BV actions for extended geometry

based on work with Jakob Palmkvist (many papers)
and Guillaume Bossard & JP ('24)

- Motivation – symmetries of gravity
- Brief intro to extended geometry
- Complexes and 1^{st} order actions for 2-derivative theories
- Summary

(Models containing) gravity exhibit global symmetries beyond isometries.

In particular, after dimensional reduction.

Examples: T-duality $O(d,d)$

U-duality $E_{n(n)}$

Ehlers, Geroch ...

<u>Examples</u> from reduction of gravity in D dim.
& D=11 supergravity

$D = d + n$

"external" "internal"

	<u>D=4 gravity</u>	<u>D-dim. gravity</u>	<u>D=11 SG</u>	
		$\vdots$	$\vdots$	
		A_{n-1}	E_n	
		$\vdots$	$\vdots$	
$d=3$	A_1	A_{D-3}	E_8	Ehlers
2	A_1^+	A_{D-3}^+	E_9	Geroch
1	A_1^{++}	A_{D-3}^{++}	E_{10}	BKL
0	A_1^{++}	A_{D-3}^{+++}	E_{11}	? [West]

Extended geometry : ("geometrisation" of duality symmetries)

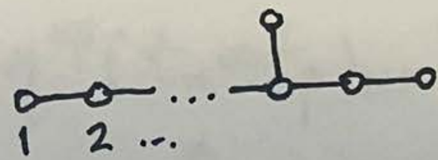
Keep dependence of internal coordinates,
extend them to fill a module of
K-M algebra G.

Dynamical fields in coset $(G \times \mathbb{R})/K(G)$
mimicking $GL(n)/SO(n)$

Choices to make:

- K-M group G
- Vector/coordinate module

Example: E_n



coordinates and vector fields in $R(-\Lambda_1)$

Example: Geroch A_1^+

$R(-\Lambda_1)$

- How does the theory "preserve" G?
- How is original model obtained?

Vectors V^M. Derivatives ∂_M

Local symmetry generated by "generalised Lie derivative"

$$\mathcal{L}_\xi V^M = \underbrace{\xi^N \partial_N V^M}_{\text{"transport"}} - \underbrace{\eta^{\alpha\beta} (t_\alpha \otimes t_\beta)_{NP}{}^{QM} \partial_Q \xi^N V^P}_{\text{"G rotation"}} + \underbrace{w\, \partial_N \xi^N V^M}_{\text{scaling}}$$

(agrees with ordinary diffeos, with Dorfmann bracket in generalised geometry).

Is this meaningful ?

when dual gravity involved: "ancillary" - will not talk about

compute and see !

$$[\mathcal{L}_\xi, \mathcal{L}_\eta] = \mathcal{L}_{[\![\xi,\eta]\!]} \quad (+ \Sigma_{\xi\eta})$$

"courant" bracket, antisymm. of Dorfmann.

$[\![\cdot,\cdot]\!]$ not Lie !

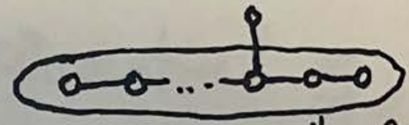
but only if $Y \partial \otimes \partial = 0$,

$$Y = -\eta^{\alpha\beta} t_\alpha \otimes t_\beta + (\lambda,\lambda) - 1 + \sigma$$

"section constraint"

Solutions to section constraint are maximal linear subspaces of minimal G-orbit in $R(\lambda)$.

Preserved by $GL(n)$ subgroup.



"gravity line"

$\longrightarrow$ back (locally) to original model.

Caveat: Global interpretation of extended space unclear.

We want to construct BV models of extended geometry.

Start with linearised theory.

This means having a complex with differential q, $q^2 = 0$. (Dual to 1-bracket of L_∞ algebra)

In addition, look for 1st order versions

- q has at most one derivative.

1st order from 2nd order

Example : YM

$$\begin{tikzcd} \Omega^0 \arrow[r, "d"] & \Omega^1 \arrow[r, "d"] & \Omega^2 & & \\ & & \Omega^{D-2} \arrow[u, "*"] \arrow[r, "d"] & \Omega^{D-1} \arrow[r, "d"] & \Omega^D \end{tikzcd}$$

$q = d + *$

homotopy transfer / elimination of algebraic field eq's

$S = \langle \Phi, q\Phi \rangle \; (+\dots)$

$$\begin{tikzcd} \Omega^0 \arrow[r] & \Omega^1 \arrow[d, "d*d"] & \\ & \Omega^{D-1} \arrow[r] & \Omega^D \end{tikzcd}$$

see talks by Maxim, Evgenia, ..

Gravity :

$$V \to V\otimes\Omega^1 \to V\otimes\Omega^2 \to \cdots$$

$$\begin{tikzcd}
\xi \arrow[r, "d"] & e \arrow[r, "d"] & T \arrow[r, "d"] & \lambda^* & \partial_{[a}T_{bc]}{}^c \arrow[l, dotted] \\
\lambda \arrow[ur, "\rho"] \arrow[r, "d"'] & \omega \arrow[ur, "\sigma"'] \arrow[r, "d"'] & e^* \arrow[ur, "\rho^*"] \arrow[r, "d"'] & \xi^* &
\end{tikzcd}$$

$$\begin{tikzcd}
 & \cdot \arrow[r, "d"] & \cdot \\
\cdot \arrow[ur, "\rho"] \arrow[r, "d"'] & \cdot \arrow[ur, "\sigma"'] &
\end{tikzcd} \quad : \quad \underline{d\circ\rho + \sigma\circ d = 0}$$

unique sol. for σ $\dashrightarrow$ hom. transfer

$$S[e] = \int T(e)\,\sigma^{-1}\,T(e)$$

Extended geometry – analogous complex

[a bit more, ancillary fields/ ghosts not displayed]

$$
\begin{tikzcd}
\cdots \arrow[r] & \xi' \arrow[r] & \xi \arrow[r] & E \arrow[r] & \Theta \arrow[r] & \lambda^* & & \\
 & & \lambda \arrow[ur] \arrow[r] & \Theta^* \arrow[ur, "\sigma"] \arrow[r] & E^* \arrow[ur] \arrow[r] & \xi^* \arrow[r] & \xi'^* \arrow[r] & \cdots
\end{tikzcd}
$$

Upper line obtained as levels in <u>tensor hierarchy algebra</u>.

(extremely useful esp. for ∞-dim G)

Only "unknown" : σ

After hom. transfer : $S_{\text{phys.}} = \frac{1}{2}\int \langle \Theta(E), \sigma^{-1}\Theta(E) \rangle$

For non-linear theory (full BV):

Need only check that the Bianchi id's used have the specific form

$$(D_M + \Theta_M) X_{NP}{}^M = 0 .$$

"small torsion"

The rest is covariantisation w.r.t. Weizenböck connection of E.

Avoids problems with not having enough torsion constraints to determine ω (but still interesting... unfolding?)

(see Falk's talk, maybe)

Conclusions & questions

- Parts of BV actions constructed for affine (and smaller) of form $S = \langle \Psi, Q\Psi \rangle + \dots$

- explicit ghost sector when $d \geq 4$ –
  ∞ sequence of ghosts for ghosts, ∞ high brackets.

- $d \leq 2$: G itself becomes enlarged. For example,
  $d=1$: New Lie algebra structure on $g^{++} \oplus R(-\lambda)$

- Relations to cosmological billiards and emergence of space?

Ευχαριστώ πολύ!