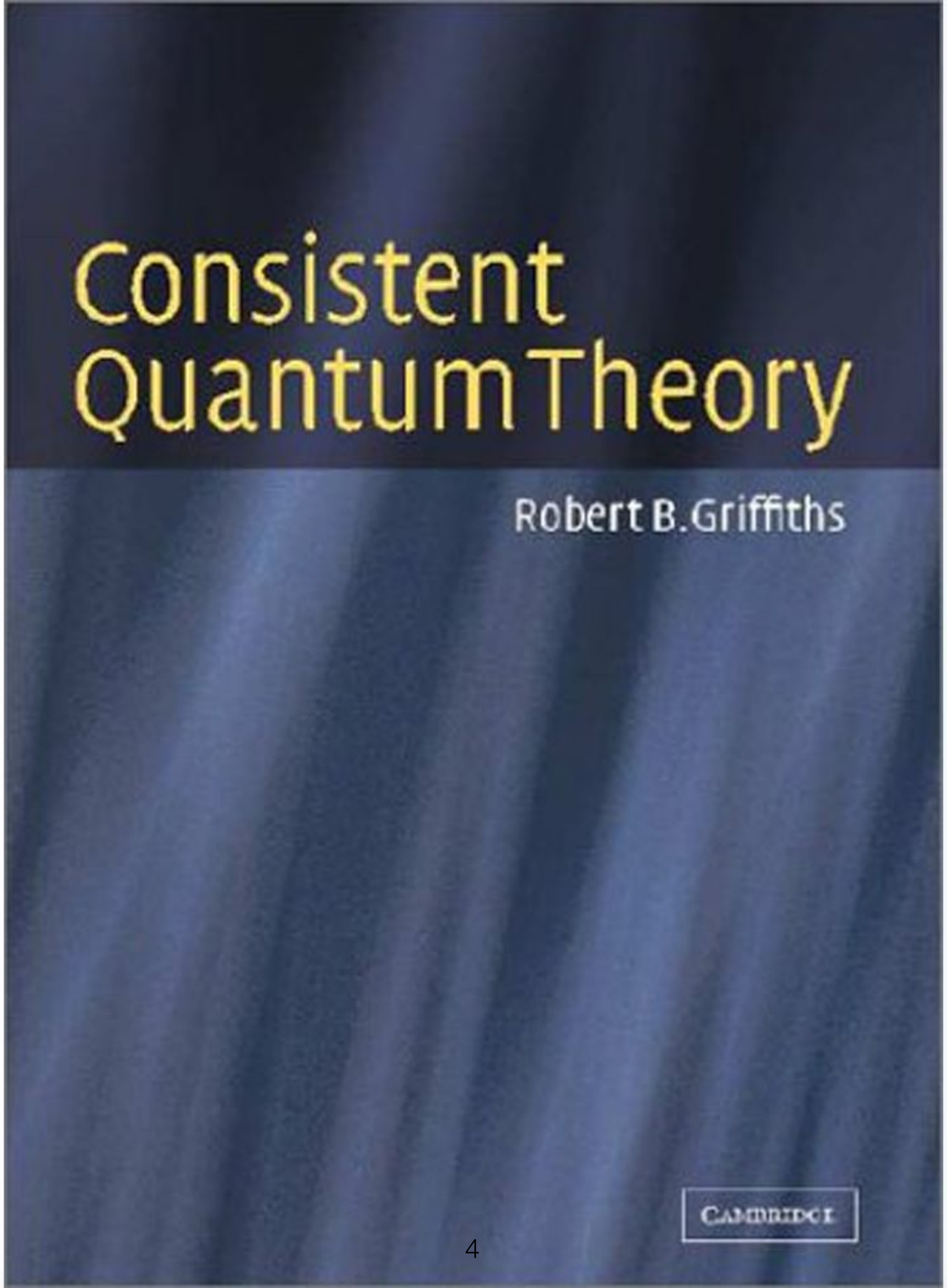
Consistent Histories

Finding the World in Quantum Theory

https://plato.stanford.edu/entries/qm-consistent-histories/

The consistent histories, also known as decoherent histories, approach to quantum interpretation is broadly compatible with standard quantum mechanics as found in textbooks. However, the concept of *measurement* by which probabilities are introduced in standard quantum theory no longer plays a fundamental role. Instead, *all* quantum time dependence is probabilistic (stochastic), with probabilities given by the Born rule or its extensions. By requiring that the description of a quantum system be carried out using a well-defined probabilistic sample space (called a “framework”) this approach resolves all the well-known quantum paradoxes of quantum foundations.

An important philosophical implication is the lack of a single universally-true state of affairs at each instant of time.


Consistent
Quantum Theory
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(1) Basics

Definitions and the Born Rule

Properties are defined by projection operators - the quantum analogue of regions in phase space.

Properties implemented by operators that commute are said to be compatible. For those, we can do logical operations.

$$Q \text{ AND } R \;\equiv\; Q \wedge R \;=\; QR$$

$$\begin{aligned} Q \text{ OR } R &= \text{NOT}\Big((\text{NOT}\, Q) \text{ AND } (\text{NOT}\, R)\Big) \\ &= 1 - \Big((1-Q)(1-R)\Big) \;=\; Q + R - QR \end{aligned}$$

$$\mathscr{H} = H_{t_f} \otimes \ldots \otimes H_{t_1} \otimes H_{t_i}$$

History Hilbert space

$$Y = F_{t_f} \otimes \ldots \otimes F_{t_1} \otimes F_{t_i}$$

History trajectory

$$\begin{aligned} T(t_b, t_a)^{-1} &= T(t_b, t_a)^{\dagger} = T(t_a, t_b) \\ T(t_c, t_b)T(t_b, t_a) &= T(t_c, t_a) \\ T(t_a, t_a) &= 1 \end{aligned}$$

Dynamics

$$K(Y^\alpha) = F^\alpha_{t_f} T(t_f, t_{f-1}) F^\alpha_{t_{f-1}} \ldots T(t_2, t_1) F^\alpha_{t_1} T(t_1, t_i) F^\alpha_{t_i}$$

$$W(Y^\alpha) = \mathrm{Tr}\, K(Y^\alpha)^\dagger K(Y^\alpha)$$

Historical weight (Probability)

These definitions and postulate generalize, and are inspired by:

$$\begin{aligned} Y^\alpha &= P_\alpha \otimes [\psi] \\ Y^0 &= 1 \otimes (1 - [\psi]) \end{aligned}$$

$$\text{Pr} = \| \, P T(t_1, t_0) \, | \psi \rangle \, \|^2$$

$$\text{Pr} = \langle \psi | \, T(t_1, t_0)^\dagger P T(t_1, t_0) \, | \psi \rangle = \text{Tr} \left(T(t_1, t_0)^\dagger P T(t_1, t_0) [\psi] \right)$$

(2) “Consistency”

Motivation, Definition, Example

$$Y_\pi = \sum_\alpha \pi_\alpha Y^\alpha$$

$$W(Y_\pi) \stackrel{?}{=} \sum \pi_\alpha W(Y^\alpha)$$

$$W(Y_\pi) = \sum_\beta \sum_\alpha \pi_\beta \pi_\alpha \,\mathrm{Tr}\, K(Y_\beta)^\dagger K(Y_\alpha)$$

$$\mathrm{Tr}\, K(Y_\beta)^\dagger K(Y_\alpha) = 0 \quad \text{for}\ \alpha \neq \beta$$

Consistent histories condition

Example of inconsistency

$$
\begin{aligned}
Y^0 &= [z^-] \otimes 1 \otimes 1 \\
Y^1 &= [z^+] \otimes [x^+] \otimes [z^+] \\
Y^2 &= [z^+] \otimes [x^+] \otimes [z^-] \\
Y^3 &= [z^+] \otimes [x^-] \otimes [z^+] \\
Y^4 &= [z^+] \otimes [x^-] \otimes [z^-]
\end{aligned}
$$

$$[z^+] = \frac{1+\sigma_3}{2} \quad \text{etc}\,.$$

$$
\begin{aligned}
\mathrm{Tr}\, K(Y^1)^\dagger K(Y^3) &= \mathrm{Tr}\frac{1+\sigma_3}{2}\frac{1+\sigma_1}{2}\frac{1+\sigma_3}{2}\frac{1-\sigma_1}{2}\frac{1+\sigma_3}{2} \\
&= \frac{1}{16}\mathrm{Tr}(1+\sigma_3+\sigma_1+i\sigma_2)(1+\sigma_3-\sigma_1-i\sigma_2) \\
&= \frac{1}{4} \neq 0
\end{aligned}
$$

(3) Frameworks

The World(s?) in Many Worlds

The *single framework rule* says that a specific quantum probabilistic model employs just one sample space and its associated event algebra, and in particular two incompatible frameworks *cannot be combined*. This rule, together with its extension to quantum dynamics, see subsection 4.2, is a central principle of histories quantum mechanics. It provides a guide for quantum reasoning that prevents one from falling into paradoxes; conversely, most quantum paradoxes are constructed by violating the single framework rule in some way.

Meta-theory

(R1) Liberty. The physicist is free to employ as many frameworks as desired when constructing descriptions of a particular quantum system, provided the principle R3 below is strictly observed.

(R2) Equality. No framework is more fundamental than any other; in particular, there is no “true” framework, no framework that is “singled out by nature”.

(R3) Incompatibility. The single framework rule: incompatible frameworks are *never* to be combined into a single quantum description. The (probabilistic) reasoning process starting from assumptions (or data) and leading to conclusions must be carried out using a *single* framework.

(R4) Utility. Some frameworks are more useful than others for answering particular questions about a quantum system.

(4) Emergence (?)

Must Frameworks be Imposed “From the Outside”?