Emergent non-invertible symmetries at strong coupling

Based on hep-th/2408.07123, with M. Del Zotto, D. Migliorati and K. Ohmori.

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Motivations

We are interested in studying how symmetries can be used to study the strong coupling regime of certain theories.

– e.g. symmetries can distinguish higgsing and confining phase.

In recent years the newly discovered "non-invertible symmetries" added a new tool to the symmetry arsenal for attacking non-perturbative regimes.

How can we use them? We are going to explore the example of 4d Adjoint QCD with 2 flavors.

Introduction

A new take on symmetries

In recent years, the concept of symmetries was extended via the new paradigm [Gaiotto, Kapustin, Seiberg, Willet '14]:

$$\text{Symmetries} \leftrightarrow \text{Topological Operators}$$

This novel approach to symmetries allowed the definition of more general "kinds" of symmetries, now dubbed *categorical symmetries*.

A regular symmetry acting on particles (0-dimensional objects) is called a 0-form symmetry.

Noether theorem associate to each symmetry a conserved current J_μ.

In a D-dimensional QFT we define the symmetry operators via

$$U(M_{D-1})_g := e^{ig \int_{M_{D-1}} \star J}$$

What if the current has more indices, $J_{\mu\nu\ldots}$?

We can define n-form symmetry!

An n-form symmetry is an operator that acts on n-dimensional objects.

Given an (n+1)-form conserved current $J_{\mu_1 \ldots \mu_{n+1}}$, we define the operators

$$U(M_{D-n-1})_g := e^{ig \int_{M_{D-n-1}} \star J}$$

As long as the current is conserved, $dJ = 0$, these operators are topological.

Example: Maxwell theory

As an example, let us consider 4d Maxwell theory without sources.

We can define two conserved currents, $dJ_e = d \star F = 0$ $J_m = dF = 0$.

The operators $U(M_2) = e^{i \int F}$ and $V(M_2) = e^{i \int \star F}$ are 2d topological operators measuring the charge of Wilson and 't Hooft lines respectively.

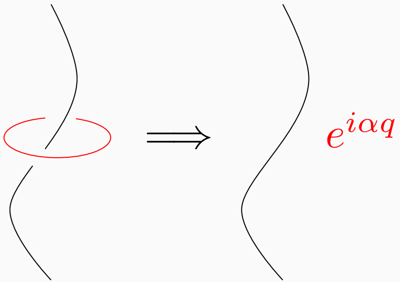

Figure 1: 3d section of the linking operation.

Example: QED

Let us now turn to 4d QED.

Here we have a $U(1)_m$ 1-form associated to $dF = 0$, while electrons break the the $U(1)_e$ since $d \star F = \star J_e$.

This theory as also an anomalous $U(1)_a$ 0-form axial $dJ_a = F \wedge F$.

We can still define a topological operator since $d(J_a - A \wedge F) = 0$ is a conserved current, but it is not gauge invariant.

$$U(M) = e^{ig \int_M J_a - A \wedge F} \xrightarrow{A \to A + \mathrm{d}\lambda} e^{ig \int_M J_a - A \wedge F} e^{ig \int \mathrm{d}\lambda \wedge F}$$

The extra phase disappears only if $g \in 2\pi\mathbb{Z}$.

However, one can make sense of the above operator for any choice of $g \in \mathbb{Q}/\mathbb{Z}$.

The "bad" gauge transformation can be canceled by stacking a 3d theory with an anomaly on the defect.

These are the so called $\mathcal{A}^{N,p}$ theories [Hsin, Lam, Seiberg '18].

The price to pay is that now the defects fuse in a non-invertible way.

$$U\bar{U} = C \tag{1}$$

Where C is a condensate.

See [Choi, Lam, Shao '22 ;Copetti, Del Zotto, Ohmori, Wang '23] for details.

Applications to AdjQCD

From $\mathcal{N} = 2$ to $\mathcal{N} = 0$

We want to apply the previous techniques to the study of AdjQCD.

We start by considering the UV theory to be $\mathcal{N} = 2$ pure $SU(n)$ and we add a small mass deformation for the adjoint scalars.

This triggers a flow that we can follow [Cordova, Dumitrescu '18; D'Hocker, Dumitrescu, Nardoni, Gerchkoviz '20].

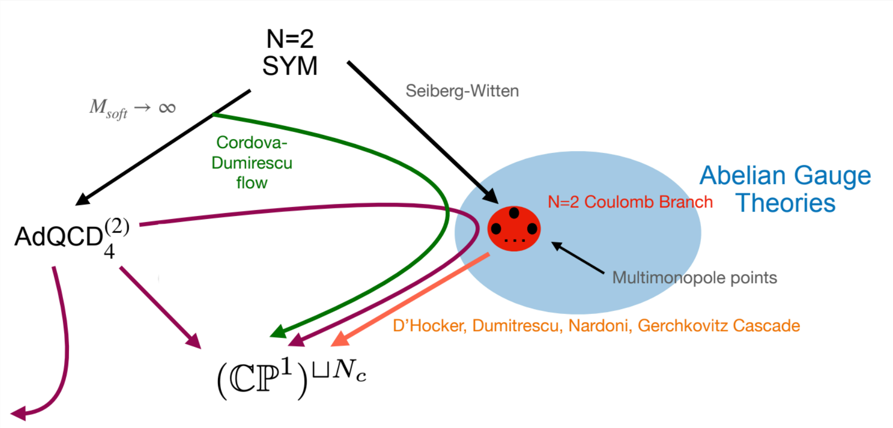

Figure 2: Different flows for $\mathcal{N} = 2$ with mass deformation.

Matching symmetries

In the $\mathbb{CP}^1$ sigma model we have:

A 1-form symmetry acting on vortices, $\pi_2(\mathbb{CP}^1) = \mathbb{Z}$, a $\mathbb{Z}_n$ subgroup can be matched with the 1-form symmetry of the abelian model.

A 0-form symmetry acting on Hopfions, $\pi_3(\mathbb{CP}^1) = \mathbb{Z}$, which is non-invertible and can match the anomalous 0-form symmetry of the abelian model [Hsin '22; Chen, Tanizaki '22].

Conclusions

Conclusions

We discussed generalized symmetries and how they work.

We applied these tools to a concrete example: AdjQCD.

What we learned is that non-invertible symmetries provides extra checks for dynamical abelianization.

These techniques are generale and can be applied to other flows.

Thank you for the attention!