Weak G_2-manifolds and scale separation in M-theory from type IIA backgrounds

Vincent Van Hemelryck

Based on [2408.16609, **VVH**]

(and [2107.00019, N. Cribiori, D. Junghans, **VVH**, T. Van Riet and T. Wrase])

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Extra dimensions in string theory

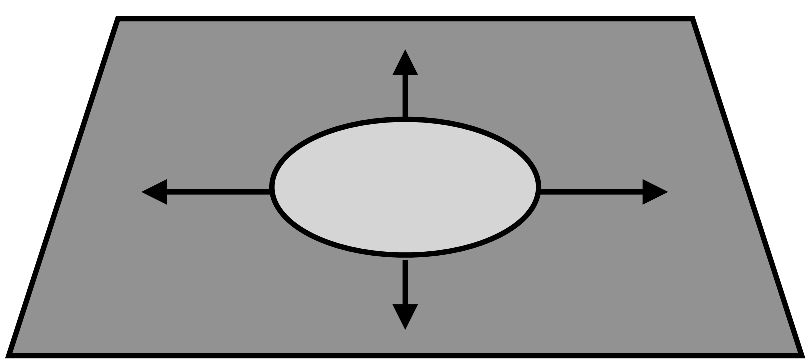
String theory best understood in 10 dimensions

Lower-dimensional theories? E.g. something like our universe

→ Hide extra dimensions: two options

Braneworlds

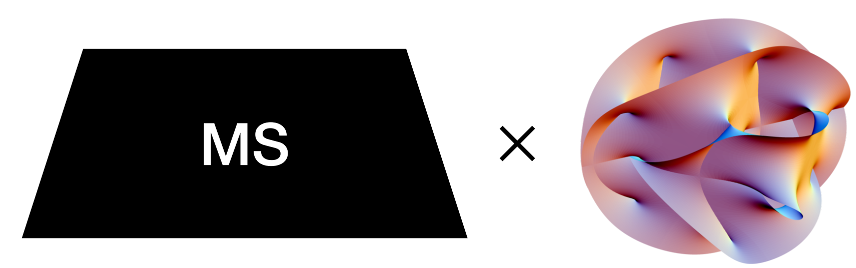
Large extra dimensions



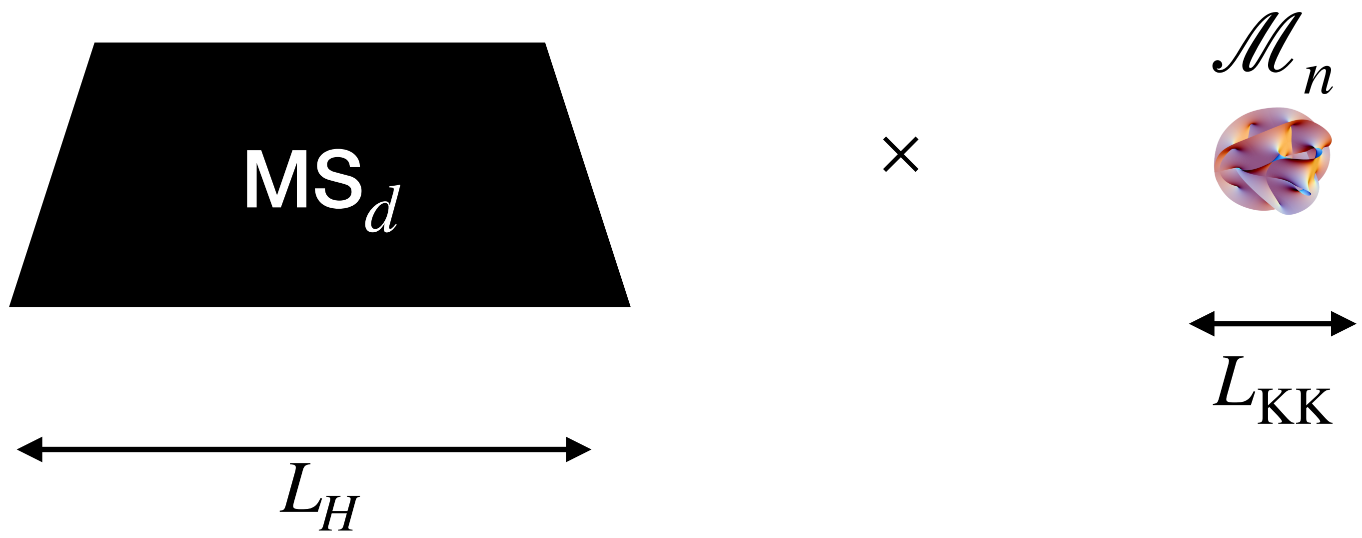
e.g. the dark bubble scenario, [Banerjee, Danielsson, Dibitetto, Giri, Schillo, 2018,…]

Flux compactifications

Small extra dimensions



Compactifications & scale separation



$$\mathrm{d}s^2_{d+n} = \mathrm{e}^{2A(y)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu \quad + \quad g_{mn} \mathrm{d}y^m \mathrm{d}y^n$$

Small dimensions amount to **scale separation:**

$$L_{\mathrm{KK}} \ll L_H$$

$$\Lambda_d \ll M^2_{\mathrm{KK}}$$

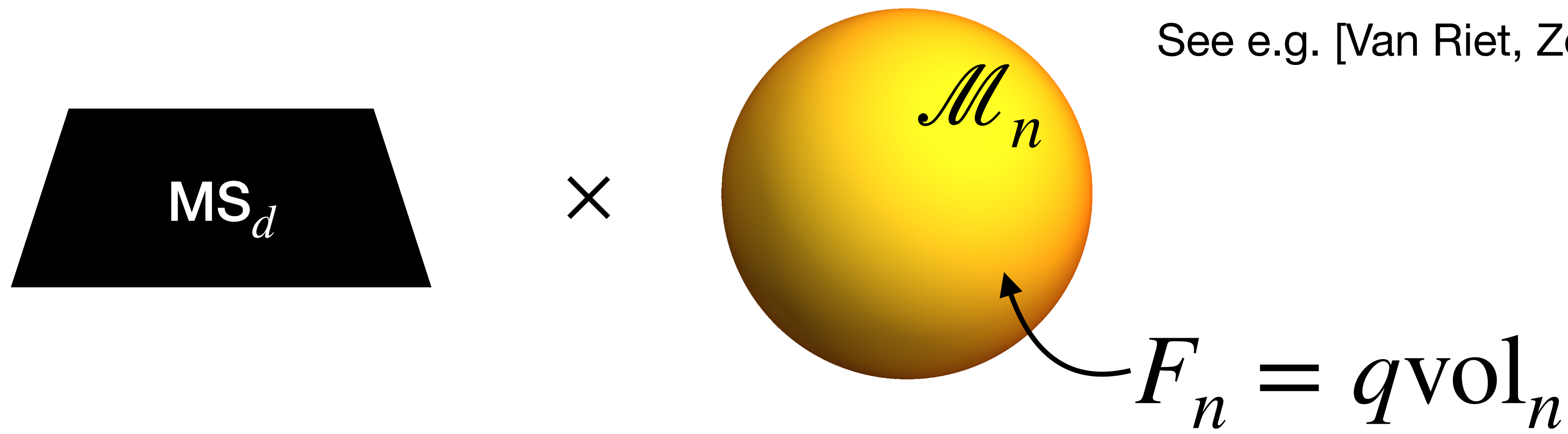
$$(\Delta_n - \lambda_l) Y_l(y) = 0$$

$$\lambda_1 = M^2_{\mathrm{KK}}$$

Desirable with full moduli stabilisation

Freund-Rubin compactifications

See e.g. [Van Riet, Zoccarato, 2023]



$$\mathrm{d}s^2_{d+n} = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \qquad + \qquad L^2 g_{nm}\mathrm{d}y^m \mathrm{d}y^n$$

Solve Einstein equations!

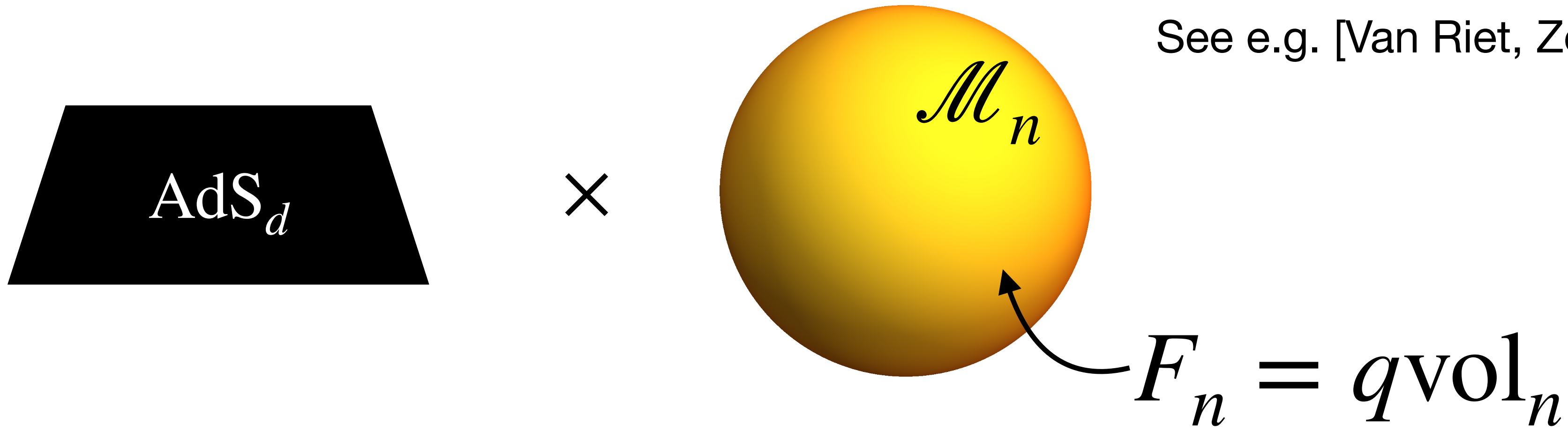
$$R_{\mu\nu} \propto -q^2 g_{\mu\nu}, \qquad R_{mn} \propto q^2 g_{mn}$$

Internal space has **positive Einstein** metric and curvatures are similar!

$$\Lambda_d \equiv R_d/d, \qquad |R_d| \sim R_n$$

Freund-Rubin compactifications

See e.g. [Van Riet, Zoccarato, 2023]



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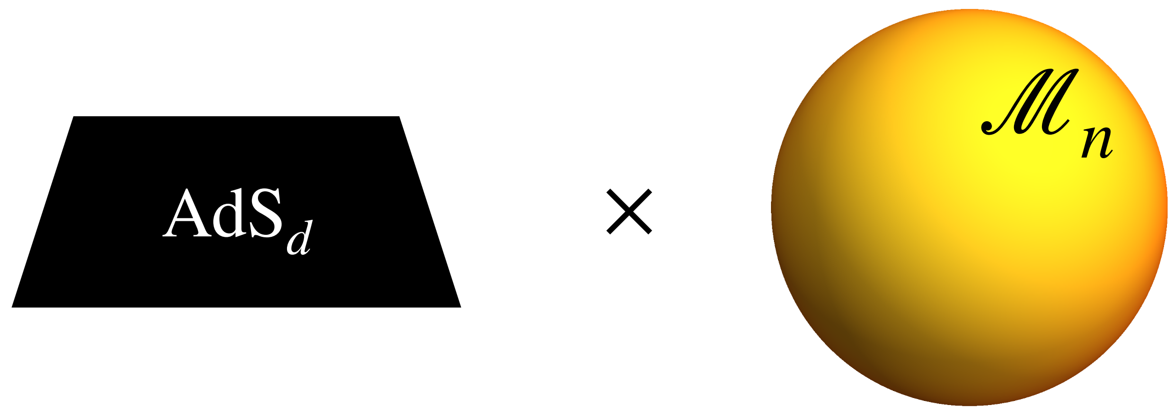

$$\Lambda_d \equiv R_d/d, \qquad |R_d| \sim R_n$$

When manifold is e.g. a sphere of radius L, then

$$R_n \sim L^{-2} \text{ and } L = L_{\text{KK}} = M_{\text{KK}}^{-1}$$

So

$$\Lambda_d \sim M_{\text{KK}}^2$$

So **no scale separation!**

Freund-Rubin compactifications

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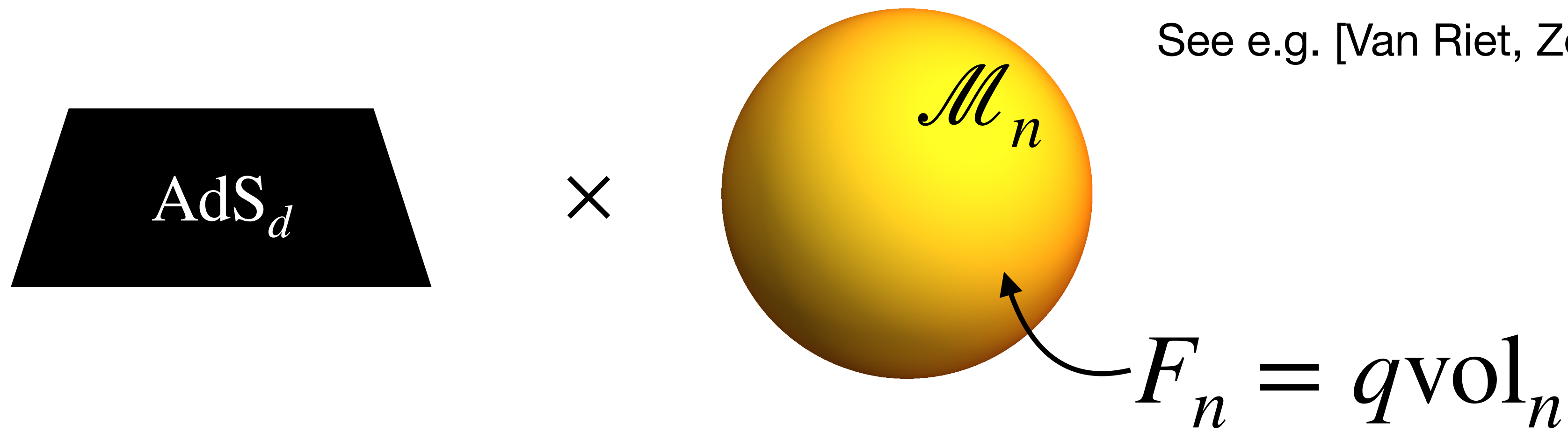

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No scale separation?

e.g. $\mathrm{AdS}_5 \times S^5$ in type IIB string theory

e.g. $\mathrm{AdS}_4 \times S^7$ or $\mathrm{AdS}_7 \times S^4$ in M-theory

No de Sitter? → a whole other issue

More generally true in string compactifications

Handful of examples with scale separation, but under criticism [DGKT, KKLT, LVS,…]

Recent arguments against scale separation for SUSY-theories with $Q \geq 8$

[Cribiori, Dall'agata, 2022]
[Bobev, David, Hong, Reys, Zhang, 2023]
[Perlmutter, 2024]

Theorems & Swampland conjectures

[Gautason, Schillo, Van Riet, Williams, 2016]
[Lüst, Palti, Vafa, 2019]
[Buratti, Calderon-Infante, Mininno, Uranga, 2020]
[De Luca, Tomasiello, 2021]

Scale separation for Freund-Rubin vacua of M-theory?

Scale separation for Freund-Rubin

for 4d compactifications of M-theory

$$R_7 \sim R_4 \sim \Lambda_4 \ll M_{\mathrm{KK}}^2$$

[Gautason, Schillo, Van Riet, Williams, 2016], see also [De Luca, Tomasiello, 2021]

$M_{\mathrm{KK}}^2 = \lambda_1 =$ lowest eigenvalue of the scalar Laplacian

$$R_7 \ll \lambda_1$$

Purely geometric condition!

Not possible according to [Collins, Jafferis, Vafa, Xu, Yau, 2022]

Examples known for negative curvature (e.g. nilmanifolds)

Supersymmetric, scale-separated 4d Freund-Rubin solution of M-theory on weak G_2-manifold

[VVH, 2024]

Lifting type IIA string theory to M-theory

Strategy

[Cribiori, Junghans, VVH, Van Riet, Wrase, 2021]

Scale-separated solution of massless type IIA

6d manifold with $SU(3)$-structure

Circle fibration →

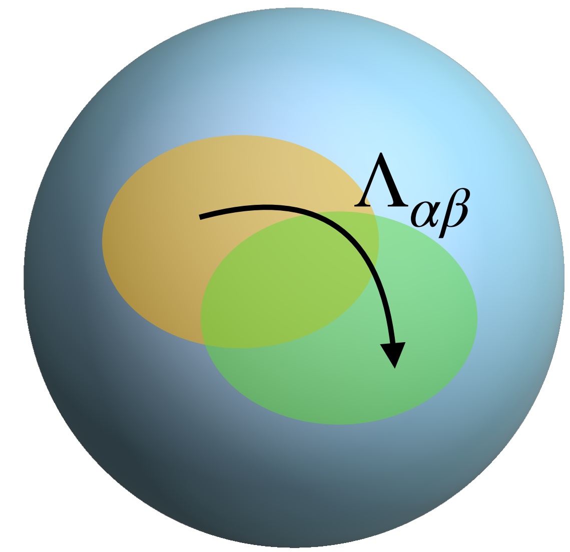
Freund-Rubin solution of M-theory

7d manifold with G_2-structure

Strongly coupled massless type IIA $\cong$ M-theory with a large circle

Compactifications on G-structure manifolds

Group structures on manifolds



$$\Lambda_{\alpha\beta} \in G \subseteq \mathrm{Gl}(d, \mathbb{R})$$

$$G = \mathrm{SU}(3)$$

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2A(y)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn} \mathrm{d}y^m \mathrm{d}y^n$$

g_{mn} → Kähler 2-form J
Holomorphic 3-form Ω

$R^m{}_{nlp}$ → Torsion classes
$\mathrm{d}J \neq 0 \,, \quad \mathrm{d}\Omega \neq 0$

Compactifications on G-structure manifolds

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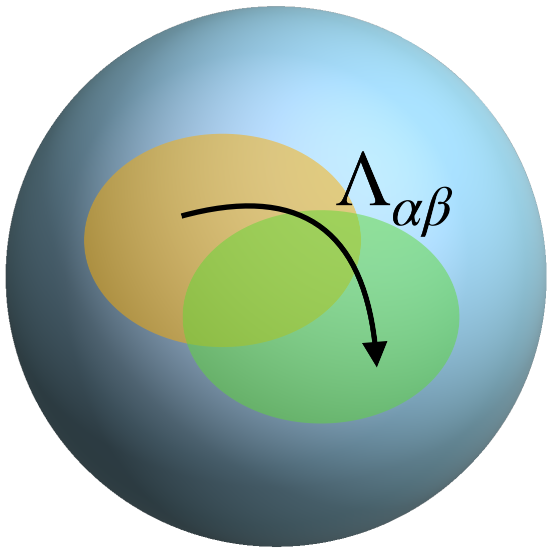


$$\Lambda_{\alpha\beta} \in G \subseteq \mathrm{Gl}(d, \mathbb{R})$$

$$G = G_2$$

$$\mathrm{d}s^2_{11} = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}\mathrm{d}y^m \mathrm{d}y^n$$

g_{mn} → 3-form Φ, 4-form $\star\,\Phi$

${R^m}_{nlp}$ → Torsion classes $\mathrm{d}\Phi \neq 0\,, \quad \mathrm{d}\star\Phi \neq 0$

Weak G_2-manifolds are positive Einstein!

Weak G_2-manifolds:

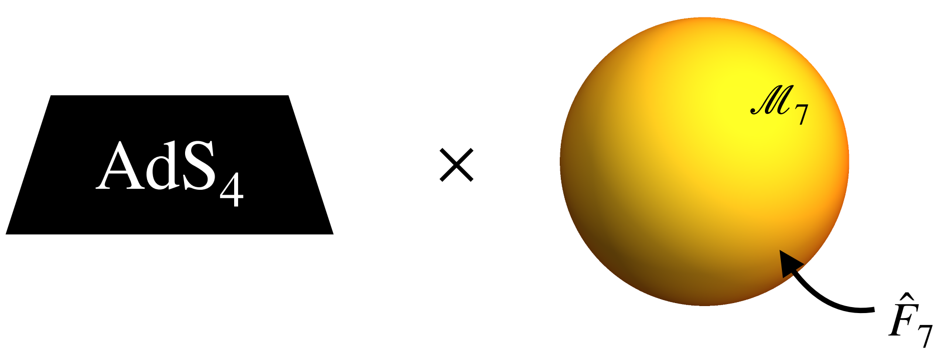
See [Friedrich, Kath, Moroianu, Semmelmann, 1997] for review

$$d\Phi = W_1 \star \Phi, \qquad d \star \Phi = 0$$

Curvature in terms of torsion class:

$$R_{mn} = \frac{3}{8} |W_1|^2 g_{mn}, \qquad R_7 = \frac{21}{8} |W_1|^2$$

Very useful for Freund-Rubin solutions!



$\mathcal{N} = 1$ SUSY for weak G_2-manifold!

Writing G_2- in terms of $SU(3)$-structures

$$\Phi = -\, v \wedge J + g_s^{-1} \mathrm{e}^{-3A} \mathrm{Im}\Omega$$

$$\star\, \Phi = g_s^{-1/3} \left(\frac{1}{2} g_s^{-1} \mathrm{e}^{-4A} J \wedge J - v \wedge \mathrm{e}^{-A} \mathrm{Re}\Omega \right)$$

$$\mathrm{d}v = -\, F_2$$

Using massless type IIA solution

$$\mathrm{d}\Phi = -\, 2\tilde{m} g_s^{1/3} \star \Phi$$

$$\mathrm{d} \star \Phi = 0$$

Weak G_2 from <u>any</u> massless IIA background*

[VVH, 2024], see also [Kaste, Minasian, Tomasiello, 2003]

Writing G_2- in terms of $SU(3)$-structures

SU(3)-structure forms

$$\Phi = -\, v \wedge J + g_s^{-1} \mathrm{e}^{-3A} \mathrm{Im}\Omega$$

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Writing G_2- in terms of $SU(3)$-structures

1-form for M-theory circle

$SU(3)$-structure forms

$$\Phi = -\nu \wedge J + g_s^{-1} e^{-3A} \mathrm{Im}\Omega$$

$$\star\Phi = g_s^{-1/3}\left(\frac{1}{2} g_s^{-1} e^{-4A} J \wedge J - \nu \wedge e^{-A} \mathrm{Re}\Omega\right)$$

$$d\nu = -F_2$$

Using massless type IIA solution

$$d\Phi = -2\tilde{m} g_s^{1/3} \star \Phi$$

$$d \star \Phi = 0$$

Weak G_2 from <u>any</u> massless IIA background*

[VVH, 2024], see also [Kaste, Minasian, Tomasiello, 2003]

Comments on scale-separated solutions

[Cribiori, Junghans, VVH, Van Riet, Wrase, 2021]

Scale-separated solution of massless type IIA

$\mathrm{AdS}_4 \times \mathscr{M}_{\mathrm{Iwasawa}}$
with F_2-, F_6-flux and O6s

→

Scale-separated Freund-Rubin solution of M-theory

$\mathrm{AdS}_4 \times \mathscr{M}_{wG_2}$
with G_7-flux

*Weak G_2 for any massless type IIA background when F_2 is closed locally:

$$\mathrm{d}F_2 = \delta_{\mathrm{D6/O6}}$$

Not possible when D6/O6 sources are smeared $\delta_{\mathrm{D6/O6}} \to j_{\mathrm{D6/O6}}$

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Local treatment of the O6-planes using perturbation theory

[Cribiori, Junghans, VVH, Van Riet, Wrase, 2021] correcting metric g_{mn},
[VVH, 2024] correcting J and Ω

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Conclusions

1. Freund-Rubin solutions only scale-separated when

   $0 < R_n \ll \lambda_1$

2. Evidence for this in M-theory from lifting a scale-separated solution of massless type IIA ($F_0 = 0$)

3. In doing so, local treatment of O6-sources is crucial

4. More work necessary to understand the full geometry

This may be the first supersymmetric, scale-separated candidate-solution of M-theory

Thank you!