Nordita Winter School 2026

Cosmological Magnetic Fields:
Generation, Observation, and Modeling

Cosmological Magnetic Fields Theory
Exercise Session 4:
Evolution of magnetic and velocity fields

UNIVERSITÉ DE GENÈVE

Swiss National Science Foundation

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Evolution of magnetic and velocity fields

Magnetic field spectra generated during radiation domination have the following large scale behavior

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$$\frac{\langle B^2 \rangle}{2} = \frac{1}{2\pi^2}\int_0^{k_*} dk\, k^2\, P_B(k)$$

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Evolution of magnetic and velocity fields

By computing the energy on a scale λ

$$B_\lambda^2 = \frac{1}{\pi^2}\int_0^\infty dk\ \ k^2\, P_B(k)\, e^{-k^2\lambda^2} = \frac{n_B+3}{2(2\pi)^{n_B+3}}\,\Gamma\left(\frac{n_B+3}{2}\right)\langle B^2\rangle\left(\frac{L}{\lambda}\right)^{n_B+3}$$

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$$B_\lambda^2\, \lambda^{n_B+3} \sim \langle B^2\rangle\, L^{n_B+3} \quad \to \langle\, B^2\,\rangle \sim L^{-(n_B+3)} \quad \to L \sim \langle\, B^2\,\rangle^{-\frac{1}{n_B+3}}$$

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Same applies to the magnetic field (from equipartition in the inertial range) $\rightarrow \langle B^2\rangle \sim \eta^{-\frac{2(n_V+3)}{n_V+5}}$

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$\rightarrow \langle v^2 \rangle \sim \langle B^2 \rangle \sim \eta^{-2p}$ $\quad p = \dfrac{n_V + 3}{n_V + 5}$ $\quad q = \dfrac{2}{n_V + 5}$ $\quad p + q = 1$

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At large scales we have

$$P_V\left(k < \frac{2\pi}{L}\right) \sim \langle v^2 \rangle L^{3+n_V} \sim \eta^{-2p} \eta^{(3+n_V)q} \sim \eta^0$$

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A) Consider fully vortical turbulence. How do $\langle v^2 \rangle, \langle B^2 \rangle, L$ scale with time?

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A) Consider fully vortical turbulence. How do $\langle v^2 \rangle, \langle B^2 \rangle, L$ scale with time?

$$n_V = 2 \rightarrow p = \frac{5}{7}, q = \frac{2}{7} \qquad \rightarrow \langle v^2 \rangle \sim \langle B^2 \rangle \sim \eta^{-10/7} \qquad \rightarrow L \sim \eta^{2/7}$$

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B) What happens to the velocity and magnetic field spectra at large scales?

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Since $n_B = n_V$ they are both constant

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C) Consider now the mixed case. How do $\langle v^2 \rangle, \langle B^2 \rangle, L$ scale with time?

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$n_V = 0 \rightarrow p = \frac{3}{5}$ and $q = \frac{2}{5}$ $\quad \rightarrow \langle v^2 \rangle \sim \langle B^2 \rangle \sim \eta^{-6/5} \qquad \rightarrow L \sim \eta^{2/5}$

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D) What happens to the velocity and magnetic field spectra at large scales?

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D) What happens to the velocity and magnetic field spectra at large scales?

P_V stays constant while P_B grows as $\eta^{4/5}$ → INVERSE CASCADE

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In the scaling laws n_V has to be replaced by $n_V^{equiv} = -2$

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$$p = \frac{1}{3} \qquad q = \frac{2}{3}$$

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$$p = \frac{1}{3} \qquad q = \frac{2}{3}$$

In the scaling laws n_V has to be replaced by $n_V^{equiv} = -2$

$$P_V\left(k < \frac{2\pi}{L}\right) \sim \langle v^2 \rangle L^{3+n_V} \sim \eta^{\frac{2\left(n_V - n_V^{equiv}\right)}{n_V^{equiv}+5}} \sim \eta^{\frac{2(n_V+2)}{3}}$$

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Exercise no. 1

Let us now consider the helical case. If helicity is conserved we have

$$h = \langle A \cdot B \rangle \sim B^2 L \sim const \qquad \rightarrow -2p + q = 0$$

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E) What happens to the velocity and magnetic spectra at large scales in the fully vortical case?

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They both grow as $\eta^{8/3}$

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E) What happens instead in the mixed case case?

The magnetic spectrum grows as $\eta^{8/3}$ while the velocity spectrum as $\eta^{4/3}$

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Summary

In the non-helical fully vortical case we have

$$\rightarrow \langle v^2 \rangle \sim \langle B^2 \rangle \sim \eta^{-10/7} \qquad \rightarrow L \sim \eta^{2/7}$$

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@ large scales

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For the 4 cases make a plot of the time evolution of P_B and P_V considering

$$P_B(k) = k^{n_B} \frac{n_B + 3}{8\pi(2\pi)^{n_B}} \langle B^2 \rangle \, L^{3+n_B} / (1 + \mathrm{k\,L})^{\mathrm{n_B + m_B}}$$

with $m_B = 11/3$