33rd Nordic Network Meeting on “String, Fields and Branes”

Mass gap puzzle for a near extremal Lifshitz black hole

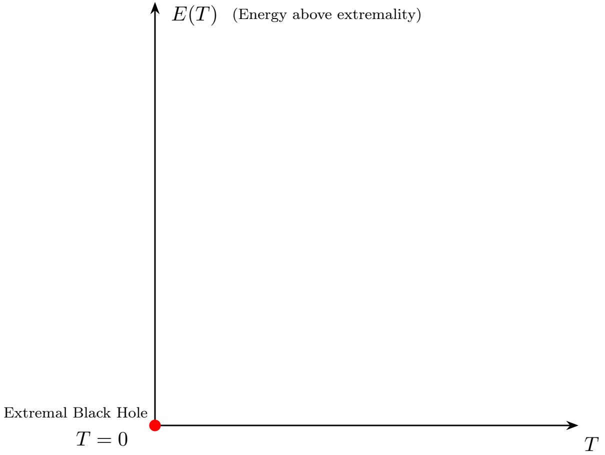
Matthias Harksen

Based on [2408.15336] with Watse Sybesma

Mass gap puzzle for Reissner–Nordström Black Holes

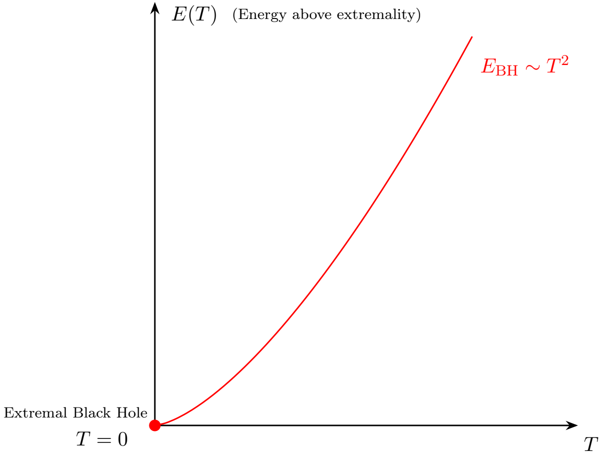
[Preskill, Schwarz, Shapere, Trivedi, Wilczek; 91]

Mass gap puzzle for Reissner–Nordström Black Holes



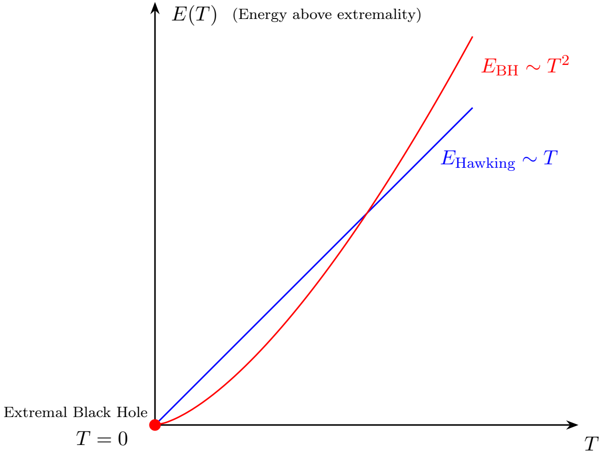
[Preskill, Schwarz, Shapere, Trivedi, Wilczek; 91]

Mass gap puzzle for Reissner–Nordström Black Holes



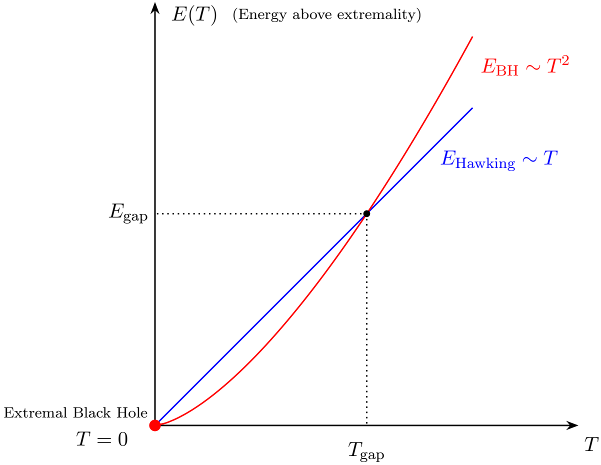
[Preskill, Schwarz, Shapere, Trivedi, Wilczek; 91]

Mass gap puzzle for Reissner–Nordström Black Holes



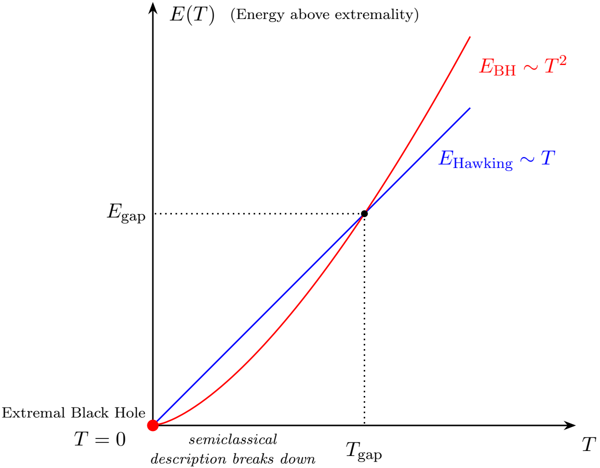
[Preskill, Schwarz, Shapere, Trivedi, Wilczek; 91]

Mass gap puzzle for Reissner–Nordström Black Holes



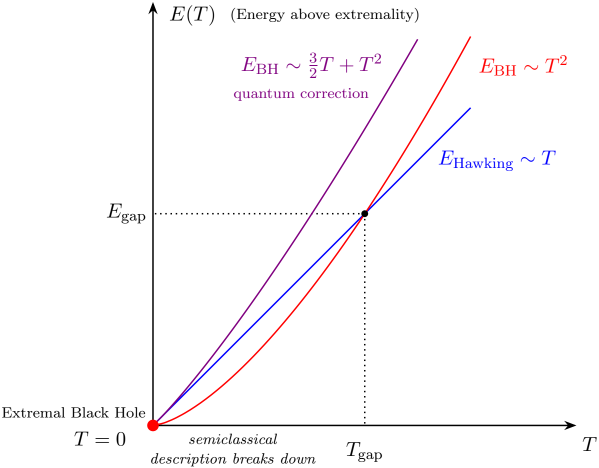
[Preskill, Schwarz, Shapere, Trivedi, Wilczek; 91]

Mass gap puzzle for Reissner–Nordström Black Holes



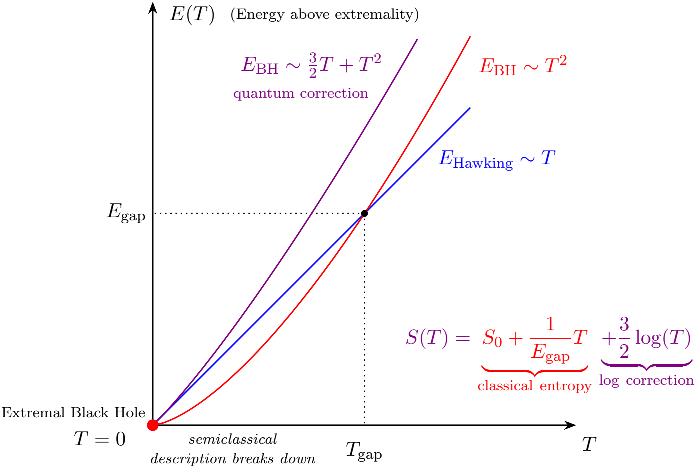
[Preskill, Schwarz, Shapere, Trivedi, Wilczek; 91]

Mass gap puzzle for Reissner–Nordström Black Holes



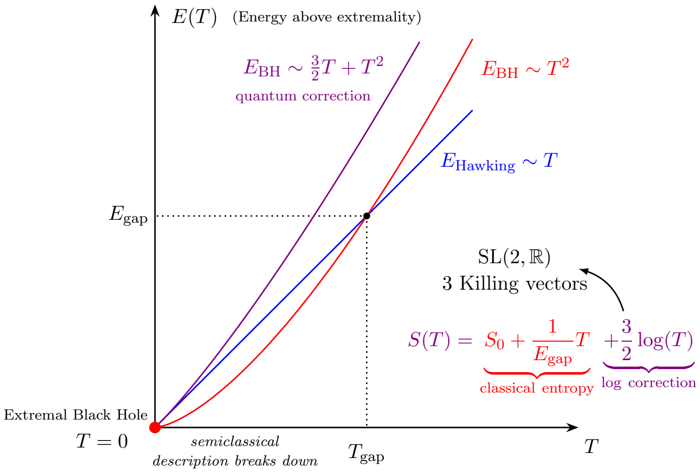
[Iliesiu, Turiaci; 20]

Mass gap puzzle for Reissner–Nordström Black Holes



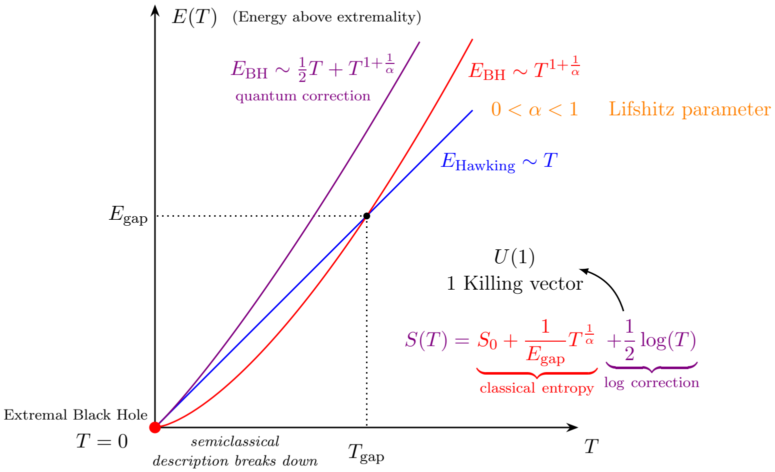
[Iliesiu, Turiaci; 20]

Mass gap puzzle for Reissner–Nordström Black Holes



[Iliesiu, Turiaci; 20]

Mass gap puzzle for Lifshitz Black Holes



[Harksen, Sybesma; 24]

From Near-Extremal Black Holes to 2D Dilaton Models

From Near-Extremal Black Holes to 2D Dilaton Models

Reissner-Nordström

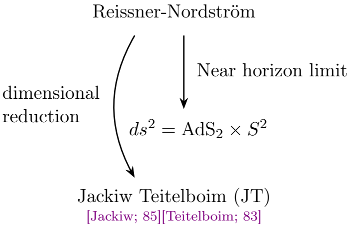
From Near-Extremal Black Holes to 2D Dilaton Models

Reissner-Nordström

↓ Near horizon limit

$ds^2 = \mathrm{AdS}_2 \times S^2$

From Near-Extremal Black Holes to 2D Dilaton Models



From Near-Extremal Black Holes to 2D Dilaton Models

Reissner-Nordström

Near horizon limit

dimensional reduction

$ds^2 = \mathrm{AdS}_2 \times S^2$

Jackiw Teitelboim (JT)
[Jackiw; 85][Teitelboim; 83]

Lifshitz Black Hole

From Near-Extremal Black Holes to 2D Dilaton Models

Reissner-Nordström

dimensional reduction

Near horizon limit

$ds^2 = \mathrm{AdS}_2 \times S^2$

Jackiw Teitelboim (JT)
[Jackiw; 85][Teitelboim; 83]

Lifshitz Black Hole

dimensional reduction
[Cadoni, Mignemi; Dec 93]
[Sybesma; 22]

Lifshitz Dilaton Model
[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

From Near-Extremal Black Holes to 2D Dilaton Models

Reissner-Nordström

dimensional reduction

Near horizon limit

$ds^2 = \mathrm{AdS}_2 \times S^2$

Jackiw Teitelboim (JT)
[Jackiw; 85][Teitelboim; 83]

Schwarzian action

$$\mathrm{Sch}(t,u) = \frac{t'''(u)}{t'(u)} - \frac{3}{2}\left(\frac{t''(u)}{t'(u)}\right)^2$$

Lifshitz Black Hole

dimensional reduction
[Cadoni, Mignemi; Dec 93]
[Sybesma; 22]

Lifshitz Dilaton Model
[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

From Near-Extremal Black Holes to 2D Dilaton Models

Reissner-Nordström

dimensional reduction

Near horizon limit

$ds^2 = \mathrm{AdS}_2 \times S^2$

Jackiw Teitelboim (JT)
[Jackiw; 85][Teitelboim; 83]

Schwarzian action

$$\mathrm{Sch}(t,u) = \frac{t'''(u)}{t'(u)} - \frac{3}{2}\left(\frac{t''(u)}{t'(u)}\right)^2$$

Lifshitz Black Hole

dimensional reduction
[Cadoni, Mignemi; Dec 93]
[Sybesma; 22]

Lifshitz Dilaton Model
[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

[Harksen, Sybesma; 24]

deformed Schwarzian action

$$\mathrm{dSch}_{\alpha}(t,u) = \mathrm{Sch}(t,u) + \frac{1}{2}(1 - \frac{1}{\alpha})\left(\frac{t''(u)}{t'(u)}\right)^2$$

From Near-Extremal Black Holes to 2D Dilaton Models

Reissner-Nordström

dimensional reduction

Near horizon limit

$ds^2 = \mathrm{AdS}_2 \times S^2$

Jackiw Teitelboim (JT)
[Jackiw; 85][Teitelboim; 83]

Schwarzian action

$$\mathrm{Sch}(t,u) = \frac{t'''(u)}{t'(u)} - \frac{3}{2}\left(\frac{t''(u)}{t'(u)}\right)^2$$

logarithmic entropy corrections
$\frac{3}{2}\log(T)$

Lifshitz Black Hole

dimensional reduction
[Cadoni, Mignemi; Dec 93]
[Sybesma; 22]

Lifshitz Dilaton Model
[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

[Harksen, Sybesma; 24]

deformed Schwarzian action

$$\mathrm{dSch}_{\alpha}(t,u) = \mathrm{Sch}(t,u) + \frac{1}{2}(1-\frac{1}{\alpha})\left(\frac{t''(u)}{t'(u)}\right)^2$$

logarithmic entropy corrections
$\frac{1}{2}\log(T)$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

$\underline{\alpha = 1}$: JT gravity [Jackiw; 85][Teitelboim; 83]

$$I_{\text{JT}} = I_{\text{Lif}}(\alpha = 1) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left[R + 4\lambda^2\right]$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

<u>$\alpha = 1$:</u> JT gravity [Jackiw; 85][Teitelboim; 83]

$$I_{\text{JT}} = I_{\text{Lif}}(\alpha = 1) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left[R + 4\lambda^2\right]$$

<u>$\alpha = 0$:</u> CGHS gravity [Callan, Giddings, Harvey, Strominger; 92]

$$I_{\text{CGHS}} = I_{\text{Lif}}(\alpha = 0) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left[R + 4(\nabla\phi)^2 + 4\lambda^2\right]$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

Can get rid of kinetic term

$$\Phi = e^{-2\phi}, \qquad g_{ab} \to \Phi^{\alpha-1} g_{ab}.$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

Can get rid of kinetic term

$$\Phi = e^{-2\phi}, \qquad g_{ab} \to \Phi^{\alpha-1} g_{ab}.$$

in which case

$$I_{\text{W-Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\left(\Phi R + 4\lambda^2\Phi^{\alpha}\right)$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

Can get rid of kinetic term

$$\Phi = e^{-2\phi}, \qquad g_{ab} \to \Phi^{\alpha-1} g_{ab}.$$

in which case

$$I_{\text{W-Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\left(\Phi R + 4\lambda^2\Phi^{\alpha}\right)$$

Not assymptotically AdS_2 ☹

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\mathrm{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

Thermodynamics:

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

Thermodynamics: [Cadoni, Mignemi; 93][Kumar, Ray; 95]

$$S \sim T^{\frac{1}{\alpha}}, \qquad M = \frac{1}{1+\alpha}TS\,.$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

Thermodynamics: [Cadoni, Mignemi; 93][Kumar, Ray; 95]

$$S \sim T^{\frac{1}{\alpha}}, \qquad M = \frac{1}{1+\alpha} TS\,.$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

Thermodynamics: [Cadoni, Mignemi; 93][Kumar, Ray; 95]

$$S \sim T^{\frac{1}{\alpha}}\,, \qquad M = \frac{1}{1+\alpha} T S\,.$$

Asymptotically Lifshitz Black Hole

$$ds^2 = -r^{2z} f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\vec{x}^2_{d-1}\,, \qquad f(r) = 1 - \left(\frac{r_h}{r}\right)^{d-1+z}\,.$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

Thermodynamics: [Cadoni, Mignemi; 93][Kumar, Ray; 95]

$$S \sim T^{\frac{1}{\alpha}}, \qquad M = \frac{1}{1+\alpha}TS\,.$$

Asymptotically Lifshitz Black Hole

$$ds^2 = -r^{2z}f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\vec{x}^2_{d-1}\,, \qquad f(r) = 1 - \left(\frac{r_h}{r}\right)^{d-1+z}\,.$$

Thermodynamics: [Taylor; 08][Taylor; 15]

$$S \sim T^{\frac{d-1}{z}}\,, \qquad M = \frac{d-1}{d-1+z}TS\,.$$

The Lifshitz Dilaton Model

[Lemos, Sá; Nov 93][Cadoni, Mignemi; Dec 93]

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\,\underbrace{e^{-2\phi}}_{\text{dilaton}}\left[R + \underbrace{4(1-\alpha)(\nabla\phi)^2}_{\text{kinetic term}} + 4\lambda^2\right]$$

Thermodynamics: [Cadoni, Mignemi; 93][Kumar, Ray; 95]

$$S \sim T^{\frac{1}{\alpha}}, \qquad M = \frac{1}{1+\alpha}TS\,.$$

Asymptotically Lifshitz Black Hole

$$ds^2 = -r^{2z}f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\vec{x}^2_{d-1}\,, \qquad f(r) = 1 - \left(\frac{r_h}{r}\right)^{d-1+z}\,.$$

Thermodynamics: [Taylor; 08][Taylor; 15]

$$S \sim T^{\frac{d-1}{z}}\,, \qquad M = \frac{d-1}{d-1+z}TS\,.$$

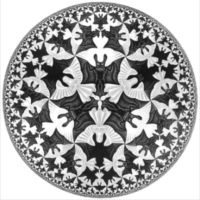
[Sybesma; 22]

$$\boxed{\alpha = \frac{z}{d-1}}$$

What is the goal?

What is the goal?

[Jackiw; 85][Teitelboim; 83]



JT / SYK

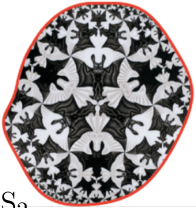
[Sachdev, Ye; 93][Kitaev; 15]

What is the goal?

[Jackiw; 85][Teitelboim; 83]

JT / SYK

[Sachdev, Ye; 93][Kitaev; 15]



$NAdS_2$

[Maldacena, Stanford, Yang; 16]

$NCFT_1$

[Maldacena, Stanford; 16]

What is the goal?

[Jackiw; 85][Teitelboim; 83]

JT / SYK

[Sachdev, Ye; 93][Kitaev; 15]

$NAdS_2$

[Maldacena, Stanford, Yang; 16]

$NCFT_1$

[Maldacena, Stanford; 16]

Schwarzian derivative

$$\mathrm{Sch}(f(z), z) = \frac{f'''(z)}{f''(z)} - \frac{3}{2}\left(\frac{f''(z)}{f'(z)}\right)^2$$

What is the goal?

[Jackiw; 85][Teitelboim; 83]

JT / SYK

[Sachdev, Ye; 93][Kitaev; 15]

$NAdS_2$

[Maldacena, Stanford, Yang; 16]

$NCFT_1$

[Maldacena, Stanford; 16]

Schwarzian derivative

$$\mathrm{Sch}\left(\frac{Az+B}{Cz+D}, z\right) = 0$$

$$\mathrm{Sch}(f(z), z) = \frac{f'''(z)}{f''(z)} - \frac{3}{2}\left(\frac{f''(z)}{f'(z)}\right)^2$$

What is the goal?

[Jackiw; 85][Teitelboim; 83]

[Sachdev, Ye; 93][Kitaev; 15]

JT / SYK

$NAdS_2$

[Maldacena, Stanford, Yang; 16]

$NCFT_1$

[Maldacena, Stanford; 16]

Schwarzian derivative

$$\text{Sch}\left(\frac{Az+B}{Cz+D}, z\right) = 0$$

$$\text{Sch}\left(\frac{af(z)+b}{cf(z)+d}, z\right) = \text{Sch}(f(z), z) = \frac{f'''(z)}{f''(z)} - \frac{3}{2}\left(\frac{f''(z)}{f'(z)}\right)^2$$

What is the goal?

[Jackiw; 85][Teitelboim; 83]

JT / SYK

[Sachdev, Ye; 93][Kitaev; 15]

NAdS_2

[Maldacena, Stanford, Yang; 16]

NCFT_1

[Maldacena, Stanford; 16]

Schwarzian derivative

$$\mathrm{Sch}\left(\frac{Az+B}{Cz+D}, z\right) = 0$$

$$\mathrm{Sch}\left(\frac{af(z)+b}{cf(z)+d}, z\right) = \mathrm{Sch}(f(z), z) = \frac{f'''(z)}{f''(z)} - \frac{3}{2}\left(\frac{f''(z)}{f'(z)}\right)^2$$

$\mathrm{SL}(2,\mathbb{R})$ symmetry

What is the goal?

[Jackiw; 85][Teitelboim; 83]

JT / [Sachdev, Ye; 93][Kitaev; 15]

NAdS_2

[Maldacena, Stanford, Yang; 16]

NCFT_1

[Maldacena, Stanford; 16]

Schwarzian derivative $\quad \mathrm{Sch}\left(\frac{Az+B}{Cz+D}, z\right) = 0$

$$\mathrm{Sch}\left(\frac{af(z)+b}{cf(z)+d}, z\right) = \mathrm{Sch}(f(z), z) = \frac{f'''(z)}{f''(z)} - \frac{3}{2}\left(\frac{f''(z)}{f'(z)}\right)^2$$

$$\mathrm{SL}(2,\mathbb{R}) \text{ symmetry} \xrightarrow[\text{this work}]{} U(1) \text{ symmetry}$$

What is the goal?

[Jackiw; 85][Teitelboim; 83]

JT / SYK

[Sachdev, Ye; 93][Kitaev; 15]

$NAdS_2$

[Maldacena, Stanford, Yang; 16]

$NCFT_1$

[Maldacena, Stanford; 16]

Schwarzian derivative

$$\text{Sch}\left(\frac{Az+B}{Cz+D}, z\right) = 0$$

$$\text{Sch}\left(\frac{af(z)+b}{cf(z)+d}, z\right) = \text{Sch}(f(z), z) = \frac{f'''(z)}{f''(z)} - \frac{3}{2}\left(\frac{f''(z)}{f'(z)}\right)^2 \boxed{+\frac{1}{2}\left(1-\frac{1}{\alpha}\right)\left(\frac{t''(u)}{t'(u)}\right)^2}$$

$SL(2,\mathbb{R})$ symmetry $\xrightarrow[\text{this work}]{}$ $U(1)$ symmetry

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\rm Lif} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM:

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM: $\delta\phi:\quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM: $\delta\phi:\quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$\delta g_{\mu\nu}:\quad \nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

$$\overset{\boxed{\text{EOM}}}{=} -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^{\mu}\nabla_{\mu}\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

EOM: $\delta\phi:\quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$\delta g_{\mu\nu}:\quad \nabla_{\mu}\nabla_{\nu}\phi - 2\alpha\nabla_{\mu}\phi\nabla_{\nu}\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM

$$= -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^{\mu}\nabla_{\mu}\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

EOM: $\delta\phi$: $\quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$\delta g_{\mu\nu}$: $\quad \nabla_{\mu}\nabla_{\nu}\phi - 2\alpha\nabla_{\mu}\phi\nabla_{\nu}\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$

$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)}, \quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}}, \quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}}, \quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM

$$= -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^\mu\nabla_\mu\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

wiggling boundary

EOM: $\delta\phi:$

$$R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$$

$\delta g_{\mu\nu}:$

$$\nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$$

$$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)}, \quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}}, \quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}}, \quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

$$\overset{\text{EOM}}{=} -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^{\mu}\nabla_{\mu}\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

wiggling boundary

EOM: $\delta\phi$: $R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$\delta g_{\mu\nu}$: $\nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$

$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)}, \quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}}, \quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}}, \quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM

$$= -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^\mu\nabla_\mu\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

wiggling boundary

EOM: $\delta\phi$: $\quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$\delta g_{\mu\nu}$: $\quad \nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$

$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)}, \quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}}, \quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}}, \quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM

$$= -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^\mu\nabla_\mu\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

EOM: $\delta\phi$: $\quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$\delta g_{\mu\nu}$: $\quad \nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$

$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)}, \quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}}, \quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}}, \quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM

$$= -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^\mu\nabla_\mu\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

wiggling boundary

$h_{uu} = \frac{1}{\epsilon^2}$

$r(u) = \frac{1}{a\epsilon t'(u)} + \mathcal{O}(\epsilon)$

$$\sqrt{h} = \frac{1}{\epsilon} + \frac{\epsilon}{2a^2}\left(\frac{t''(u)}{t'(u)}\right)^2 - \frac{b}{2}t'(u)^{1+\frac{1}{\alpha}}\epsilon^{\frac{1}{\alpha}} + \mathcal{O}(\epsilon^3)$$

EOM: $\delta\phi$: $$R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$$

$\delta g_{\mu\nu}$: $$\nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$$

$$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)}, \quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}}, \quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}}, \quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM

$$= -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^\mu\nabla_\mu\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

wiggling boundary

$$h_{uu} = \frac{1}{\epsilon^2}$$

$$r(u) = \frac{1}{a\epsilon t'(u)} + \mathcal{O}(\epsilon)$$

$$\sqrt{h} = \frac{1}{\epsilon} + \frac{\epsilon}{2a^2}\left(\frac{t''(u)}{t'(u)}\right)^2 - \frac{b}{2}t'(u)^{1+\frac{1}{\alpha}}\epsilon^{\frac{1}{\alpha}} + \mathcal{O}(\epsilon^3)$$

$$K = a + \frac{1}{a}\text{Sch}(t,u)\epsilon^2 + \frac{ab}{2\alpha}t'(u)^{1+\frac{1}{\alpha}}\epsilon^{1+\frac{1}{\alpha}} + \mathcal{O}(\epsilon^4)$$

EOM: $\delta\phi$: $\quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$\delta g_{\mu\nu}$: $\quad \nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$

$$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)}, \quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}}, \quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}}, \quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM

$$= -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^\mu\nabla_\mu\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

wiggling boundary

$h_{uu} = \frac{1}{\epsilon^2}$

$r(u) = \frac{1}{a\epsilon t'(u)} + \mathcal{O}(\epsilon)$

$$\sqrt{h} = \frac{1}{\epsilon} + \frac{\epsilon}{2a^2}\left(\frac{t''(u)}{t'(u)}\right)^2 - \frac{b}{2}t'(u)^{1+\frac{1}{\alpha}}\epsilon^{\frac{1}{\alpha}} + \mathcal{O}(\epsilon^3)$$

$$K = a + \frac{1}{a}\text{Sch}(t,u)\epsilon^2 + \frac{ab}{2\alpha}t'(u)^{1+\frac{1}{\alpha}}\epsilon^{1+\frac{1}{\alpha}} + \mathcal{O}(\epsilon^4)$$

$$-2(1-\alpha)n^\mu\nabla_\mu\phi = \left(\frac{1}{\alpha}-1\right)a - \frac{1-\alpha}{2a\alpha}\left(\frac{t''(u)}{t'(u)}\right)^2\epsilon^2 - \frac{ab}{2}\left(\frac{1}{\alpha}-1\right)t'(u)^{1+\frac{1}{\alpha}}\epsilon^{1+\frac{1}{\alpha}} + \mathcal{O}(\epsilon^4)$$

EOM: $\delta\phi:\quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$\delta g_{\mu\nu}:\quad \nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$

$$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)},\quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}},\quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}},\quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

$$\overset{\text{EOM}}{=} -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^\mu\nabla_\mu\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

wiggling boundary

$b \to \epsilon^{1-\alpha} b$

$t(u) \to \epsilon^{\alpha-1} t(u)$

$h_{uu} = \frac{1}{\epsilon^2}$

$r(u) = \frac{1}{a\epsilon t'(u)} + \mathcal{O}(\epsilon)$

$$\sqrt{h} = \frac{1}{\epsilon} + \frac{\epsilon}{2a^2}\left(\frac{t''(u)}{t'(u)}\right)^2 - \frac{b}{2}t'(u)^{1+\frac{1}{\alpha}}\epsilon^{\frac{1}{\alpha}} + \mathcal{O}(\epsilon^3)$$

$$K = a + \frac{1}{a}\text{Sch}(t,u)\epsilon^2 + \frac{ab}{2\alpha}t'(u)^{1+\frac{1}{\alpha}}\epsilon^{1+\frac{1}{\alpha}} + \mathcal{O}(\epsilon^4)$$

$$-2(1-\alpha)n^\mu\nabla_\mu\phi = \left(\frac{1}{\alpha}-1\right)a - \frac{1-\alpha}{2a\alpha}\left(\frac{t''(u)}{t'(u)}\right)^2\epsilon^2 - \frac{ab}{2}\left(\frac{1}{\alpha}-1\right)t'(u)^{1+\frac{1}{\alpha}}\epsilon^{1+\frac{1}{\alpha}} + \mathcal{O}(\epsilon^4)$$

EOM: $\delta\phi:$ $$R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$$

$\delta g_{\mu\nu}:$ $$\nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$$

$$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)}, \quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}}, \quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}}, \quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM

$$= -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^\mu\nabla_\mu\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

$$\sqrt{h} = \tfrac{1}{\epsilon} + \mathcal{O}(\epsilon^3)$$

$$K = a + \left(\tfrac{1}{a}\text{Sch}(t,u) + \tfrac{ab}{2\alpha}t'(u)^{1+\frac{1}{\alpha}}\right)\epsilon^2 + \mathcal{O}(\epsilon^4)$$

$$-2(1-\alpha)n^\mu\nabla_\mu\phi = \left(\tfrac{1}{\alpha} - 1\right)a - (1-\alpha)\left(\tfrac{ab}{2\alpha}t'(u)^{1+\frac{1}{\alpha}} + \tfrac{1}{2a\alpha}\left(\tfrac{t''(u)}{t'(u)}\right)^2\right)\epsilon^2 + \mathcal{O}(\epsilon^4)$$

EOM: $\delta\phi: \quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$$\delta g_{\mu\nu}: \quad \nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$$

$$ds^2 = f(r)dt^2 + \tfrac{dr^2}{f(r)}, \quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}}, \quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}}, \quad a = \tfrac{2\alpha\lambda}{\sqrt{1+\alpha}}$$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{Lif}} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R+4(1-\alpha)(\nabla\phi)^2+4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K-\frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM

$$= -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^\mu\nabla_\mu\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

$$= -\int du\,\frac{1}{t'(u)^{\frac{1}{\alpha}}}\left(\frac{ab}{2}\,t'(u)^{1+\frac{1}{\alpha}} + \frac{1}{a}\text{Sch}(t,u) + \frac{(1-\frac{1}{\alpha})}{2a}\left(\frac{t''(u)}{t'(u)}\right)^2\right) + \mathcal{O}(\epsilon)$$

$b \to \epsilon^{1-\alpha} b$

$t(u) \to \epsilon^{\alpha-1} t(u)$

$h_{uu} = \frac{1}{\epsilon^2}$

$r(u) = \frac{1}{a\epsilon t'(u)} + \mathcal{O}(\epsilon)$

$$\sqrt{h} = \tfrac{1}{\epsilon} + \mathcal{O}(\epsilon^3)$$

$$K = a + \left(\tfrac{1}{a}\text{Sch}(t,u) + \tfrac{ab}{2\alpha}t'(u)^{1+\frac{1}{\alpha}}\right)\epsilon^2 + \mathcal{O}(\epsilon^4)$$

$$-2(1-\alpha)n^\mu\nabla_\mu\phi = \left(\tfrac{1}{\alpha}-1\right)a - (1-\alpha)\left(\tfrac{ab}{2\alpha}t'(u)^{1+\frac{1}{\alpha}} + \tfrac{1}{2a\alpha}\left(\tfrac{t''(u)}{t'(u)}\right)^2\right)\epsilon^2 + \mathcal{O}(\epsilon^4)$$

EOM: $\delta\phi$: $\quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$\delta g_{\mu\nu}$: $\quad \nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$

$$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)},\quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}},\quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}},\quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$$

Deriving the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\rm Lif} = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left(R+4(1-\alpha)(\nabla\phi)^2+4\lambda^2\right) - \underbrace{\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(K-\frac{2\lambda}{\sqrt{1+\alpha}}\right)}_{\text{Gibbons-Hawking-York + counter term}}$$

EOM

$$= -\int_{\partial\mathcal{M}} du\sqrt{h}e^{-2\phi}\left(-2(1-\alpha)n^\mu\nabla_\mu\phi + K - \frac{2\lambda}{\sqrt{1+\alpha}}\right)$$

$$= -\int du\frac{1}{t'(u)^{\frac{1}{\alpha}}}\left(\frac{ab}{2}t'(u)^{1+\frac{1}{\alpha}} + \frac{1}{a}\mathrm{Sch}(t,u) + \frac{(1-\frac{1}{\alpha})}{2a}\left(\frac{t''(u)}{t'(u)}\right)^2\right) + \mathcal{O}(\epsilon)$$

$$\overset{\epsilon\to 0}{=} \boxed{-\alpha M\int t'(u)du - \frac{1}{a}\int du\frac{1}{t'(u)^{\frac{1}{\alpha}}}\mathrm{dSch}_\alpha(t,u)}$$

wiggling boundary

$(t(u),r(u))$

$b\to\epsilon^{1-\alpha}b$

$t(u)\to\epsilon^{\alpha-1}t(u)$

$h_{uu}=\frac{1}{\epsilon^2}$

$r(u)=\frac{1}{a\epsilon t'(u)}+\mathcal{O}(\epsilon)$

$$\sqrt{h} = \frac{1}{\epsilon} + \mathcal{O}(\epsilon^3)$$

$$K = a + \left(\frac{1}{a}\mathrm{Sch}(t,u) + \frac{ab}{2\alpha}t'(u)^{1+\frac{1}{\alpha}}\right)\epsilon^2 + \mathcal{O}(\epsilon^4)$$

$$-2(1-\alpha)n^\mu\nabla_\mu\phi = \left(\frac{1}{\alpha}-1\right)a - (1-\alpha)\left(\frac{ab}{2\alpha}t'(u)^{1+\frac{1}{\alpha}} + \frac{1}{2a\alpha}\left(\frac{t''(u)}{t'(u)}\right)^2\right)\epsilon^2 + \mathcal{O}(\epsilon^4)$$

EOM: $\delta\phi:\quad R + (4(1-\alpha)\nabla^2\phi - 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2 = 0$

$\delta g_{\mu\nu}:\quad \nabla_\mu\nabla_\nu\phi - 2\alpha\nabla_\mu\phi\nabla_\nu\phi - \left(\nabla^2\phi - (1+\alpha)(\nabla\phi)^2 + \lambda^2\right)g_{\mu\nu} = 0$

$$ds^2 = f(r)dt^2 + \frac{dr^2}{f(r)},\quad f(r) = (ar)^2 - b(ar)^{1-\frac{1}{\alpha}},\quad e^{-2\phi} = (ar)^{\frac{1}{\alpha}},\quad a = \frac{2\alpha\lambda}{\sqrt{1+\alpha}}$$

Remarks on the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t] = -\alpha M \int t'(u) du - \frac{1}{a} \int du \frac{1}{t'(u)^{\frac{1}{\alpha}}} \text{dSch}_{\alpha}(t, u)$$

Remarks on the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\mathrm{dSch}}[t] = \underbrace{-\alpha M \int t'(u) du}_{\text{free energy term}} - \frac{1}{a} \int du \frac{1}{t'(u)^{\frac{1}{\alpha}}} \mathrm{dSch}_{\alpha}(t, u)$$

Remarks on the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t] = \underbrace{-\alpha M \int t'(u)du}_{\text{free energy term}} - \frac{1}{a}\int du \frac{1}{t'(u)^{\frac{1}{\alpha}}} \text{dSch}_{\alpha}(t,u)$$

where $\quad \text{dSch}_{\alpha}(t,u) = \text{Sch}(t,u) + \frac{1}{2}(1-\frac{1}{\alpha})\left(\frac{t''(u)}{t'(u)}\right)^2$

Remarks on the deformed Schwarzian [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t] = \underbrace{-\alpha M \int t'(u)du}_{\text{free energy term}} - \frac{1}{a} \int du \frac{1}{t'(u)^{\frac{1}{\alpha}}} \text{dSch}_{\alpha}(t,u)$$

where $\quad \text{dSch}_{\alpha}(t,u) = \text{Sch}(t,u) + \frac{1}{2}(1 - \frac{1}{\alpha}) \left(\frac{t''(u)}{t'(u)}\right)^2$

(JT) $\quad$ $\underline{\alpha = 1:}$ $\qquad \text{dSch}_{\alpha=1}(t,u) = \text{Sch}(t,u)$

Classical Partition Function [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t] = -\alpha M \int t'(u) du - \frac{1}{a} \int du \frac{1}{t'(u)^{\frac{1}{\alpha}}} \text{dSch}_{\alpha}(t, u)$$

Classical Partition Function [Harksen, Sybesma; 24]

$$I_{\mathrm{dSch}}[t] = -\alpha M \int t'(u)du - \frac{1}{a}\int du \frac{1}{t'(u)^{\frac{1}{\alpha}}} \mathrm{dSch}_{\alpha}(t,u)$$

EOM:

$$\frac{d}{du}\left(\frac{\mathrm{dSch}_{\alpha}}{t'(u)^{1+\frac{1}{\alpha}}}\right) = 0 \qquad \Longrightarrow \qquad t(u) = \begin{cases} au+b & \text{if } 0<\alpha<1 \\ \frac{au+b}{cu+d} & \text{if } \alpha = 1 \end{cases}$$

Classical Partition Function [Harksen, Sybesma; 24]

$$I_{\mathrm{dSch}}[t] = -\alpha M \int t'(u)du - \frac{1}{a}\int du \frac{1}{t'(u)^{\frac{1}{\alpha}}}\mathrm{dSch}_{\alpha}(t,u)$$

EOM:

$$\frac{d}{du}\left(\frac{\mathrm{dSch}_{\alpha}}{t'(u)^{1+\frac{1}{\alpha}}}\right) = 0 \qquad \Longrightarrow \qquad t(u) = \begin{cases} au+b & \text{if } 0 < \alpha < 1 \\ \frac{au+b}{cu+d} & \text{if } \alpha = 1 \end{cases}$$

On-shell action:

$$\mathcal{I}_{\mathrm{dSch}} = I_{\mathrm{dSch}}[t=u] = -\alpha M \beta = \beta F \sim \beta^{-\frac{1}{\alpha}}$$

Classical Partition Function [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t] = -\alpha M \int t'(u)du - \frac{1}{a}\int du \frac{1}{t'(u)^{\frac{1}{\alpha}}}\text{dSch}_{\alpha}(t,u)$$

EOM: $$\frac{d}{du}\left(\frac{\text{dSch}_{\alpha}}{t'(u)^{1+\frac{1}{\alpha}}}\right) = 0 \qquad \Longrightarrow \qquad t(u) = \begin{cases} au+b & \text{if } 0<\alpha<1 \\ \frac{au+b}{cu+d} & \text{if } \alpha = 1 \end{cases}$$

On-shell action: $$\mathcal{I}_{\text{dSch}} = I_{\text{dSch}}[t=u] = -\alpha M\beta = \beta F \sim \beta^{-\frac{1}{\alpha}}$$

Classical partition function: $$Z_{\text{cl}}(\beta) = e^{-\mathcal{I}_{\text{Lif}}} = e^{S_0 + \alpha C \beta^{-\frac{1}{\alpha}}}$$

Classical Partition Function [Harksen, Sybesma; 24]

$$I_{\mathrm{dSch}}[t] = -\alpha M \int t'(u) du - \frac{1}{a} \int du \frac{1}{t'(u)^{\frac{1}{\alpha}}} \mathrm{dSch}_{\alpha}(t,u)$$

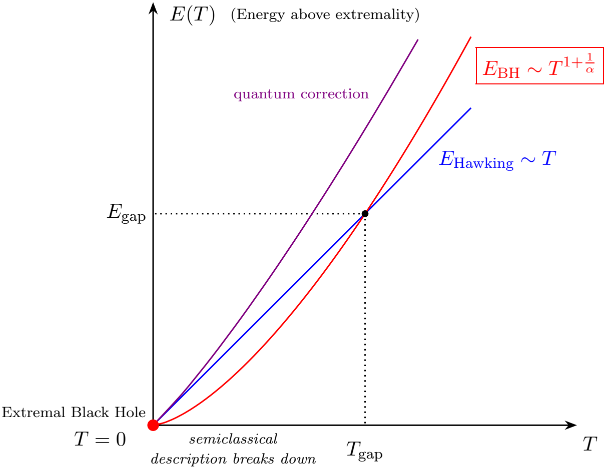
EOM: $$\frac{d}{du}\left(\frac{\mathrm{dSch}_{\alpha}}{t'(u)^{1+\frac{1}{\alpha}}}\right) = 0 \qquad \Longrightarrow \qquad t(u) = \begin{cases} au + b & \text{if } 0 < \alpha < 1 \\ \frac{au+b}{cu+d} & \text{if } \alpha = 1 \end{cases}$$

On-shell action: $$\mathcal{I}_{\mathrm{Lif}} = \mathcal{I}_{\mathrm{dSch}} = I_{\mathrm{dSch}}[t=u] = -\alpha M \beta = \beta F \sim \beta^{-\frac{1}{\alpha}}$$

Classical partition function: $$Z_{\mathrm{cl}}(\beta) = e^{-\mathcal{I}_{\mathrm{Lif}}} = e^{S_0 + \alpha C \beta^{-\frac{1}{\alpha}}}$$

Energy above extremality: $$E_{\mathrm{BH}}(\beta) = C\beta^{-(1+\frac{1}{\alpha})}$$

Classical Partition Function [Harksen, Sybesma; 24]



Logarithmic Correction [Harksen, Sybesma; 24]

Logarithmic Correction [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t = u + \epsilon(u)]$$

Logarithmic Correction [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t = u + \epsilon(u)] = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{1}{2a\alpha} \int_0^\beta du\, \epsilon''(u)^2 + \mathcal{O}(\epsilon^3)$$

Logarithmic Correction [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t = u + \epsilon(u)] = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{1}{2a\alpha} \int_0^\beta du\, \epsilon''(u)^2 + \mathcal{O}(\epsilon^3)$$

$$\epsilon(u) = \frac{\beta}{2\pi} \sum_{n\neq 0} \left(\epsilon_n^{(R)} + i\epsilon_n^{(I)}\right) e^{-\frac{2\pi}{\beta} inu}$$

Logarithmic Correction [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t = u + \epsilon(u)] = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{1}{2a\alpha}\int_0^\beta du\, \epsilon''(u)^2 + \mathcal{O}(\epsilon^3)$$

$\epsilon(u) = \frac{\beta}{2\pi}\sum_{n\neq 0}\left(\epsilon_n^{(R)} + i\epsilon_n^{(I)}\right) e^{-\frac{2\pi}{\beta} inu}$

$$= -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{2\pi^2}{\beta\alpha a}\sum_{n\neq 0} n^4 \left(\epsilon_n^{(R)}\epsilon_n^{(R)} + \epsilon_n^{(I)}\epsilon_n^{(I)}\right)$$

Logarithmic Correction [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t = u + \epsilon(u)] = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{1}{2a\alpha} \int_0^\beta du\, \epsilon''(u)^2 + \mathcal{O}(\epsilon^3)$$

$$\boxed{\epsilon(u) = \frac{\beta}{2\pi} \sum_{n\neq 0} \left(\epsilon_n^{(R)} + i\epsilon_n^{(I)}\right) e^{-\frac{2\pi}{\beta} inu}} \quad = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{2\pi^2}{\beta\alpha a} \sum_{n\neq 0} n^4 \left(\epsilon_n^{(R)}\epsilon_n^{(R)} + \epsilon_n^{(I)}\epsilon_n^{(I)}\right)$$

$$Z_{\text{Q}}(\beta) \approx \int_{\text{Diff}(S^1)/U(1)} [\mathcal{D}t]_{\text{Lif}}\, e^{-I_{\text{dSch}}[t]}$$

Logarithmic Correction [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t = u + \epsilon(u)] = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{1}{2a\alpha}\int_0^\beta du\,\epsilon''(u)^2 + \mathcal{O}(\epsilon^3)$$

$$\boxed{\epsilon(u) = \frac{\beta}{2\pi}\sum_{n\neq 0}\left(\epsilon_n^{(R)} + i\epsilon_n^{(I)}\right)e^{-\frac{2\pi}{\beta}inu}} \quad = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{2\pi^2}{\beta\alpha a}\sum_{n\neq 0} n^4\left(\epsilon_n^{(R)}\epsilon_n^{(R)} + \epsilon_n^{(I)}\epsilon_n^{(I)}\right)$$

$$[\mathcal{D}t]_{\text{JT}} = \prod_{n\geq 2} 4\pi n(n-1)(n+1)\,d\epsilon_n^{(R)}d\epsilon_n^{(I)}$$

$$[\mathcal{D}t]_{\text{Lif}} = \prod_{n\geq 1} 4\pi n\,d\epsilon_n^{(R)}d\epsilon_n^{(I)}$$

$$Z_{\text{Q}}(\beta) \approx \int_{\text{Diff}(S^1)/U(1)} [\mathcal{D}t]_{\text{Lif}}\, e^{-I_{\text{dSch}}[t]}$$

Logarithmic Correction [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t = u + \epsilon(u)] = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{1}{2a\alpha}\int_0^\beta du\, \epsilon''(u)^2 + \mathcal{O}(\epsilon^3)$$

$\epsilon(u) = \frac{\beta}{2\pi}\sum_{n\neq 0}\left(\epsilon_n^{(R)} + i\epsilon_n^{(I)}\right)e^{-\frac{2\pi}{\beta}inu}$

$$= -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{2\pi^2}{\beta\alpha a}\sum_{n\neq 0} n^4 \left(\epsilon_n^{(R)}\epsilon_n^{(R)} + \epsilon_n^{(I)}\epsilon_n^{(I)}\right)$$

$[\mathcal{D}t]_{\text{JT}} = \prod_{n\geq 2} 4\pi n(n-1)(n+1) d\epsilon_n^{(R)} d\epsilon_n^{(I)}$

$[\mathcal{D}t]_{\text{Lif}} = \prod_{n\geq 1} 4\pi n\, d\epsilon_n^{(R)} d\epsilon_n^{(I)}$

$$Z_{\text{Q}}(\beta) \approx \int_{\text{Diff}(S^1)/U(1)} [\mathcal{D}t]_{\text{Lif}}\, e^{-I_{\text{dSch}}[t]}$$

$$= e^{\alpha C \beta^{-\frac{1}{\alpha}}} \prod_{n\geq 1} 4\pi n \int d\epsilon_n^{(R)} d\epsilon_n^{(I)} e^{-\frac{2\pi^2 n^4}{\beta\alpha a}\left(\epsilon_n^{(R)}\epsilon_n^{(R)} + \epsilon_n^{(I)}\epsilon_n^{(I)}\right)}$$

Logarithmic Correction [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t = u + \epsilon(u)] = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{1}{2a\alpha} \int_0^\beta du\, \epsilon''(u)^2 + \mathcal{O}(\epsilon^3)$$

$$\boxed{\epsilon(u) = \frac{\beta}{2\pi} \sum_{n\neq 0} \left(\epsilon_n^{(R)} + i\epsilon_n^{(I)}\right) e^{-\frac{2\pi}{\beta} inu}} \quad = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{2\pi^2}{\beta\alpha a} \sum_{n\neq 0} n^4 \left(\epsilon_n^{(R)}\epsilon_n^{(R)} + \epsilon_n^{(I)}\epsilon_n^{(I)}\right)$$

$$[\mathcal{D}t]_{\text{JT}} = \prod_{n\geq 2} 4\pi n(n-1)(n+1) d\epsilon_n^{(R)} d\epsilon_n^{(I)}$$

$$[\mathcal{D}t]_{\text{Lif}} = \prod_{n\geq 1} 4\pi n\, d\epsilon_n^{(R)} d\epsilon_n^{(I)}$$

$$Z_{\text{Q}}(\beta) \approx \int_{\text{Diff}(S^1)/U(1)} [\mathcal{D}t]_{\text{Lif}}\, e^{-I_{\text{dSch}}[t]}$$

$$= e^{\alpha C \beta^{-\frac{1}{\alpha}}} \prod_{n\geq 1} 4\pi n \int d\epsilon_n^{(R)} d\epsilon_n^{(I)} e^{-\frac{2\pi^2 n^4}{\beta\alpha a}\left(\epsilon_n^{(R)}\epsilon_n^{(R)} + \epsilon_n^{(I)}\epsilon_n^{(I)}\right)}$$

$$= \frac{1}{4\pi^{3/2}\sqrt{a\alpha\beta}} e^{\alpha C \beta^{-\frac{1}{\alpha}}} \quad \Longrightarrow \quad S(T) \sim S_0 + \frac{1}{E_{\text{gap}}} T^{\frac{1}{\alpha}} + \frac{1}{2}\log(T)$$

Logarithmic Correction [Harksen, Sybesma; 24]

$$I_{\text{dSch}}[t = u + \epsilon(u)] = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{1}{2a\alpha}\int_0^\beta du\, \epsilon''(u)^2 + \mathcal{O}(\epsilon^3)$$

$$\boxed{\epsilon(u) = \frac{\beta}{2\pi}\sum_{n\neq 0}\left(\epsilon_n^{(R)} + i\epsilon_n^{(I)}\right)e^{-\frac{2\pi}{\beta}inu}} \quad = -\alpha C \beta^{-\frac{1}{\alpha}} + \frac{2\pi^2}{\beta\alpha a}\sum_{n\neq 0} n^4\left(\epsilon_n^{(R)}\epsilon_n^{(R)} + \epsilon_n^{(I)}\epsilon_n^{(I)}\right)$$

$$[\mathcal{D}t]_{\text{JT}} = \prod_{n\geq 2} 4\pi n(n-1)(n+1)d\epsilon_n^{(R)}d\epsilon_n^{(I)}$$

$$[\mathcal{D}t]_{\text{Lif}} = \prod_{n\geq 1} 4\pi n\, d\epsilon_n^{(R)}d\epsilon_n^{(I)}$$

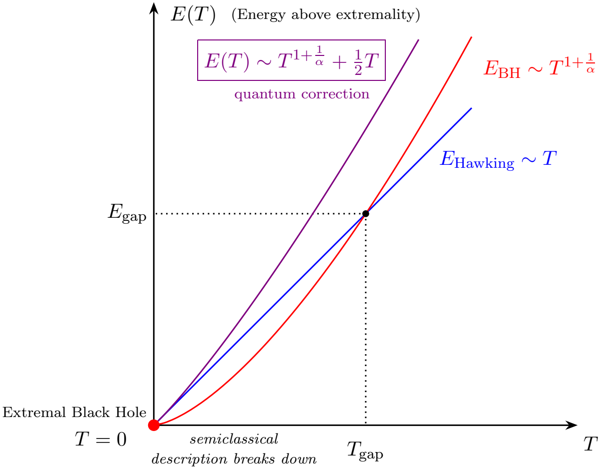
$$Z_{\text{Q}}(\beta) \approx \int_{\text{Diff}(S^1)/U(1)} [\mathcal{D}t]_{\text{Lif}}\, e^{-I_{\text{dSch}}[t]}$$

$$= e^{\alpha C \beta^{-\frac{1}{\alpha}}}\prod_{n\geq 1} 4\pi n \int d\epsilon_n^{(R)} d\epsilon_n^{(I)} e^{-\frac{2\pi^2 n^4}{\beta\alpha a}\left(\epsilon_n^{(R)}\epsilon_n^{(R)} + \epsilon_n^{(I)}\epsilon_n^{(I)}\right)}$$

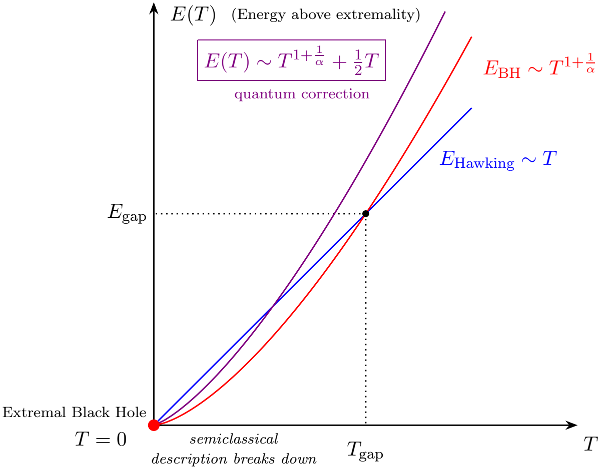
$$= \frac{1}{4\pi^{3/2}\sqrt{a\alpha\beta}}e^{\alpha C\beta^{-\frac{1}{\alpha}}} \quad \Longrightarrow \quad S(T) \sim S_0 + \frac{1}{E_{\text{gap}}}T^{\frac{1}{\alpha}} + \frac{1}{2}\log(T)$$

$$E(T) \sim T^{1+\frac{1}{\alpha}} + \frac{1}{2}T$$

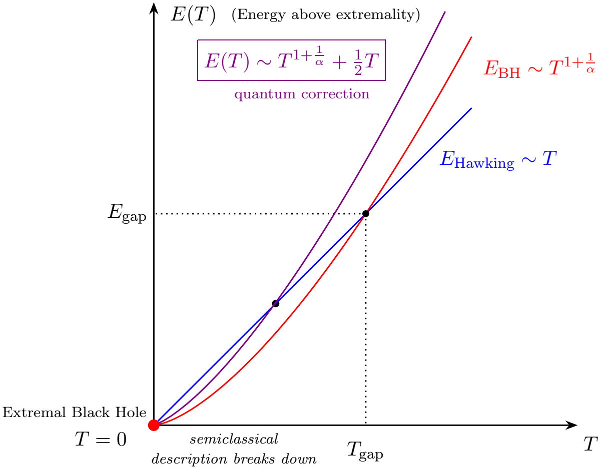
Logarithmic Correction [Harksen, Sybesma; 24]



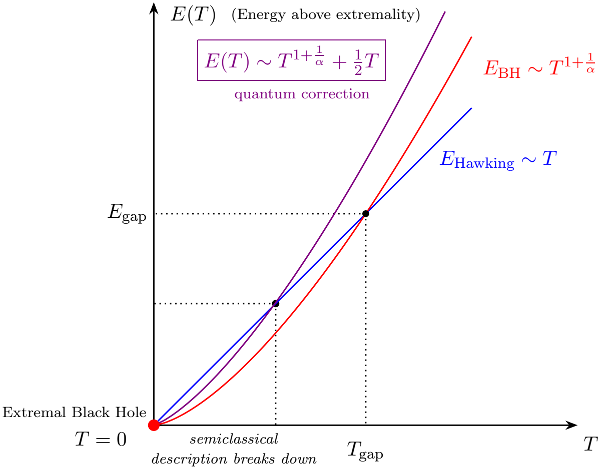
Logarithmic Correction [Harksen, Sybesma; 24]



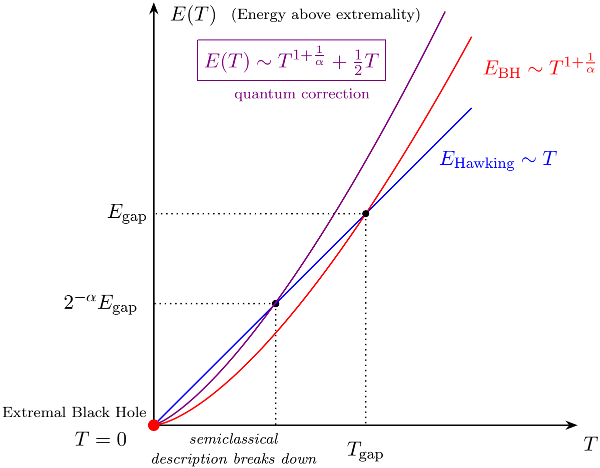
Logarithmic Correction [Harksen, Sybesma; 24]



Logarithmic Correction [Harksen, Sybesma; 24]



Logarithmic Correction [Harksen, Sybesma; 24]



Summary and Conclusion

Summary and Conclusion

☐ Examined the $2D$ dilaton model:

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left[R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right] .$$

which arises in the dimensional reduction of a Lifshitz black hole.

Summary and Conclusion

☐ Examined the $2D$ dilaton model:

$$I_{\mathrm{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left[R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right] .$$

which arises in the dimensional reduction of a Lifshitz black hole.

△ Derived a deformed Schwarzian action

$$I_{\mathrm{dSch}}[t] = -\alpha M\int t'(u)du - \frac{1}{a}\int du\frac{1}{t'(u)^{\frac{1}{\alpha}}}\mathrm{dSch}_{\alpha}(t,u) .$$

which breaks the $\mathrm{SL}(2,\mathbb{R})$ symmetry down to $U(1)$.

Summary and Conclusion

□ Examined the $2D$ dilaton model:

$$I_{\text{Lif}}(\alpha) = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-2\phi}\left[R + 4(1-\alpha)(\nabla\phi)^2 + 4\lambda^2\right] .$$

which arises in the dimensional reduction of a Lifshitz black hole.

△ Derived a deformed Schwarzian action

$$I_{\text{dSch}}[t] = -\alpha M\int t'(u)du - \frac{1}{a}\int du\frac{1}{t'(u)^{\frac{1}{\alpha}}}\text{dSch}_{\alpha}(t,u)\,.$$

which breaks the $\text{SL}(2,\mathbb{R})$ symmetry down to $U(1)$.

◯ Computed a logarithmic correction to the entropy

$$S(T) = S_0 + \frac{1}{E_{\text{gap}}}T^{\frac{1}{\alpha}} + \frac{1}{2}\log(T)\,.$$

and lowers the mass-gap scale.