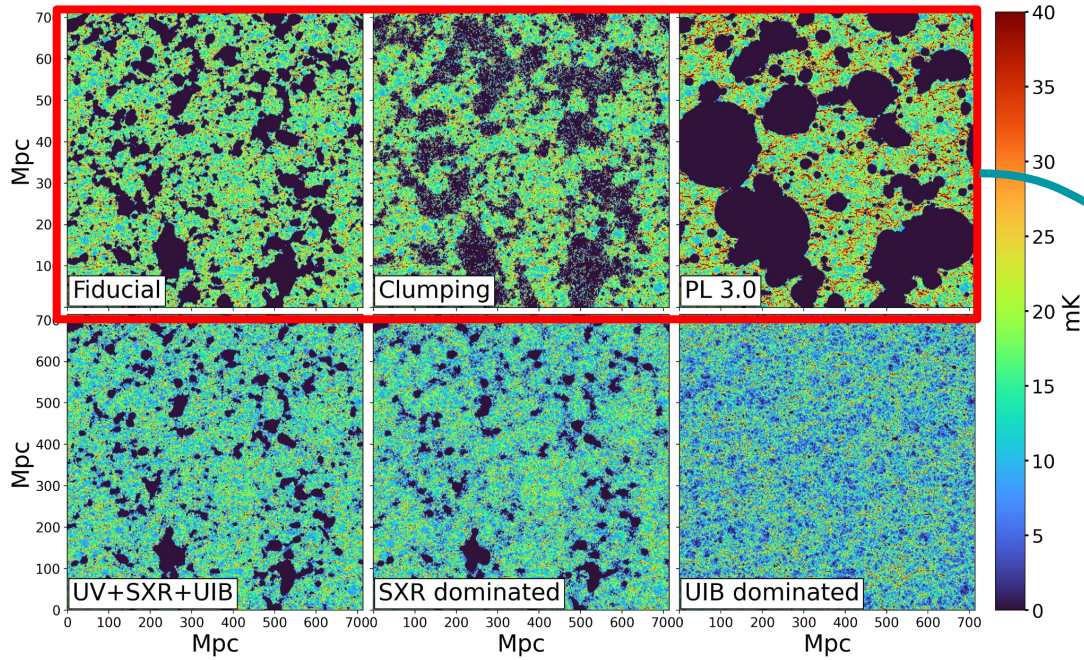


# Impact of the sources of reionization on the 21-cm bispectrum



- Impact of various **reionization morphologies** on the **21-cm bispectrum**
- To what extent the **21-cm bispectrum** can distinguish between different reionization morphologies

# Reionization scenarios

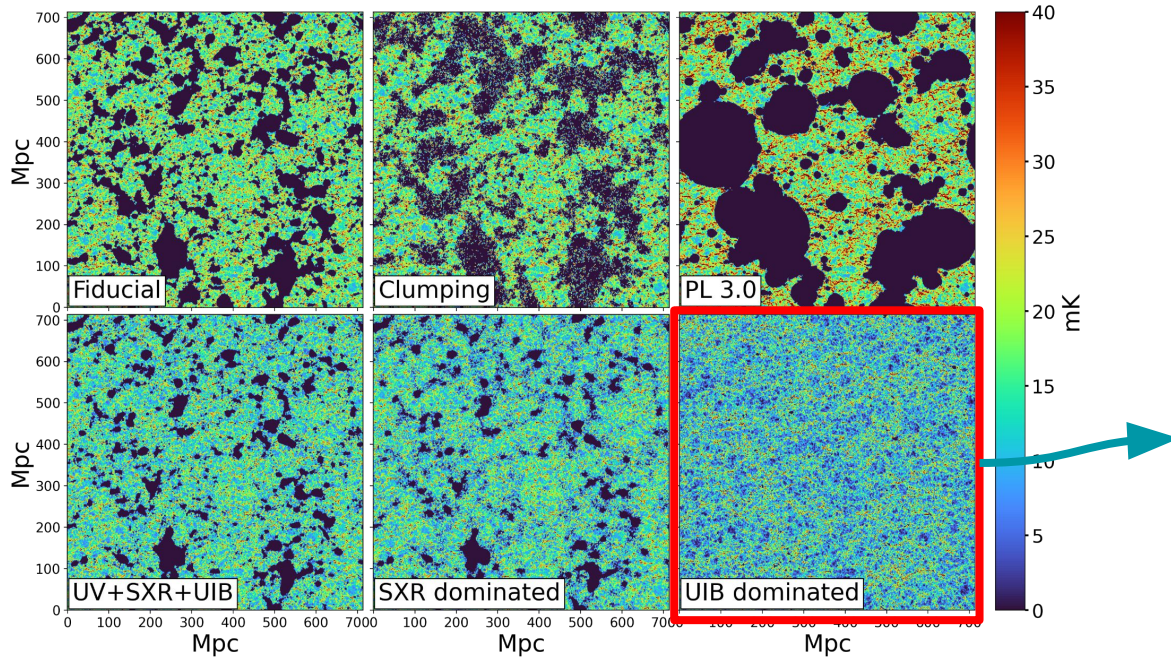


**Inside-out** reionization scenarios

Noble, SM et al. 2024, arXiv: 2406.03118



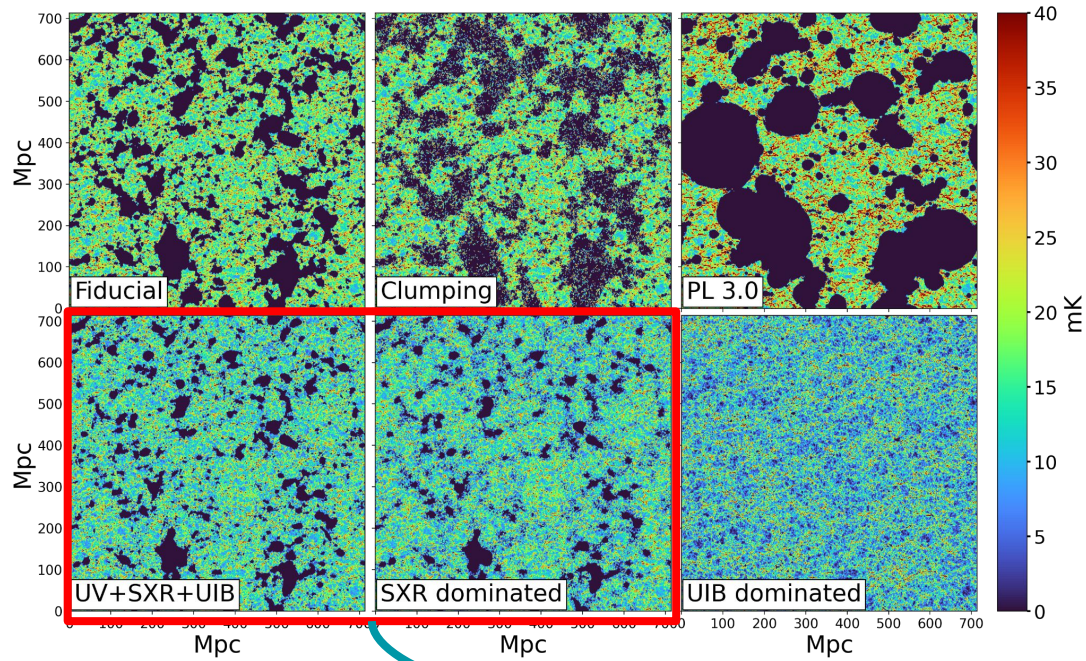
# Reionization scenarios



**Outside-in reionization scenario**

Noble, SM et al. 2024, arXiv: 2406.03118

# Reionization scenarios

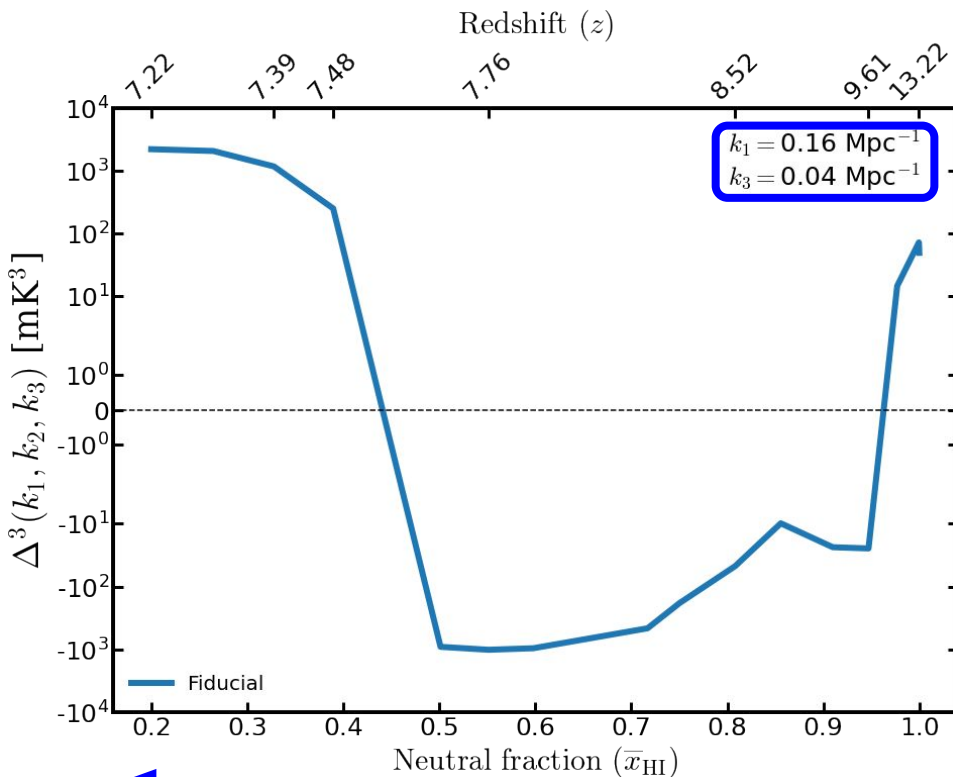


Combination of **inside-out** and **outside-in**

Noble, SM et al. 2024, arXiv: 2406.03118

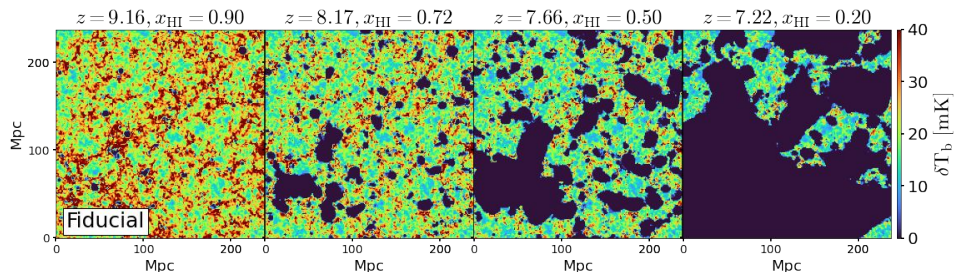


# Evolution of squeezed-limit bispectrum



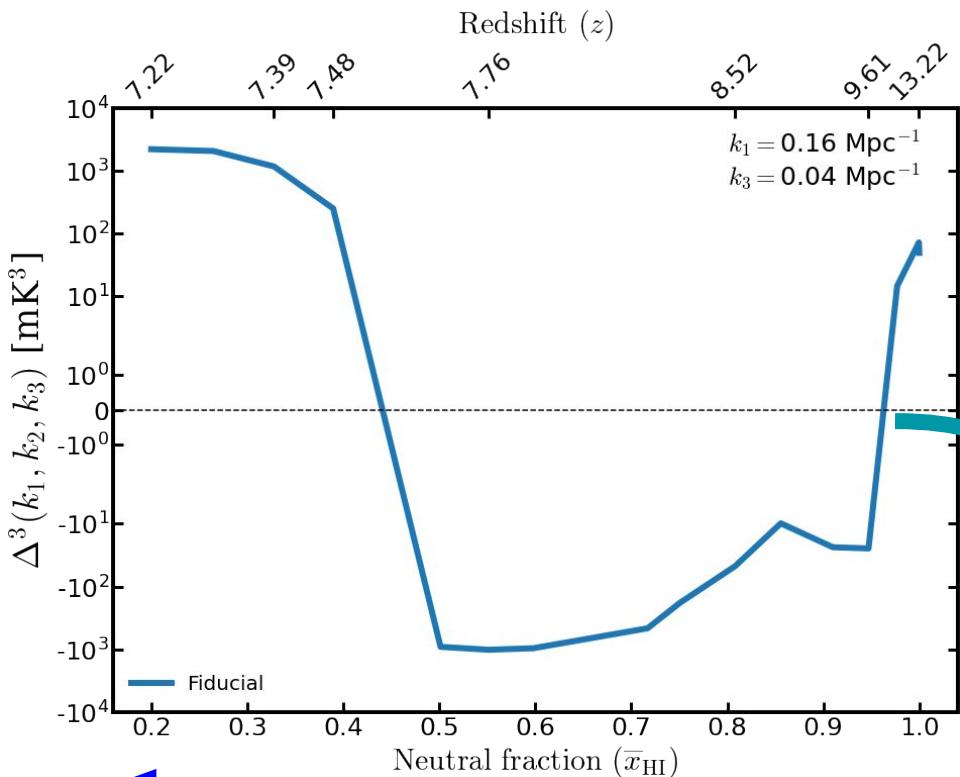
Early stage

Late stage



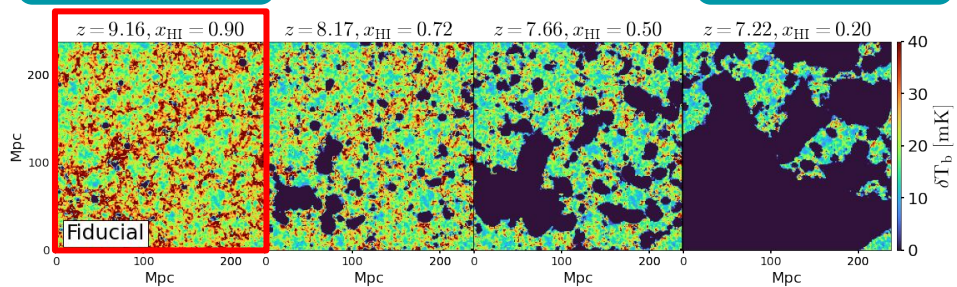
Noble, SM et al. 2024, arXiv: 2406.03118

# Evolution of squeezed-limit bispectrum



Early stage

Late stage

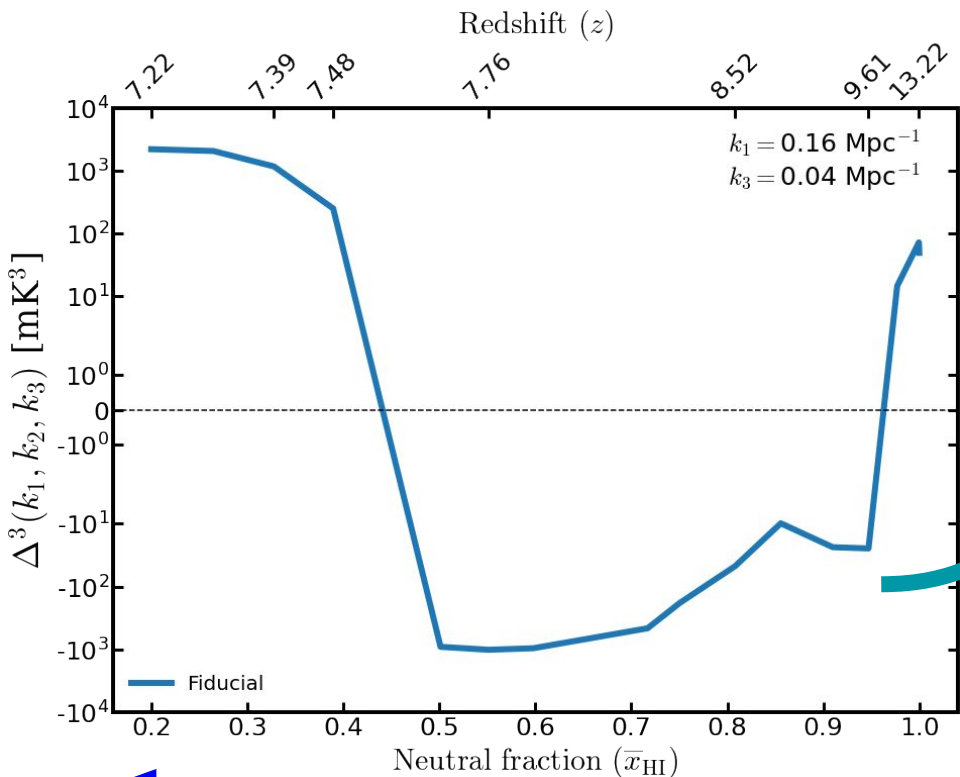


Change in the sign of the bispectrum

Noble, SM et al. 2024, arXiv: 2406.03118

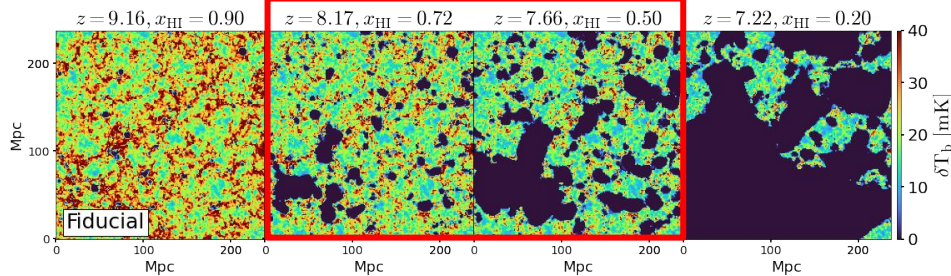
Progress of reionization

# Evolution of squeezed-limit bispectrum



Early stage

Late stage

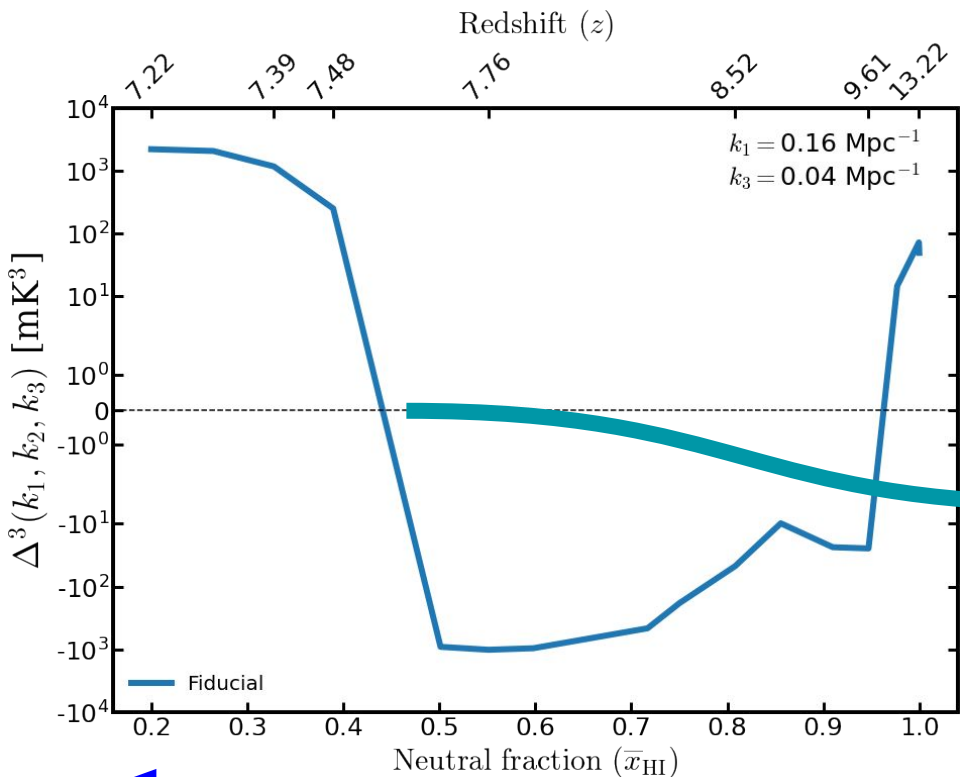


Magnitude of the 21-cm bispectrum increases

Progress of reionization

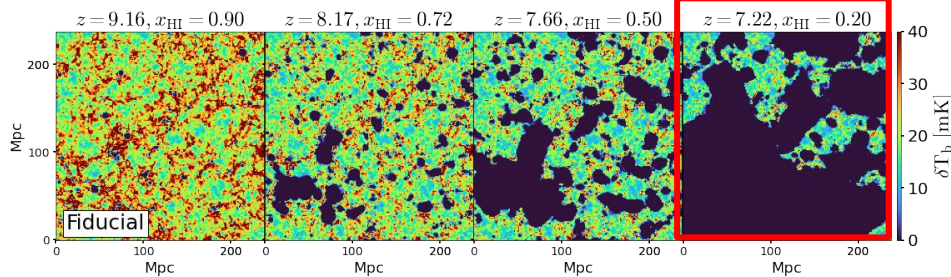
Noble, SM et al. 2024, arXiv: 2406.03118

# Evolution of squeezed-limit bispectrum

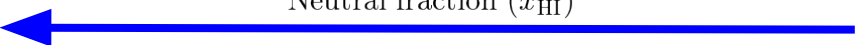


Early stage

Late stage



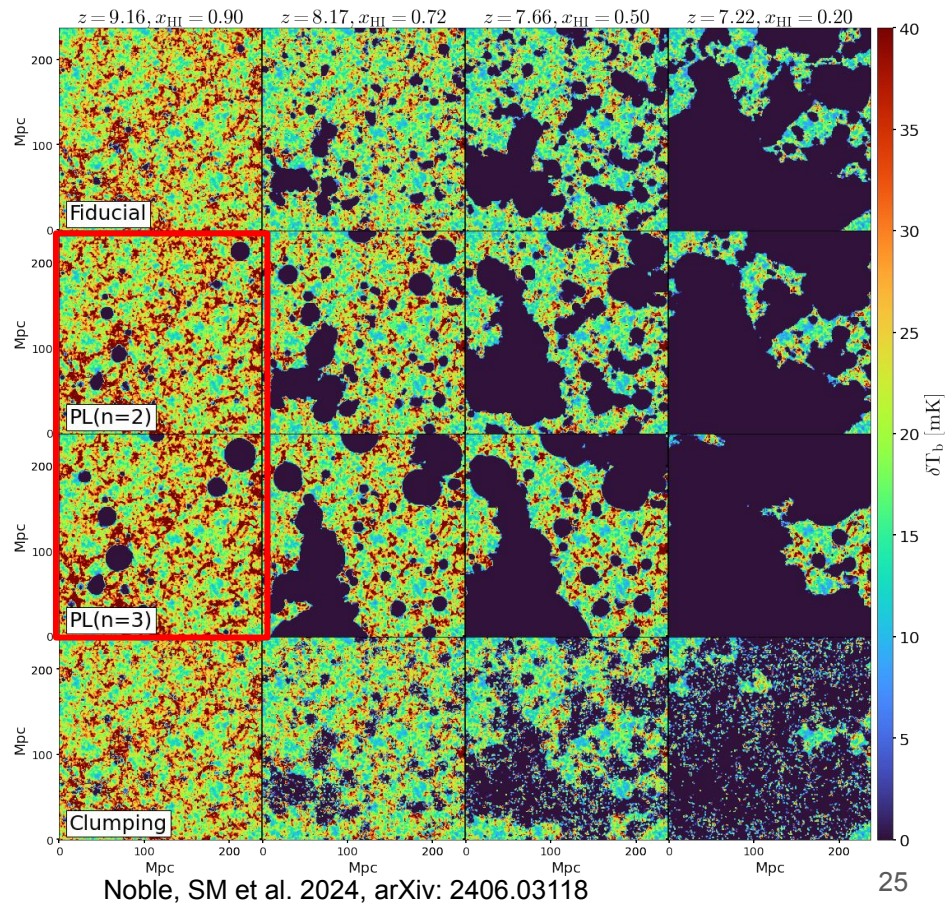
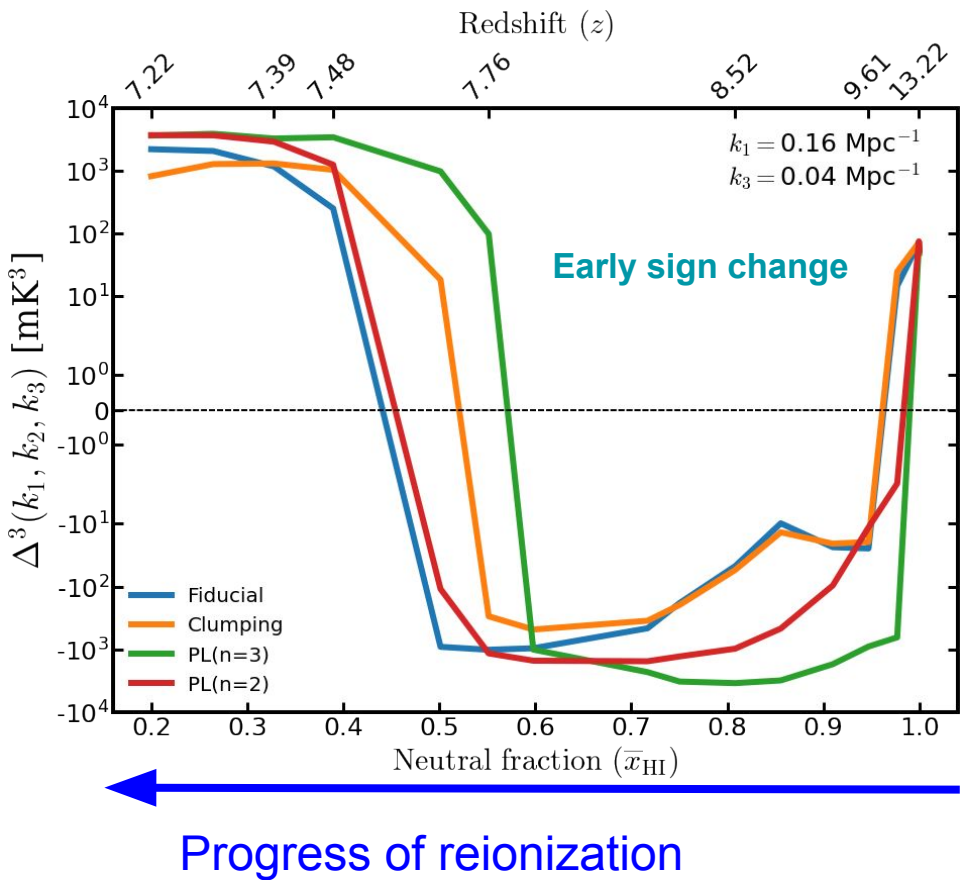
Second sign change in the 21-cm bispectrum



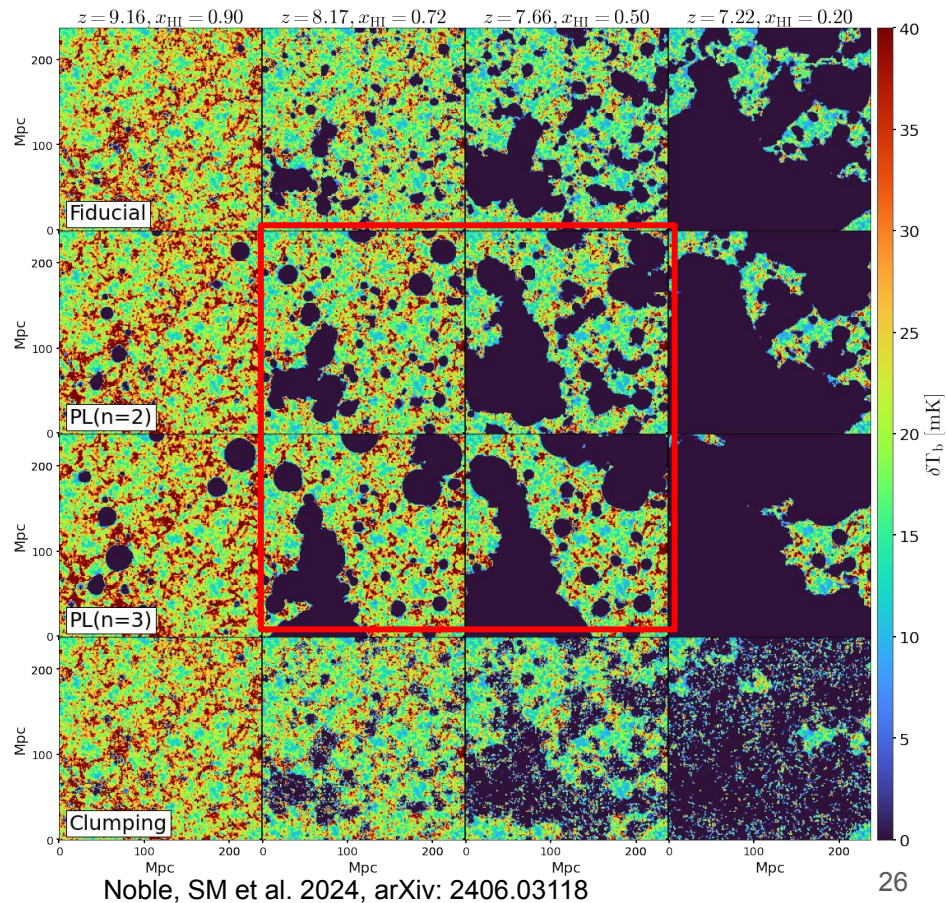
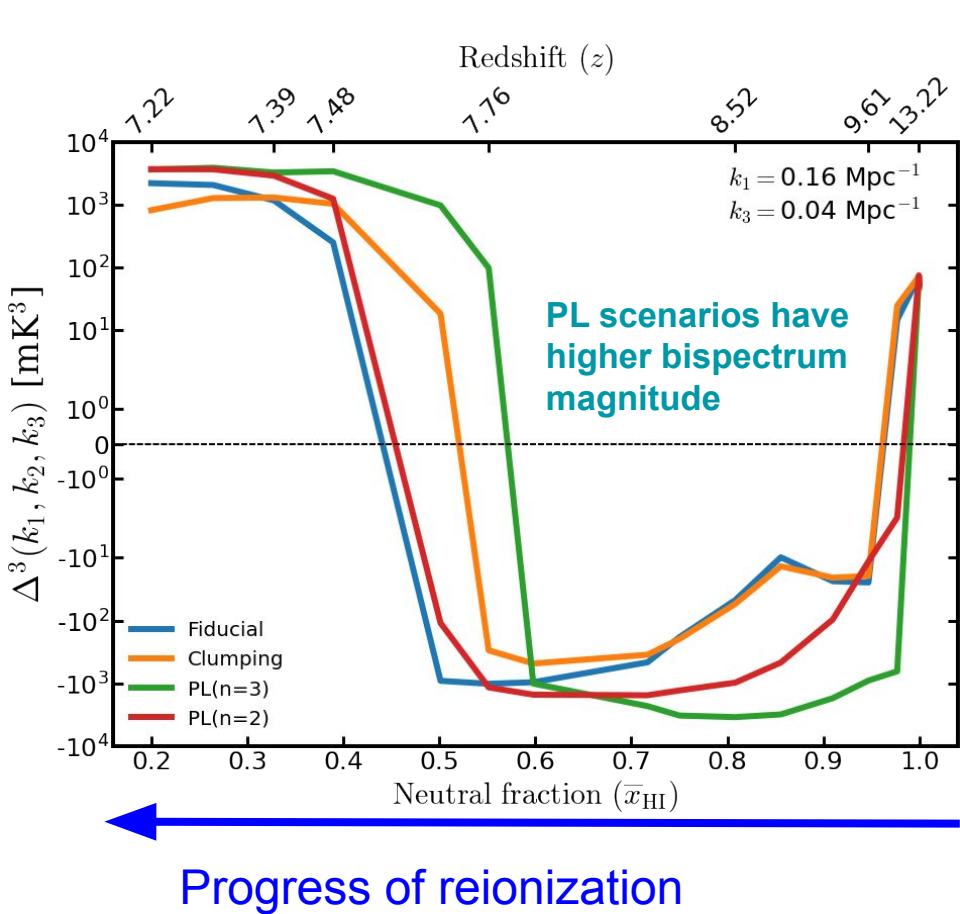
Progress of reionization



# Evolution of squeezed-limit bispectrum: Inside-out scenarios

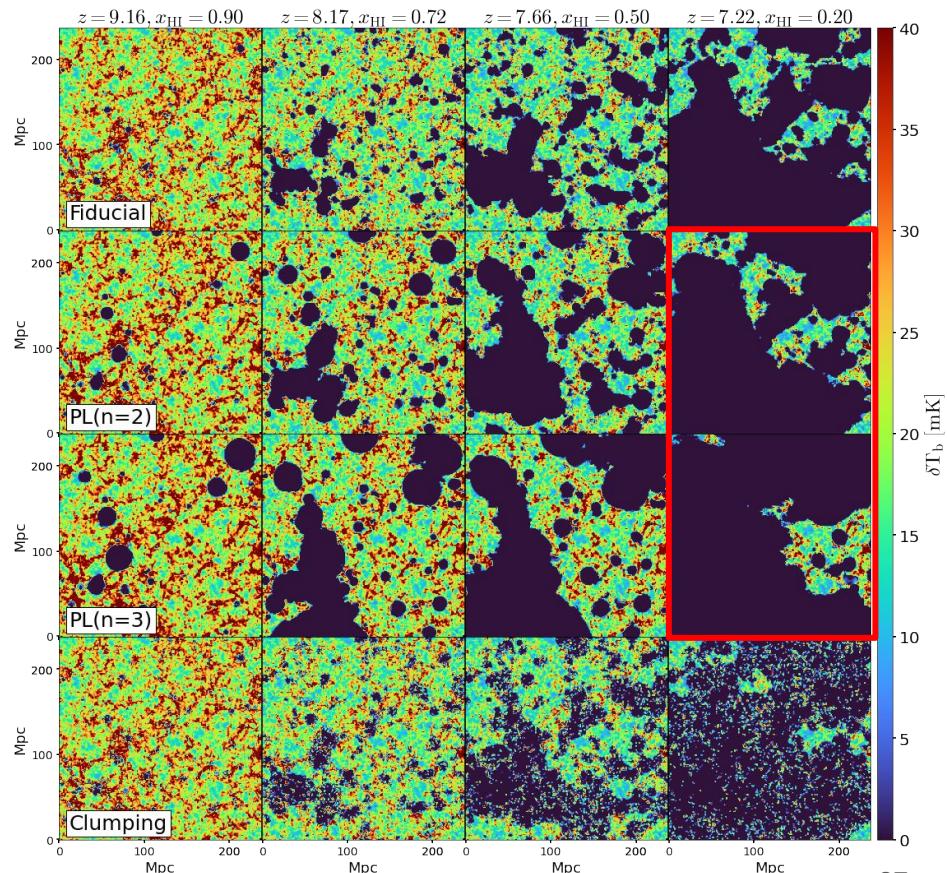
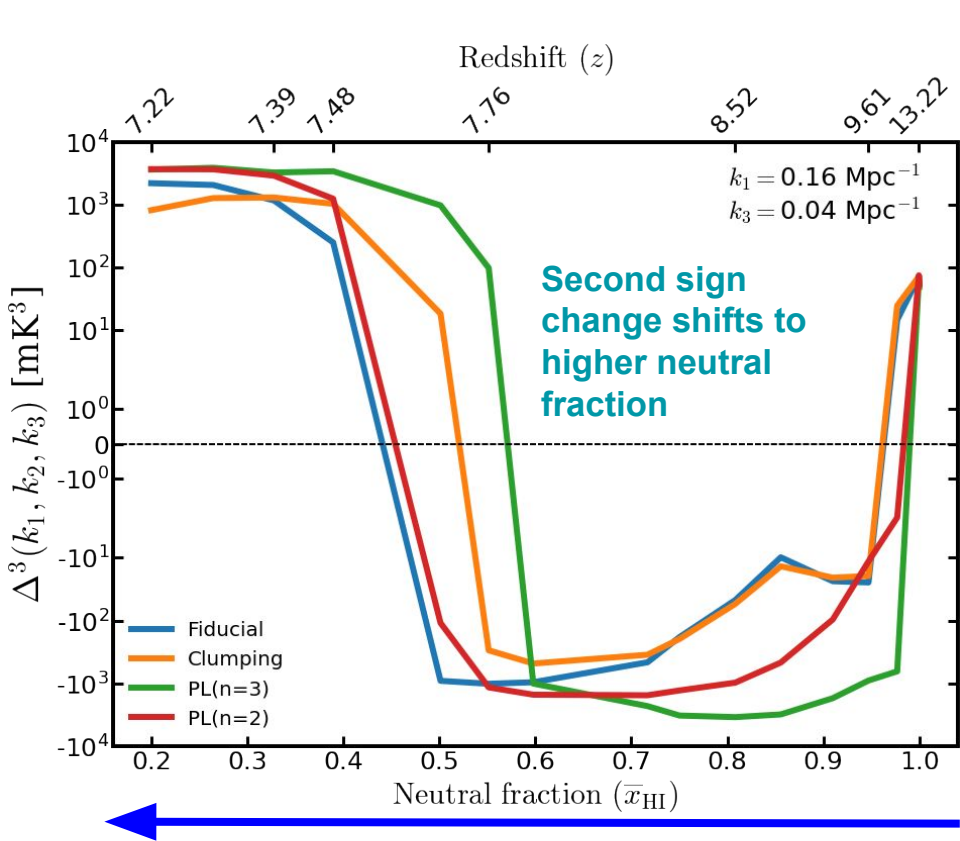


# Evolution of squeezed-limit bispectrum: Inside-out scenarios



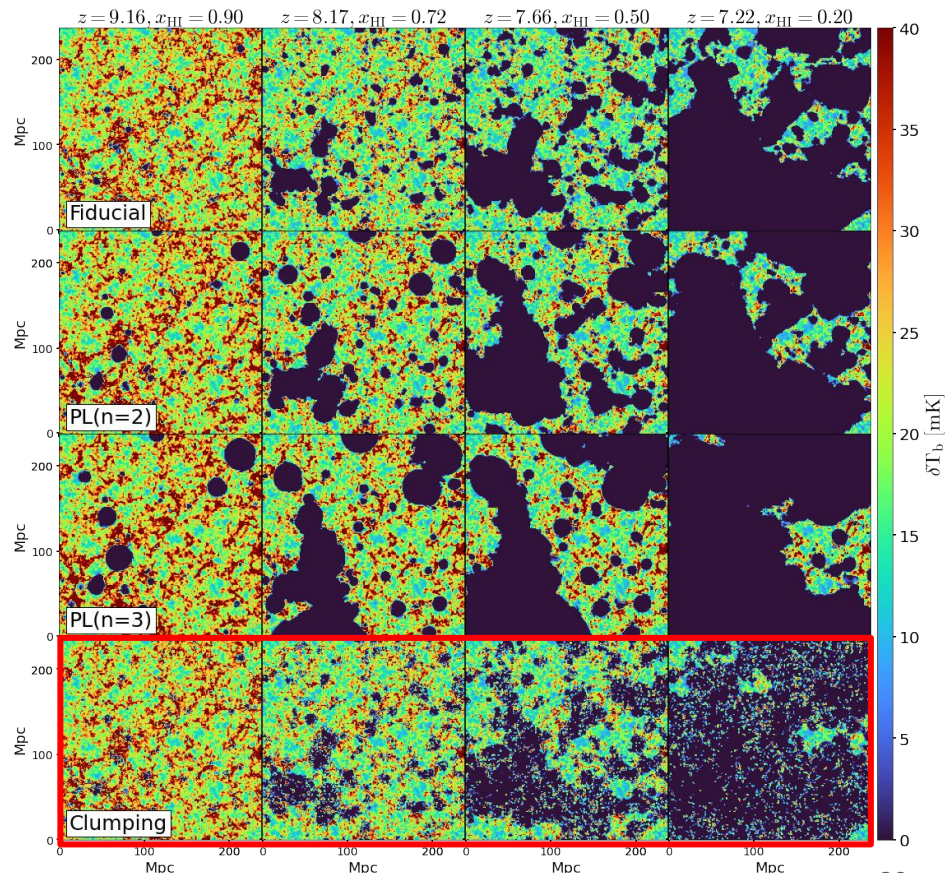
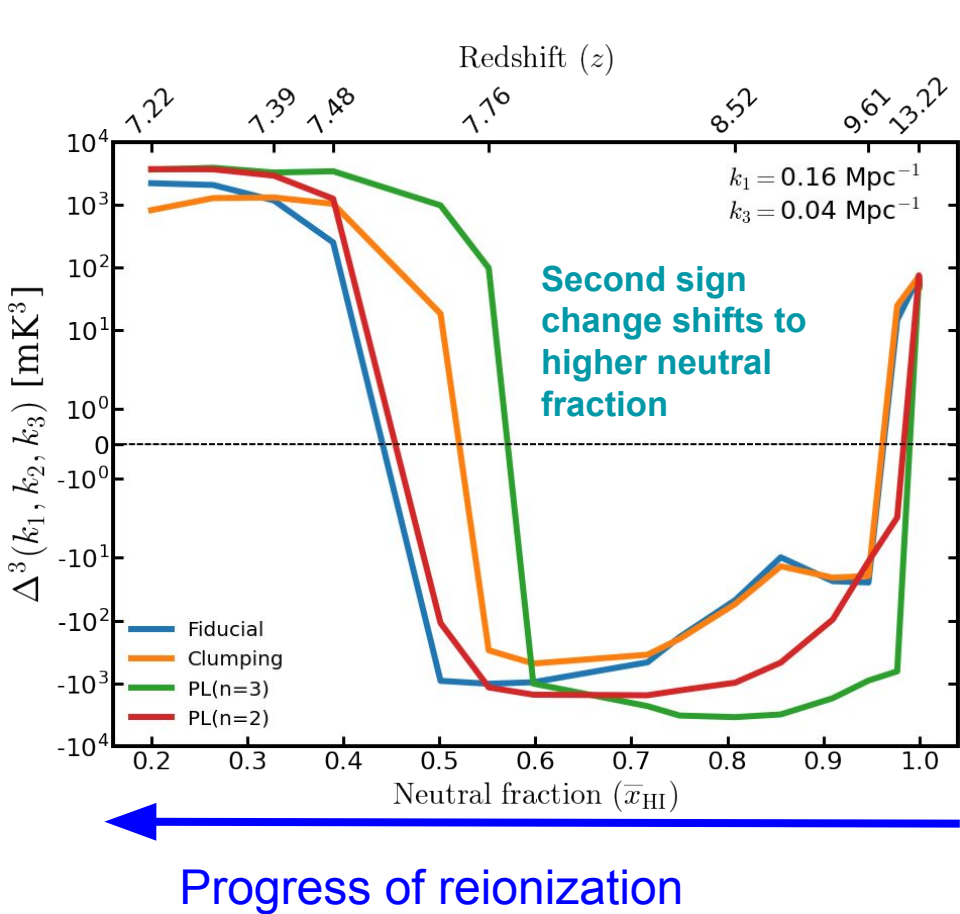


# Evolution of squeezed-limit bispectrum: Inside-out scenarios



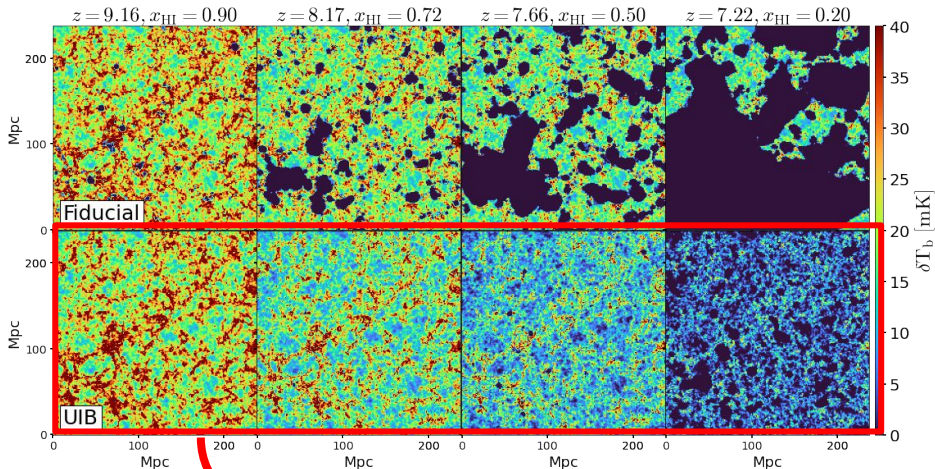
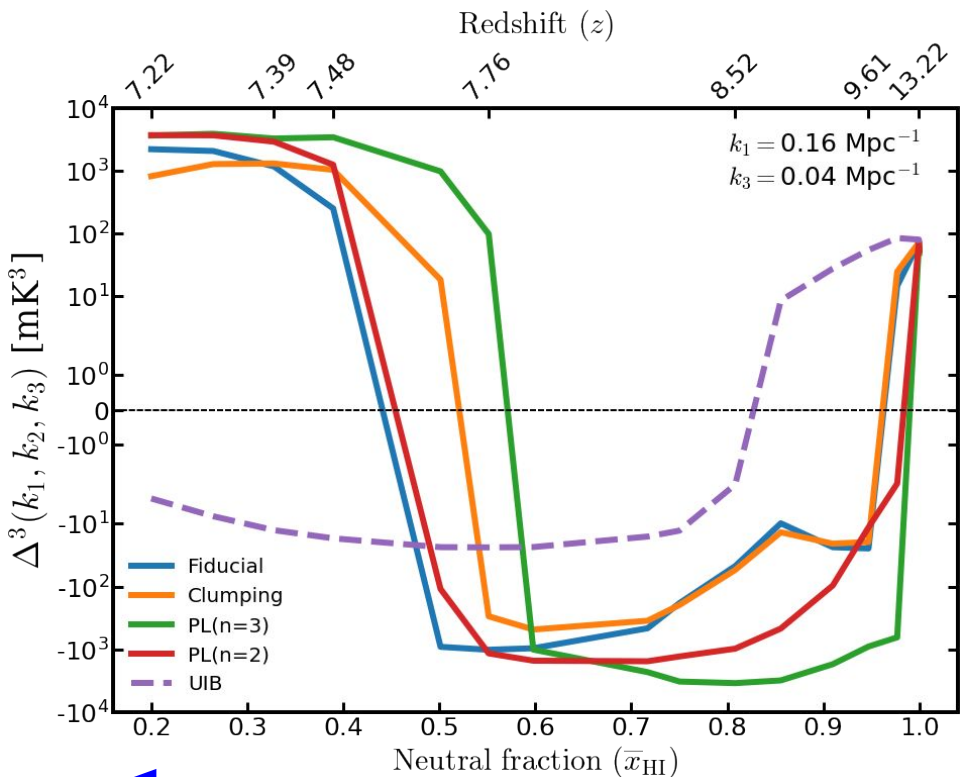
Progress of reionization

# Evolution of squeezed-limit bispectrum: Inside-out scenarios





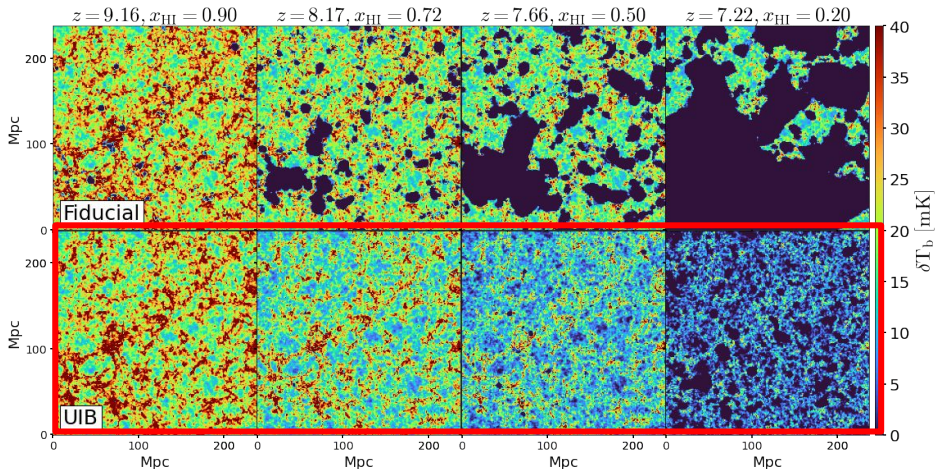
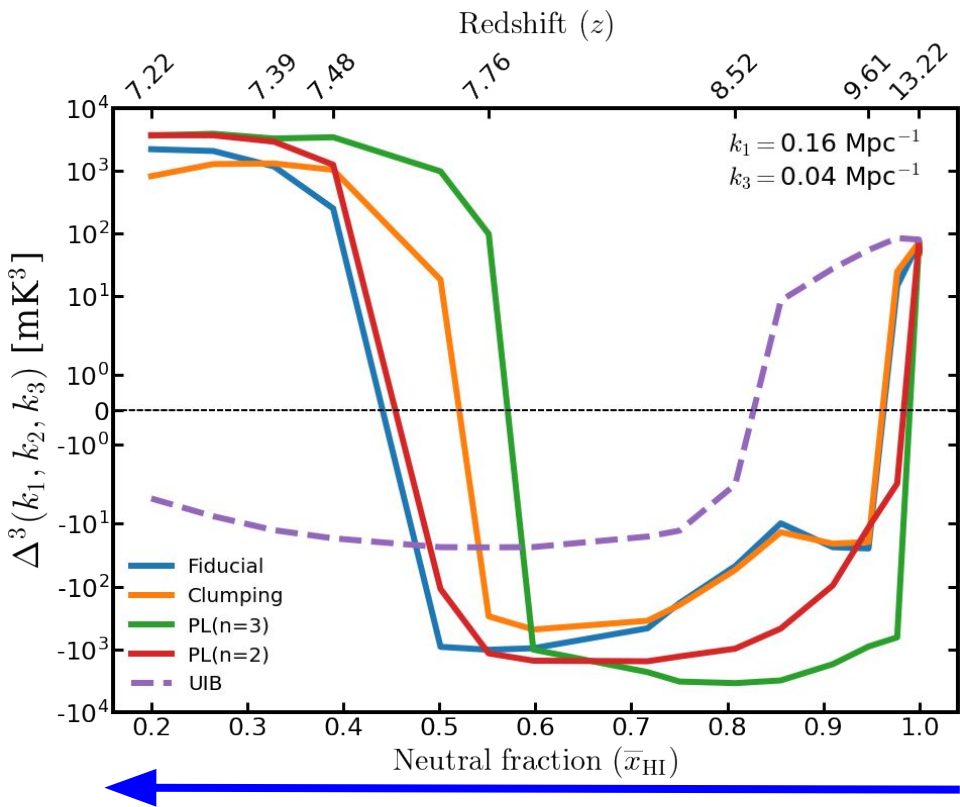
# Evolution of squeezed-limit bispectrum: Outside-in scenario



**Outside-in reionization scenario**

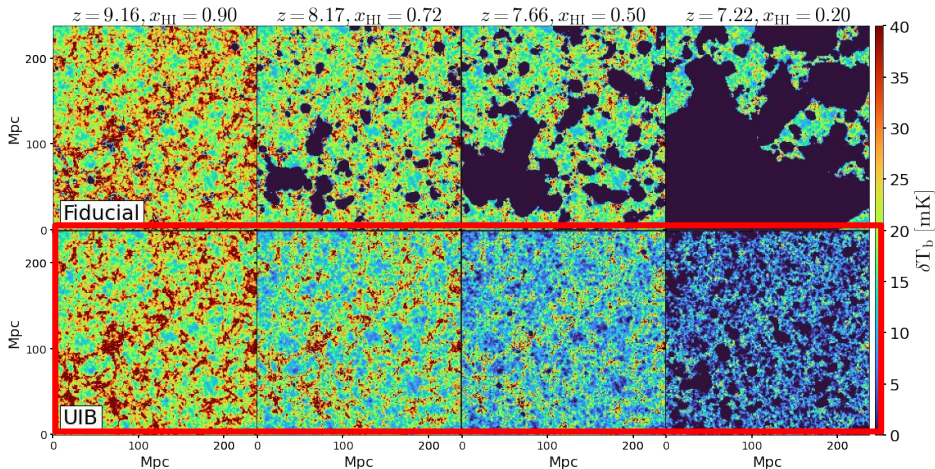
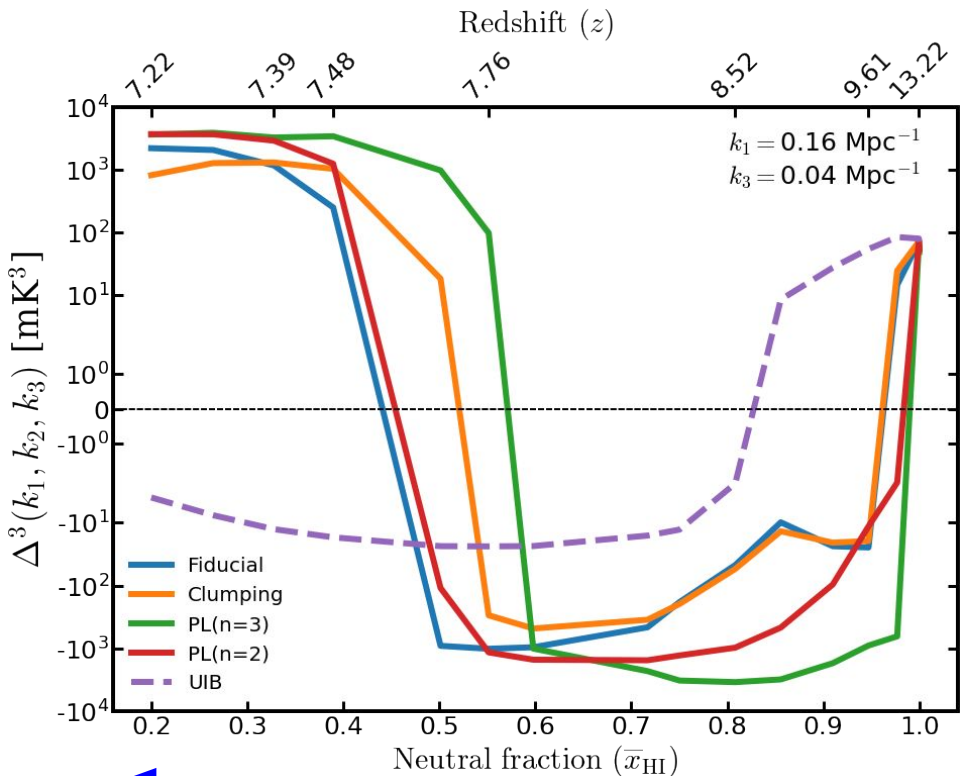
Noble, SM et al. 2024, arXiv: 2406.03118

# Evolution of squeezed-limit bispectrum: Outside-in scenario



- First change in the sign of bispectrum happens at a later stage of reionization
- Magnitude of the bispectrum is lower than inside-out scenarios

# Evolution of squeezed-limit bispectrum: Outside-in scenario

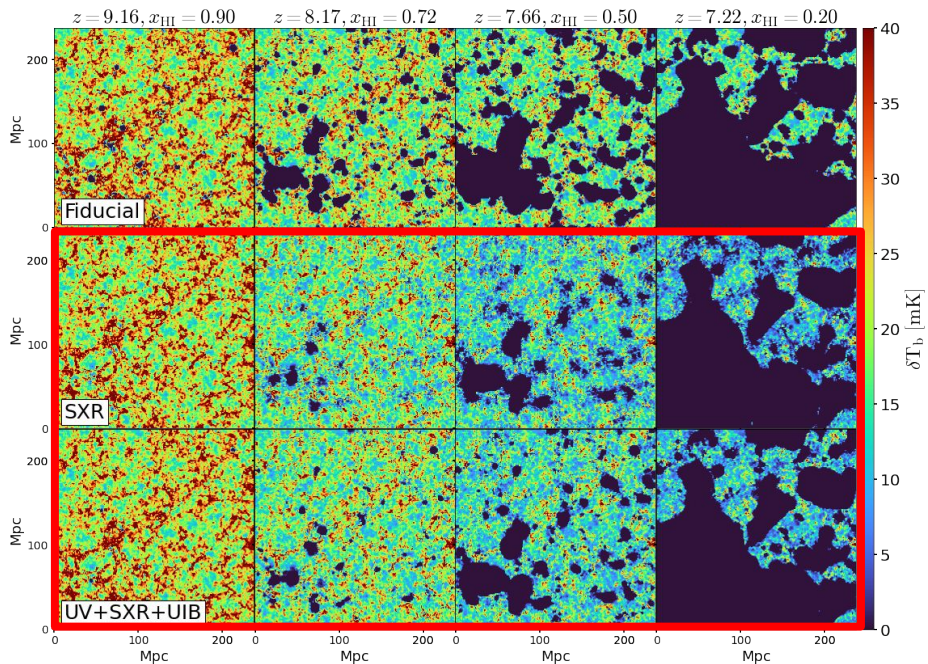
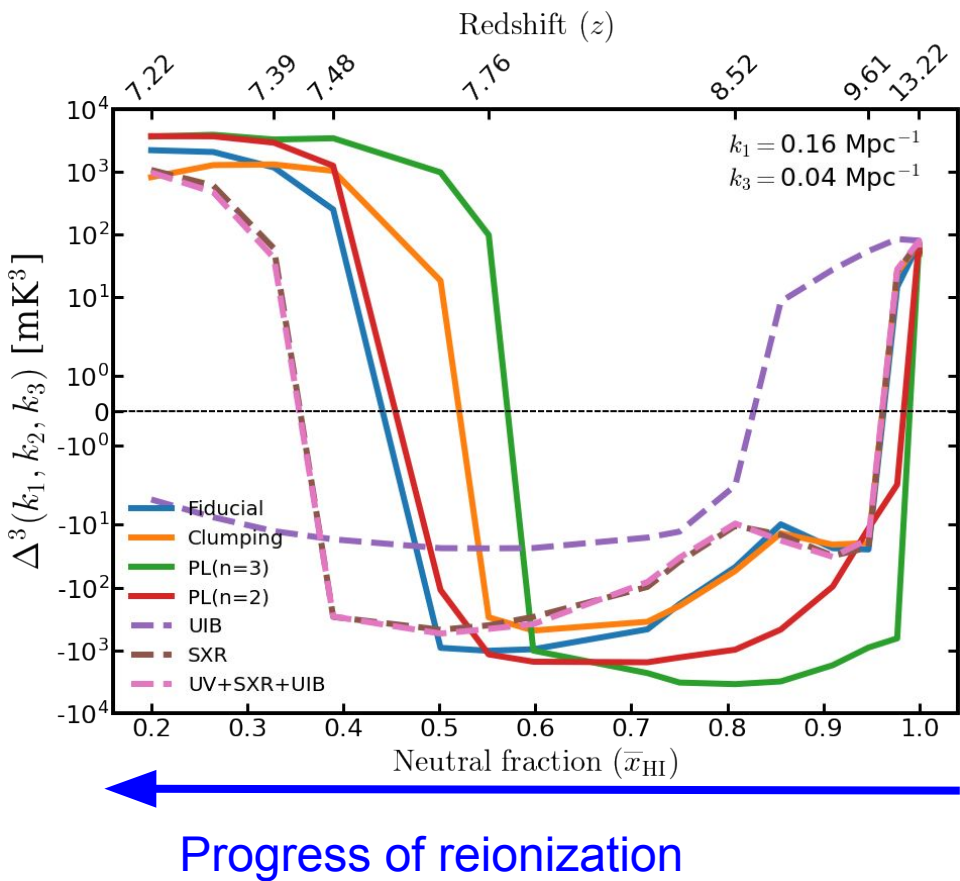


- First change in the sign of bispectrum happens at a late stage of reionization
- Magnitude of the bispectrum is lower than inside-out scenarios
- No change in the **sign of bispectrum** at late stage of reionization

Progress of reionization

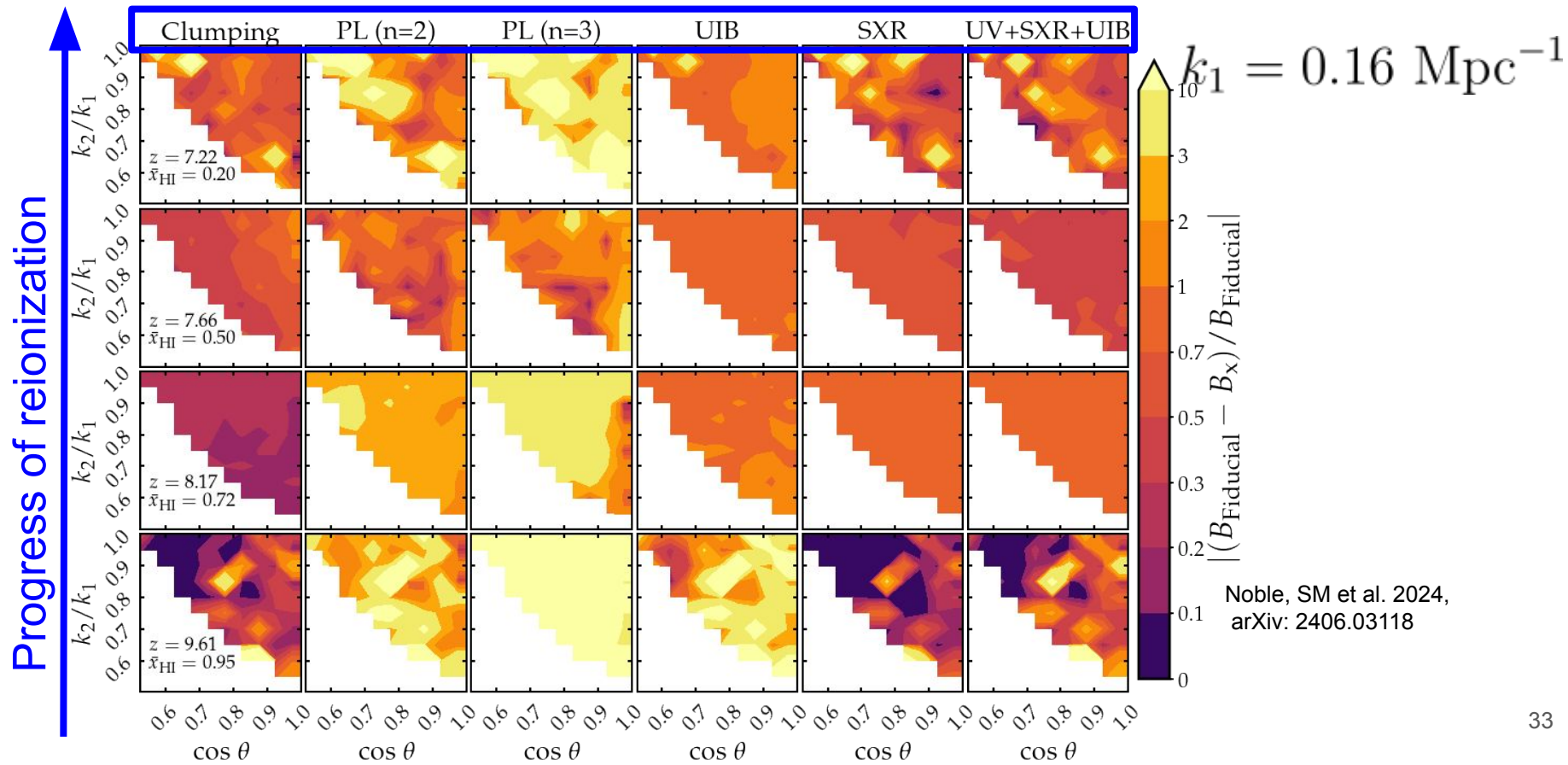


# Evolution of squeezed-limit bispectrum: Combination of inside-out and outside-in

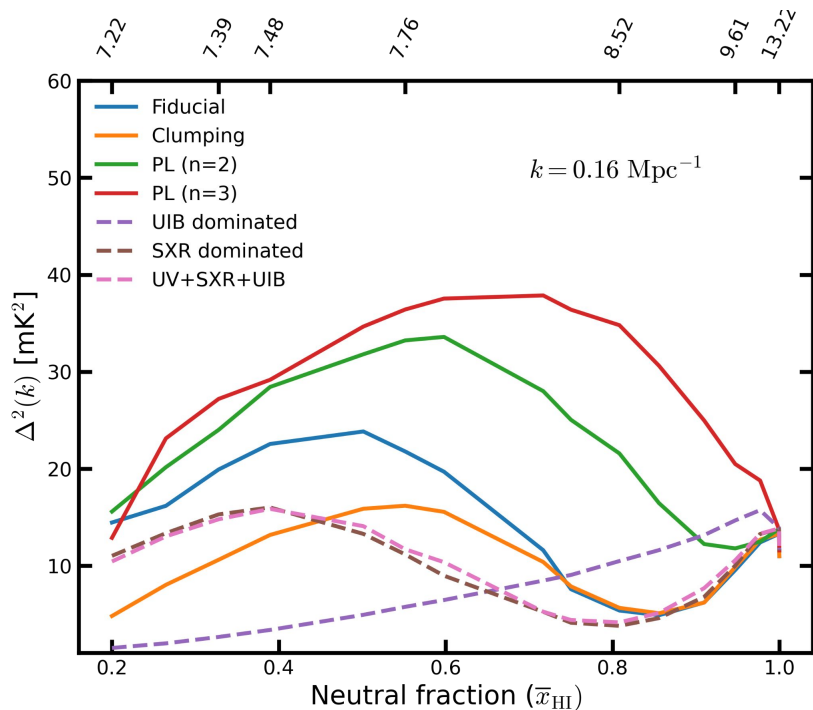


**Second sign change shifts to lower neutral fraction**

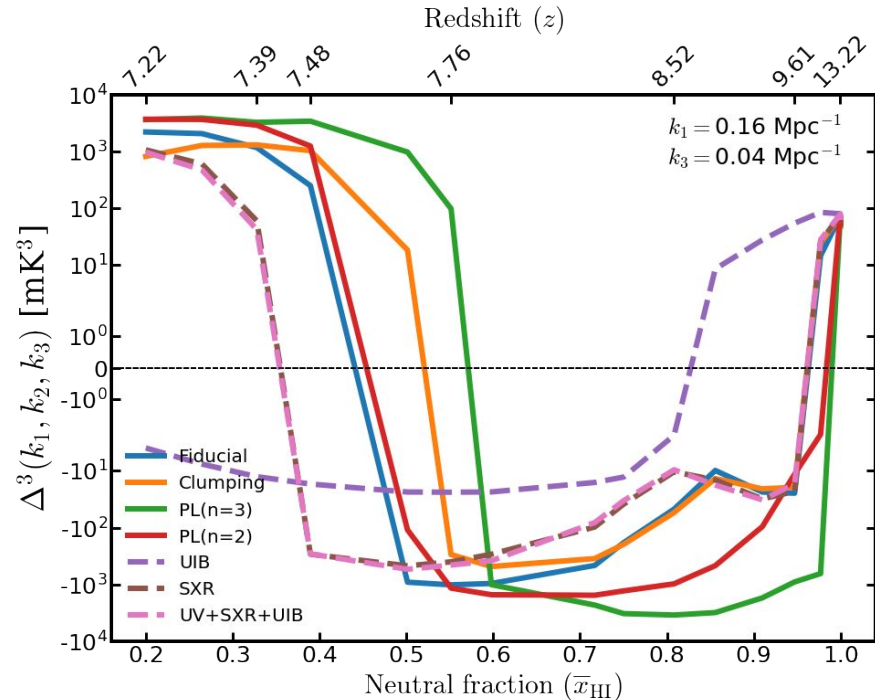
# Magnitude differences in 21-cm bispectrum between different reionization scenarios w. r. t. Fiducial



# Power spectrum vs bispectrum

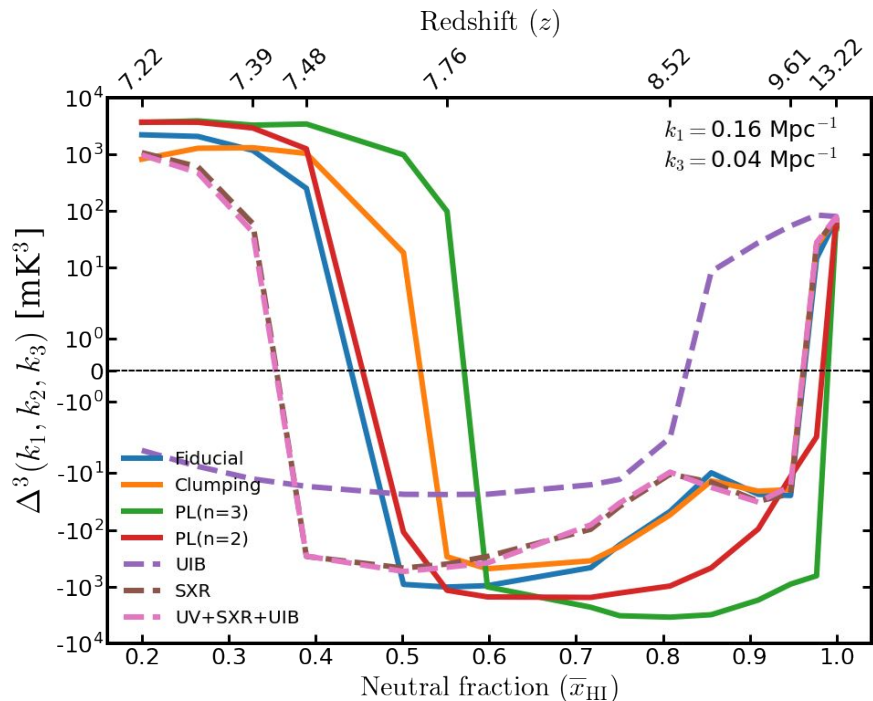


Majumdar et al. 2016, arXiv: 1509.07518



Noble, ..., SM, .. et al. 2024, arXiv: 2406.03118

# Summary I



- **21-cm bispectrum can capture the time evolving non-Gaussianity in different reionization scenarios.**
- **It can distinguish between different reionization scenarios.**
- **It can distinguish reionization scenarios better than the 21-cm power spectrum thanks to its sign and sequence of sign changes.**

**Noble,..., SM et al. 2024, arXiv: 2406.03118**

# Cosmic Dawn 21-cm bispectrum

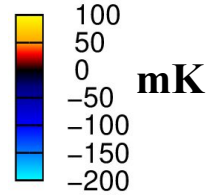


To identify which IGM process is dominating at what cosmic time (during CD) using the 21-cm bispectrum

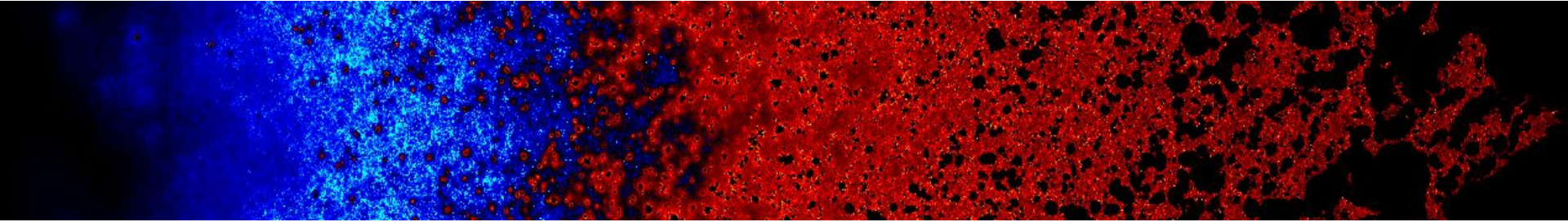


Disentangling the Ly $\alpha$  coupling and X-ray heating

# HI 21-cm Brightness Temperature Fluctuations



$z$



$$\delta T_b(\mathbf{r}, z) = 27 x_{\text{HI}}(\mathbf{r}, z) \left(1 - \frac{T_{\text{CMB}}(z)}{T_{\text{S}}(\mathbf{r}, z)}\right) (1 + \delta_b(\mathbf{r}, z)) \left(\frac{\Omega_b h^2}{0.023}\right) \left(\frac{0.15}{\Omega_m h^2} \frac{1+z}{10}\right)^{1/2} \text{ mK}$$

IGM physics makes the spatial and temporal fluctuations of the CD-EoR 21-cm signal  
**highly non-Gaussian!**

# Simulations

- **Dark matter N-body simulation** → To generate dark matter distributions.

J. Harnois-Déraps+ 2013



- **Radiative transfer simulation** → GRIZZLY → To generate CD 21-cm maps.

Ghara+ 2015, 2018

**Raghunath Ghara**

Postdoc at the University of Pennsylvania



# Simulations

## Different CD scenarios

Processes \ Scenarios	Model-a <sub>0</sub>	Model-a	Model-b	Model-c
Ly $\alpha$ -coupling	Yes	Yes	Saturated	Yes
X-ray heating	No	No	Yes	Yes
Ionization	No	Yes	Yes	Yes



**Simplistic scenarios**



Only one single physical process dominates the 21-cm fluctuations



**Realistic scenario**



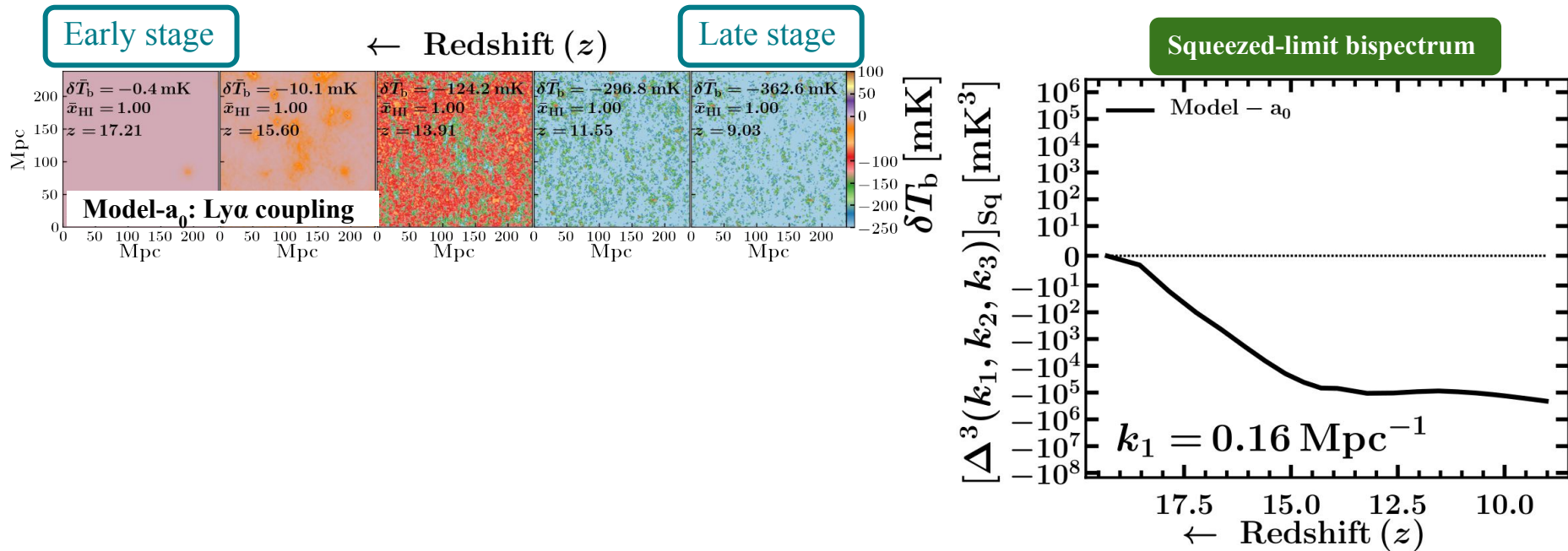
All three physical processes incorporated self-consistently

### Importance of first three scenarios:

Used to identify the unique signature of each IGM process on the 21-cm bispectrum

⇒ Helps in explaining the bispectrum from **Model-c, the most realistic scenario.**

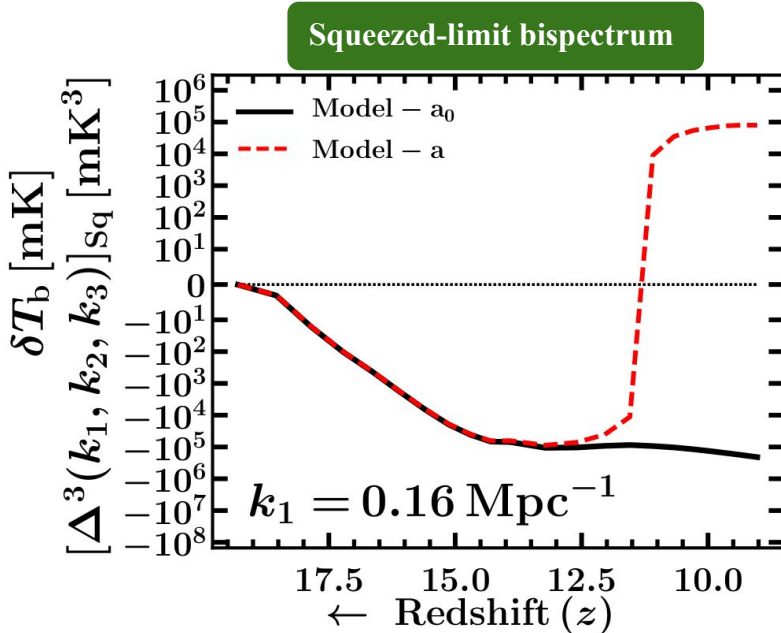
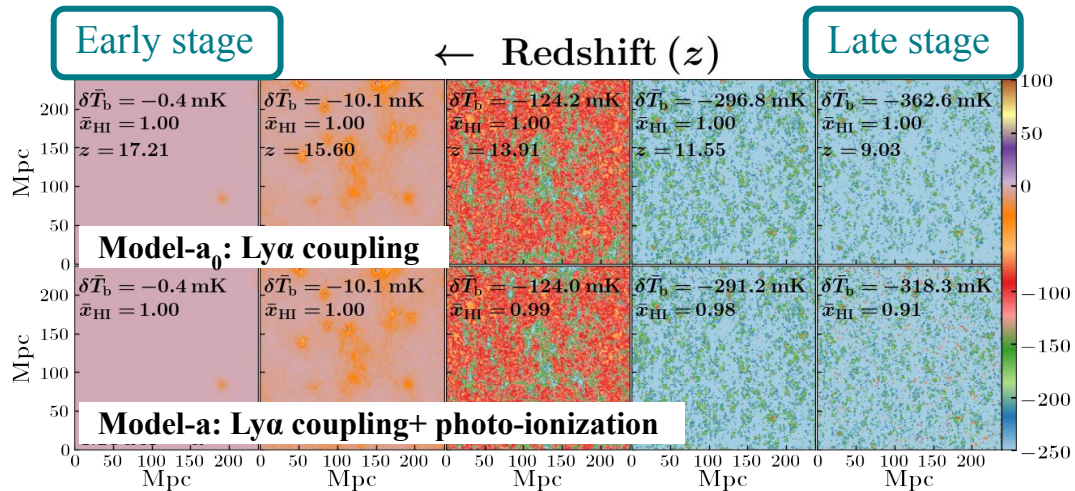
# 21-cm bispectrum as a probe of IGM physics during CD



**Magnitude:** Monotonically increasing

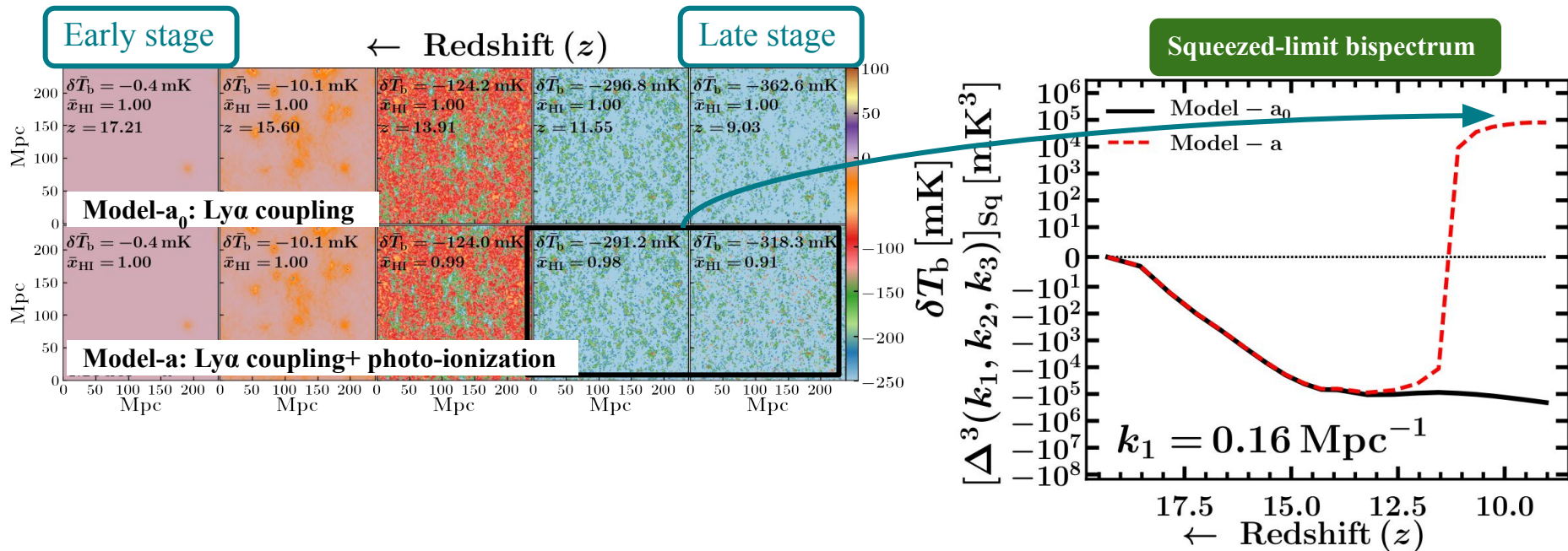
**Sign:** Negative during the entire CD

# 21-cm bispectrum as a probe of IGM physics during CD



Bispectrum for Model-a agrees well with Model- $a_0$  until  $z \sim 13$

# 21-cm bispectrum as a probe of IGM physics during CD



Bispectrum for Model-a agrees well with Model-a<sub>0</sub> until  $z \sim 13$

At  $z < 13$ , sign reversal.

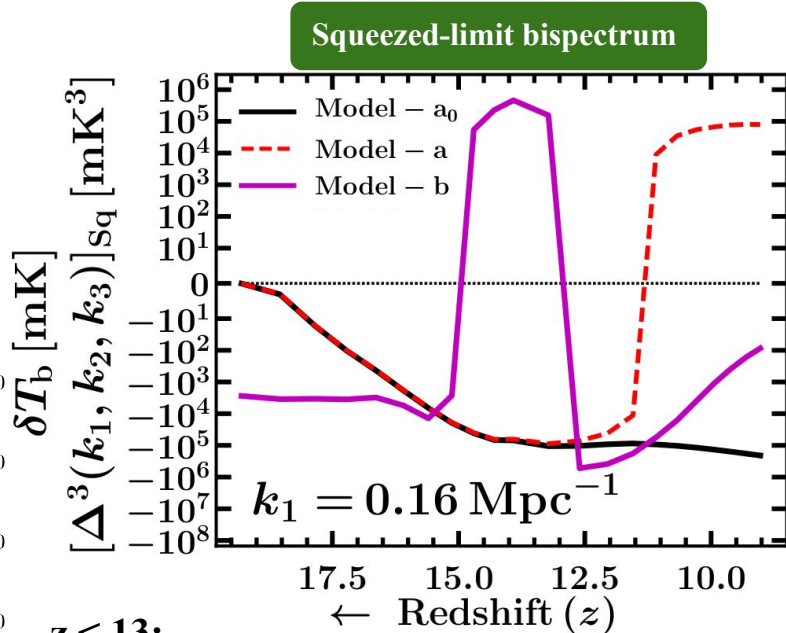
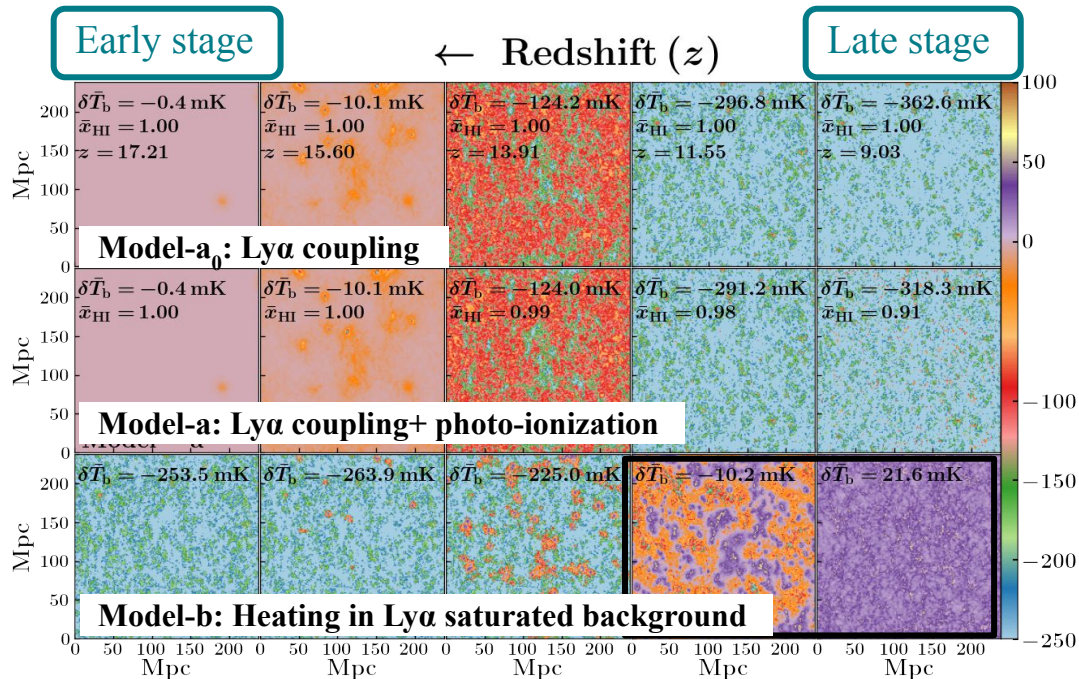








# 21-cm bispectrum as a probe of IGM physics during CD



$z < 13$ :

Further heating  $\Rightarrow$  percolation  $\Rightarrow$  a single large connected cluster of heated regions.



Negative bispectrum as signal fluctuations are dominated by the leftover absorption regions in a heated background.

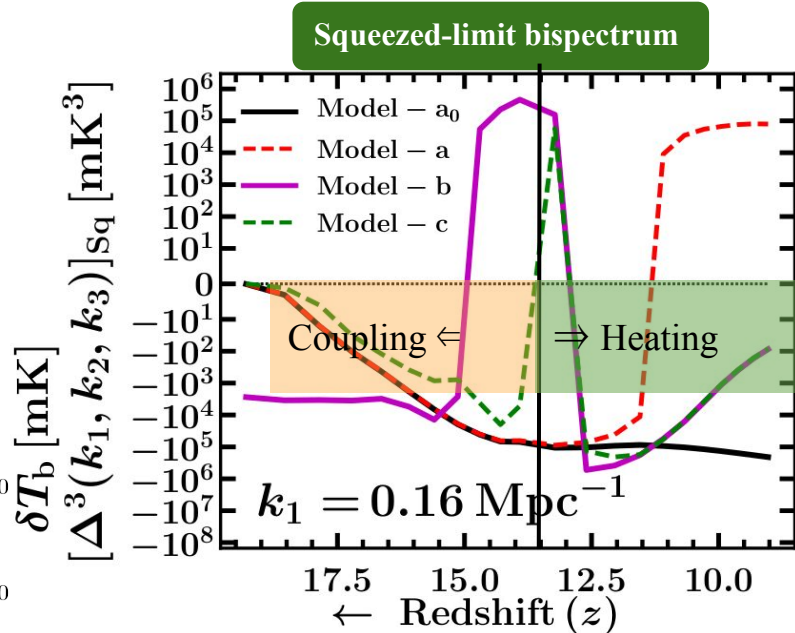
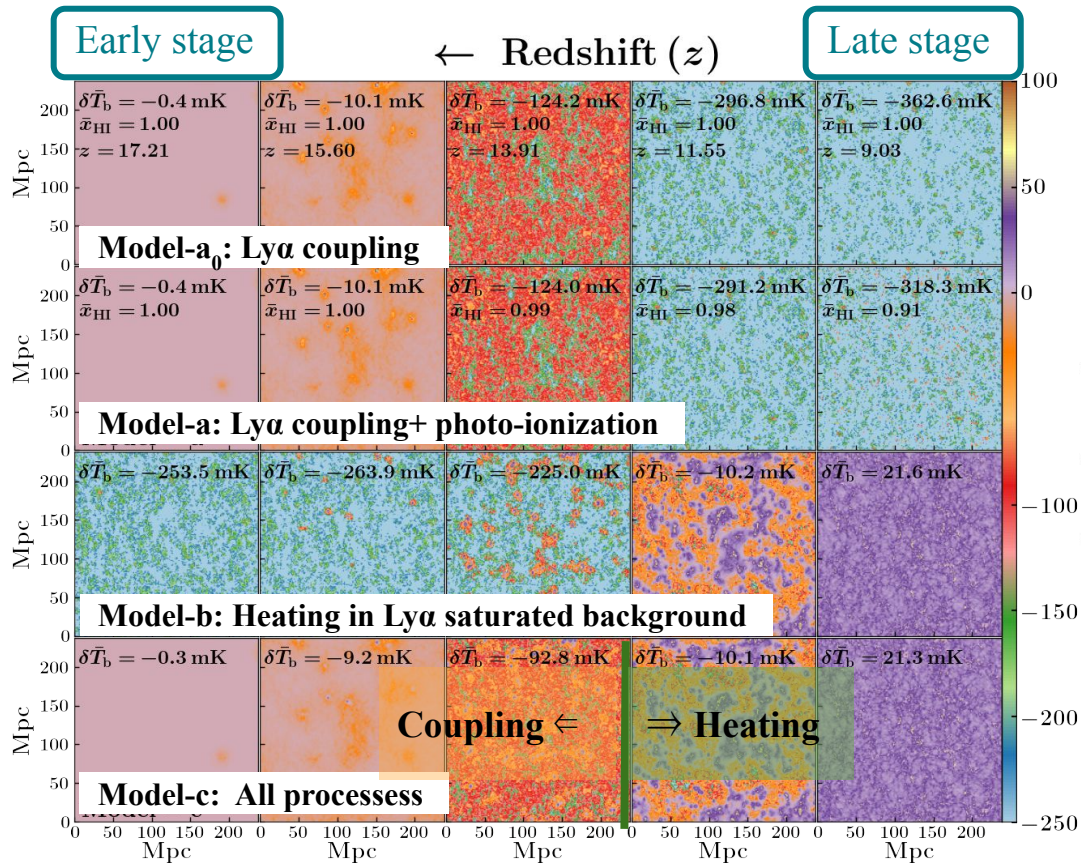








# 21-cm bispectrum as a probe of IGM physics during CD

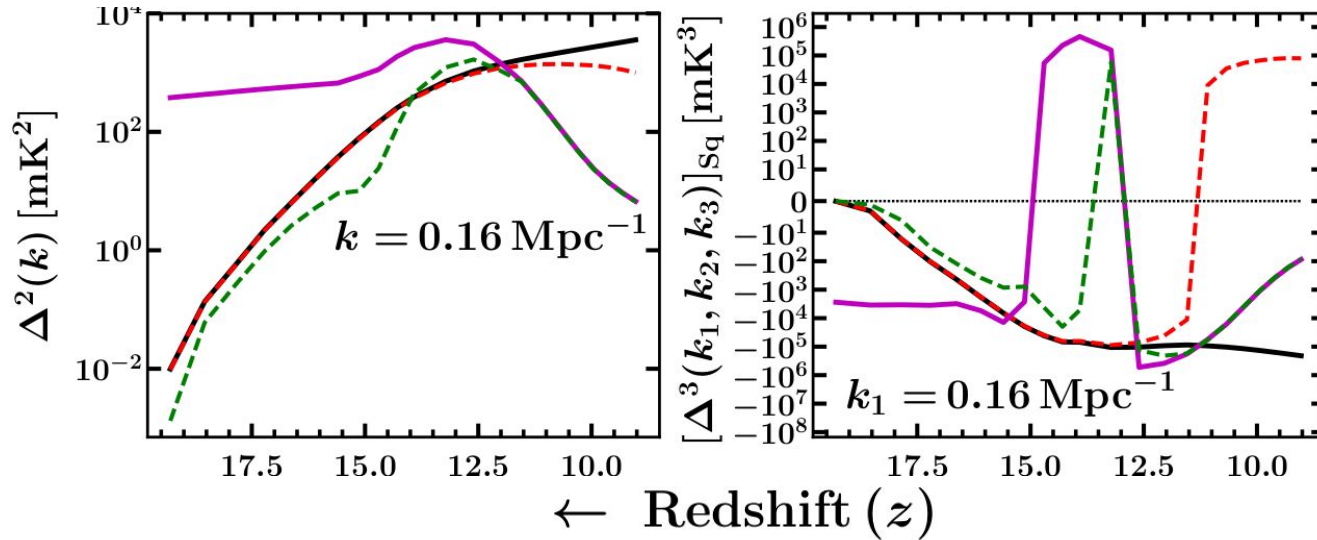


Initially Model-c follows Models- $a_0$  and  $a$   
 $\Rightarrow$  Ly $\alpha$  coupling dominates until  $z \sim 14$

For  $z < 14$  Model-c follows Model-b  $\Rightarrow$   
 X-ray heating dominates



## Power spectrum vs bispectrum



Bispectrum via its sign and sign changes can conclusively tell us which IGM process dominates the 21-cm fluctuations at what cosmic time.

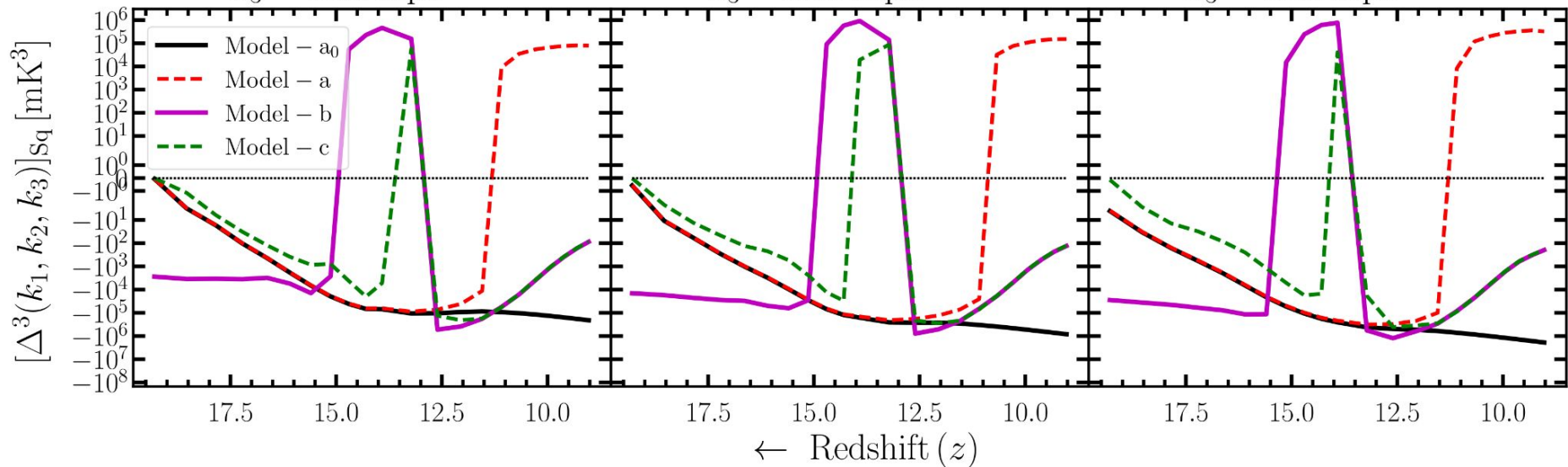
Power spectrum  $\Rightarrow$  always +ve  $\Rightarrow$  it is difficult to unequivocally identify these transitions on the basis of the power spectrum alone.

## Robustness of the bispectrum

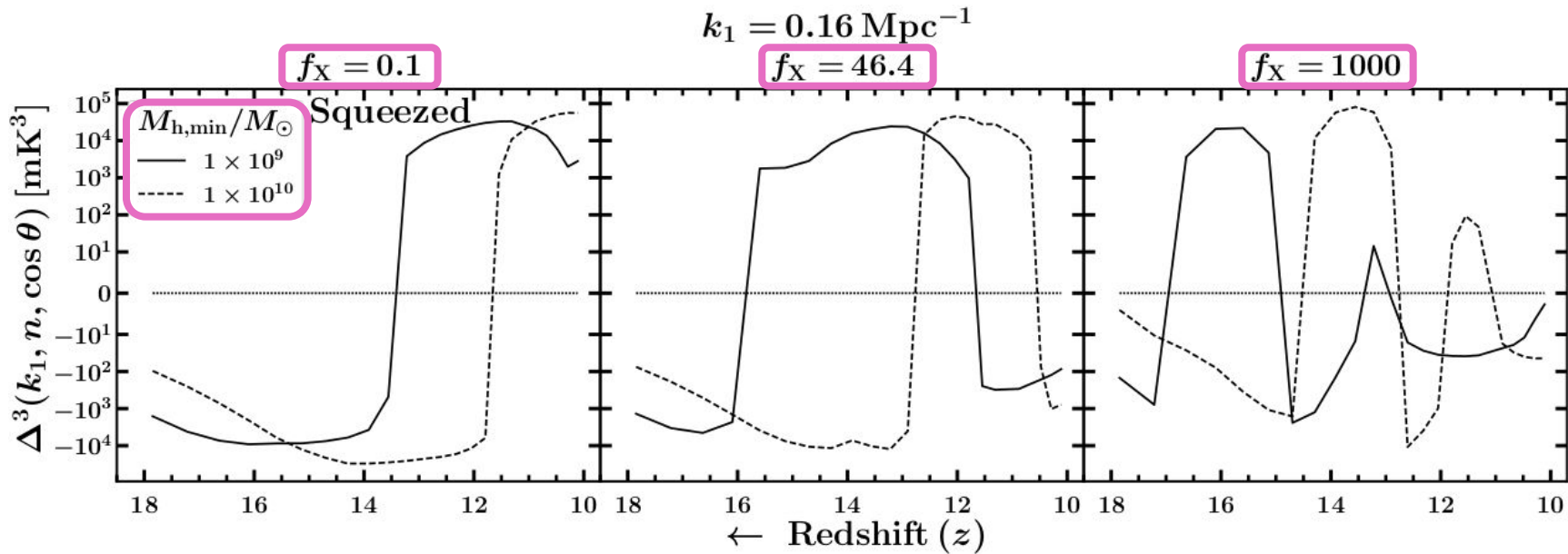
$$k_1 = 0.16 \text{ Mpc}^{-1}$$
$$k_3 = 0.05 \text{ Mpc}^{-1}$$

$$k_1 = 0.23 \text{ Mpc}^{-1}$$
$$k_3 = 0.08 \text{ Mpc}^{-1}$$

$$k_1 = 0.34 \text{ Mpc}^{-1}$$
$$k_3 = 0.11 \text{ Mpc}^{-1}$$

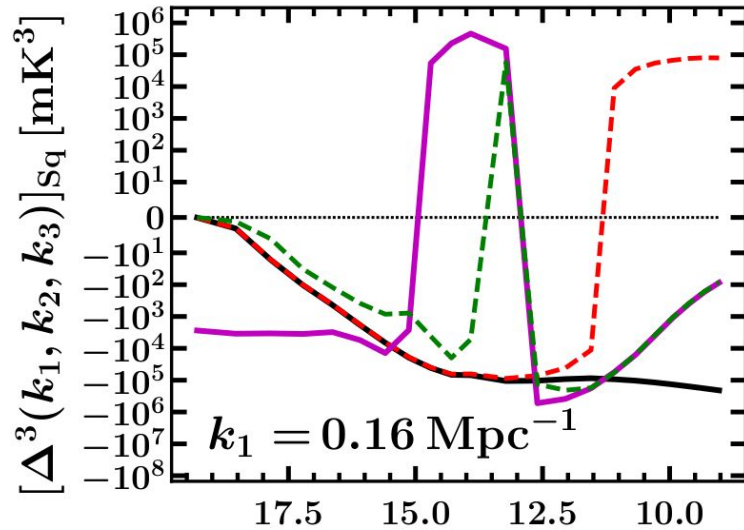


# Impact of different source models



Kamran,..., SM et al 2022, arXiv: 2207.09128

## Summary II

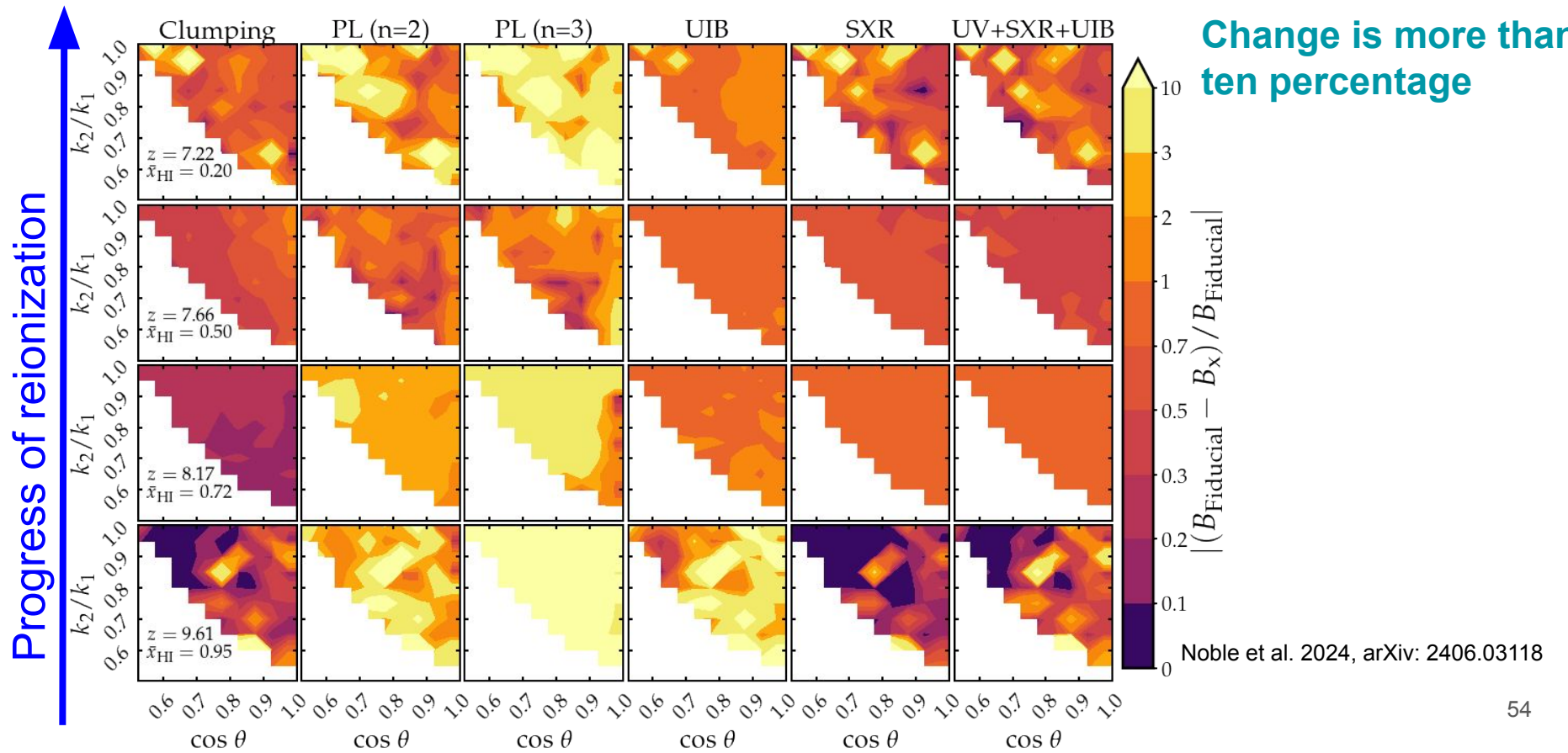


- CD 21-cm signal is highly non-Gaussian which the bispectrum statistic can potentially capture.
- The bispectrum can probe the IGM physics that sources the non-Gaussianity in the signal.
- The bispectrum features can be used as a confirmative test for the 21-cm observations using the next-generation telescopes such as SKA.

**The sign of the bispectrum can tell us the relative contrast of the fluctuations in the 21-cm signal with respect to its background.**

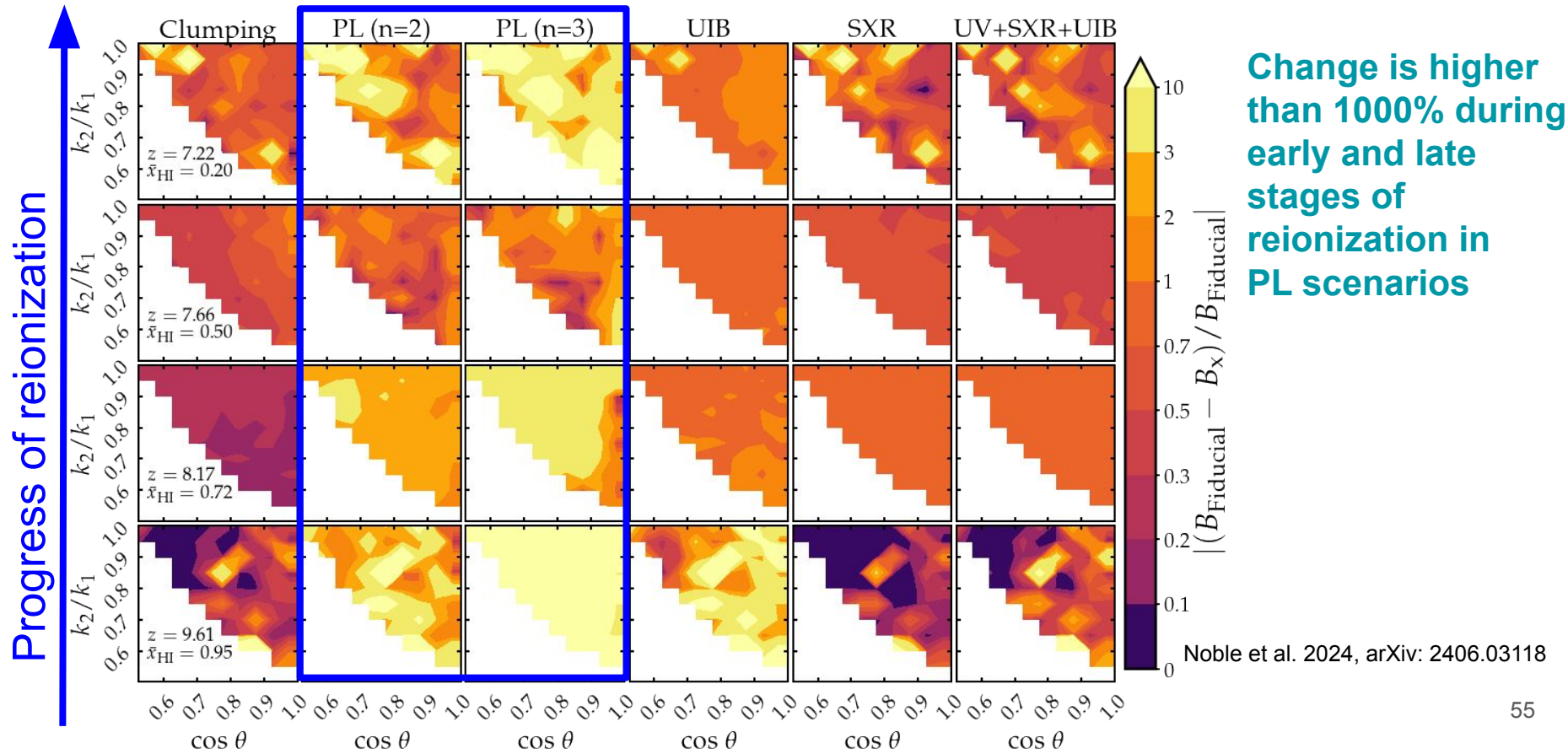
**The sign, shape and the sequence of sign change in the bispectrum works as a smoking gun for the dominant physical processes in the IGM.**

# Magnitude differences in 21-cm bispectrum between different reionization scenarios w. r. t. Fiducial

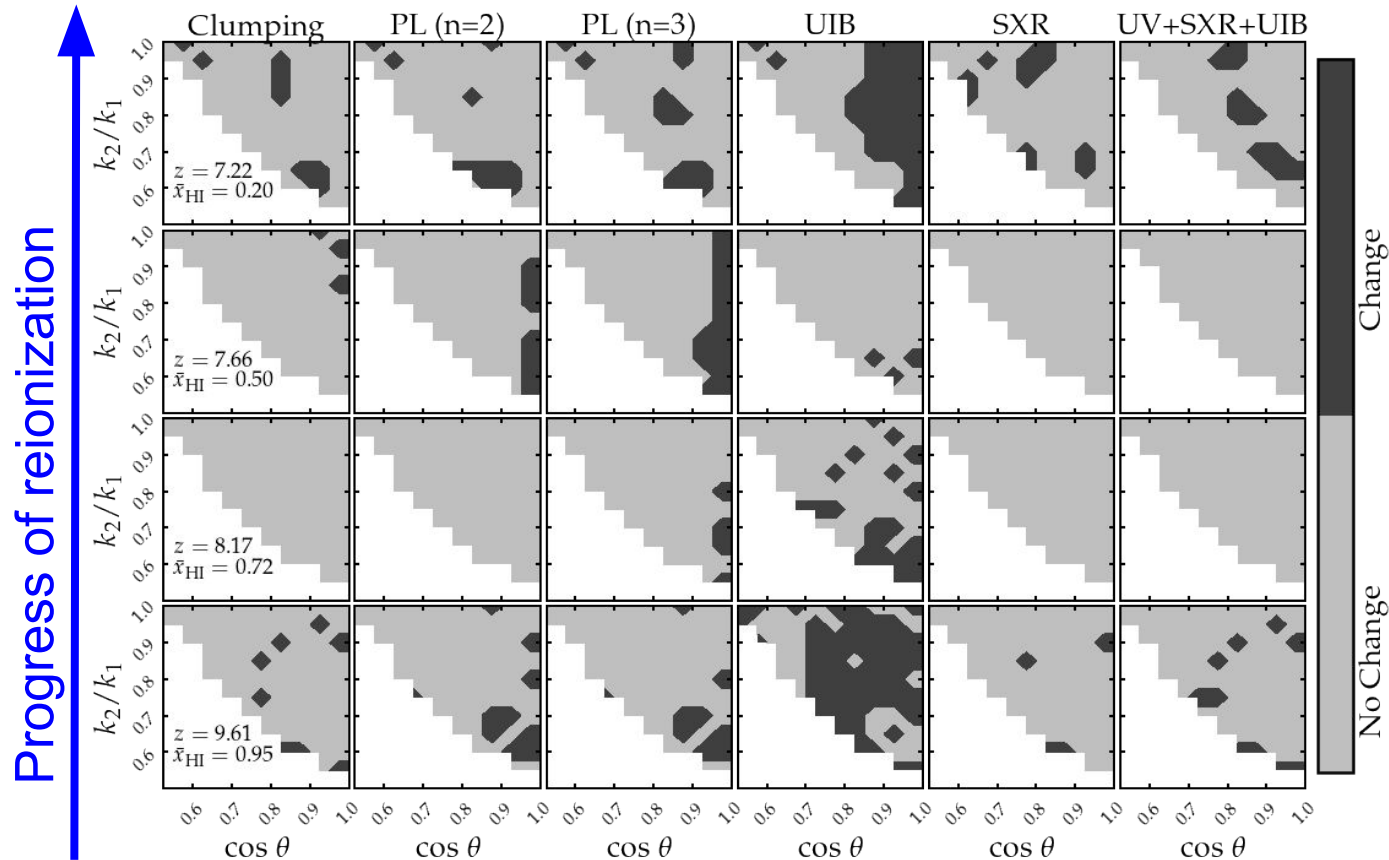




# Magnitude differences in 21-cm bispectrum between different reionization scenarios w. r. t. Fiducial

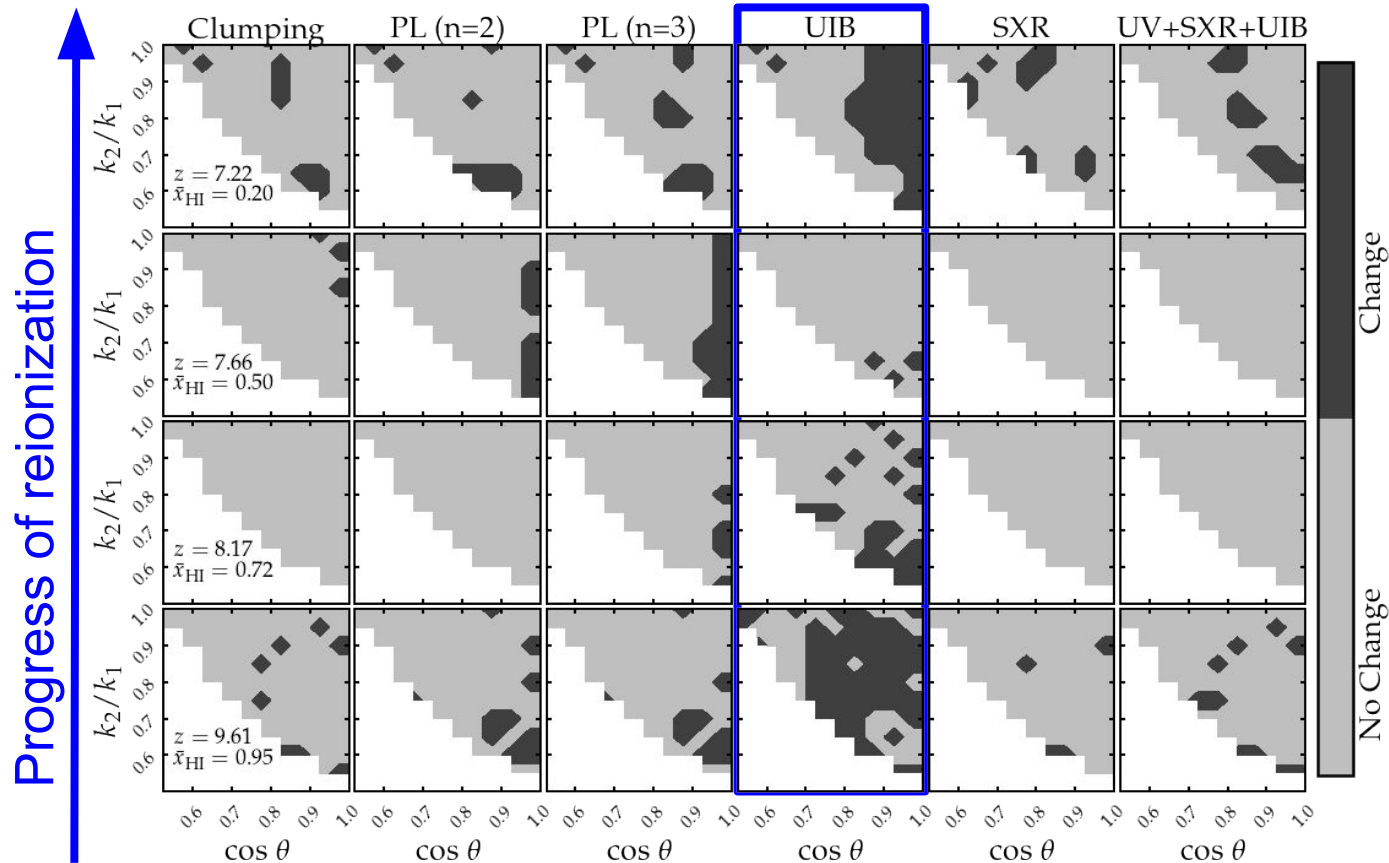


# Sign differences in 21-cm bispectrum between different reionization scenarios w. r. t. Fiducial



Noble et al. 2024, arXiv: 2406.03118

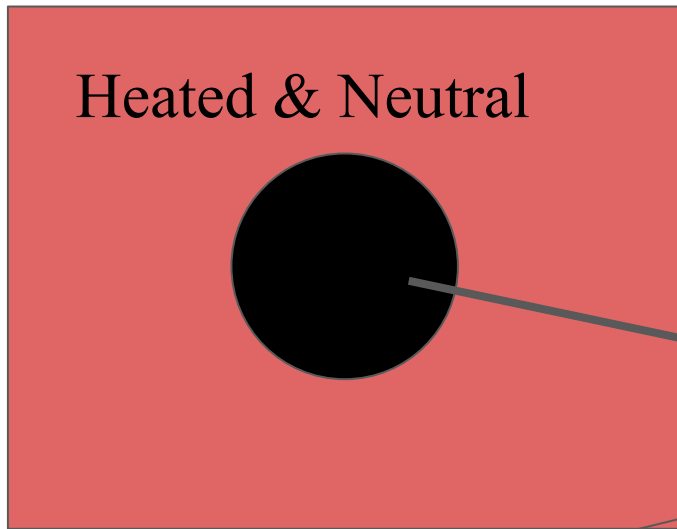
# Sign differences in 21-cm bispectrum between different reionization scenarios w. r. t. Fiducial



Change is highest  
in UIB scenario

Noble et al. 2024, arXiv: 2406.03118

## Interpretation

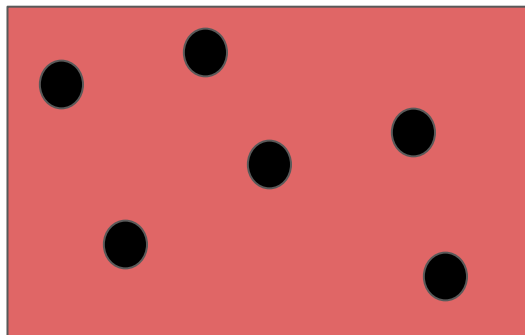


$$\Delta_{21} \propto -W(kR) \left( \sum \exp\{i\mathbf{k} \cdot \mathbf{r}\} \right).$$

Ionized

Many non-overlapping  
ionized regions

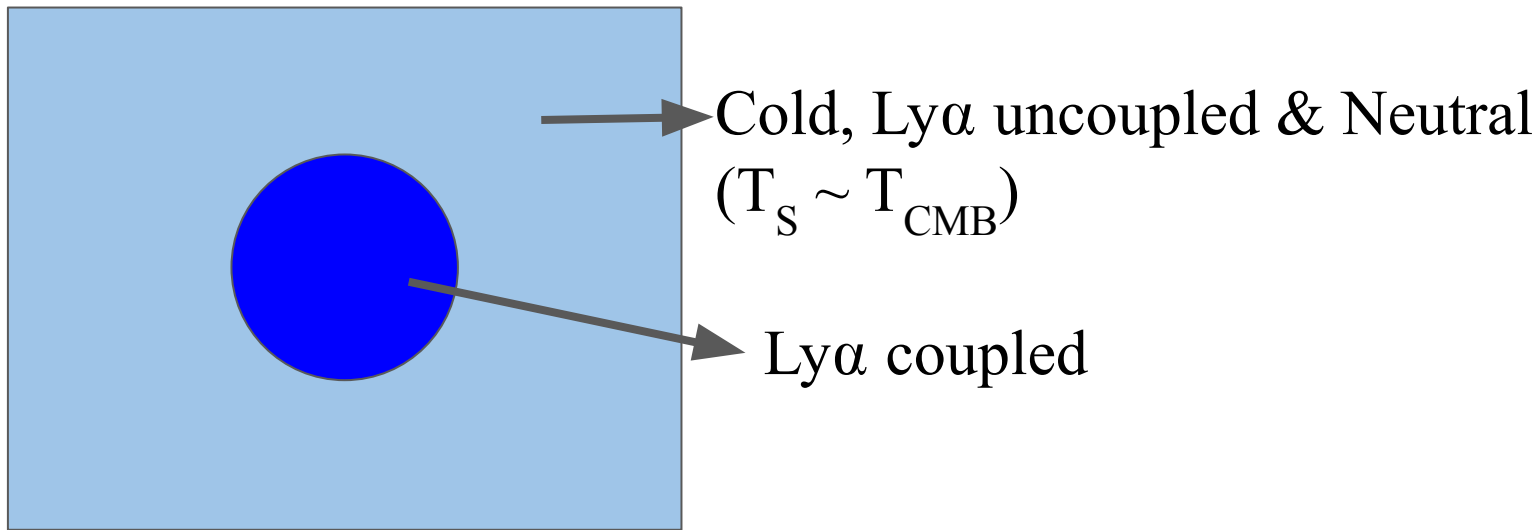
Large scale signal -ve



Bharadwaj & Ali 2004  
Bharadwaj & Pandey 2005



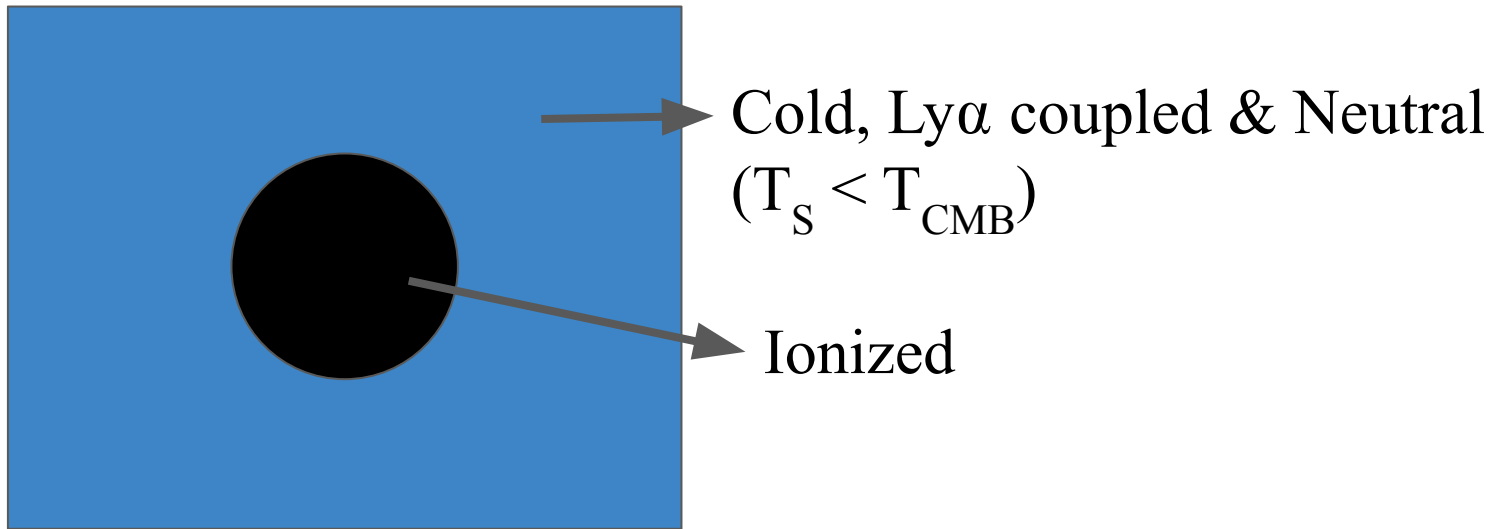
## Interpretation



Strong -ve signal in a weak -ve background

Large scale signal -ve

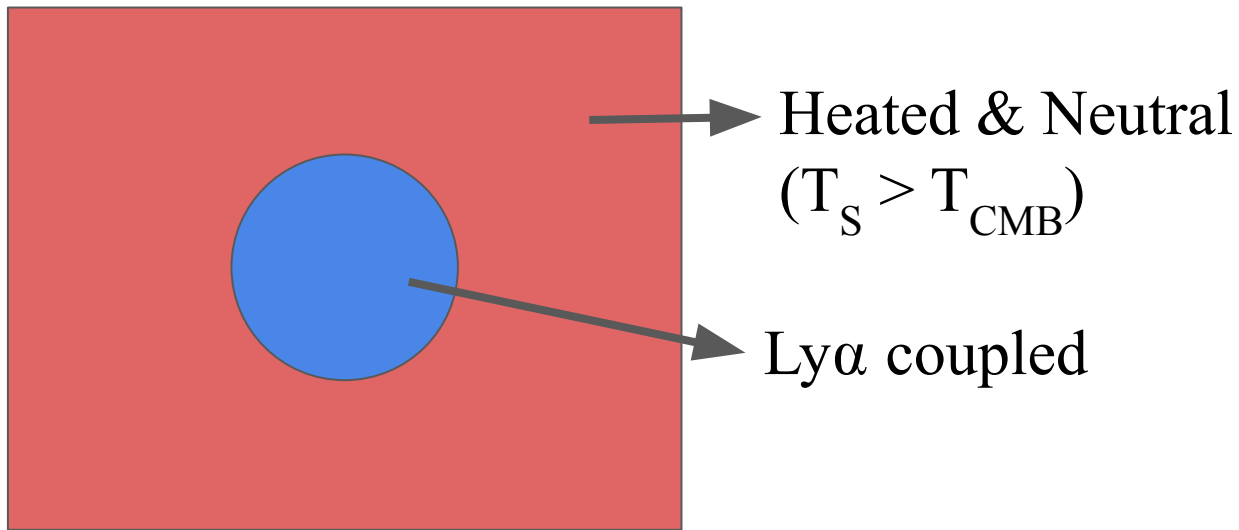
## Interpretation



zero signal in a -ve background

Large scale signal +ve

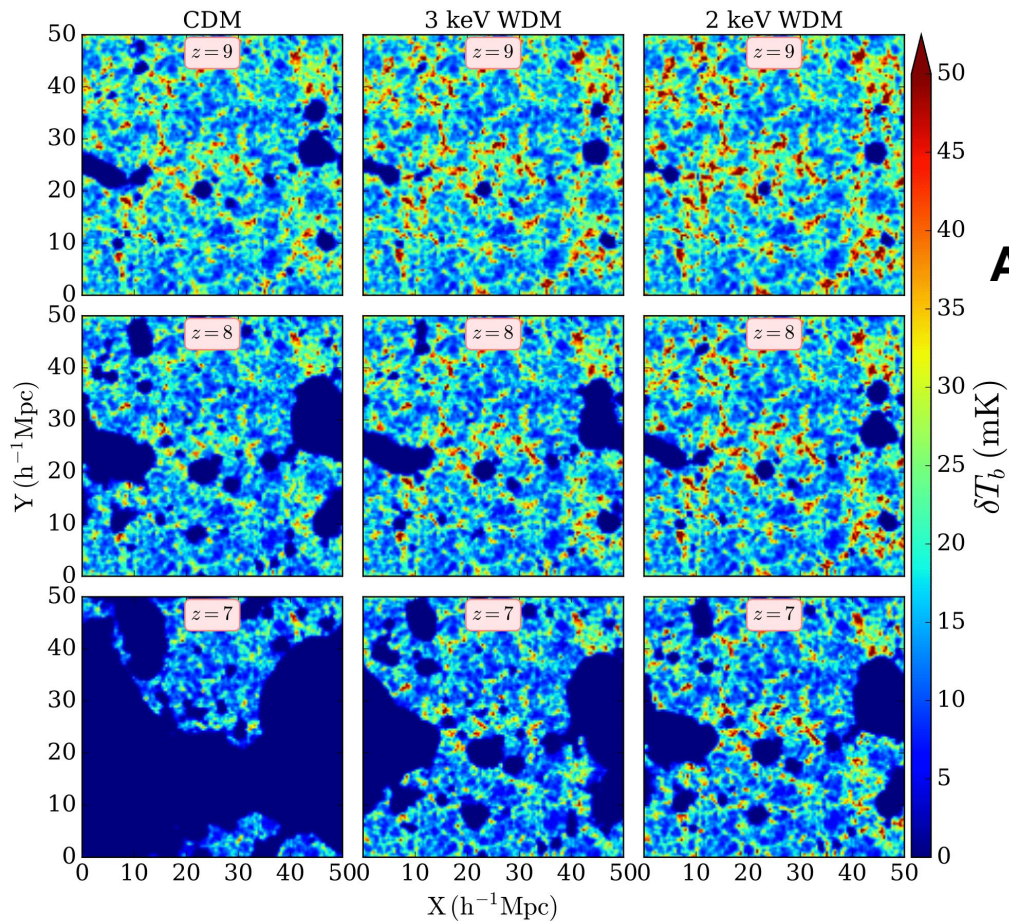
## Interpretation



-ve signal in a +ve background

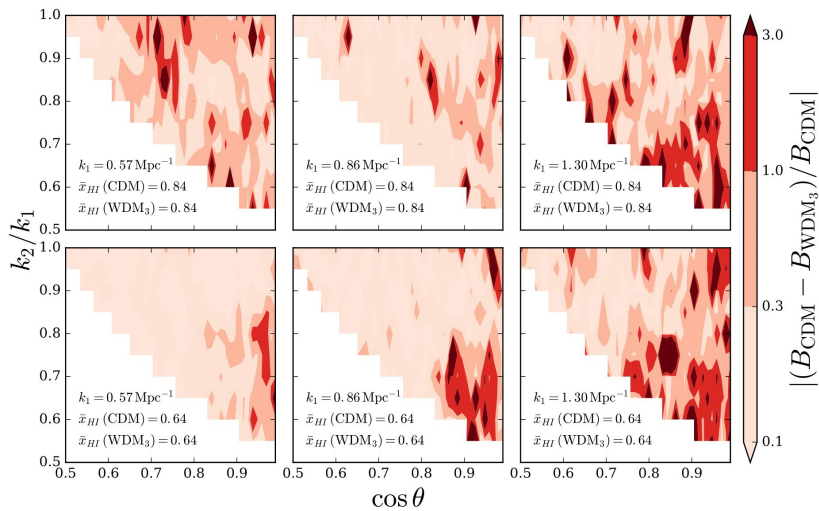
Large scale signal -ve

# Impact of dark matter models on 21-cm signal bispectrum

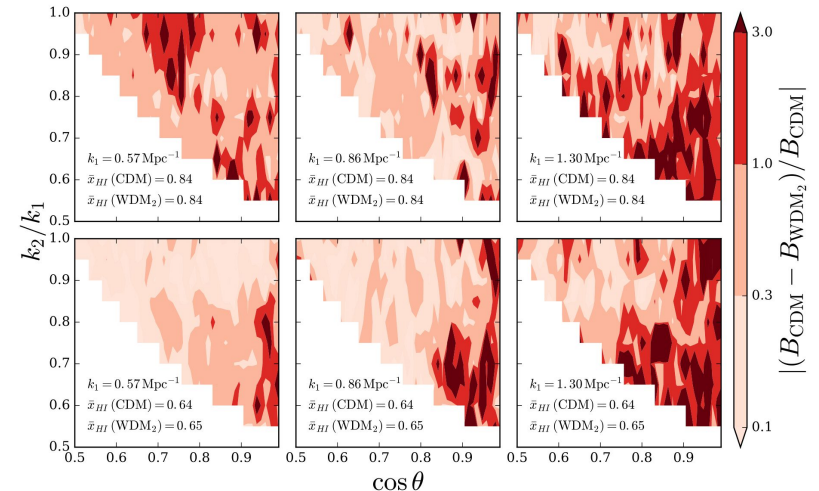


**Astrophysical parameters remain fixed**

Saxena et al. 2020, MNRAS, 497, 2941



**Relative differences between the 21-cm bispectra for WDM and CDM models varies between 10% – 300% for all unique k-triangles**



Saxena et al. 2020, MNRAS, 497, 2941