Nordita Winter School 2026

Cosmological Magnetic Fields:
Generation, Observation, and Modeling

Cosmological Magnetic Fields Theory Exercise Session 2: Conductivity in the early Universe

Based on Ghosh, Brandenburg, Caprini, Neronov, Vazza [2510.26918]

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↑

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The conductivity of electrons is determined by the process with the smallest mean free path between collisions

$\tau_{e,C}$ increases with temperature while $\tau_{e,T}$ stays constant $\rightarrow$ at high temperatures $\tau_{e,C} > \tau_{e,T}$ hence Thomson determines the conductivity of electrons

$$\tau_{e,C} = \frac{3m_e^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)} \qquad \tau_{e,T} = \frac{3m_e^2}{8\pi\, e^4\, n_\gamma}$$

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B) Above which temperature does Thomson determine the conductivity of the electrons?

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C) Which process determines the conductivity of the protons at high temperatures?

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Consider temperatures above recombination and below the electron mass $T_{rec} < T < m_e$

$T_{rec} \simeq 0.32\ eV$, $m_e \simeq 0.511\ MeV$

In this temperature range we have $T_b \simeq T_\gamma \equiv T$ and $X_e(T) \approx 1$

Moreover we have $\ln \Lambda_c \approx 30$, $\frac{n_b}{n_\gamma} \equiv \eta_b \approx 6 \times 10^{-10}$

C) Which process determines the conductivity of the protons at high temperatures?

Thomson, for the same reason as for the electrons

$$\tau_{e,C} = \frac{3 m_e^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)}$$

$$\tau_{p,C} = \frac{3 m_p^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)}$$

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D) Above which temperature does Thomson determine the conductivity of the protons?

$$\tau_{e,C} = \frac{3m_e^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)}$$

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When $\tau_{p,C} > \tau_{p,T}$

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$m_p \simeq 938\ MeV$

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$m_p \simeq 938\ MeV$

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E) At high temperature which between electrons and protons give the dominant contribution to the total conductivity?

$$\tau_{e,C} = \frac{3 m_e^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)} \qquad \tau_{e,T} = \frac{3 m_e^2}{8\pi\, e^4\, n_\gamma}$$

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The specie with largest τ/m

$$\tau_{e,C} = \frac{3m_e^{1/2}\ T_b^{3/2}}{4\sqrt{2\pi}\ e^4 \ln \Lambda_c\, n_b X_e(T)}$$

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The specie with largest τ/m → both determined by Thomson at high temperatures

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The specie with largest τ/m → both determined by Thomson at high temperatures $[\tau_{p,T}/m_p]/[\tau_{e,T}/m_e] = m_p/m_e$

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$\rightarrow$ protons dominate at high temperature

$$\tau_{e,C} = \frac{3 m_e^{1/2}\ T_b^{3/2}}{4\sqrt{2\pi}\ e^4 \ln \Lambda_c\ n_b X_e(T)}$$

$$\tau_{p,C} = \frac{3 m_p^{1/2}\ T_b^{3/2}}{4\sqrt{2\pi}\ e^4 \ln \Lambda_c\ n_b X_e(T)}$$

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F) Consider the temperature range in which the protons conductivity is determined by Coulomb, while the electrons conductivity is determined by Thomson. Below which temperature do the electrons dominate?

$$\tau_{e,C} = \frac{3m_e^{1/2}\ T_b^{3/2}}{4\sqrt{2\pi}\ e^4 \ln \Lambda_c\, n_b X_e(T)}$$

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F) Consider the temperature range in which the protons conductivity is determined by Coulomb, while the electrons conductivity is determined by Thomson. Below which temperature do the electrons dominate?

When $\frac{\tau_{e,T}}{m_e} > \frac{\tau_{p,C}}{m_p}$

$$\tau_{e,C} = \frac{3m_e^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)}$$

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$$\tau_{e,T} = \frac{3m_e^2}{8\pi\, e^4\, n_\gamma}$$

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$$\tau_{e,C} = \frac{3m_e^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)} \qquad \tau_{e,T} = \frac{3m_e^2}{8\pi\, e^4\, n_\gamma}$$

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When $\frac{\tau_{e,T}}{m_e} > \frac{\tau_{p,C}}{m_p} \rightarrow \frac{3m_e}{8\pi\, e^4\, n_\gamma} > \frac{3m_p^{-1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)} \rightarrow T < \left(\frac{\ln \Lambda_c}{\sqrt{2\pi}} \eta_b X_e \frac{m_e}{m_p}\right)^{2/3} m_p \simeq 23\ eV$

$$\tau_{e,C} = \frac{3m_e^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)} \qquad \tau_{e,T} = \frac{3m_e^2}{8\pi\, e^4\, n_\gamma}$$

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Exercise no. 2

Consider temperatures above recombination and below the electron mass

$$T_{rec} < T < m_e$$

$\simeq 0.32\ eV$ $\simeq 0.511\ MeV$

In this temperature range we have $T_b \simeq T_\gamma \equiv T$ and $X_e(T) \approx 1$

Moreover we have $\ln \Lambda_c \approx 30$, $\frac{n_b}{n_\gamma} \equiv \eta_b \approx 6 \times 10^{-10}$

A) Create a plot of the evolution of the total conductivity σ with the temperature for $T_{rec} < T < m_e$

$$\tau_{e,C} = \frac{3 m_e^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)}$$

$$\tau_{p,C} = \frac{3 m_p^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)}$$

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A) Create a plot of the evolution of the total conductivity σ with the temperature for $T_{rec} < T < m_e$

B) Identify in the plot the temperature regimes in which the conductivity is dominated respectively by:
Thomson for the protons; Coulomb for the protons; Thomson for the electrons; Coulomb for the electrons .

$$\tau_{e,C} = \frac{3 m_e^{1/2}\, T_b^{3/2}}{4\sqrt{2\pi}\, e^4 \ln \Lambda_c\, n_b X_e(T)} \qquad \tau_{e,T} = \frac{3 m_e^2}{8\pi\, e^4\, n_\gamma}$$

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