

# FROM THE DELAY OF THE PHOTOELECTRON WAVEPACKET TO THE RECONSTRUCTION OF THE DENSITY MATRIX

Eva Lindroth

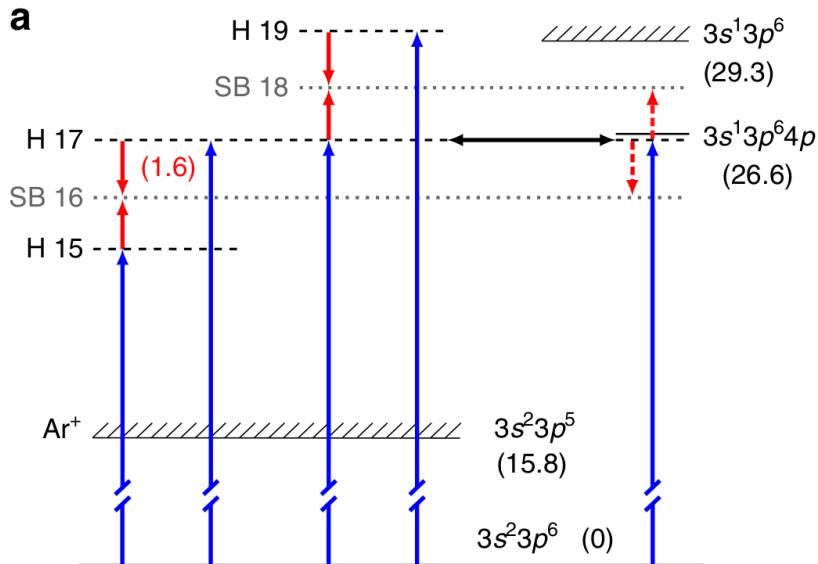
Stockholm University

Quantum Connections, 20 June 2024

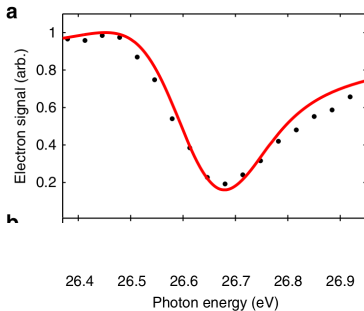
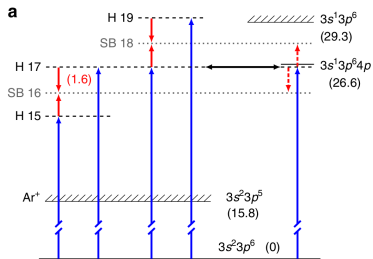


- Resonances: When the sideband symmetry breaks.
- the Kraken method - check the relation between the two-photon measurement and the desired one-photon information.

# RABBIT OVER A RESONANCE: ARGON EXAMPLE

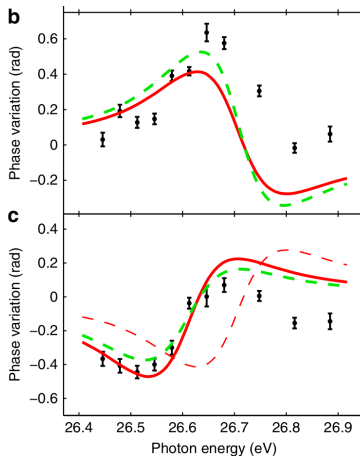
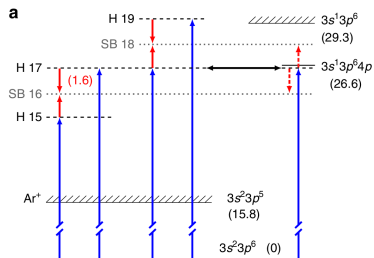


# RABBIT OVER A RESONANCE: ARGON EXAMPLE



Kotur et al DOI:  
10.1038/ncomms10566

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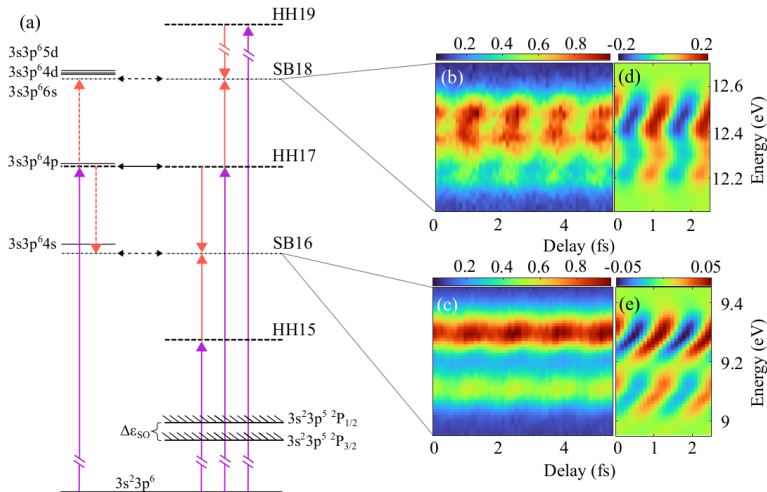
SB 16

SB 18

Kotur et al DOI:  
10.1038/ncomms10566

# RABBIT OVER A RESONANCE: ARGON EXAMPLE

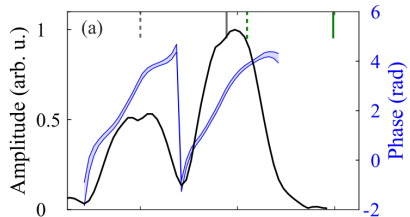
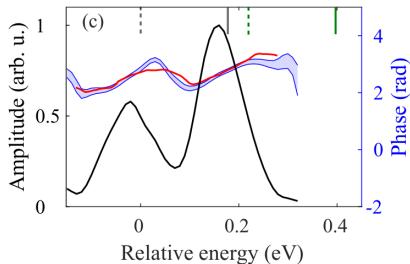
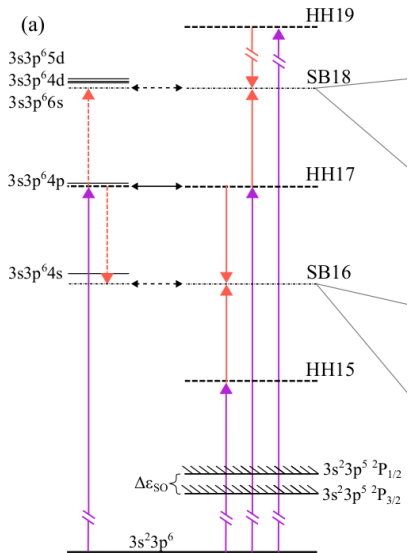
RAINBOW RABBIT TECHNIQUE, NARROW BANDWIDTH PROBE (10 NMM)



Luo et al, to be published

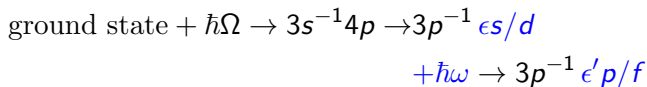
# RABBIT OVER A RESONANCE: ARGON EXAMPLE

RAINBOW RABBIT TECHNIQUE, NARROW BANDWIDTH PROBE (10 NMM)

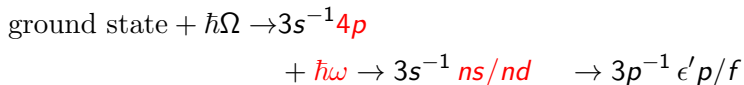


# WHY IS THE SIDEBAND SYMMETRY BROKEN?

- The “usual” **continuum-continuum** path for the 2nd photon:



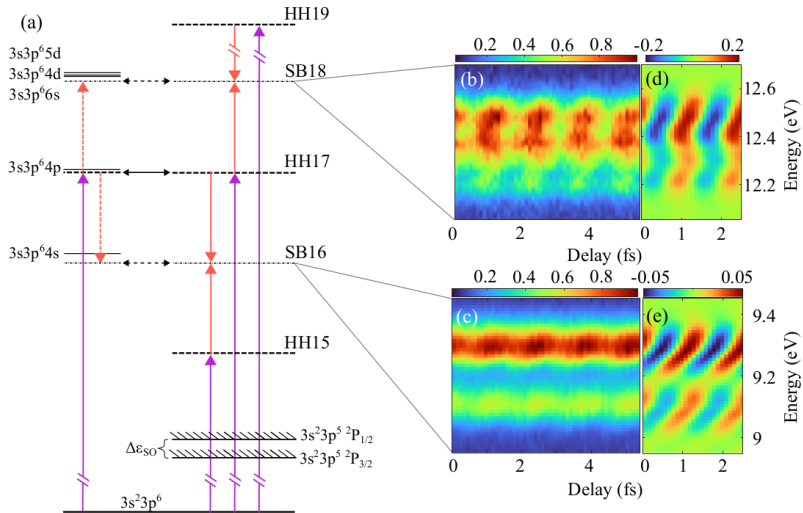
- The “**bound-bound**” path:



- does not have to be “on-shell”

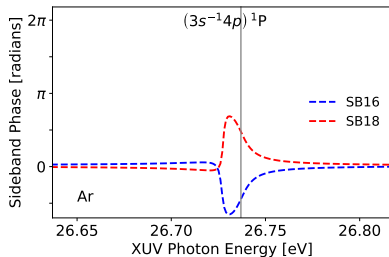


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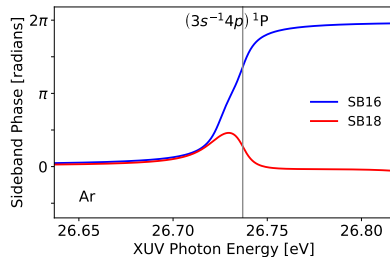
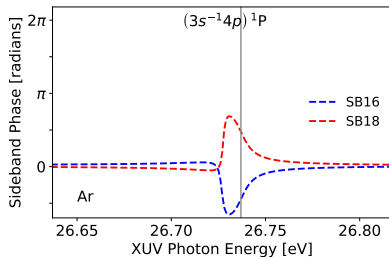
Luo et al, to be published

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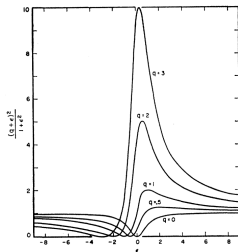


- The “usual” continuum-continuum path for the 2nd photon

- + the “bound-bound” part

# WHY IS THE SIDEBAND SYMMETRY BROKEN?

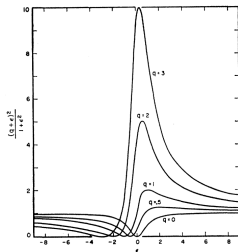
$$\sigma = \sigma_0 \frac{(\epsilon + q)^2}{1 + \epsilon^2} + \sigma_{bg}$$
$$\epsilon = \frac{E - E_r}{\Gamma/2}$$



- Fano PR 124, 1866 (1961)
- Asymmetric line profiles quantified by  $q$

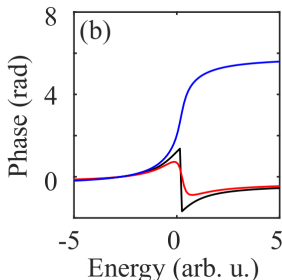
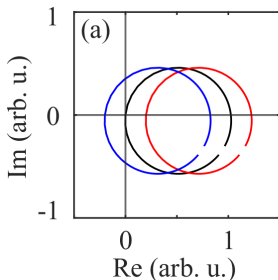
# WHY IS THE SIDEBAND SYMMETRY BROKEN?

$$\sigma = \sigma_0 \frac{(\epsilon + q)^2}{1 + \epsilon^2} + \sigma_{bg}$$
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- Fano PR 124, 1866 (1961)
- Asymmetric line profiles quantified by  $q$

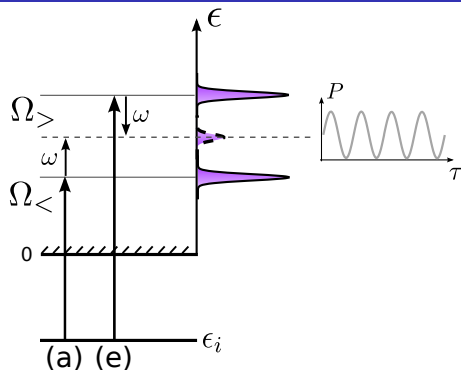
$$A(\epsilon) = A_0 \frac{\epsilon + q}{\epsilon + i} + A_{bg}$$



- Resonances: When the sideband symmetry breaks.
- the Kraken method - check the relation between the two-photon measurement and the desired one-photon information.

# PHOTOIONIZATION DELAY

CALCULATIONS CAN DISENTANGLE THE CONTRIBUTION FROM THE SECOND PHOTON



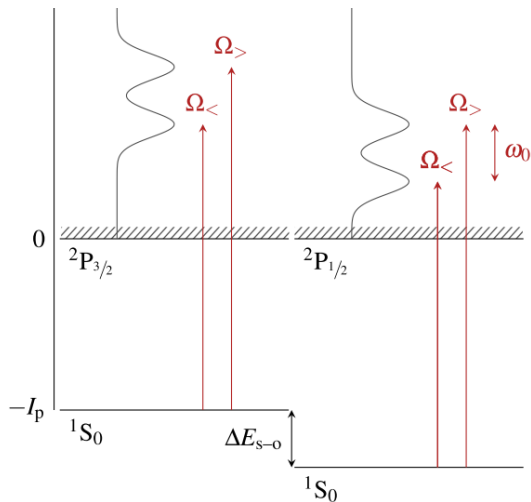
$$\begin{aligned} e^{i\eta_{emi}} e^{-i\eta_{abs}} &= \\ e^{i2\omega(\eta_{emi}-\eta_{abs})/2\omega} &= \\ e^{i2\omega(\hbar\Delta\eta/\Delta E)} & \\ \rightarrow e^{i2\omega\tau_A} & \end{aligned}$$

Laser-induced sideband signal:

$$P \sim |M_{abs\omega} + M_{emi\omega}|^2 \sim A + B \cos[2\omega(\tau - \tau_{GD} - \tau_A)]$$

where  $\tau_{GD} \approx (\phi_> - \phi_<)/2\omega$  is the group delay of the attopulse

# BUT IF WE DO NOT HAVE PURE QUANTUM STATE?



- A **pure quantum state** can be described by a state vector
- But often we have a **mixed state**

Because:

- experimental imperfections
- **unobserved** degrees of freedom

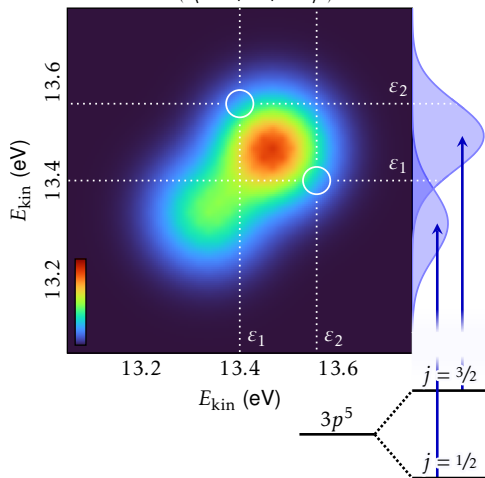
from Carlström et al. J. Phys. B51,015201



# MEASURE THE PHOTOELECTRON

BUT NOT THE IONIC STATES (EXAMPLE: AR)

$$\text{abs}(\langle \epsilon_1 | \hat{\rho} | \epsilon_2 \rangle)$$



The **pure state**:

$$|\Psi_{ion,e^-}\rangle = \sum_j \int d\epsilon c_j(\epsilon) |j, \epsilon\rangle$$

Reduced **density matrix**:

Trace over ionic states,  
and over photoelectron  
angular momenta ( $\ell_j$ ).

$$\hat{\rho}(\epsilon_1, \epsilon_2) = \text{tr}(|\Psi_{ion,e^-}\rangle\langle\Psi_{ion,e^-}|)$$

- Can  $\hat{\rho}$  be reconstructed experimentally?

# KRAKEN

## QUANTUM STATE TOMOGRAPHY

Attempts for continuum electron Quantum State Tomography

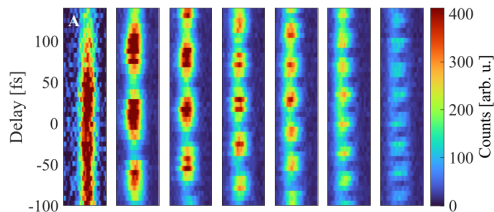
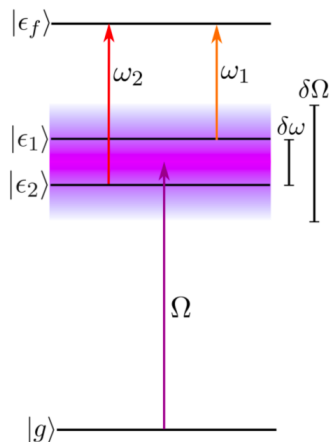
- Mixed-FROG: Bourassin-Bouchet et al, PRX 10, 031048 2020
- SQUIRRELS: Priebe et al, Nature Photonics 11,793, 2017
- **KRAKEN**

**K**vanttilståndets tomografi av **A**ttosekunder **E**lektronvågpaket  
(quantum state tomography of attosecond electron wave packets)

- Is it possible to reconstruct the density matrix?
- recall David Busto's discussion!
- How can theory contribute?



# KRAKEN - THE EXPERIMENT



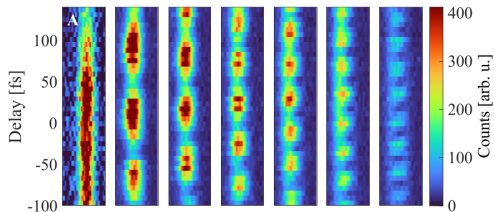
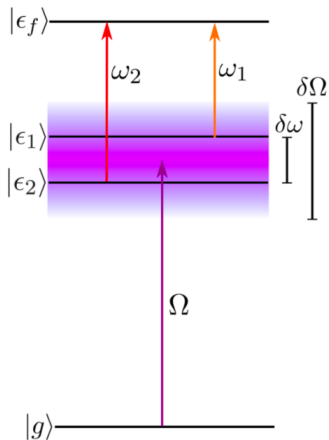
Photoelectron yield oscillates with  $\delta\omega$  when  $\tau$  is varied.  $T = 100$  fs  $\hbar\delta\omega = 41$  meV

- $A_{\delta\omega}(\epsilon_f)$  and  $\phi_{\delta\omega}(\epsilon_f)$  are **extracted**.

Laurell et al arxiv.org:  
2309.13945

$$A_{\delta\omega}(\epsilon_f) \sim |\hat{\rho}(\epsilon_1, \epsilon_2)|$$
$$\phi_{\delta\omega}(\epsilon_f) \sim \arg[\hat{\rho}(\epsilon_1, \epsilon_2)]$$

# KRAKEN - THE EXPERIMENT

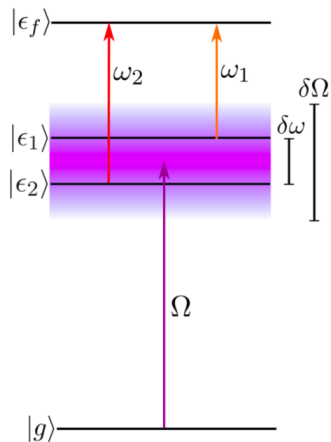


Photoelectron yield oscillates with  $\delta\omega$  when  $\tau$  is varied

- Obtain **one photon information** from a **two photon measurement**.

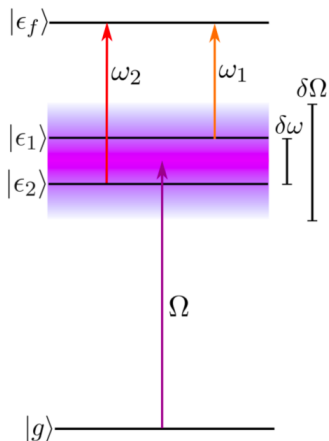
Laurell et al arxiv.org:  
2309.13945

$$A_{\delta\omega}(\epsilon_f) \sim |\hat{\rho}(\epsilon_1, \epsilon_2)|$$
$$\phi_{\delta\omega}(\epsilon_f) \sim \arg[\hat{\rho}(\epsilon_1, \epsilon_2)]$$



- The density matrix after the XUV photon can be **calculated**

$$\hat{\rho}_{XUV}(\epsilon_1, \epsilon_2) \sim \sum_{l, l_j} \left\{ M_{l, \epsilon_1, l_j}^{(1)*} A_{XUV}^*(\Omega = E_l + \epsilon_1) \times M_{l, \epsilon_2, l_j}^{(1)} A_{XUV}(\Omega = E_l + \epsilon_2) \right\}$$



- The experimental reconstruction after the XUV photon can also be **calculated**

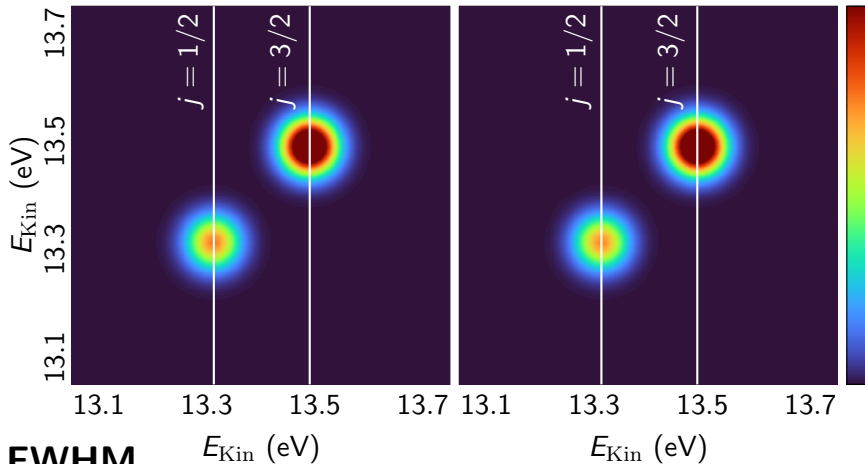
$$\hat{\rho}(\epsilon_1, \epsilon_2) \sim \sum_{l, l_j} \left\{ \left[ \sum_{l'_{j'}} M_{\epsilon_1, l'_{j'} \rightarrow \epsilon_f, l_j}^{(2)} A_{XUV}(\Omega = E_l + \epsilon_1) \right]^* \times \left[ \sum_{l'_{j'}} M_{\epsilon_2, l'_{j'} \rightarrow \epsilon_f, l_j}^{(2)} A_{XUV}(\Omega = E_l + \epsilon_2) \right] \right\}$$

- and compared with  $\hat{\rho}_{XUV}(\epsilon_1, \epsilon_2)$

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



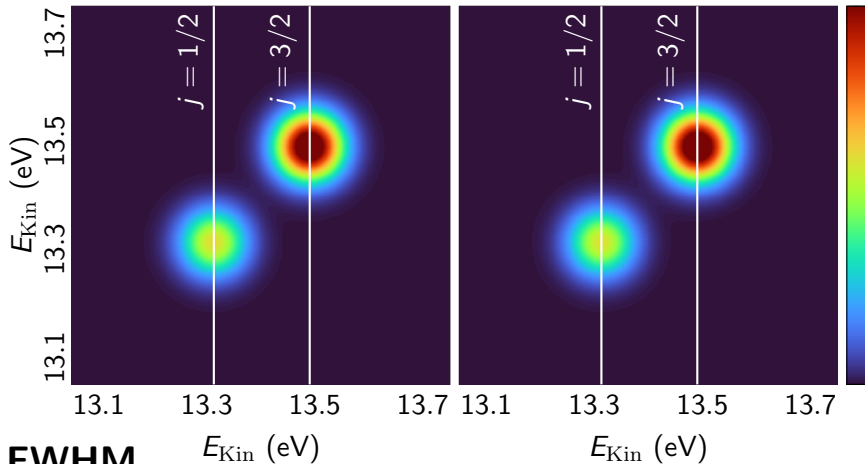
**FWHM**

0.08eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



**FWHM**

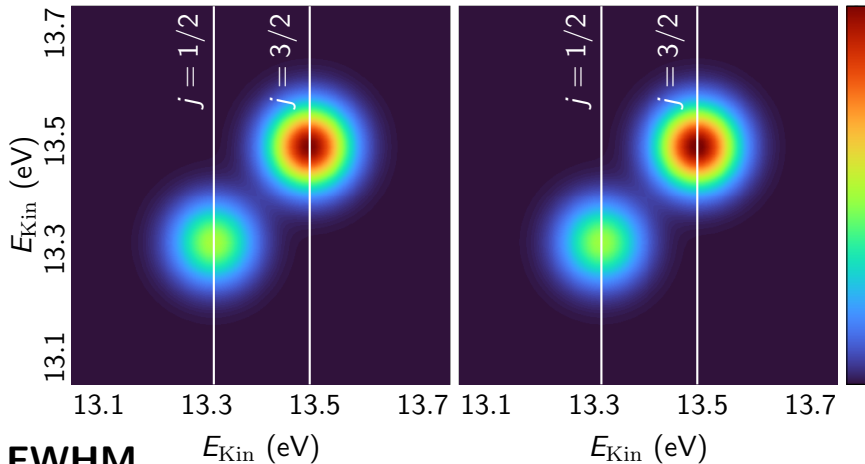
0.10eV



# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



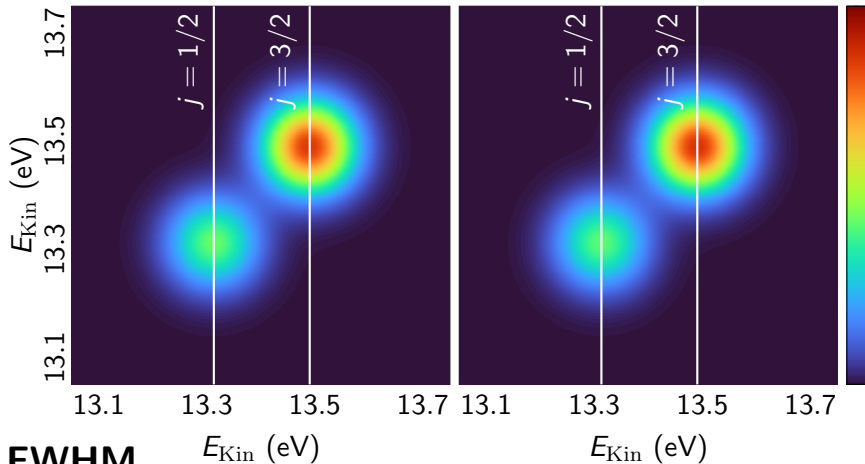
**FWHM**

0.12eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



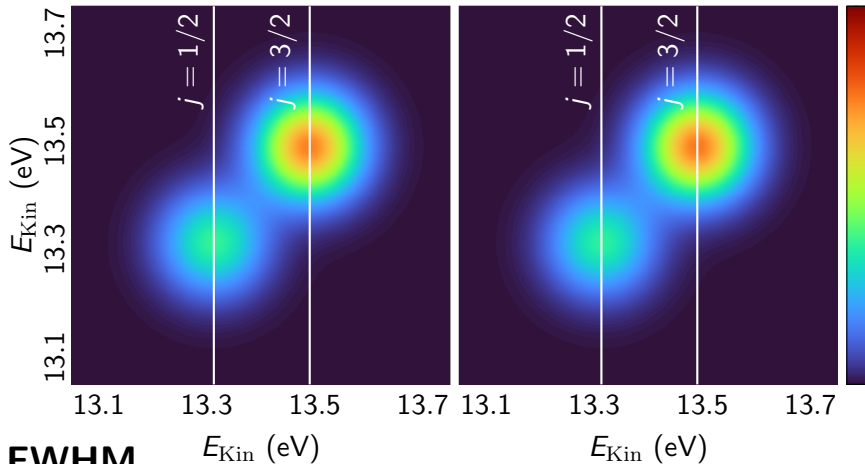
**FWHM**

0.14eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



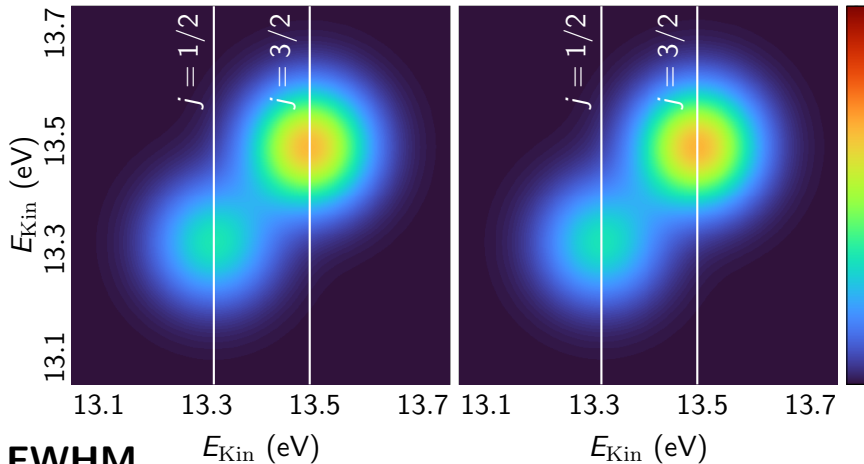
**FWHM**

0.16eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



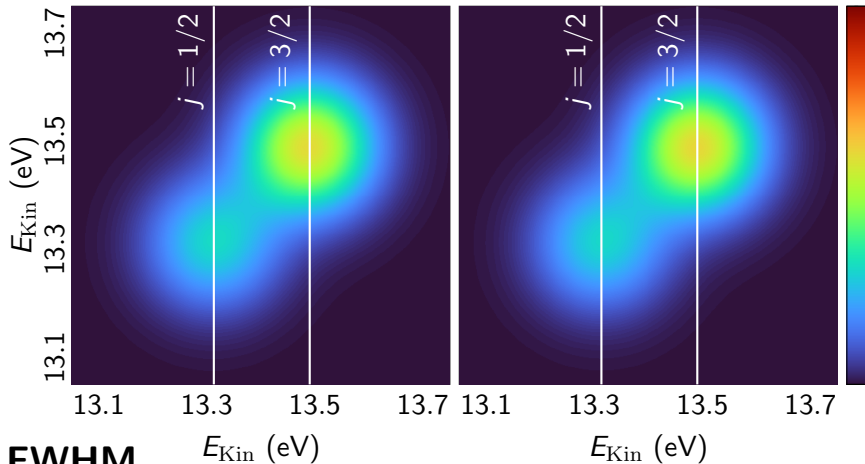
**FWHM**

0.18eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



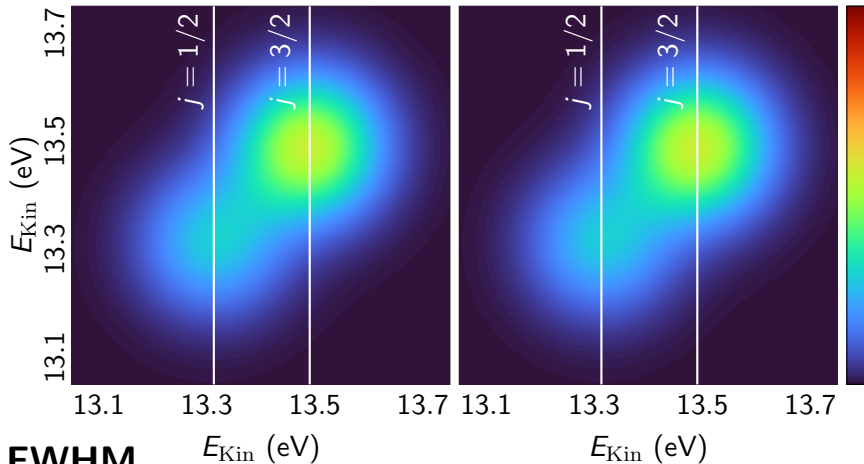
**FWHM**

0.20eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



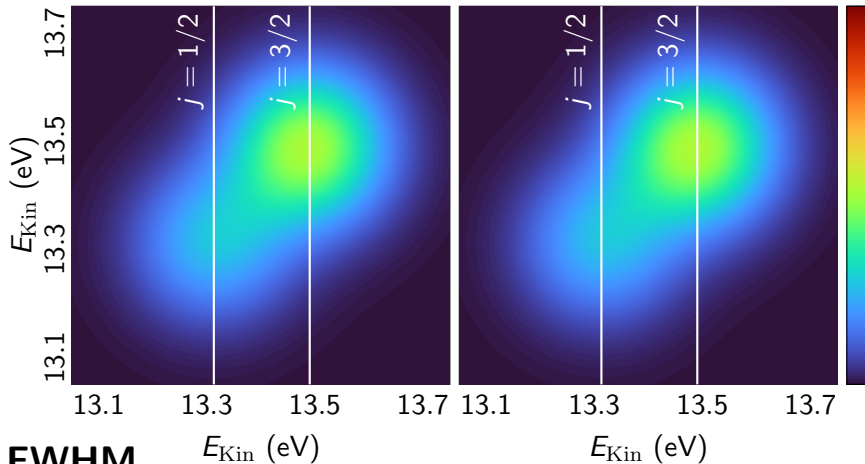
**FWHM**

0.22eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



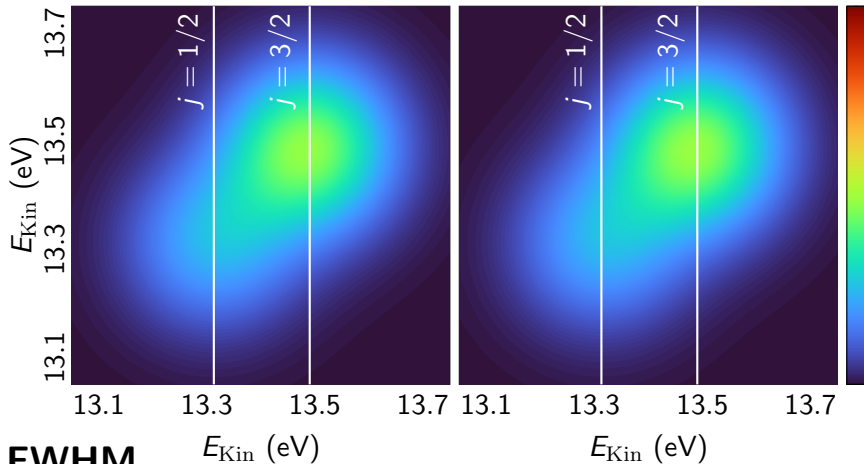
**FWHM**

0.24eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



**FWHM**

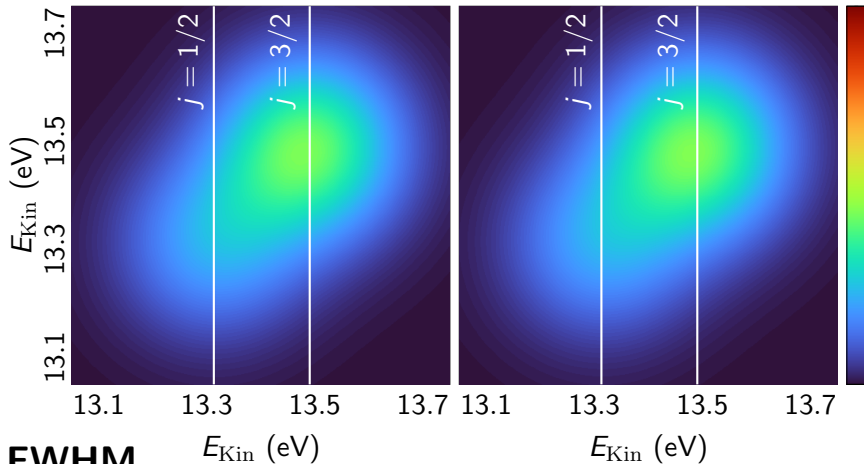
0.26eV



# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



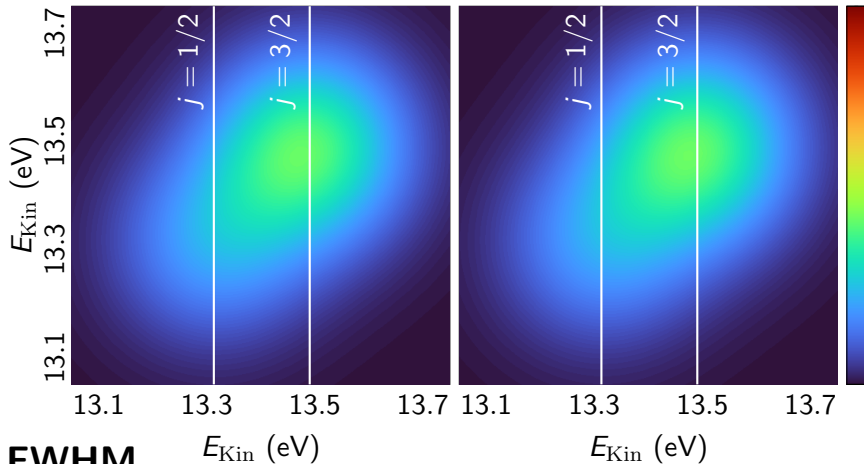
**FWHM**

0.28eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



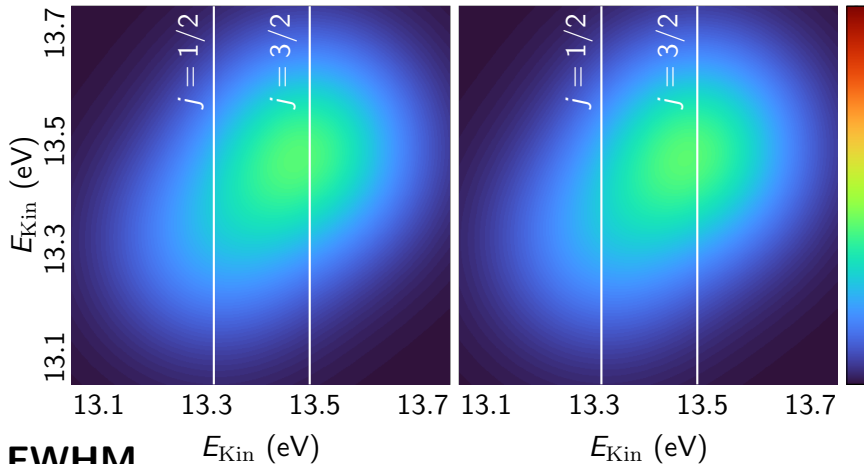
**FWHM**

0.30eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



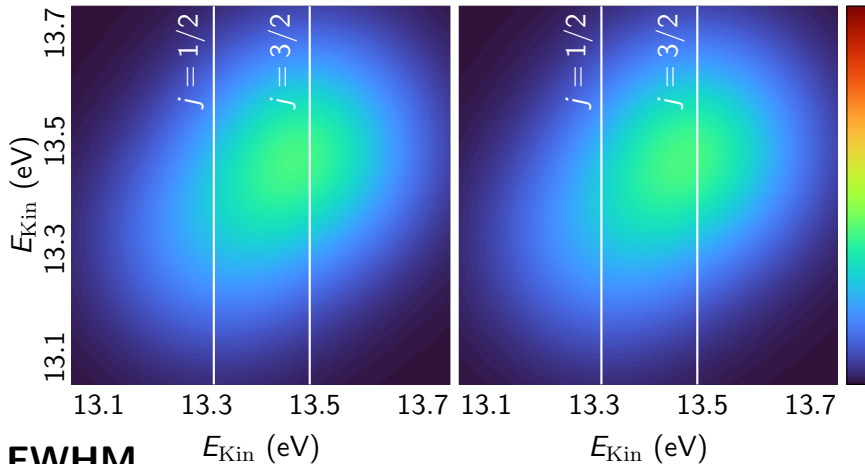
**FWHM**

0.32eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



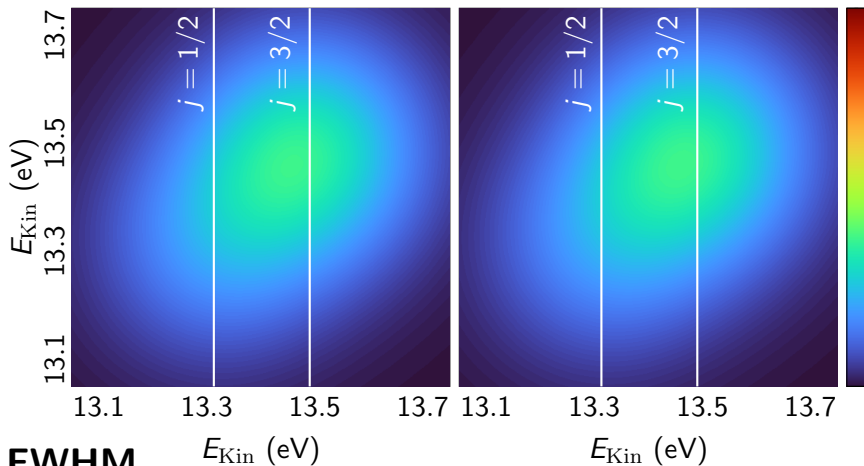
**FWHM**

0.34eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

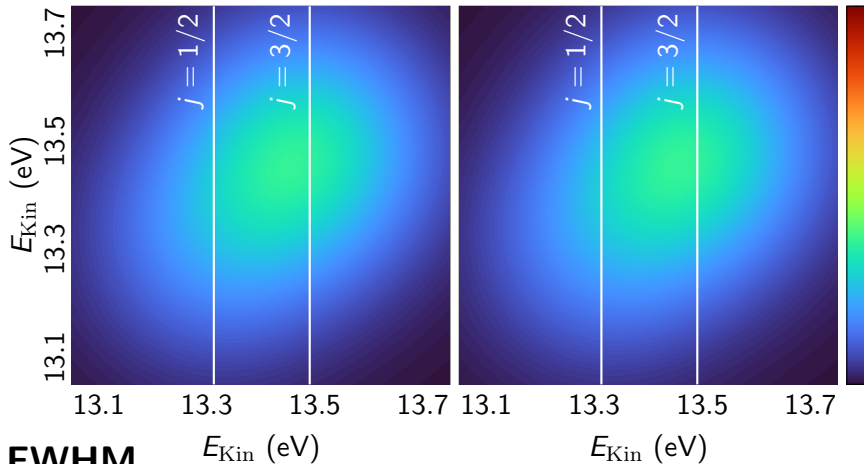
KRAKEN reconstruction



# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

KRAKEN reconstruction



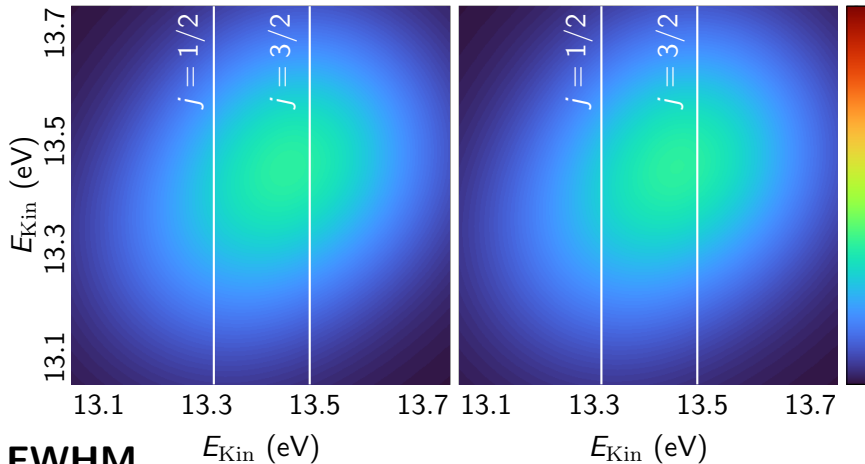
**FWHM**

0.38eV

# THE DENSITY MATRIX BY FWHM ( $A_R$ FS=0.18 eV)

One-photon density matrix

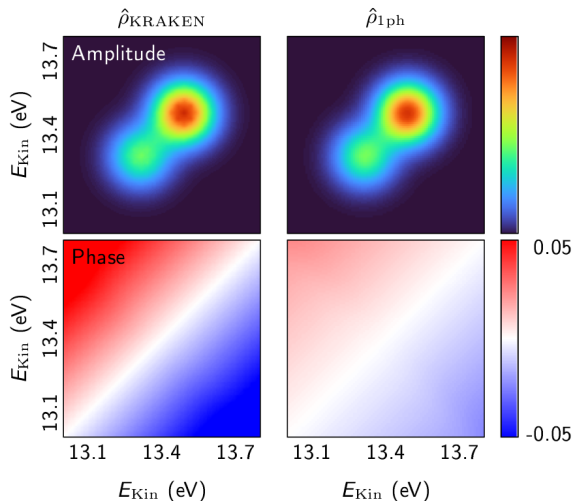
KRAKEN reconstruction



**FWHM**

0.40eV

# KRAKEN RECONSTRUCTION VERSUS ONE PHOTON

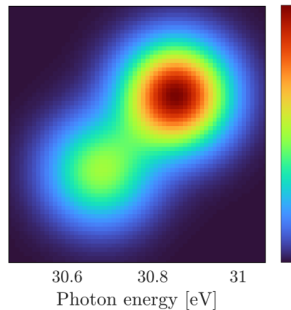
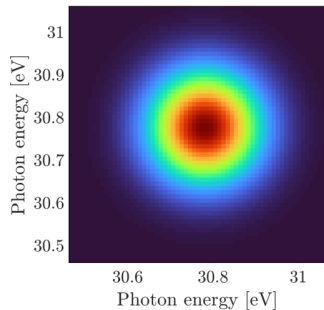
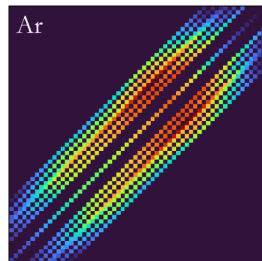
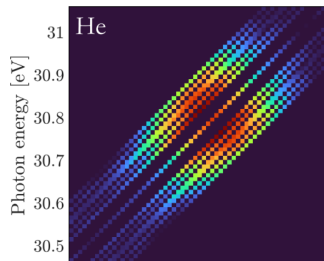


- flat part of the continuum
- 2nd photon contribution nearly constant over the interval
- 2nd photon contribution very similar in the two channels

FWHM 0.14 eV, Argon



# KRAKEN RECONSTRUCTION VERSUS EXPERIMENT



# PURITY

$$\hat{\rho} = A\hat{\rho}_{1/2} + B\hat{\rho}_{3/2}$$

Normalization  $A + B = 1$

Purity:  $\gamma = \text{tr}(\hat{\rho}^2)$

$$\gamma = A^2 + B^2 + 2AB\text{tr}(\hat{\rho}_{1/2}\hat{\rho}_{3/2})$$

- non-rel. limit  $B = 2A$
- $FWHM \ll \Delta_{FS}$   
 $\rightarrow \gamma = A^2 + B^2 = 5/9$

# PURITY

$$\hat{\rho} = A\hat{\rho}_{1/2} + B\hat{\rho}_{3/2}$$

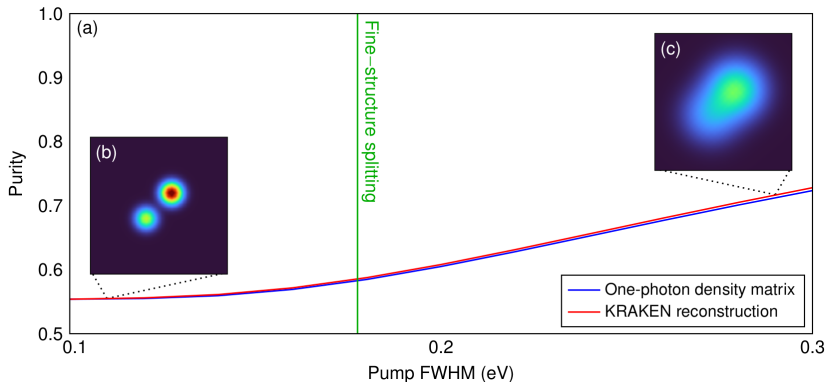
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$$\hat{\rho} = A\hat{\rho}_{1/2} + B\hat{\rho}_{3/2}$$

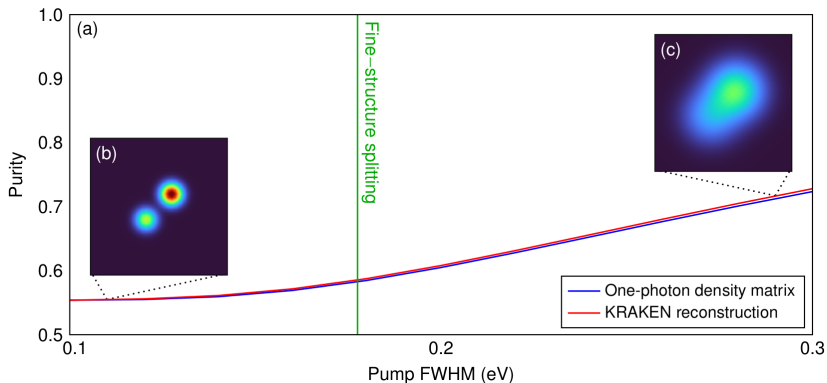
$$\text{Normalization } A + B = 1$$

$$\text{Purity: } \gamma = \text{tr}(\hat{\rho}^2)$$

$$\gamma = A^2 + B^2 + 2AB\text{tr}(\hat{\rho}_{1/2}\hat{\rho}_{3/2})$$

- non-rel. limit  $B = 2A$

- $FWHM \ll \Delta_{FS}$   
 $\rightarrow \gamma = A^2 + B^2 = 5/9$



$$B \approx 1.98 A, B \approx 2.05 A$$

Purity (FWHM 0.2 meV)

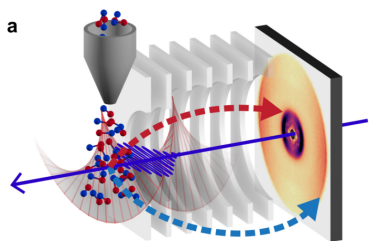
	Helium	Argon
Experiment	$0.94 \pm 0.06$	$0.65 \pm 0.02$
Theory	1.0	0.61

- How can the experimental purity be so high?
- compensate for detector response He purity:  $0.87 \rightarrow 0.94$

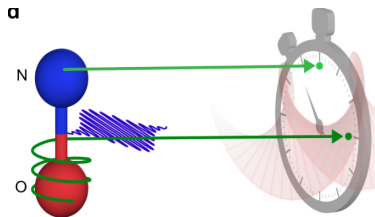
# New Developments and Emerging Possibilities

# X-RAY ATTOSECOND DELAYS

XFEL + IR-STREAKING LASER



T. Driver et al

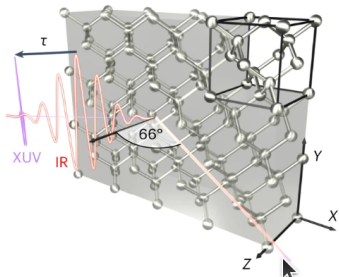


arXiv:2402.12764

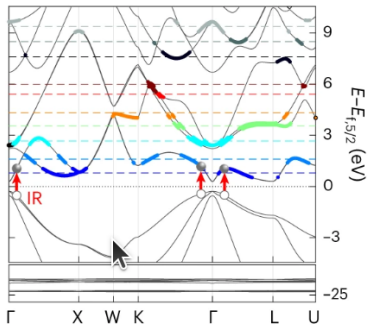
- Free-electron lasers: Higher (X-ray) energies, K-shell ionization
- Angular streaking
- Compare electrons from Oxygen ( $E_B \sim 500$  eV) and Nitrogen ( $E_B \sim 400$  eV) in NO.
- up to 700 as delays in NO near O K-shell threshold

# SOLID STATE

LIGHT PULSES TO MANIPULATE THE ELECTRO-OPTICAL PROPERTIES OF A SOLID



Inzani et al

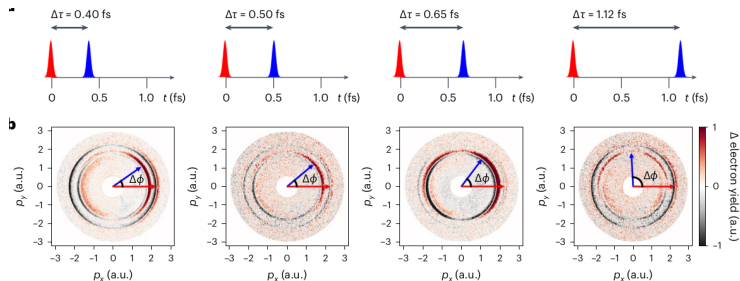


Nature Photonics 17,1059 (2023)

- Light-driven excitation in monocrystalline germanium
- Study of sub cycle response to intense ( $8 \text{ TW}/\text{cm}^{-2}$ ) IR pulse
- Probed by the attosecond pulse
- Ultrafast transient, as well as long-lasting, features observed.



# ATTO SECOND PUMP & PROBE

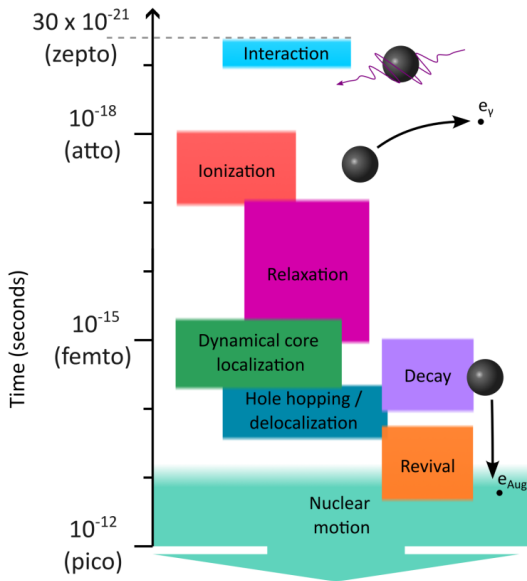


Guo et al, Nature photonics

<https://doi.org/10.1038/s41566-024-01419-w>

- Two FEL pulses 370 eV and 740 eV. FWHM  $\sim 1$  fs
- + Circularly polarized IR -field  $\rightarrow$  Angular Streaking
- maps time to angle
- Looks at Carbon 1s ionization in several molecules.
- Post collisional effects

# THE FUTURE



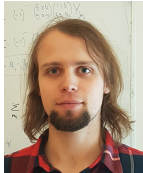
# ACKNOWLEDGEMENTS



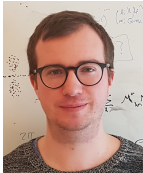
Marcus Dahlström  
**Anne l'Huillier** and her group



Jimmy  
Vinblad



Johan  
Sörngård



Anton  
Ljungdahl



Luca  
Argenti,  
Orlando



Leon  
Pettersson



Soumyajit  
Saha