Strings near black holes are Carrollian

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Strings near black holes?

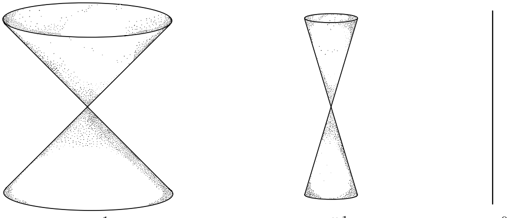
Two a priori unrelated questions:

Q1: How do strings behave near the horizon of a black hole?

Q2: What is Carrollian string theory?

A taste of Carrollian symmetry & geometry

- Physically $c \to 0$; hence "ultra-local"



- Carroll boosts transformation act as

$$t \to t' = t + b_i x^i \,, \qquad \vec{x} \to \vec{x}' = \vec{x}$$

- Arises at the infinities of asymptotically flat spacetime; also related to the geometry on null hypersurfaces and therefore black hole horizons; obtained as the $c \to 0$ limit of Lorentzian geometry

Metric $g_{\mu\nu}$ replaced by $(v^\mu, h_{\mu\nu})$ satisfying $v^\mu h_{\mu\nu} = 0$

[Duval, Gibbons, Horvathy, '14; Hartong, '15; Figueroa-O'Farrill, EH, Prohazka, Salzer, '21; Hansen, Obers, Oling, Søgaard, '21]

Plan

1. The string Carroll expansion for non-extremal black holes
2. Near-horizon strings are Carrollian
3. Magnetic and electric Carroll strings

The near-horizon geometry of a Schwarzschild black hole

- The four-dimensional Schwarzschild metric is

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$

- Want to zoom into the near-horizon region $r \sim 2GM =: r_h$, so define a new radial coordinate $\mathscr{r}$ via

$$r = r_h + \frac{1}{r_h}\epsilon \mathscr{r}^2$$

Positive dim'less parameter that goes to zero as we approach the horizon

- This leads to

$$ds^2 = r_h^2 d\Omega^2 + \epsilon\left[-\frac{\mathscr{r}^2}{r_h^2}(dt)^2 + 4d\mathscr{r}^2 + 2\mathscr{r}^2 d\Omega^2\right] + \epsilon^2\left[\frac{\mathscr{r}^4}{r_h^2}(dt)^2 + \frac{4\mathscr{r}^2}{r_h^2}d\mathscr{r}^2 + \frac{\mathscr{r}^4}{r_h^2}d\Omega^2\right] + \mathcal{O}(\epsilon^3)$$

- This has the form of a *string Carroll expansion*

The string Carroll expansion

- Consider a vielbein decomposition of a metric g

$$g(c^2) = -c^2 E^0(c^2)E^0(c^2) + c^2 E^1(c^2)E^1(c^2) + E^i(c^2)E^i(c^2)$$

- Expand vielbeine in powers of c^2 as $E(c^2) = e + c^2 e_{(1)} + \cdots$:

$$g(c^2) = e^i e^i + c^2(-e^0 e^0 + e^1 e^1) + 2c^2 e^i e^i_{(1)} + \cdots = h + c^2\tau + c^2\Phi + \cdots$$

where $h = e^i e^i$, $\tau = -e^0 e^0 + e^1 e^1$ and $\Phi = 2e^i e^i_{(1)}$

- To match with the near-horizon expansion, we identify

$$\boxed{\epsilon \equiv c^2}$$

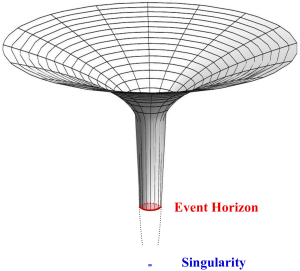
- Comparing with our near-horizon expansion, we find

$$h_{\mu\nu} dx^\mu dx^\nu = r_h^2 d\Omega^2\,, \qquad \tau_{\mu\nu} dx^\mu dx^\nu = \underbrace{-\frac{\mathfrak{r}^2}{r_h^2} dt^2 + 4 d\mathfrak{r}^2}_{\text{2d Rindler spacetime}}\,, \qquad \Phi_{\mu\nu} dx^\mu dx^\nu = 2\mathfrak{r}^2 d\Omega^2$$

- This is a *generic* feature: the near-horizon region of any non-extremal black hole looks like a string Carroll expansion

(Non-)extremal black holes and Rindler spacetime

- Extremal black holes generically have a Lorentzian near-horizon structure with an AdS_2 throat [Kunduri, Lucietti, Reall, '07]
- Non-extremal black holes admit a string Carroll expansion which contain a 2d Rindler space fibered over a 2d base



[Li, '21]

A Rindler reminder:

- Worldline of body with const. acceleration in X-direction as function of proper time t and rapidity αt

$$T = x \sinh(\alpha t)\,, \qquad X = x \cosh(\alpha t)\,, \qquad x = \text{const.}$$

with ranges $0 < X < \infty\,, \quad -X < T < X$

- Leads to $ds^2 = -dT^2 + dX^2 = -(\alpha x)^2 dt^2 + dx^2$

Warmup: magnetic and electric Carroll scalars

The simplest Carroll-invariant field theories are obtained by taking the $c \to 0$ limit of a (free) relativistic scalar field theory

$$\hat{\mathcal{L}} = \frac{1}{2c^2}(\partial_t \phi)^2 - \frac{1}{2}(\partial_i \phi)^2$$

This limit can be taken in two ways:

- The "electric" Carroll scalar: as $c \to 0$, the first term above dominates

$$\hat{\mathcal{L}} = \frac{1}{c^2}\mathcal{L}_{\mathsf{e}} + \mathcal{O}(1)\,, \qquad \mathcal{L}_{\mathsf{e}} = \frac{1}{2}(\partial_t \phi)^2$$

- The "magnetic" Carroll scalar: perform a HS transformation to obtain

$$\hat{\mathcal{L}} = -\frac{c^2\chi^2}{2} + \chi\partial_t\phi - \frac{1}{2}(\partial_i\phi)^2 \xrightarrow{c\to 0} \chi\partial_t\phi - \frac{1}{2}(\partial_i\phi)^2 =: \mathcal{L}_{\mathsf{m}}$$

[de Boer, Hartong, Obers, Sybesma, Vandoren, '21]

The relativistic string and its Carrollian expansion

The phase space Lagrangian for string theory is

$$\mathcal{L} = \dot{X}^\mu P_\mu - \frac{1}{2} e \left(g^{\mu\nu}(X) P_\mu P_\nu + T^2 g_{\mu\nu}(X) X'^\mu X'^\nu \right) - u X'^\mu P_\mu$$

where $X^\mu = x^\mu + \epsilon y^\mu + z^\mu + \mathcal{O}(\epsilon^3)$ and $u = u_{(0)} + \mathcal{O}(\epsilon)$.

Two choices for the ϵ-scaling of e:

Mag.: $e = \mathcal{O}(1) \Rightarrow P = \mathcal{O}(1)$, leading to

$$\mathcal{L} = \epsilon^{-1} \mathcal{L}_{\text{m,LO}} + \mathcal{L}_{\text{m,NLO}} + \epsilon \mathcal{L}_{\text{m,NNLO}} + \mathcal{O}(\epsilon^2)$$

Elec.: $e = \mathcal{O}(\sqrt{\epsilon}) \Rightarrow P = \mathcal{O}(\sqrt{\epsilon})$, leading to

$$\mathcal{L} = \sqrt{\epsilon} \mathcal{L}_{\text{e,LO}} + \mathcal{O}(\epsilon^{3/2})$$

The magnetic Carrollian string

The magnetic Carrollian string has a "traditional" Polyakov formulation and may *also* be obtained by expanding the Polyakov Lagrangian

$$\mathcal{L}_{\mathsf{P}} = -\frac{T}{2}\sqrt{-\gamma}\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X)$$

with $\gamma_{\alpha\beta} = \gamma_{(0)\alpha\beta} + \epsilon\gamma_{(2)\alpha\beta} + \mathcal{O}(\epsilon^2)$

- ϵ-expanded metric $g_{\alpha\beta}(X) = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X)$ includes "Taylor terms"

  $$g_{\alpha\beta}(X) = h_{\alpha\beta}(x) + \epsilon\hat{\Phi}(x,y) + \mathcal{O}(\epsilon^2)$$

  where

  $$h_{\alpha\beta}(x) = \partial_\alpha x^\mu \partial_\beta x^\nu h_{\mu\nu}(x)\,,$$

  $$\hat{\Phi}(x,y) = \tau_{\alpha\beta}(x) + \Phi_{\alpha\beta}(x) + 2h_{\mu\nu}(x)\partial_{(\alpha}x^\mu\partial_{\beta)}y^\nu + \partial_\alpha x^\mu \partial_\beta x^\nu y^\rho \partial_\rho h_{\mu\nu}(x)$$

- This implies that NB: Lorentzian WS

  $$\mathcal{L}_{\mathsf{P}} = \mathcal{L}_{\mathsf{P\text{-}LO}} + \epsilon\mathcal{L}_{\mathsf{P\text{-}NLO}} + \mathcal{O}(\epsilon^2)\,, \qquad \mathcal{L}_{\mathsf{P\text{-}LO}} = -\frac{T}{2}\sqrt{-\gamma_{(0)}}\gamma_{(0)}^{\alpha\beta}h_{\alpha\beta}(x)$$

cf. NR strings [Hartong, EH, '21; '22; '24]

The electric Carroll string

- In contrast to the magnetic Carrollian string, the electric string cannot be obtained from the Polyakov string since it is not quadratic in transverse momenta as $\epsilon \to 0$

$\Rightarrow$ The closest we can get to a Polyakov-like formulation is

$$\mathcal{L}_{\mathsf{e}} = -T^2\mathbb{E}\left(-\mathbb{v}^\alpha\partial_\alpha X^\mu\tilde{P}_\mu + h_{\mu\nu}\mathbb{e}^\alpha\mathbb{e}^\beta\partial_\alpha X^\mu\partial_\beta X^\nu\right)+\frac{\mathbb{E}}{2}\tau_{\mu\nu}\mathbb{v}^\alpha\mathbb{v}^\beta\partial_\alpha X^\mu\partial_\beta X^\nu$$

- This theory has a Carrollian WS with frame $(\mathbb{v}^\alpha, \mathbb{e}^\alpha)$; $\mathbb{E}$ is the measure

- This theory is a combination of electric Carrollian scalars in the longitudinal directions and magnetic Carrollian scalars in the transverse directions

See also [Gomis, Yan, '23; Blair, Lahnsteiner, Obers, Yan, '23 & '24]

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Take home messages

1. The near-horizon geometry of 4d non-extremal black holes takes the form of a string Carroll expansion

2. Near-horizon strings are Carrollian

3. There are two types of Carrollian strings: magnetic and electric, which have, respectively, Lorentzian and Carrollian worldsheets

Thank you for your attention