An eye for an i saturates the two-point functions

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The discovery of AdS/CFT correspondence definitively resolved the black hole information paradox

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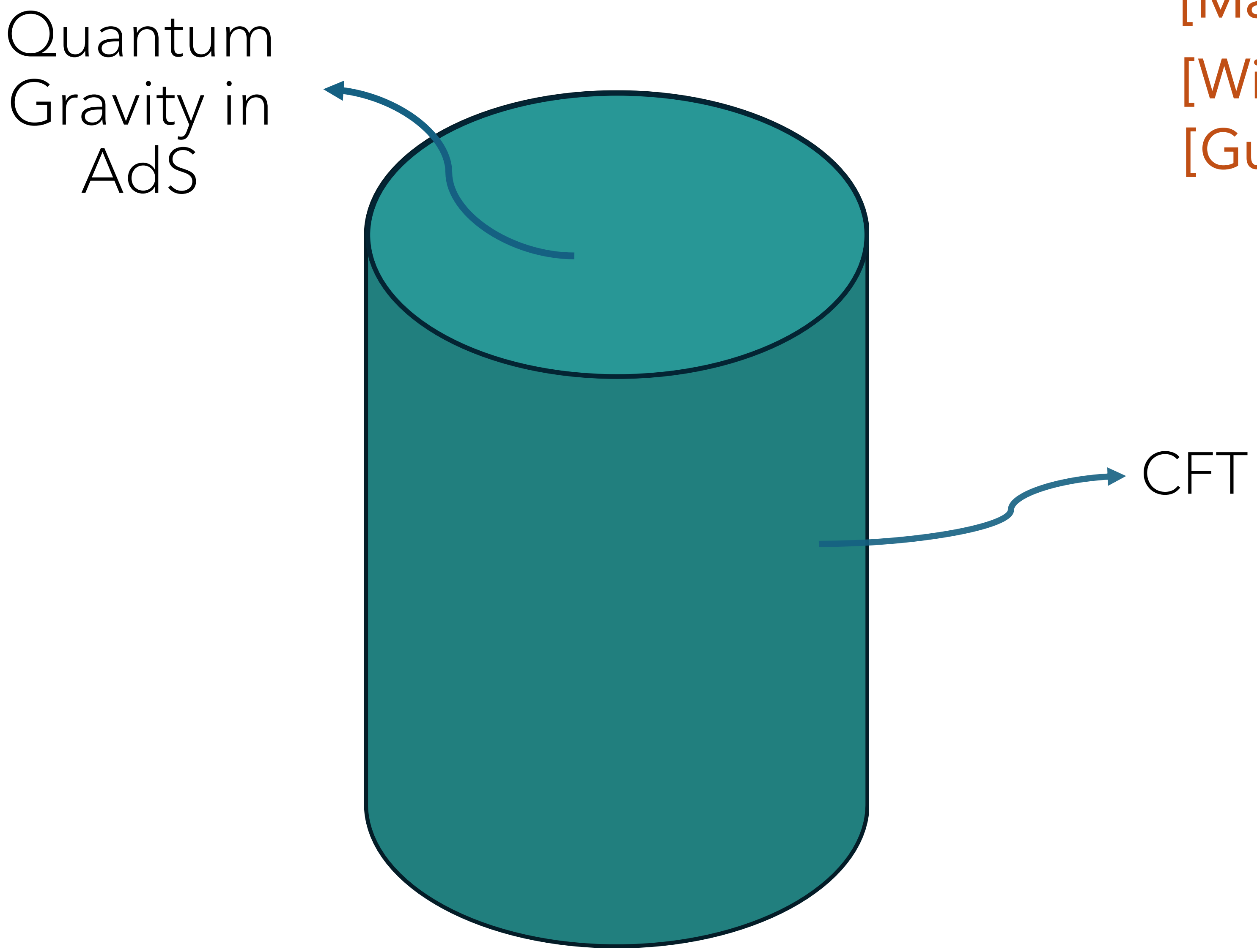
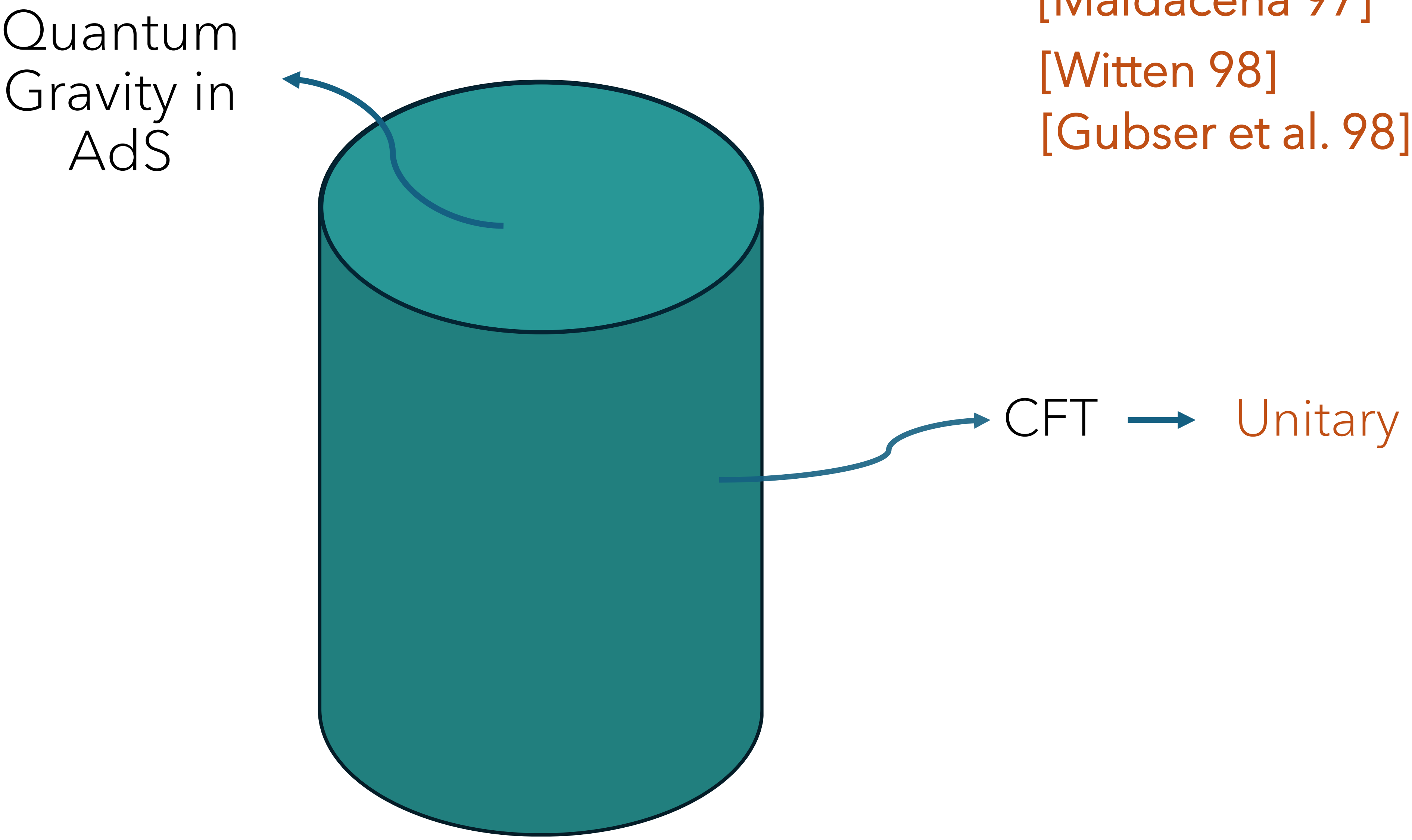
albeit in a rather trivial way.



[Maldacena 97]
[Witten 98]
[Gubser et al. 98]



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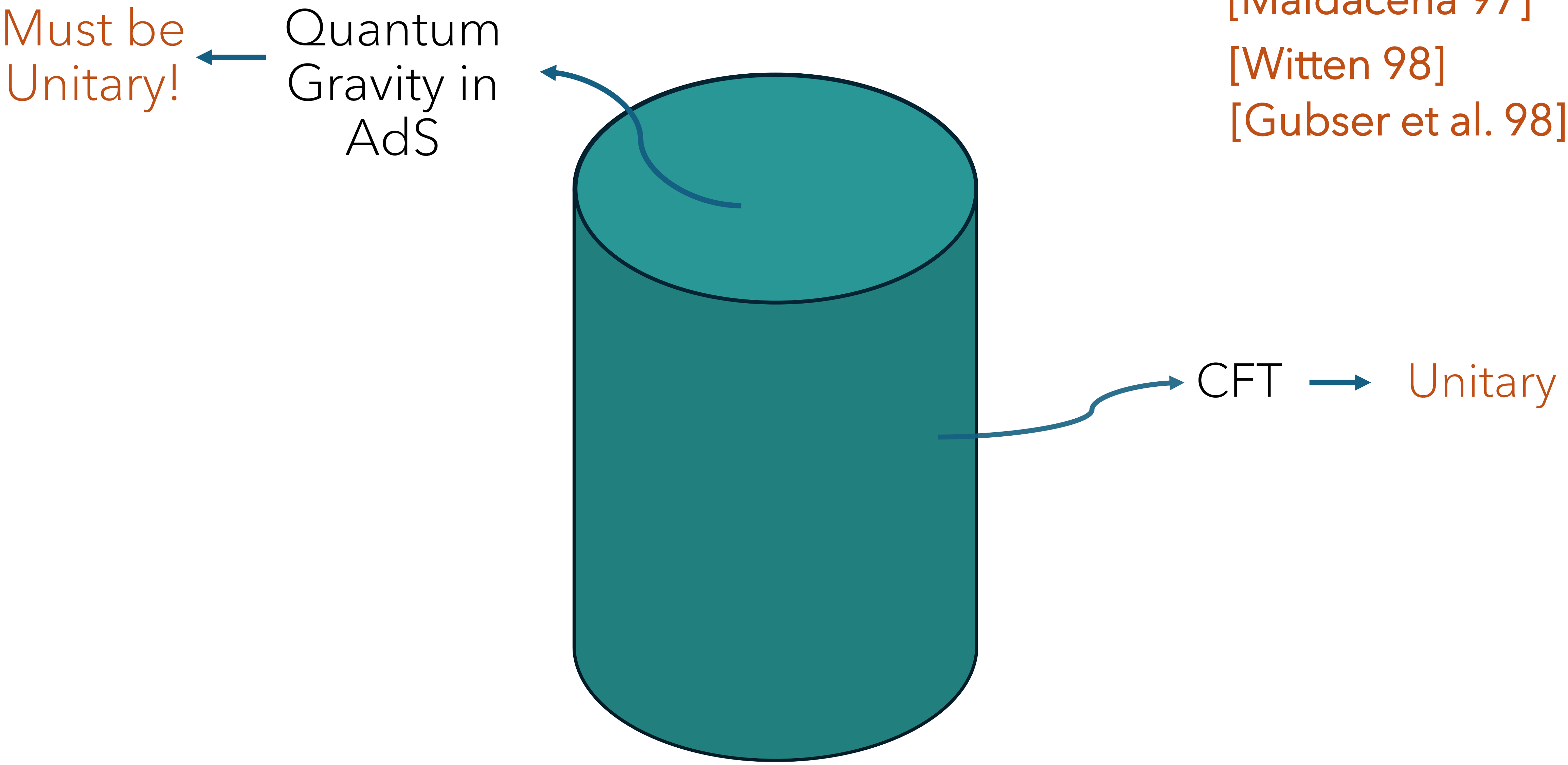

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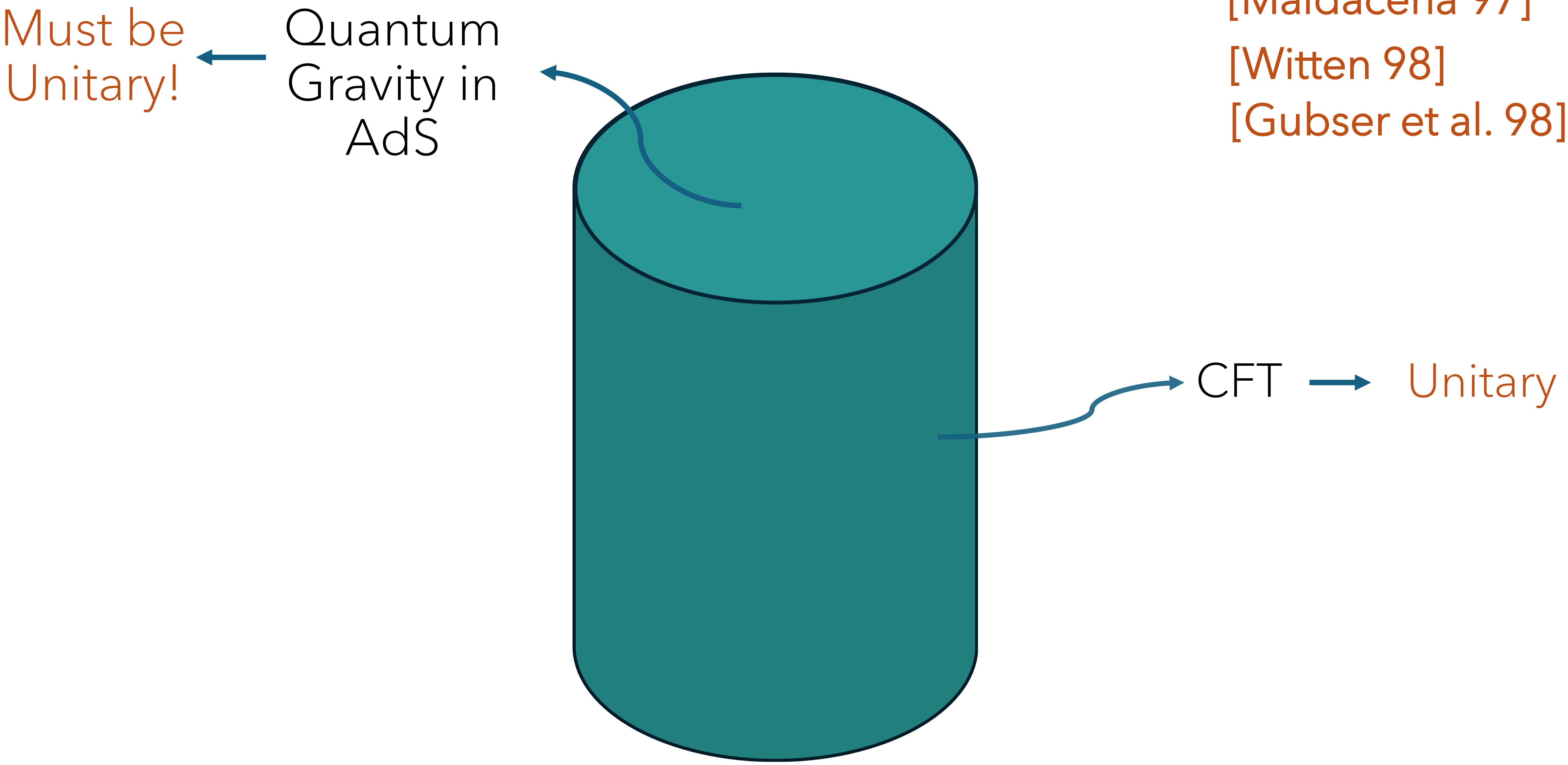

Fine-grained information is preserved in the bulk

[Susskind Thorlacius Uglum 93]

The principle of black hole complementarity helps us understand the semiclassical limit.

[Susskind Thorlacius Uglum 93]

The principle of black hole complementarity helps us understand the semiclassical limit.

For "simple" observers, QFT on the black hole background is a good enough description.

Quantities that probe the finer structure of the bulk quantum gravity Hilbert space require us to go beyond perturbation theory around the black hole spacetime.

2pt functions in TFD

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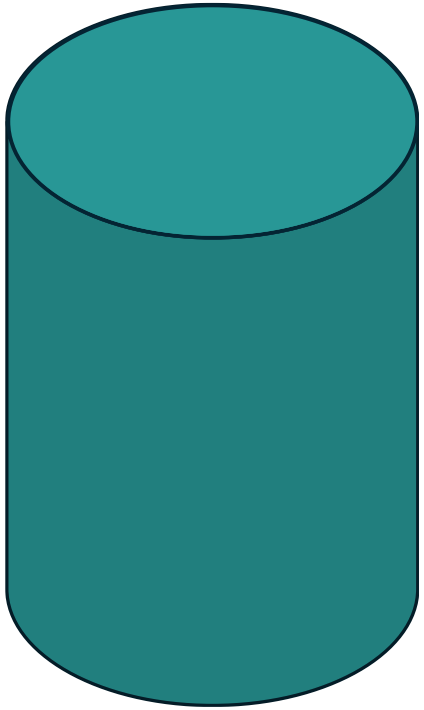
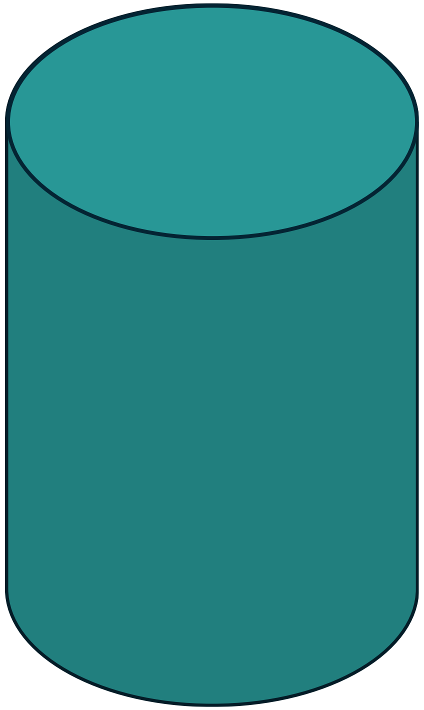


CFT_L

$R \times S^{d-1}$



CFT_R

$$|TFD_\beta\rangle = \sum_n e^{-\frac{\beta}{2}n}|n\rangle_L |n\rangle_R$$

Thermofield double state (TFD)

2pt functions in TFD

Let $O_{L,R}$ be operators of scaling dimension Δ. We can define two-sided two-point function as follows:

$$G_2(\tau) = \langle TFD_\beta | O_L(-\tau) O_R(\tau) | TFD_\beta \rangle$$

The 2pt functions, at late times, oscillate erratically around a constant value.

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In the *coarse-grained, semiclassical* limit, the 2pt functions generically decay **exponentially**!

The exponential decay is closely related to the existence of a black hole in the dual bulk relation.

Decay rate $\rightarrow$ Quasi-normal modes

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Decay rate $\rightarrow$ Quasi-normal modes

Maldacena's Information Paradox

[Maldacena 01]

Corrected Semiclassical limit

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Black holes are chaotic objects. [Shenker Stanford 13] [Cotler et al. 16] [Maldacena et al. 15]

The emergence of chaos requires the energy levels to display *level repulsion*, and coarse-graining over states removes this information. [McDonald Kaufman 79] [Berry 81] [Bohigas et al. 84]

We can fix this by treating the semiclassical limit as a random matrix theory (RMT).

When we add the universal RMT corrections, called the sine-kernel, the 2pt functions stops decaying and saturates to a constant!

The saturation occurs at $\tau \sim e^{S_0}$.

Late-time Expansion

Let us zoom in on the saturation by taking the limit:

$$\tau \to \infty, \quad S_0 \to \infty, \quad \tau e^{-S_0} = \text{const}$$

τ scaling limit

We find that

$$G_2(\tau) = \sum_{g=1}^{\infty} \tau^{2g-1} e^{-(2g-1)S_0} Z_{2g-1}$$

where

$$Z_{2g-1} \simeq \begin{cases} \frac{-1}{(2g-2)(2g-1)} \oint_0 \frac{\mathrm{d}E}{2\pi \mathrm{i}} e^{-\beta E} \; e^{-m\ell(E)} \; e^{-(2g-1)S(E)} & g > 1 \\ \int_{E_{\min}}^{\infty} \mathrm{d} \; e^{-\beta E} \; e^{-m\ell(E)} \; e^{-S(E)} & g = 1 \end{cases}$$

Can we reproduce this series from a bulk calculation?

Bulk calculation reduces to a geodesic calculation!

Geodesics in JT (d=1 case)

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The metric is given by

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)}\,, \qquad f(r) = \frac{r^2 - r_h^2}{L^2}$$

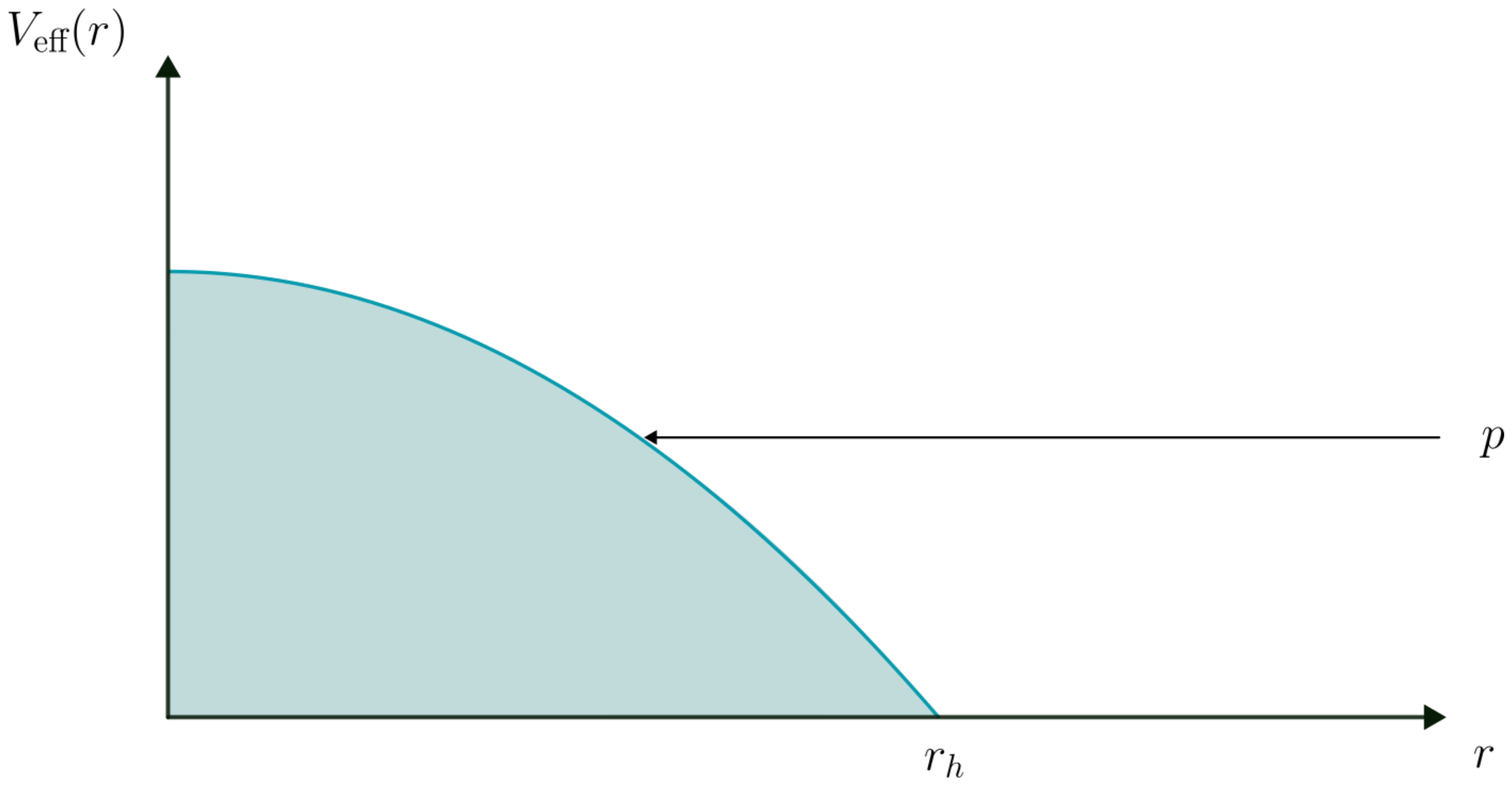
The geodesic equations are given by

$$f(r)\dot{t} = p$$

Conserved quantity

$$\dot{r}^2 = f(r) + p^2$$

The dot is w.r.t. the proper time λ along the curve.


$V_{\mathrm{eff}}(r)$
p
r_h
r

Quantizing the geodesics

Quantizing the geodesics

We can study the geodesics by treating it as a 2d particle with the action

$$I = m \int d\lambda \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}$$

Note that the action is imaginary for spacelike geodesics.

Let us look at the boundary-to-boundary propagator of this particle:

$$\left\langle r_{\mathrm{UV}}; -\tau - \frac{i\beta}{2} \middle| r_{\mathrm{UV}}; \tau \right\rangle = \int \mathcal{D}r\mathcal{D}t \ e^{iI}$$

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In the semiclassical approximation, we can evaluate the path integral using a saddle point approximation.

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In the semiclassical approximation, we can evaluate the path integral using a saddle point approximation.

Only classical saddle points contribute $\longrightarrow$ Propagator decays exponentially!

Constrained instantons

When we quantize the 2d particle, there will be a non-zero probability of the particle tunnelling through the effective potential.

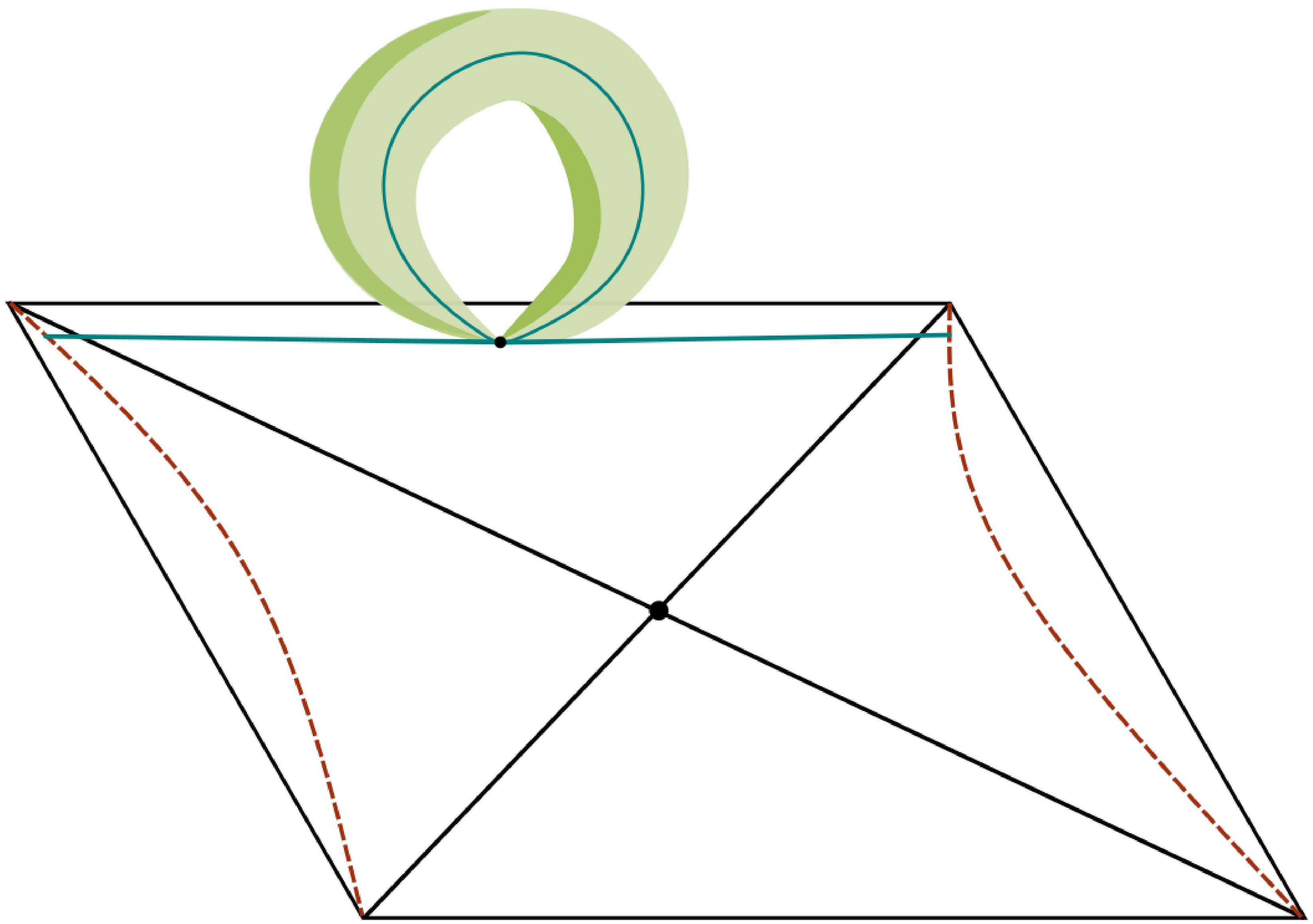
The tunnelling solutions can be obtained by Wick rotating the proper time.

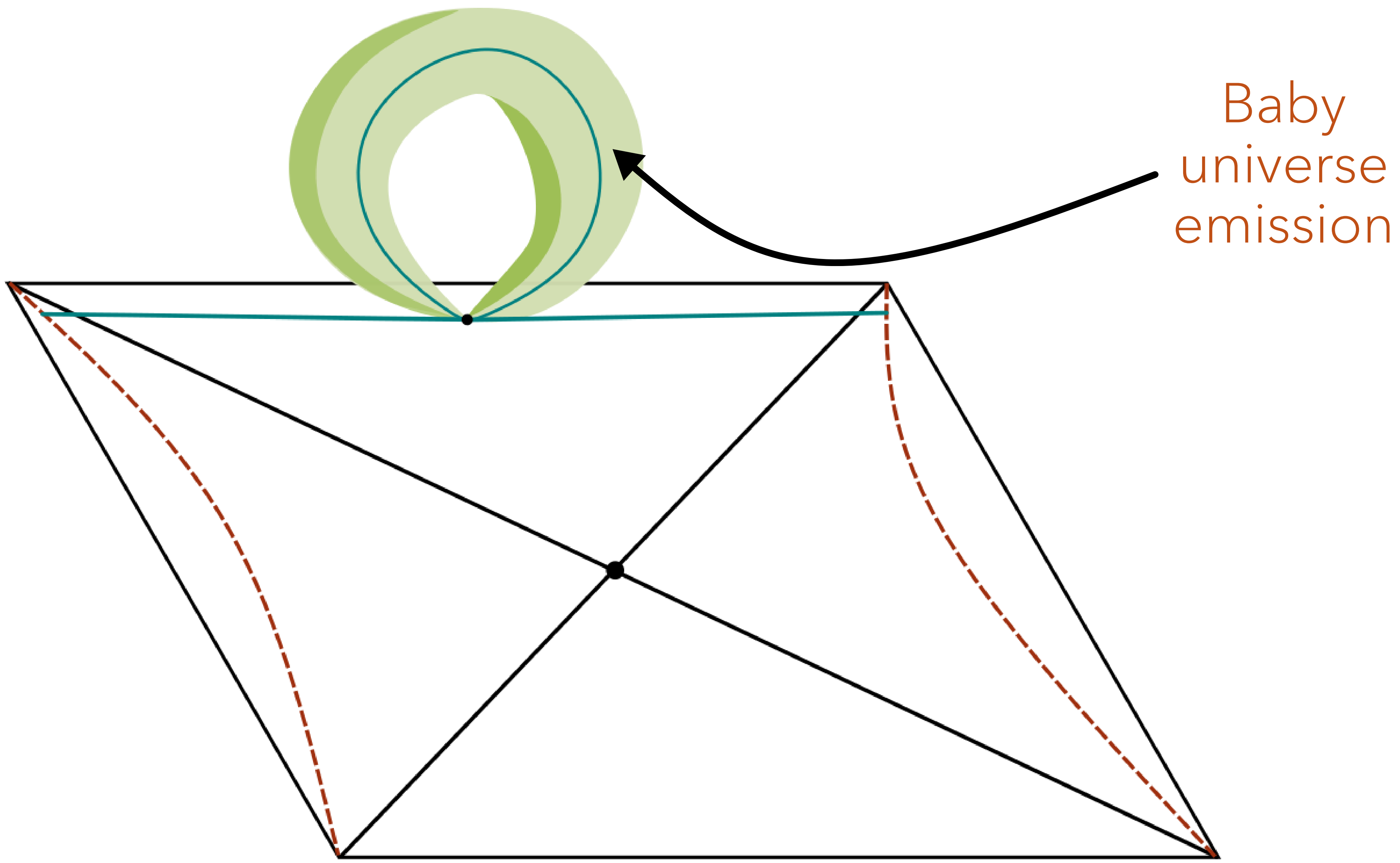
Wick rotating the proper time,

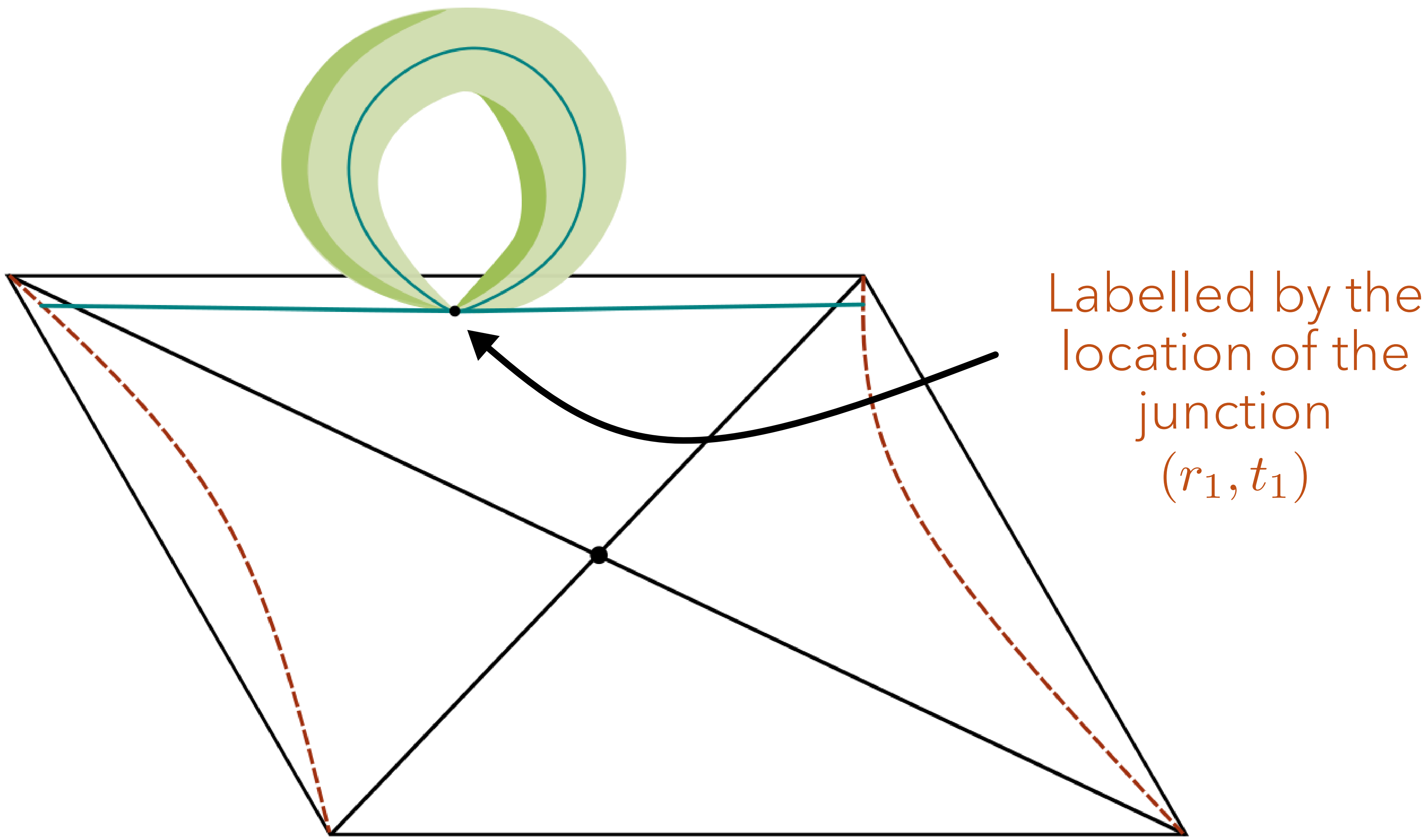
$$\lambda = -i\alpha$$

we get tunnelling solutions.

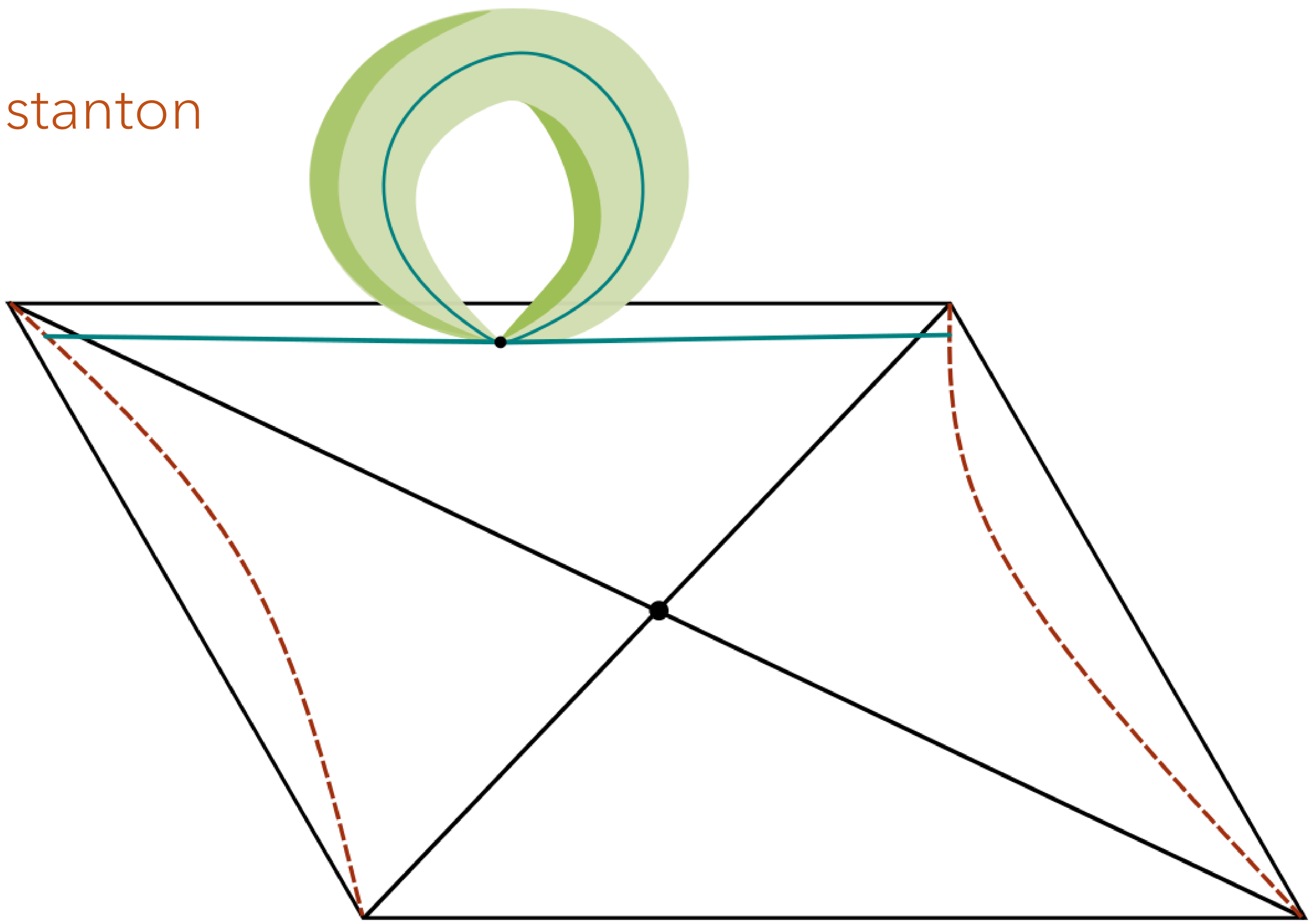
The solutions do not anchor to the cutoff surface in the asymptotic region!




Baby
universe
emission


Labelled by the
location of the
junction
(r_1, t_1)

1- instanton



These configurations are called constrained instantons.

These configurations can be shown to contribute to the path integral by the insertion of a constraint. [Affleck 1981]

Semiclassical propagator

In the semiclassical limit, we have

$$\left\langle r_{\mathrm{UV}}; -\tau - \frac{i\beta}{2} \middle| r_{\mathrm{UV}}; \tau \right\rangle = \sum_{\mathrm{saddles}} e^{iI} = \quad G_2(\tau)$$

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Plugging in everything we get

One-loop fluctuations around the solution

$$\begin{aligned} G_2(\tau) \; &= e^{im\ell_{\mathrm{cl}}(\tau)} + \int_{r_{\mathrm{min}}}^{r_{\mathrm{UV}}} dr_1 \mu(r_1) \int_0^{\tau} dt_1 \; e^{im\ell_1} \hat{Z}_1 \\ &+ \int_{r_{\mathrm{min}}}^{r_{\mathrm{UV}}} dr_1 dr_2 dr_3 \mu(r_1)\mu(r_2)\mu(r_3) \int_0^{\tau} dt_1 dt_2 dt_3 \; e^{im\ell_3} \hat{Z}_3 + \; \cdots \end{aligned}$$

Massaging the expression, and taking the τ-scaling limit, we get

$$G_2(\tau) = \sum_{g=1}^{\infty} \tau^{2g-1} e^{-(2g-1)S_0} Z_{2g-1}$$

with

$$Z_{2g-1} \simeq \begin{cases} \frac{1}{(2g-2)(2g-1)} \int_{E_{\min}}^{\infty} dE \;\; \mu(E) \;\; e^{-m\ell(E)} \; e^{-(2g-1)S(E)} & g > 1 \\ \int_{E_{\min}}^{\infty} dE \;\; \mu(E) \;\; e^{-m\ell(E)} \; e^{-S(E)} & g = 1 \end{cases}$$

Relation to Schwinger effect

[Schwinger 51]

Increasing the electric field *E* results in the unsuppressed production of e^-e^+ pairs

The boundary time plays the role of *E* and the baby universes play the role of the e^-e^+ pair.

Worldline Instantons → [Dunne Schubert 05]

Discussion

The calculation will go through in higher dimensions.

We have found a systematic way of including baby universe/ higher topology corrections that works in any theory of gravity.

Thank you for listening!

Backup Slides

The 2pt functions, at late times, oscillate erratically around a constant value.

In the coarse-grained semiclassical limit, we have

$$\sum_n \qquad \rightarrow \qquad \int \mathrm{d}n \rho(n)$$

and

$$c \rightarrow \infty \quad , \quad \Delta \rightarrow \infty \text{ with } \Delta/c \rightarrow 0$$

In the limit:

$$\tau \to \infty, \quad S_0 \to \infty, \quad \tau e^{-S_0} = \mathrm{const}$$ τ scaling limit

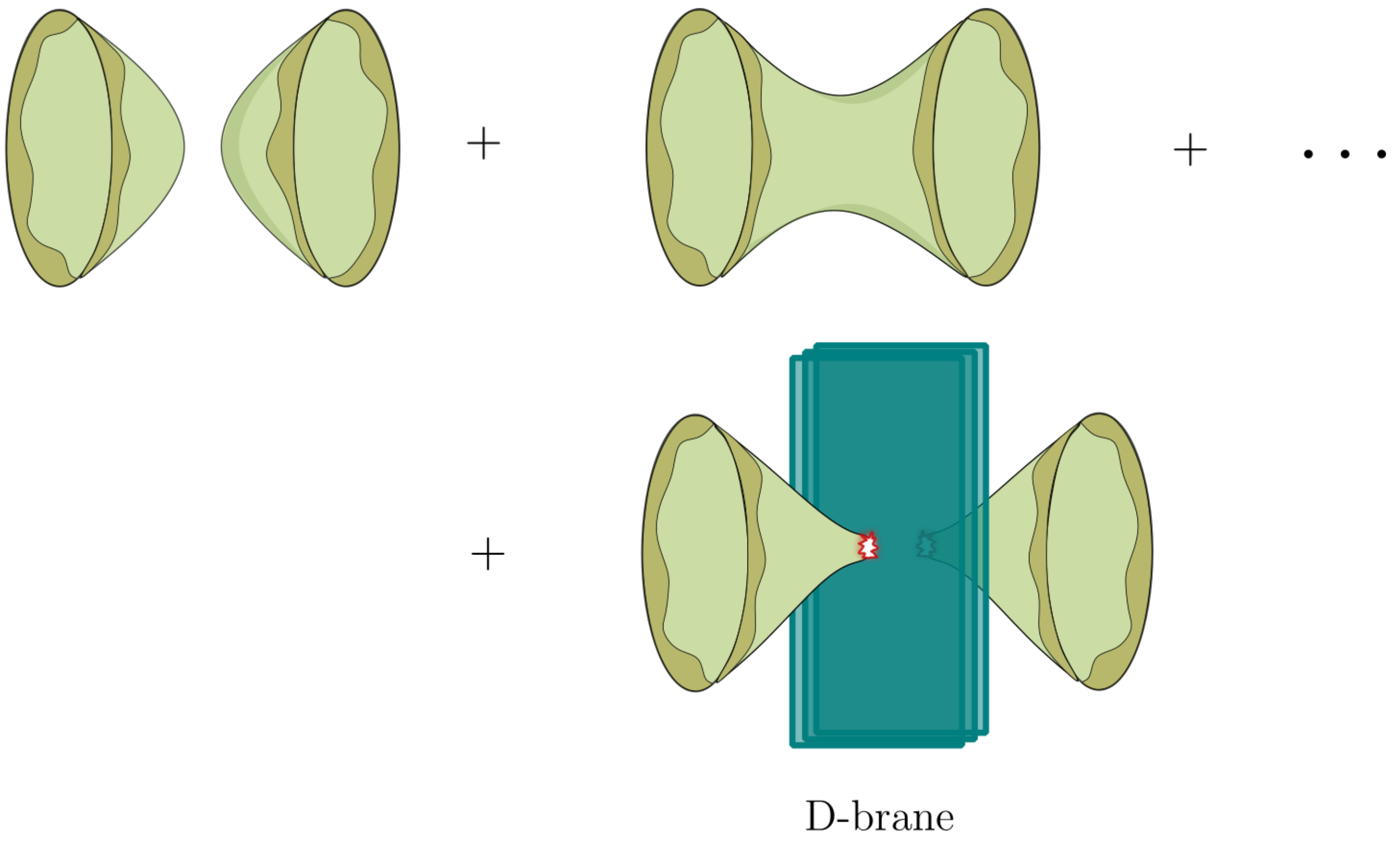
The density of states picks up the following ***universal*** correction.

$$\langle \rho(n)\rho(m) \rangle = \langle \rho(n) \rangle \langle \rho(m) \rangle \underbrace{- \frac{\sin^2\left(\pi \left\langle \rho(\bar{E}) \right\rangle (n-m)\right)}{\left(\pi\left(n-m\right)\right)^2} + \left\langle \rho(\bar{E}) \right\rangle \delta\left(n-m\right)}_{\text{Sine-kernel}}$$

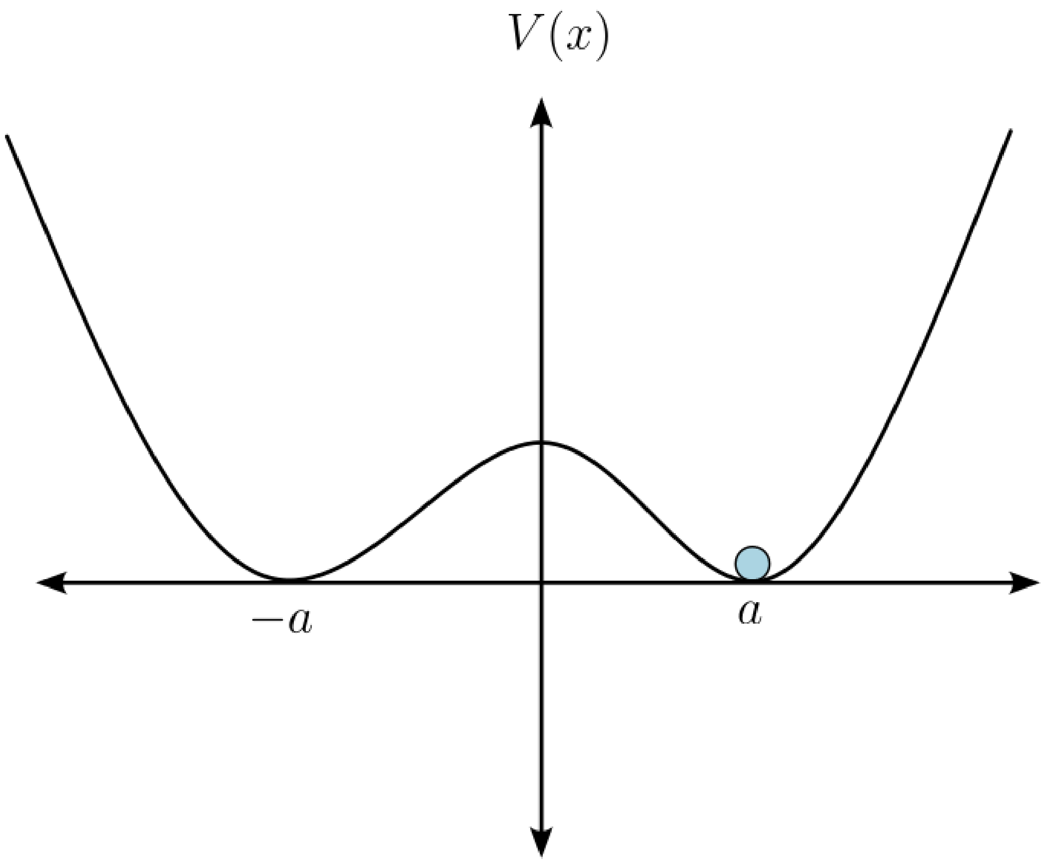
JT gravity

Let us look at the d=1 case. The bulk theory reduces to JT gravity. Using the explicit matrix model description, we have

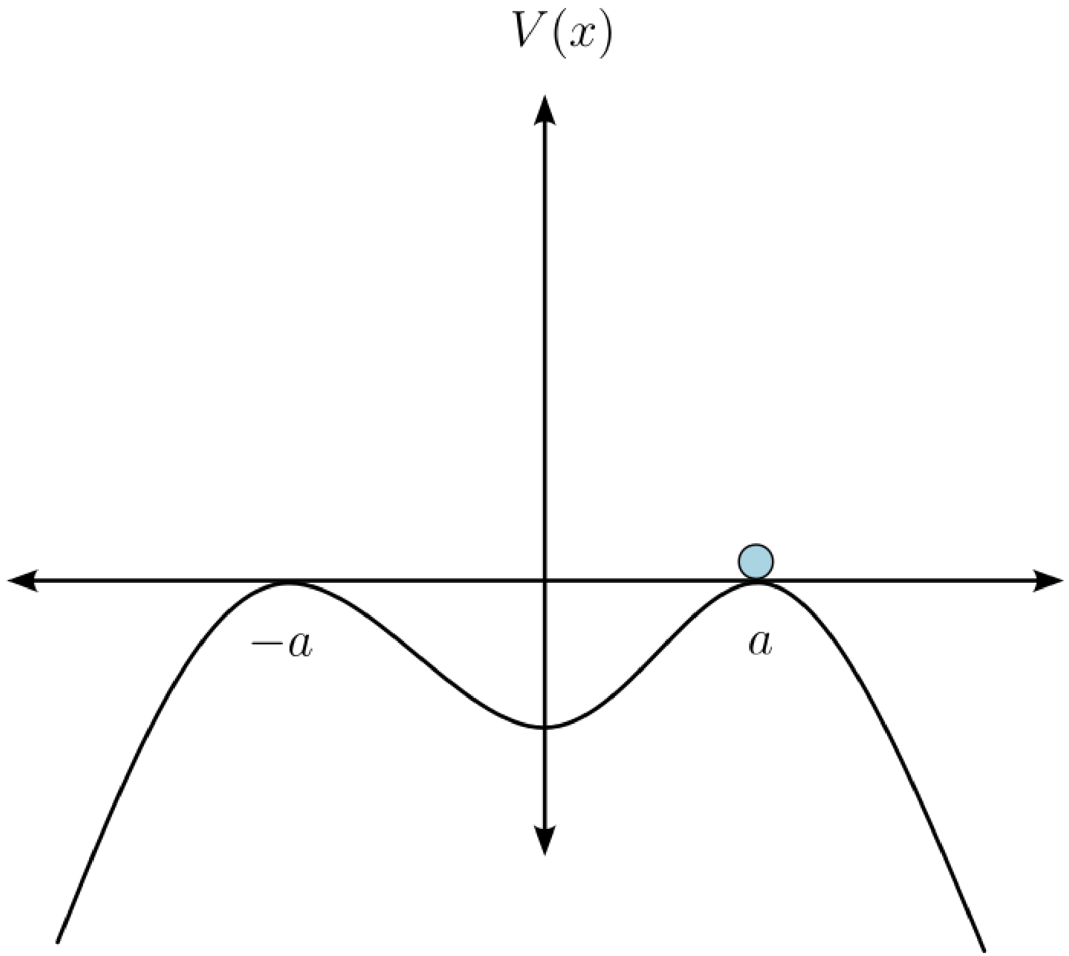
$$\langle \rho(E_1)\rho(E_2)\rangle =$$



[Coleman 1977]
[Callan, Coleman 1977]



$t \to -i\tau$



Length of trajectories

The classical geodesics have the length

$$\ell_{\rm cl} = \int \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} d\lambda - 2iL \log\left(\frac{r_{\rm UV}}{r_h}\right)$$

$$= \frac{2ir_h}{L}\tau$$

Independent of τ

The (2n-1)-instanton will have the length

$$\ell_{2n-1} = 2\pi i(2n-1)L + 2iL \ \text{arccosh}\left(\frac{r_{\rm UV}}{r_{\rm min}}\right) - 2iL\left(\sum_{j=1}^{2n-1}(-1)^{j+1}\text{arccosh}\left(\frac{r_j}{r_{\rm min}}\right)\right)$$

$$- 2iL \log\left(\frac{r_{\rm UV}}{r_h}\right)$$

These hybrid configurations are not true saddle points of the length functional.

These configurations are called constrained instantons.

These configurations can be shown to contribute to the path integral by the insertion of a constraint.

QFT [Affleck 1981] [Frishman Yankielowicz 1979]

Gravity [Cotler Jensen 2020]

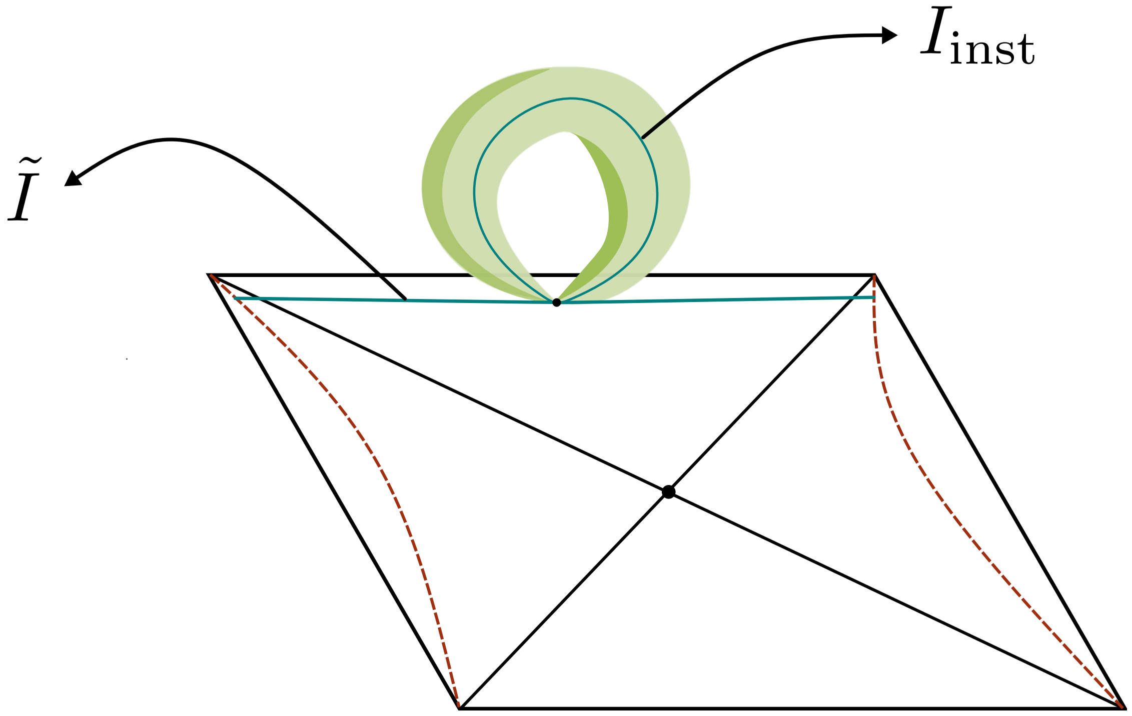
Cosmology [Hawking Turok 1998]

Picking up constrained instantons

Consider two points in the Penrose diagram symmetric w.r.t the t=0 line. We will label these points by (r_1, t_1).

We can resolve the identity as follows

$$1 = \int dr_1 dt_1 \mu(r_1, t_1) \delta\left(I - (I_{\text{inst}} + \tilde{I})\right)$$



Inserting this into the path integral, we get

$$\int \mathcal{D}r\mathcal{D}t \; e^{iI} = \int dr_1 dt_1 \mu(r_1, t_1) \int \mathcal{D}r\mathcal{D}t \; \delta\left(I - (I_{\text{inst}} + \tilde{I})\right) e^{iI}$$

$$= \int dr_1 dt_1 \mu(r_1, t_1) \int d\sigma \int \mathcal{D}r\mathcal{D}t \; e^{iI + i\sigma\left(I - I_{\text{inst}} + \tilde{I}\right)} \, .$$

Let us look at the saddle points of the new path integral. When we vary all the parameters, we will get the equation

$$\sigma = 0$$

However, if we keep (r_1, t_1) fixed and vary the remaining parameters, we obtain our hybrid instanton configurations.