The Equivariant B Model*

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w/ G. Festuccia and M. Zabzine

*a.k.a. “The B model for perverts”, *M. Zabzine*

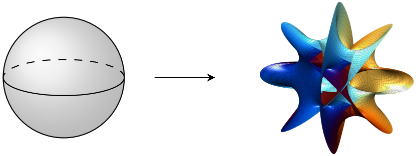
Review: The B Model

The B Model

topological twist of 2d $\mathcal{N} = (2,2)$ NLSM using $U(1)_{\mathrm{A}}$ [Witten '91]

$\rightsquigarrow$ topological SM w/ supersymmetry δ, $\delta^2 = 0$

governs maps $\Phi : \Sigma \to X$, here:



"physical" observables related to holomorphic quantities (e.g. $H^\bullet(X, \bar{\partial})$)

correlators probe moduli space of complex structures on X

dual to A model: complex structure $\longleftrightarrow$ Kähler structure

The B Model

local formulation: $\Phi \quad \rightsquigarrow \quad \phi^i(\vartheta, \varphi),\ \phi^{\bar{i}}(\vartheta, \varphi)$

other fields: $\rho^i,\ \eta^{\bar{i}},\ \theta^{\bar{i}},\ \Sigma^i,\ \beta^{\bar{i}} \quad (\delta\phi^i = 0,\ \delta\phi^{\bar{i}} = \eta^{\bar{i}},\ \delta\rho^i = \mathrm{d}\phi^i, \dots)$

B model Lagrangian: $\quad \mathcal{L}_B = \delta\mathcal{V}, \quad \mathcal{V} = \mathrm{i}g_{i\bar{i}}\rho^i \wedge \star\mathrm{d}\phi^{\bar{i}} - \frac{1}{2}\Sigma^i\theta_i$

has fermionic zero-modes in $\eta^{\bar{i}}$ and $\theta_i \quad \rightsquigarrow \quad \mathcal{Z}_B = 0$

insert observables to soak them up $\quad \rightsquigarrow \quad H^\bullet(X, \bar{\partial})$

compute via localisation: $\quad$ locus $=$ {const. maps ϕ_0}

Adding Superpotentials W

consider $W : X \to \mathbb{C}$, holomorphic in ϕ

Landau-Ginzburg: $\mathcal{L}_B + \frac{1}{2}\partial_i W \Sigma^i + \frac{\mathrm{i}}{4} D_k \partial_l W \rho^k \wedge \rho^l + \delta(\frac{1}{2}\partial_{\bar{i}} \widetilde{W} \theta^{\bar{i}})$ [Vafa '91]

lifts zero-modes $\rightsquigarrow$ $\mathcal{Z}_{\mathrm{LG}} \neq 0$

observables: $\mathcal{R} = \{F(\phi)\}/(\partial_i W)$ chiral ring ($\delta\theta_i = \partial_i W$)

e.g. structure constants of $\mathcal{R}$: $C_{abc} = \langle f_a(\phi) f_b(\phi) f_c(\phi) \rangle$ (f_a generators)

compute via localisation: locus $= \{$const. maps ϕ_0 s.t. $\partial_i W(\phi_0) = 0\}$

Equivariance

Why Equivariance?

equivariance provides refinement of theory; e.g. $\mathcal{Z} \rightsquigarrow \mathcal{Z}(\epsilon)$

can simplify computations; e.g. $\mathcal{Z}_{\mathrm{Nek}}$

allows for more general observables $\mathcal{O} = \mathcal{O}(\vartheta)$

examples: – superpotential $W(\phi, \vartheta)$ (this talk)

– complex structure deformation $\mu(\phi, \bar{\phi}, \vartheta)$ (see preprint)

(construct funky theories: $B/\bar{B}$, see outlook)

The Equivariant B Model

SUSY and Lagrangian

isometry $U(1) \circlearrowright S^2$ via $v = \epsilon\partial_\varphi$, $\kappa = g_{S^2}(v, \cdot)$

$\delta_\epsilon = \delta + v^\mu \delta_\mu \quad \rightsquigarrow \quad \delta_\epsilon^2 = -2\mathrm{i}\mathcal{L}_v$ cf. [Yagi '14]

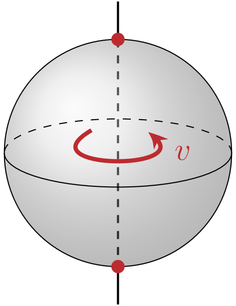
$\delta^2 = 0$

acts as e.g. $\delta_\epsilon \phi^i = \iota_v \rho^i$, $\delta_\epsilon \eta^{\bar{i}} = -2\mathrm{i}\mathcal{L}_v \phi^{\bar{i}}$

$\delta\phi^i = 0$, $\delta\eta^{\bar{i}} = 0$,

Lagrangian: $\delta_\epsilon \mathcal{V} = \mathcal{L}_B = \delta\mathcal{V}$

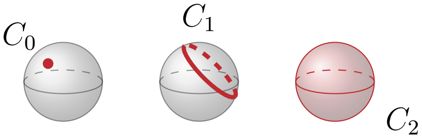
$\rightsquigarrow$ B model invariant under family of deformations



Observables

build observables in twisted theory via descent equations: [Witten '88]

$$\delta\mathcal{O}_0 = 0, \quad \mathrm{d}\mathcal{O}_0 = \delta\mathcal{O}_1, \quad \mathrm{d}\mathcal{O}_1 = \delta\mathcal{O}_2$$



C_i i-cycle: $\delta \int_{C_i} \mathcal{O}_i = \int_{C_i} \mathrm{d}\mathcal{O}_{i-1} = 0$

equivariance: $\delta_\epsilon \mathcal{O}_0 = \iota_v \mathcal{O}_1 \ , \quad \delta_\epsilon \mathcal{O}_1 = \iota_v \mathcal{O}_2 - 2id\mathcal{O}_0 \ , \quad \delta_\epsilon \mathcal{O}_2 = -2id\mathcal{O}_1$

take any polyform $\omega_0 + \omega_2 \in \Omega^*_\Sigma$ such that $2\mathrm{id}\omega_0 = \iota_v \omega_2$

$$\rightsquigarrow \quad \tilde{\mathcal{O}}_0 = \omega_0(\vartheta)\mathcal{O}_0, \quad \tilde{\mathcal{O}}_1 = \omega_0(\vartheta)\mathcal{O}_1, \quad \tilde{\mathcal{O}}_2 = \omega_0(\vartheta)\mathcal{O}_2 + \omega_2(\vartheta)\mathcal{O}_0$$

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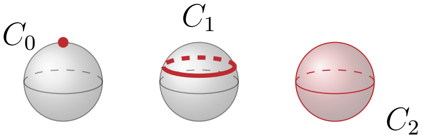

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$$\rightsquigarrow \quad \tilde{\mathcal{O}}_0 = \omega_0(\vartheta)\mathcal{O}_0, \quad \tilde{\mathcal{O}}_1 = \omega_0(\vartheta)\mathcal{O}_1, \quad \tilde{\mathcal{O}}_2 = \omega_0(\vartheta)\mathcal{O}_2 + \omega_2(\vartheta)\mathcal{O}_0$$

Example: Superpotential

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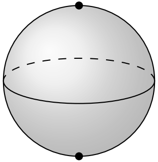
generalised superpotential: $W(\phi) \rightsquigarrow W(\phi, \vartheta)$

$$W_N = W_1 + W_2$$



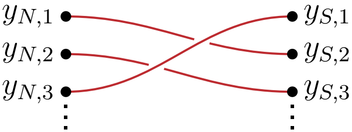
$$W_S = W_1 - W_2$$

$\mathcal{O}_0 = \frac{\mathrm{i}}{2} W(\phi, \vartheta) \overset{\text{descent eqns}}{\rightsquigarrow} \mathcal{O}_2 = \ldots$

new action: $S_B + \int_\Sigma \mathcal{O}_2$

e.g. $W(\phi, \vartheta) = W_1(\phi) + \cos(\vartheta) W_2(\phi), \quad \partial W_{N,S}(y_{N,S}) = 0$

localisation $\rightsquigarrow$ locus $= \Big\{ \phi_0(\vartheta)$ s.t.



$\Big\}$

Example: Superpotential

compute correlators of $f(\phi)$ at N & S:

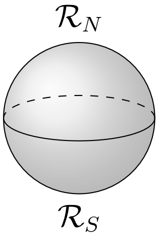
two chiral rings $\mathcal{R}_N,\ \{f_i(\phi)\}$ & $\mathcal{R}_S,\ \{\tilde{f}_i(\phi)\}$

"topological metric" and structure constants:

$$\nu^N_{ij} = \langle f_i(\phi)|_{\vartheta=0} f_j(\phi)|_{\vartheta=0}\rangle$$

$$(C^N)^i{}_{jk} = (\nu_N^{-1})^{il}\langle f_l(\phi)|_{\vartheta=0} f_j(\phi)|_{\vartheta=0} f_k(\phi)|_{\vartheta=0}\rangle$$

can do more: $\psi^i{}_j = (\nu_N^{-1})^{ik}\langle f_k(\phi)|_{\vartheta=0}\tilde{f}_j(\phi)|_{\vartheta=\pi}\rangle$



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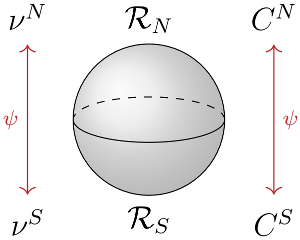
$\rightsquigarrow$ ψ gives ring homomorphism $\mathcal{R}_N \to \mathcal{R}_S$



Summary and Outlook

Sumary

introduced equivariant deformation of B model:

standard B model	equivariant B model
$\delta^2 = 0$	$\delta_\epsilon^2 = -2\mathrm{i}\mathcal{L}_v$
$\mathcal{L}_B = \delta(\mathrm{i}g_{i\bar{i}}\rho^i \wedge \star\mathrm{d}\phi^{\bar{i}} - \frac{1}{2}\Sigma^i\theta_i)$	$\mathcal{L}_B = \delta_\epsilon(\mathrm{i}g_{i\bar{i}}\rho^i \wedge \star\mathrm{d}\phi^{\bar{i}} - \frac{1}{2}\Sigma^i\theta_i)$
$W(\phi) \rightsquigarrow \mathcal{R}$	$W(\phi, \vartheta) \rightsquigarrow \mathcal{R}_{N,S}$
locus $= \{$const. $\phi_0 \vert \partial W(\phi_0) = 0\}$	locus $= \{$tracking sol.$\}$

$\rightsquigarrow$ equivariance yields interesting extension

Outlook: $B/\bar{B}$

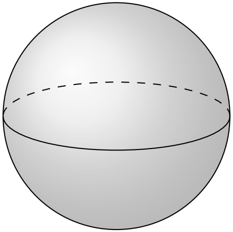
different twist of $\mathcal{N} = (2,2)$ NLSM:

B model: $\rho^i,\ \Sigma^i,\quad \eta^{\bar{i}},\ \theta^{\bar{i}},\ \beta^{\bar{i}}$

$\bar{B}$ model: $\rho^{\bar{i}},\ \Sigma^{\bar{i}},\quad \eta^i,\ \theta^i,\ \beta^i$

equivariance $\rightsquigarrow$ theory $\mathcal{T}[\vartheta]$ s.t.

$$\mathcal{T}[0] = B$$



$$\mathcal{T}[\pi] = \bar{B}$$

X can be any Kähler manifold

generalised version of topological/anti-topological fusion