# Rotation, magnetic fields and superconductivity

**Maxim Chernodub**

Institut Denis Poisson, CNRS, Tours, France
Department of Physics, West University of Timișoara, Romania

**Questions: (i) can we imagine objects with negative moment of inertia?**
**(ii) can we get an electromagnetically superconducting state of the vacuum?**
**(iii) do time-crystalline states exist within the Standard Model?**

**Review of the Barnett polarization/magnetization effects at all scales:**

**non-relativistic: ferromagnetic (iron),**
**electronic (in a metal fluid),**
**nuclear (protons in water)**
**relativistic: hadronic ($\Lambda$ hyperons),**
**quarks and gluons (in quark-gluon plasma)**

**Unusual consequences of vacuum polarization effects due to strong magnetic field.**

Everything rich in fascinating phenomena and, sometimes, very complicated in structure often (and, almost, always) starts from a very simple, basic equation that can often be written in a single line.

**We will discuss Dirac equation, Yang-Mills theory (QCD), Weinberg-Salam model, lattice Wilson action**

# (Very) brief history

## “Rotation, magnetic fields and superconductivity”

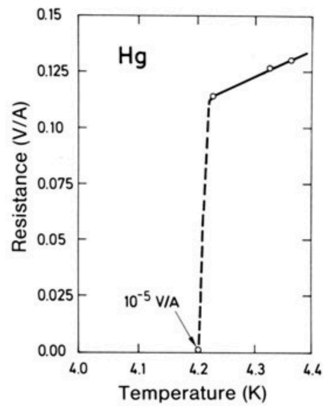
### - Rotation

- Circular motion is one of the simplest mechanical motions observed in Nature, characterized by an object moving along the circumference of a circle.

- Early humans observed the daily rising and setting of the sun, as well as rolling stones downhill, which involved rotational motion.

### - Magnetic field

- The earliest observations of magnetic fields by humans date back to ancient China and Greece, where the natural properties of lodestones ($Fe_3O_4$) were discovered (6th-4th centuries BC). The philosopher Thales of Miletus noted “*lodestones have souls, because they attract iron*”.




### - Superconductivity

- This phenomenon was discovered on Saturday (April 8, 1911) by the Dutch physicist Heike Kamerlingh Onnes. When mercury was cooled to 4.2 K, its electrical resistance abruptly dropped to zero.

**We discuss interplay between these phenomena in certain (extreme) conditions.**

# Motivation(*)

“Rotation, magnetic fields and superconductivity”

**Motivation is catalyzed by puzzles, and puzzles are often revealed by posing provocative (and, seemingly, absurd) questions.**

(*) If you do not understand something, please try to explain the puzzle to someone else. Giving lectures is one of the best ways to achieve this aim.

## - Rotation

The moment of inertia $I$ measures a mechanical resistance of a physical body to changes in its rotational motion about a specific axis (how much torque is required for a given angular acceleration needed to set an object in rotation).

For a discrete system of particles, the moment of inertia $I$ about a given axis is defined as

$$I = \sum_{i=1}^{n} m_i r_i^2$$

- $m_i$ is the mass of the $i$-th particle,
- $r_i$ is the perpendicular distance of the $i$-th particle from the axis of rotation.

In classical mechanics, the masses of particles are positively defined quantities, $m_i > 0$, implying, automatically, that the classical moment of inertia is a positive quantity, $I > 0$.

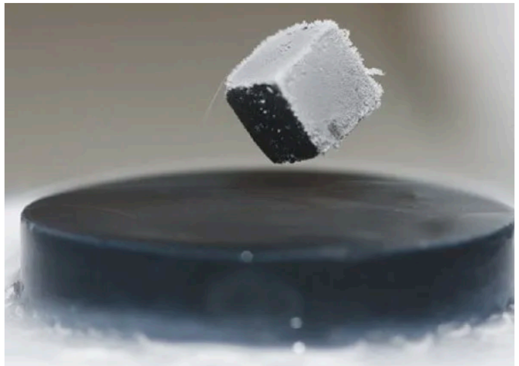
**Question(s): can we imagine a closed physical system with a negative moment of inertia in the true thermodynamic equilibrium state? Time crystals?**

**(conventional thermodynamics seems to say “no” but please wait a bit).**

**Hint: non-perturbative (chromo)magnetic fields will play an interesting role here.**

# Motivation

## - Superconductivity



- Superconductivity is a quantum mechanical phenomenon in which a material exhibits zero electrical resistance and expulsion of magnetic fields (the Meissner effect) below certain critical temperature.

- In a superconducting state, electric current flows without energy loss, making superconductors highly efficient for applications like powerful electromagnets, magnetic resonance imaging (MRI),  and potentially lossless power transmission.

- Electrons pair up to form Cooper pairs. In conventional superconductors, this pairing is mediated by lattice vibrations (phonons). Cooper pairs condense into a quantum ground state that can be described by a single macroscopic wave function.

**Question(s):**

- Do we really need a material (= something made of matter) to support an electrically superconducting state?

- Can we make a superconductor out of vacuum (a state devoid of matter = nothing)?
  What are “phonons” then? If it is nothing, how can it transfer something (electric charge)?

- Can, in certain sense, “virtual particles” serve as “Cooper pairs” that support a dissipationless superconducting state?

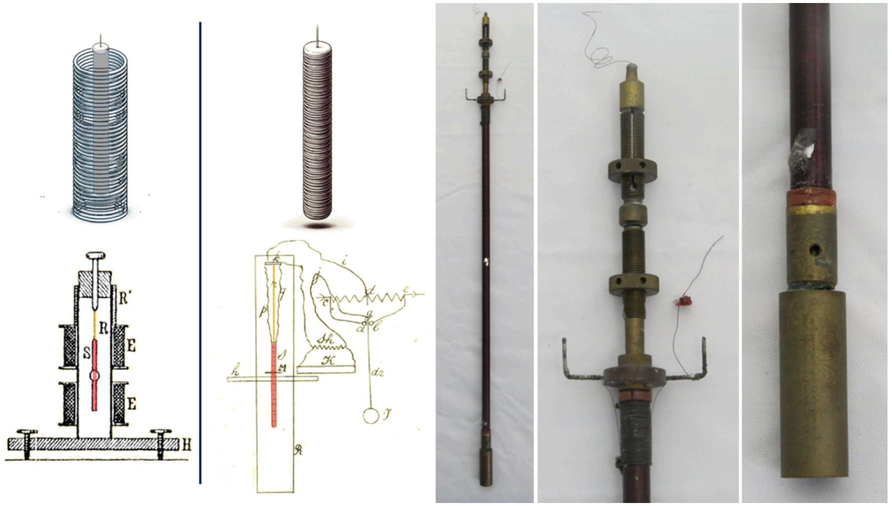
(conventional wisdom seems to say multiple “no” but please wait a bit)

**Hint:** magnetic fields will play an interesting role here.

# The Einstein-de Haas effect: some history

**Physics.** — *"Experimental proof of the existence of Ampère's molecular currents."* By Prof. A. EINSTEIN and Dr. W. J. DE HAAS. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the meeting of April 23, 1915).




The experiments were made in the winter of 1914/15 in "Physikalisch-Technische Reichsanstalt", a metrology laboratory in Berlin.

The coupling of mechanical rotation and magnetization was first proposed by Owen Richardson earlier:

A MECHANICAL EFFECT ACCOMPANYING MAGNETIZATION.

BY O. W. RICHARDSON.

PRINCETON, N. J., December 23, 1907.

Phys. Rev. (Series I) **26**, 248 – Published 1 March 1908

LE FIGARO

Élections législatives 2024 JO Paris 2024 Éditions locales

## Une expérience unique d'Albert Einstein découverte dans les réserves d'un musée près de Lyon

Par **Justin Boche**

SUIVRE

Publié le 19/03/2024

Alfonso San Miguel et Bernard Pallandre, tous deux bénévoles au musée Ampère de Poleymieux-au-Mont-d'Or (dans la métropole de Lyon), ont fait une découverte extraordinaire. Dans les réserves de ce lieu dédié au célèbre physicien lyonnais, André-Marie Ampère, les deux hommes ont découvert l'unique version complète et authentique de l'expérience réalisée en 1915 par Albert Einstein et Wander de Haas sur les travaux moléculaires d'Ampère qui ont permis de comprendre le mouvement des électrons autour du noyau des atomes.

# Magnetism and rotation: the Einstein-de Haas effect

**Physics.** — *"Experimental proof of the existence of Ampère's molecular currents."* By Prof. A. EINSTEIN and Dr. W. J. DE HAAS. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the meeting of April 23, 1915).

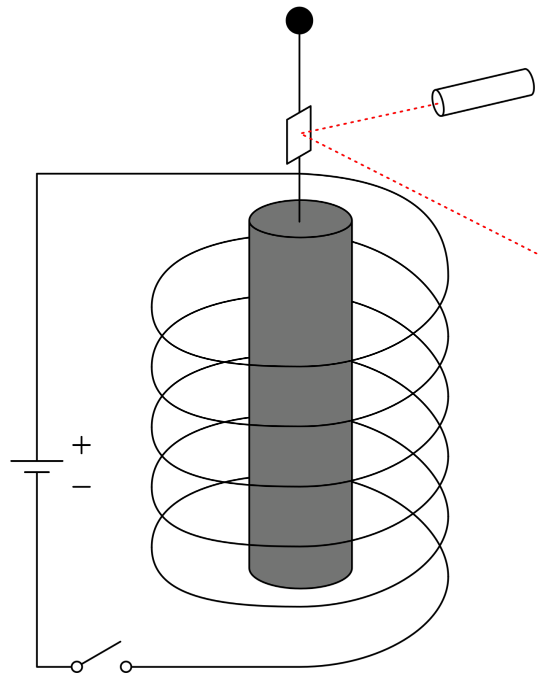
## Why the title includes "Ampère currents"?

... AMPÈRE succeeded in doing so by his celebrated hypothesis of currents circulating around the molecules without encountering any resistance.

...

it is difficult to conceive a circulation of electricity free from all resistance and therefore continuing for ever. Indeed, according to MAXWELL's equations circulating electrons must lose their energy by radiation; the molecules of a magnetic body would therefore gradually lose their magnetic moment. Nothing of the kind having ever been observed, the hypothesis seems irreconcilable with a general validity of the fundamental laws of electromagnetism.

The magnetic molecule behaves as a gyroscope whose axis coincides wich the direction of the magnetisation. Every change of magnetic state involves an alteration of the orientation of the gyroscopes and of the moment of momentum of the magnetic elements. In virtue of the law of conservation of moment of momentum the change of "magnetic" moment of momentum must be compensated by an equal and opposite one in the moment of momentum of ponderable matter. The magnetisation of a body must therefore give rise to a couple, which makes the body rotate.



[image from Wikipedia]

Gyromagnetic ratio $\gamma$ for a particle with the electric charge $e$ and mass $m$

$$\boldsymbol{\mu} = g\frac{e}{2m}\mathbf{S}$$

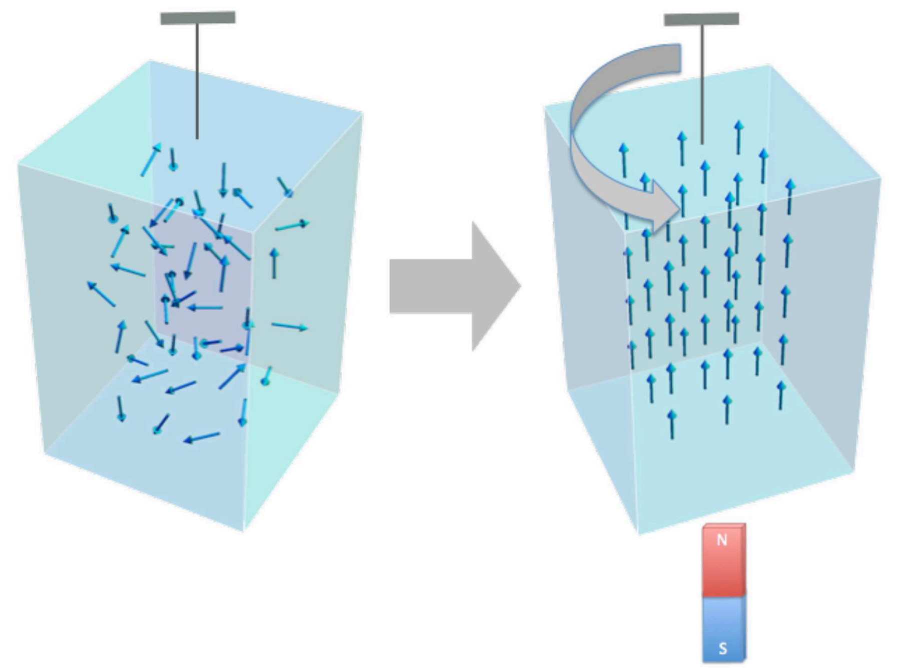
spin

generally, total angular momentum

magnetic moment

$g$-factor ($g_2 \simeq 2$ for an electron)

# Experimental setup: rotation by magnetization




**Einstein-de Haas effect.** Modulation of magnetization of iron by applying the external magnetic filed, magnetic angular momentum is changed. As a result, the mechanical angular momentum is induced for compensating the modulation of the annular momentum.

- **Objectives:** 1) To demonstrate that changes in magnetization can induce mechanical rotation.

gyromagnetic ratio

change in magnetization $\quad \Delta\boldsymbol{\mu} = \gamma \Delta \boldsymbol{J} \quad$ change in angular momentum

2) measurement of the gyromagnetic ratio:

$$\gamma = \frac{\Delta\mu}{\Delta J}$$

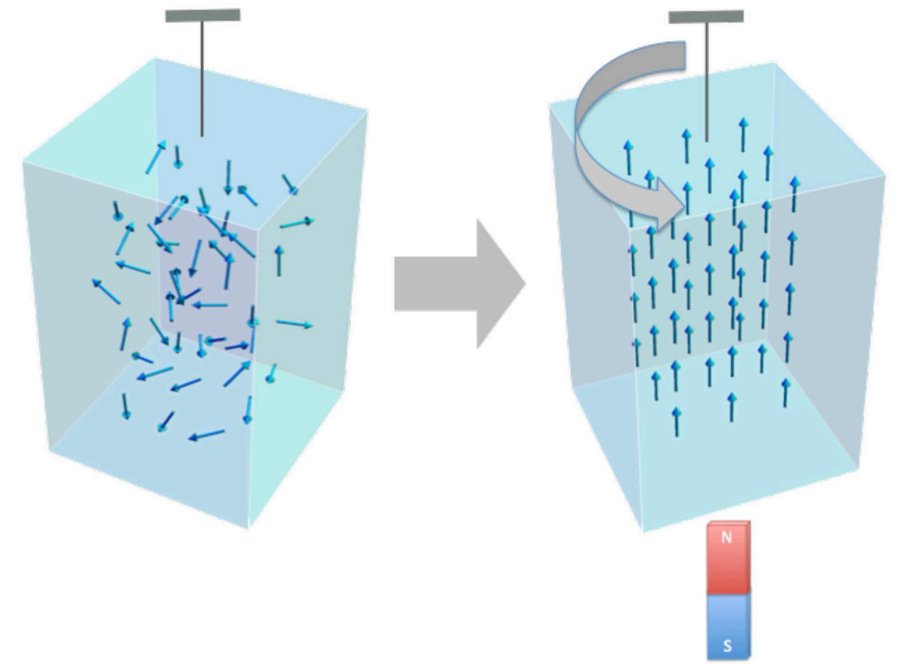
- **Experimental setup:** A cylindrical ferromagnetic rod suspended by a thin wire, placed within a solenoid.

- **Procedure:** When the magnetic field in the solenoid is altered, the magnetization of the rod changes, resulting in a measurable torsional motion of the rod.
- **Spin and orbital angular momenta contribute.** For pure iron, 96% comes from spin polarization of electrons and 4% are due to polarization of their orbital momenta.

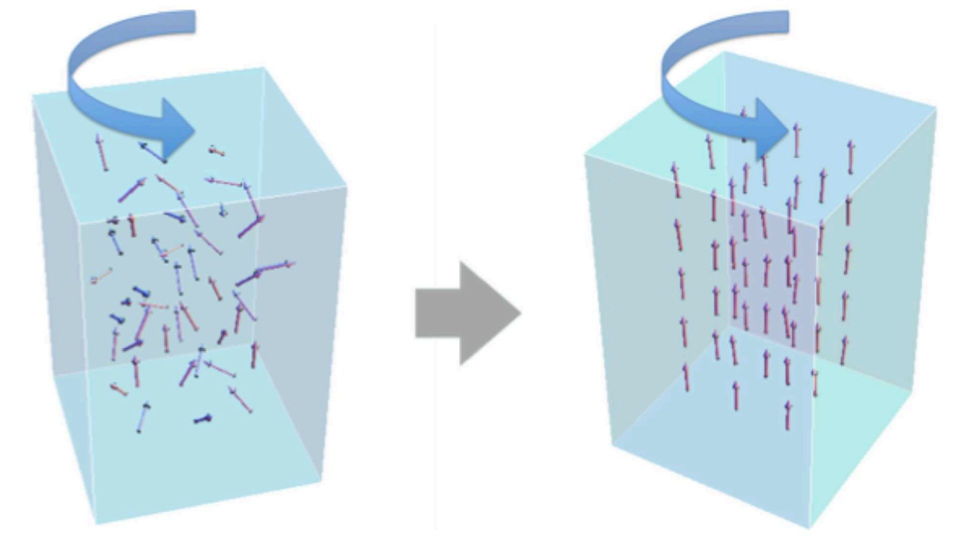
[images in Einstein-de Haas and Barnett sections are from the review by Matsuo, Ieda, Maekawa, Frontiers in Physics 3, 54 (2015)]

# Magnetization by rotation: The Barnett effect

## Coupling between mechanical rotation and spin orientation




**Einstein-de Haas effect.** Modulation of magnetization of iron by applying the external magnetic filed, magnetic angular momentum is changed. As a result, the mechanical angular momentum is induced for compensating the modulation of the annular momentum.



**Barnett effect.** Magnetization is induced by applying mechanical rotation since an effective magnetic field, emerges in a rotating body.

**Magnetization due to rotation:** $M = \chi\Omega/\gamma$

**Effective magnetic field:** $B_\Omega = \Omega/\gamma$

$\chi$ is the magnetization susceptibility of the medium

$\gamma$ is the gyromagnetic ratio

**The Barnett effect is a reciprocal phehomenon to the Einstein-de Haas effect**

# The Barnett effect

October, 1915 Vol. VI., No. 4

PHYSICAL REVIEW.

MAGNETIZATION BY ROTATION.[1]

By S. J. Barnett.

The Physical Laboratory,
The Ohio State University.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.
...

## - Classical mechanics.

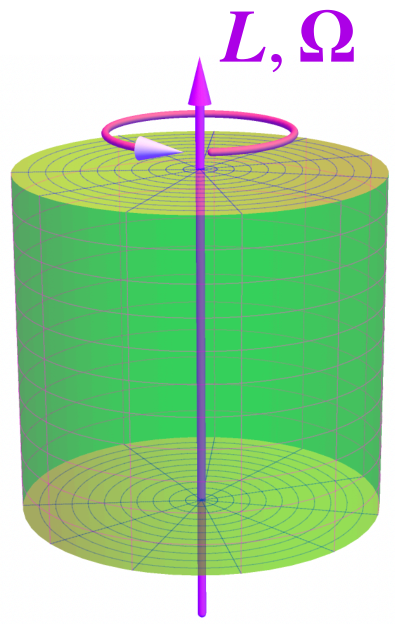
Consider a classical object possessing a moment of inertia $I$, rotating with an angular velocity $\Omega$. The angular (orbital) momentum is $L = I\Omega$ .

The rotational energy of the system:

$$E = \frac{L^2}{2I}$$

A small change in the angular momentum, $L \to L + \Delta L$, with $\Delta L \ll L$, leads to the change in the rotational energy:

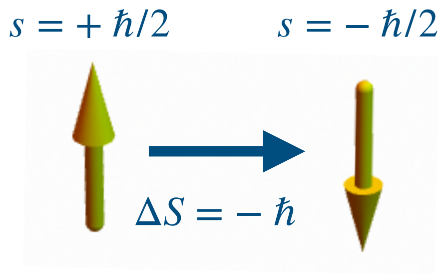
$$\Delta E = \frac{L}{I}\Delta L \equiv \Omega \Delta L$$



# The Barnett effect: a conservation law

## - Conservation of angular momentum

1. Consider an electron with spin $s = \hbar/2$ pointed along $\mathbf{\Omega}$.




2. Suppose that spin of an electron flips to the opposite direction:

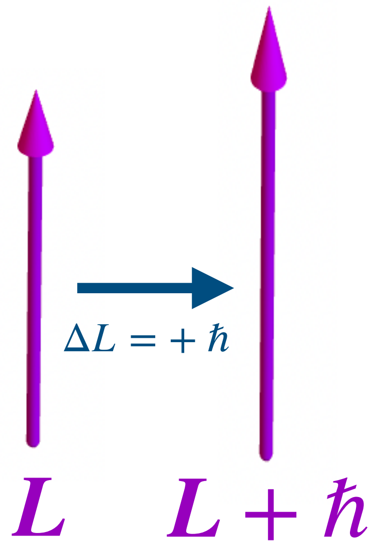
$$s_{\uparrow} = +\hbar/2 \;\; \rightarrow \;\; s_{\downarrow} = -\hbar/2$$

3. The change in spin: $\Delta S = -\hbar$

4. The angular total momentum $J$ is conserved: $\Delta J = \Delta L + \Delta S = 0$

5. The change in the orbital momentum $\Delta L = +\hbar$

6. The change in energy $(\Delta E = \Omega \Delta L)$ due to spin flip is




**remarkable formula:** $$\Delta E = \hbar\Omega$$

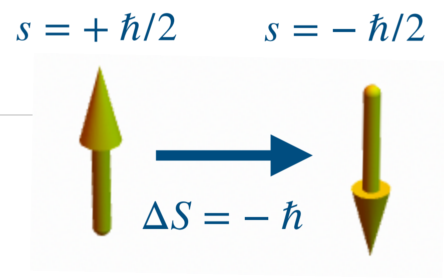
change in "**classical**"(*) rotational energy

Planck constant

angular frequency of rotation of a **classical** body

(*) obviously, the spin flip is a quantum effect

# The Barnett effect: thermodynamics (I)




## - Thermodynamics

Boltzmann distribution, probability:

$$W \propto \exp\left(-\frac{E}{k_B T}\right)$$

$$\Delta E = \hbar\Omega$$

Ratio of spin-↑ and spin-↓ probabilities:

$$\frac{W_\uparrow}{W_\downarrow} = \exp\left(-\frac{\Delta E}{k_B T}\right)$$

Average spin:

$$\langle s \rangle = s_0 \tanh \frac{\hbar\Omega}{2k_B T}$$

Magnetization - for a single spin: $\langle \mu \rangle = \gamma \langle s \rangle$;

for an ensemble of $N$ spins, $M = N\langle \mu \rangle$:

$$M = M_0 \tanh \frac{\hbar\Omega}{2k_B T}$$

Normalization:

$$W_\uparrow + W_\downarrow = 1$$

where $s_0 = \hbar/2$

Gyromagnetic ratio:

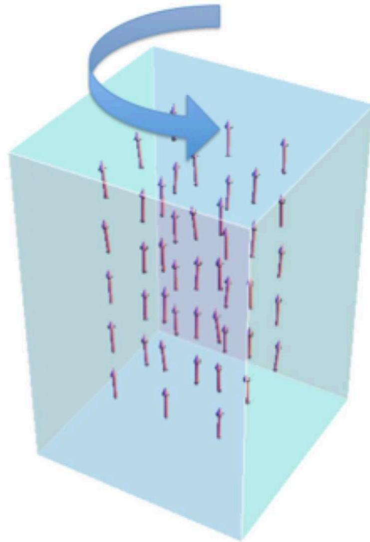
$$\gamma = g\frac{e}{2m}$$

where $M_0$ is a maximal magnetization in a fully polarized state with all spins pointed upwards.

# The Barnett effect: thermodynamics (II)

Practical applications: $\Omega = 2\pi \times 10^4\,\mathrm{Hz}$ and $T = 300\,\mathrm{K}$, one gets $\dfrac{\hbar\Omega}{2k_B T} \sim 10^{-9} \ll 1$

**(actually, this ratio in quark-gluon plasma it is also a small number)**



**The Barnett magnetization:**

$$M = M_0 \frac{\hbar\Omega}{2k_B T}$$

**Analogy with an effective magnetic field.**

Energy shift due to spin flip in a rotating system: $\Delta E_\Omega = \hbar\Omega$

Change in magnetic moment due to spin flip: $\Delta\mu = \gamma\Delta s \equiv \gamma\hbar$

Energy shift due to spin flip in background of magnetic field: $\Delta E_B = B\Delta\mu = \hbar\gamma B$

Formal similarity, $\Delta E_\Omega = \Delta E_B$, leads to an identification: $\Omega = \gamma B_\Omega$

**Effective Barnett magnetic field:**

$$B_\Omega = \Omega/\gamma$$

**Despite rotation can create a magnetization equal to the magnetization created by the effective Barnett magnetic field $B_\Omega$, the rotating sample does not really generate the magnetic field $B_\Omega$ in its interior.**

**Below, we will talk about the Barnett effect in extreme relativistic environements**

# Rotation ≡ magnetic field ?

In the field of condensed matter physics, considerations of gyromagnetic phenomena associated with the spin of the particle $s \neq 0$, often use Larmor's theorem to state that the influence of rotation on a system is equivalent to the application of a magnetic field $B_\Omega = \Omega/\gamma$, with $\gamma$ representing the particle's gyromagnetic ratio.

Example:

**a. Larmor's Theorem in Quantum Mechanics**

According to both classical theory and quantum mechanics, the effect of a uniform magnetic field on a system of charged particles may be shown to be equivalent to a rotation of the coordinate system.

REVIEWS OF MODERN PHYSICS
VOLUME 34, NUMBER 2 APRIL, 1962

**Theory of Gyromagnetic Effects and Some Related Magnetic Phenomena**

S. P. HEIMS*
*Brandeis University, Waltham, Massachusetts*
AND
E. T. JAYNES†
*Washington University, St. Louis, Missouri*

As we have just seen, this statement appears formally due similarity of the (linear) contributions to the Hamiltonian coming from the energy associated with

- the magnetic moment $\boldsymbol{\mu}$ of the particle in the background magnetic field $\boldsymbol{B}$

$$\delta H = -\boldsymbol{\mu} \cdot \boldsymbol{B}$$

- the angular momentum $\boldsymbol{J}$ in the background rotating with the angular velocity $\boldsymbol{\Omega}$

$$\delta H = -\boldsymbol{J} \cdot \boldsymbol{\Omega}$$

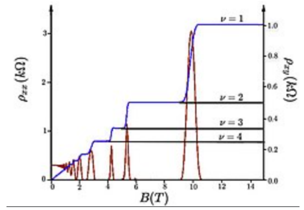
- as $\boldsymbol{\mu} \propto \boldsymbol{J}$ then $\boldsymbol{B} \propto \boldsymbol{\Omega}$

# Rotation $\neq$ magnetic field

In general case (relativistic system, fast rotation or strong field), rotation **cannot** be treated as (an artificial) magnetic field.

Reasons:

1. **Ground state degeneracy**: the number of states at the lowest Landau level is proportional to the magnetic field strength, $N_{\mathrm{LLL}} = |eB|/(2\pi)$. No such degeneracy for a rotating system, where the number of states at the ground state level is largely insensitive to the angular frequency $\Omega$.



**cf. Quantum Hall effect**

2. **Dimensional reduction**: in strong magnetic field, the motion of particles is of a one-dimensional nature as the energy gap between the ground state and the next excited state increases. It is not the case for rotation.

**cf. Aurora Borealis**

3. **Charge (a)symmetry**: for particles and antiparticles, the polarization effect due to rotation is the same, whereas the magnetic background field acts oppositely on them. For example, a neutral system of spinful particles can be magnetized by a background magnetic field but not by global rotation.

Rotation


Magnetic field

# The Dirac equation: non-relativistic limit

The Dirac equation for an electron in electromagnetic background:

$$\left[\gamma^\mu c(p_\mu - eA_\mu) - mc^2\right]\Psi = 0$$

for a 4-spinor $\Psi = \begin{pmatrix}\chi \\ \varphi\end{pmatrix}$ where $\chi$ and $\varphi$ are 2-spinors

The Dirac equation can be rewritten in a form

$$i\hbar\frac{\partial}{\partial t}\Psi = H_D\Psi$$

that resembles the non-relativistic Schrödinger equation
with the Dirac Hamiltonian

$$H_D = \beta mc^2 + c\boldsymbol{\alpha}\cdot\boldsymbol{\pi} + eA_0$$

Non-relativistic limit (the Foldy—Wouthuysen transformation),
an expansion in series of $1/m^2$, writing $\Psi = \Psi' e^{-imc^2 t/\hbar}$, or
repeatedly performing a unitary tranformation $\mathcal{O} = c\boldsymbol{\alpha}\cdot\boldsymbol{\pi}$

$$H' = U_F H_0 U_F^\dagger - i\hbar U_F \partial_t U_F^\dagger$$ with $U_F = \exp(-i\beta\mathcal{O}/2mc^2)$

[L.L. Foldy, S.A.Wouthuysen, Phys. Rev. 78, 29 (1950); S. Tani, Prog Theor Phys. 6, 267 (1951)]

Notations:

$$p_\mu = -i\hbar\frac{\partial}{\partial x^\mu}$$

3-momentum:

$$\boldsymbol{\pi} = \boldsymbol{p} - e\boldsymbol{A}$$

The background gauge potential:

$$A^\mu = (A^0, \boldsymbol{A})$$

Gamma matrices

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

$$\gamma^0 = \beta, \quad \gamma^i = \beta\alpha^i$$

$(i = 1,2,3)$

$$\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}$$

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

# The non-relativistic Dirac (Pauli) equation: polarization effects

Non-relativistic Hamiltonian has the following form

(written for a lower 2-spinor component of the Dirac 4-spinor)

$$H_e^{(1/m^2)} = \frac{\pi^2}{2m} - eA_0 - \frac{e\hbar}{2m}\sigma \cdot \mathbf{B} - \frac{e\lambda}{\hbar}\sigma \cdot (\pi \times \mathbf{E})$$

the spin-orbit coupling

$$\lambda = \hbar^2/4\mathrm{m}^2c^2$$

**Kinetic energy**

**Electrostatic potential energy**

**Zeeman coupling**

**Spin-orbit coupling**

Spin polarization in a background magnetic field $\boldsymbol{B}$

## Spin-dependent velocity

$$\mathbf{v} = \frac{1}{i\hbar}[\mathbf{r}, H_e] = \frac{\pi}{m} - \frac{e\lambda}{m}\sigma \times \mathbf{E}$$

spin

auxiliary corner (needed for the hydro Barnett Effect)

**(inverse) Spin-Hall effect**

a parameter: "spin Hall conductivity"

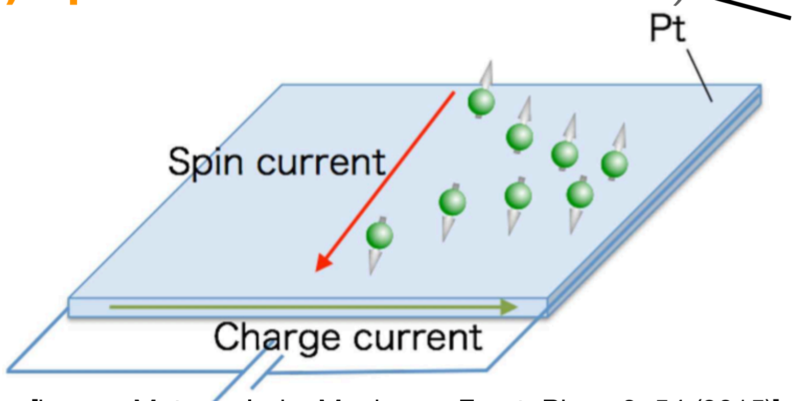
**The spin Hall effect:** $\boldsymbol{J}_s = \sigma_{\mathrm{sH}} \boldsymbol{E} \times \boldsymbol{s}$

spin current

**The inverse spin Hall effect:** $\boldsymbol{J}_c = \theta_{\mathrm{ISHE}} \boldsymbol{J}_s \times \boldsymbol{s}$

charge current

a parameter: "spin Hall angle"




[image: Matsuo, Ieda, Maekawa, Front. Phys. 3, 54 (2015)]

[M.I. Dyakonov, V.I.Perel, Phys. Lett. 35A,459 (1971); J.E. Hirsch, Phys Rev Lett. 83, 1834 (1999)]

# The non-relativistic Dirac (Pauli) equation in co-rotating frame

The Dirac equation for an electron in electromagnetic background:

$$\left[\gamma^{\mu}c(p_{\mu} - eA_{\mu}) - mc^2\right]\Psi = 0$$

now rewrite the Hamiltonian in the co-rotating frame

spin operator

$$\Sigma = \frac{1}{2}\begin{pmatrix} \sigma & O \\ O & \sigma \end{pmatrix}$$

coordinate transformation

$$d\mathbf{r}' = d\mathbf{r} + (\Omega \times \mathbf{r})\, dt, \;\; dt' = dt$$

$$\bar{H}_D = \beta mc^2 + (c\alpha \underbrace{- \mathbf{\Omega} \times \mathbf{r}}_{\text{rotation velocity}}) \cdot \pi + qA_0 \underbrace{- \hbar \mathbf{\Omega} \cdot \mathbf{\Sigma}}_{\text{Spin-rotation coupling}}$$

Non-relativistic limit (the Foldy—Wouthuysen—Tani transformation):

$$\bar{H}_e^{(1/m)} = \frac{\pi^2}{2m} - eA_0 - \underbrace{\mathbf{r} \times \pi}_{\text{orbital momentum}} \cdot \mathbf{\Omega} - \frac{e\hbar}{2m}\sigma \cdot (\mathbf{B} + \mathbf{B}_{\Omega})$$

Zeeman coupling

Barnett effect

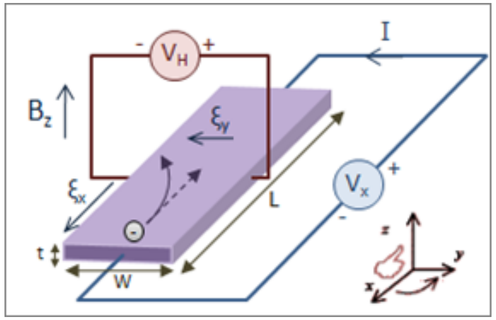
**The effective Barnett field** $\mathbf{B}_{\Omega} = m\mathbf{\Omega}/e$

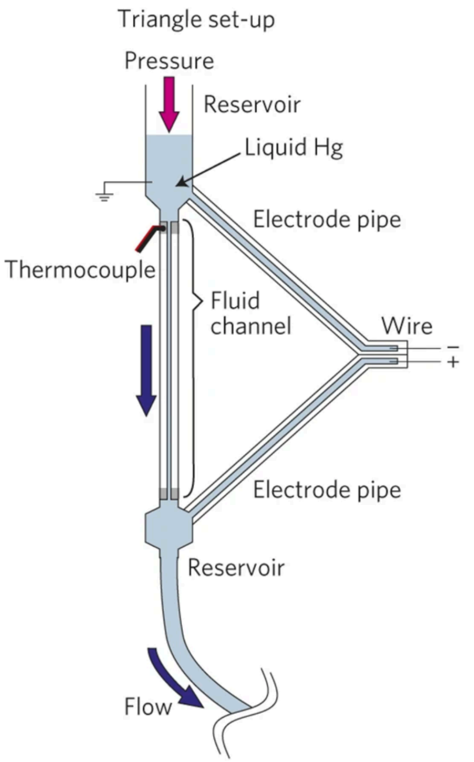
aligns the spins along the rotation axis

**to be discussed in more detail later**

# Polarization of electrons via vorticity in fluids

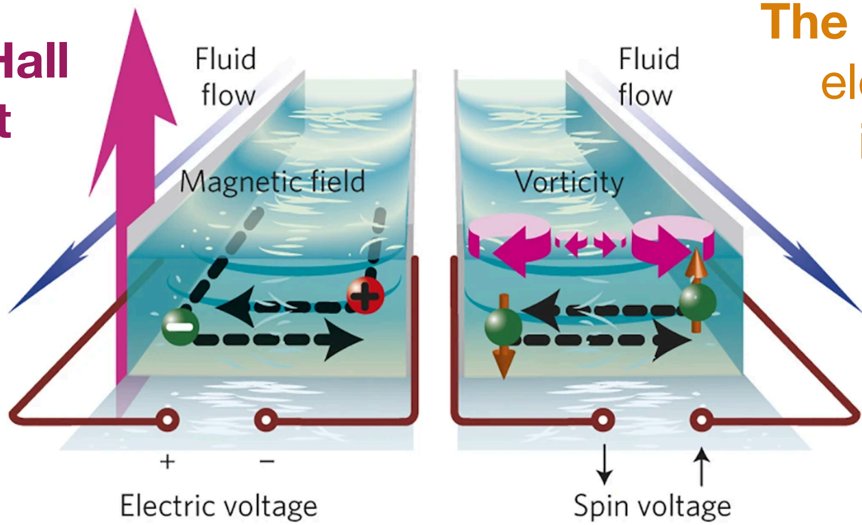
**The electron spin polarization due to the Barnett effect has been observed in a vortical fluid: a flowing metallic mercury.**



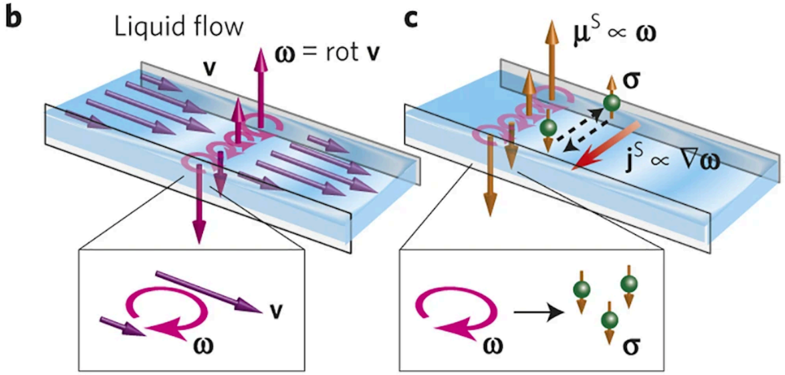

[R. Takahashi et al., Nat. Phys. 12, 52 (2016)]

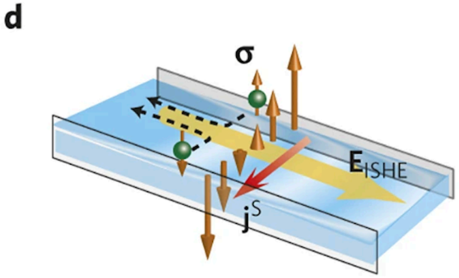
**The Hall effect**




**The Barnett effect** of electrons observed in flowing vortical metallic mercury

revealed via the (inverse) spin-Hall effect




$$J_c = \theta_{\mathrm{ISHE}} J_s \times s$$

Vorticity generation: the Poiseuille flow of the flowing mercury

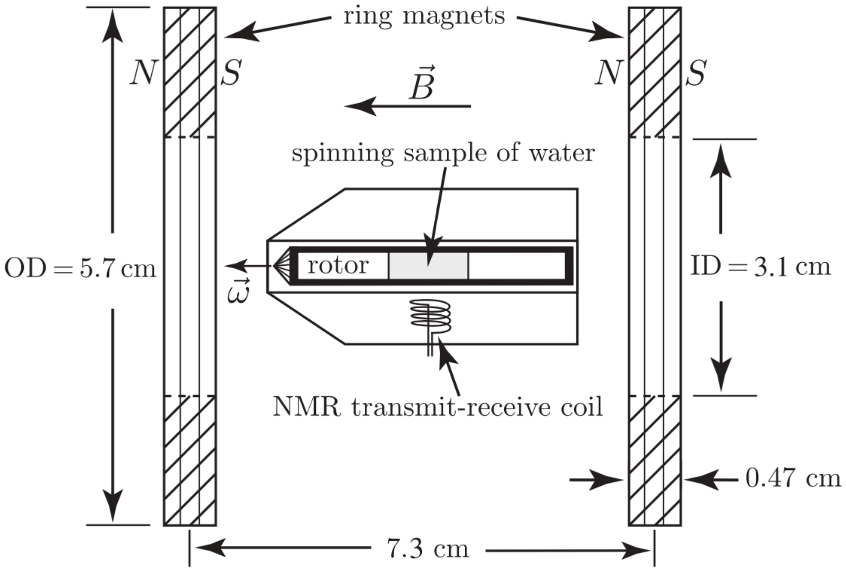
**hydrodynamics**

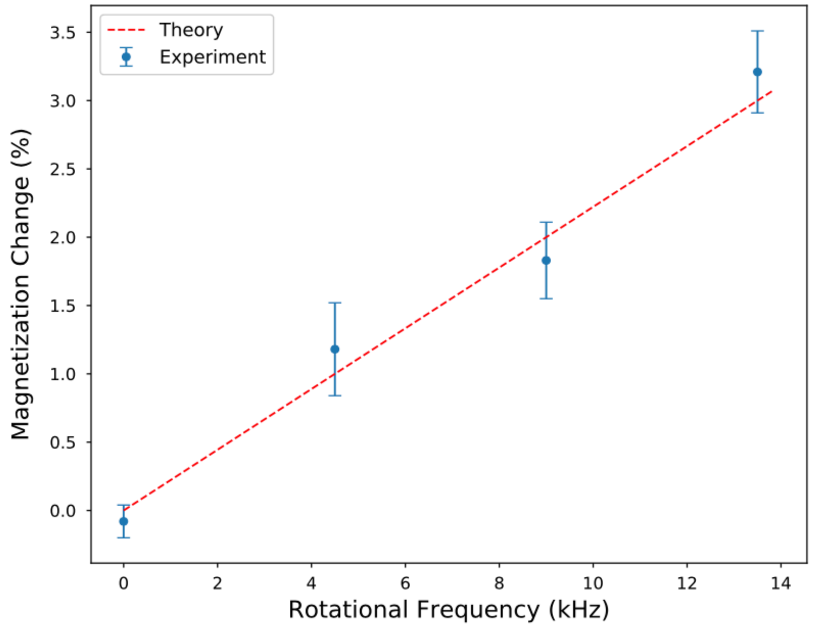
**The Barnett effect**

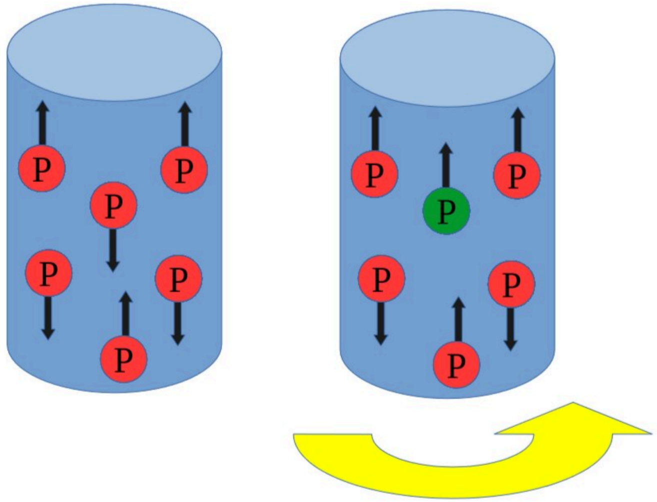
Measurement: The inverse spin Hall effect.

**spin-orbit**

# Nuclear Barnett Effect found in water



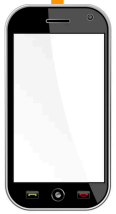

The nuclear Barnett effect by rotating a sample of water at rotational speeds up to 13.5 kHz and observed a change in the polarization of the protons in the sample that is proportional to the frequency of rotation.

*Arabgol and Sleator, PRL 122, 177202 (2019)*

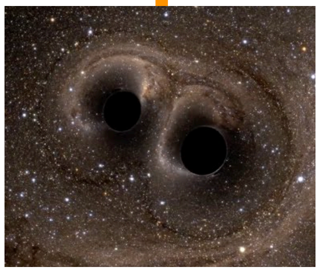
# Let us make mechanical rotation extreme

$10^2$ Hz — $10^4$ Hz — $10^6$ Hz — $10^{21}$ Hz

revolutions per second

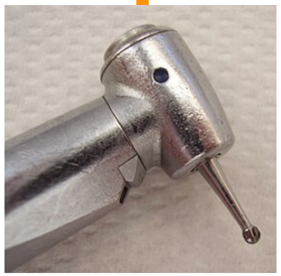


a smartphone vibration motor



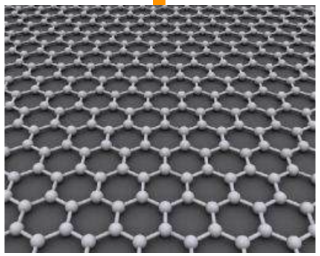
first-ever detected binary black hole merger event GW150914

(a peak frequency of gravitational waves)



frequency of revolutions of a modern air turbine dental drill

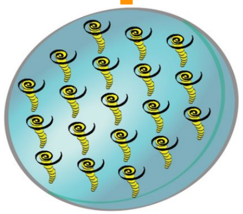
(also Nuclear Barnett Effect)



levitated spinning graphene flakes in an electric ion trap

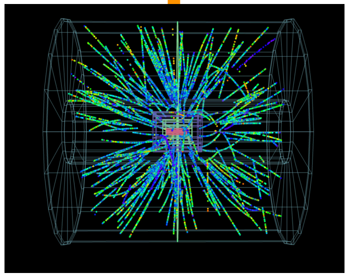
*B.E.Kane, Phys. Rev. B 82, 115441 (2010)*

“fastest-spinning macroscopic object ever”



nanodroplets of superfluid helium

*L. F. Gomez et al. Science 345, 906 (2014)*



Quark-Gluon plasma created at Relativistic Heavy-Ion Collider (RHIC)

*STAR Collaboration, Nature 62, 548 (2017)*

**“most vortical fluid ever”**

# The most vortical fluid ever observed

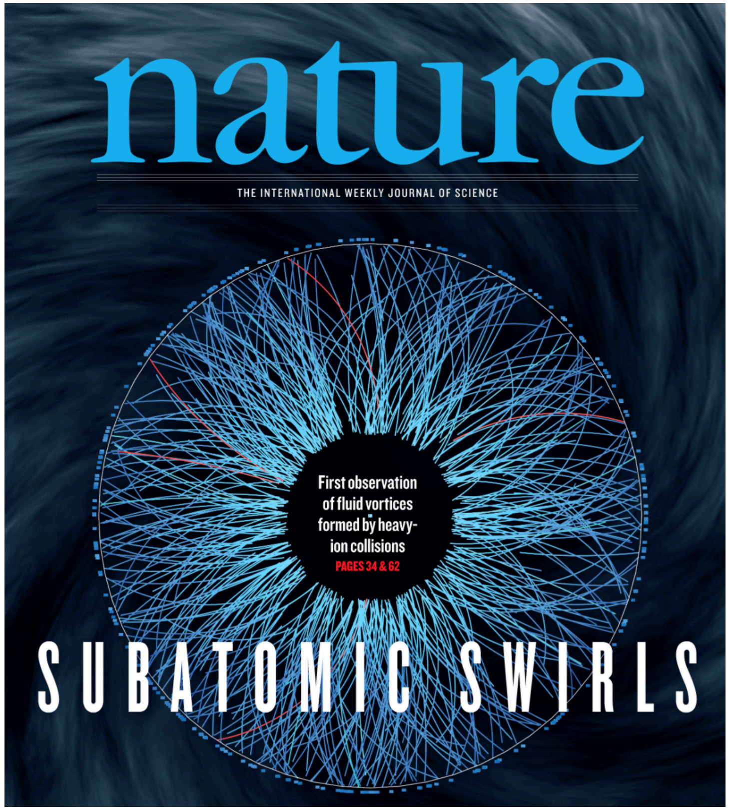
**The experimental result for the vorticity:**

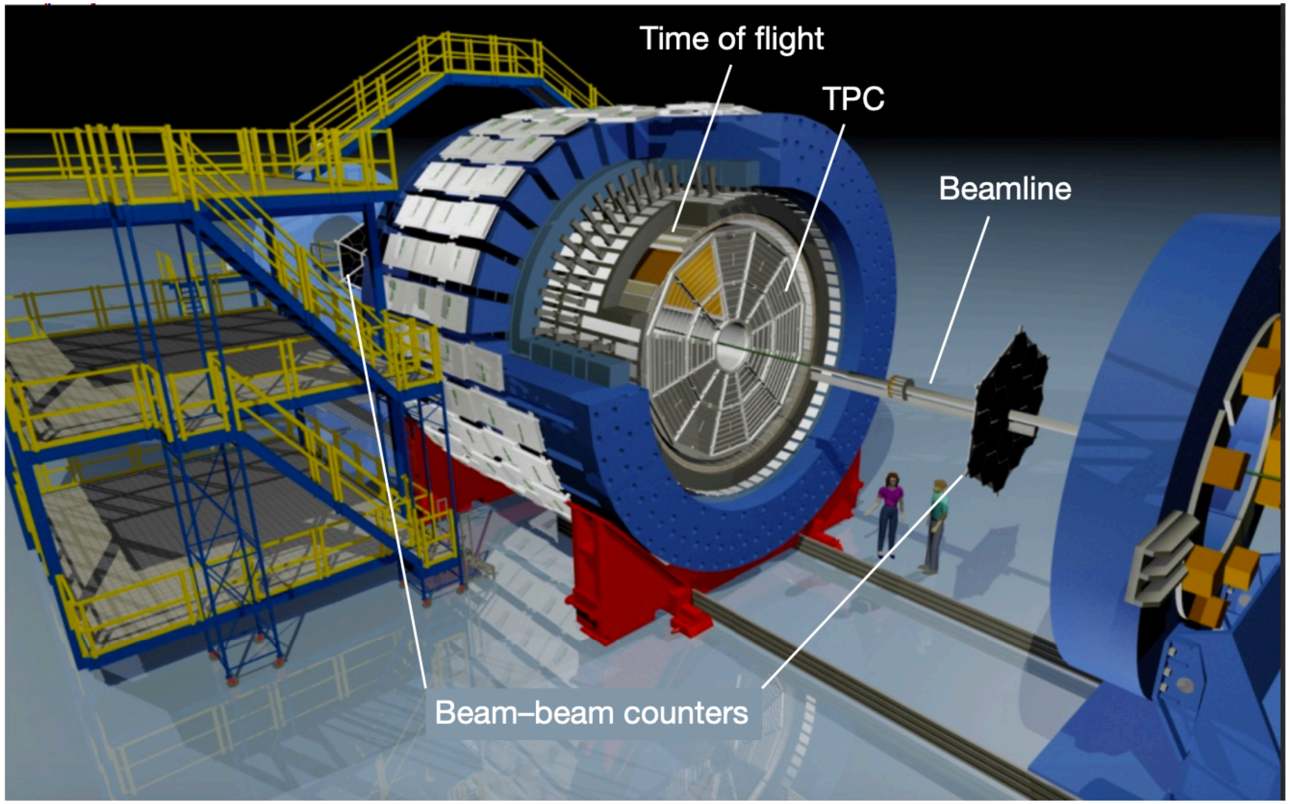
$$\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$$

**Is it large (in the QCD scale)? No!** $\omega \simeq 6.6\,\mathrm{MeV}$

**Other scales. Critical temperature:** $T_c \simeq 155\,\mathrm{MeV} \simeq 2 \times 10^{12}\,\mathrm{K}$

**time scale of collision:** $1\,\mathrm{fm/s} \simeq 3.3 \times 10^{-24}\,\mathrm{s}$ **(yoctoseconds)**




The STAR Collaboration, Nature 62, 548 (2017)

# Quark-Gluon plasma

**Quark-Gluon Plasma (QGP) is a state of matter, in which quarks and gluons, the fundamental constituents of protons and neutrons, are no longer confined within individual nucleons but are free to move within a larger volume.**

## Why it is interesting?

### QGP is believed to have existed shortly after the Big Bang

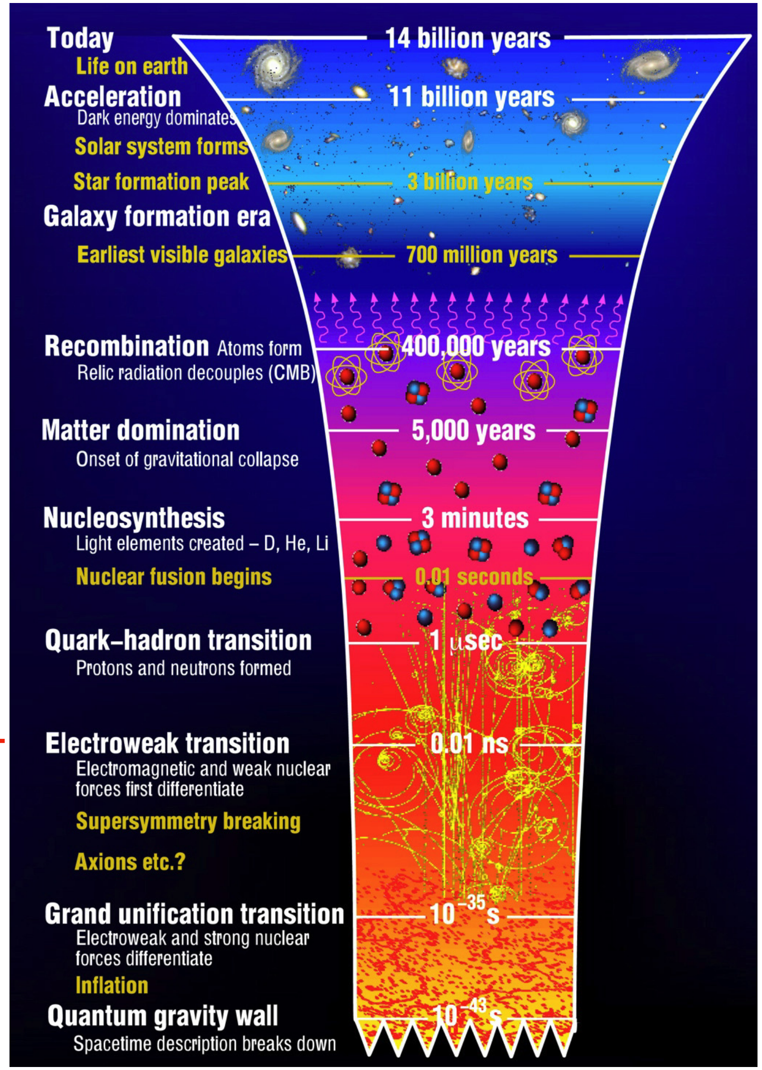
3. A few microseconds and later: Hadronic matter regime.

2. First microsecond, $10^{-6}$ s:
 the Universe cools to temperatures $150\,\mathrm{MeV} \sim 2 \times 10^{12}\,\mathrm{K}$.
 Hadronization takes place (a smooth crossover transition).

1. A fraction of a nanosecond, $10^{-(11\ldots12)}$ s, after the Big Bang:
 the Universe enters the QGP epoch right after
 the Electroweak phase transition.

The interactions between quarks and gluons are described by QCD, the theory of strong interactions.

QGP Epoch

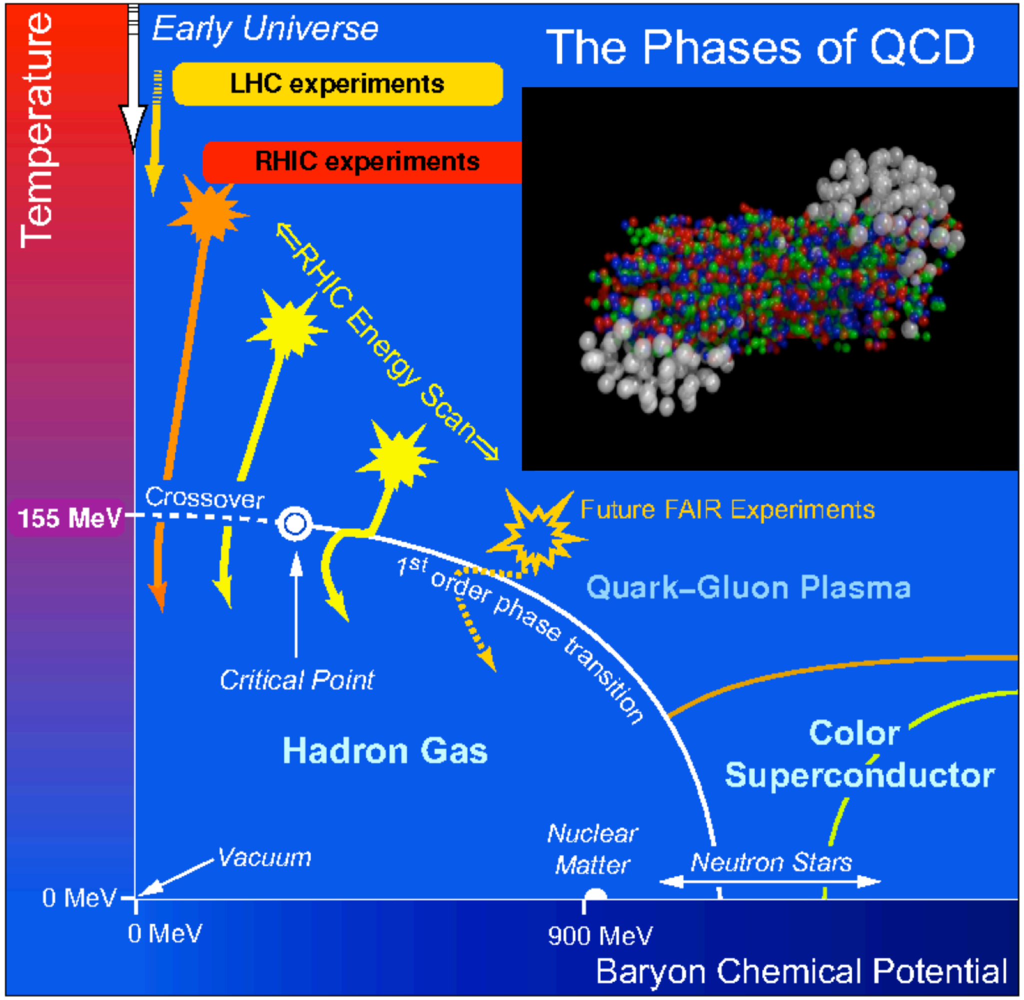



[Centre for Theoretical Cosmology, Cambridge, UK]

# Phase diagram of QCD

**1) Hot quark-gluon plasma phase and cold hadron phase constitute, basically, one single phase because they are separated by a nonsingular transition ("crossover").**

**2) The color superconducting phases at high baryonic chemical potential $\mu$ were extensively studied theoretically**
[currently, they are out of reach of both lattice simulations and Earth-based experiments]




From a BNL webpage

**3) The LHC and RHIC experiments probe low baryon density physics. One can safely take $\mu$ = 0 in further discussions.**

# Finite-temperature structure of QCD

QCD Lagrangian:

gluons quarks

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^{\mu\nu a} F_{\mu\nu a} + \bar{\psi}\left(i \not{D} - M\right)\psi$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

$SU(3)$ color gauge symmetry

$N_c = 3$ colors; $N_c^2 - 1 = 8$ gluons

$$D^\mu \equiv \partial^\mu + igA_a^\mu \lambda^a/2$$

$N_f = 2 + 1$ species of quarks

$u, d, s, \ldots$ quarks

Two most important physical phenomena in low-$T$ phase:

1) Quark confinement (comes from the gluon sector):

No quarks and gluons in the physical spectrum;

The physical degrees of freedom are hadrons (mesons and baryons)

2) Chiral symmetry breaking (effect on quarks due to dynamics of gluons):

The source of hadron masses (mesons and baryons are massive)

# Mass gap generation: a non-perturbative problem

**Yang-Mills theory (1954): simple** (but nonlinear) **Lagrangian, complicated puzzles**

$$\mathscr{L} = -\frac{1}{4} F^{a,\mu\nu} F^{a}_{\mu\nu}$$

**field strength**

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc}\, A^b_\mu\, A^c_\nu$$

**for gluon fields** $A_\mu = A^a_\mu T^a$

$T^a$ are generators of $su(3)$ Lie algebra

$$\mathrm{tr}\left(T^a\, T^b\right) = \tfrac{1}{2}\delta^{ab} \qquad \left[T^a,\, T^b\right] = i\, f^{abc}\, T^c$$

$SU(3)$ local (gauge) symmetry:

$$A_\mu(x) \to A'_\mu(x) = U(x) A_\mu(x) U^\dagger(x) + \tfrac{i}{g}(\partial_\mu U(x)) U^\dagger(x)$$

$$F_{\mu\nu}(x) \to F'_{\mu\nu}(x) = U(x) F_{\mu\nu}(x) U^\dagger(x)$$

**This symmetry is not broken: neither explicitly, nor spontaneously, nor anomalously**

**equations of motion**

$$\partial^\mu F^a_{\mu\nu} + g\, f^{abc}\, A^{\mu b}\, F^c_{\mu\nu} = 0$$

## A puzzle:

- **Yang-Mills Lagrangian has no mass parameters but all its exitations are massive (a mass gap phenomenon)**

The Millennium Prize Problems

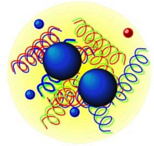
Unsolved

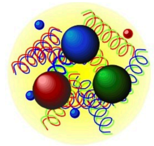
Yang-Mills & The Mass Gap

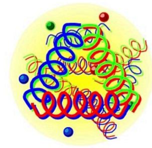
Experiment and computer simulations suggest the existence of a “mass gap” in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Clay Mathematics Institute

# Non-perturbative physics


meson


baryon


glueball?

[image from a BNL page]

## Puzzle 1:

**Mass gap generation: all physical particles in QCD —hadrons (mesons, baryons) and also glueballs (gluonic excitations in pure Yang-Mills theory) — are massive. There are no massless physical states at low-temperature phase.**

**However, the classical Yang-Mills theory contains no massive parameters (!)**

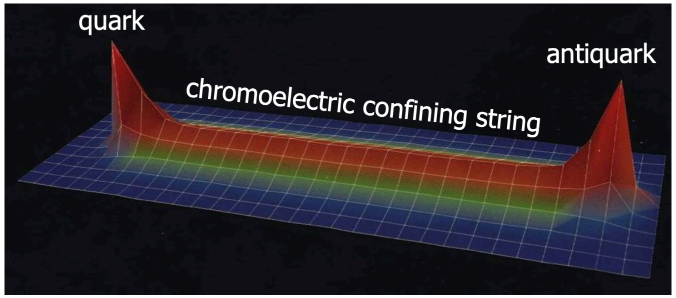
**In QCD, bare masses of light *u* and *d* quarks (a few MeV) are much smaller than the proton mass (a state of three quarks) which is about one GeV.**

## Puzzle 2:

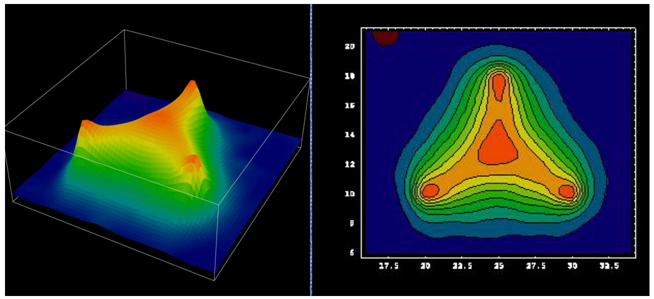
**Color confinement: in the low-temperature hadronic phase, the elementary are colorless states. No elementary gluons and quarks are observed. This effect is ascribed to be due to formation of a confining QCD string, seen in lattice first-principle simulations but not understood theoretically.**

**examples of numerical sumulations, energy density profiles**

confining string between static quarks



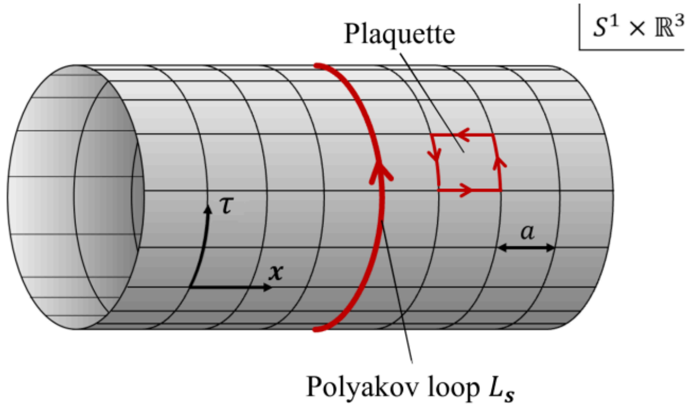

confining string inside a static “proton”



# Quark confinement (I): order parameter

The relevant order parameter is the Polyakov loop:

$$L(x) = \frac{1}{3}\mathrm{Tr}\,\mathcal{P}\exp\left[i\int_0^{1/T} \mathrm{d}\tau\, A_4(\vec{x},\tau)\right]$$




[image from ArXiv:2212.13874]

1) defined at finite temperature $T$ in the Euclidean space-time (after a Wick rotation, $t \to -it = \tau \equiv x_4$)

2) related to the free energy $F_q$ of a single quark:

the length of the compactified dimension is given by temperature

$$\langle L \rangle = \exp\left(-F_q/T\right)$$

$$L_\tau = \frac{1}{T}$$

3) order parameter:

$$\text{Confinement}: \begin{cases} \langle L \rangle = 0 \quad , & \text{low } T \\ \langle L \rangle \neq 0 \quad , & \text{high } T \end{cases}$$

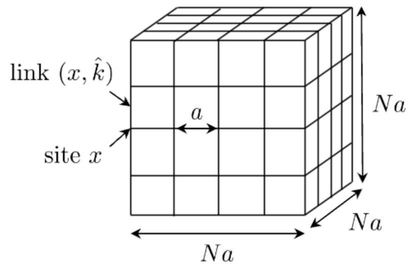
**no free quarks**

**free quarks exist**

**Cannot be computed analytically. → First-principle numerical simulations.**

# Non-Abelian lattice gauge theory

**Discretization**: Space-time is represented as a lattice, making the theory adapted to numerical simulations.

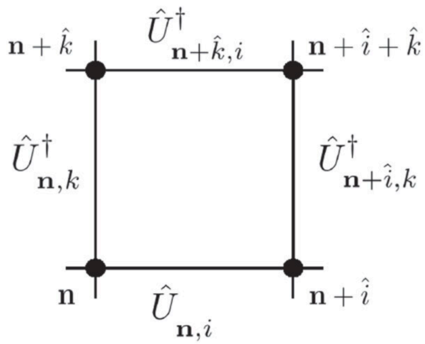



**Euclidean Space-Time**: Transition to a Euclidean metric to make statistical simulations possible.

**Link Variables**: $U_\mu(x) \in SU(N)$ elements are parallel transporters between sites.

**Plaquette Operator**: Defines the elementary gauge-invariant quantity:

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)$$




**Lattice action**:

$$S = \beta \sum_{x,\mu,\nu} \left( 1 - \frac{1}{N} \text{Re Tr}\, U_{\mu\nu}(x) \right)$$

$\beta = 2N/g^2$ is the lattice coupling expressed via a bare continuum coupling $g$.

**Gauge Transformation on the Lattice**:

$$U_\mu(x) \to U'_\mu(x) = V(x) U_\mu(x) V^\dagger(x+\hat{\mu})$$

$V(x) \in SU(N)$ is the gauge transformation matrix

# Continuum limit

$U_\mu(x) \in SU(N)$ - lattice gauge field
$a$ - lattice spacing
$g$ - continuum gauge coupling
$A_\mu(x) = A_\mu^a(x) T^a$ - Continuum gauge field

**Link variable:**

$$U_\mu(x) = \exp\left(iagA_\mu(x)\right)$$

**Small-$a$ expansion:**

$$U_\mu(x) = \exp\left(iagA_\mu(x)\right) \approx 1 + iagA_\mu(x) - \frac{a^2g^2}{2}A_\mu^2(x) + \mathcal{O}(a^3)$$

**Plaquette Operator:**

$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)$$

**In the continuu limit, $a \to 0$:**

(1) $$U_{\mu\nu}(x) \approx \exp\left(ia^2gF_{\mu\nu}(x)\right) \approx 1 + ia^2gF_{\mu\nu}(x) - \frac{a^4g^2}{2}F_{\mu\nu}^2(x) + \mathcal{O}(a^6)$$

(2) $$\text{Re Tr}\, U_{\mu\nu}(x) \approx N - \frac{a^4g^2}{2}\text{Tr}\, F_{\mu\nu}^2(x)$$

(3) $$S \approx \frac{\beta a^4 g^2}{2N}\sum_{x;\mu>\nu}\text{Tr}\, F_{\mu\nu}^2(x)$$

(4) $$\sum_x a^4 \to \int d^4x$$

**Continuum limit:**

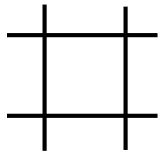
$$S_E \to \frac{\beta g^2}{4N}\int d^4x\, \text{Tr}\, F_{\mu\nu}^2(x) = \frac{1}{4}\int d^4x\, (F_{\mu\nu}^a)^2$$

$$\beta \to 2N/g^2$$

# Physical quantities and the beta function

We have a model with the action:

$$S = \beta \sum_{x,\mu,\nu} \left( 1 - \frac{1}{N} \mathrm{Re\,Tr}\, U_{\mu\nu}(x) \right)$$



on a hypercubic lattice with $SU(3)$ matrix link fields ($N = 3$) and a single coupling constant $\beta$. The model has no dimensionful parameters.

**How can we obtain a physical mass of something (in MeV, say)?** 1) fix $\beta$

2) calculate a correlation function between two lattice points $m = x/a$ and $n = y/a$:

$$\langle \mathcal{O}^\dagger(x)\mathcal{O}(y) \rangle \sim \exp\{-m_{\mathrm{lat}}(\beta)\,|n-m|\,\} + \ldots = \exp\{-m_{\mathrm{phys}}(\beta)\,|x-y|\,\} + \ldots$$

3) the operator $\mathcal{O}$ should correspond to a known physical mass of an excitation:

$$m_{\mathrm{phys}}[\mathrm{MeV}] = \frac{m_{\mathrm{lat}}(\beta)}{a(\beta)}$$

4) get $a = a(\beta)$

5) in fact, $a = a(\beta)$, is described by the beta function (running charge):

$a\Lambda_L = R(\beta)$ with known $\Lambda_L$ [in MeV]

$$R(\beta) = \left(\frac{\beta}{2Nb_0}\right)^{b_1/2b_0^2} \exp[-\beta/4Nb_0]$$

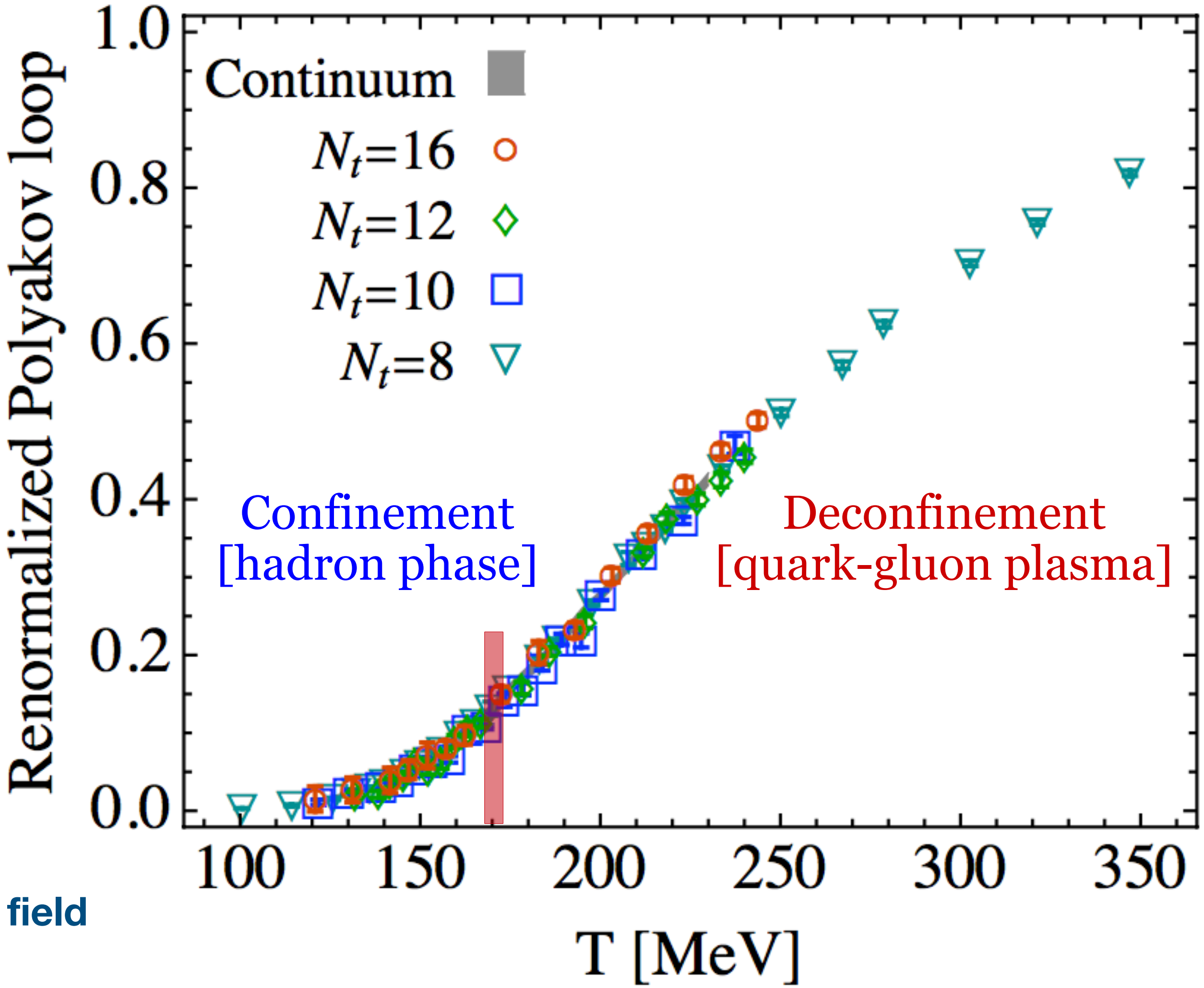
$$b_0 = \frac{11N}{48\pi^2} \quad , \quad b_1 = \frac{34}{3}\left(\frac{N}{16\pi^2}\right)^2$$

6) measure anything (any correlator), now we know $a = a(\beta)$ in fm (or 1/MeV).

# Quark confinement (II): lattice simulations

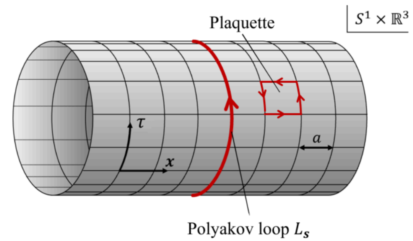
## The expectation value of the Polyakov loop vs. temperature

$$\langle L \rangle = \exp\left(-F_q/T\right)$$




Smooth transition (crossover)

**QCD with physical quark masses in the continuum limit**




$$T = \frac{1}{L_\tau} = \frac{1}{N_\tau a(\beta)}$$

“temperature equals to the inverse length of the compactified time direction”

**no magnetic field**
**zero density**
**no rotation**

[adapted from Borsanyi et al, JHEP 1009 (2010) 073, ArXiv:1005.3508]

# Chiral symmetry breaking (I): The symmetry

Left/right quarks, for each flavor *f* : $\psi_f = \psi_{Lf} + \psi_{Rf}$

$$\psi_{Lf} = \frac{1}{2}(1-\gamma_5)\psi_f \, ; \qquad \psi_{Rf} = \frac{1}{2}(1+\gamma_5)\psi_f$$

projectors

Lagrangian:

$$\mathcal{L}_q = i\bar{\psi}_L \not{D} \psi_L + i\bar{\psi}_R \not{D} \psi_R - \bar{\psi}_L M \psi_R - \bar{\psi}_R M \psi_L$$

If the quark masses are zero, $M = 0$, then the global internal continuous symmetries of QCD are as follows:

$$\psi_L \to \Omega_L \psi_L$$
$$\psi_R \to \Omega_R \psi_R$$

with $\Omega_{L,R} \in U(N_f)_{L,R}$

The global symmetry group: $U(N_f)_L \times U(N_f)_R$

# Chiral symmetry breaking (II): order parameter

The order parameter of the chiral symmetry is the chiral condensate:

$$\langle \bar{\psi}\psi \rangle \equiv \langle \bar{\psi}_L \psi_R \rangle + \langle \bar{\psi}_R \psi_L \rangle$$

In the hadronic phase of QCD the chiral condensate is nonzero and the chiral symmetry subgroup

Baryon symmetry (unbroken)

$$U(N_f)_L \times U(N_f)_R \simeq SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_A$$

Axial symmetry (broken by an anomaly)

is broken spontaneously:

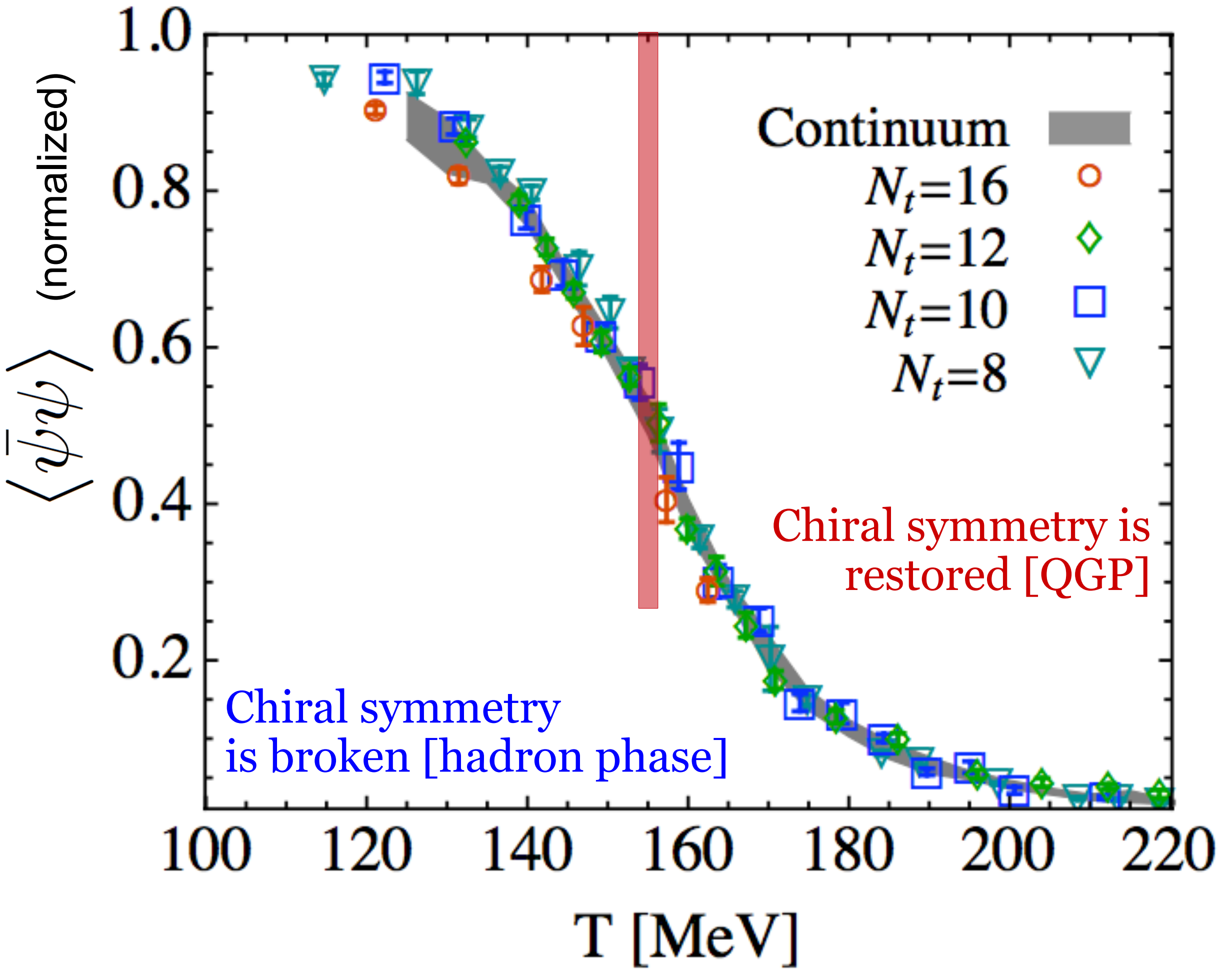
$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_{L+R}$$

so that the allowed transformations are as follows:

$$\psi_L \rightarrow \Omega \psi_L \text{ and } \psi_R \rightarrow \Omega \psi_R \text{ with } \Omega \in SU(N_f)_{L+R}$$

# Chiral symmetry breaking (III): lattice results

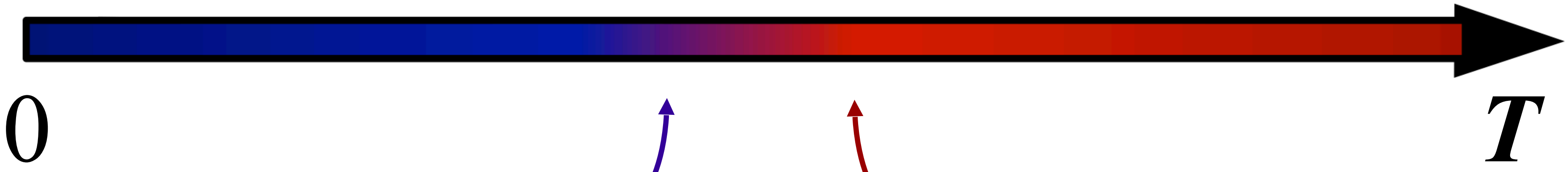
## Chiral condensate vs. temperature (crossover transition):




QCD with physical quark masses in the continuum limit

[adapted from Borsanyi et al, JHEP 1009 (2010) 073, ArXiv:1005.3508]

# Physical picture (confinement + chiral symmetry)

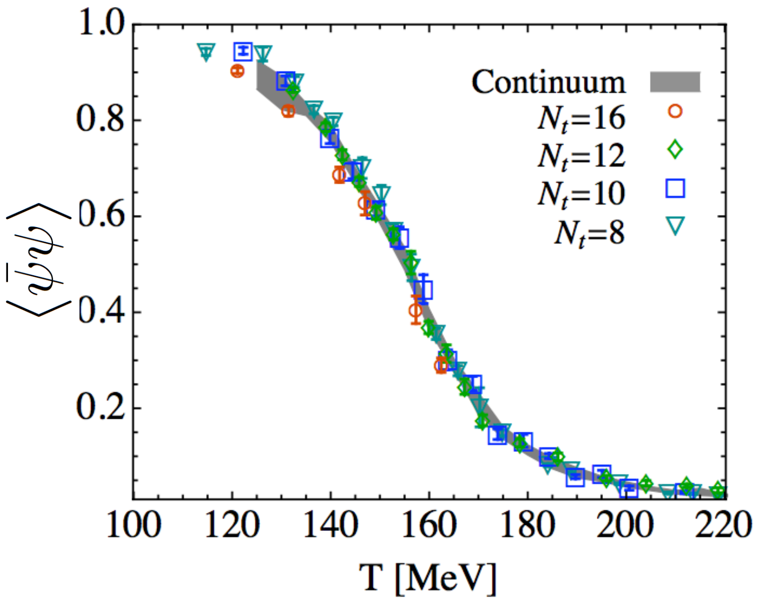


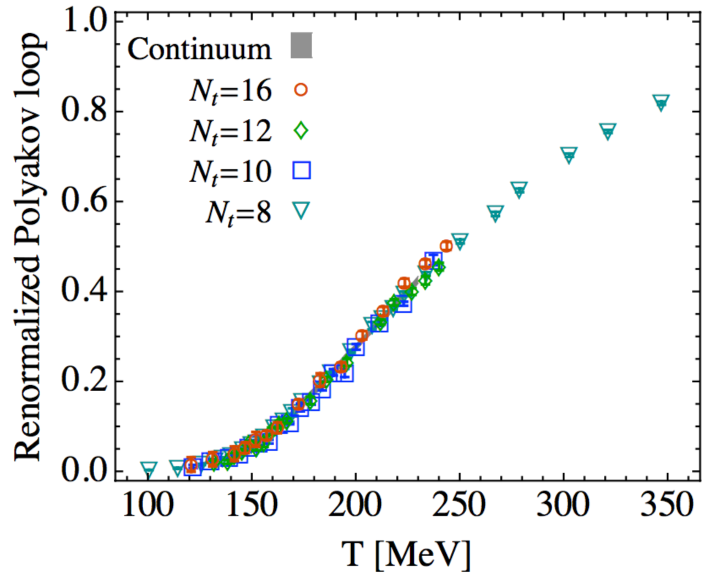

Restoration of
the chiral symmetry

Deconfinement
transition

$T_{\text{chiral}} \approx 155\,\text{MeV}$

$T_{\text{deconf}} \approx 170\,\text{MeV}$

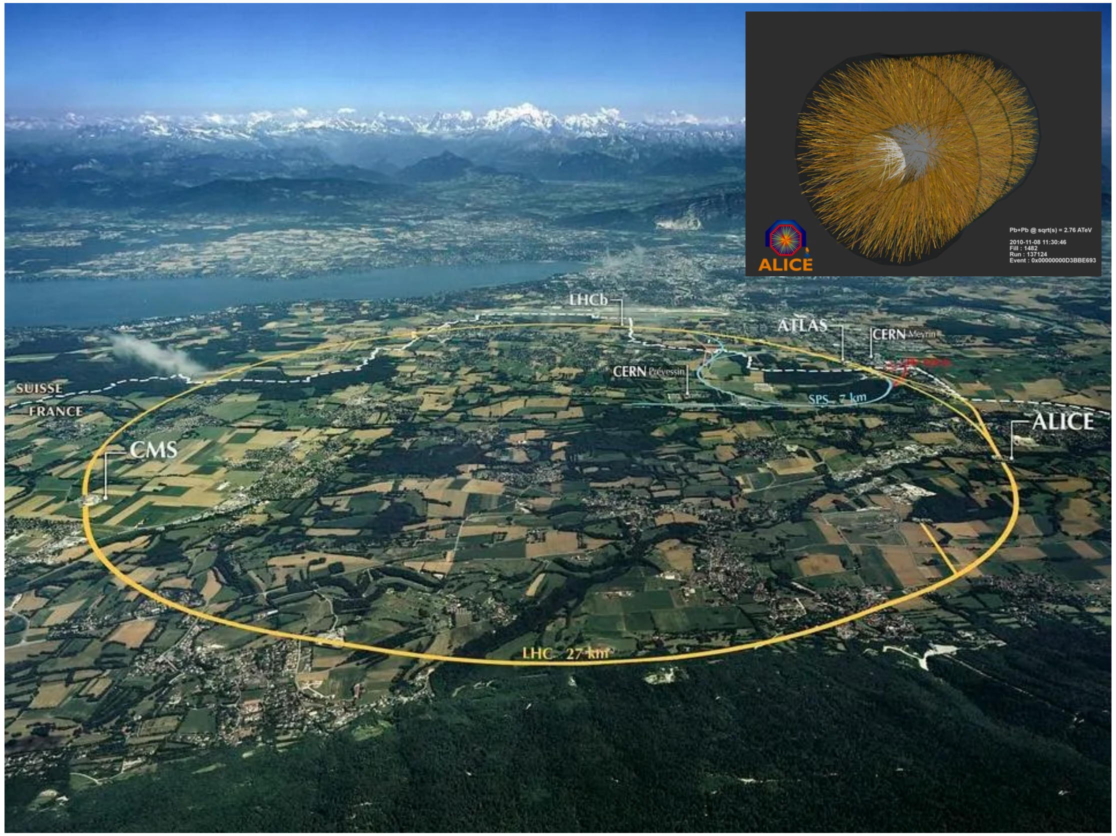


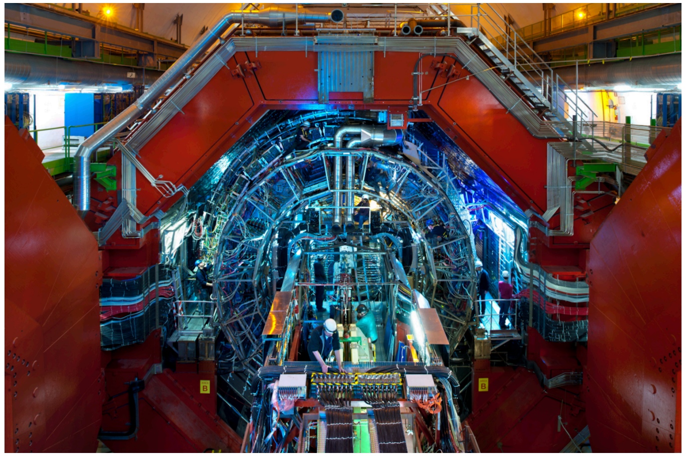



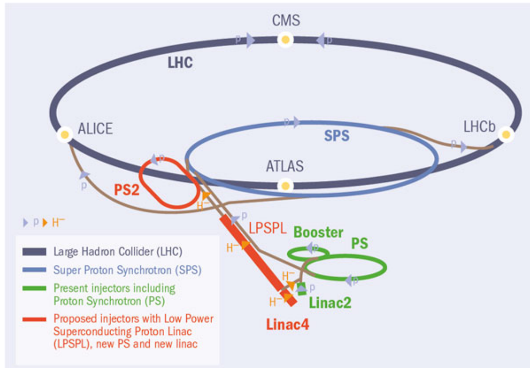

**No thermodynamic transition (a smooth crossover)**

What happens with this picture in (rotating) plasma?

# How do we create QGP plasma?



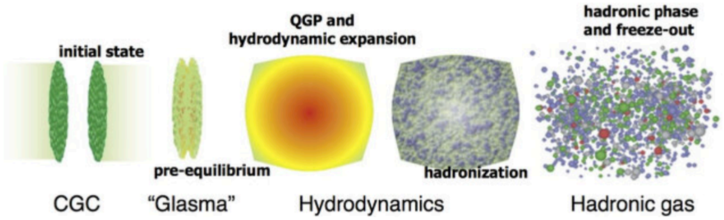


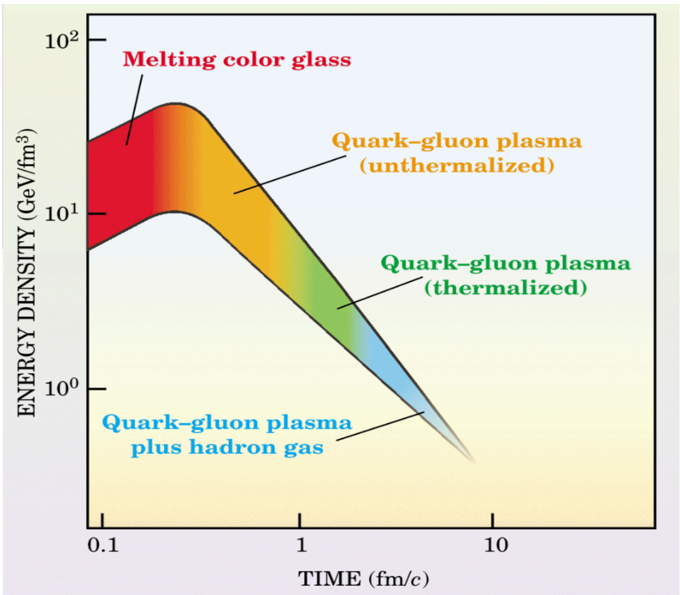

Geneva, 10 February 2000. “At a special seminar, spokespersons from the experiments on CERN Heavy Ion programme presented compelling evidence for the existence of a new state of matter in which quarks, instead of being bound up into more complex particles such as protons and neutrons, are liberated to roam freely.”

[https://home.cern/news/press-release/cern/new-state-matter-created-cern]

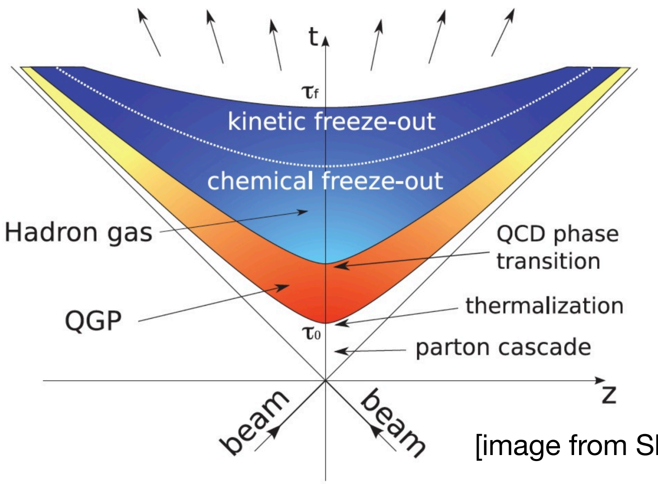
The volume occupied by the quark-gluon plasma formed in heavy-ion collisions is only a few times larger than a nucleus. Heavy ions are used to form a bulk system that is significantly larger than the confinement volume characteristic of a hadron.

# Stages of heavy-ion collisions




[L. McLerran, Physics Today 56, 10, 48 (2003)]




[image from Shusu Shi, BNL thesis, 2010]

**Glasma Formation (hypothesized)**: highly non-equilibrium dynamics of strong chromoelectric and chromomagnetic fields in longitudinal (along the beam) direction. The fields are ordered over the spatial extent of the collision region (coherence).

**Formation of Quark-Gluon Plasma**:
Quarks and gluons exist freely in a hot, dense medium
A high degree of thermalization and collective behavior.

**The QGP expands rapidly and cools down.** The system maintains a state of local thermal equilibrium. The temperatures remain high enough to support a deconfined state of the QGP.

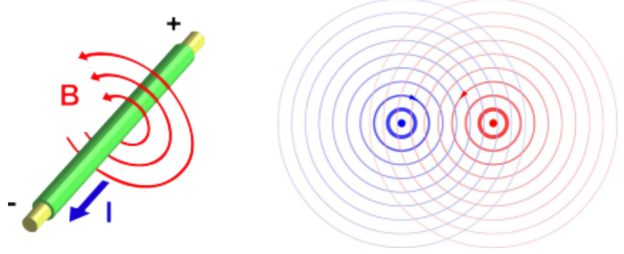
**Chemical Freeze-Out**: As the system continues to expand and cool, it reaches a temperature at which inelastic collisions between particles become rare. At this point, the relative abundances of different particle species (such as the ratios of baryons to mesons) are set.

**Kinetic Freeze-Out**: After chemical freeze-out, the system continues to cool and expand, leading to kinetic freeze-out. This is the stage at which elastic collisions also cease, and particles free-stream to the detectors.

# Noncentral relativistic heavy-ion collisions

## generate both magnetic field and angular momentum

Electromagnetism at work:




**Strong magnetic field**

$$B \sim 10^{14}\,\mathrm{T}$$

STAR Collaboration, Phys. Rev. X 14, 011028 (2024)

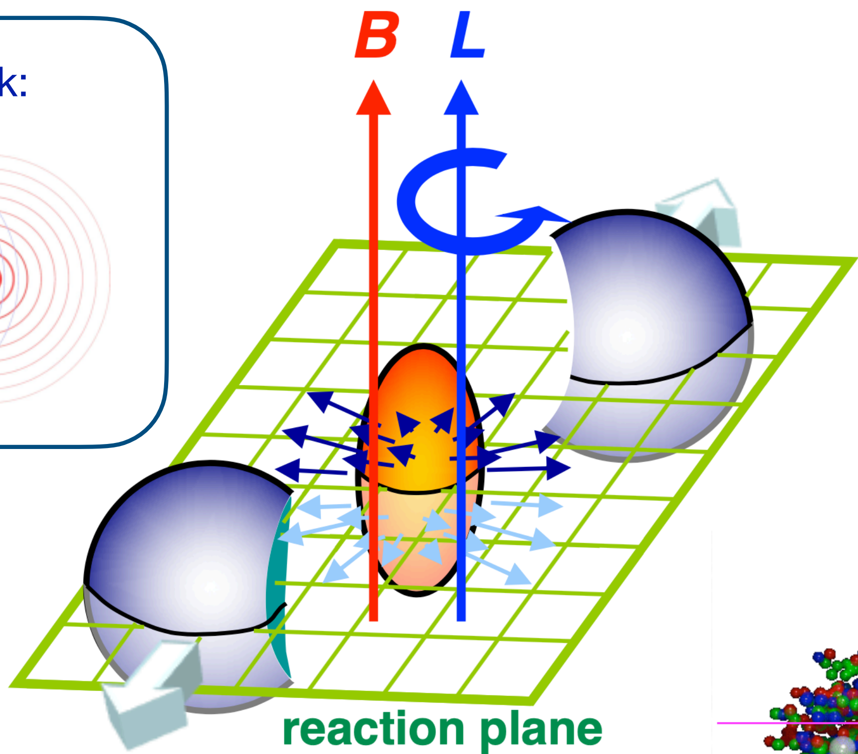
$$eB \sim m_\pi^2 \quad (\tau \sim 0.2 \text{ fm})$$

(?)

et early times of the collision

the effects of magnetic fields may be small (under discussion)

D. Kharzeev, L. McLerran, and H. Warringa, Nucl.Phys.A803, 227 (2008);
McLerran and Skokov, Nucl. Phys. A929, 184 (2014)



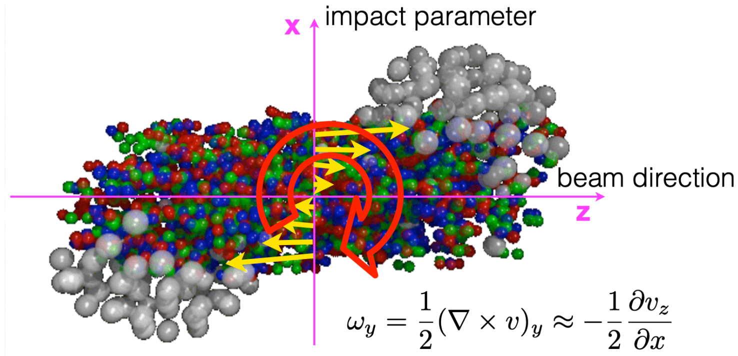

Classical mechanics at work

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \sim 10^6 \hbar$$

**a fraction gets into the QGP:**

$$L_{\mathrm{QGP}} \sim 10^{3-4} \hbar$$

**Large orbital angular momentum**



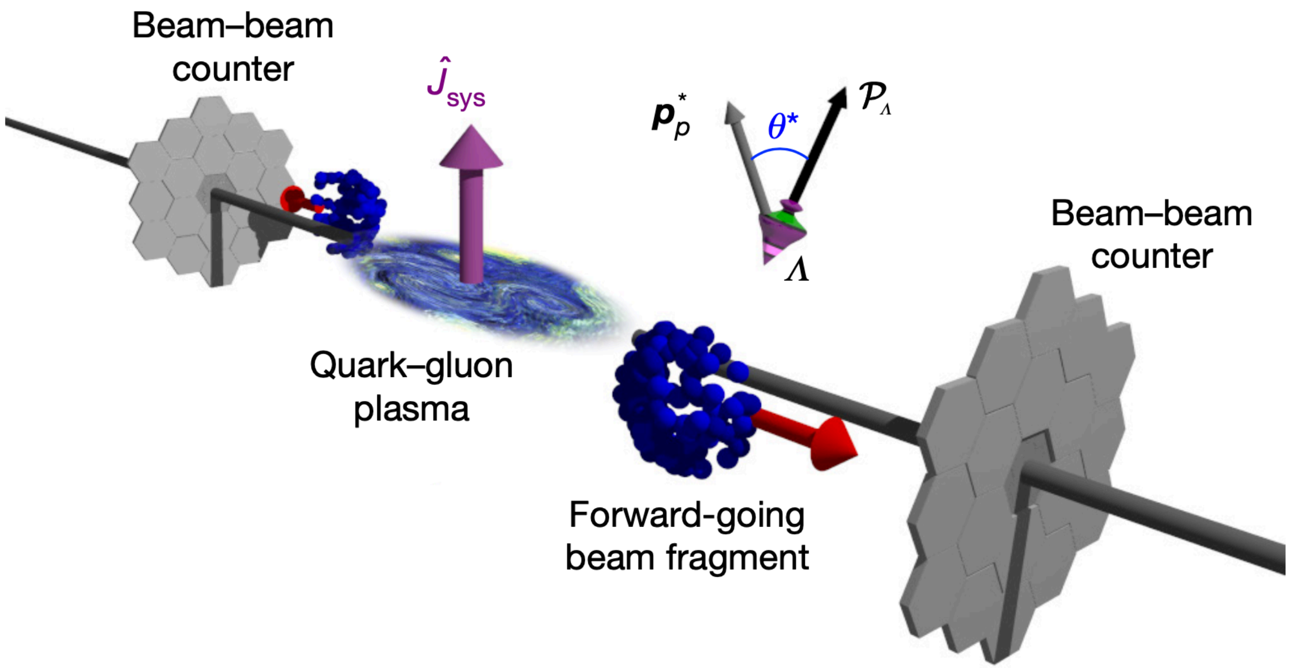

$$\omega_y = \frac{1}{2}(\nabla \times v)_y \approx -\frac{1}{2}\frac{\partial v_z}{\partial x}$$

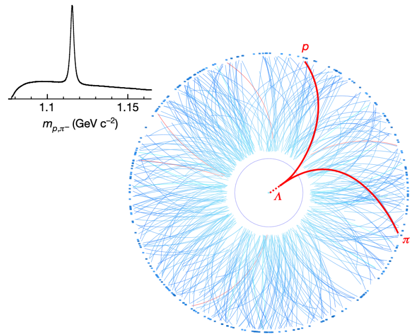
**strong vorticity**

Z.-T. Liang and X.-N. Wang, PRL94, 102301 (2005);
S. Voloshin, nucl-th/0410089 (2004)

# How to measure the polarization?

**The observed hyperon spin polarization ignited much interest.**





$$\Lambda \to p + \pi^-$$

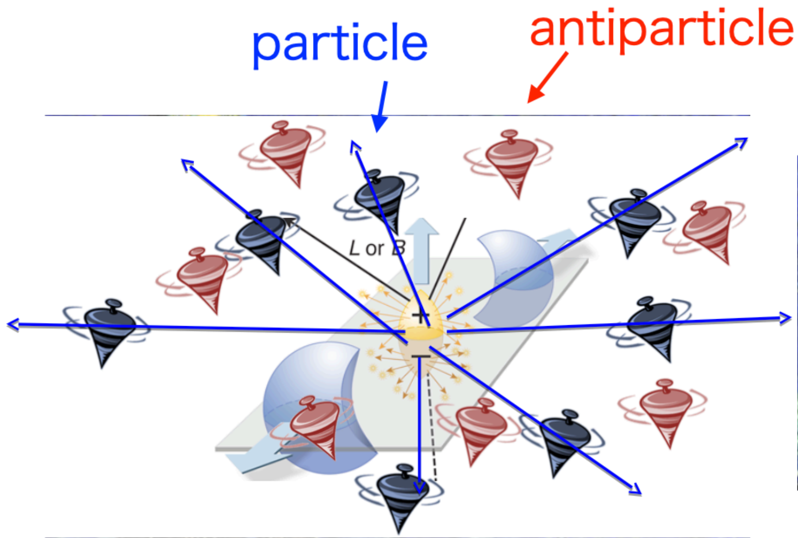
(BR: 63.9%, c$\tau$ ~7.9 cm)

**Overview (including experimental status): "Polarization and Vorticity in the Quark–Gluon Plasma", F. Becattini, M. A. Lisa, Ann.Rev.Nucl.Part.Sci. 70, 395 (2020)**

**Overview of the theoretical models: "Vorticity and Spin Polarization in Heavy Ion Collisions: Transport Models", X.-G. Huang, J. Liao, Q. Wang, X.-L. Xia, Lect.Notes Phys. 987, 281 (2021)**

# How to measure the vorticity?

the vorticity could be probed via quark's spin polarization




The mechanism:

1) orbital angular momentum
of the rotating quark-gluon plasma
is transferred to the particle spin

2) both particles and anti-particles
are polarized in the same way
(spin polarization is not sensitive to the particle charge)

3) The vorticity may be measured via the polarization of the produced particles

**Which particles? Hyperons!**

**The mechanism is the quark/hadronic Barnett effect**

# How do we infer the (global) vorticity of plasma?

## In non-relativistic quantum-mechanics

**mean spin vector:**

$$\boldsymbol{S} = \langle \hat{\boldsymbol{S}} \rangle \equiv \mathrm{Tr}\,(\hat{\rho}\hat{\boldsymbol{S}})$$

**mean polarization:**

$$\boldsymbol{P} = \langle \hat{\boldsymbol{S}} \rangle / S$$

**Statistical operator:**

$$\hat{\rho} = \frac{1}{Z}\exp\left(-\hat{H}/T + \boldsymbol{\omega}\cdot\boldsymbol{J}/T\right)$$

Hamiltonian vorticity

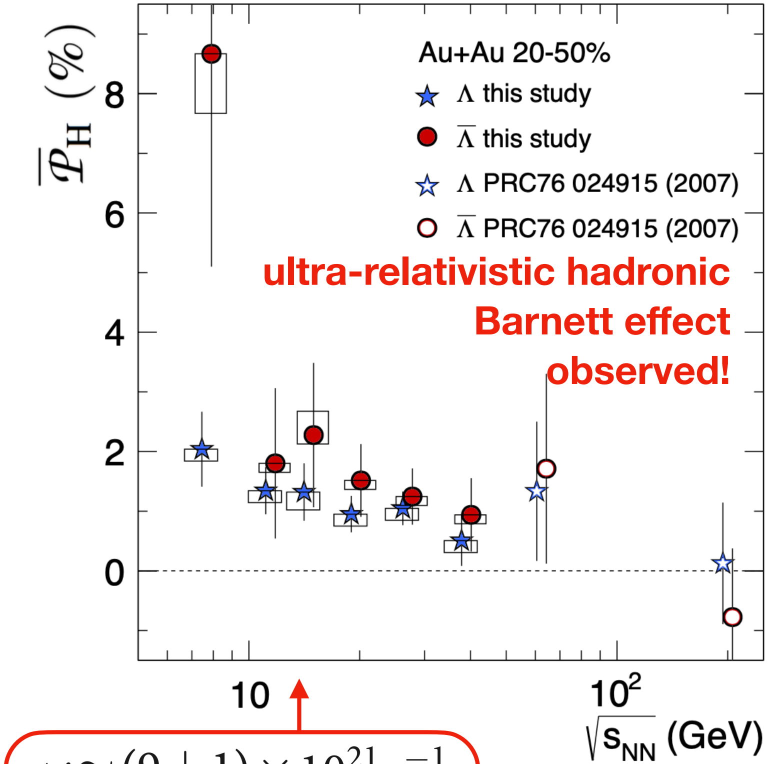
total angular momentum

$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$

orbital + spin

angular velocity $\omega$ plays the role of a chemical potential
for the angular momentum $J$ (and for the spin $S$ in particular)

STAR, Nature 548, 62 (2017)




$\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$

**Mean spin along the vorticity vector:**

$$\boldsymbol{S} = \frac{\boldsymbol{\omega}}{\omega}\frac{\sum_{s_z=-S}^{S} s_z e^{s_z\omega/T}}{\sum_{s_z=-S}^{S} e^{s_z\omega/T}}$$

**For $S = 1/2$ particles (for example, for $\Lambda$):**

$$\boldsymbol{S} = \frac{1}{2}\boldsymbol{P} \simeq \frac{1}{4}\frac{\boldsymbol{\omega}}{T}$$

**Vorticity from polarization:**

$$\omega = k_B T\left(\overline{\mathcal{P}}_{\Lambda'} + \overline{\mathcal{P}}_{\overline{\Lambda}'}\right)/\hbar$$

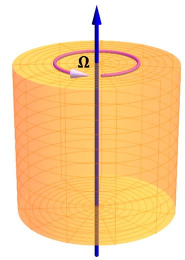
**The Barnett effect in ferromagnets:**

$$M = M_0 \frac{\hbar\Omega}{2k_B T}$$

**Notice: vorticity vs. angular velocity**

The velocity field for a uniformly rotating solid body: $\boldsymbol{v} = \boldsymbol{\Omega}\times\boldsymbol{r}$

Vorticity: $\boldsymbol{\omega} = \frac{1}{2}\nabla\times\boldsymbol{v} = \boldsymbol{\Omega}$




[more details, e.g.: F. Becattini et al, Phys.Rev.C 95 (2017) 5, 054902]

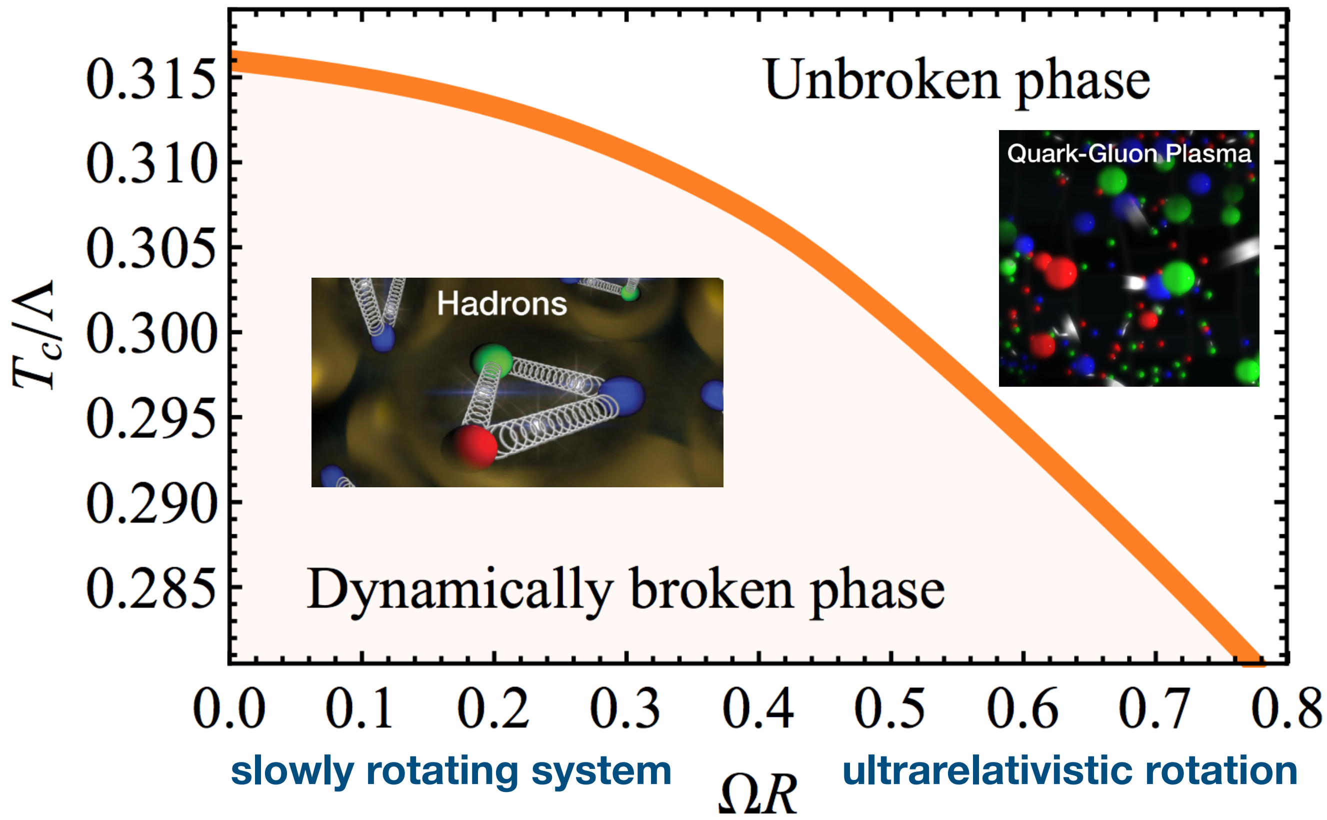
# Phase diagram at finite temperature

## All(*) infrared effective models indicate that

(*) all natural, not specially fine-tuned

## rotation decreases the critical temperature of the chiral phase transition

The critical temperature of the chiral symmetry breaking transition




slowly rotating system

ultrarelativistic rotation

Effective models say that uniform rotation should restore the chiral symmetry

Holographic approaches [B. McInnes, Nucl.Phys. B911 (2016) 173], NJL models [H.-L. Chen, K. Fukushima, X.-G. Huang, K. Mameda, Phys.Rev. D93 (2016) 104052], [Y. Jiang, J. Liao, Phys.Rev.Lett. 117 (2016), 192302]; M.Ch. and Shinya Gongyo, JHEP 01, 136 (2017)

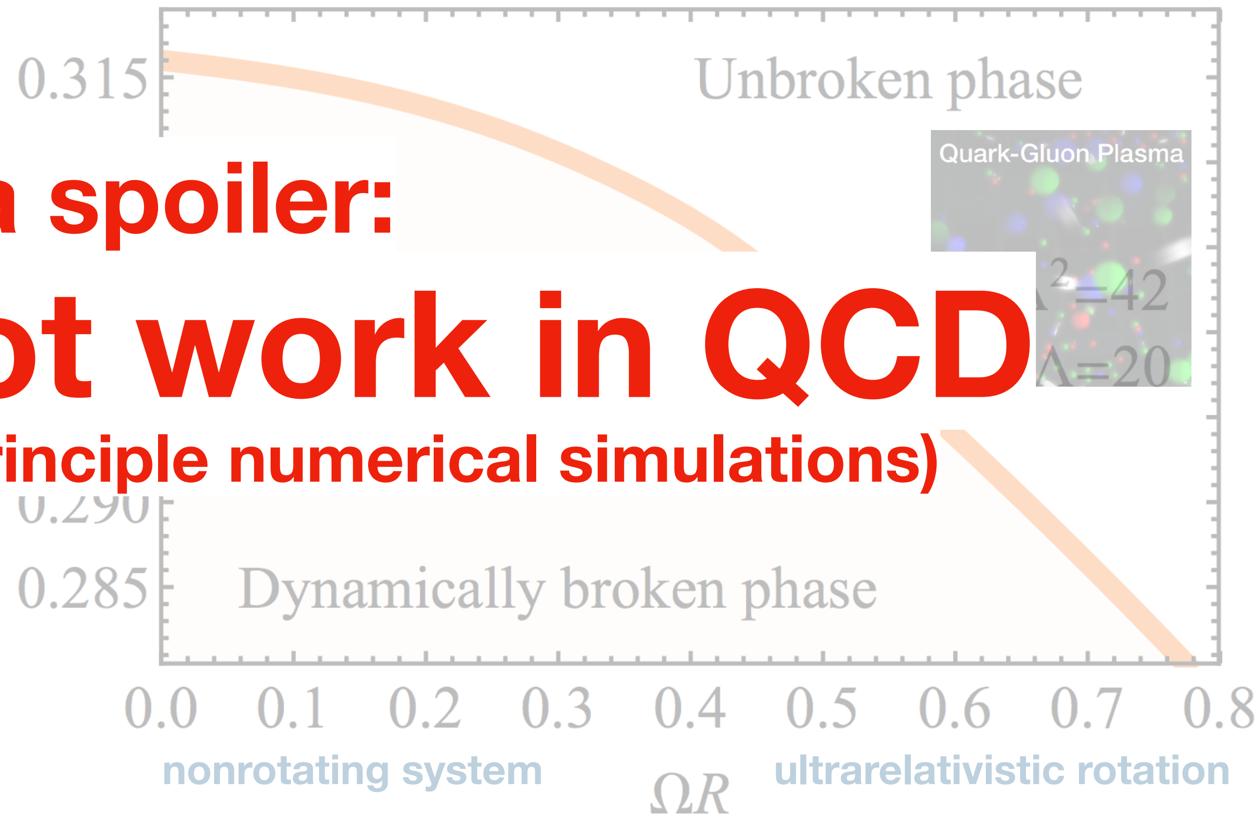
# Phase diagram at finite temperature

## All(*) infrared effective models indicate that

(*) all natural, not specially fine-tuned

## rotation decreases the critical temperature of the chiral phase transition

The critical temperature of the chiral symmetry breaking transition




nonrotating system

ultrarelativistic rotation

Effective models say that uniform rotation should restore the chiral symmetry

**a spoiler:**

# does not work in QCD

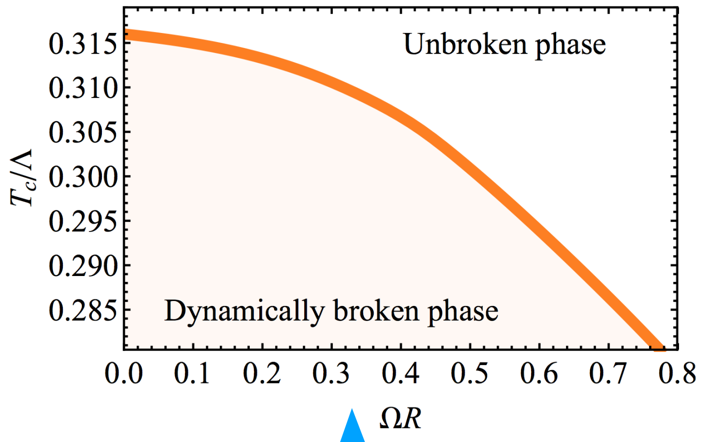
**(recent first-principle numerical simulations)**

Holographic approaches [B. McInnes, Nucl.Phys. B911 (2016) 173], NJL models [H.-L. Chen, K. Fukushima, X.-G. Huang, K. Mameda, Phys.Rev. D93 (2016) 104052], [Y. Jiang, J. Liao, Phys.Rev.Lett. 117 (2016), 192302]; M.Ch. and Shinya Gongyo, JHEP 01, 136 (2017)

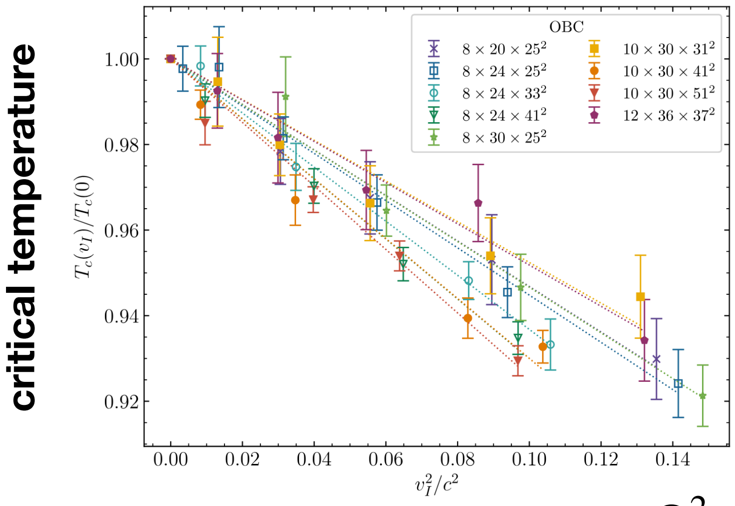
## What is the mechanism?

# Evidence of the failure of our undertsanding

**theory at real-valued rotation**

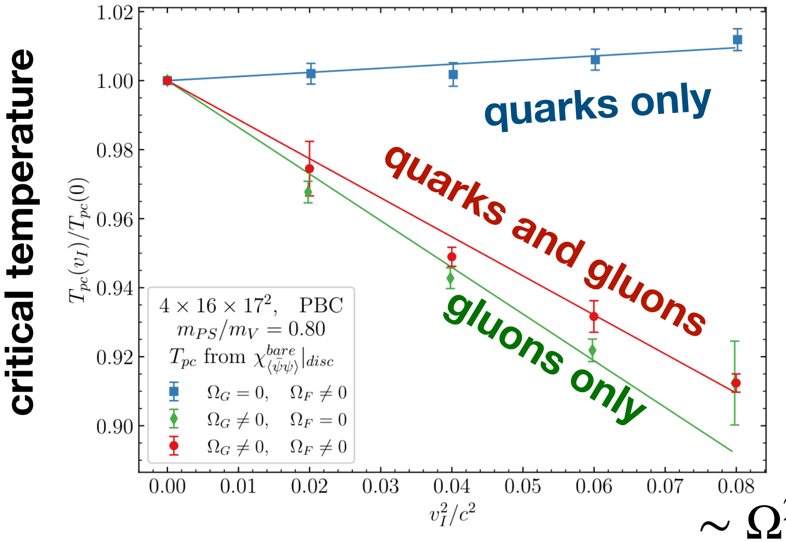



V.V. Braguta et al, Phys.Rev.D 103, 094515 (2021)




$\sim \Omega_I^2$

**who rotates?**

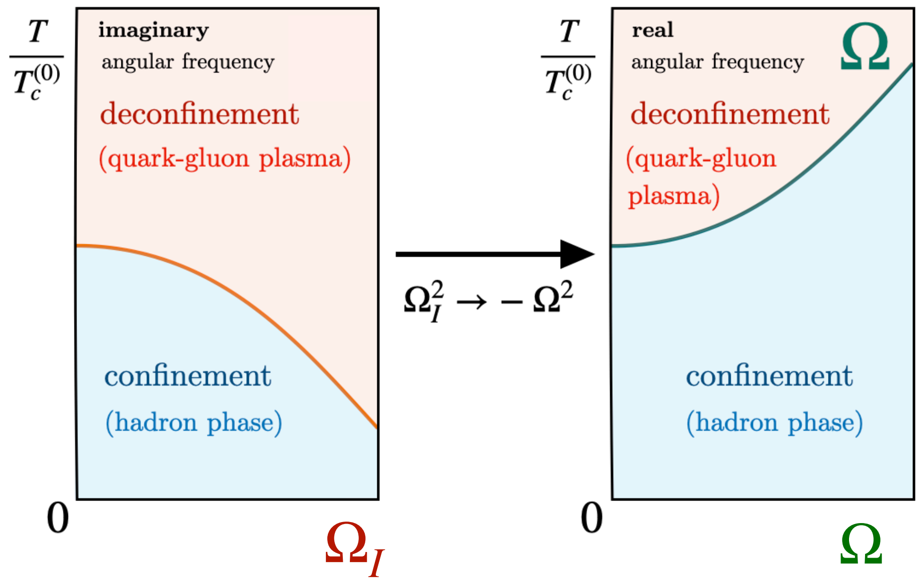



$\sim \Omega_I^2$

[V.V. Braguta, A. Kotov, A. Roenko, D. Sychev, ArXiv:2212.03224, also J.-C. Yang, X.-G. Huang, ArXiv: 2307.05755]

**do not match at all**

**imaginary rotation** **real rotation**



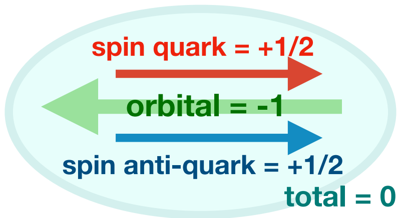
# A would-be quark Barnett effect in vortical QCD matter

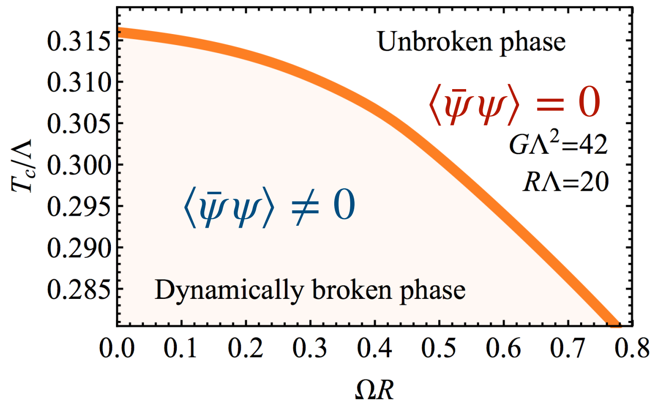
## "Uniform rotation restores the chiral symmetry"

### What could be the mechanism?

The chiral condensate is a spin-0 object

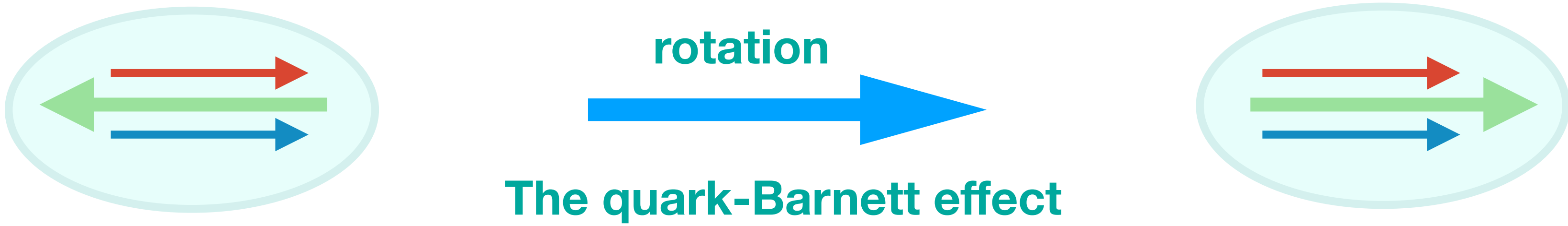
$$\langle \bar{\psi}\psi \rangle = -\frac{\sigma}{2G}$$




the chiral condensate (a quark-anti-quark pairing state with $L = S = 1$ but $J = 0$)

**The Barnett effect should polarize both the spin of a quark and the spin of an anti-quark along the axis of rotation**




**spin-0** **spin-2**

**Suppression effect on the (pseudo)scalar pairing states: The chiral condensate should be destroyed by rotation due to a quark analogue of the Barnett effect**


[Y. Jiang, J. Liao, Phys.Rev.Lett. 117 (2016), 192302]

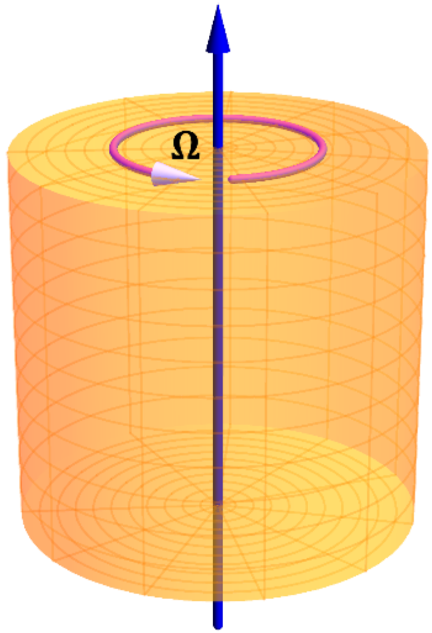
# How do we rotate the rotating medium?
# Free uniformly rotating fermions inside a cylinder

Take free fermions, put them in a cylinder and set them in rotation.

Causality requirement: rotational velocity of fermions should be smaller than the speed of light ($c = 1$).

Put the system in an infinitely high cylinder of radius $R$. The angular frequency $\Omega$ (with $\Omega > 0$ here) is thus bounded:

$$\Omega R \leqslant 1$$




In order to confine the fermions inside the cylinder one should impose an appropriate boundary condition for the fermionic field.

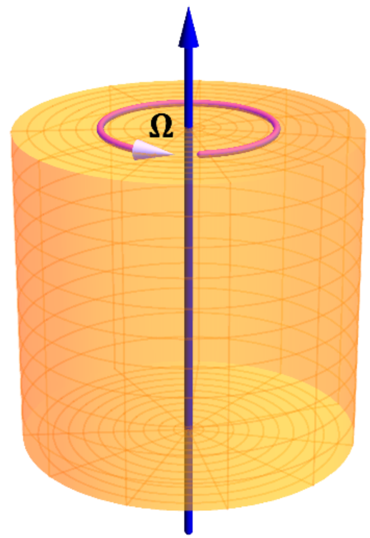
[Following V. E. Ambrus, E. Winstanley, Phys.Rev. D93, 104014 (2016); M.Ch. and Shinya Gongyo, JHEP 01, 136 (2017)]

# Coordinates

Cylindrical coordinates:

$$x \equiv (x_0, x_1, x_2, x_3) = (t, \rho \sin\varphi, \rho\cos\varphi, z)$$

Laboratory frame vs. corotating frame



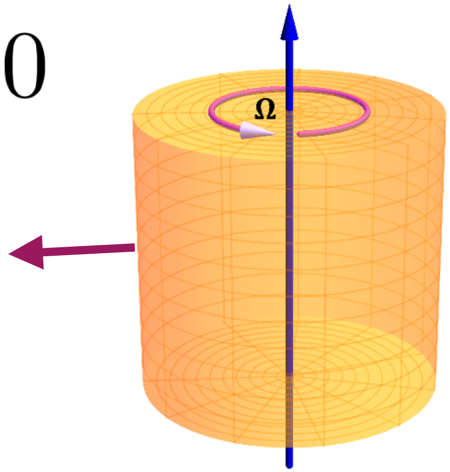

| | |
|---|---|
| *time coordinate* | $t = t_{\mathrm{lab}}$ |
| *radial coordinate* | $\rho = \rho_{\mathrm{lab}}$ |
| *height coordinate* | $z = z_{\mathrm{lab}}$ |
| *angular coordinate* | $\varphi = [\varphi_{\mathrm{lab}} - \Omega t]_{2\pi}$ |

only the azimuthal angular coordinate feels the rotation

# Boundary conditions

We impose the MIT conditions at the boundary of the cylinder:

$$\left[i\gamma^\mu n_\mu(\varphi) - 1\right]\psi(t,z,\rho,\varphi)\Big|_{\rho=R} = 0$$




where

$$n_\mu(\varphi) = (0, R\cos\varphi, -R\sin\varphi, 0)$$

is the vector normal to the boundary.

The normal component of vector (electric) fermionic current

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

it carries electric (vector) charge,
not allowed to communicate
with the exterior

vanishes at each point of the boundary:

$$j_{\boldsymbol{n}} \equiv \boldsymbol{j}\boldsymbol{n} \equiv -j^\mu n_\mu = 0 \qquad \text{at} \quad \rho = R$$

# Curvilinear coordinates in corotating frame

Physics is described by an eigensystem of the Dirac equation formulated in the corotating (not laboratory!) frame.

→ The laboratory system is the flat Minkowski spacetime
→ The corotating system, given by the transformation,

$$t = t_{\mathrm{lab}} \quad \rho = \rho_{\mathrm{lab}} \quad z = z_{\mathrm{lab}} \quad \varphi = [\varphi_{\mathrm{lab}} - \Omega t]_{2\pi}$$

is described by the metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

← we use here the Cartesian coordinate system

corresponding to the line element of the curved space-time:

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = \left(1-\rho^2\Omega^2\right) dt^2 - 2\rho^2 \Omega dt d\varphi - d\rho^2 - \rho^2 d\varphi^2 - dz^2$$

# Curvilinear coordinates in corotating frame

Despite the metric has nontrivial elements,

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

the physical space is still physically flat.

$$R_{\mu\nu\alpha\beta} \equiv 0$$

All components of the Riemann tensor are zero.

Basically, what we did corresponds to a diffeomorphism ≡ a change of coordinates.

However, we can call this space as "curved" since the metric is nontrivial, certain Christoffel symbols are also nonzero.

# Dirac equation in curved space

The Dirac equation in a space with a non-flat metric is

$$\left[i\gamma^{\mu}\left(\partial_{\mu}+\Gamma^{\mu}\right)-M\right]\psi=0$$

where $\Gamma_{\mu}=-\frac{i}{4}\omega_{\mu ij}\sigma^{ij}$ is the affine (spin) connection with

$$\omega_{\mu\hat{i}\hat{j}}=g_{\alpha\beta}e^{\alpha}_{\hat{i}}\left(\partial_{\mu}e^{\beta}_{\hat{j}}+\Gamma^{\beta}_{\nu\mu}e^{\mu}_{\hat{j}}\right)$$ and $$\sigma^{\hat{i}\hat{j}}=\frac{i}{2}\left[\gamma^{\hat{i}},\gamma^{\hat{j}}\right]$$

Indices with the hats = laboratory frame; indices without hats = curved frame.

Christoffel connection:

$$\Gamma^{\lambda}_{\mu\nu}=\tfrac{1}{2}g^{\lambda\sigma}\left(g_{\sigma\nu,\mu}+g_{\mu\sigma,\nu}-g_{\mu\nu,\sigma}\right)$$

vierbein

Gamma matrices in the curved space-time, $\gamma^{\mu}=e^{\mu}_{\hat{i}}\gamma^{\hat{i}}$

fulfill the anticommutation relation: $\{\gamma^{\mu},\gamma^{\nu}\}=2g^{\mu\nu}$

# Vierbein, spin connection, matrices in rotation

The vierbein $e^{\mu}_{\hat{i}}$ is the “inverse square root” of the metric:

$$\underset{\text{flat metric}}{\underset{\uparrow}{\eta_{\hat{i}\hat{j}}}} = \underset{\text{curved metric}}{\underset{\uparrow}{g_{\mu\nu}}} e^{\mu}_{\hat{i}} e^{\nu}_{\hat{j}}$$

In the corotating frame the only nonzero components of the vierbein

$$e^{t}_{\hat{t}} = e^{x}_{\hat{x}} = e^{y}_{\hat{y}} = e^{y}_{\hat{y}} = 1, \qquad e^{x}_{\hat{t}} = y\Omega, \qquad e^{y}_{\hat{t}} = -x\Omega$$

The nonzero components of the Christoffel symbol are

$$\Gamma^{y}_{tx} = \Gamma^{y}_{xt} = \Omega, \quad \Gamma^{x}_{ty} = \Gamma^{x}_{yt} = -\Omega, \quad \Gamma^{x}_{tt} = -x\Omega^2, \quad \Gamma^{y}_{tt} = -y\Omega^2$$

One nonzero component of the spin connection $\Gamma_t = -\dfrac{i}{2}\Omega\,\sigma^{\hat{x}\hat{y}}$

Gamma matrices:

$$\gamma^{t} = \gamma^{\hat{t}}, \qquad \gamma^{x} = y\Omega\gamma^{\hat{t}} + \gamma^{\hat{x}}, \qquad \gamma^{y} = -x\Omega\gamma^{\hat{t}} + \gamma^{\hat{y}}, \qquad \gamma^{z} = \gamma^{\hat{z}}$$

# The Dirac equation in a rotating frame

The Dirac equation:

**Effect of rotation is expected:** $H \to H - \boldsymbol{\Omega} \cdot \boldsymbol{J}$

$$\left[\gamma^{\hat{t}}\left(i\partial_t + \Omega J_z\right) + i\gamma^{\hat{x}}\partial_x + i\gamma^{\hat{y}}\partial_y + i\gamma^{\hat{z}}\partial_z - M\right]\psi = 0$$

where

$$\hat{J}_z = -i\left(-y\partial_x + x\partial_y\right) + \frac{1}{2}\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \equiv -i\partial_\varphi + \frac{1}{2}\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

is the *z*-component of the total angular momentum and

$$\sigma^{\hat{x}\hat{y}} = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}$$

in the Dirac representation of the gamma matrices:

$$\gamma^{\hat{t}} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \quad \gamma^{\hat{i}} = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

# Solutions: quantum numbers

The wave function

4-spinor

$$U_j = \frac{1}{2\pi} e^{-i\widetilde{E}t + ik_z z} u_j(\rho, \varphi)$$

is characterized by the quantum numbers:

$$j = (k_z, m, l, \mathrm{sign}(E))\,, \qquad m \in \mathbb{Z}\,, \qquad l = 1, 2, \ldots\,, \qquad k_z \in \mathbb{R}$$

distinguishes particle/antiparticle

angular momentum quantum number (a projection to *z* axis)

radial excitation quantum number

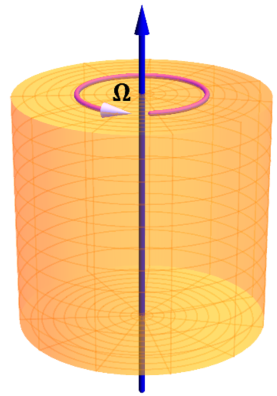
momentum along the *z* axis

The eigenstate of the *z*-component of the angular momentum:

$$\hat{J}_z U_j = \mu_m U_j \qquad \mu_m = m + \frac{1}{2}$$

# Solutions: energy in laboratory/rotating frames

Energy in the corotating frame:




$$\widetilde{E}_j = E_j - \Omega\left(m + \frac{1}{2}\right) \equiv E_j - \Omega\mu_m$$

Energy in the laboratory frame:

$$E_j \equiv E_{ml}(k_z, M) = \pm\sqrt{k_z^2 + \frac{q_{ml}^2}{R^2} + M^2}$$

where $q_{ml}$ is the $l^{\mathrm{th}}$ positive root of the equation

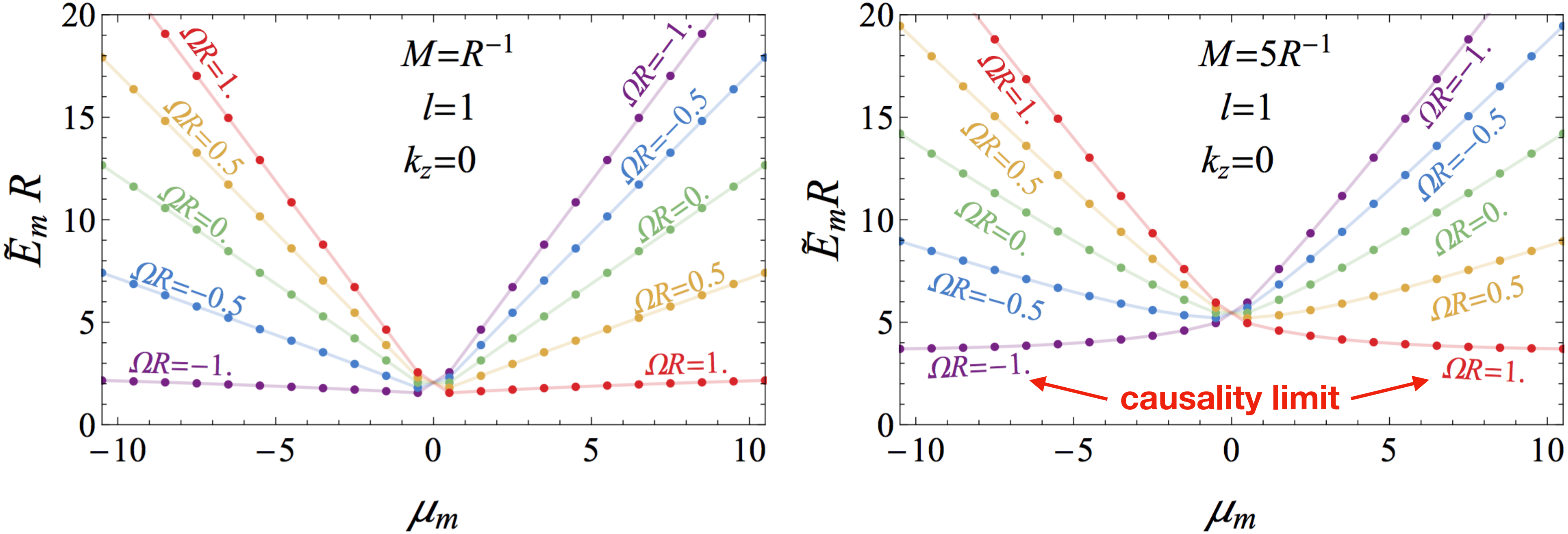
$$\mathrm{j}_m^2(q) + \frac{2MR}{q}\mathrm{j}_m(q) - 1 = 0 \quad \text{with} \quad \mathrm{j}_m(x) = \frac{J_m(x)}{J_{m+1}(x)}$$

the boundary condition: $j_{\boldsymbol{n}} \equiv \boldsymbol{j}\boldsymbol{n} \equiv -j^\mu n_\mu = 0 \qquad \mathrm{at} \quad \rho = R$

[V. E. Ambruş, E. Winstanley, Phys.Lett.B 734 (2014) 296; Phys.Rev.D 93 (2016) 10, 104014]

# Energy spectrum: examples

**Lowest energy levels in co-rotating frame**

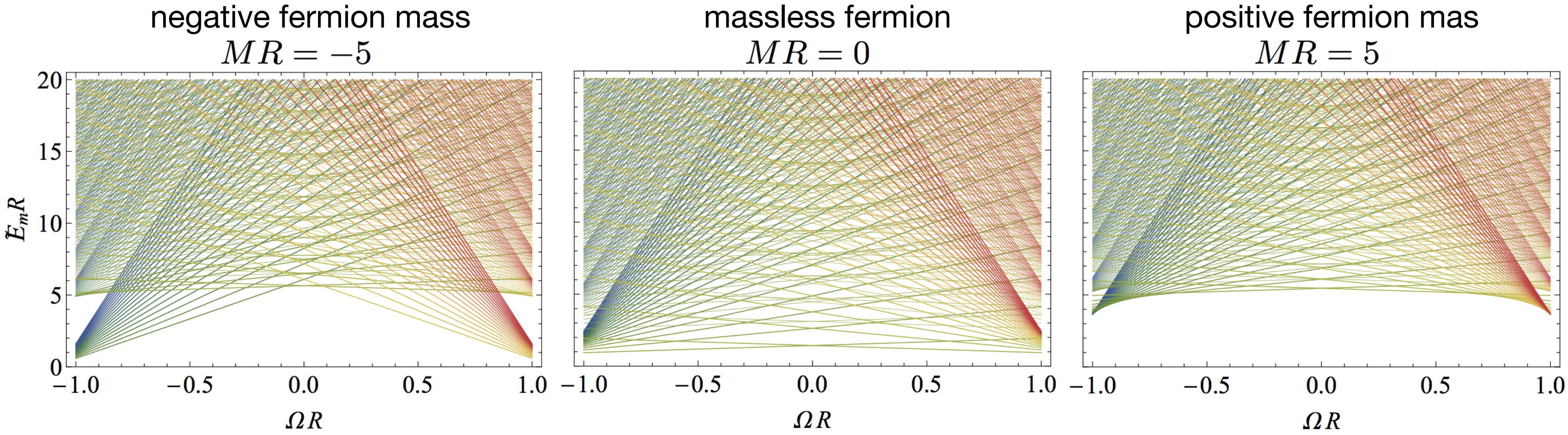



$$\hat{J}_z \psi = \mu_m \psi$$

Features of spectrum:

- convex as function of total angular momentum $\mu_m = m + \frac{1}{2}$
- linear at large angular momentum
- symmetric with respect to a simultaneous inversion of the angular frequency $\Omega$ and the angular momentum $\mu_m$

# Energy spectrum vs. angular momentum




Tower-like structure of the spectrum.

Each tower corresponds to a fixed radial number $l$.

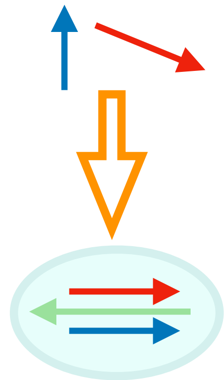
Blue color: positive total angular momentum.
Green color: angular momentum close to zero.
Red color: negative total angular momentum.

No degeneracy at the lowest rotational "Landau level":
rotation is different from applying magnetic field

# Interacting theory: NJL model

Natural question: what is the effect of the rotations on the chiral symmetry breaking in an interacting fermionic theory?

**A relativistic generalization of the Bardeen Cooper Schrieffer (1957) model of superconductivity:**



Consider the Nambu-Jona-Lasinio model:

self-interaction of fermions

$$S_{\mathrm{NJL}} = \int_{\mathrm{V}} d^4x \sqrt{-\det\left(g_{\mu\nu}\right)}\, \mathcal{L}_{\mathrm{NJL}}\left(\bar{\psi}, \psi\right),$$

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}\left[i\gamma_\mu\left(\partial^\mu + \Gamma^\mu\right) - m_0\right]\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi} i\gamma_5\psi\right)^2\right]$$

rotation

The bare fermion mass will be set to zero for simplicity. In this case the Lagrangian is chirally invariant: $\psi \to e^{i\gamma_5\omega}\psi \;\; ; \;\; \bar{\psi} \to \bar{\psi}e^{i\gamma_5\omega}$

In a certain region of the temperature *T* and the coupling *G*, the model experiences the spontaneous breaking of the chiral symmetry by developing a nonzero dynamical mass of the fermion.

# NJL model: mean field analysis

Take the partition function

$$\mathcal{Z} = \int D\psi D\bar{\psi} \exp\left\{ i \int_V d^4x \, \mathcal{L}_{\mathrm{NJL}} \right\}$$

Insert the identity

$$1 = \int D\pi D\sigma \exp\left\{ -\frac{i}{2G} \int_V d^4x \left[ \left(\sigma + G\bar{\psi}\psi\right)^2 + \left(\pi + G\bar{\psi}i\gamma_5\psi\right)^2 \right] \right\}$$

Cancel the four-fermionic term

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}\left[ i\gamma_\mu \left(\partial^\mu + \Gamma^\mu\right) - m_0 \right]\psi + \frac{G}{2}\left[ \left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\psi\right)^2 \right]$$

Integrate over the fermions

$$\mathcal{Z} = \int D\pi D\sigma \exp\left\{ -\frac{i}{2G} \int_V d^4x \left(\sigma^2 + \pi^2\right) + \ln \mathrm{Det}\left[ i\gamma_\mu \left(\partial^\mu + \Gamma^\mu\right) - m_0 - \left(\sigma + i\gamma_5 \pi\right) \right] \right\}$$

the fermionic determinant

# NJL model: chiral rotation

$$\mathcal{Z} = \int D\pi D\sigma \exp\left\{-\frac{i}{2G}\int_V d^4x\left(\sigma^2+\pi^2\right) + \ln \mathrm{Det}\left[i\gamma_\mu\left(\partial^\mu+\Gamma^\mu\right) - m_0 - \left(\sigma + i\gamma_5\pi\right)\right]\right\}$$

the fermionic determinant

Assume that $\sigma$ and $\pi$ are space-independent fields

Perform the chiral rotation

$$\sigma + i\gamma_5\pi \to e^{-i\gamma_5\theta}\left(\sigma + i\gamma_5\pi\right)e^{-i\gamma_5\theta} = \tilde{\sigma}(\sigma,\pi)$$

Get the new scalar field $|\tilde{\sigma}| = \sqrt{\sigma^2+\pi^2}$

… and the simplified determinant:

$$\mathrm{Det}\left[i\gamma_\mu\left(\partial^\mu+\Gamma^\mu\right) - \left(\sigma + i\gamma_5\pi\right)\right] = \mathrm{Det}\left[e^{-i\gamma_5\theta}\left\{i\gamma_\mu\left(\partial^\mu+\Gamma^\mu\right) - \left(\sigma + i\gamma_5\pi\right)\right\}e^{-i\gamma_5\theta}\right]$$

$$= \mathrm{Det}\left[i\gamma_\mu\left(\partial^\mu+\Gamma^\mu\right) - e^{-i\gamma_5\theta}\left(\sigma + i\gamma_5\pi\right)e^{-i\gamma_5\theta}\right] = \mathrm{Det}\left[i\gamma_\mu\left(\partial^\mu+\Gamma^\mu\right) - \tilde{\sigma}\right],$$

denote below $\tilde{\sigma} \to \sigma$

# NJL model → effective bosons model

$$\mathcal{Z} = \int D\pi D\sigma \exp\left\{ -\frac{i}{2G}\int_V d^4x \left(\sigma^2 + \cancel{\pi^2}\right) + \ln \mathrm{Det}\left[i\gamma_\mu\left(\partial^\mu + \Gamma^\mu\right) - \cancel{\mu\gamma}_0 - \left(\sigma + i\cancel{\gamma_5\pi}\right)\right]\right\}$$

Density of the Helmholtz free energy (the thermodynamic potential):

$$\widetilde{F}(\sigma, \pi) = \frac{\sigma^2}{2G} + V(\sigma)$$

The potential induced by the vacuum fermion loop:

$$V(\sigma) = -\frac{i}{\mathrm{Vol}_4} \ln \mathrm{Det}\left[i\gamma_\mu\left(\partial^\mu + \Gamma^\mu\right) - \sigma\right] \equiv -\frac{i}{\mathrm{Vol}_4} \mathrm{tr} \ln\left[\left(i\partial_t + \Omega\hat{J}_z\right)^2 + \vec{\partial}^2 - \sigma^2\right]$$

the effect of the rotation

the angular momentum about the axis of rotation

$$\hat{J}_z = -i\left(-y\partial_x + x\partial_y\right) + \frac{1}{2}\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \equiv -i\partial_\varphi + \frac{1}{2}\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

# NJL model: effective potential

$$V(\sigma;T,\Omega) = \underbrace{V_{\text{vac}}(\sigma)}_{\text{vacuum}} + \underbrace{V_{\text{rot}}(\sigma;T,\Omega)}_{\text{matter}}$$

The matter part:

coupling between global rotation $\Omega$, quarks orbital and spin angular momenta

$$\mu_m = m + \frac{1}{2}$$

$$V_{\text{rot}}(\sigma;T,\Omega) = -\frac{T}{\pi R^2}\sum_{m\in\mathbb{Z}}\sum_{l=1}^{\infty}\int\frac{dk_z}{2\pi}$$

$$\times\left[\ln\left(1+e^{-\frac{E_{ml}(k_z,\sigma)-\Omega\mu_m}{T}}\right)+\ln\left(1+e^{-\frac{E_{ml}(k_z,\sigma)+\Omega\mu_m}{T}}\right)\right]$$

Thermal occupation numbers depend on the fermionic energy calculated in the rotating frame

$$n(T,\Omega) = \left(1+e^{-\frac{E-\Omega\mu_m}{T}}\right)^{-1}$$

The divergent vacuum energy depends neither on temperature nor on the rotation velocity

$$V_{\text{vac}}(\sigma) = -\frac{1}{\pi R^2}\sum_{m\in\mathbb{Z}}\sum_{l=1}^{\infty}\int\frac{dk_z}{2\pi}E_{ml}(k_z,\sigma)$$

# Rotation and occupation numbers

The angular frequency Ω works as a “chemical potential” for the angular momentum $\mu_m$

$$n(T,\Omega) = \left(1 + e^{-\frac{E-\Omega\mu_m}{T}}\right)^{-1}$$

Angular momentum operator:

$$\hat{J}_z = -i\left(-y\partial_x + x\partial_y\right) + \frac{1}{2}\begin{pmatrix}\sigma_3 & 0 \\ 0 & \sigma_3\end{pmatrix} \equiv -i\partial_\varphi + \frac{1}{2}\begin{pmatrix}\sigma_3 & 0 \\ 0 & \sigma_3\end{pmatrix}$$

with the eigenvalues:

$$\hat{J}_z\psi = \mu_m\psi\,, \qquad \mu_m = m + \frac{1}{2}$$

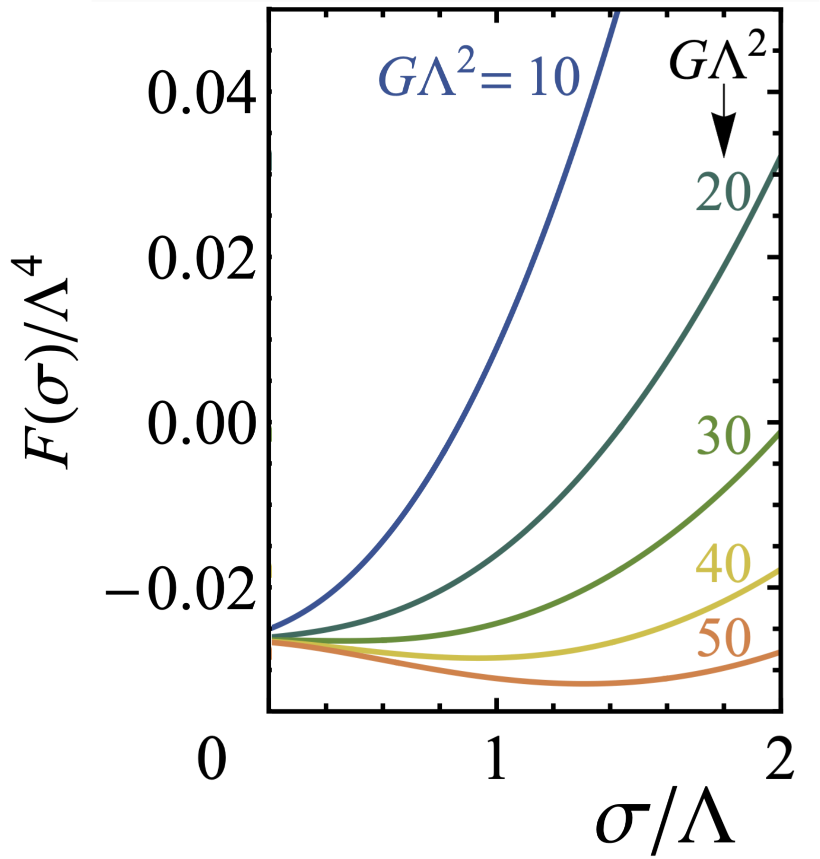
# Interacting theory: NJL model

The value of condensate $\sigma$ in the ground state is determined by the global minimum of the free energy:

$$\widetilde{F}(\sigma) = \frac{\sigma^2}{2G} + V_{\mathrm{vac}}(\sigma) + V_{\mathrm{rot}}(\sigma; T, \Omega)$$

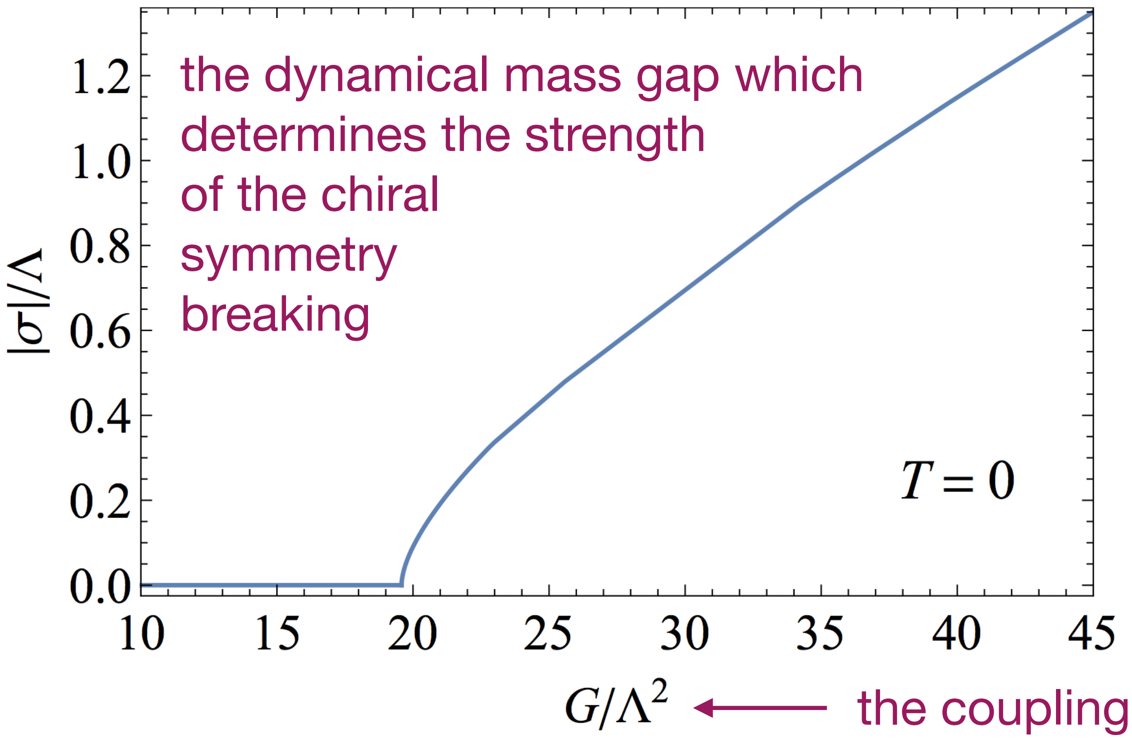
The (nonzero) condensate $\sigma$ corresponds to the value of a dynamically generated fermionic mass.

The fermion acquires mass and the chiral symmetry gets (dynamically) broken.



# Spontaneous symmetry breaking at $T = 0$

## Mass gap generation at zero temperature in unbounded space:




← the coupling of the model

$$\mathcal{L}_{\text{NJL}} = \bar{\psi}\left[i\gamma_\mu\left(\partial^\mu + \Gamma^\mu\right) - m_0\right]\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\psi\right)^2\right]$$

The model is not renormalizable (but very successful in describing chiral properties of QCD including low energy spectra of hadrons and their decays).

Here $\Lambda$ is the ultraviolet cutoff which plays the role of a physical parameter of the model.

Bounded case: At zero temperature $T = 0$ the fermions are insensitive to a uniform rotation. In particular, the mass gap does not depend on the angular frequency $\Omega$.

→ **"Cold vacuum does not rotate".**

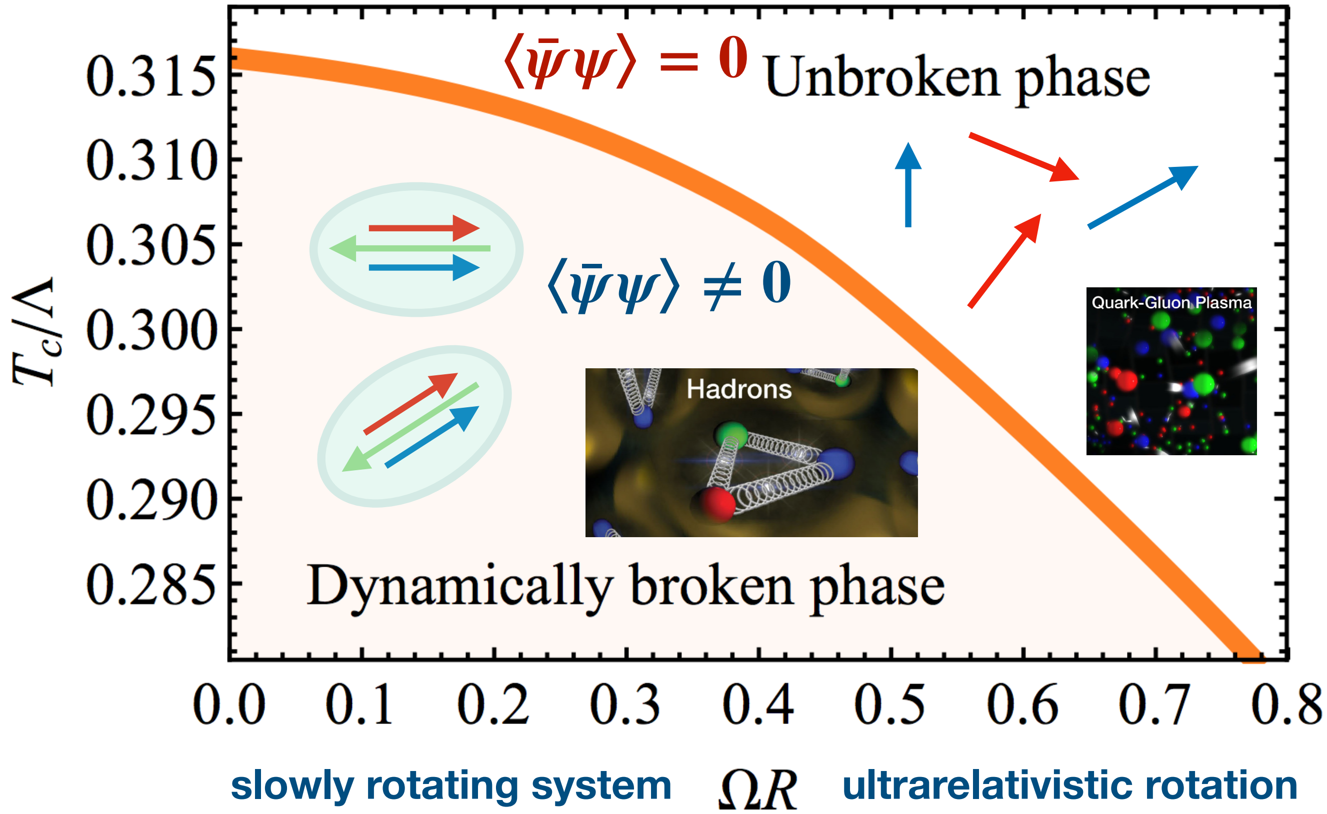
**causality constraint:**
**corotating energy is always positive**

$$n(T, \Omega) = \left(1 + e^{-\frac{E - \Omega\mu_m}{T}}\right)^{-1}$$

# Phase diagram at finite temperature from the NJL model

“rotation decreases the critical temperature of the chiral phase transition”

The critical temperature of the chiral symmetry breaking transition

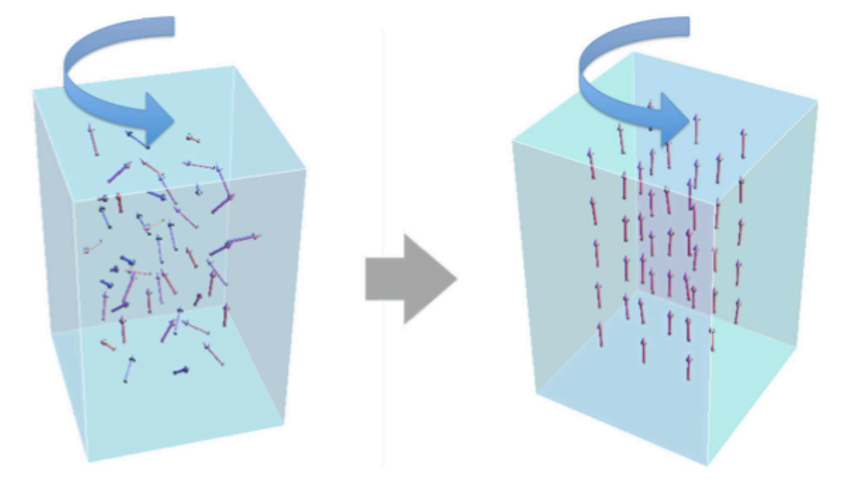



This effective model says that uniform rotation should restore the chiral symmetry

This approach, which intrinsically relies on the quark Barnett effect does not work in QCD (recent lattice simulations)

# A short summary of the previous lectures

## Coupling between mechanical rotation and spin orientation



**Barnett effect.** Magnetization is induced by applying mechanical rotation since an effective magnetic field, emerges in a rotating body.

**Effective magnetic field:** $B_\Omega = \Omega/\gamma$

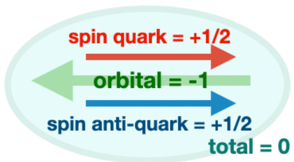
$\gamma$ is the gyromagnetic ratio

**We discussed the Barnett effects for**

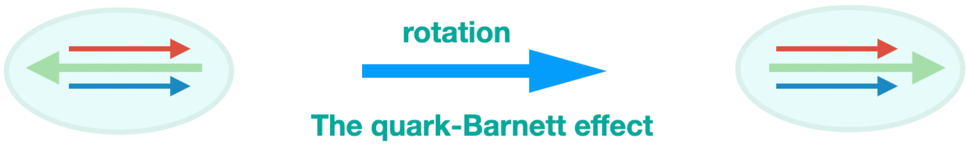
- **magnetic moments in a solid ferromagnetic,**
- **electrons in a liquid metal,**
- **for nuclei in rotating liquid (protons in water),**
- **in ultrarelativistic hadronic physics ($\Lambda$ hyperons)**
- **for quarks in quark-gluon plasma**

**It does not work for quarks!**

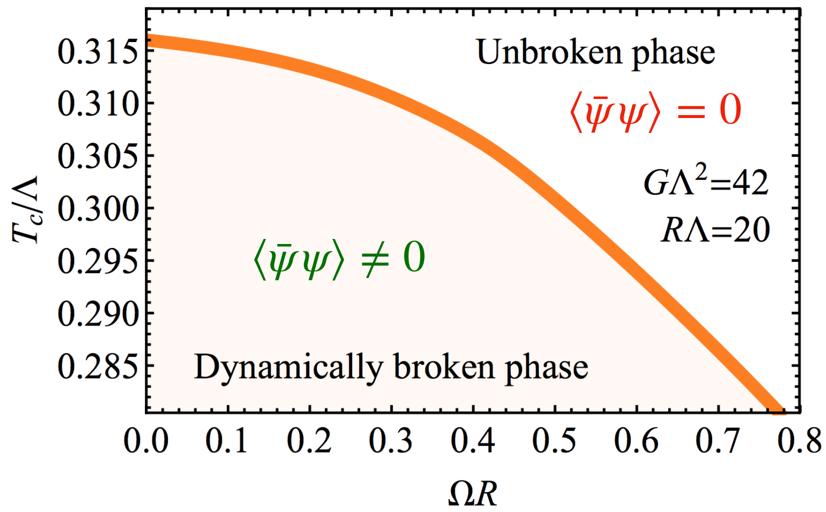
$$\langle\bar\psi\psi\rangle = -\frac{\sigma}{2G}$$




the chiral condensate (a quark-anti-quark pairing state with $L = S = 1$ but $J = 0$)

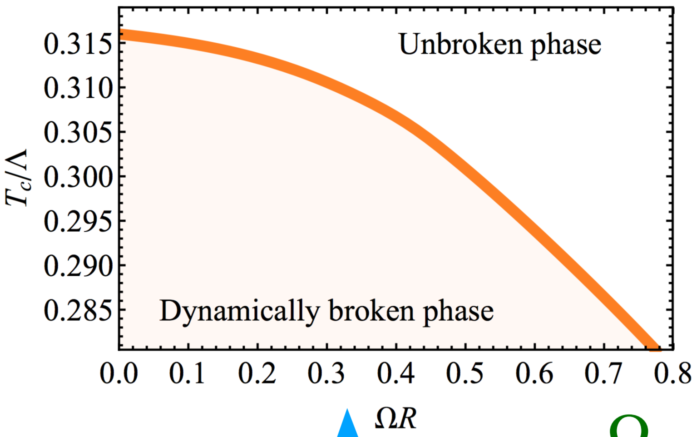



$\langle\bar\psi\psi\rangle \neq 0$ $\qquad$ $\langle\bar\psi\psi\rangle = 0$



# Evidence of the failure of our undertsanding

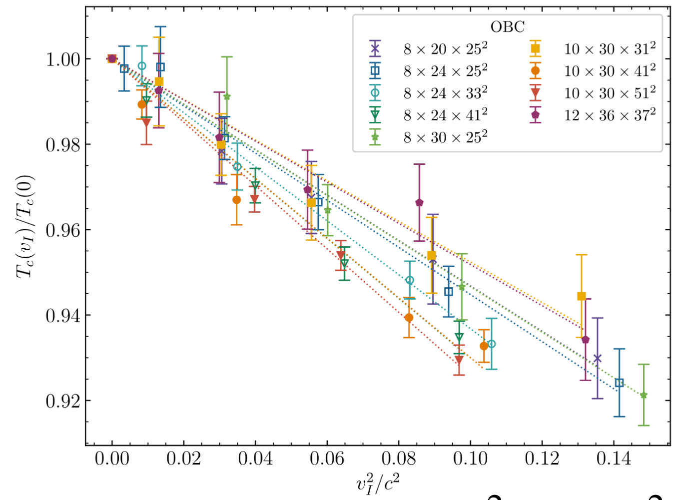
**theory at real-valued rotation**




$\Omega$

V.V. Braguta et al, Phys.Rev.D 103, 094515 (2021)

critical temperature

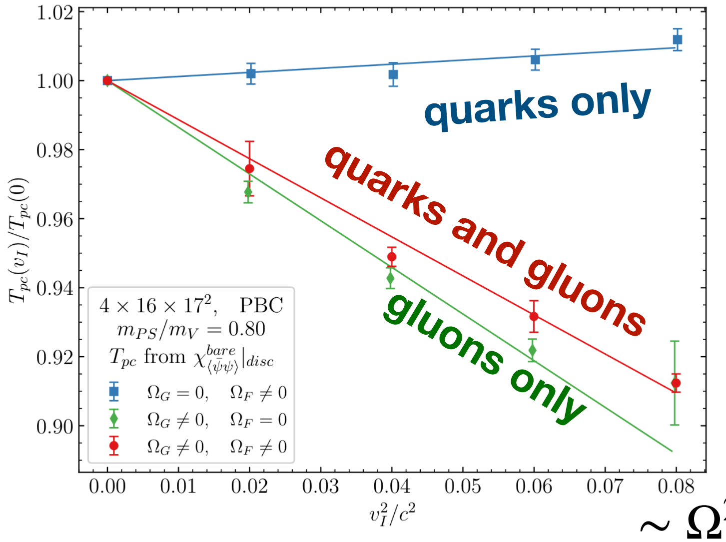



$\sim \Omega_I^2 = -\Omega^2$

**do not match at all**

**who rotates?**

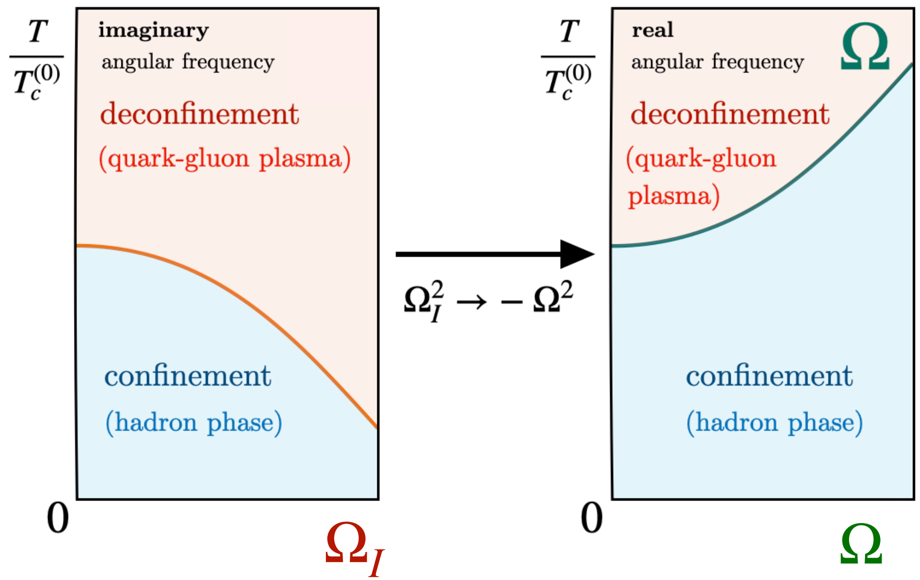
critical temperature




$\sim \Omega_I^2$

[V.V. Braguta, A. Kotov, A. Roenko, D. Sychev, ArXiv:2212.03224,
also J.-C. Yang, X.-G. Huang, ArXiv: 2307.05755]

**imaginary rotation** **real rotation**



# Who spoils the quark Barnett effect in QCD? Gluons (?!!!)

**Thus, the Barnett effect should imply that the spin of gluons gets polarized due to the orbital motion caused by vorticity (rotation) of plasma as a whole.**

**However, what is the spin of a gluon?**

**The decompositon of the gluon angular momentum presents several challenges.(*)**

## The Jaffe-Manohar decomposition:

total angular momentum

$$\mathbf{J} = \mathbf{S}_q + \mathbf{L}_q + \mathbf{S}_g + \mathbf{L}_g$$

$$\mathbf{S}_q = \int d^3x\, \psi^\dagger \frac{\boldsymbol{\Sigma}}{2} \psi$$

quark spin

$$\mathbf{L}_q = \int d^3x\, \psi^\dagger \mathbf{x} \times (-i\mathbf{D})\, \psi$$

quark orbital momentum

$$\mathbf{S}_g = \int d^3x\, \mathbf{E}^a \times \mathbf{A}^a$$

gluon spin

$$\mathbf{L}_g = \int d^3x\, E^{ai} (\mathbf{x} \times \mathbf{D}) A_i^a$$

gluon orbital momentum

**Hopeless??**

$\boldsymbol{\Sigma}$ - the quark spin matrices, $\boldsymbol{D}$ - the covariant derivative,
$\boldsymbol{E}^a$ - the chromoelectric field, $\boldsymbol{A}^a$ - the gluon vector potential

**Intuitively understood**
**but not a gauge-invariant separation!**

[R. L. Jaffe, A. Manohar, Nuclear Physics B, 337, 509 (1990)]

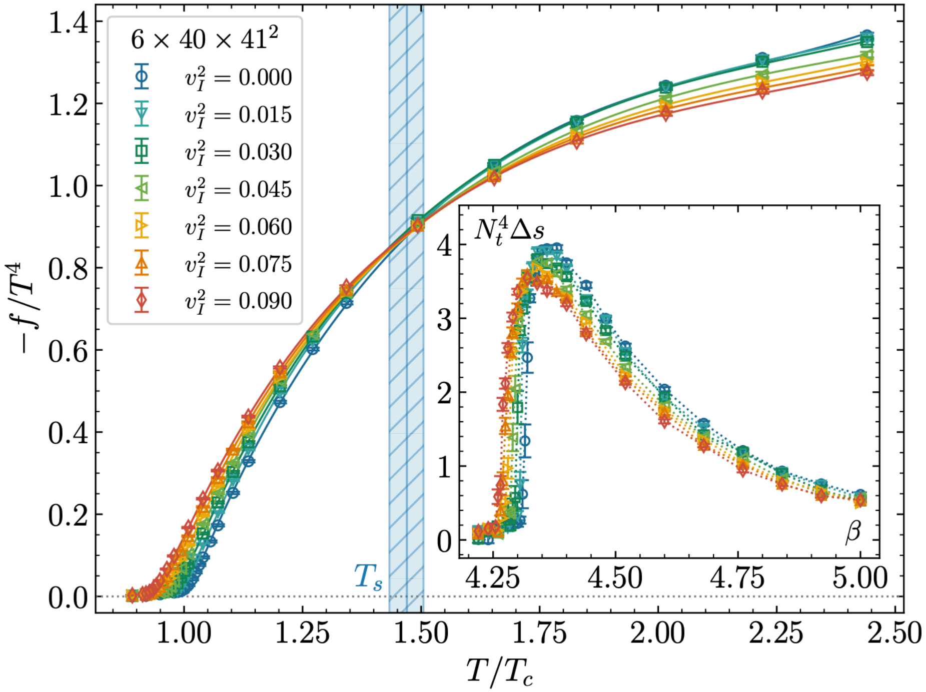
A gauge-invariant but non-local alternative: [X. Ji, Phys. Rev. Lett., 78, 610 (1997)]

(*) **The proton spin crisis**. In 1988, the European Muon Collaboration (EMC) conducted deep inelastic scattering (DIS) experiments that aimed to measure the spin structure of the proton. The EMC found that the quark spins contributed only a small fraction, about 20-30%, of the proton's total spin (significantly lower than expected). The planned Electron-Ion Collider (EIC), is expected to offer unprecedented precision in studying the proton spin structure.

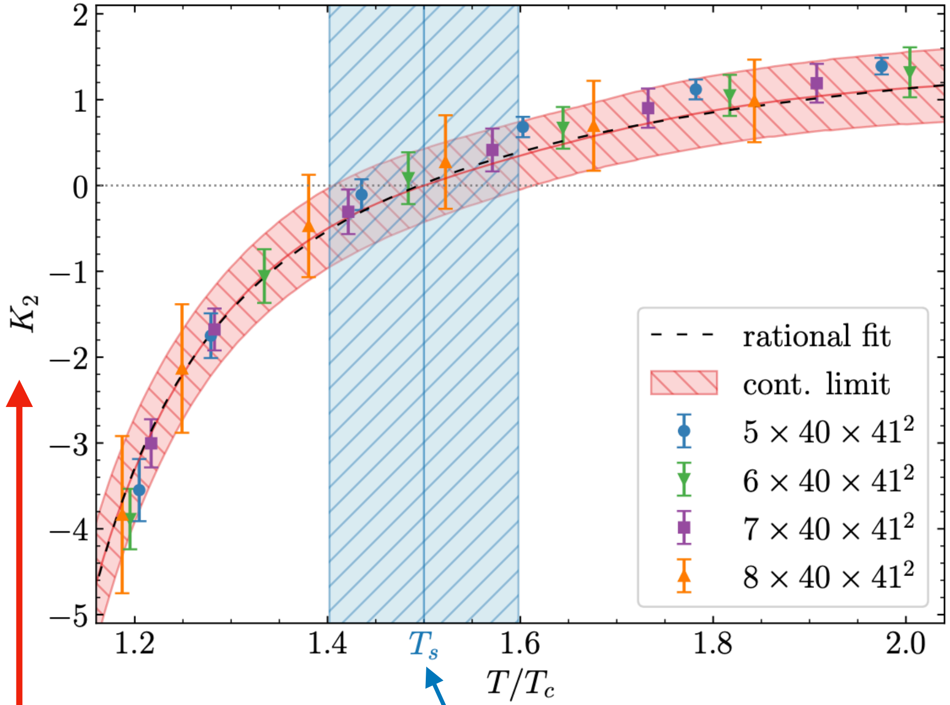
[EMC, *Phys.Lett.B* 206, 364 (1988)]

## Exotic behavior of critical temperature: a result of an exotic Barnett effect for gluons?

### Free energy density in SU(3) Yang-Mills theory




### The dimensionless moment of inertia




**negative moment of inertia**

**supervortical temperature**

$$T_s = 1.50\,(10)\,T_c$$

$$f(T, v_I) = f_0(T)\left(1 - \frac{1}{2}K_2(T)v_I^2\right)$$

linear imaginary velocity at the boundary $v_I = \Omega_I R$

$$I(T) \equiv \lim_{\Omega \to 0} I(T, \Omega) = -K_2(T)F_0(T)R^2$$

notice that $F_0(T) < 0$

$$K_2^{(\text{fit})}(T) = K_2^{(\infty)} - \frac{c}{T/T_c - 1}$$

$$K_2^{(\infty)} = 2.23(39)$$

for a free particle: $K_2 = 2$

[Braguta et al, Phys. Lett. B 852, 138604 (2024)]

[V. E. Ambruş et al., Phys.Rev.D 108,085016 (2023)]

## Mechanism behind the negativity of gluonic moment of inertia?

The moment of inertia can be obtained from the free energy in the co-rotating frame:

$$F(T,R,\Omega) = F_0(T,R) - \frac{1}{2} I(T,R)\Omega^2$$

**Let's spin the gluons!**

The free energy in the co-rotating frame:

$$F = -T \ln \int DA e^{iS}$$

where $S$ is the Yang-Mills action in the co-rotating frame:

$$S = -\frac{1}{2g_{\mathrm{YM}}^2} \int d^4x \sqrt{-g}\, g^{\mu\nu} g^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta}$$

with the metric

$$g_{\mu\nu} = \begin{pmatrix} 1 - r_\perp^2 \Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

curved metric (with $R_{\mu\nu\alpha\beta} = 0$)

The action in the co-rotating frame is quadratic in the angular frequency $\Omega$:

$$S = S_0 + S_1\Omega + \frac{S_2}{2}\Omega^2$$

$$S = -\frac{1}{2g_{\rm YM}^2}\int d^4x\,\sqrt{-g}\,g^{\mu\nu}g^{\alpha\beta}F^a_{\mu\alpha}F^a_{\nu\beta}$$

$$g_{\mu\nu} = \begin{pmatrix} 1-r_\perp^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

with

**chromoelectric fields**

$$S_1 = \frac{1}{g_{YM}^2}\int d^4x\left[xF^a_{yx}F^a_{xt} + xF^a_{yz}F^a_{zt} - yF^a_{xy}F^a_{yt} - yF^a_{xz}F^a_{zt}\right]$$

**standard "mechanical" contribution**

**chromomagnetic fields**

$$S_2 = -\frac{1}{g_{YM}^2}\int d^4x\left[r_\perp^2(F^a_{xy})^2 + y^2(F^a_{xz})^2 + x^2(F^a_{yz})^2 + 2xyF^a_{xz}F^a_{zy}\right]$$

**chromo-magnetic contribution**

chromomagnetic contribution only

Moment of inertia:

$$I^{\rm gl} = \lim_{\Omega\to 0}\left[-\frac{1}{\Omega}\left(\frac{\partial F}{\partial \Omega}\right)_T\right] = \lim_{\Omega\to 0}\left[-\left(\frac{\partial^2 F}{\partial \Omega^2}\right)_T\right]$$

where

$$F = -T\ln\int DA\,e^{iS}$$

for a good smooth $F = F(\Omega)$

$\Longrightarrow$ $S_2$ **will contribute!!!**

# Decomposition of the moment of inertia: the mechanical part

Moment of inertia of the gluon plasma can be decomposed into two parts:

$$I^{\mathrm{gl}} = I^{\mathrm{gl}}_{\mathrm{mech}} + I^{\mathrm{gl}}_{\mathrm{magn}}$$

(nonlocal)
**standard mechanical**
(exists for quark and gluons)

(local)
**non-trivial chromomagnetic**
(gluons are special! No such term for quarks)

---

**The standard mechanical part:**

$$I^{\mathrm{gl}}_{\mathrm{mech}} = T \left\langle\!\left\langle S_1^2 \right\rangle\!\right\rangle_T = \frac{1}{T} \left\langle\!\left\langle \left(\boldsymbol{n} \cdot \boldsymbol{J}^{\mathrm{gl}}\right)^2 \right\rangle\!\right\rangle_T$$

total angular momentum of gluons

$$J_i^{\mathrm{gl}} = \frac{T}{2} \int_V d^4x \, \epsilon_{ijk} M^{jk}_{\mathrm{gl}}(x) \qquad i, j = 1, 2, 3$$

or: $$\vec{\mathrm{J}}_g = \int d^3x [\vec{\mathrm{x}} \times (\vec{\mathrm{E}} \times \vec{\mathrm{B}})]$$

the local angular momentum of gluons

$$M^{ij}_{\mathrm{gl}}(\boldsymbol{x}) = x^i T^{j0}_{\mathrm{gl}}(\boldsymbol{x}) - x^j T^{i0}_{\mathrm{gl}}(\boldsymbol{x})$$

gluonic stress-energy tensor

$$T^{\mu\nu}_{\mathrm{gl}} = F^{a,\mu\alpha} F^{a,\nu}{}_{\alpha} - (1/4)\eta^{\mu\nu} F^{a,\alpha\beta} F^a_{\alpha\beta}$$

a Belinfante-improved form
(symmetric, gauge invariant, and conserved)

thermal expectation value

$$\langle\!\langle \mathcal{O} \rangle\!\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0}$$

"cold vacuum cannot be set into rotation"

**"Mechanical part of the moment of inertia with respect to an axis $n$ is the susceptibility of the projection of total angular momentum on the axis $n$."**

## Decomposition of the moment of inertia: the chromomagnetic part

$$I^{\rm gl}_{\rm magn} = T\left\langle\!\left\langle S_2 \right\rangle\!\right\rangle_T = \int_V d^3x \Big[ \left\langle\!\left\langle (\boldsymbol{B}^a \cdot \boldsymbol{x}_\perp)^2 \right\rangle\!\right\rangle_T + \left\langle\!\left\langle (\boldsymbol{B}^a \cdot \boldsymbol{n})^2 \right\rangle\!\right\rangle_T \boldsymbol{x}_\perp^2 \Big]$$

chromomagnetic field:

$$B_i^a = \frac{1}{2}\epsilon^{ijk} F^a_{jk}$$

distance to the axis of rotation:

$$\boldsymbol{x}_\perp = \boldsymbol{x} - \boldsymbol{n}(\boldsymbol{n}\cdot\boldsymbol{x})$$

In the static limit, $\Omega \to 0$, the space is $O(3)$ isotropic:

$$\left\langle\!\left\langle B_i^a B_j^a \right\rangle\!\right\rangle_T = \frac{1}{3}\delta_{ij} \left\langle\!\left\langle (\boldsymbol{B}^a)^2 \right\rangle\!\right\rangle_T$$

The chromomagnetic contribution to the moment of inertia is proportional
to the thermal part of the chromomagnetic condensate:

$$I^{\rm gl}_{\rm magn} = \frac{2}{3}\int_V d^3x\, \boldsymbol{x}_\perp^2 \left\langle\!\left\langle (\boldsymbol{B}^a)^2 \right\rangle\!\right\rangle_T$$

Compare to the formula from classical mechanics:

$$I_{\rm class} = \int_V d^3x\, \boldsymbol{x}_\perp^2\, \rho(\boldsymbol{x})$$

"classical" mass density

$$\rho(\boldsymbol{x}) \to \tfrac{2}{3}\left\langle\!\left\langle (\boldsymbol{B}^a)^2 \right\rangle\!\right\rangle_T$$

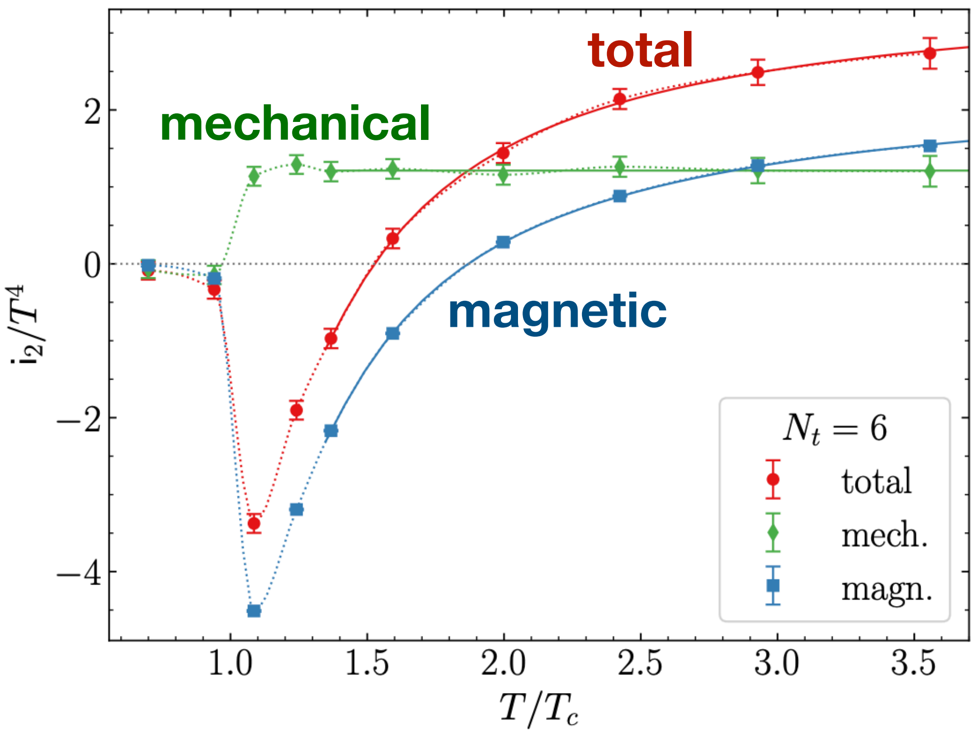
## Mechanism behind the negativity of gluonic moment of inertia?

Melting of the gluon condensate, $\langle\langle (\boldsymbol{B}^a)^2 \rangle\rangle_T < 0$ !

Gluon condensate melts at $T \gtrsim T_c$, and the moment of inertia receives a negative contribution

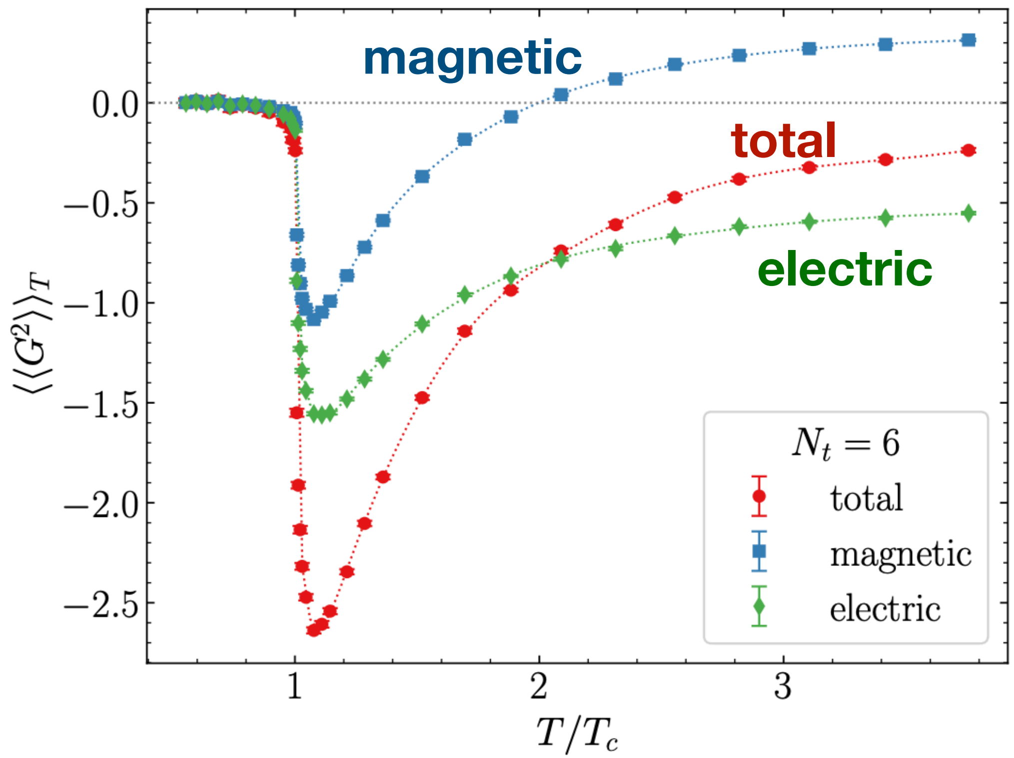
$$F(T, R_\perp, \Omega) = F_0(T, R_\perp) - V \sum_{k=1}^{\infty} \frac{\mathrm{i}_{2k}(T)}{(2k)!} R_\perp^{2k} \Omega^{2k} \equiv F_0 - \frac{I}{2}\Omega^2 + O(\Omega^4)$$

**specific (normalized) moment of inertia**




$$I^{\mathrm{gl}} = I^{\mathrm{gl}}_{\mathrm{mech}} + I^{\mathrm{gl}}_{\mathrm{magn}}$$

**(normalized) gluon condensates**

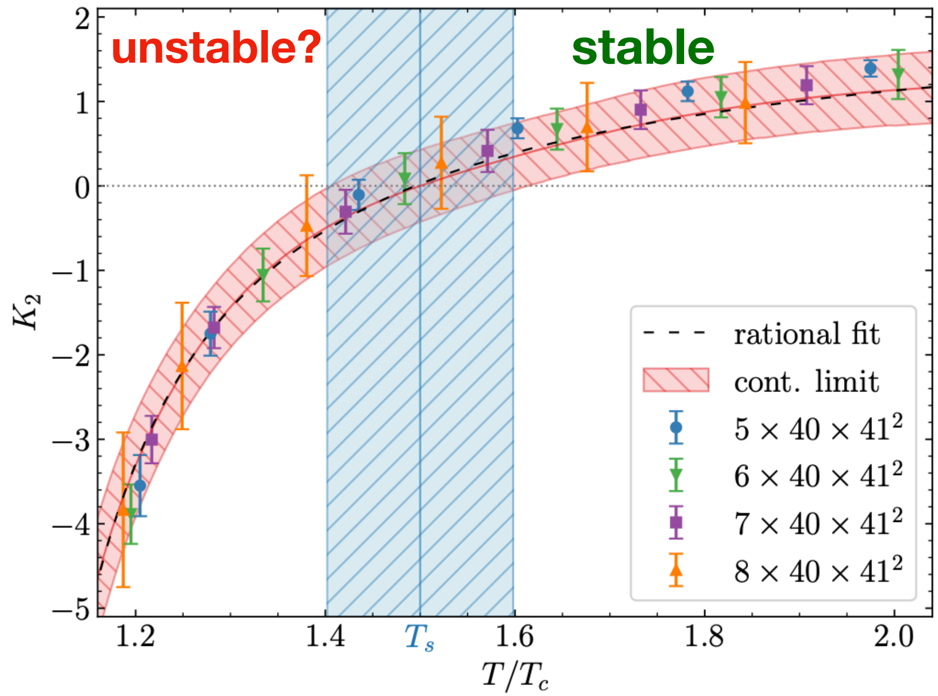



$$I^{\mathrm{gl}}_{\mathrm{magn}} = \frac{2}{3} \int_V d^3x\, \boldsymbol{x}_\perp^2 \left\langle\!\left\langle (\boldsymbol{B}^a)^2 \right\rangle\!\right\rangle_T$$

# Negative moment of inertia: instability of rigid rotation?

**Thermodynamic equilibrium:**

$$\delta E - T\delta S - \boldsymbol{\Omega}\delta \boldsymbol{J} > 0$$

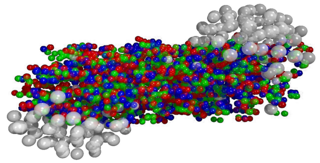



**For rotating system: all eigenvalues of the inverse Weinhold metric**

$$g^{(W),\mu\nu} = -\frac{\partial^2 f(T,\boldsymbol{\Omega})}{\partial X_\mu \partial X_\nu}\,, \qquad X_\mu = (T, \Omega_i)$$

**should be positively defined:**

$$C_J > 0\,, \qquad C_J = T\left(\frac{\partial S}{\partial T}\right)_J$$ ← **specific heat** $C_V = \frac{\langle E^2\rangle - \langle E\rangle^2}{k_B T^2}$



$$\mathrm{spec}(I^{ij}) > 0\,, \qquad I^{ij} = \left(\frac{\partial J^i}{\partial \Omega_j}\right)_T$$ ← **tensor of moments of inertia**

**In our notations:** $K_2(T) > 0$ ← **condition of thermodynamic stability**

**Emerges also in spinning black holes**

[B.F. Whiting, J. W. York Jr, PRL 61, 1336 (1988); T. Prestidge, PRD 61 (2000), 084002;
H. S. Reall, PRD 64 (2001), 044005; R. Monteiro, M. J. Perry, H. E. Santos, PRD 80 (2009), 024041]

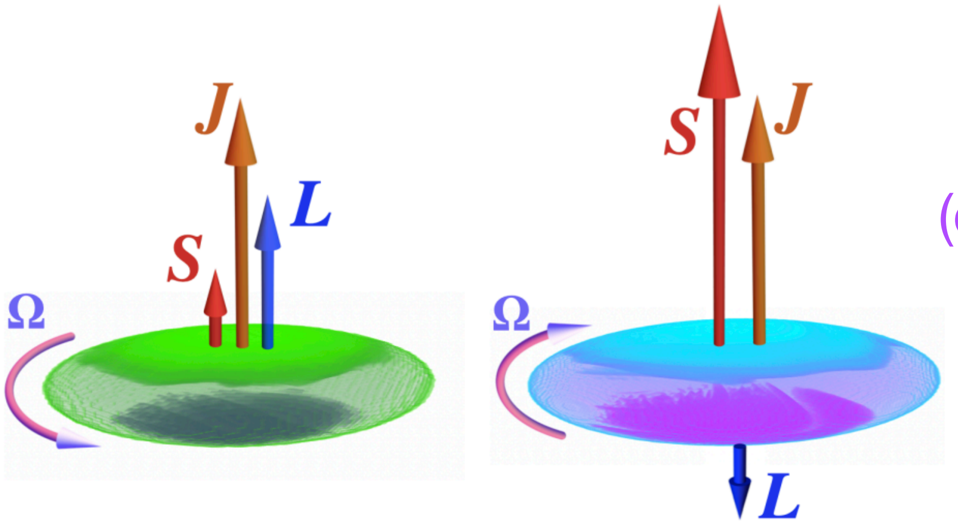
# Physical picture: a negative Barnett effect for gluons?

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$$

total angular momentum = orbital part + spin part

ordinary fluid (gas)

$$\boldsymbol{S} = \kappa\,\boldsymbol{\Omega}$$
$$\kappa > 0$$




(quark) gluon plasma

$$\boldsymbol{S} = \kappa\,\boldsymbol{\Omega}$$
$$\kappa < 0$$

**Barnett** **negative Barnett**

1) gluon spins $\boldsymbol{S}$ are over-polarized by rotation leading to $\boldsymbol{S} \| \boldsymbol{J}$ with $S > J$

2) since $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$, the $L$ must take a negative value, $L < 0$, so do $\Omega < 0$

3) one arrives to $S > 0$ and $\Omega < 0$, leading to the negative Barnett effect

$$\boldsymbol{S} = \kappa\,\boldsymbol{\Omega} \text{ with } \kappa < 0$$

**open question: any link to the proton spin crisis?**

[Braguta et al, ArXiv: 2310.16036]

# Time crystals with negative moment of inertia?

Time crystals were proposed as states the break translational time symmetry in thermodynamic equilibrium both in quantum and classical systems.

F. Wilczek, Phys. Rev. Lett.109, 160401 (2012); A. Shapere, F. Wilczek, Phys. Rev. Lett. 109, 160402 (2012)

The **no-go** theorems challenge their existence in closed thermodynamic systems.

Ph. Nozieres, Europhysics Letters 103, 57008 (2013);
P. Bruno, Phys. Rev. Lett. 111, 070402 (2013);
H. Watanabe, M. Oshikawa, Phys. Rev. Lett. 114, 251603 (2015).

The discrete-time crystals were experimentally revealed as sub-harmonic, out-of-equilibrium states in open systems subjected to periodic external driving.

J. Zhang et al., Nature 543, 217 (2017); S. Choi et al, Nature 543, 221 (2017);
D. V. Else, C. Monroe, Ch. Nayak, N. Y. Yao, Ann.Rev. of Cond. Matt. Phys. 11, 467 (2020);
M. P. Zaletel, M. Lukin, Ch. Monroe, Ch. Nayak, F. Wilczek, N. Y. Yao, Rev. Mod. Phys. 95, 031001 (2023).

**A roadblock may present an opportunity to discover a new path forward.**

A loophole in the “no-go”? **Assume that the free energy is not an analytical function of the angular momentum.**

Is it even possible? How to compose such a physical system?
Recipe: Use some negative moment of inertia.

# Simple thermodynamics of rotation

## Notations:

$F$ - free energy in the laboratory (inertial) reference frame.

$\tilde{F}$ - free energy in the co-rotating (non-inertial) reference frame.

$\boldsymbol{\Omega}$ - angular velocity; $\boldsymbol{J}$ - total angular momentum

## Basic relations:

Angular momentum:

$$\boldsymbol{J} = -\left(\frac{\partial \widetilde{F}}{\partial \boldsymbol{\Omega}}\right)_T$$

determined via the free energy in the co-rotating reference frame

Angular velocity:

$$\boldsymbol{\Omega} = \left(\frac{\partial F}{\partial \boldsymbol{J}}\right)_T$$

expressed via the free energy in the laboratory reference frame

The Legendre transform:

$$F = \widetilde{F} + \boldsymbol{\Omega} \cdot \boldsymbol{J}$$

Moment of inertia $I$:

$$I(\Omega) = -\frac{1}{\Omega}\left(\frac{\partial \widetilde{F}}{\partial \Omega}\right)_T$$

is the proportionality coefficient:

$$\boldsymbol{J} = I\boldsymbol{\Omega}$$

determines how the angular momentum $\boldsymbol{J}$ depends on the angular velocity $\boldsymbol{\Omega}$

# How thermodynamics works: example

Consider a classical solid body rotating about one of its principal axes.
The moment of inertia, $I$, is independent of the angular frequency $\boldsymbol{\Omega}$.

**The ground state (●) is the global minimum of the free energy in the laboratory frame $F = F(J)$, considered as a function of the angular momentum $J$.**

$$\boldsymbol{J} = -\left(\frac{\partial \widetilde{F}}{\partial \boldsymbol{\Omega}}\right)_T \qquad F = \widetilde{F} + \boldsymbol{\Omega}\cdot\boldsymbol{J} \qquad \boldsymbol{\Omega} = \left(\frac{\partial F}{\partial \boldsymbol{J}}\right)_T$$

The free energy in the co-rotating reference frame:

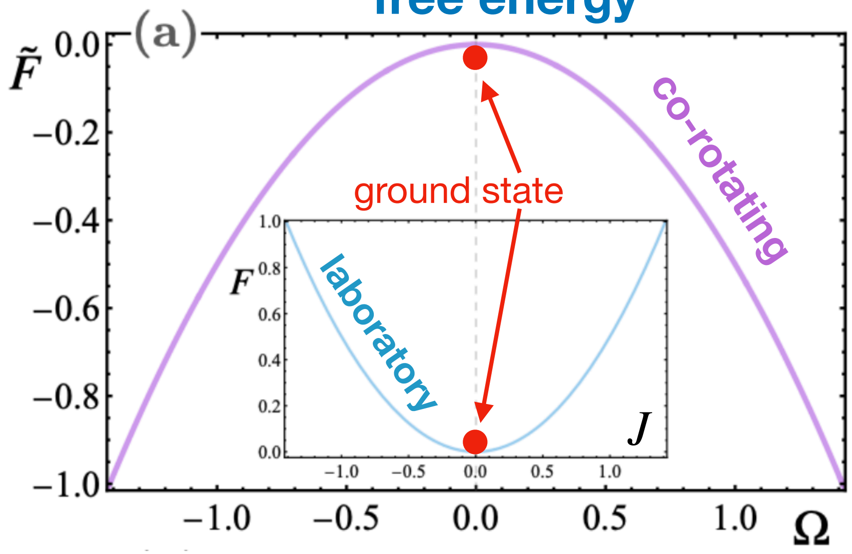
The angular momentum:

The free energy in the laboratory reference frame:

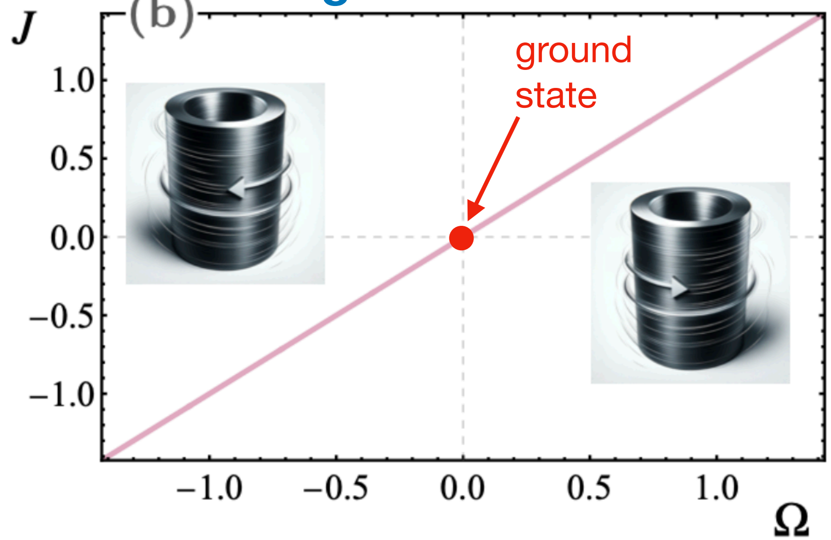
The thermodynamic ground state

$$\widetilde{F} = -\frac{I}{2}\Omega^2 \Longrightarrow J = I\Omega \Longrightarrow F(J) = \frac{J^2}{2I} \Longrightarrow \boldsymbol{J = 0}$$

**free energy**




**angular momentum**



# A system with a negative moment of inertia (I)

Instead of the ordinary classical expression for the co-rotating free energy:

$$\widetilde{F} = -\frac{I}{2}\Omega^2$$

with a positive moment of inertia, $I > 0$

we take:

$$\widetilde{F}_{\mathrm{tc}} = -\frac{\kappa_2}{2}\Omega^2 - \frac{\kappa_4}{4}\Omega^4$$

with a negative prefator:

$$\kappa_2 = -|\kappa_2| < 0$$

The moment of inertia $I$:

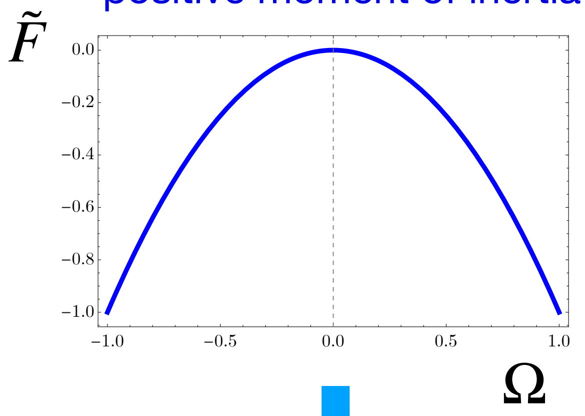
$$I(\Omega) = -\frac{1}{\Omega}\left(\frac{\partial \widetilde{F}}{\partial \Omega}\right)_T$$

takes a negative value at $\Omega \to 0$:

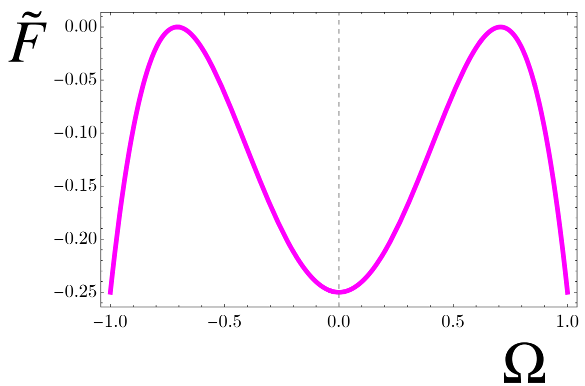
$$I(\Omega) = \kappa_2 + \kappa_4 \Omega^2$$

**What is the ground state of this system?**

positive moment of inertia




negative moment of inertia



# A system with a negative moment of inertia (II)

Introduce the dimensionless variables:

$$\omega = \frac{\Omega}{\Omega_0}, \qquad j = \frac{J}{J_0}, \qquad \mathsf{i} = \frac{I}{I_0}, \qquad \overset{(\sim)}{f} = \frac{\overset{(\sim)}{F}}{F_0}$$

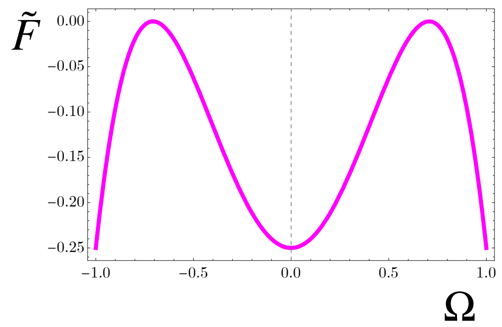
angular frequency — angular momentum — moment of inertia — free energies in lab. and rot. frames

where

$$\Omega_0 = \sqrt{\frac{|\kappa_2|}{\kappa_4}}, \qquad J_0 = \kappa_4 \Omega_0^3, \qquad I_0 = |\kappa_2|, \qquad F_0 = \frac{\kappa_2^2}{4\kappa_4}$$

are characteristic parameters coming from the free energy:

$$\widetilde{F}_{\mathrm{tc}} = -\frac{\kappa_2}{2}\Omega^2 - \frac{\kappa_4}{4}\Omega^4 - \frac{\kappa_2^2}{4\kappa_4}$$




an inessential normalization constant

remember that
$\kappa_2 = -|\kappa_2| < 0$

# An object with a negative moment of inertia (III)

Free energy in the co-rotating frame:

$$\tilde{f}_{\mathrm{tc}} = -(\omega^2 - 1)^2$$

thermodynamics

$$\boldsymbol{J} = -\left(\frac{\partial \widetilde{F}}{\partial \boldsymbol{\Omega}}\right)_T$$

Angular momentum:

$$j = \omega(\omega^2 - 1)$$

a solution of

Legendre transform

$$F = \widetilde{F} + \boldsymbol{\Omega} \cdot \boldsymbol{J}$$

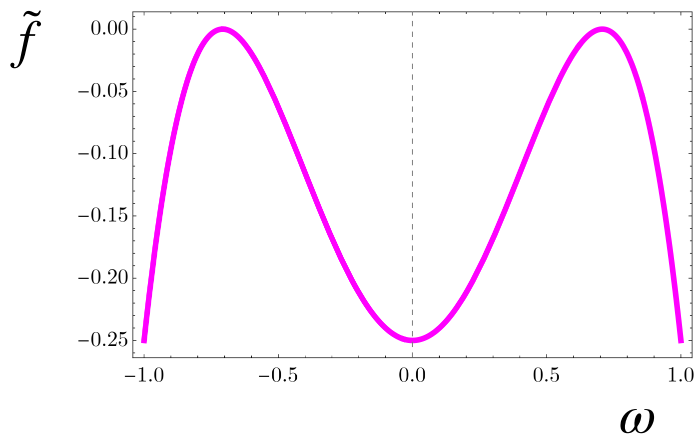
Free energy in the laboratory frame:

$$f_{\mathrm{tc}}(j) = 3\omega^4(j) - 2\omega^2(j) - 1$$

$$\omega = \omega(j)$$

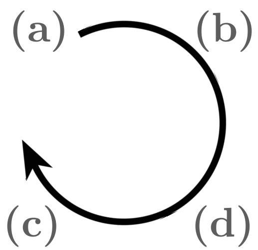
The ground state is the global minimum of the free energy with respect to the angular momentum $j$.

Let us apply this set of the thermodynamic transformations to the following free energy:



# An object with a negative moment of inertia (IV)

Let us circulate our attention clockwise!

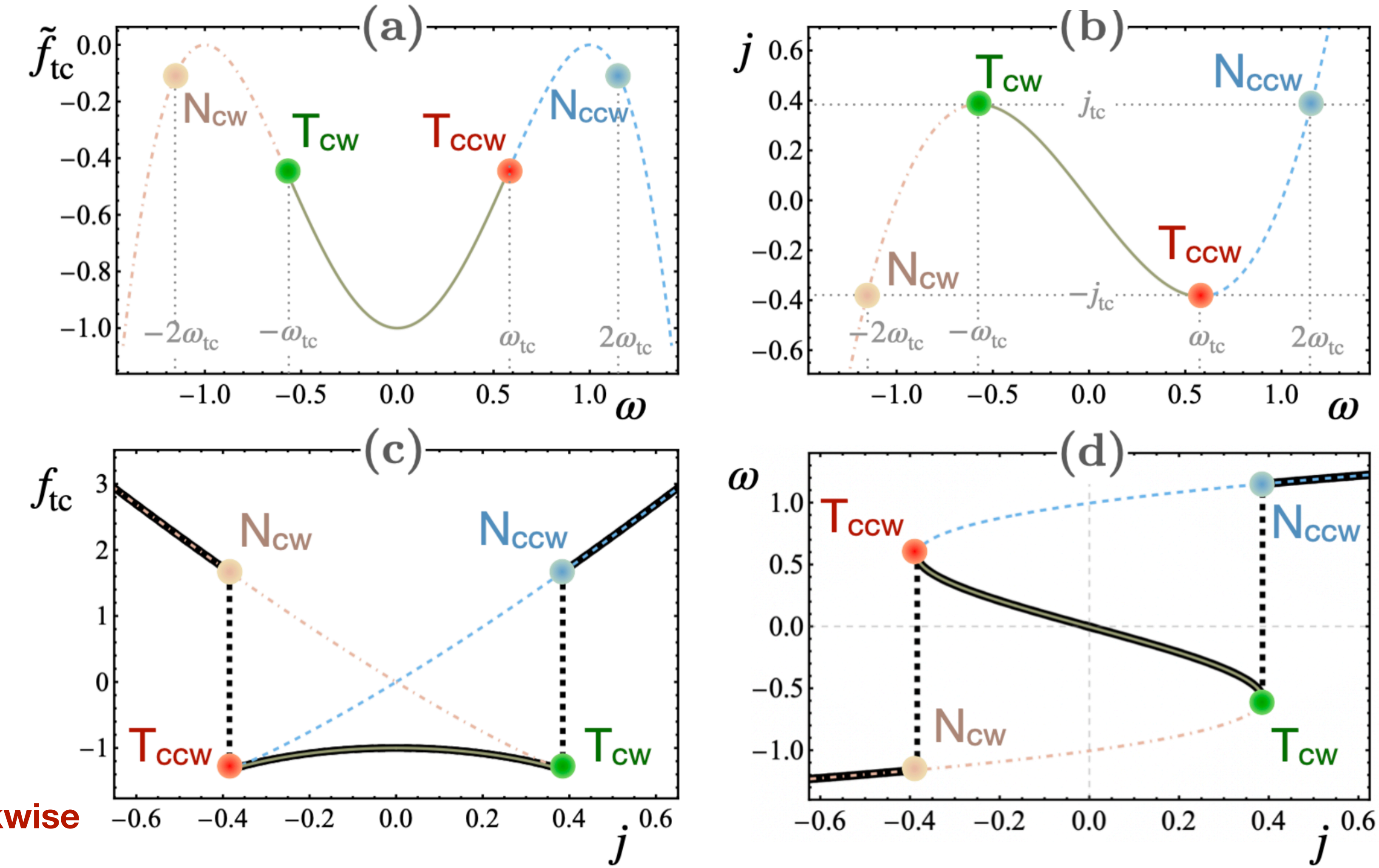



**co-rotating free energy**

$$\tilde{f}_{\rm tc} = -(\omega^2 - 1)^2$$

**angular momentum**

$$j = \omega(\omega^2 - 1)$$




$N_{cw}$ and $N_{ccw}$ are ordinary excited (not ground) states that are partners of

$T_{ccw}$ and $T_{cw}$ — time-crystalline ground states (!)

**CCW: counterclockwise**

**CW: clockwise**

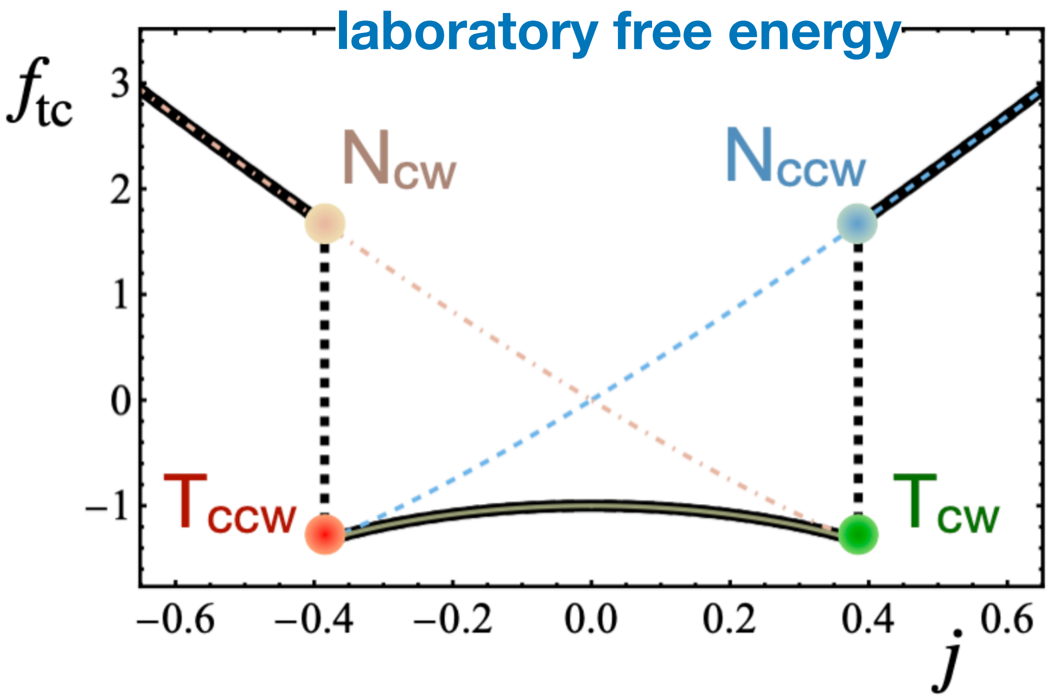
**laboratory free energy**

$$f_{\rm tc}(j) = 3\omega^4(j) - 2\omega^2(j) - 1$$

**angular frequency**

$$\omega = \omega(j)$$

# An object with a negative moment of inertia (V)




| | $T_{ccw}$ | $T_{cw}$ | $N_{ccw}$ | $N_{cw}$ |
|---|---|---|---|---|
| $\omega/\omega_{tc}$ | $1$ | $-1$ | $2$ | $-2$ |
| $j/j_{tc}$ | $-1$ | $1$ | $1$ | $-1$ |

$$\omega_{tc} = \frac{1}{\sqrt{3}} = 0.577\ldots, \qquad j_{tc} = \frac{2}{3\sqrt{3}} = 0.385\ldots$$

The time-crystalline ground states $T_{ccw}$ and $T_{cw}$ are true global minima of the free energy in the laboratory reference frame as the function of the angular momentum $J$.

**Thus, the negative moment of inertia supports the emergence of mechanical time crystals.**

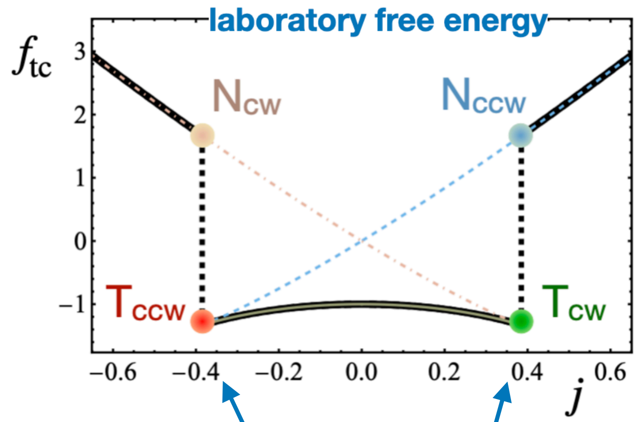
**The no-go theorems are bypassed due to the non-analyticity of the free energy in the laboratory reference frame.**

Notice the large mass (energy) gap between the time-crystalline ground states and their excited normal counterparts (e.g., $T_{cw}$ vs. $N_{ccw}$).

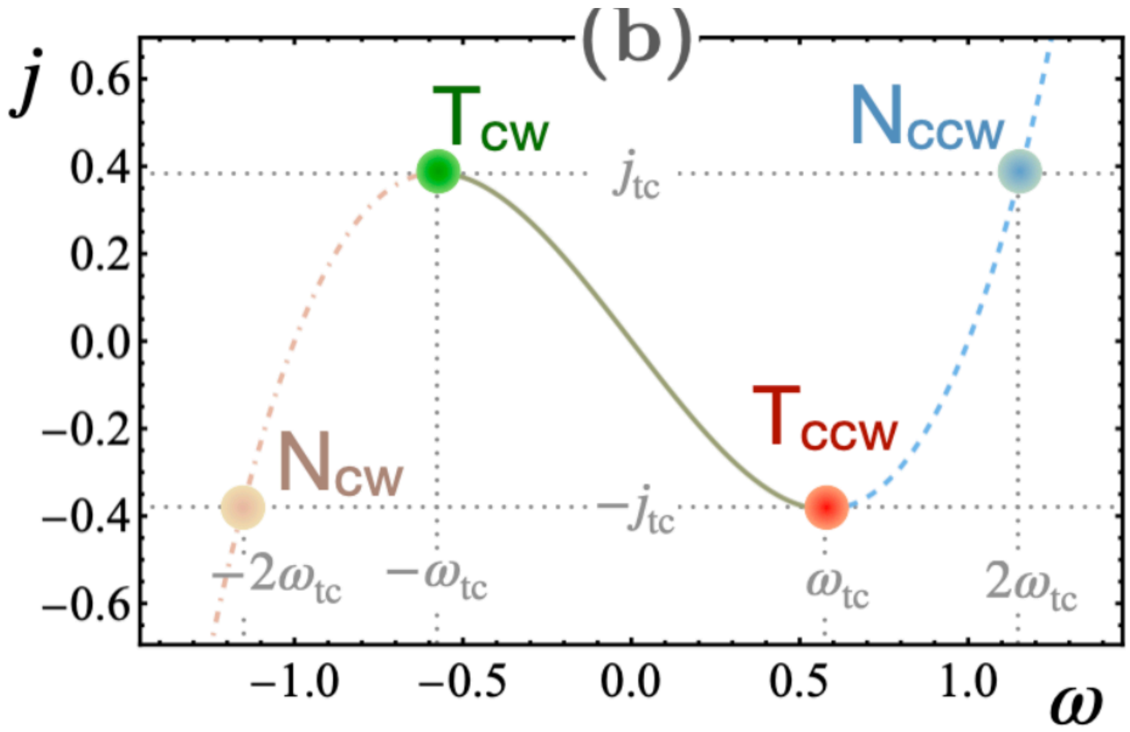
# Time-crystalline states vs. thermodynamics wisdom

**"in the thermodynamic ground state, the moment of inertia, similarly to the specific heat, cannot be negative"**

Thermodynamics




angular momentum




angular frequency

each **ground state** corresponds to a **non-zero angular frequency** of permanent rotation

The moment of inertia at both time-crystalline ground states vanishes:

| | $T_{ccw}$ | $T_{cw}$ | $N_{ccw}$ | $N_{cw}$ |
|---|---|---|---|---|
| $\omega/\omega_{tc}$ | 1 | −1 | 2 | −2 |
| $j/j_{tc}$ | −1 | 1 | 1 | −1 |

we identify
$\delta J = I(\Omega)\delta\Omega$

$$\mathrm{i} = \left.\frac{\partial j}{\partial \omega}\right|_{\omega=\pm\omega_{tc}} = 0$$

**No contradiction with Thermodynamics!**

# Some take-aways from Part 1:

**The Barnett effects appear at all scales:**

**for magnetic moments in a solid ferromagnetic, for electrons in a liquid metal, for nuclei in rotating liquid (protons in water), in ultrarelativistic environment of hadronic physics ($\Lambda$ hyperons), for quarks and gluons in quark-gluon plasma**

**All effects are understood everywhere, except for theory of fundamental strong interactions, QCD. The strongly interacting medium <u>fails</u> to follow the conventional wisdom of the Barnett effects.**

**The Barnett effect for gluons is surprising: it seems to operate in the opposite way than it should based on our understanding of the gluon as a poinlike vector particle.(*)**

## Gluonic medium is a time crystal?

(*) Surely, gluons are not free. Not at all (strongly interacting particles that generate a confining force between quarks and setting the confinement for themselves). Therefore, our intuition may fail here.

# A “large g-factor” effect: Polarization of virtual W-bosons by strong magnetic field

One could also call this part of the talk as:

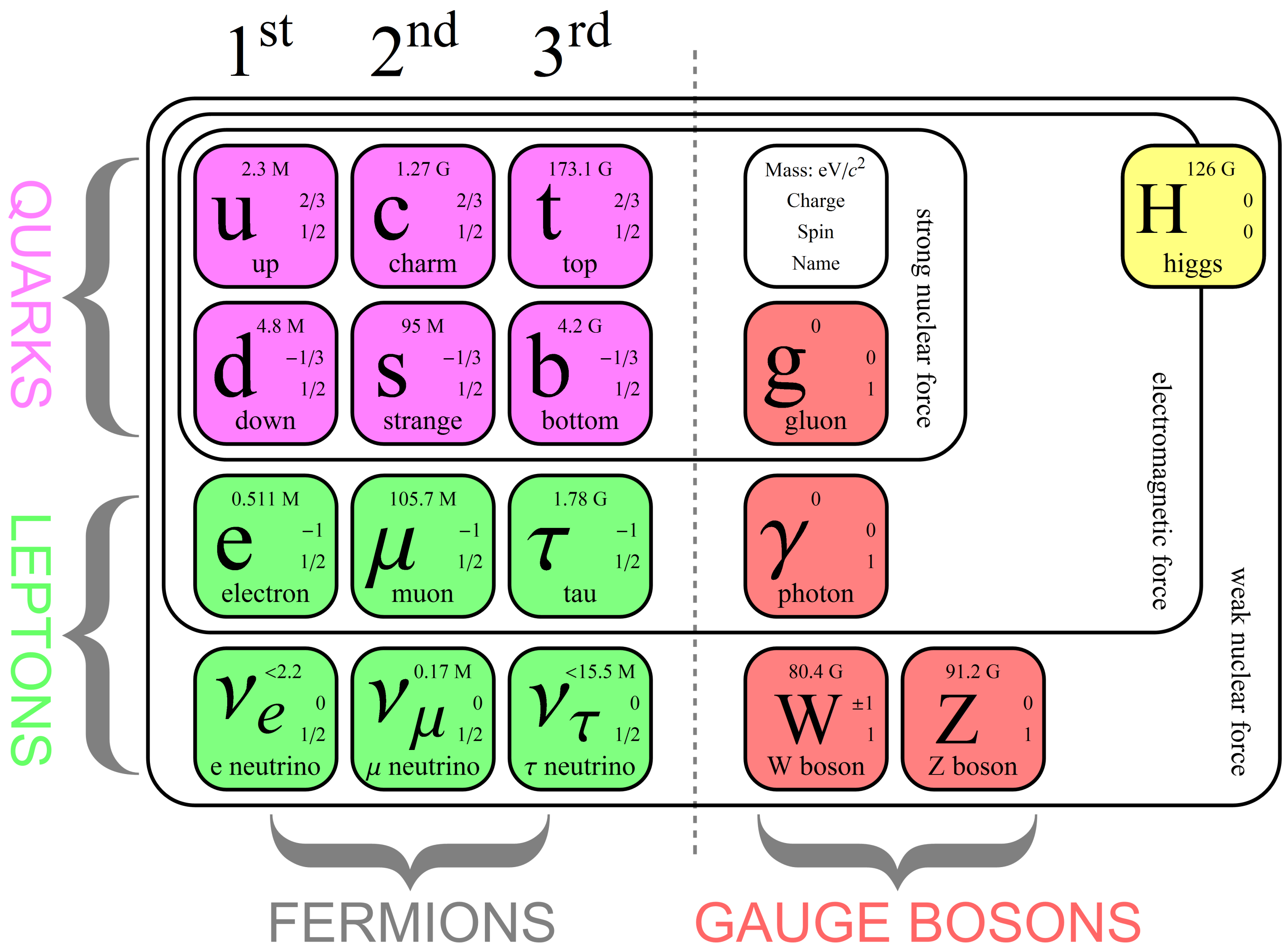
**Solidifying and evaporating vortex solid liquid**
**— made, by the way, from nothing* —**
**possessing superconductivity and superfluidity**
**at the same time****
**and all that requires just one simple**
**ingredient: magnetic field*** …**

*) yes, vacuum is the most “nothing” of all available nothings

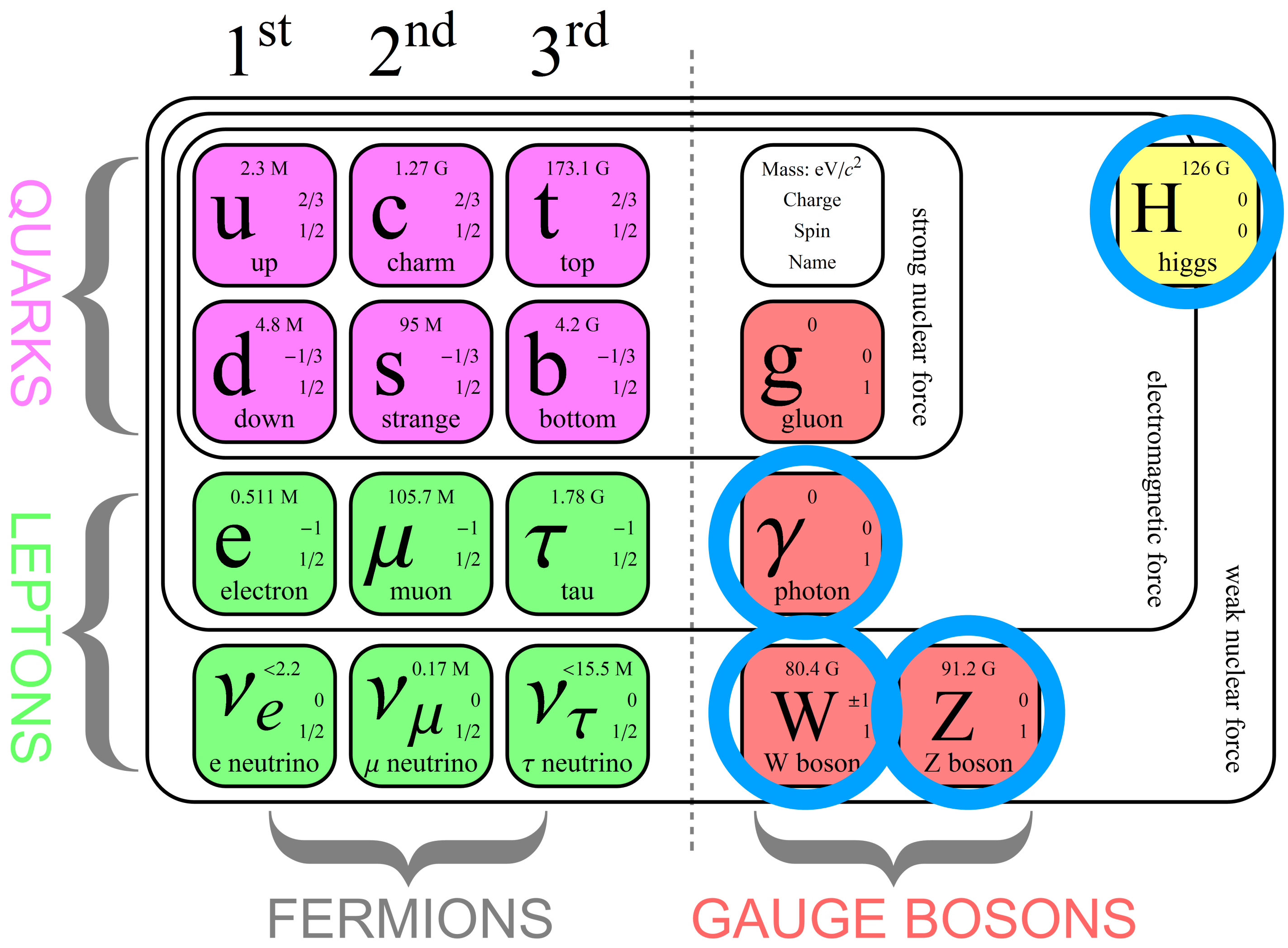
**) yes, the transport should be <u>dissipationless</u>

***) disclaimer: to create all that we need really strong magnetic field

# Elementary particles in the Standard Model



# Electroweak Sector of the Standard Model




# Bosonic part of the Electroweak Sector

# Bosonic part of the Electroweak Sector

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_X \to \mathrm{U}(1)_{\mathrm{em}} \qquad \langle \Phi \rangle = (0, v)^T$$

symmetry breaking

Lagrangian:

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda (|\Phi|^2 - v^2/2)^2$$

coupling

expectation value

Potential on Higgs $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$

Higgs doublet interacts with $W^a_\mu$ and $X_\mu$

$$D_\mu = \partial_\mu - i g \tau^a W^a_\mu / 2 - i g' X_\mu / 2$$

electroweak couplings (charges)

$$e = g \sin\theta = g' \cos\theta$$

The Weinberg angle $\theta \simeq 29°$

electric charge

$U(1)_X$ vector gauge field:

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

$SU(2)_L$ vector gauge fields:

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \epsilon^{abc} W^b_\mu W^c_\nu$$

$$\boxed{\begin{aligned} W^3_\mu &= \sin\theta A_\mu + \cos\theta Z_\mu \\ X_\mu &= \cos\theta A_\mu - \sin\theta Z_\mu \end{aligned}}$$

symmetry breaking

$W_\mu$ — W-bosons (**massive** vector),
$Z_\mu$ — Z-boson (**massive** vector),
$A_\mu$ — photon (massless vector),
$\Phi$ —Higgs particle (**massive** scalar)

# Bosonic part of the Electroweak Sector

Parameters are well known and fixed:

$$e \approx 0.303 \qquad m_H \approx 125.3 \text{ GeV}$$

$$g \approx 0.642 \qquad m_Z \approx 91.2 \text{ GeV}$$

$$g' \approx 0.344 \qquad m_W \approx 80.4 \text{ GeV}$$

$$\sin^2\theta_W \approx 0.223$$

A finite-temperature crossover to
a symmetry-restored phase:

$$T_c = 159.5 \pm 1.5\,\text{GeV}$$

[M. D'Onofrio and K. Rummukainen, Phys.Rev.D 93 (2016) 2, 025003]

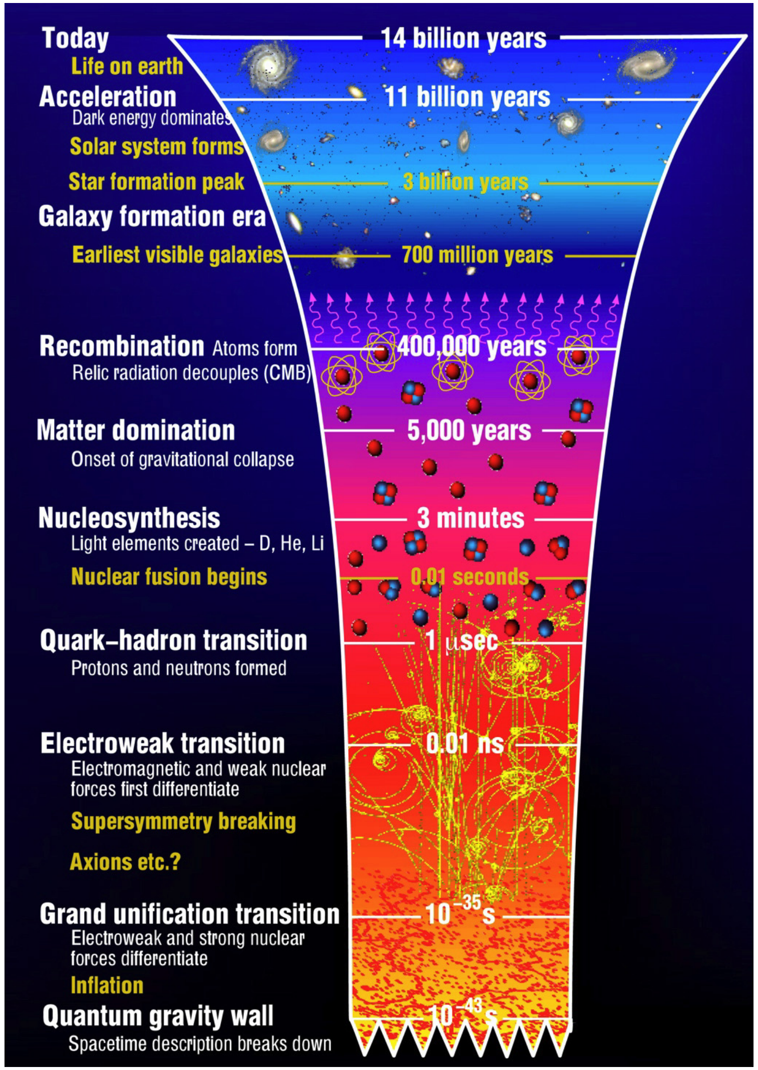
---

2. First picosecond, $10^{-12}$ s:
the Universe cools to temperatures $160\,\text{GeV} \sim 2 \times 10^{15}\,\text{K}$
and enters the quark-gluon plasma epoch

1. A fraction of a nanosecond, $10^{-(32\ldots36)}$ s, after the Big Bang:
the Universe enters the Electroweak epoch at the start/during/right after the end of the inflation period.

**Electroweak Epoch is believed to have existed shortly after the Big Bang**

EW Epoch




[Centre for Theoretical Cosmology, Cambridge, UK]

# Standard model and magnetic fields

The Universe is a magnetized place.

What happens with the Electroweak sector at high magnetic fields?

Nuclear Physics B90 (1975) 203–220

**TRANSITION ELECTROMAGNETIC FIELDS IN PARTICLE PHYSICS**

Abdus SALAM
*International Centre for Theoretical Physics, Trieste, Italy and Imperial College, London, England*

J STRATHDEE
*International Centre for Theoretical Physics, Trieste, Italy*

We present a computation of one-loop effective potentials for elementary systems placed in a strong magnetic or a laser-produced electromagnetic environment. This permits a determination in principle of a hierarchy of transition field strengths, for which the systems concerned may (for appropriate values of the parameters in the theory) make transitions from a spontaneously broken asymmetric phase to one of restored symmetry

## Higgs sector

Suggested:
Restoration of electroweak symmetry for the magnetic fields of the order of the scalar boson mass.

Volume 62B, number 4 PHYSICS LETTERS 21 June 1976

**SYMMETRY BEHAVIOUR IN EXTERNAL FIELDS**

A.D. LINDE
*P.N Lebedev Physical Institute, Moscow, USSR*

Received 12 April 1976

It is noted that in most of the gauge theories with neutral currents symmetry restoration takes place not due to a magnetic field but due to massive vector fields created simultaneously by the magnetic field sources. Symmetry behaviour in the theories without neutral currents in the presence of magnetic and laser fields and the problem of dynamical symmetry restoration are also discussed

# Standard model and magnetic fields

## Vacuum Structures in {Weinberg-Salam} Theory

V.V. Skalozub (BITP, Kiev)
Jun, 1986

24 pages
Published in: *Sov.J.Nucl.Phys.* 45 (1987) 1058, *Yad.Fiz.* 45 (1987) 1708-

The vacuum structures of electroweak interactions in magnetic field H = const at finite temperature T ≠ 0 are found, which appear due to the evolution of tachyonic instability in the W-boson spectrum.

## *W*-boson sector:

Tachyonic instability and formation of the a phase with a *W*-condensate.

Volume 214, number 4 PHYSICS LETTERS B 1 December 1988

## ANTI-SCREENING OF LARGE MAGNETIC FIELDS BY VECTOR BOSONS

J. AMBJØRN and P. OLESEN
*The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

Received 2 July 1988

In the SO(3) model with massive vector bosons we show that for magnetic fields exceeding $m_W^2/e$ there is condensation of W's. This condensation is characterized by anti-screening. Near the critical field we show that the condensate is a lattice of vortex lines.

Vacuum **superconductivity** and **superfluidity** [M.Ch. et al, PRD 80, 054503 (2009); Phys.Rev.D 88 (2013) 065006]

?

# Negative lattice results are finite temperature

## The electroweak phase transition in a magnetic field

K. Kajantie [a,b,1], M. Laine [a,b,2], J. Peisa [c,3], K. Rummukainen [d,4], M. Shaposhnikov [a,5]

[a] Theory Division, CERN, CH-1211 Geneva 23, Switzerland
[b] Department of Physics, P.O. Box 9, 00014 University of Helsinki, Finland
[c] Department of Physics, University of Wales Swansea, Singleton Park, Swansea SA2 8PP, UK
[d] NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Received 7 September 1998; accepted 9 December 1998

### 6. Conclusions

We have found that for Higgs mass $m_H \gtrsim 80$ GeV, even magnetic fields up to $H_Y/T^2 \sim 0.3$ ($b \sim 0.6$) do not suffice to make the transition be of first order: there is only a crossover. This is in contrast to the perturbative estimates in [5,6]. Moreover, we do not observe any sign of the exotic phase with broken translational invariance proposed by Ambjørn and Olesen for these magnetic fields: all the gauge-invariant operators and correlation lengths we have studied behave qualitatively as without a magnetic field, even though the solution of the classical equations of motion has a vortex structure with a $W^{\pm}$-condensate. We conclude that fluctuations are strong enough to remove the non-trivial structure for the parameter values studied.

# Scales of magnetic field in (particle) (astro)physics - I

**1 T — Reference scale**

(T = Tesla)   1 T = $10^4$ G   (G = Gauss)


loudspeaker

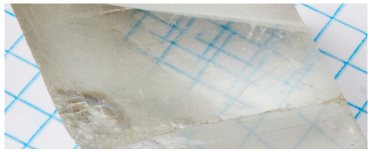

NMR imaging

---

**$10^9$ T — QED scale; the Schwinger limit**

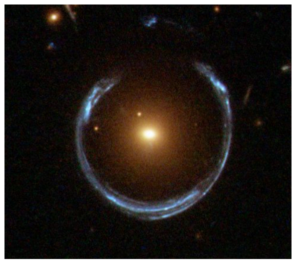
$$B^{\mathrm{QED}} = \frac{m_{\mathrm{e}}^2}{e} \simeq 4 \times 10^9\,\mathrm{T}$$

— vacuum acquires optical birefringence properties



SL Adler, Annals Phys. 67, 599 (1971)

— vacuum can act as a "magnetic lens" which is able to distort and magnify images

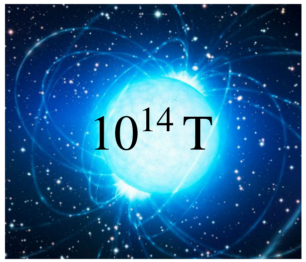


NJ Shaviv, JS Heyl, Y. Lithwick, MNRAS 306, 333 (1999) [astro-ph/9901376]

(similar to gravitational lens)


magnetar surfaces

SA Olausen, VMKaspi, "The McGill magnetar catalog" AP SS 212, 6 (2015) [arXiv:1309.4167]




cores of magnetars

D Lai and SL Shapiro AJ 383, 745 (1991)
CY Cardall, M Prakash, JM Lattimer AJ 554, 322 (2001) [astro-ph/0011148]

Images: Physics Today, Wikipedia, free resources

# Scales of magnetic field in (particle) (astro)physics - II

## $10^{16}$ T — QCD scale

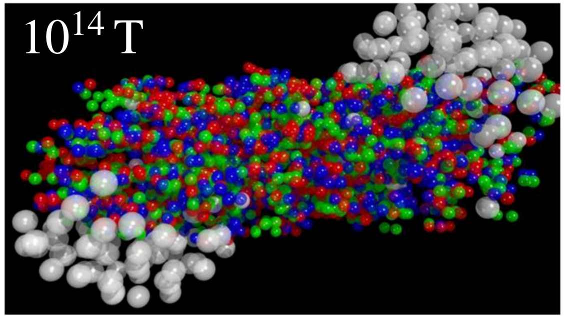
$$B^{\mathrm{QCD}} = \frac{m_p^2}{e} \sim 10^{16}\,\mathrm{T}$$

— magnetic catalysis (enhancement of chiral symmetry breaking)

SP Klevansky, RH Lemmer, Phys. Rev. D 39, 3478 (1989);
KG Klimenko, Z. Phys. C 54, 323 (1992);
great review: IA Shovkovy, Lect. Notes Phys. 871, 13 (2013).

— vacuum superconductivity?

MN Ch., Phys. Rev. D 82, 085011 (2010); PRL 106, 142003 (2011)




transient fields $(10^{-24}\,\mathrm{s})$
in heavy-ion collisions

V Skokov, A Yu Illarionov, V Toneev,
Int. J. Mod. Phys. A 24, 5925 (2009);
WT Deng, XG Huang,
Phys. Rev. C 85, 044907 (2012)

---

## $10^{20}$ T — EW scale

$$B^{\mathrm{EW}} = \frac{m_W^2}{e} \sim 10^{20}\,\mathrm{T}$$

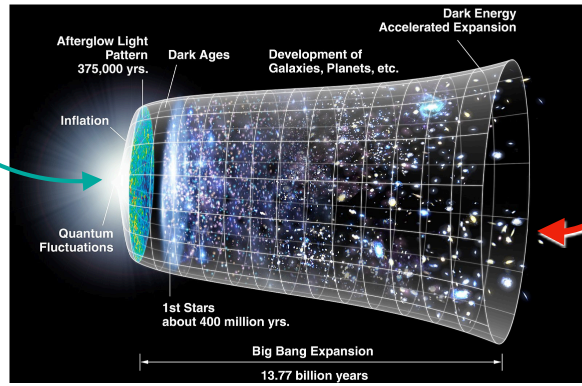
— change in vacuum structure

A Salam and JA Strathdee, Nucl. Phys. B 90, 203 (1975);
AD Linde, Phys. Lett. B 62, 435 (1976)
VV Skalozub, Sov. J. Nucl. Phys. 28, 1 45, 6 (1987)
J Ambjorn, P Olesen, Phys. Lett. B 214, 565 (1988);
J Ambjorn, P Olesen, Nucl. Phys. B 315, 606 (1989)

Early Universe?

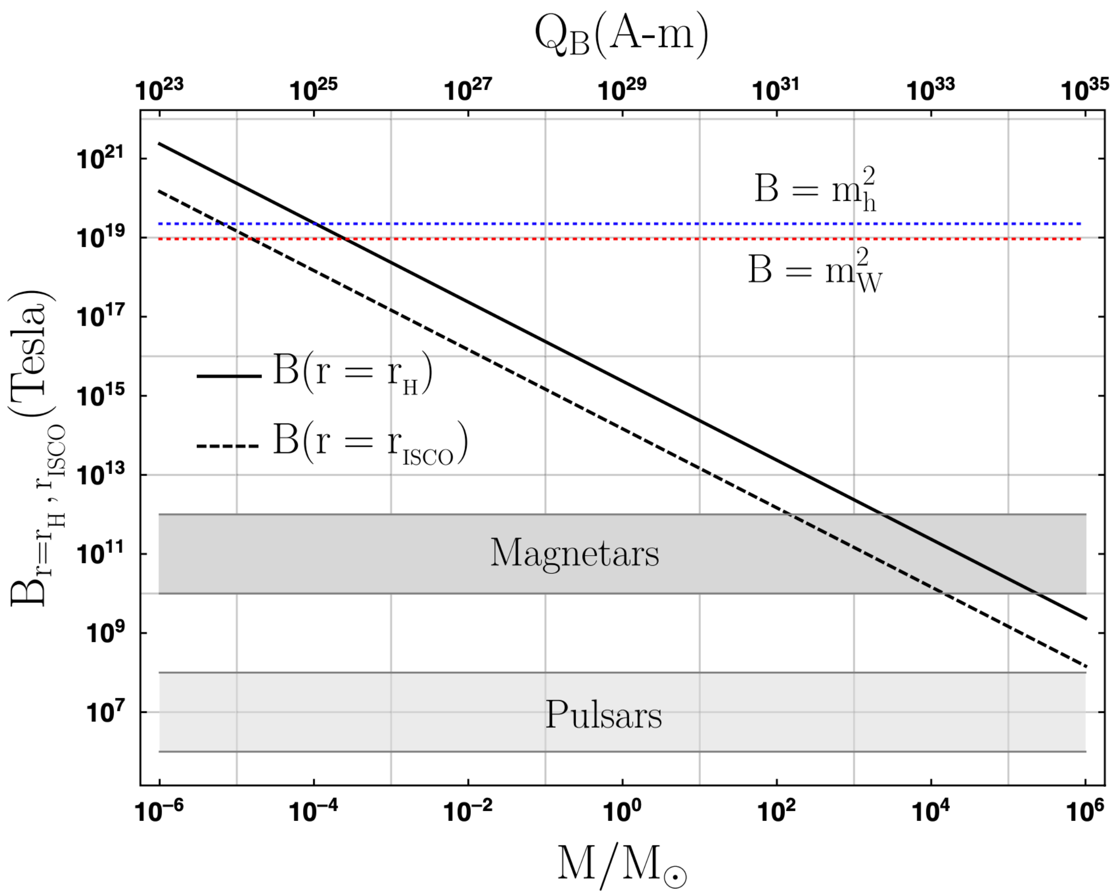
T Vachaspati, PLB 265, 258 (1991);
D Grasso, HR Rubinstein,
Phys. Rept. 348, 163 (2001)

you are here




Images: BNL, Physics Today

# Scales of magnetic field in particle/astro-physics - III

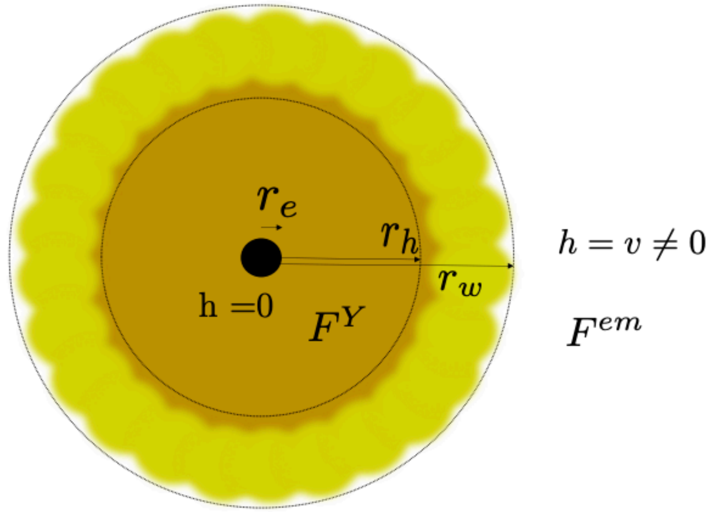



**Astrophysical hints for magnetic black holes**


Diptimoy Ghosh,* Arun Thalapillil,† and Farman Ullah‡



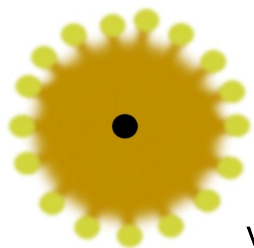
PHYSICAL REVIEW D **103**, 023006 (2021)


## quantum atmospheres of magnetized black holes

black-hole mass

$$M_{BH} \simeq (1/3) M_{\oplus}$$



black-hole radius

$$R \simeq 1\,\mathrm{cm}$$

vortex (superconducting) atmosphere

$$r \simeq 1\,\mathrm{mm}$$

J Maldacena, JHEP 04, 079 (2021)

magnetic field in the atmosphere

$$B \simeq 10^{21}\,\mathrm{T}$$

# A free charged (spinful) relativistic particle in magnetic field

- Energy of a relativistic particle in the external magnetic field $B_{\text{ext}}$:

$$\varepsilon^2_{n,s_z}(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\text{ext}} + m^2$$

momentum along the magnetic field axis

nonnegative integer number

$g = 2$

projection of spin on the magnetic field axis

(the external magnetic field is directed along the *z*-axis)

**Instability for quantum numbers:**

$$p_z = 0; \;\; n = 0; \;\; s_z = +1$$

**Critical magnetic field:**

$$eB_c = m^2$$

**For *W* bosons (if we disregard interactions):** $M_W^2(B) = M_W^2 - |eB|$

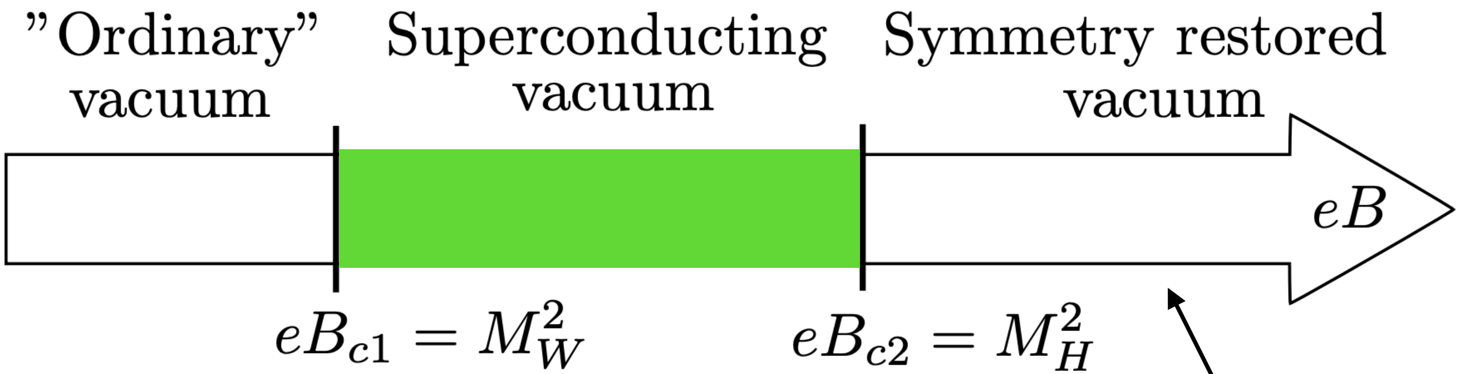
**The critical field is:**

$$B_c^{\text{EW}} = \frac{M_W^2}{e} \simeq 1.1 \times 10^{20}\,\text{T}$$

**Electroweak vacuum should become unstable toward *W* condensation!**

# What theory says about the phase structure?

(Weinberg-Salam model in strong magnetic field at T=0)




**Inhomogeneous phase**

**made of a vortex crystal**

(the aim of this talk)

symmetry restored phase

A Salam and JA Strathdee, Nucl. Phys. B 90, 203 (1975);

AD Linde, Phys. Lett. B 62, 435 (1976)

with remnants of the vortex lattice

P Olesen, Phys. Lett. B 268, 389 (1991);

J Van Doorsselaere, PRD, 88, 025013 (2013)

**Our Aim No. 1: Check this phase structure**

---

**EW Lagrangian:** $\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda \left( |\Phi|^2 - v^2/2 \right)^2$

$$D_\mu = \partial_\mu - ig\tau^a W^a_\mu/2 - ig' X_\mu/2$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu$$

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

**Ordinary vacuum, symmetry breaking:**

$$SU(2)_L \times U(1)_X \to U(1)_{\rm em}$$

**Particles:**

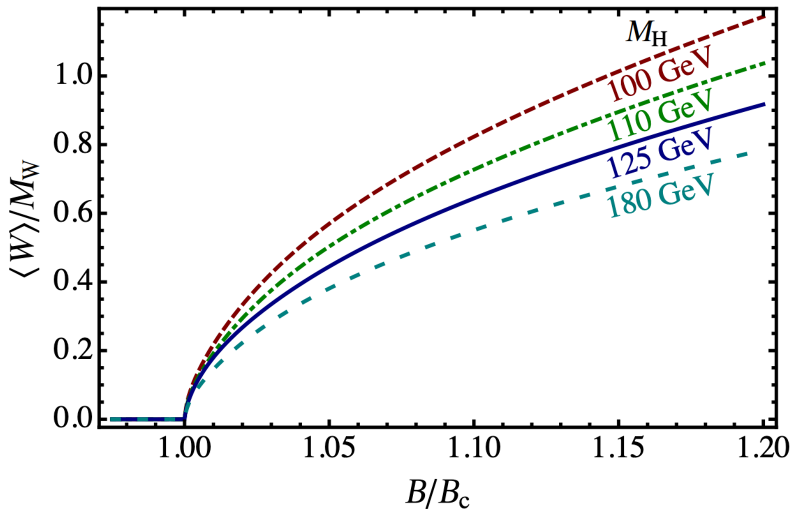
- $W_\mu$ — W-bosons (**massive** vector),
- $Z_\mu$ — Z-boson (**massive** vector),
- $A_\mu$ — photon (massless vector),
- $\Phi$ —Higgs particle (**massive** scalar)

# Superconducting phase, what to expect (theory)

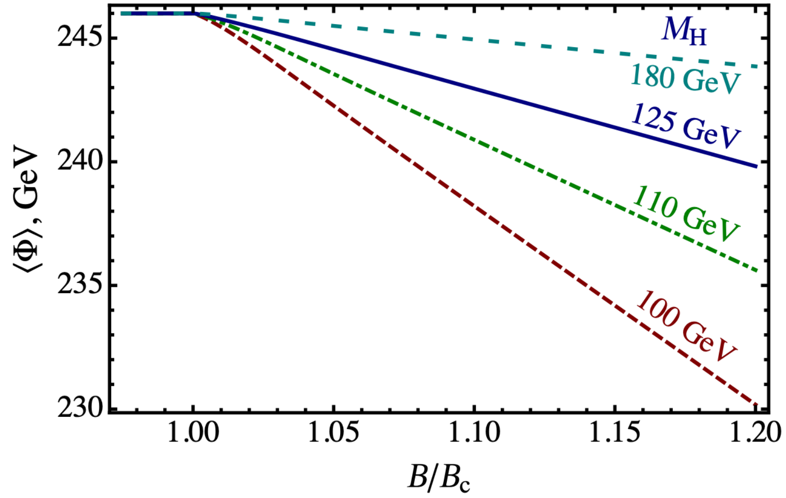
## Solution of classical equations of motion (at a set of Higgs masses)

### Transition at the vicitiny of the first critical field: $B_c = M_W^2/e$

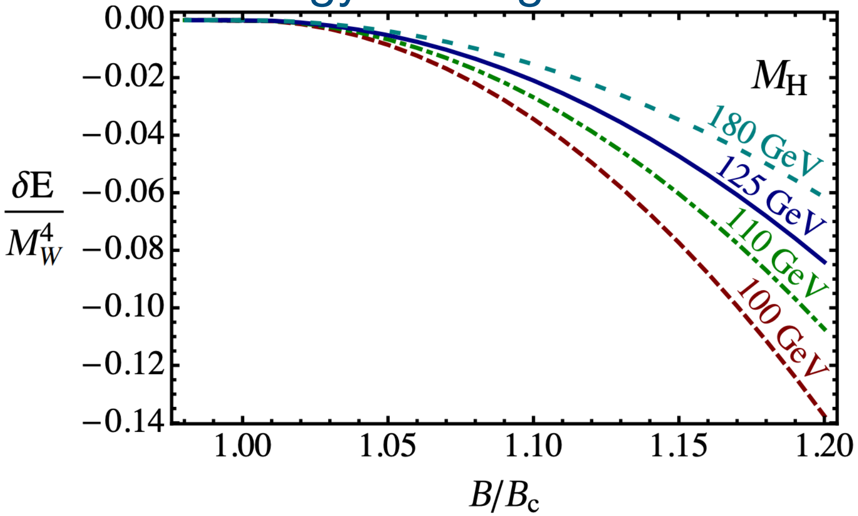
### W-boson condensate



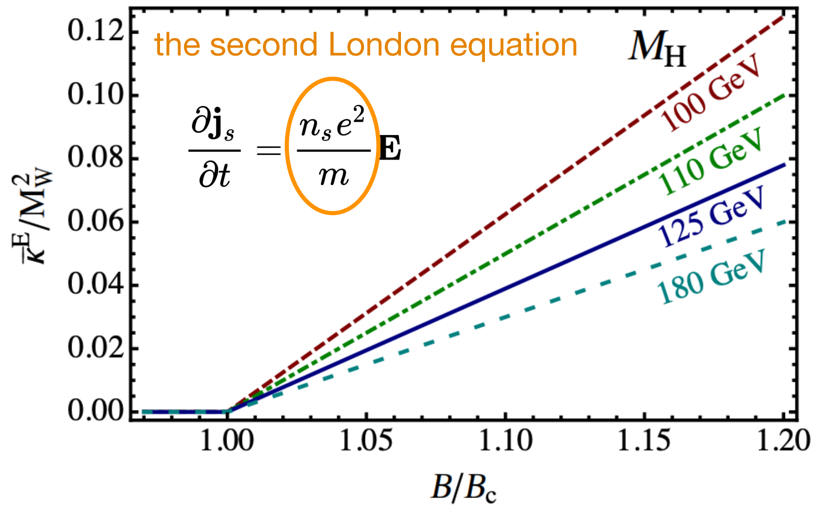

### Higgs condensate




### Energy of the ground state




### Density of superconducting "pairs"




**Second order phase transition**

[J Van Doorsselaere, H Verschelde, M.Ch., PRD 88, 065006 (2013)]

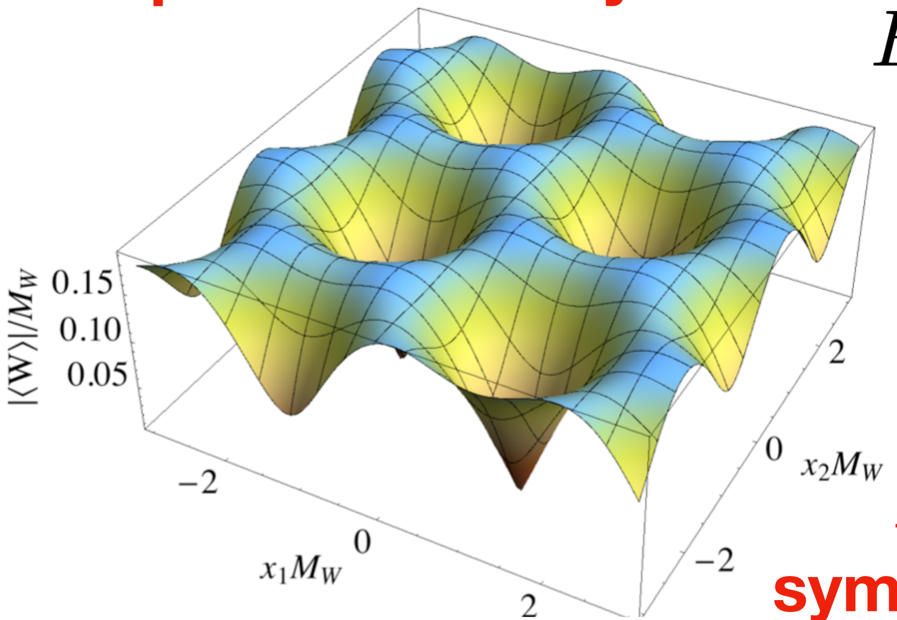
# Superconducting phase, inhomogeneity (theory)

## Superconductivity

**Hexagonal vortex lattice**

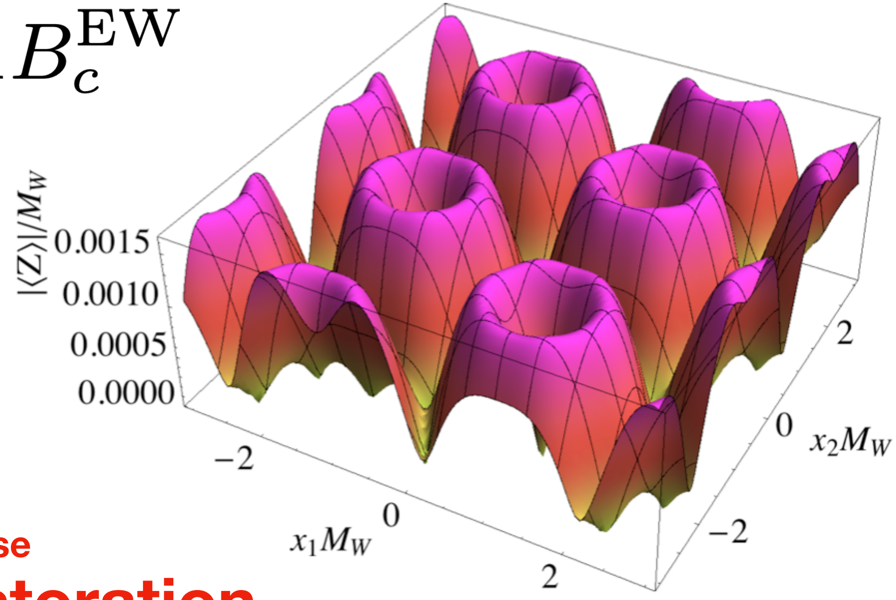
$$B = 1.01 B_c^{\mathrm{EW}}$$

## Superfluidity




W-boson condensate

**— vanishes in the vortex core**



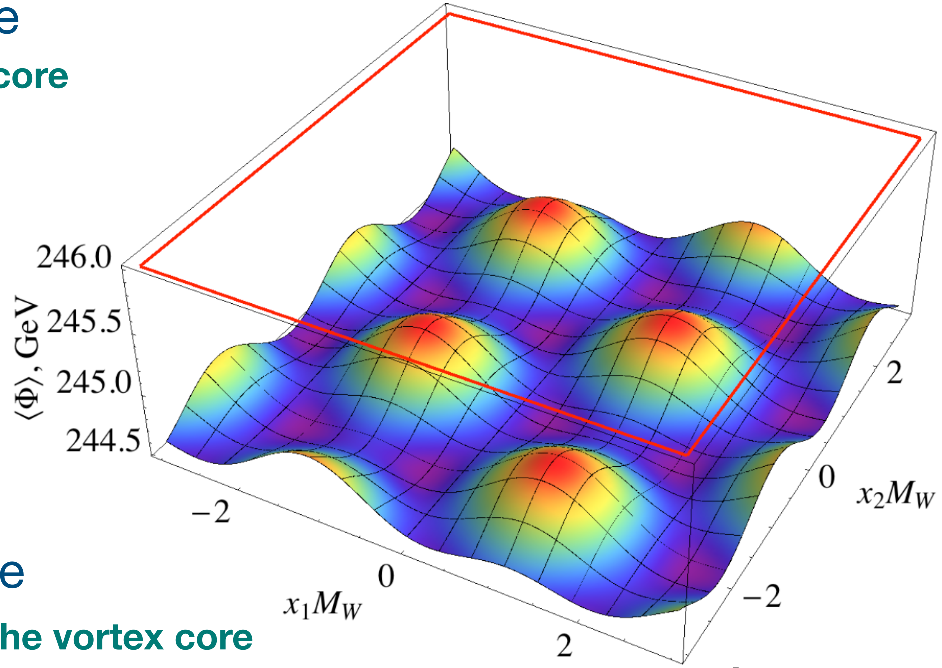

Z-boson condensate

**— vanishes in the vortex core and at an "equidistant manifold" in between the vortices;**

**— gets enhanced at intermediate distances**

tendency to cause

## symmetry restoration



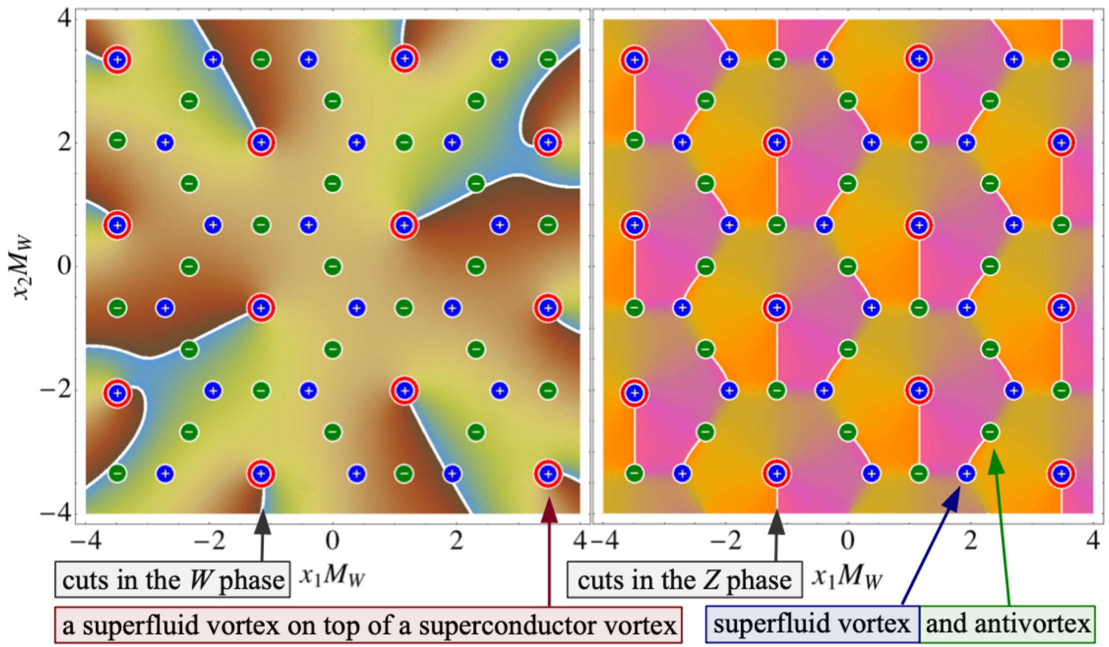

Higgs condensate

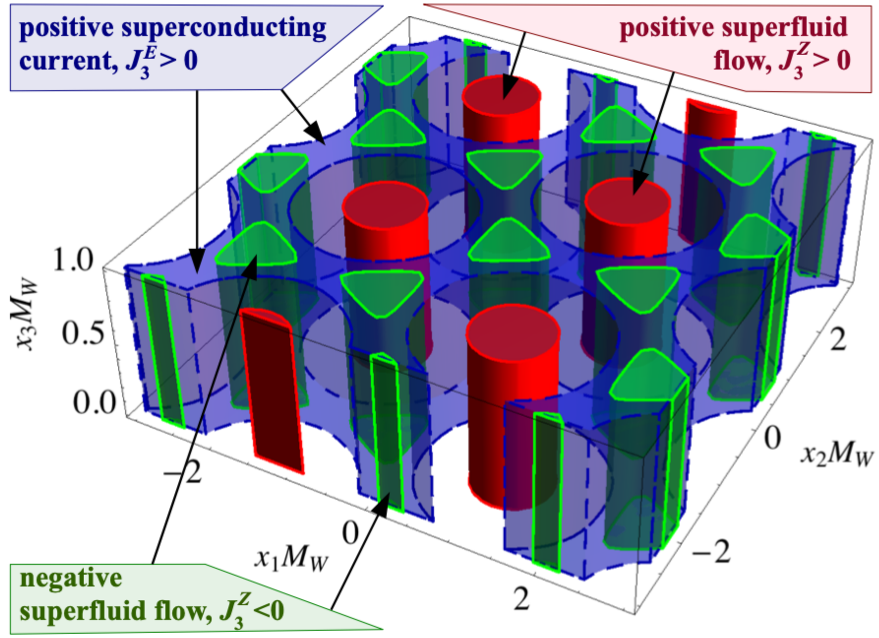
**— gets enhanced in the vortex core**

[J Van Doorsselaere, H Verschelde, M.Ch., PRD 88, 065006 (2013)]

# Superconducting phase, inhomogeneity (theory)

Vortex structure in superconducting (W) and superfluid (Z) condensates




[Jos Van Doorsselaere, Henri Verschelde, M.Ch., Phys. Rev. D 88, 065006 (2013)]

**Visually (and distantly) similar but physically very different from the Abrikosov lattice in type-2 superconductors**

Theoretical expectations based on classical equations of motion:

— Magnetic field leads to condensation of charged W bosons

— Condensation of the W's leads to a condensation of neutral Z bosons

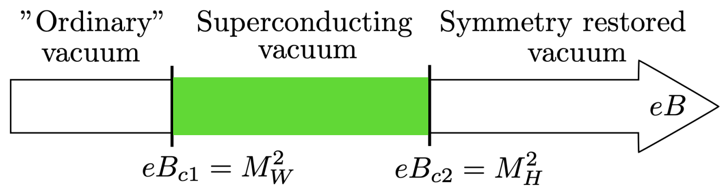
→ **Coexisting superconducting and superfluid condensates**

**Our Aim No. 2: Check the nature of the (superconducting? - check) phase**

# Mean Higgs condensate in (hyper)magnetic field

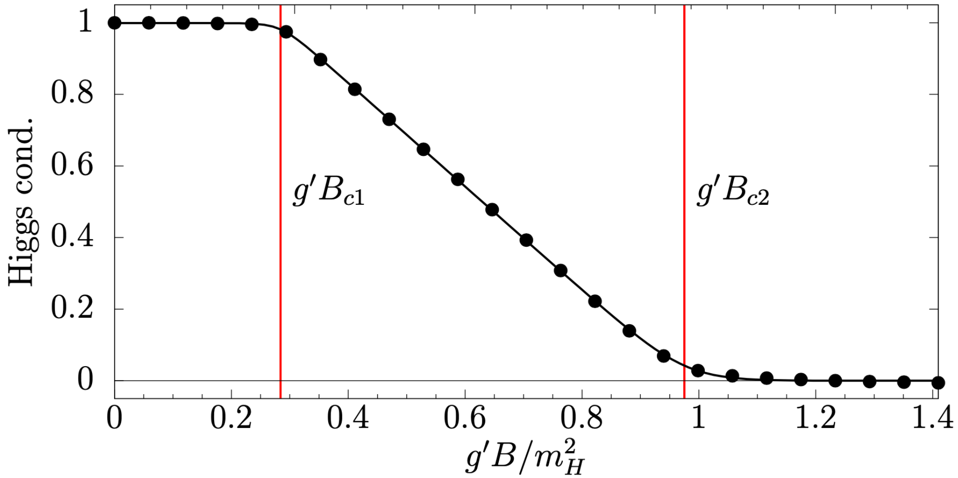
[V. Goy et al, Phys.Rev.Lett. 130 (2023) 11, 111802]

**theory:**

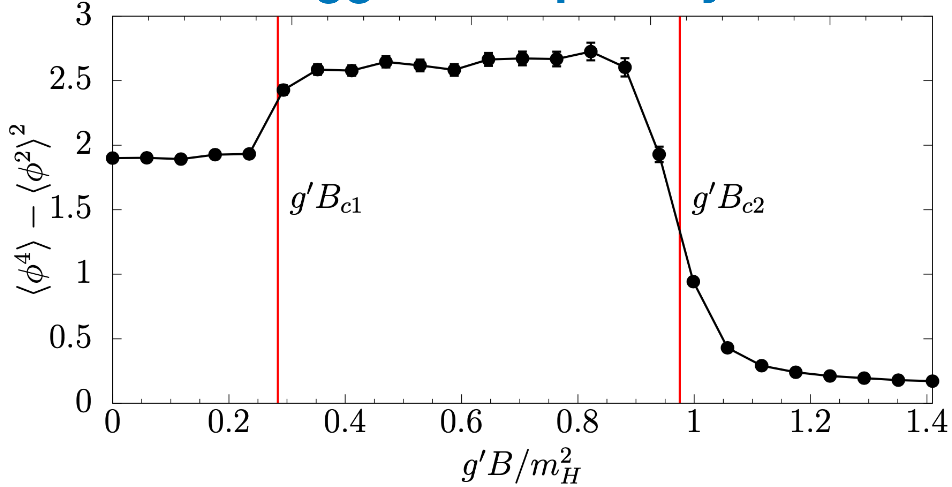



**lattice simulations:**

**Higgs condensate** (normalized)




**Higgs susceptibility** (normalized)




**Result 1. Two phase transitions** (as predicted by theory) **located at:**

**First transition:** $eB_{c1} \simeq 0.68(5)m_W^2$ (theory: $eB_{c1} = m_W^2$)

**Second transition:** $eB_{c2} \simeq 0.99(2)m_H^2$ (theory: $eB_{c2} = m_H^2$)

V. Skalozub, M. Bordag, IJMPA 15 (2000) 349

**Result 2. The strength: both transitions are smooth crossovers, no singularity.**

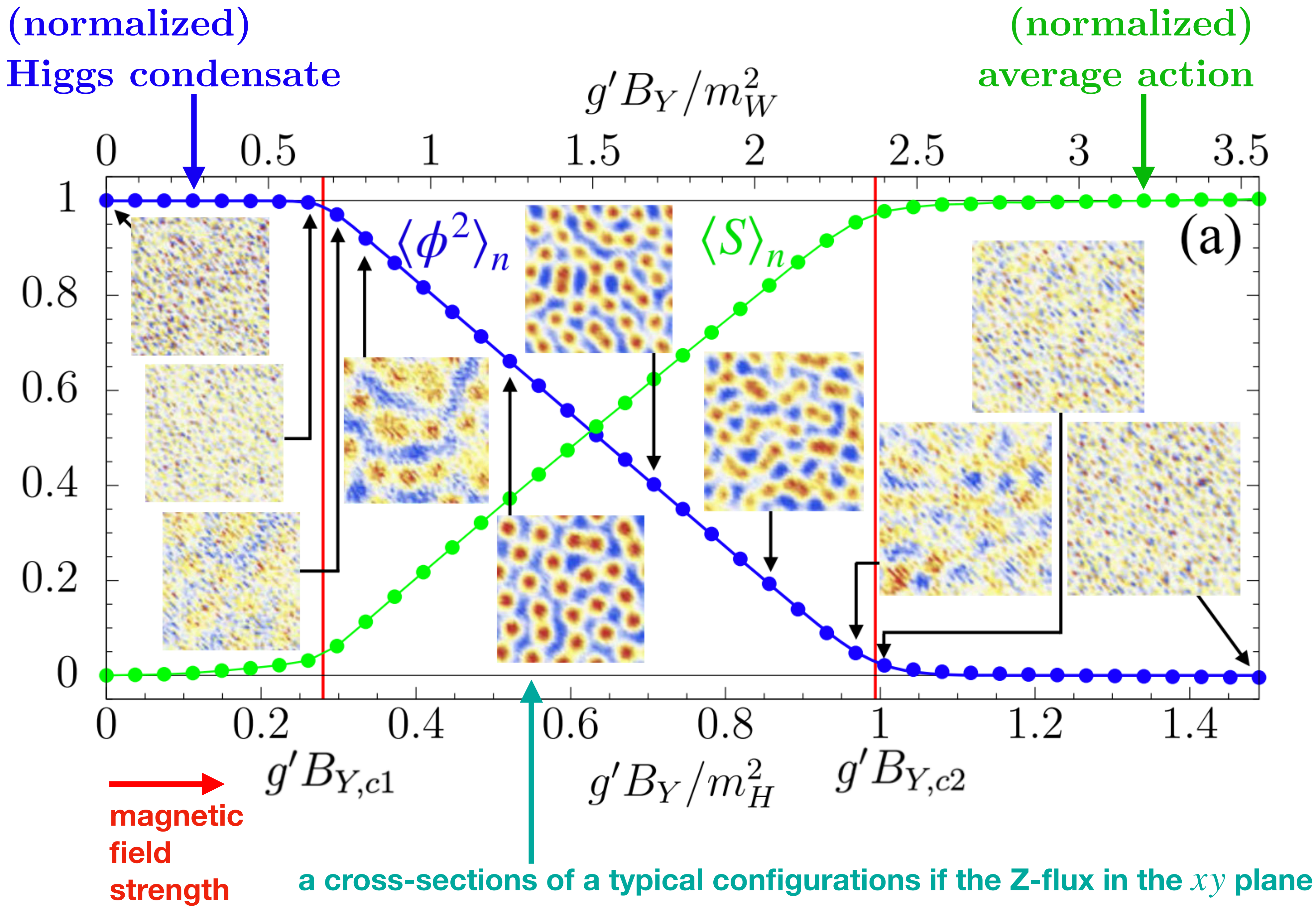
**Expectations,** classical approach: the transition is of the **second** order

perturbation theory: the transition is of the **first** order

**Reality,** first-principle simulations: the transition is of the **infinite** order (crossover)

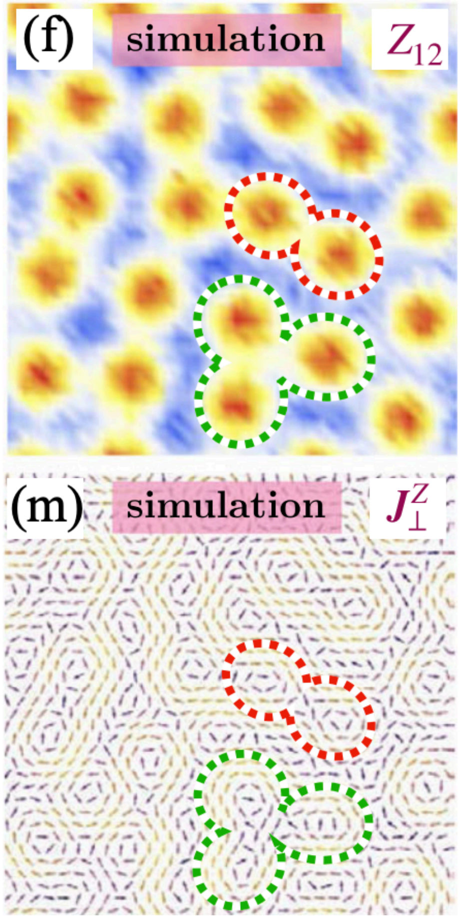
**Result 3. The high-field phase $(B > B_{c2})$: symmetry-restored phase,** OK with theory.

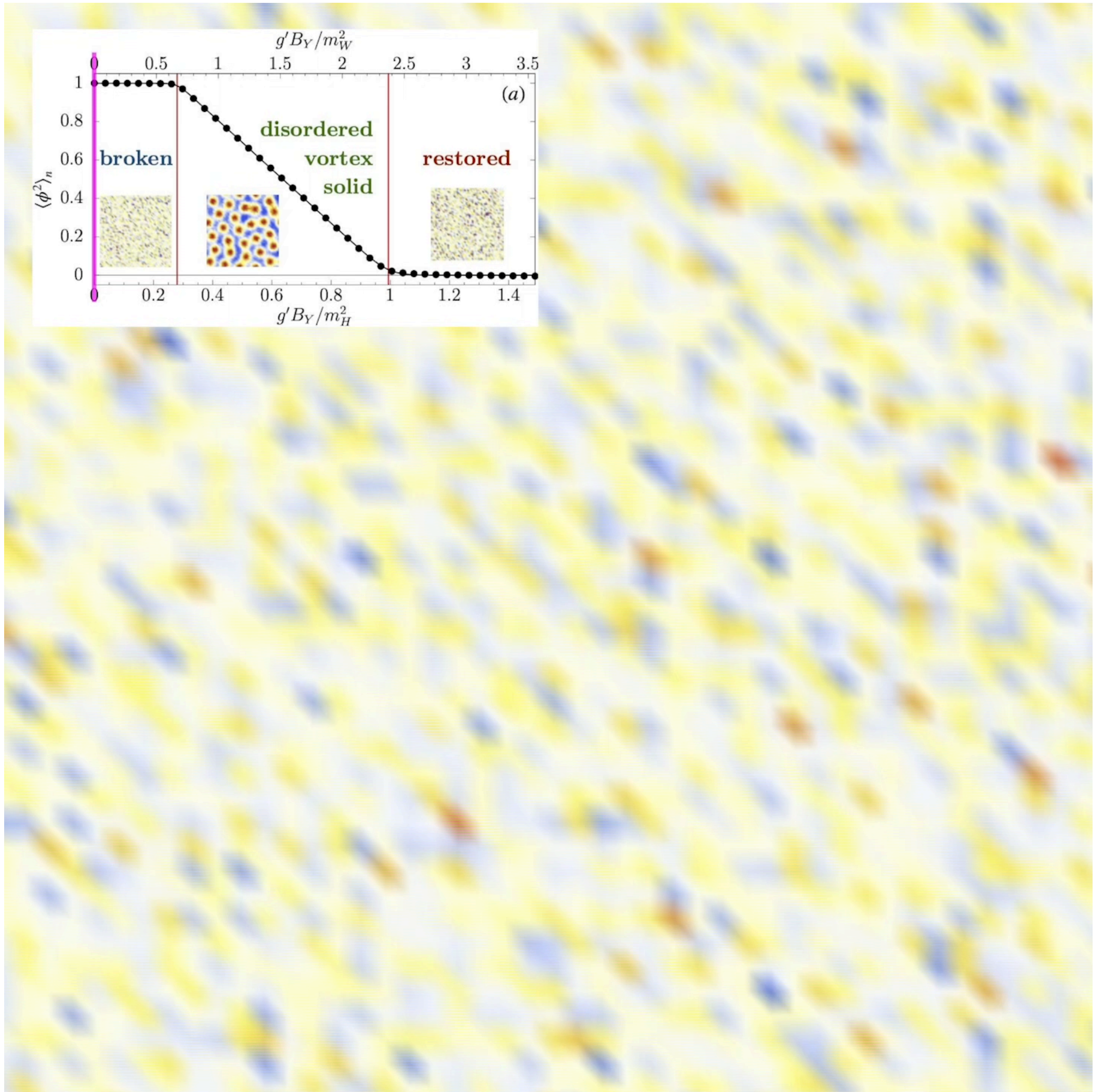
# General view - I



# General view - II

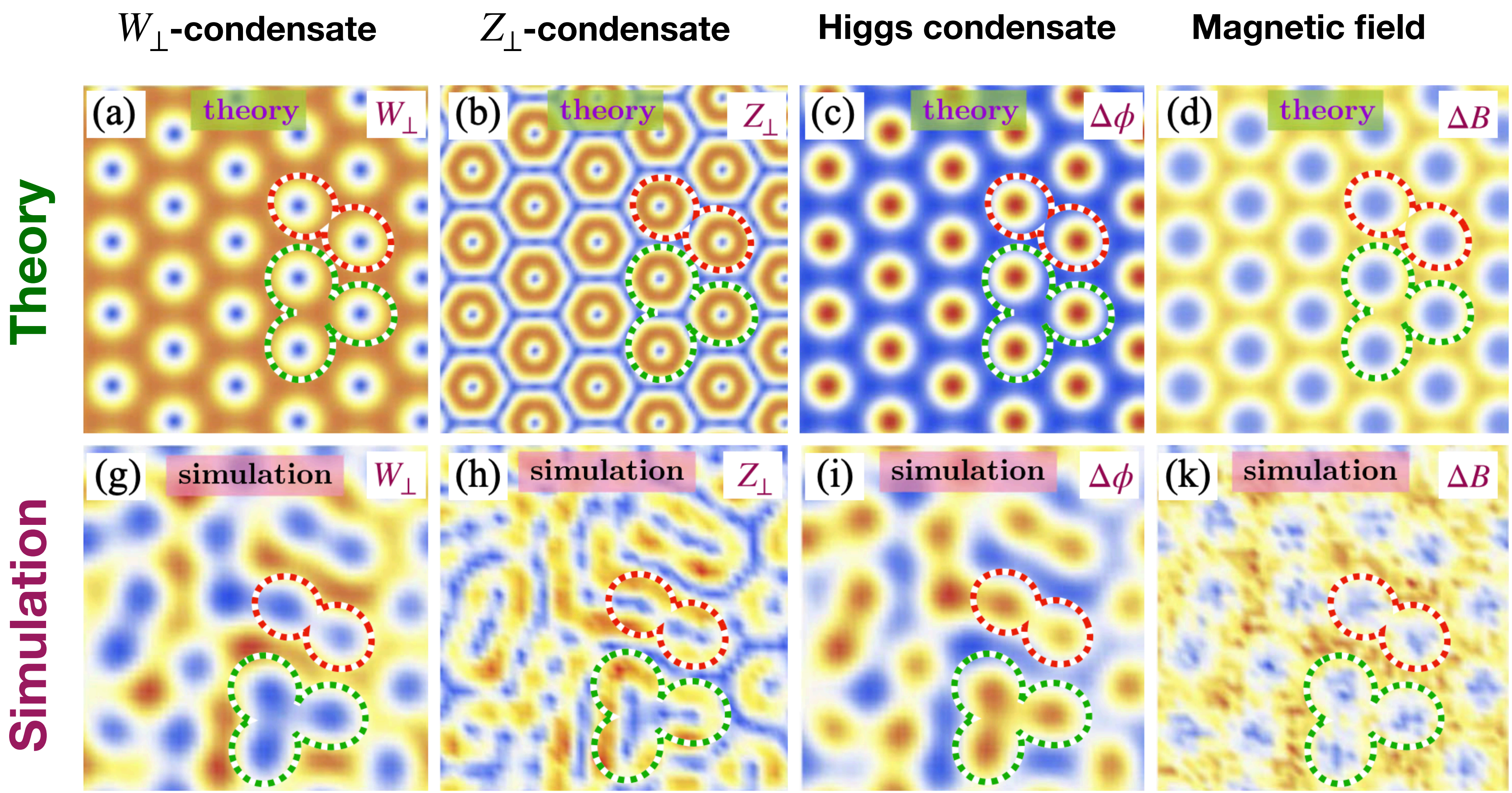
## $Z$-vortex condensate




a cross-section of a typical configuration in the $xy$ plane

# General view - III



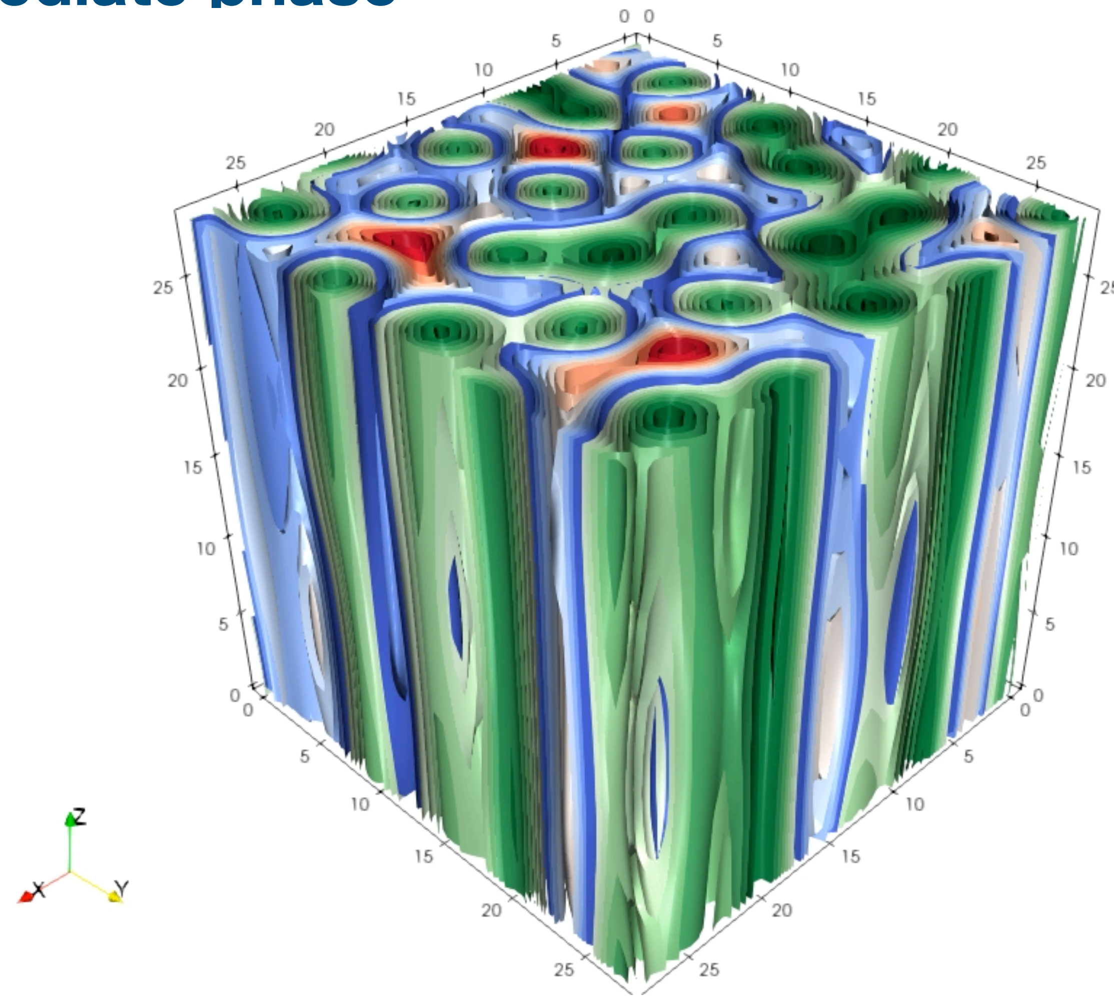

Not a "usual" type-II superconductor: magnetic field is strong outside the vortex cores and it is suppressed inside the vortices!

a cross-section of a typical configuration in the $xy$ plane

# Nature of the intermediate phase

$$eB = 1.1 M_W^2$$

$$B_{c1} < B < B_{c2}$$



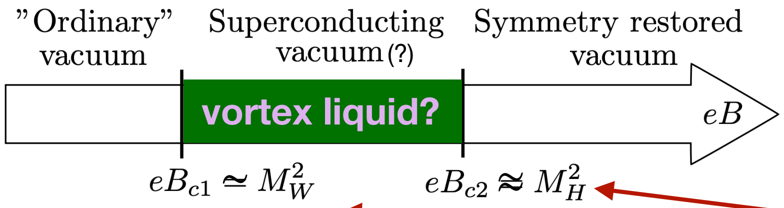

The blue (green) surfaces denote the equipotential surfaces  of the **W condensate (the Higgs condensate)**.

The lines denote the lines of the hypermagnetic field.

**Result 4. No crystalline order for vortices (presumably, due to quantum fluctuations).**
(Classical) theory predicts the hexagonal vortex solid. Not OK with theory.
**The vacuum presumably becomes a liquid made of vortices.**

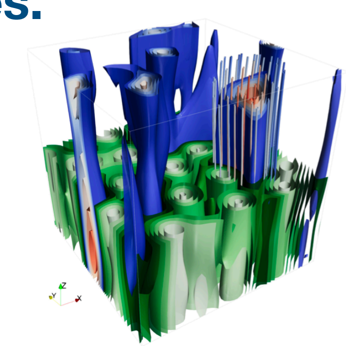
# Conclusions

1. **The polarization effects associated with a large $g$-factor ($g = 2$) of the W-bosons make a vacuum a liquid (a disordered solid) vortex matter that presumably has superconducting-superfluid properties.**




smooth crossovers

$$B^{\mathrm{EW}} = \frac{m_W^2}{e} \sim 10^{20}\,\mathrm{T}$$



2. **The polarization effects appear also in rotating/vortical environments (the Barnett effect):**

   **The Barnett effects appear at many scales:**

   **for magnetic moments in a solid ferromagnetic, for electrons in a liquid metal, for nuclei in rotating liquid (protons in water), in ultrarelativistic environment of hadronic physics ($\Lambda$ hyperons), for quarks and gluons in quark-gluon plasma**

**All effects are understood everywhere except in the theory of strong interactions, QCD. The strongly interacting medium <u>fails</u> to follow the conventional wisdom of the Barnett effects.**

**The Barnett effect for gluons is surprising: hot gluons in quark-gluon plasma seem to possess a negative moment of inertia.**

**Gluonic medium is a time crystal?**