Åbo Akademi University

HELSINKI INSTITUTE OF PHYSICS

Dynamics of first-order phase transitions in holographic theories

Oscar Henriksson

Åbo Akademi University &

Helsinki Institute of Physics

First-order phase transitions

Rich physics!

- Bubble nucleation, spinodal decomposition, *critical phenomena*, ...

Important in all areas of physics! Example: **Cosmology**

- Occured in early universe?? → bubble collisions produce grav. waves → sign of BSM physics!!

Dynamics of transitions include complicated out-of-equilibrium behavior

- **Difficult** to study, even at weak coupling...
- **Holographic duality** → window into these processes at **strong coupling**!

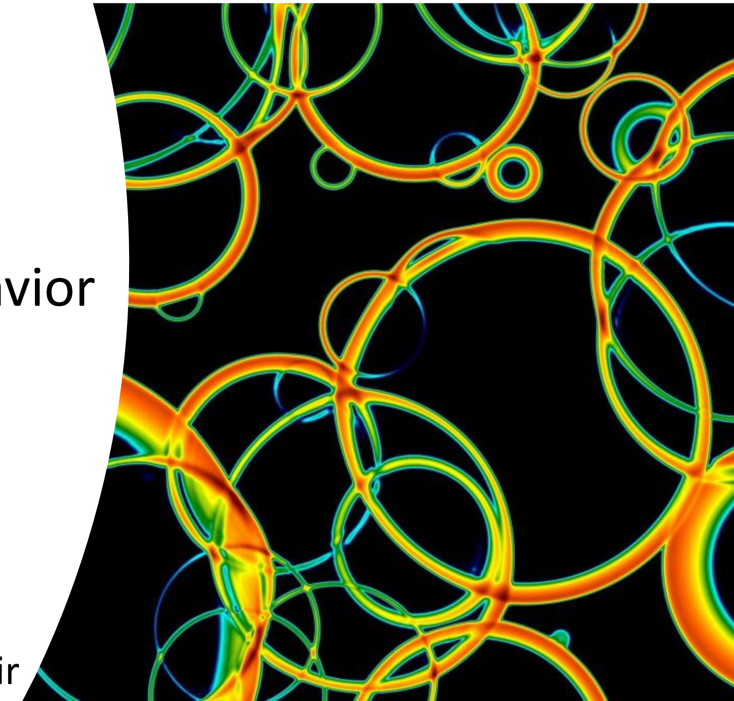
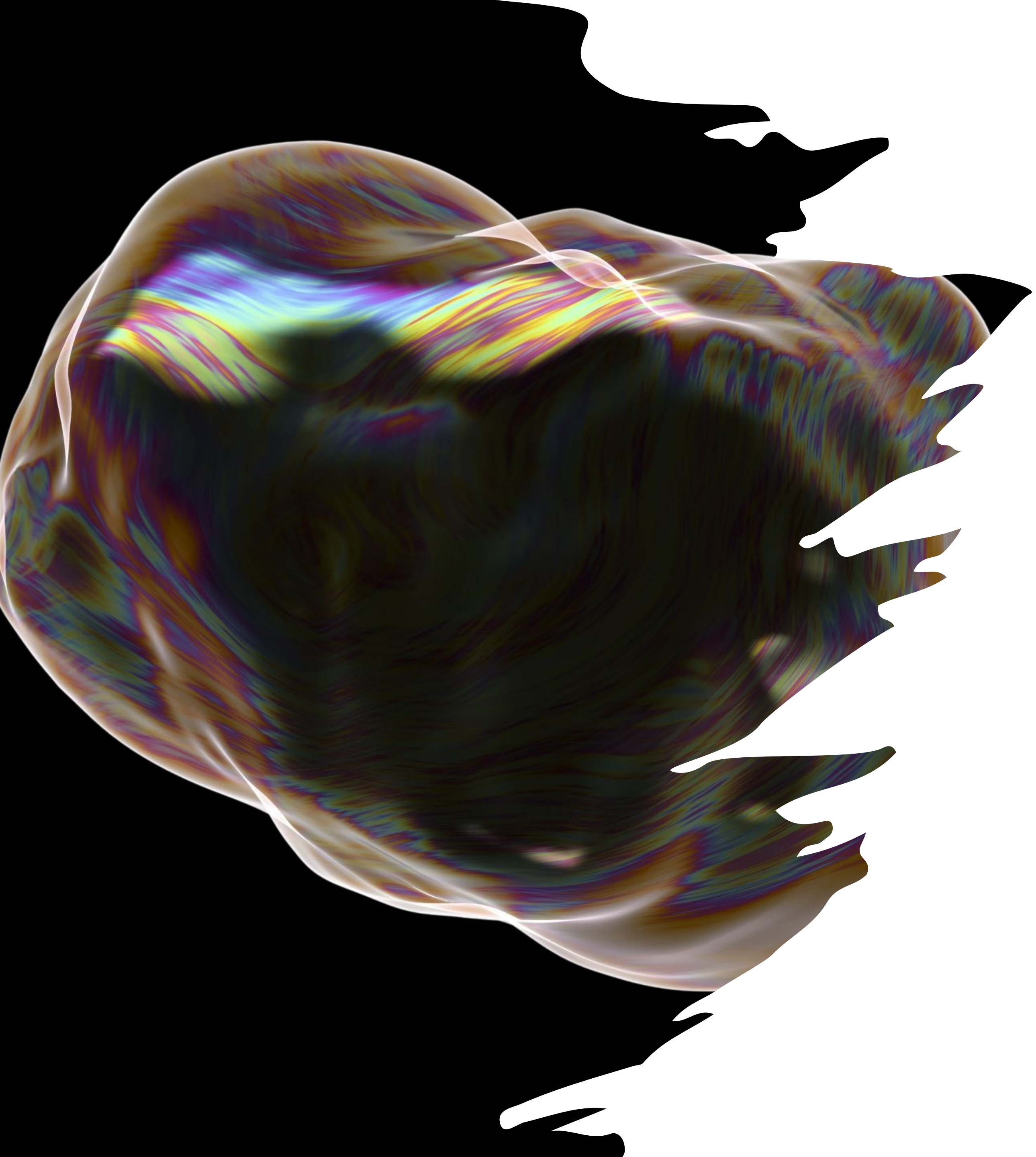


Figure by David Weir



Outline

1. A **simple holographic model** for studying phase transitions
2. First order transitions and **bubble nucleation**
3. When nucleation is suppressed – **approaching the *spinodal***

Based on…

- 2109.13784 and 2110.14442 with Fëanor Reuben Ares, Mark Hindmarsh, Carlos Hoyos & Niko Jokela.
- 2406.15297, with Alessio Caddeo, Carlos Hoyos & Mikel Sanchez-Garitaonandia.

A **simple** holographic toy model

Minimalistic bottom-up theory:

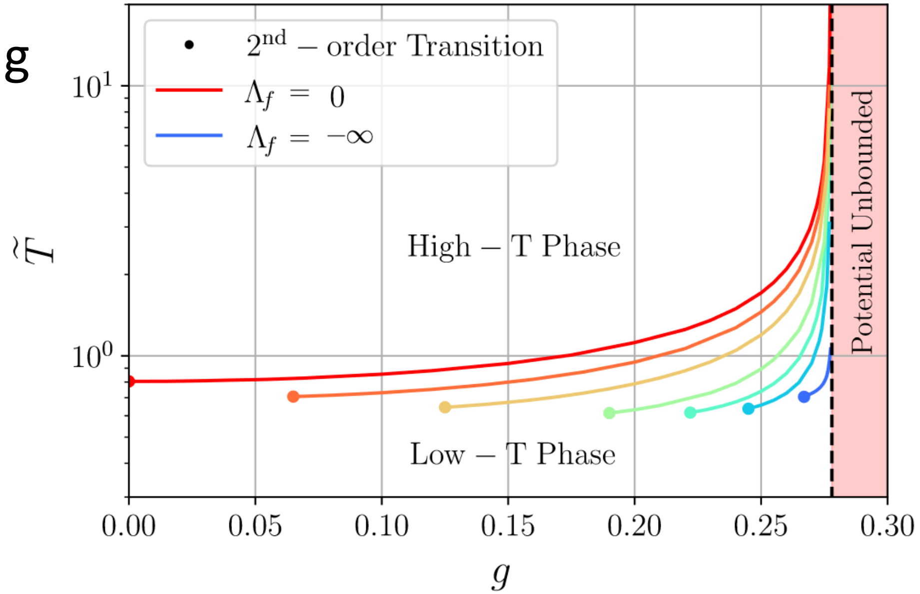
$$S \sim \int \mathrm{d}^{\mathrm{D}}x\,\sqrt{-g}\left\{R - \Lambda - \left(\partial_\mu \phi\right)^2 - m^2\phi^2\right\}$$

- ϕ dual to "order parameter" Ψ.
- Dimensionality Δ of Ψ determined by m^2; use **alternate quantization**.

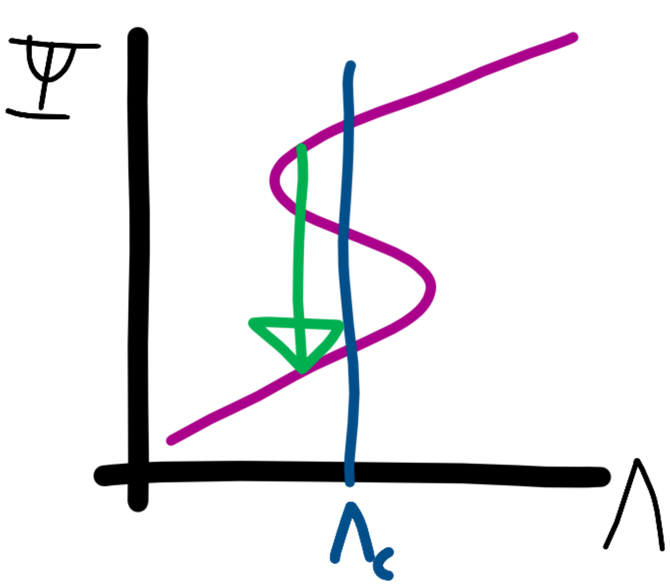
Deform dual CFT with **multi-trace operators** to create interesting phase structure:

$$S_{CFT} \rightarrow S_{CFT} + \int d^{D-1}x\left\{\Lambda\Psi + \frac{f}{2}\Psi^2 + \frac{g}{3}\Psi^3 + \cdots\right\}$$

- Easy in holography: changing **boundary conditions** in AdS
- Can get 1st and 2nd order transitions, and crossovers! →



First-order transitions & bubble nucleation



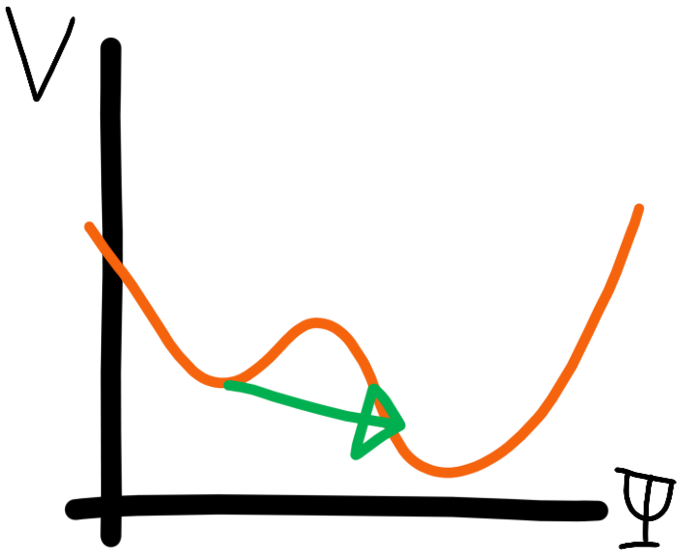
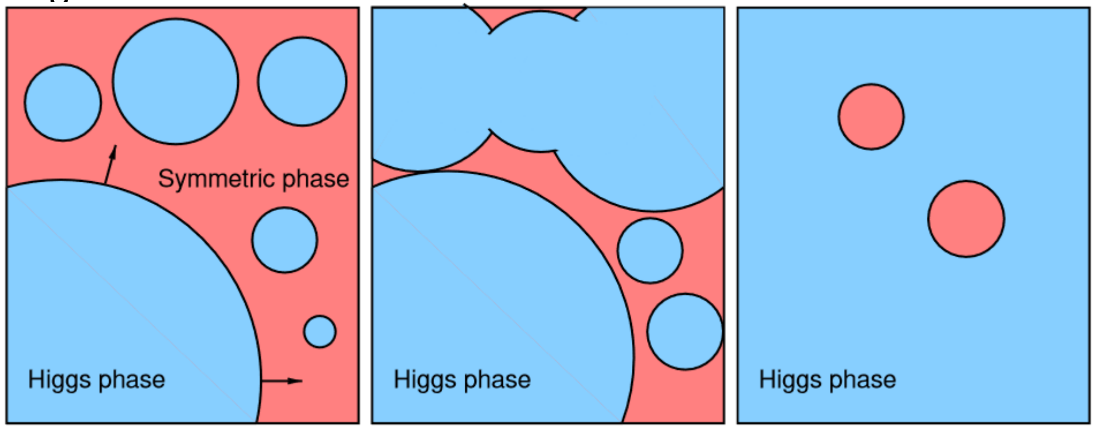
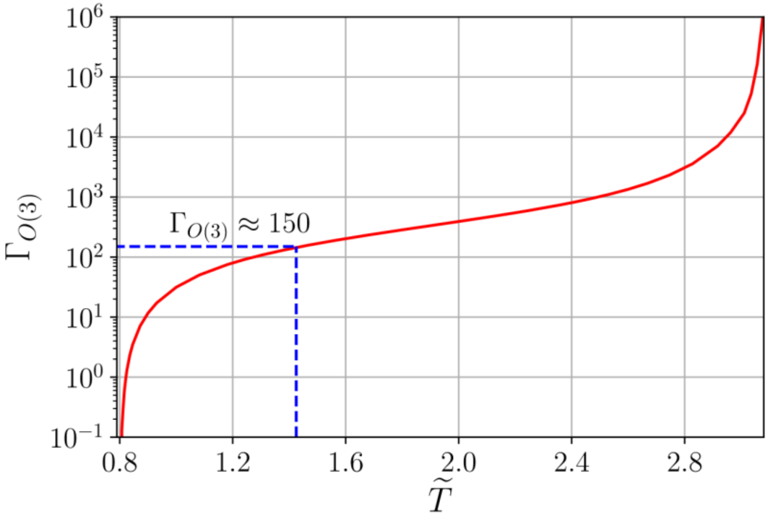


Figure from Hindmarsh et al. 2008.09136



Callan, Coleman ('77);
Linde ('81):

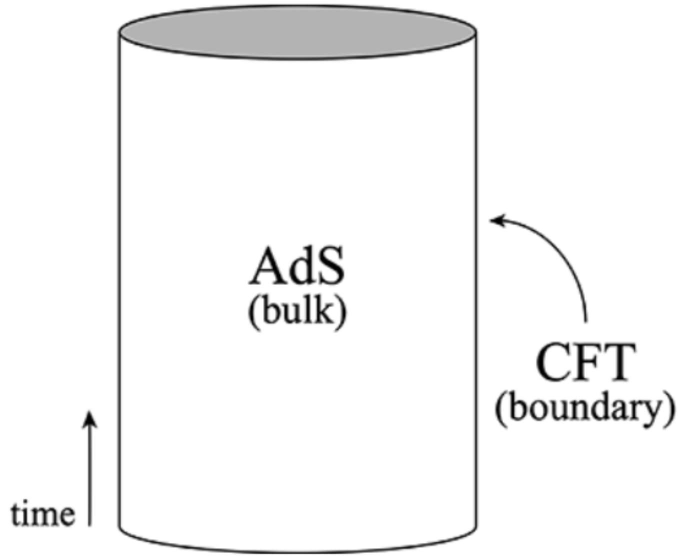
1. Find **critical bubble** from effective action
2. Bubble action gives nucleation rate →



Bubbles in **holographic theories**

In 2109.13784 & 2110.14442, we used **multi-trace model**

$$S \sim \int \mathrm{d}^5 x \sqrt{-g}\left\{R - \Lambda - \left(\partial_\mu \phi\right)^2 - m^2\phi^2\right\}$$

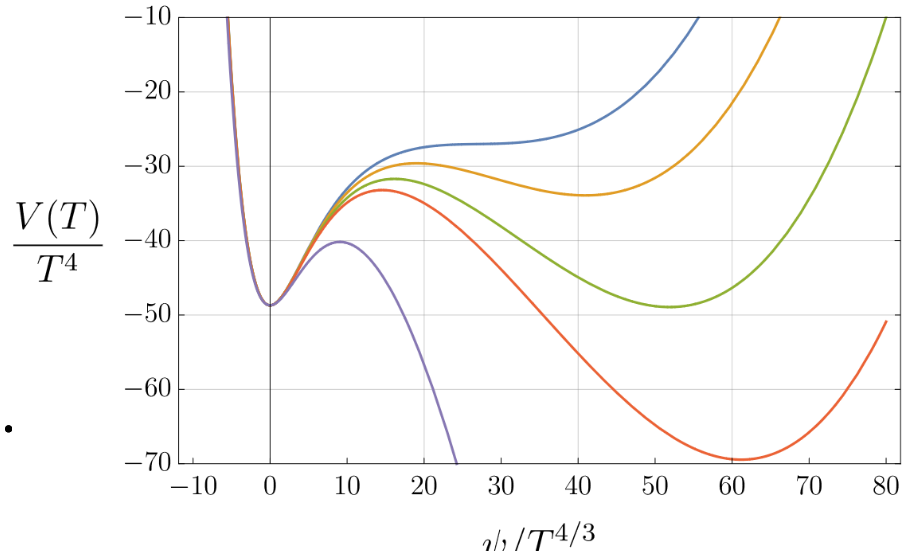


We computed field theory **effective action** for operator Ψ, in a **derivative expansion**:

$$\Gamma[\Psi] = -N^2 \int d^4x \left\{V(\Psi) + \frac{1}{2} Z(\Psi)(\nabla\Psi)^2 + \cdots\right\}$$

→Bubble solutions from $\frac{\delta\Gamma}{\delta\Psi} = 0$ (ODE – easy! ☺)

→Nucleation rates, "quasi-equilibrium" parameters, etc.



What if bubble nucleation is suppressed?

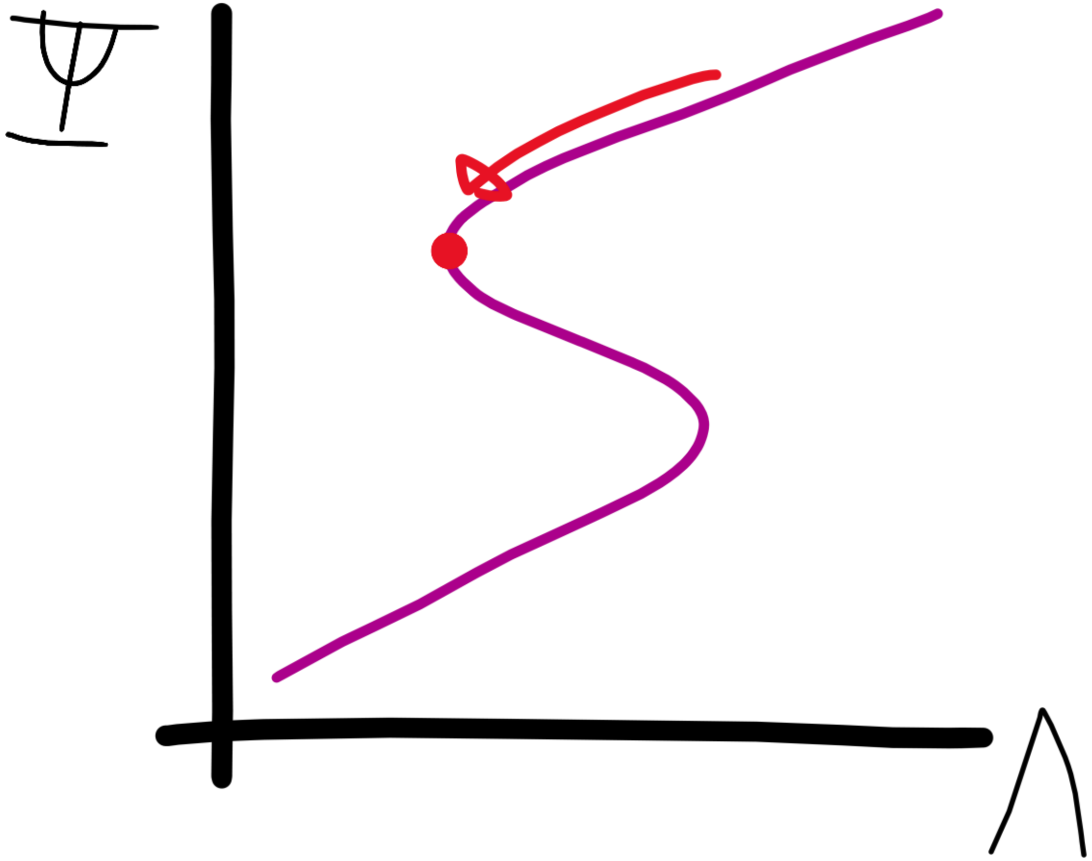
Spinodal approach

If nucleation suppressed
(as is the case for $N \to \infty$, or generically in mean-field limit)

→ approach **spinodal point**

…then what?

(Here, we only discuss **approaching** the spinodal point along the metastable branch. Not **spinodal decomposition** which occurs when starting on unstable branch)



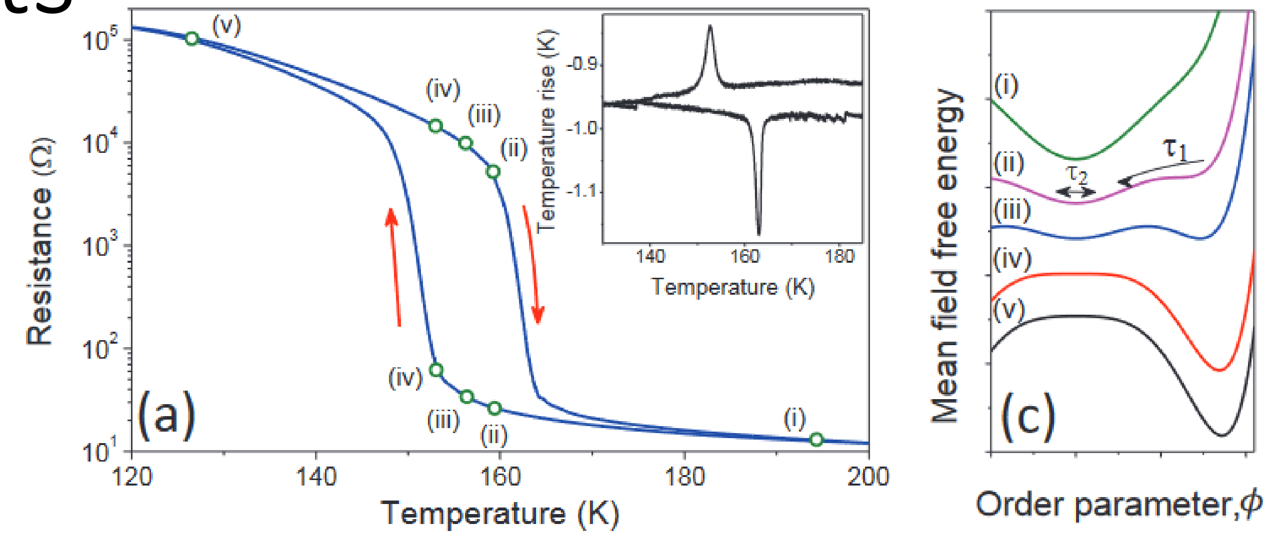
Spinodal & critical points

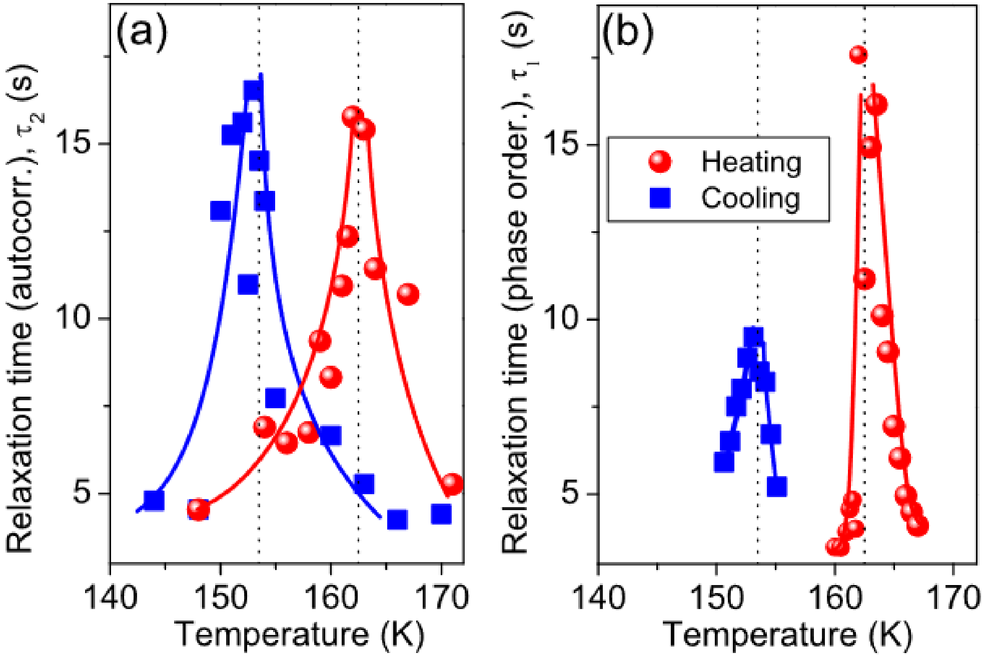
Spinodal points share similarities with **second order critical points**!

- Susceptibilities diverge
- Critical behavior, scaling

Effects of spinodals can be seen in cond-mat **experiments**!

→e.g. Kundu et al. 2023 on V_2O_3





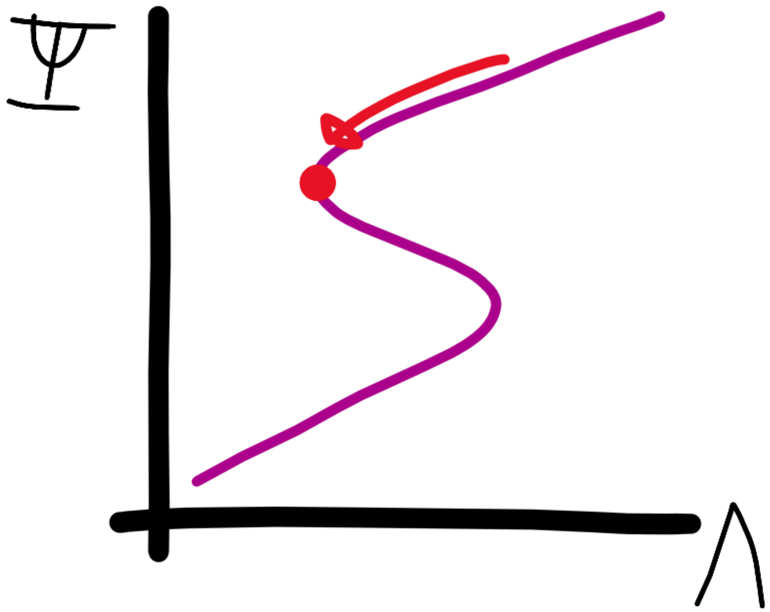
Spinodal & critical points

When approaching critical/spinodal point…

…order parameter goes as $\Delta\Psi \sim |\Delta T|^{\beta}$

…correlation length diverges as $\xi \sim |\Delta T|^{-\nu}$

…relaxation time diverges as $\tau \sim |\Delta T|^{-z\nu}$



”Critical slowing down”

→Equilibrium state changes fast, system **cannot keep up**!

→System thrown out of equilibrium, no matter how slow the cooling

Spinodal & critical points

Assume **cooling linearly in time** near critical/spinodal point: $\frac{T_c - T}{T_c} = \frac{t}{\tau_Q}$

Define t_* as time when Ψ departs from equilibrium by some (small) ϵ: $\left|\frac{\Delta\Psi}{\Psi}\right| < \epsilon$

System will deviate from equilibrium when $\partial_t \Delta\Psi \sim \tau^{-1}$ →

$$t_* \sim \tau_Q^{\frac{z\nu-\beta}{1+z\nu-\beta}} \quad , \qquad \left|\frac{T_c - T_*}{T_c}\right| \sim \tau_Q^{\frac{1}{\beta - z\nu - 1}}$$

Compare with **Kibble-Zurek-mechanism**, concerned with domain formation during a 2nd order transition; there, the *"freeze-out time"* $\hat{t}$ scales as

$$\hat{t} \sim \tau(\hat{t}) \quad \rightarrow \quad \hat{t} \sim \tau_Q^{\frac{z\nu}{1+z\nu}}$$

Order	z	β	ν
1st	2	1/2	1/4
2nd	2	1/3	1/3

Approaching spinodal/critical point

Our **multi-trace model** lets us study 1st and 2nd order transitions.

→For **numerical simplicity**, take scalar field to be homogeneous **probe** on top of AdS-Schwarzschild.

→Work in **4D gravity** with dual operator $\Delta = 3/4$

$$S \sim \int \mathrm{d}^4x \sqrt{-g} \left\{ R - \Lambda - \frac{1}{\mathrm{N}} \left[\left(\partial_\mu \phi \right)^2 - m^2 \phi^2 \right] \right\}$$

Make **BCs time-dependent** → approach spinodal by **changing multi-trace couplings**.

→For *slow* time-dependence this can be equivalent to cooling/heating system.

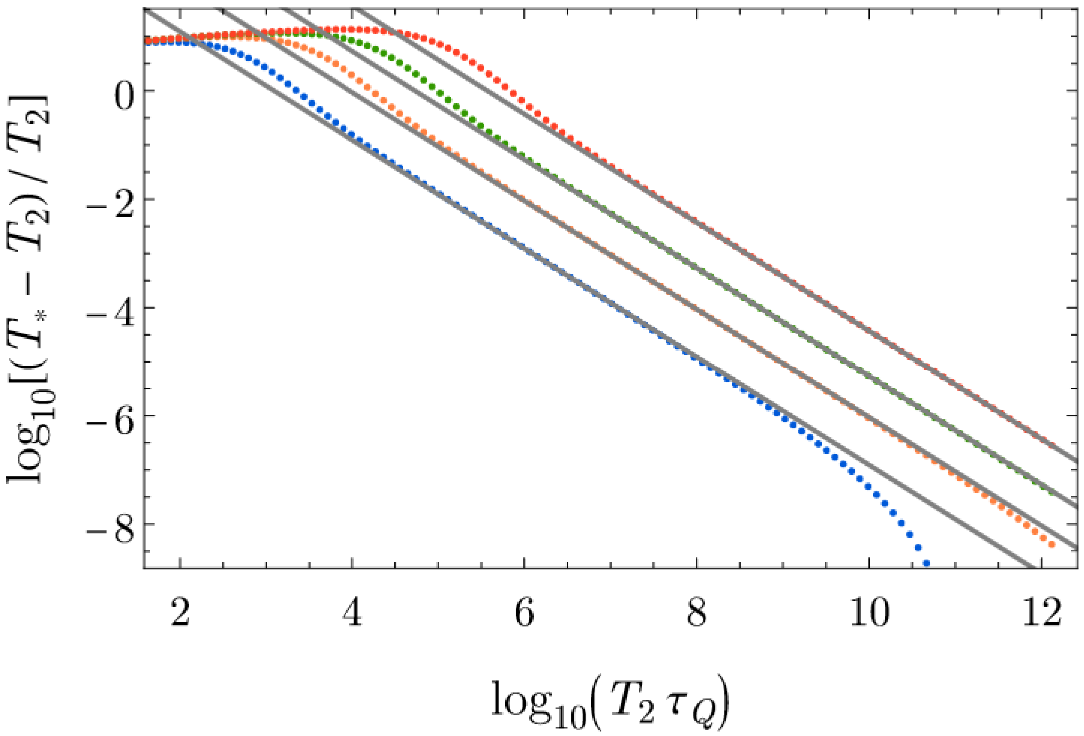
In 2406.15297 we…

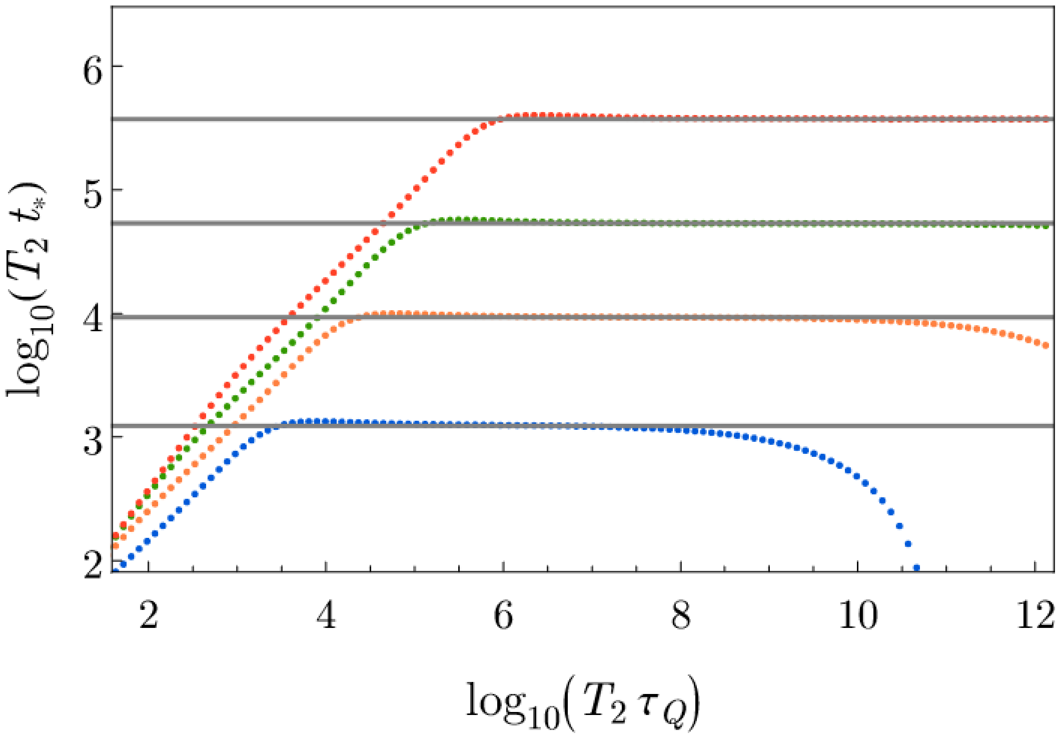
- assume homogeneous scalar field → PDE in time and holographic radial direction
- simulate approaches to spinodal and critical points
- study resulting scaling behavior

Numerical results – first-order transition

→We find good agreement with predicted dynamic critical exponents!

$$\left|\frac{T_c - T_*}{T_c}\right| \sim \tau_Q^{-1} \quad , \quad t_* \sim \tau_Q^0$$

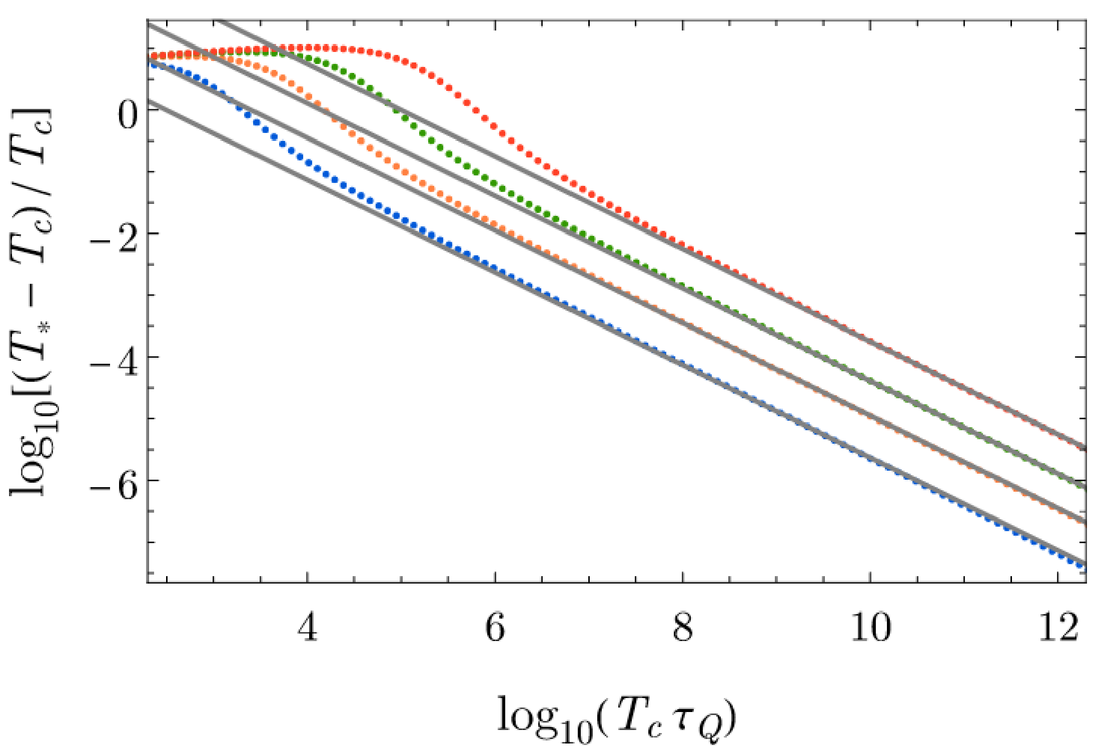


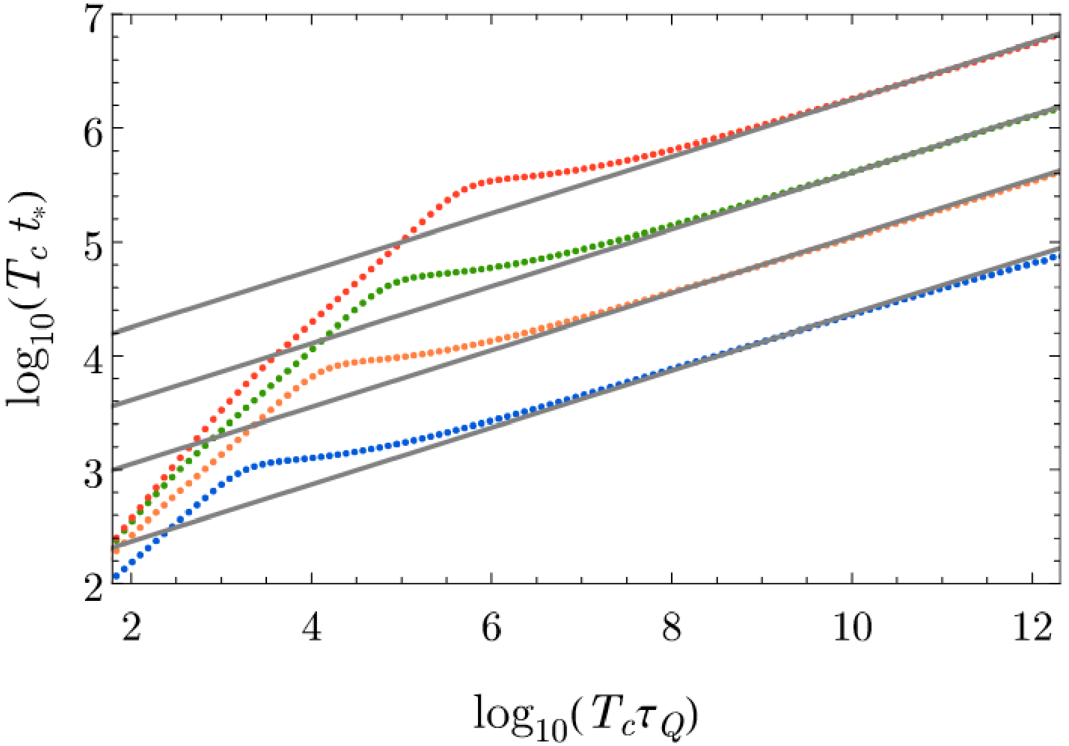


Numerical results – second-order transition

→We find good agreement with predicted dynamic critical exponents!

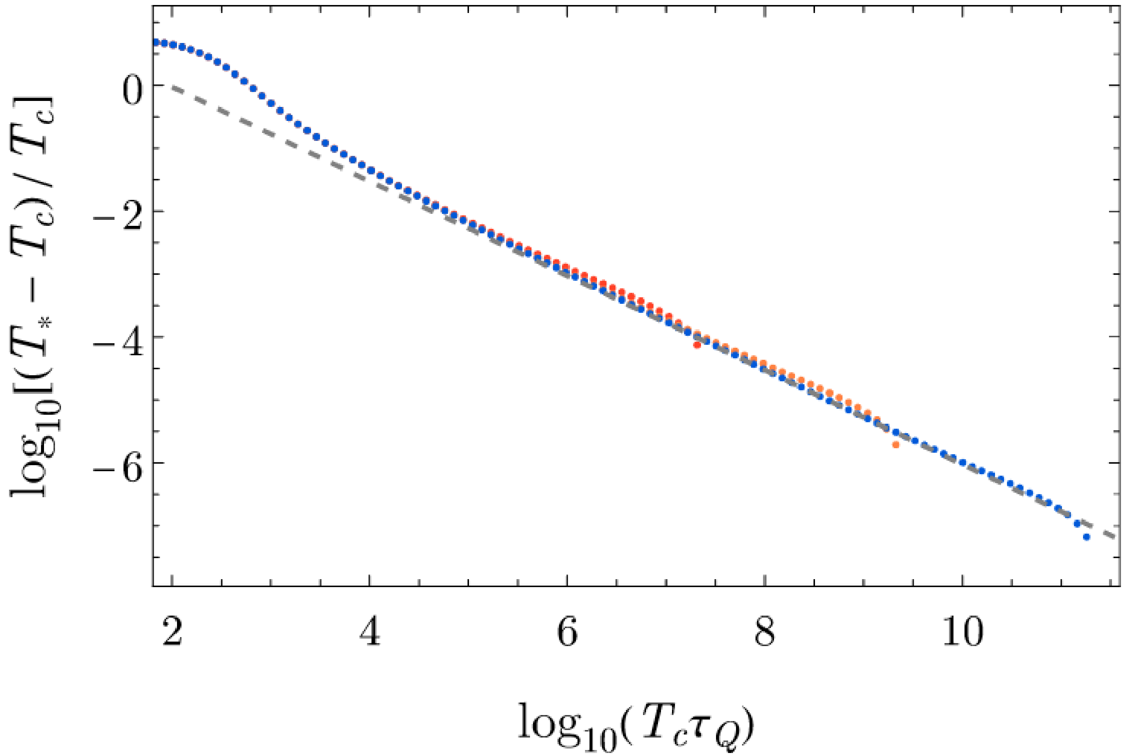
$$\left|\frac{T_c - T_*}{T_c}\right| \sim \tau_Q^{-3/4} \quad , \quad t_* \sim \tau_Q^{1/4}$$

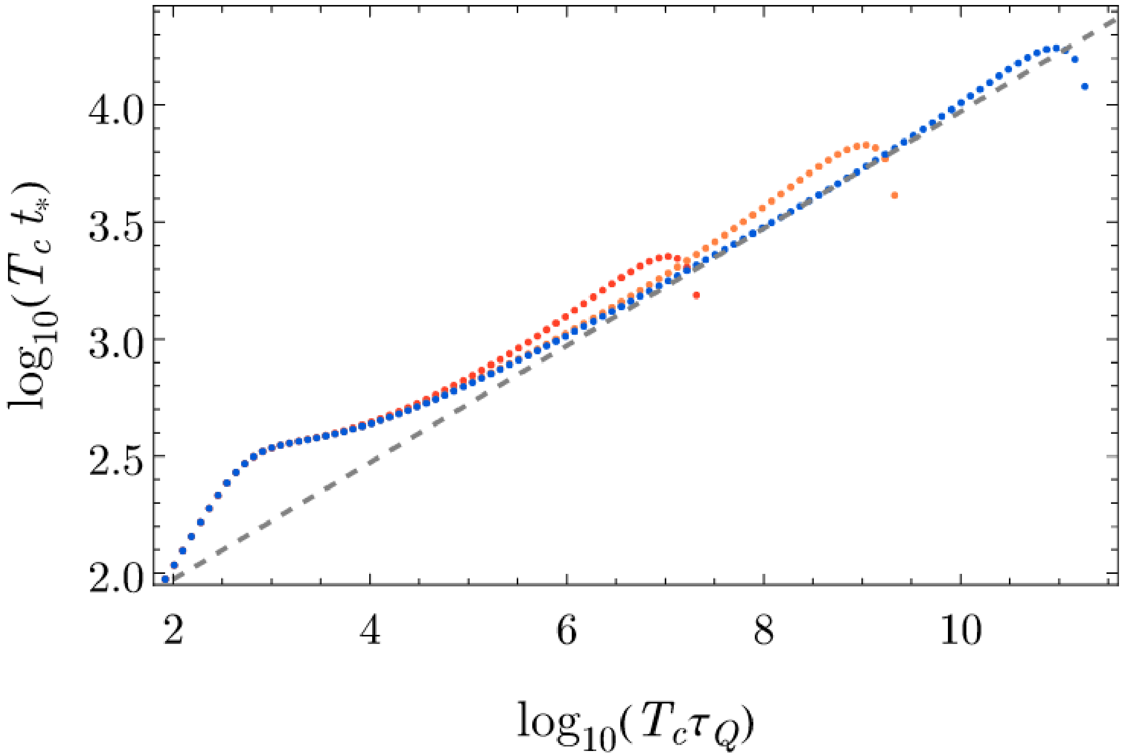




Strong crossover – traces of the critical point

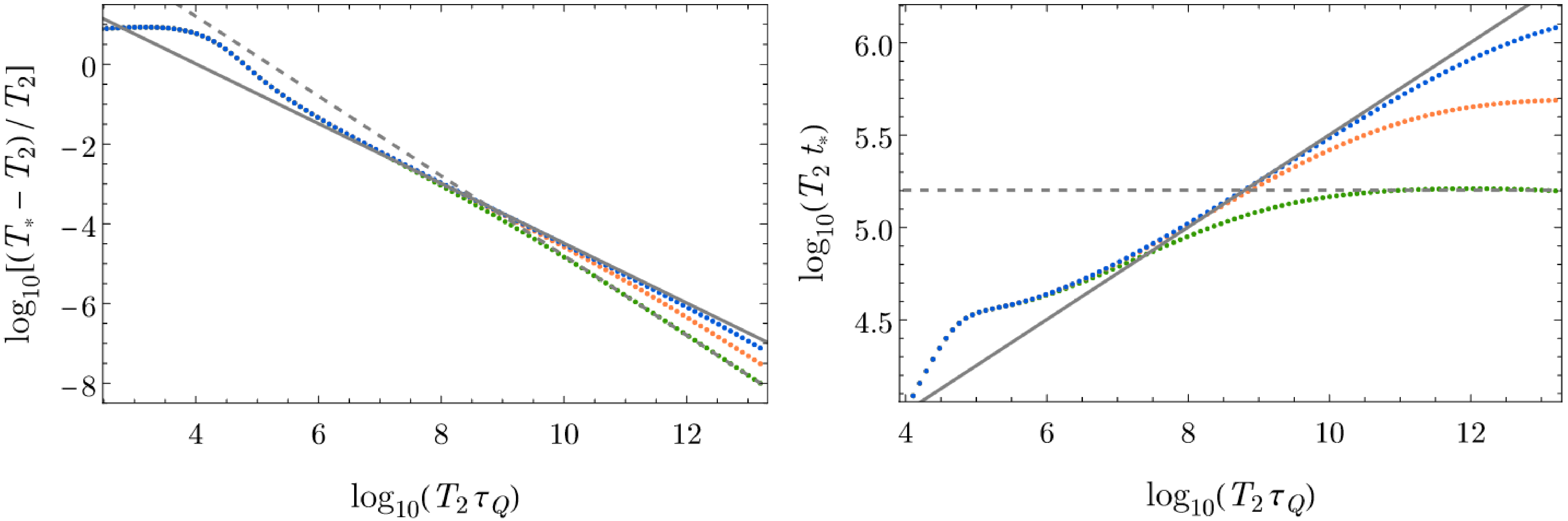
At a *strong crossover* – close to a 2nd order critical point – we see **traces of the critical exponents**!





Weak first-order transitions

For **weak first-order** transitions – close to a 2nd order critical point – we see traces of **both critical and spinodal exponents**.



Summary

- **1st order PTs** typically proceed through **bubble nucleation**
  - →Can treat in holography by computing QFT effective action in derivative expansion
- When nucleation is *supressed*, approach **spinodal point**
  - →Critical phenomena similar to 2nd order transitions
  - →System ”thrown out of” equilibrium
  - →We derived and numerically studied dynamic critical exponents
  - →Also saw scaling for 2nd order transitions, crossovers
- All this within a **simple holographic model** where **multi-trace deformations** tune phase structure
- Much left to do! One example: add **spatial dependence** to probe model
  - →Study defect formation; compare with Kibble-Zurek
  - →Real-time simulations of nucleation