Toward Double Copy on Arbitrary Backgrounds

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Based on 2405.10016 [Ilderton, Lindved]

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Outline

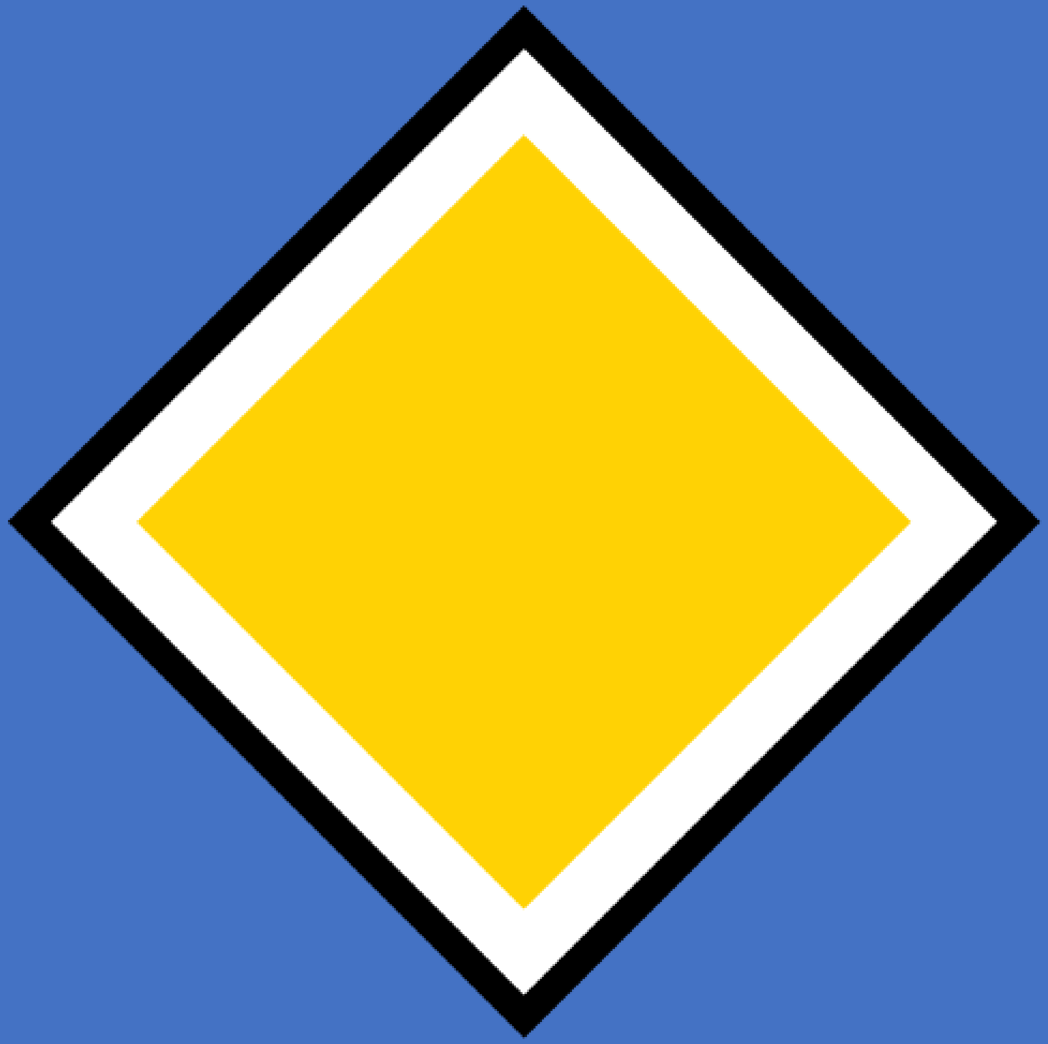
- Quantum field theories in backgrounds
- Overview of the double copy
- The double copy of background field theories
- An example construction

Background Field Theory

Physical Setup

- Classical gauge field in a flat spacetime
- Simplest processes for massive particles
  - Geodesics $1 \to 1$
  - Pair production $0 \to 2$
- Background field as gluons in the initial state $|\alpha\rangle$

$$\langle\alpha|\hat{A}_\mu(x)|\alpha\rangle = A_\mu^{\mathrm{b.g.}}(x)$$



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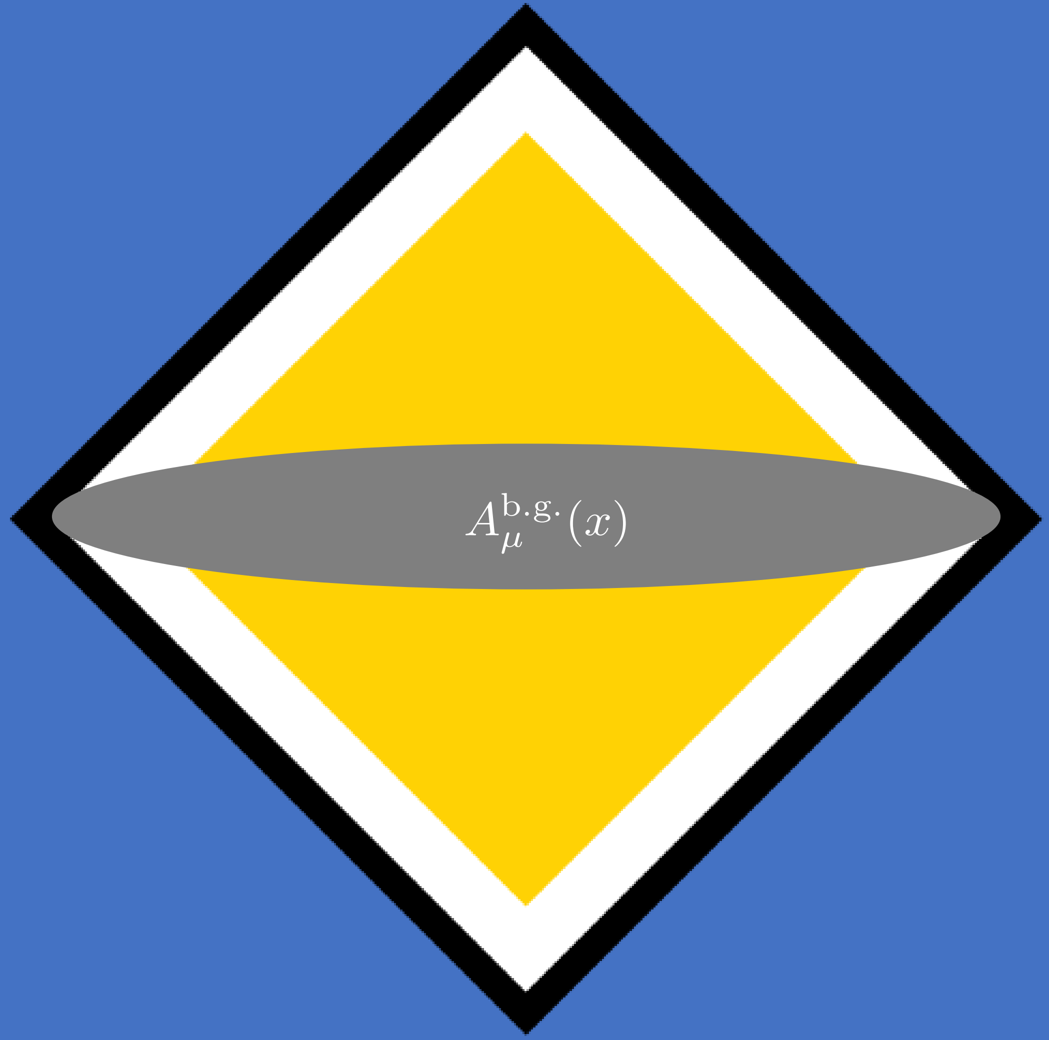

Background Field Theory

Coherent States

- Created by displacement operator,

$$|\alpha\rangle = D(\alpha)|0\rangle,$$
$$D(\alpha) = \exp\left(\int_k \alpha(k)a^\dagger(k) - \bar{\alpha}(k)a(k)\right).$$

- Amplitudes given by

$$\langle \text{out}, \alpha|S|\alpha, \text{in}\rangle = \langle \text{out}|D^\dagger(\alpha)SD(\alpha)|\text{in}\rangle$$

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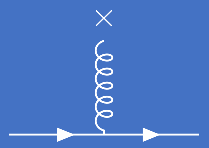
Background Field Theory

- New scattering matrix

$$D^\dagger(\alpha)S[A]D(\alpha) = S[A + A^{\mathrm{b.g.}}]$$

- Leads to new Feynman rules:



$$\sim g \int d^4k\, A^{\mathrm{b.g.}}_\mu(k)$$

- Possible to instead treat background exactly

BCJ Double Copy

Gauge Theory

$$\mathcal{A}^{(L)} \sim \sum_i \int \mathrm{d}^{LD}\ell \, \frac{c_i n_i}{D_i}$$

[Bern, Carrasco, Johansson]

BCJ Double Copy

Gauge Theory

Gravity Theory

$$\mathcal{A}^{(L)} \sim \sum_i \int \mathrm{d}^{LD}\ell \, \frac{c_i n_i}{D_i}$$

$c_i \rightarrow \tilde{n}_i$

$$\mathcal{M}^{(L)} \sim \sum_i \int \mathrm{d}^{LD}\ell \, \frac{\tilde{n}_i n_i}{D_i}$$

[Bern, Carrasco, Johansson]

Classical Double Copy

Kerr-Schild Metric

- The Kerr-Schild metric is

$$g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu$$

with k_μ null.

- Vacuum Einstein equations

$$R_{\mu\nu} = 0$$

[Monteiro, O'Connell, White '14]

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Classical Electromagnetic Field

- Define an electromagnetic field as

$$A_\mu = \phi k_\mu$$

- Automatically solves Maxwell's equations

$$\partial_\mu F^{\mu\nu} = 0$$

[Monteiro, O'Connell, White '14]

Background Double Copy

- Is there a way to double copy background theories?

$$\mathcal{L}_{\text{Gauge}}[A + A^{\text{b.g.}}] \qquad \longrightarrow \qquad \mathcal{L}_{\text{Gravity}}[h + h^{\text{b.g.}}]$$

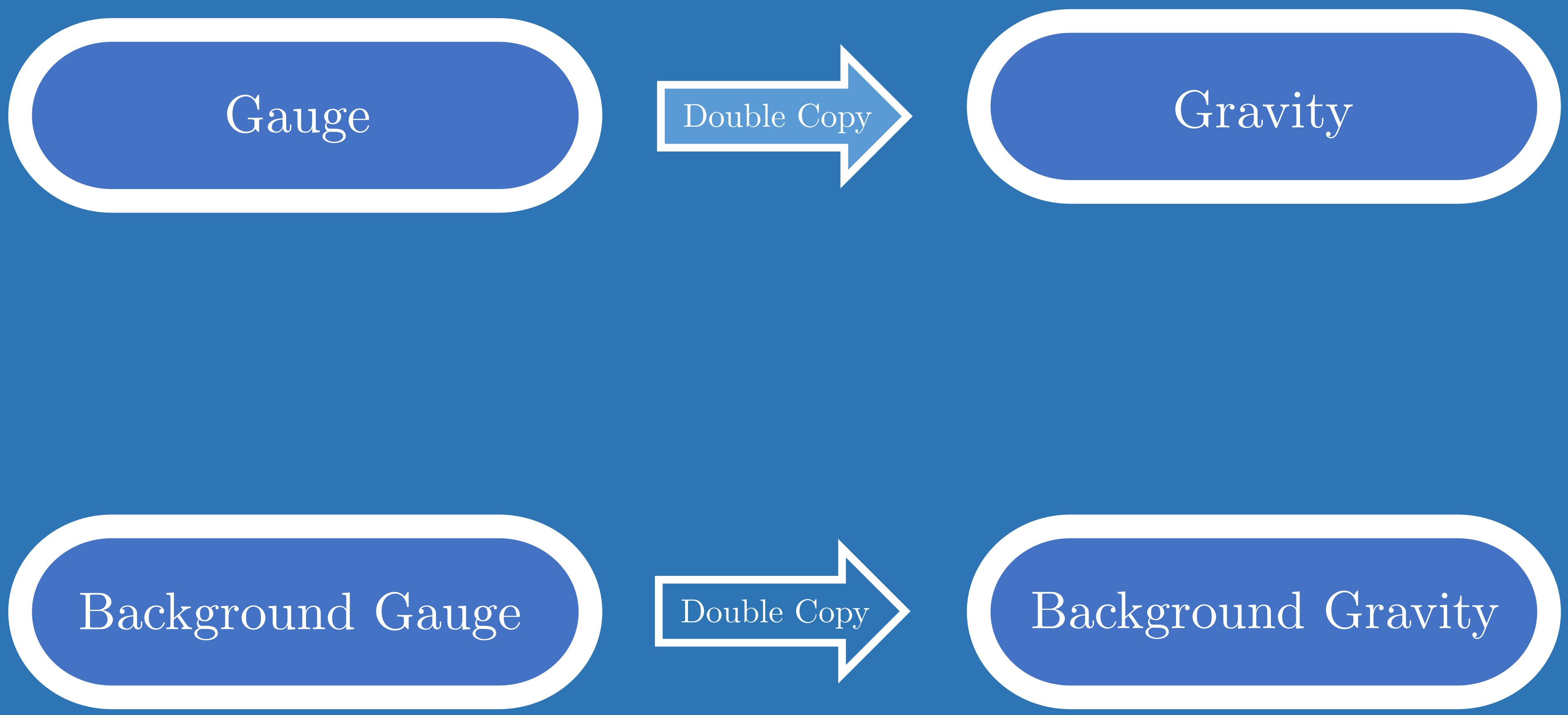
- How are the two theories related?

- What is the relation between the background metric and gauge field?

$$A_{\mu}^{\text{b.g.}} \qquad \longrightarrow \qquad h_{\mu\nu}^{\text{b.g.}}$$

- Are non-perturbative phenomena captured by such a double copy map?

Target Gravity Theory



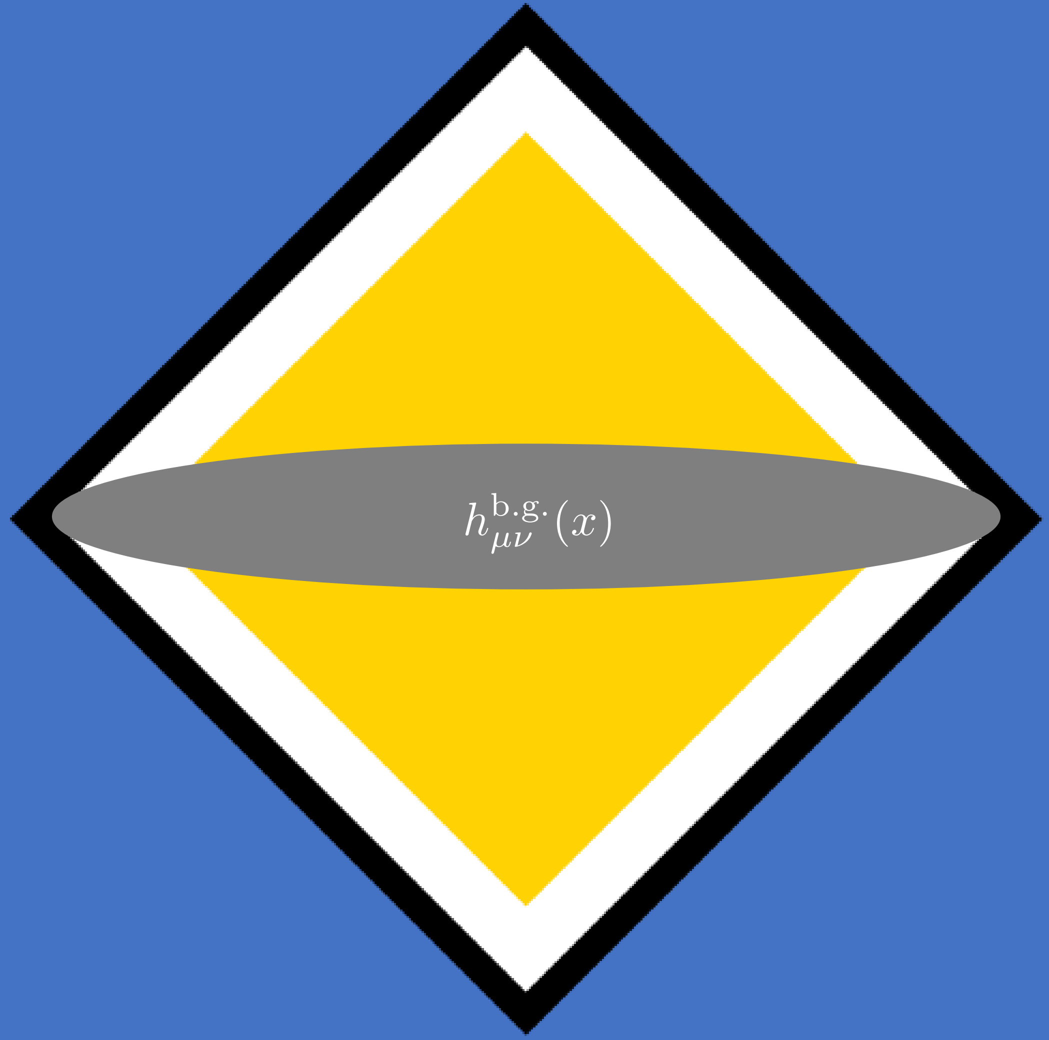
Background Metric

Background Metric

- Perturbation around Minkowski

- Coherent state of gravitons $|\beta\rangle$

$$\langle\beta|\hat{h}_{\mu\nu}(x)|\beta\rangle = h^{\text{b.g.}}_{\mu\nu}(x)$$



Background Double Copy

Scalar QCD

- Described by Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + (D_\mu \varphi)^\dagger D^\mu \varphi - m^2 \varphi^\dagger \varphi.$$

- Complex scalar φ coupled to a gauge field

$$A_\mu(x) \equiv A^a_\mu(x) \mathsf{T}^a.$$

Background Scalar QCD

- Consider fluctuations around a background,

$$A_\mu \to A^{\text{b.g.}}_\mu + A_\mu.$$

- We completely neglect quantum fluctuations,

$$\mathcal{L}[A] \to \mathcal{L}[A^{\text{b.g.}}]$$

- Scalar moving in a background $A^{\text{b.g.}}_\mu \equiv A_\mu$,

$$\mathcal{L} = (D_\mu \varphi)^\dagger D^\mu \varphi - m^2 \varphi^\dagger \varphi\,.$$

Target Gravity Theory

Double Copy of Scalar QCD

- Gravity theory with extra massless scalars:
  - dilaton ϕ
  - axion χ
- Same scalar coupled to graviton and dilaton

$$\mathcal{L} = \mathcal{L}_{\text{Grav.}} + \sqrt{-g}e^{\phi}\,\frac{g^{\mu\nu}\partial_\mu\varphi^\dagger\partial_\nu\varphi - m^2\varphi^\dagger\varphi}{\left(1-\frac{\kappa^2}{32}\varphi^\dagger\varphi\right)^2}$$

[Plefka, Shi, Wang '22]

Background Axio-Dilaton Gravity

- Replace h, ϕ, χ with background fields
- Assuming a strong background, we find a Lagrangian for a scalar in a background:

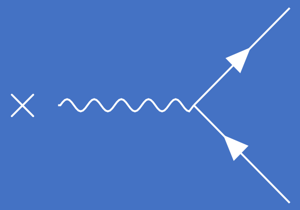
$$\mathcal{L} = \sqrt{-g}\,e^{\phi}\left(g^{\mu\nu}\partial_\mu\varphi^\dagger\partial_\nu\varphi - m^2\varphi^\dagger\varphi\right)$$

Amplitudes

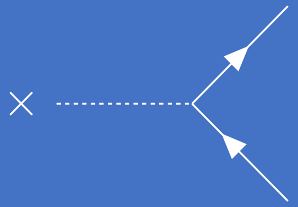
- Compare 2-point amplitudes in perturbation theory

$$\mathcal{L}_{\mathrm{Gravity}} = \sqrt{-g}\, e^{\phi} \left(g^{\mu\nu} \partial_\mu \varphi^\dagger \partial_\nu \varphi - m^2 \varphi^\dagger \varphi \right)$$

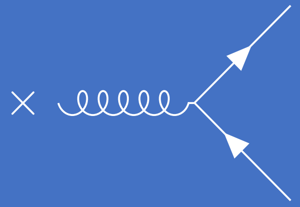
$$\mathcal{L}_{\mathrm{sQCD}} = (D_\mu \varphi)^\dagger D^\mu \varphi - m^2 \varphi^\dagger \varphi$$



$$\sim \kappa \int d^4k\, h^{\mu\nu}(k)$$



$$\sim \kappa \int d^4k\, \phi_{(1)}(k)$$



$$\sim g \int d^4k\, \mathsf{T}^a A^a_\mu(k)$$

Amplitudes at LO

- LO amplitudes

  - Scalar QCD: $$i\mathcal{A}_{(1)} = -(2g)\int \hat{\mathrm{d}}^4k\,\hat{\delta}^4(p+q-k)A^{a\mu}(k)\mathsf{T}^a_{ij}\left(p-\tfrac{1}{2}k\right)_\mu$$

  - Gravity: $$i\mathcal{M}_{(1)} = \kappa\int \hat{\mathrm{d}}^4k\,\hat{\delta}^4(p+q-k)\Big[h^{\mu\nu}(k)(p-\tfrac{1}{2}k)_\mu(p-\tfrac{1}{2}k)_\nu+\underbrace{\tfrac{1}{4}R_{(1)}(k)+\tfrac{1}{2}k^2\phi_{(1)}(k)}_{=0}\Big]$$

    if dilaton obeys EoM

- We need to

  1. replace colour factors with a kinematic structure, $c_i \rightarrow n_i$

  2. relate the background metric to the background gauge field, $A^{a\mu} \longrightarrow h^{\mu\nu}$

LO Background Double Copy

- LO amplitudes
  - Scalar QCD: $$iA_{(1)} = -(2g)\int \hat{\mathrm{d}}^4k\,\hat{\delta}^4(p+q-k)A^{a\mu}(k)\mathsf{T}^a_{ij}\left(p-\tfrac{1}{2}k\right)_\mu$$
  - Gravity: $$i\mathcal{M}_{(1)} = \kappa\int \hat{\mathrm{d}}^4k\,\hat{\delta}^4(p+q-k)h^{\mu\nu}(k)\left(p-\tfrac{1}{2}k\right)_\mu\left(p-\tfrac{1}{2}k\right)_\nu$$
- Let $g\to\kappa/2,$ and replace $\mathsf{T}^a_{ij}\to-\Phi^{-1}_{ab}(k)A^{b\nu}(k)\left(p-\tfrac{1}{2}k\right)_\nu$
- Identify the metric perturbation $h^{\mu\nu}(k) = A^{\mu a}(k)\Phi^{-1}_{ab}(k)A^{\nu b}(k)$
- "Convolutional double copy" form

[Anastasiou, Borsten, Duff, Hughes, Nagy '14]

All Orders Background Double Copy

Conjecture

- A gauge amplitude can be written

$$i\mathcal{A}_{(m)} = g^m \int_m \frac{\prod_{s=1}^m A^{\mu_s a_s}(k_s)}{D_m} N_{\mu_1\ldots\mu_m} \left(\prod_{s=1}^m t^{a_s}\right)_{ij}$$

- Make the replacement

$$\left(\prod_{s=1}^m t^{a_s}\right)_{ij} \rightarrow \left(\prod_{s=1}^m \Phi^{-1}_{a_s b_s}(k_s) A^{\nu_s b_s}(k_s)\right) N_{\nu_1\ldots\nu_m}$$

- This gives the amplitude

$$i\mathcal{M}_{(m)} = \left(\frac{\kappa}{2}\right)^m \int_m \frac{\prod_{s=1}^m h^{\mu_s \nu_s}(k_s)}{D_m} N_{\mu_1\ldots\mu_m} N_{\nu_1\ldots\nu_m}$$

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Colour-Kinematic Numerators

- With multi-indices $\mathbf{a} = (a_1, \ldots, a_s)$, define

$$c_{\mathbf{a}} \equiv \left(\prod_{s=1}^m t^{a_s}\right)_{ij}, \quad n_{\mathbf{a}} \equiv \left(\prod_{s=1}^m A^{\mu_s a_s}(k_s)\right) N_{\mu_1\ldots\mu_m}$$

- With these variables, the amplitudes are

$$i\mathcal{A}_{(m)} = g^m \sum_{\mathbf{a}} \int_m \frac{n_{\mathbf{a}} c_{\mathbf{a}}}{D_m}$$

$$i\mathcal{M}_{(m)} = \left(\frac{\kappa}{2}\right)^m \sum_{\mathbf{a},\mathbf{b}} \int_m \frac{n_{\mathbf{a}} \Phi^{-1}_{\mathbf{ab}} n_{\mathbf{b}}}{D_m}$$

Summary and Outlook

- Background double copy
- Explicit construction in specific theory
- Is there a general background double copy scheme?
- Are non-perturbative effects captured by the double copy?