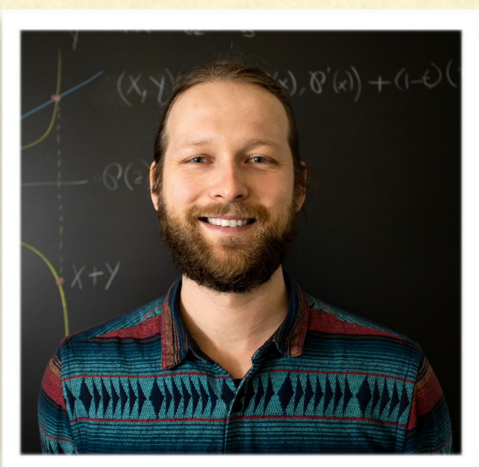
Regge spectroscopy of higher twist states in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory

Rob Klabbers, Michelangelo Preti, and István M. Szécsényi



PRL 132, 191601
(arXiv:2307.15107)

33rd Nordic Meetings on "Strings, Fields and Branes"

30th of October 2024

UNIMORE
UNIVERSITÀ DEGLI STUDI DI MODENA E REGGIO EMILIA

SIMONS CENTER FOR GEOMETRY AND PHYSICS

UNIVERSITÀ DI TORINO

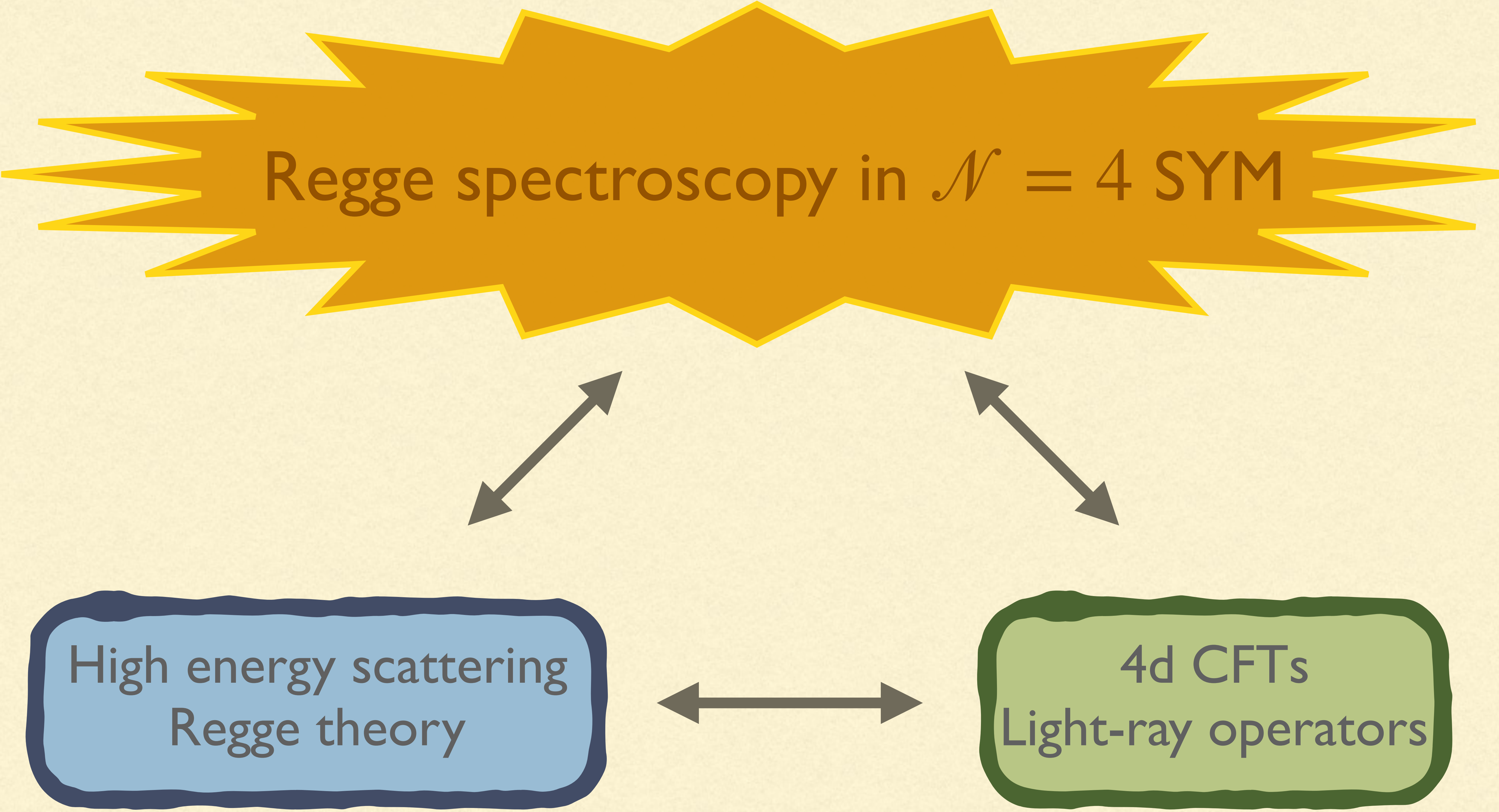
NORDITA
The Nordic Institute for Theoretical Physics

High energy scattering
Regge theory

High energy scattering
Regge theory

↔

4d CFTs
Light-ray operators


Regge spectroscopy in $\mathcal{N} = 4$ SYM
High energy scattering
Regge theory
4d CFTs
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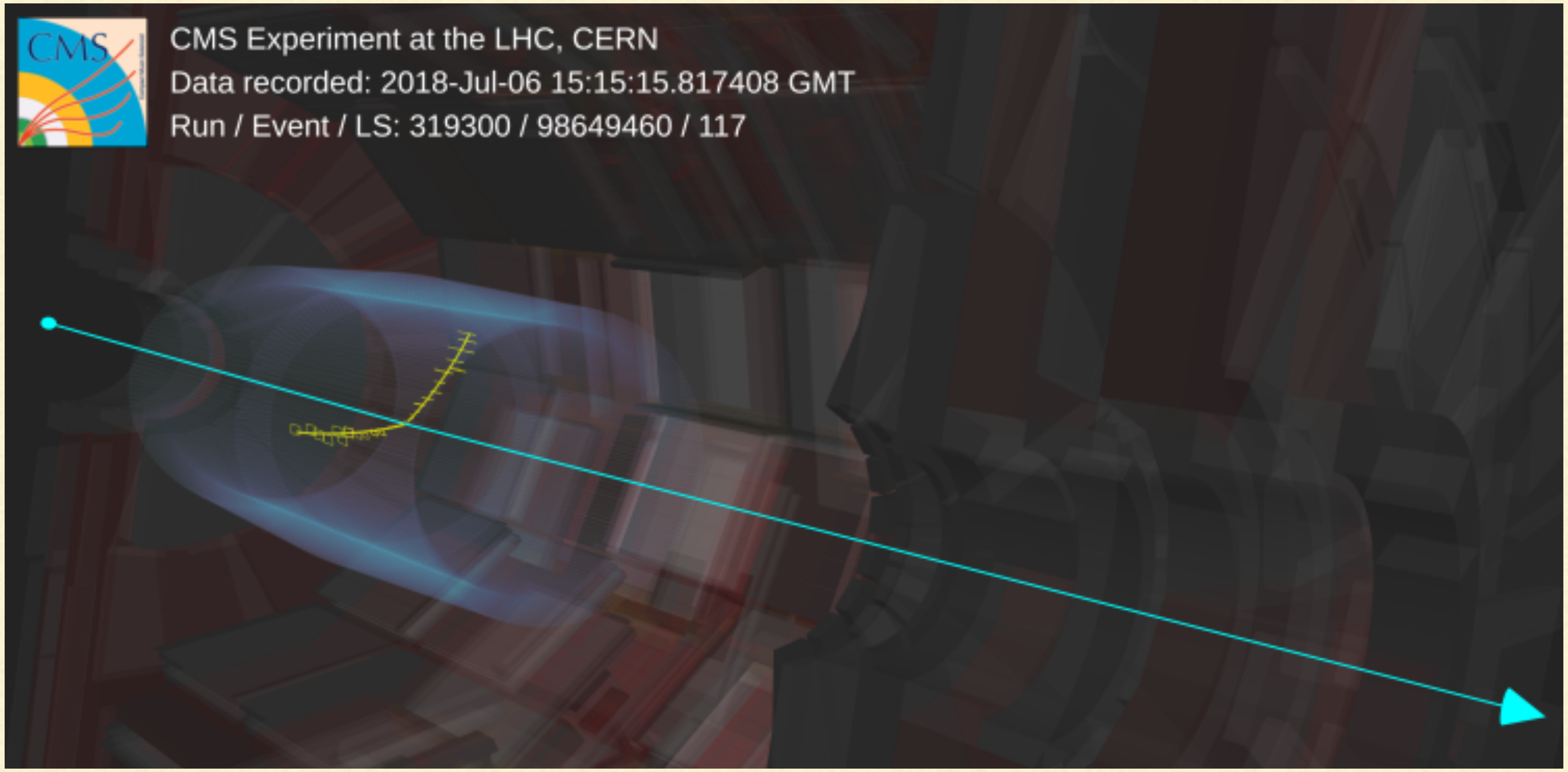
High energy scattering



[CMS Collaboration, "In search of the strong interaction: the pomeron" Phys Rev D, in press, '23]

Kinematic regime: $s \gg 1\,, \quad t < 0$

→ Power law behaviour

Regge theory

Tullio Regge

Regge theory

Tullio Regge

Gufram
(Radical period
furniture manufacturer)

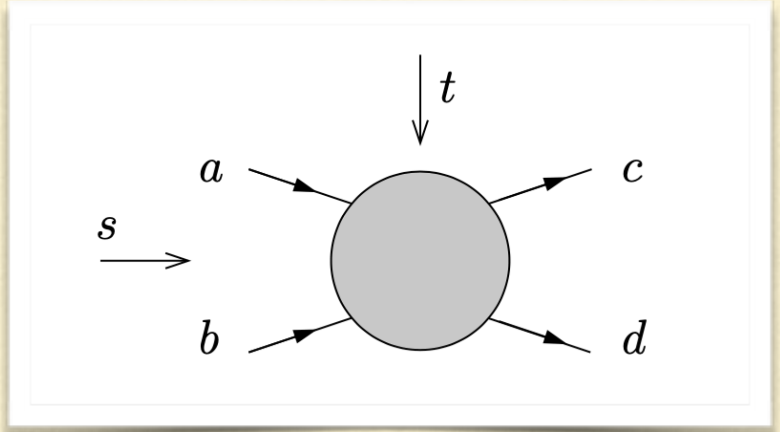
1968 Detecma seat

"transformed a mathematical quartic function into a volume with intentionally ergonomic characteristics"

Regge theory

Tullio Regge

[Regge, '59]

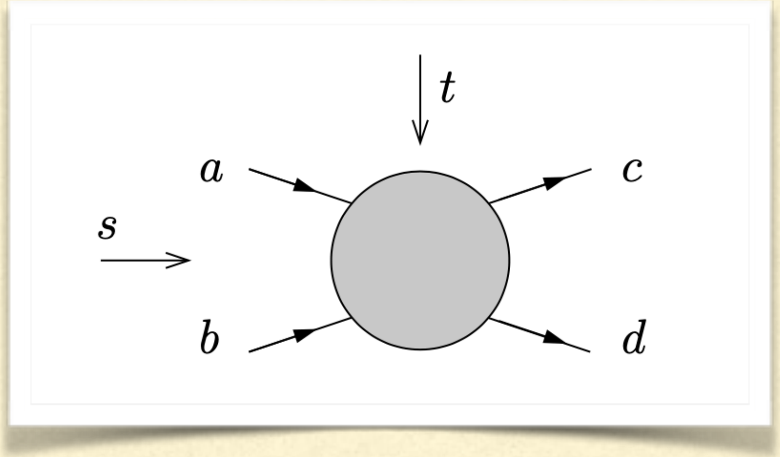


$$\mathscr{A}^{ab}_{ab}$$

Regge theory

Tullio Regge

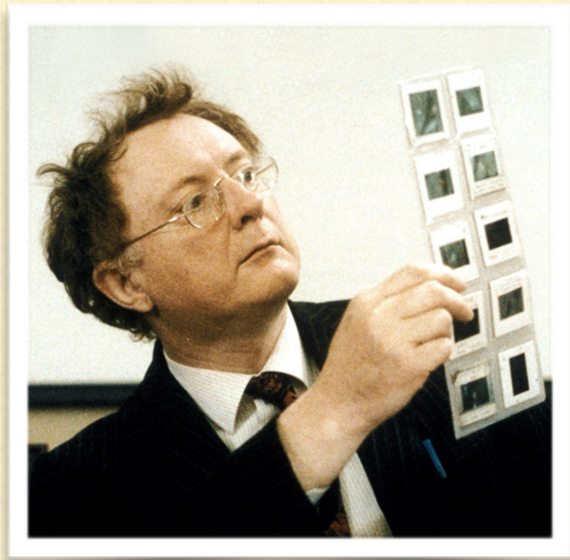
[Regge, '59]



$$\mathscr{A}^{ab}_{ab}$$

- Partial wave expansion
- Analytic continuation in spin

Regge theory



Tullio Regge

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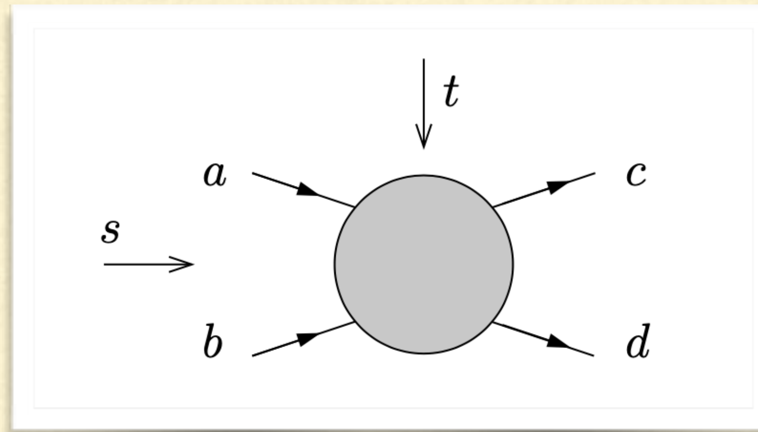

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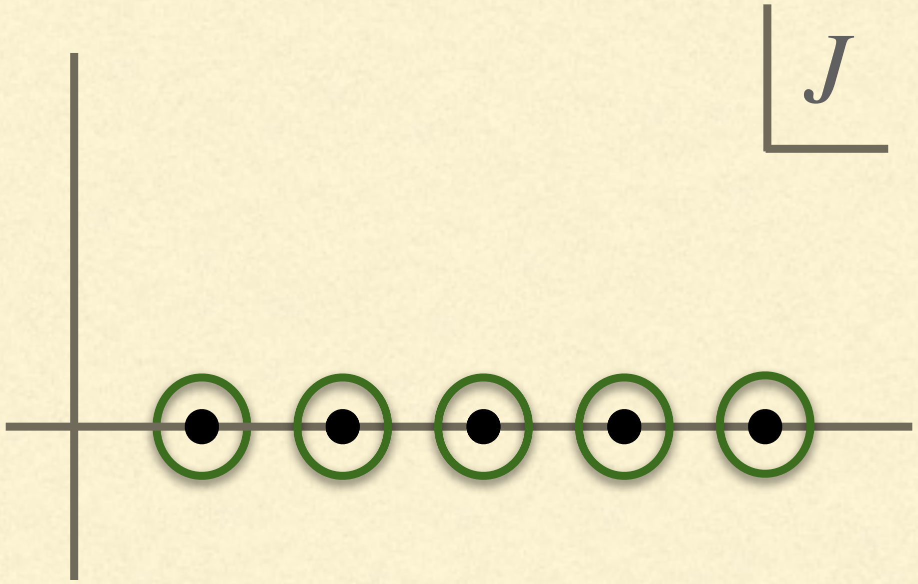
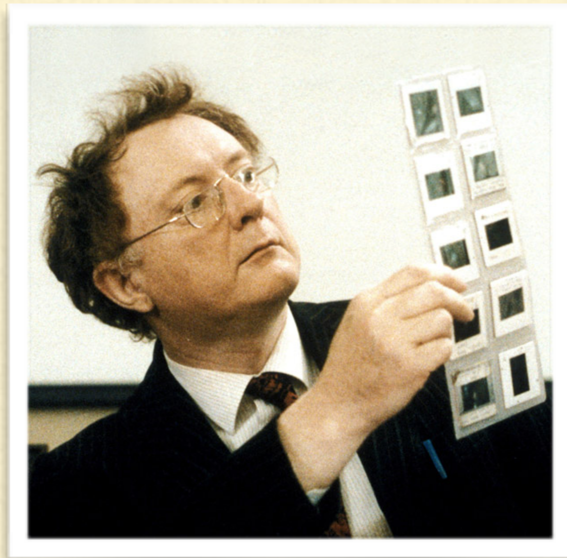
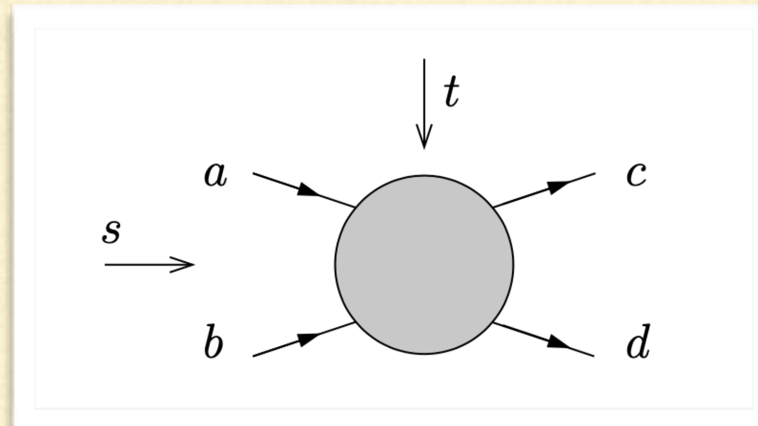


Regge theory



Tullio Regge

[Regge, '59]



$$\mathscr{A}^{ab}_{ab}$$

→ Partial wave expansion

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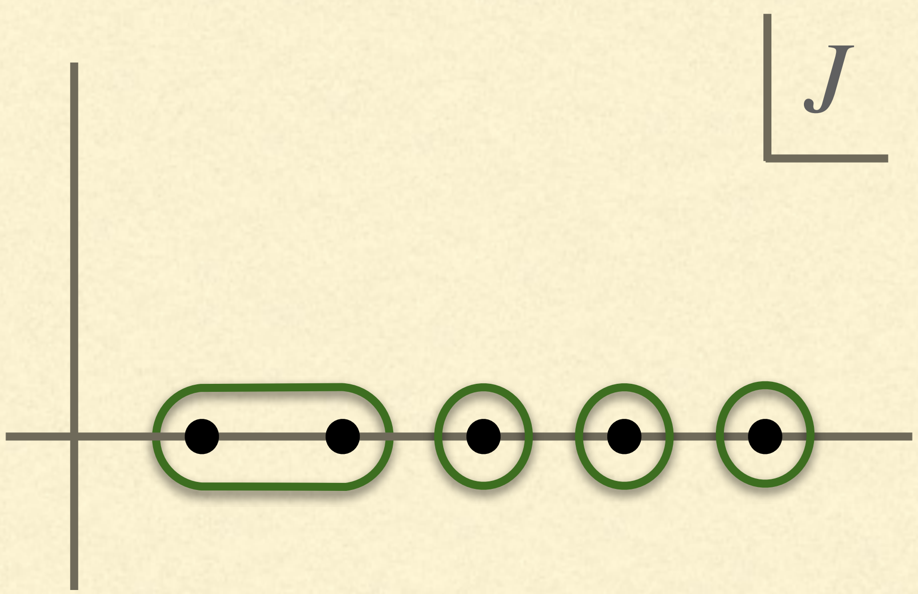
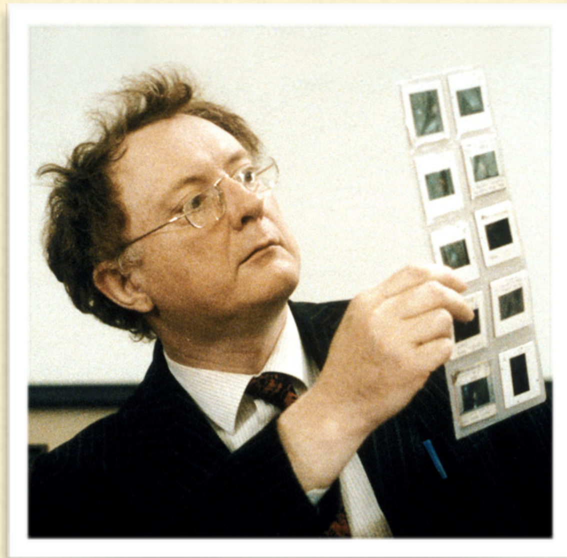


Regge theory



Tullio Regge

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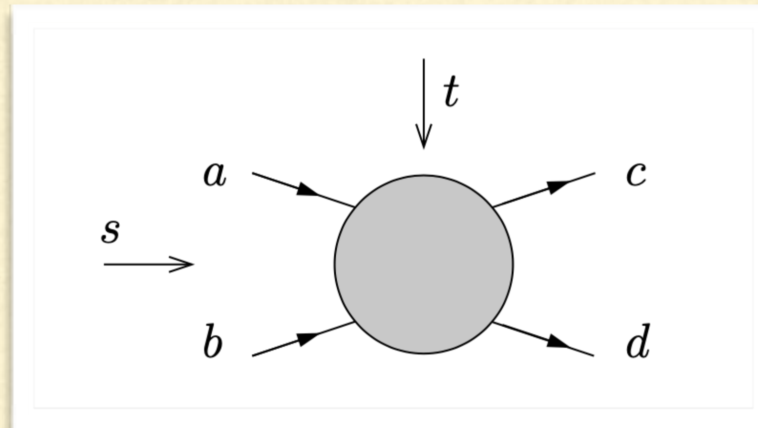
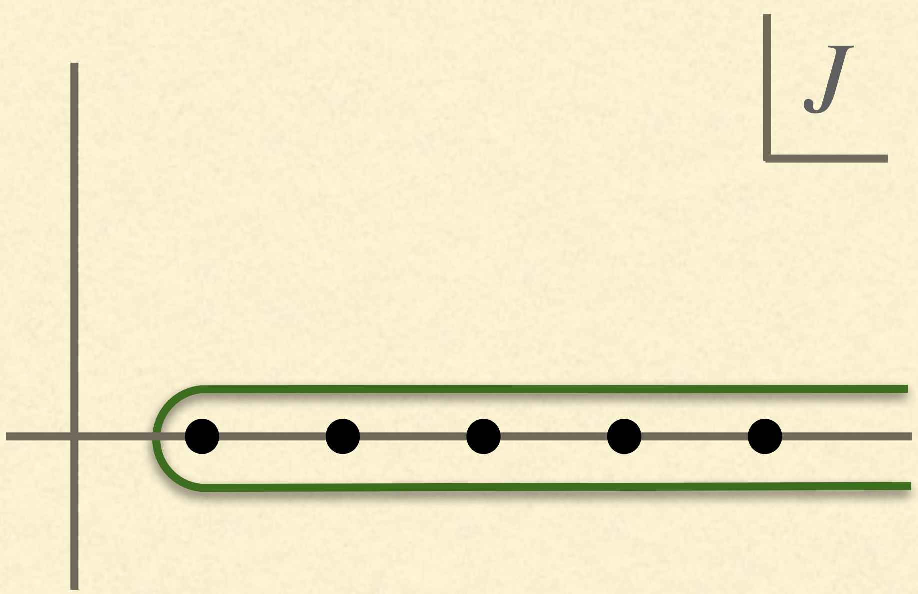


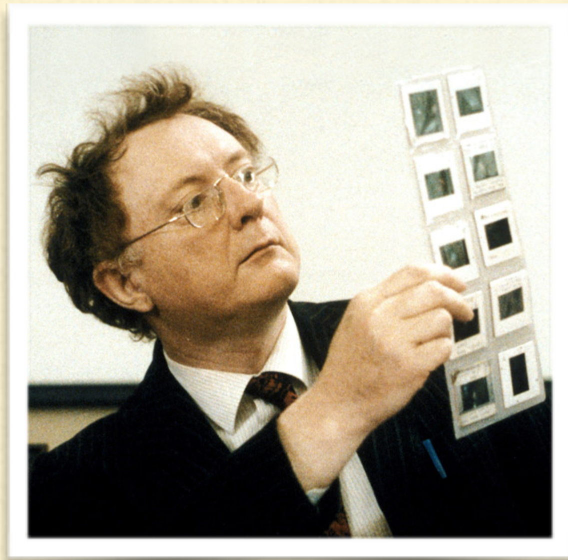
$$\mathscr{A}^{ab}_{ab}$$

→ Partial wave expansion

→ Analytic continuation in spin



Regge theory



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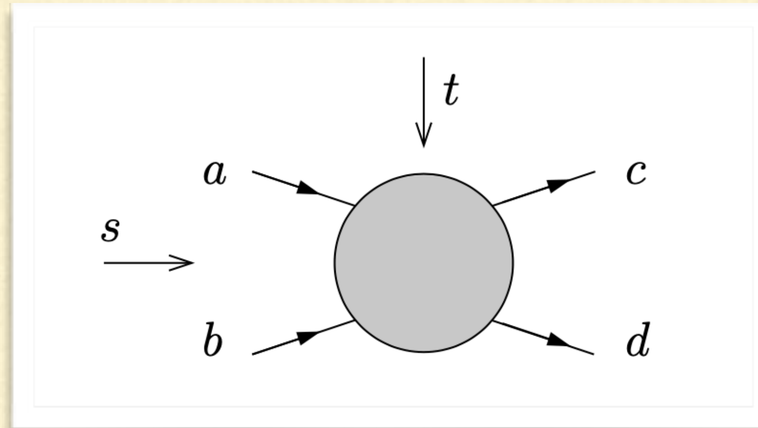

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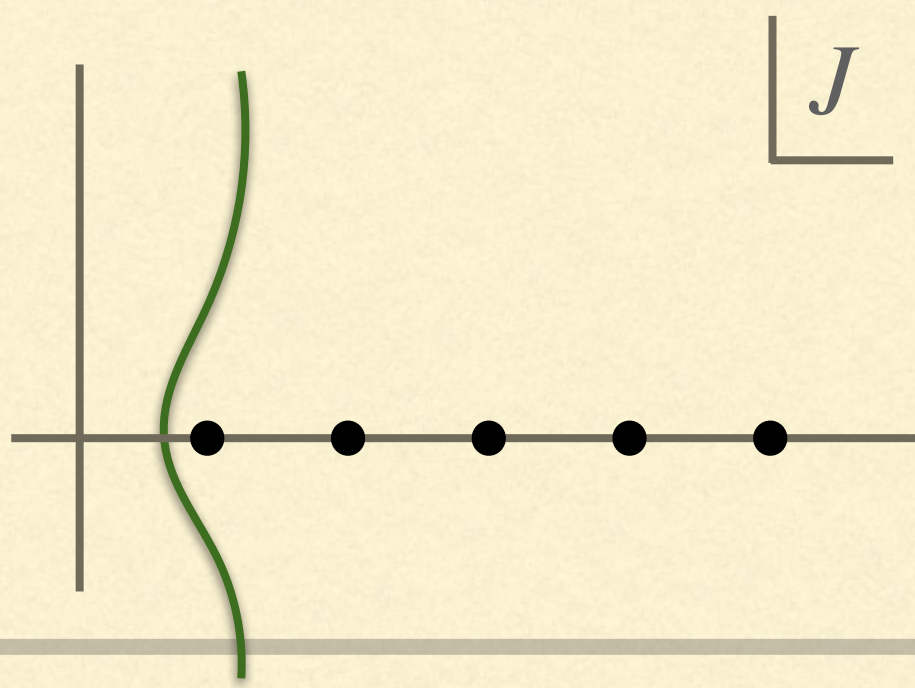
→ Partial wave expansion

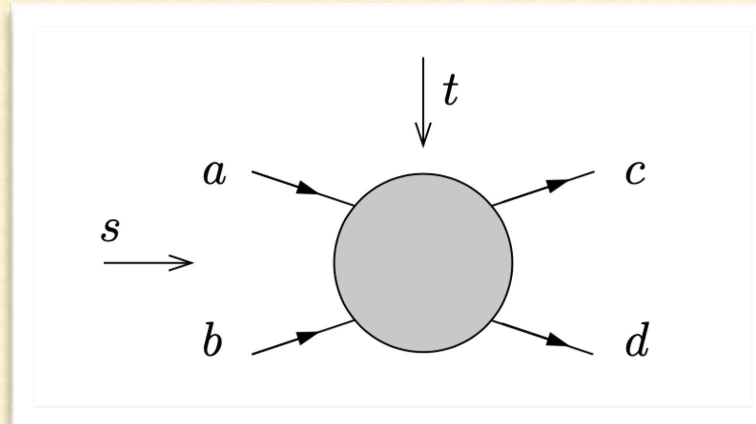
→ Analytic continuation in spin



Regge theory

Tullio Regge

[Regge, '59]



$$\mathscr{A}^{ab}_{ab}$$

- Partial wave expansion
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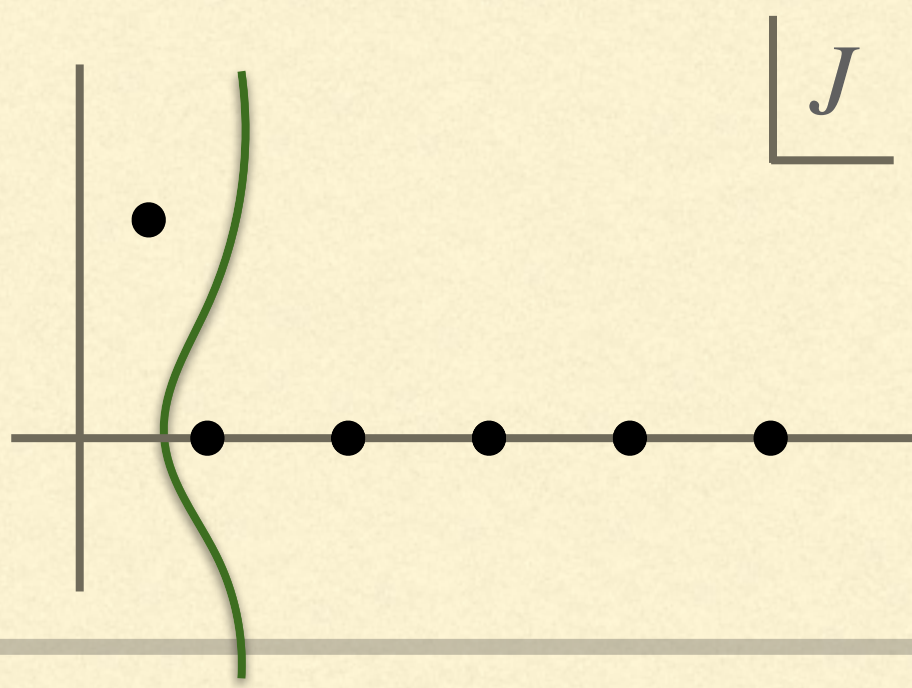
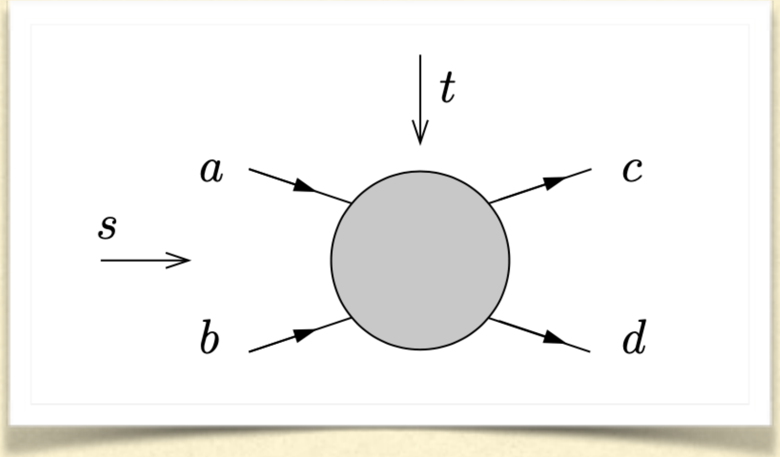


Regge theory

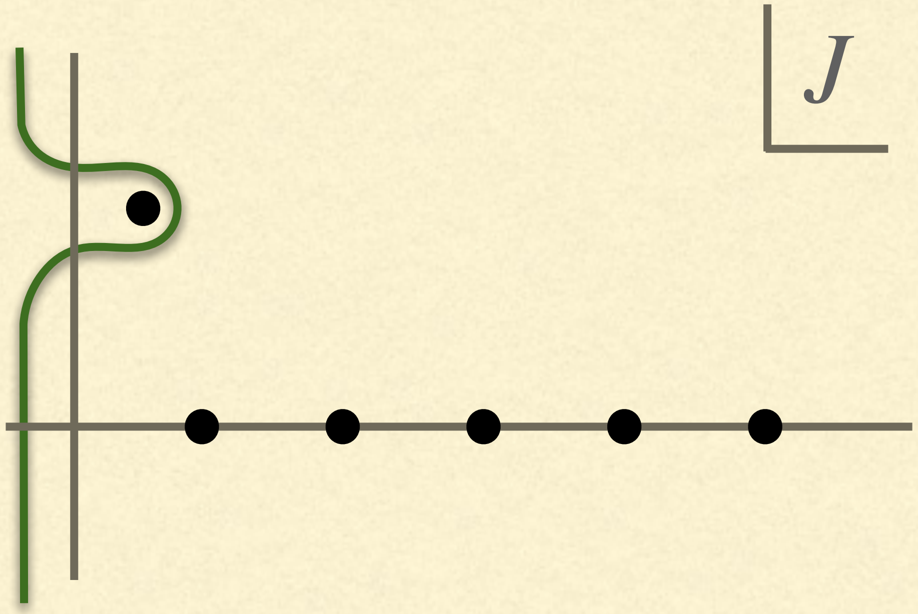
Tullio Regge

[Regge, '59]



$$\mathscr{A}_{ab}^{ab}$$

- Partial wave expansion
- Analytic continuation in spin

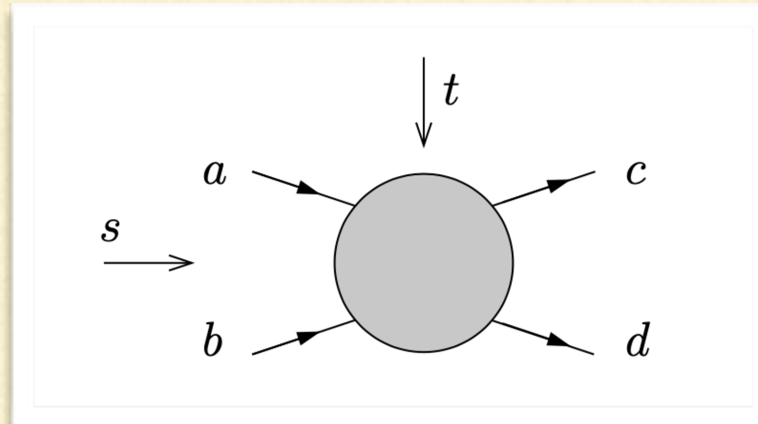


$$\mathscr{A}_{ab}^{ab} \sim s^{\alpha(t)}$$

Regge theory

Tullio Regge

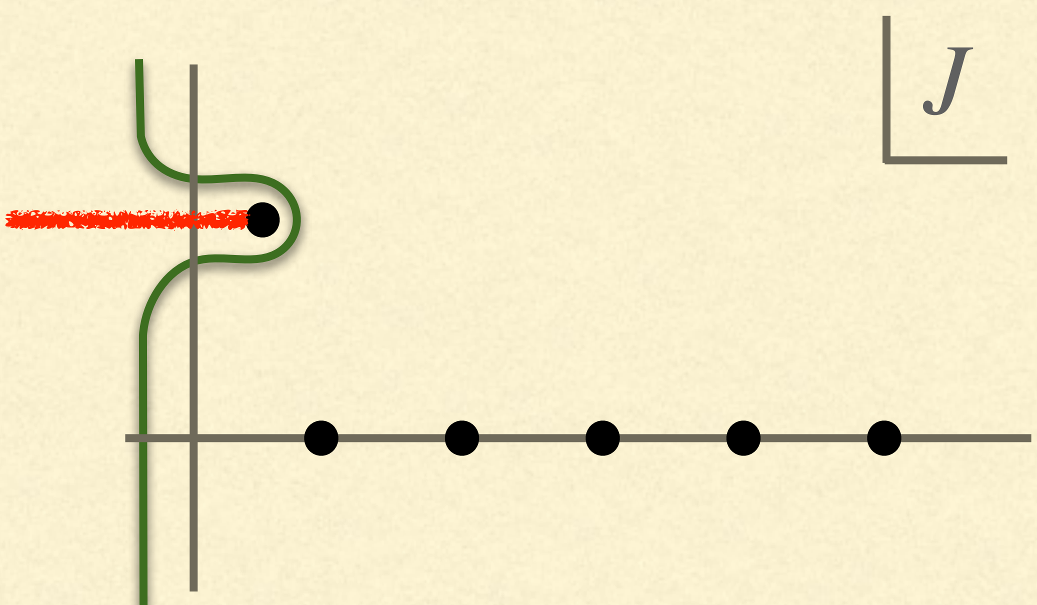
[Regge, '59]



$$\mathscr{A}_{ab}^{ab}$$

→ Partial wave expansion

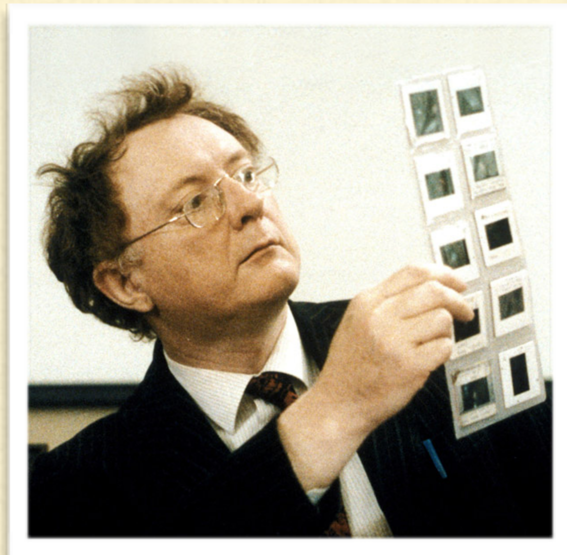
→ Analytic continuation in spin



$$\mathscr{A}_{ab}^{ab} \sim s^{\alpha(t)}$$

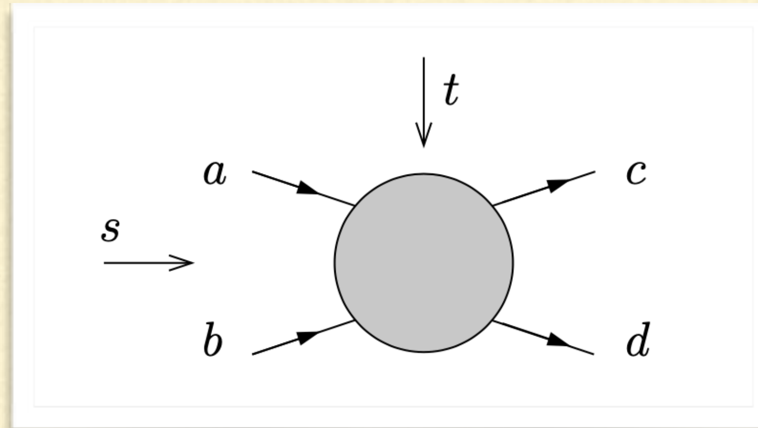
$$\mathscr{A}_{ab}^{ab} \sim s^{\alpha(t)} \log(s)^{-\gamma(t)}$$

Regge theory



Tullio Regge

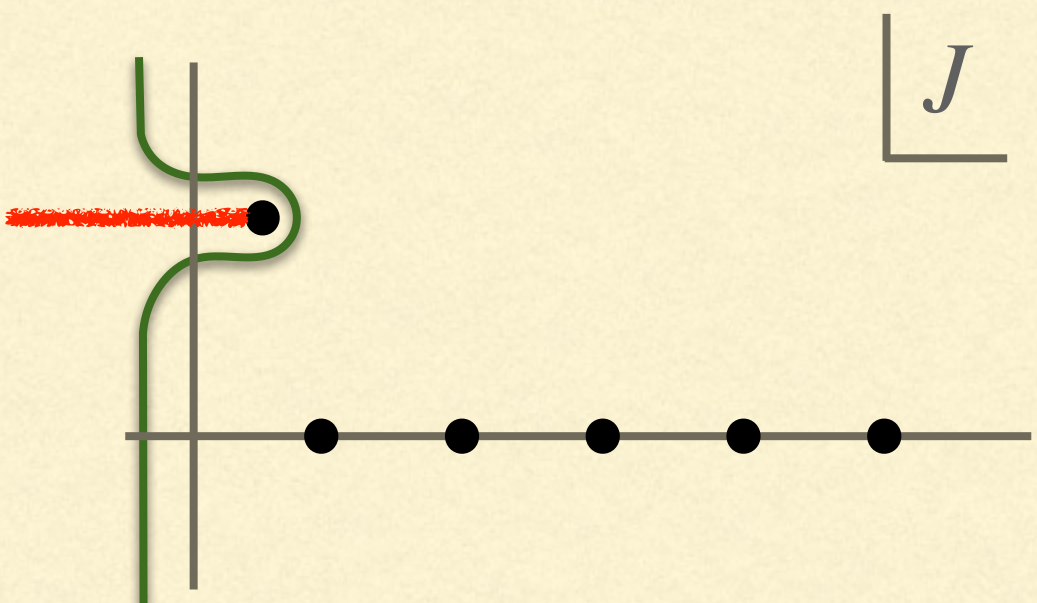
[Regge, '59]



$$\mathscr{A}_{ab}^{ab}$$

→ Partial wave expansion

→ Analytic continuation in spin



$$\mathscr{A}_{ab}^{ab} \sim s^{\alpha(t)}$$

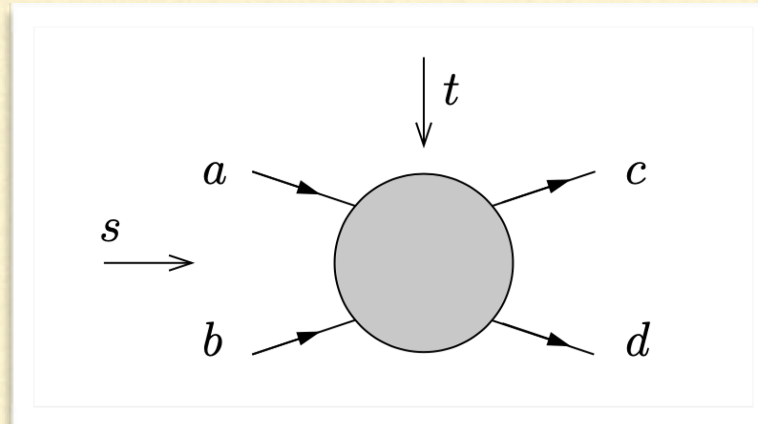
$$\mathscr{A}_{ab}^{ab} \sim s^{\alpha(t)} \log(s)^{-\gamma(t)}$$

$$\sigma_{\text{TOT}} \sim s^{\alpha(0)-1}$$

Regge theory

Tullio Regg

[Regge, '59]

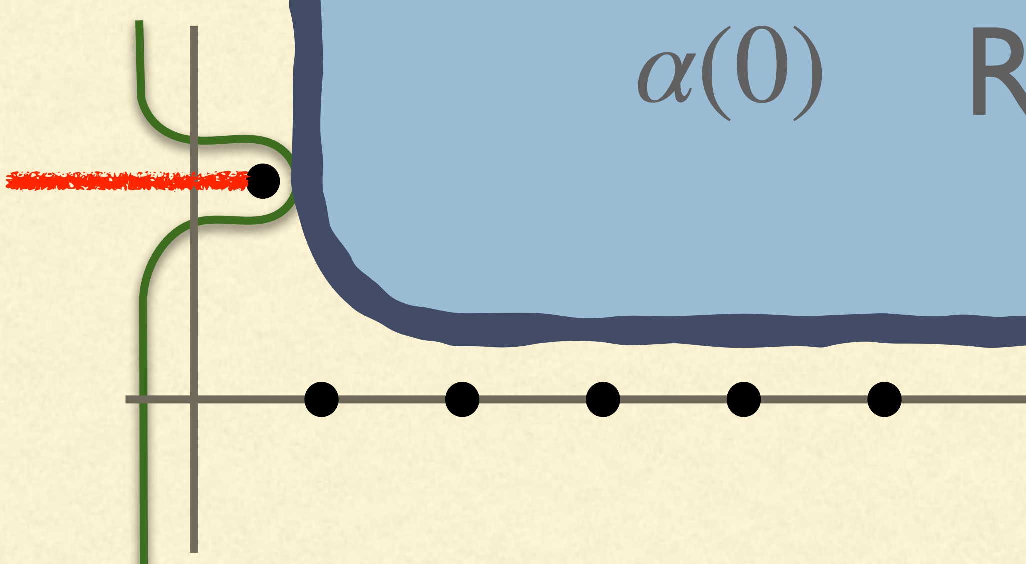


$\mathscr{A}^{ab}_{ab}$

$\alpha(t)$ Regge trajectory

$\alpha(0)$ Regge intercept

spin

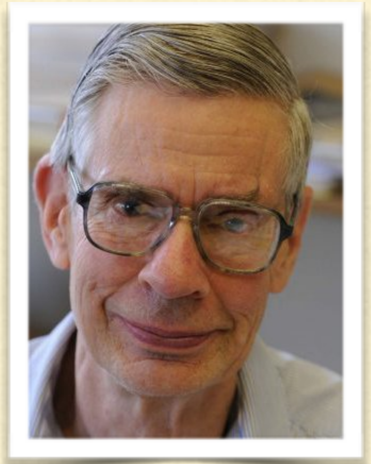


$\gamma(t)$

$\sigma_{\mathrm{TOT}} \sim s^{\alpha(0)-1}$

Chew-Frautschi plot

Geoffrey Chew



Steven Frautschi

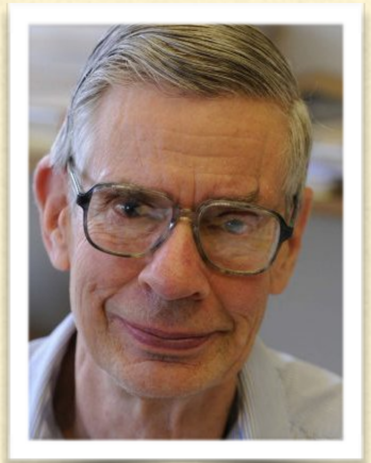
[Chew, Frautschi, ’61]

For $t > 0, \quad \alpha(t_*) \in \mathbb{Z}$

→ Resonances

Chew-Frautschi plot

Geoffrey Chew

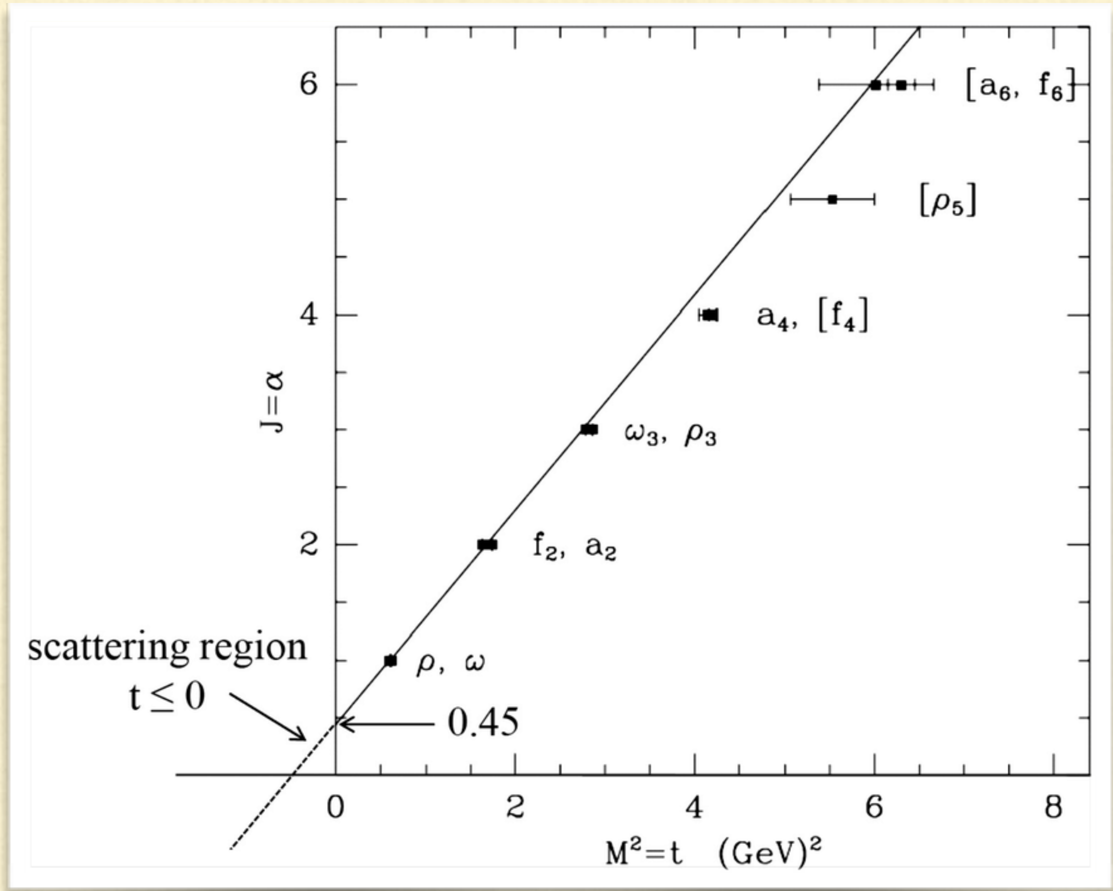


Steven Frautschi

[Chew, Frautschi, '61]

For $t > 0, \quad \alpha(t_*) \in \mathbb{Z}$

⟶ Resonances



[Amaldi, "60 years of CERN experiments . . . ", '15]

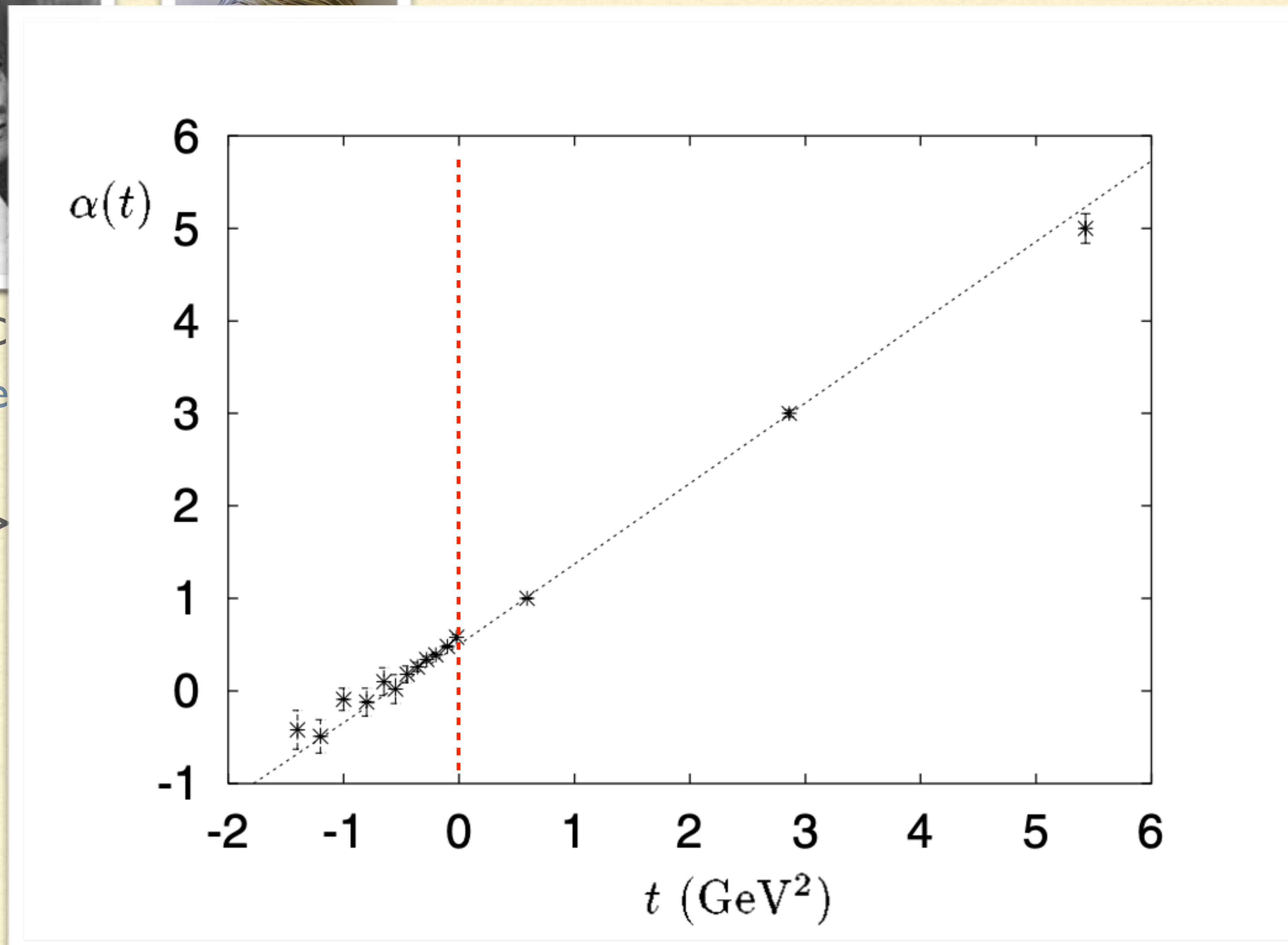
$$\alpha(t) = 0.45 + 0.9t$$

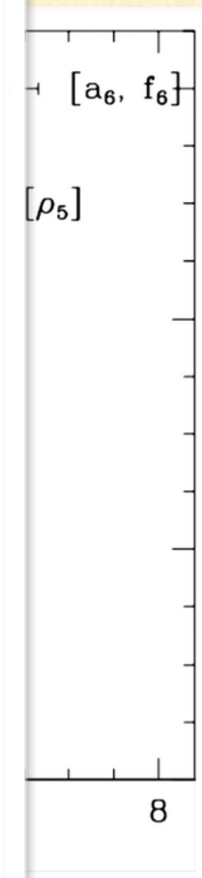
Chew-Frautschi plot

Geoffrey C

[Che

For $t >$





…”, ’15]

[Donnachie, Dosch, Landshoff, Nachtmann, “Pomeron Physics and QCD”, ’02]

Pomeron

Isaak Pomeranchuk

[Pomeranchuk '61]

$$\frac{\sigma^{pp}_{\mathrm{TOT}}}{\sigma^{p\bar{p}}_{\mathrm{TOT}}} \xrightarrow{s \to \infty} 1$$

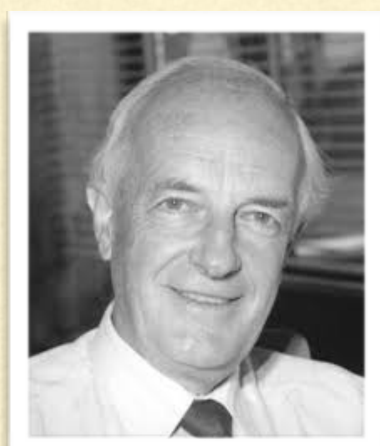
Pomeron

Isaak Pomeranchuk

[Pomeranchuk '61]

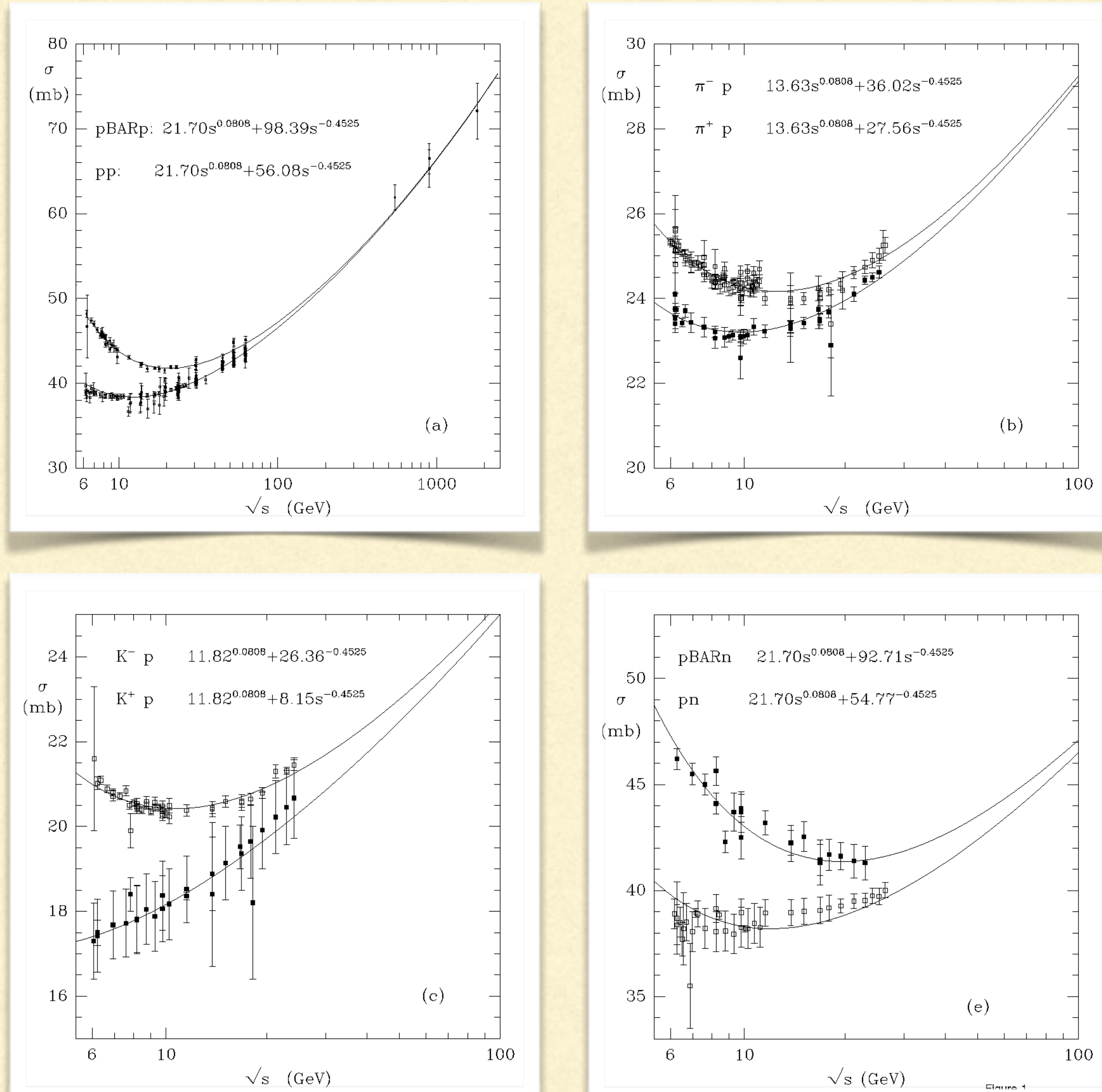
$$\frac{\sigma^{pp}_{\mathrm{TOT}}}{\sigma^{p\bar{p}}_{\mathrm{TOT}}} \xrightarrow{s \to \infty} 1$$

[Donnacchie, Landshoff, '92]



Sandy Donnacchie

Peter Landshoff



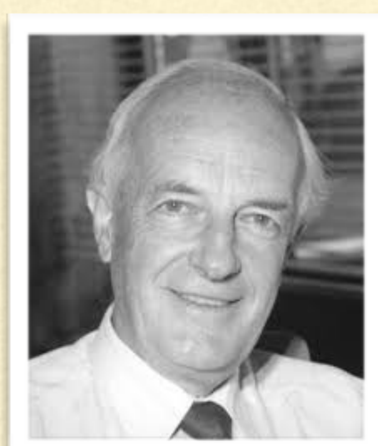
Pomeron

Isaak Pomeranchuk

[Pomeranchuk '61]

$$\frac{\sigma^{pp}_{\mathrm{TOT}}}{\sigma^{p\bar{p}}_{\mathrm{TOT}}} \xrightarrow{s \to \infty} 1$$

[Donnacchie, Landshoff, '92]



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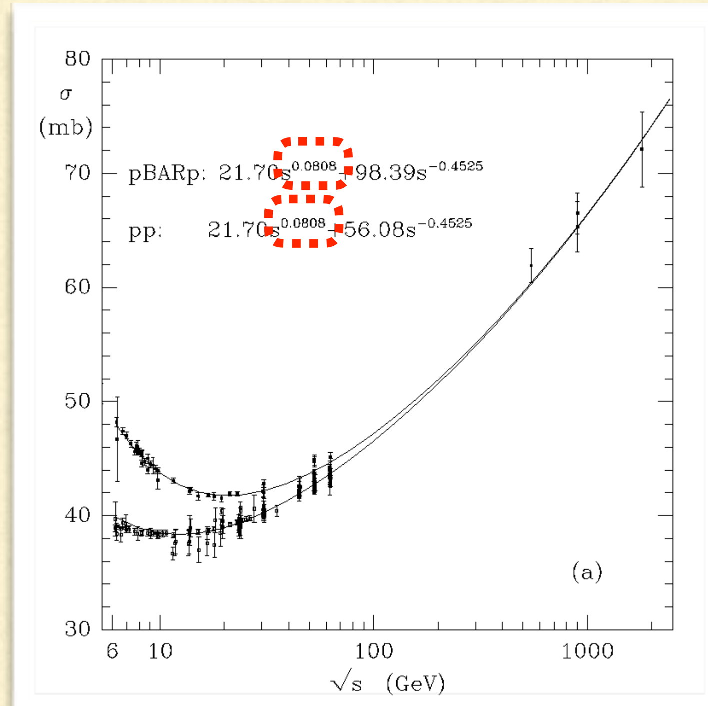
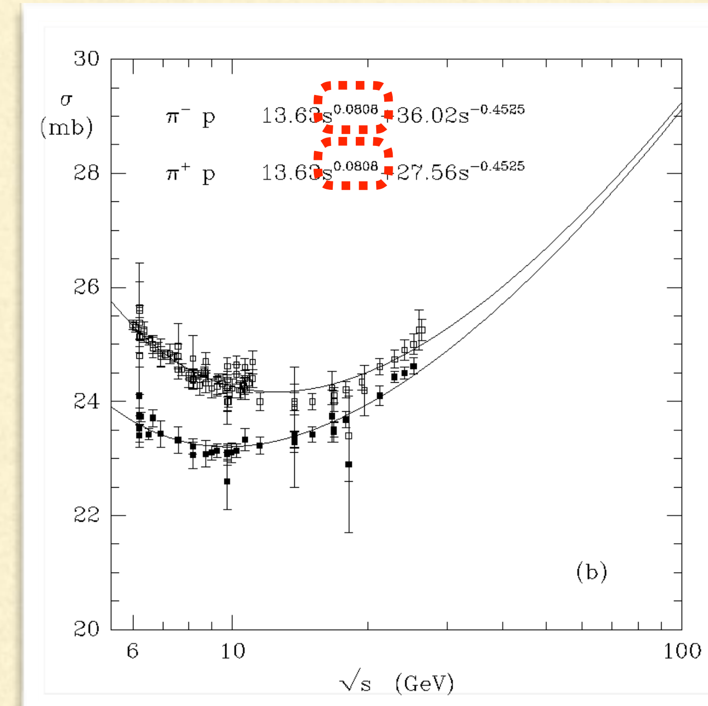
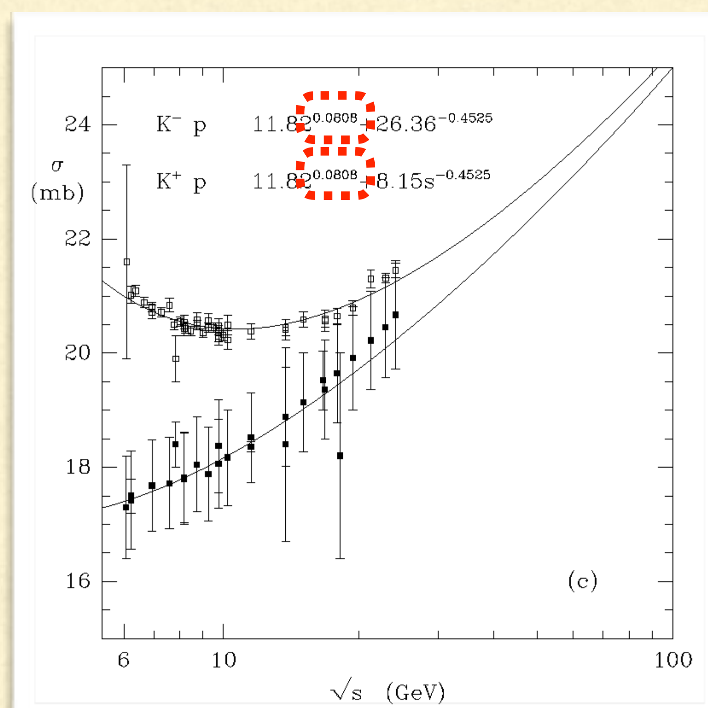
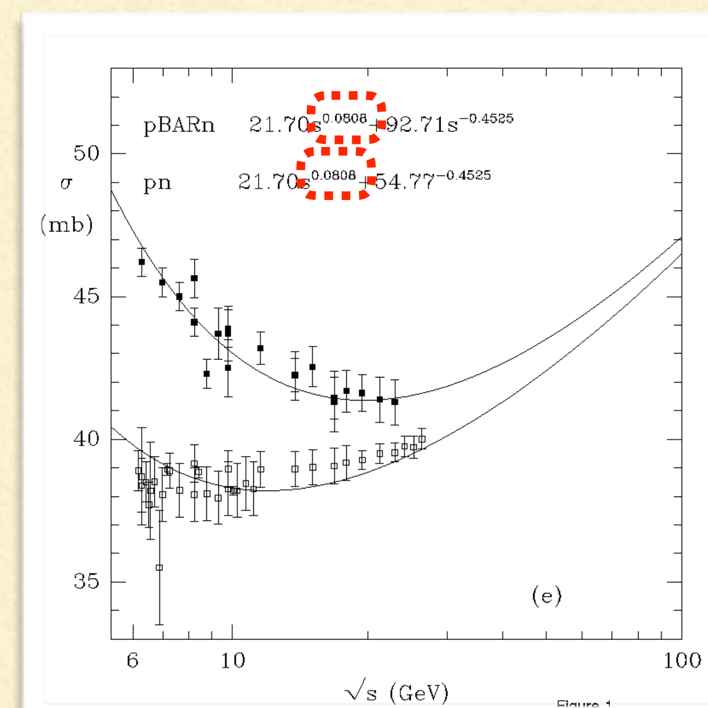






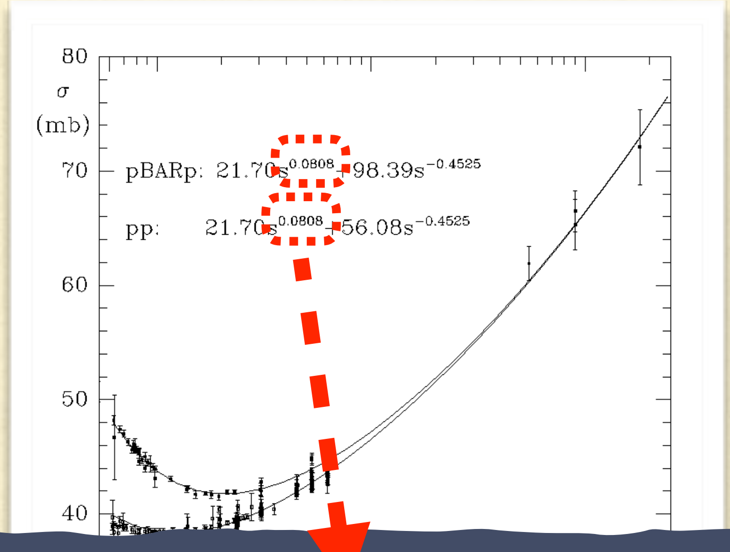


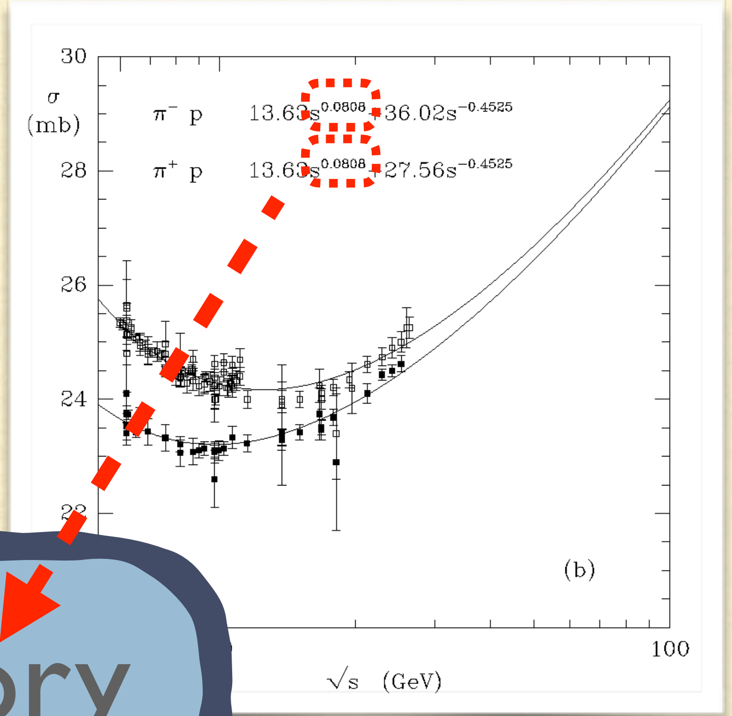
Pomeron

Isaak Pomeranchuk

[Pomeranchuk '61]

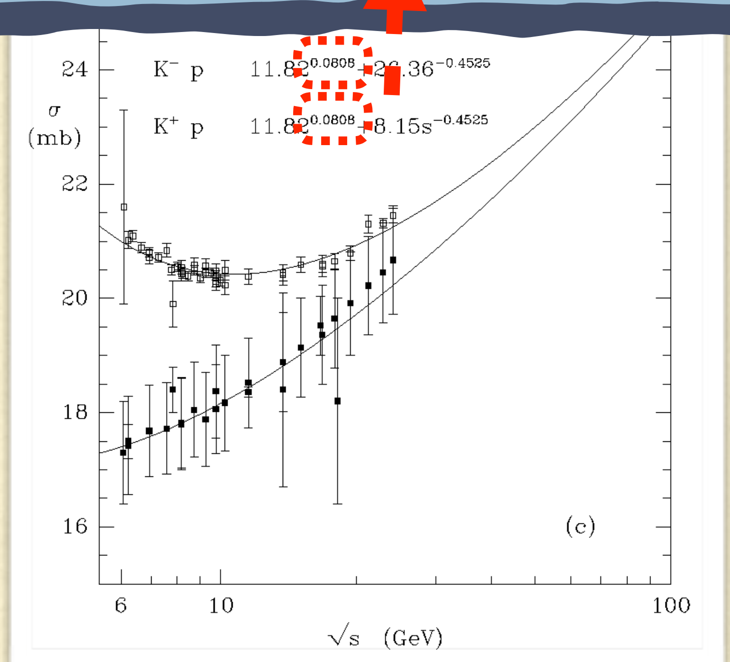
$$\frac{\sigma_{\rm TOT}^{pp}}{\sigma_{\rm TOT}^{p\bar{p}}} \to 1 \qquad s \to \infty$$

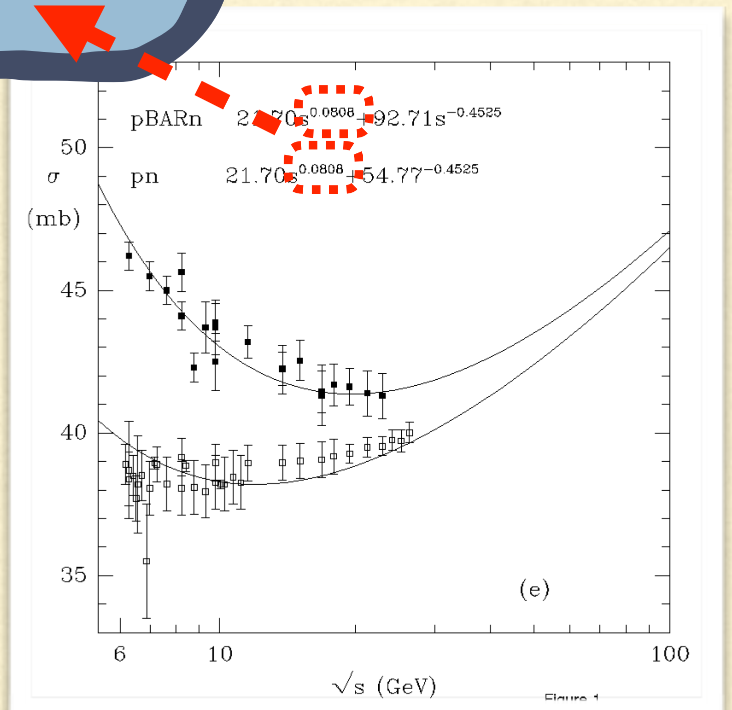


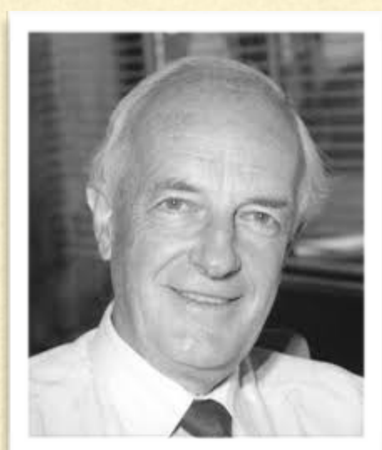


$\alpha(t) = 1.08 + 0.25t$ Pomeron trajectory

[Donnacchie, Landshoff, '92]







Sandy Donnacchie

Peter Landshoff

Odderon

$$\lim_{s \to \infty} \left(\sigma_{\text{TOT}}^{pp} - \sigma_{\text{TOT}}^{p\bar{p}} \right) = ?$$

Odderon

$$\lim_{s\to\infty}\left(\sigma^{pp}_{\mathrm{TOT}}-\sigma^{p\bar{p}}_{\mathrm{TOT}}\right)=\,?$$

Odderon: hypothetical trajectory with $C=-1$

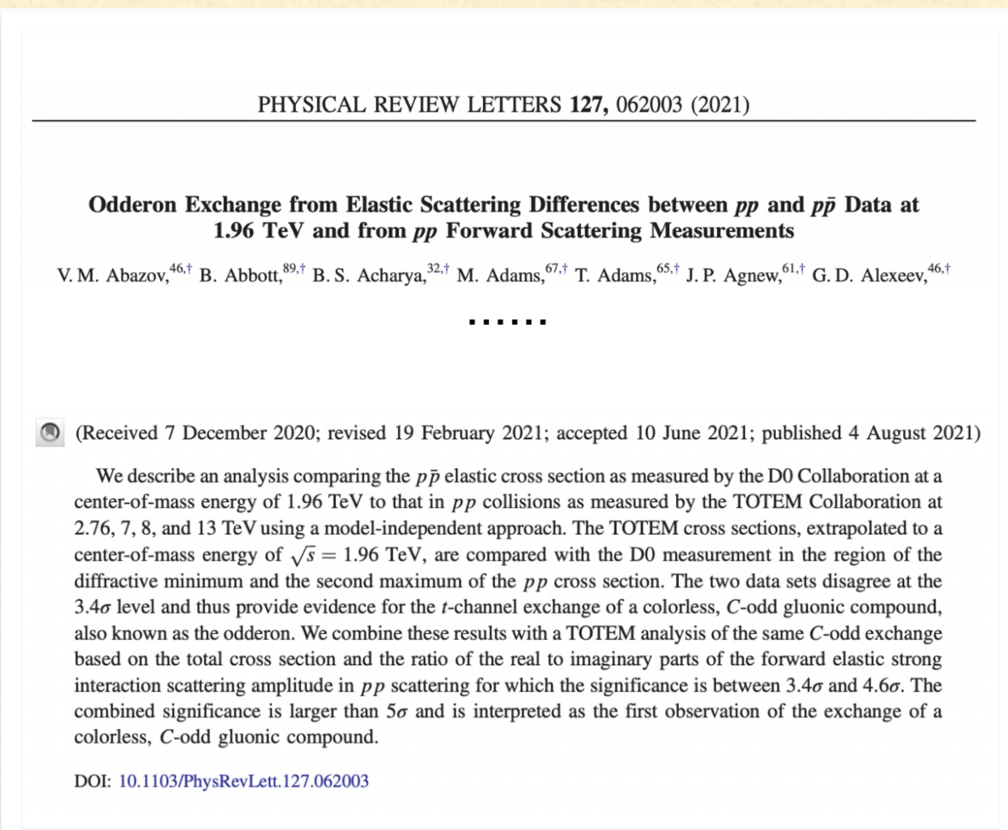
[Łukaszuk, Nicolescu, ’73]

Odderon

$$\lim_{s\to\infty}\left(\sigma^{pp}_{\mathrm{TOT}}-\sigma^{p\bar{p}}_{\mathrm{TOT}}\right)=\,?$$

Odderon: hypothetical trajectory with $C = -1$

[Łukaszuk, Nicolescu, '73]


PHYSICAL REVIEW LETTERS **127,** 062003 (2021)

Odderon Exchange from Elastic Scattering Differences between pp and $p\bar{p}$ Data at 1.96 TeV and from pp Forward Scattering Measurements

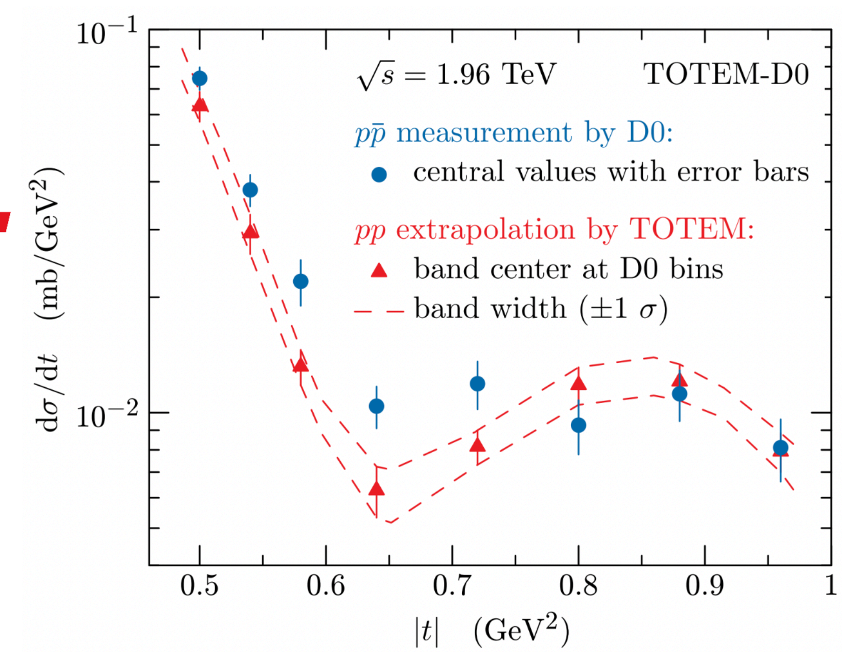
V. M. Abazov,[46,†] B. Abbott,[89,†] B. S. Acharya,[32,†] M. Adams,[67,†] T. Adams,[65,†] J. P. Agnew,[61,†] G. D. Alexeev,[46,†]

……

(Received 7 December 2020; revised 19 February 2021; accepted 10 June 2021; published 4 August 2021)

We describe an analysis comparing the $p\bar{p}$ elastic cross section as measured by the D0 Collaboration at a center-of-mass energy of 1.96 TeV to that in pp collisions as measured by the TOTEM Collaboration at 2.76, 7, 8, and 13 TeV using a model-independent approach. The TOTEM cross sections, extrapolated to a center-of-mass energy of $\sqrt{s} = 1.96$ TeV, are compared with the D0 measurement in the region of the diffractive minimum and the second maximum of the pp cross section. The two data sets disagree at the 3.4σ level and thus provide evidence for the t-channel exchange of a colorless, C-odd gluonic compound, also known as the odderon. We combine these results with a TOTEM analysis of the same C-odd exchange based on the total cross section and the ratio of the real to imaginary parts of the forward elastic strong interaction scattering amplitude in pp scattering for which the significance is between 3.4σ and 4.6σ. The combined significance is larger than 5σ and is interpreted as the first observation of the exchange of a colorless, C-odd gluonic compound.

DOI: 10.1103/PhysRevLett.127.062003



[Abazov et al. (TOTEM, D0) '21]

Odderon

$$\lim_{s\to\infty}\left(\sigma^{pp}_{\mathrm{TOT}}-\sigma^{p\bar{p}}_{\mathrm{TOT}}\right)=\,?$$

Odderon: hypothetical trajectory with $C = -1$

PHYSICAL RE

Odderon Exchange from Elasti
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V. M. Abazov,[46,†] B. Abbott,[89,†] B. S. Achar

(Received 7 December 2020; revised 1

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DOI: 10.1103/PhysRevLett.127.062003

Physics Letters B 831 (2022) 137199

Contents lists available at ScienceDirect

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ELSEVIER

Lack of evidence for an odderon at small t

A. Donnachie[a], P.V. Landshoff[b,*]

[a] University of Manchester, United Kingdom of Great Britain and Northern Ireland
[b] University of Cambridge, United Kingdom of Great Britain and Northern Ireland

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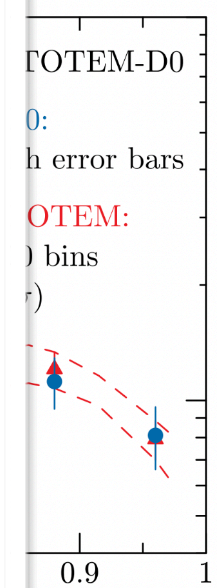
ABSTRACT

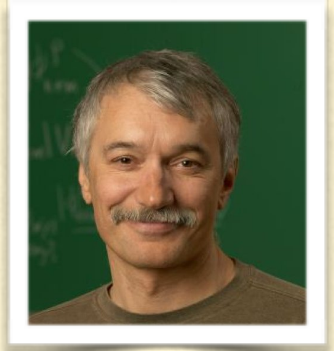
It is fundamental that the phase of an elastic scattering amplitude is related to its energy variation. We repeat a previous fit to pp and $p\bar{p}$ elastic scattering data from 13 to 13000 GeV, taking better account of the very high accuracy of the 13 TeV data. The conclusion remains that there is no evidence for the existence of an odderon in the small-t data.



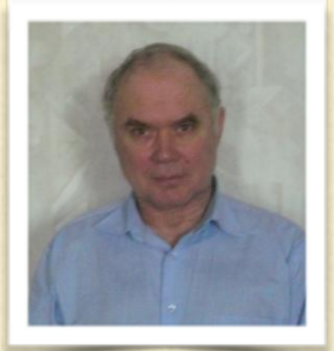
[Abazov et al. (TOTEM, D0) '21]

Reggeons in high-energy QCD

In the Regge limit QCD is conformal



Ian Balitsky



Victor S. Fadin

Eduard A. Kuraev

Lev N. Lipatov

BFKL Pomeron

[Balitsky, Lipatov, Fadin, Kuraev ’76,’77]

Connection to integrability

[Faddeev, Korchemsky ’95], [Korchemsky ’95]

Odderon: Janik-Wosiek solution

[Janik, Wosiek ’99]

Bartels-Lipatov-Vacca Solution

[Bartels, Lipatov, Vacca ’99]

Conformal Regge theory

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Conformal Field Theory → No massive particles
No asymptotic states

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No asymptotic states

? ? ?

What is scattering in CFT?

? ? ?

Conformal Regge theory

[Costa, Goncalves, Penedones '12]

Massive QFT

CFT

Conformal Regge theory

[Costa, Goncalves, Penedones '12]

Massive QFT

CFT

$$\mathscr{A}_{ab}^{ab}$$

Conformal Regge theory

[Costa, Goncalves, Penedones '12]

Massive QFT | CFT

$\mathscr{A}_{ab}^{ab} \longrightarrow \langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle$

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Regge limit:

$s \gg 1, t < 0$

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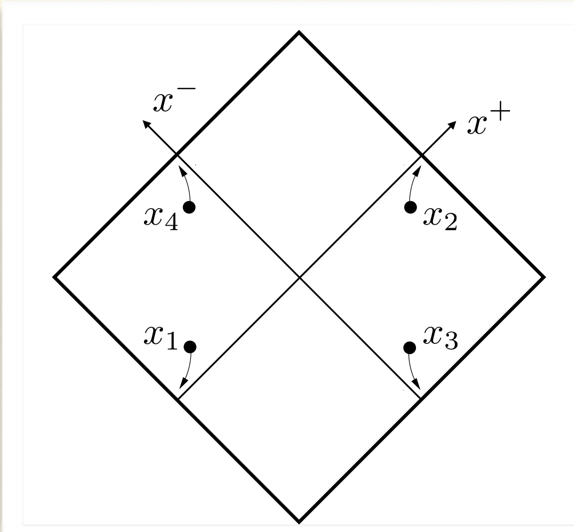
Regge limit:

$s \gg 1, t < 0$

→



Conformal Regge theory

[Costa, Goncalves, Penedones '12]

Massive QFT

CFT

t

Conformal Regge theory

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Massive QFT		CFT
t	$\longrightarrow$	Δ

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Conformal Regge theory

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t	$\longrightarrow$	Δ
Bound states: m_*^2	$\longrightarrow$	Local operators: Δ_i
Trajectory: $\alpha(t)$	$\longrightarrow$	$S(\Delta)$

Conformal Regge theory

[Costa, Goncalves, Penedones '12]

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t	$\longrightarrow$	Δ
Bound states: m_*^2	$\longrightarrow$	Local operators: Δ_i
Trajectory: $\alpha(t)$	$\longrightarrow$	$S(\Delta)$
Intercept: $\alpha(0)$		

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Bound states: m_*^2	$\longrightarrow$	Local operators: Δ_i
Trajectory: $\alpha(t)$	$\longrightarrow$	$S(\Delta)$
Intercept: $\alpha(0)$	$\longrightarrow$	Minima of $S(\Delta)$ $S_0 = S(\Delta = 0)$

Conformal Regge theory

[Costa, Goncalves, Penedones '12]

Massive QFT

CFT

$$\mathscr{A}^{ab}_{ab} \sim s^{\alpha(t)} \quad \longrightarrow \quad \frac{\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle}{\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \langle \mathcal{O}_3 \mathcal{O}_3 \rangle} \sim 1 + \sum_{\mathbb{O}} C_{\mathcal{O}_1 \mathcal{O}_2 \mathbb{O}}(S_0) C_{\mathcal{O}_3 \mathcal{O}_4 \mathbb{O}}(S_0) e^{t(S_0 - 1)}$$

[Caron-Huot '17]
[Kravchuk, Simmons-Duffin '18]
[Caron-Huot, Kologlu, Kravchuk, Meltzer Simmons-Duffin '22]
[Balitsky, Radyushkin '97]
[Balitsky, Kazakov, Sobko '13]

Conformal Regge theory

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Light-ray operators

[Caron-Huot '17]
[Kravchuk, Simmons-Duffin '18]
[Caron-Huot, Kologlu, Kravchuk, Meltzer Simmons-Duffin '22]
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$\mathcal{N} = 4$ supersymmetric Yang-Mills

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4D super-conformal theory

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Integrable in the planar limit

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Maximal transcendentality
[Kotikov, Lipatov, '01]

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} → Light-ray operators

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→ Light-ray operators

→ QCD Reggeons

$PSU(2,2|4)$ symmetry: $(J_1, J_2, J_3 | \Delta, S, S_2)$

$\Delta = \tau + S + \gamma(S)$

$$\gamma(S) = \sum_{i=1}^{\infty} \gamma_n(S) g^{2n}$$

$\mathcal{N} = 4$ supersymmetric Yang-Mills

4D super-conformal theory

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τ → Twist

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1. Fix τ and $(J_1, J_2, J_3 \,|\, S_2)$
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2. Analytical continuation (QSC)
   $\Delta(S) \leftrightarrow S(\Delta)$

$\mathcal{N} = 4$ supersymmetric Yang-Mills

Strategy:

1. Fix τ and $(J_1, J_2, J_3 \,|\, S_2)$
   + other quantum numbers

2. Analytical continuation (QSC)
   $\Delta(S) \leftrightarrow S(\Delta)$

3. Analyse around the intercept

Quantum Spectral Curve

$\mathbf{P}_a(u), \mathbf{Q}_i(u) \quad a, i = 1,\ldots 4$

[Gromov, Kazakov, Leurent, Volin '13]
[Alfimov, Gromov, Sizov '18]
[Marboe, Volin '18], …

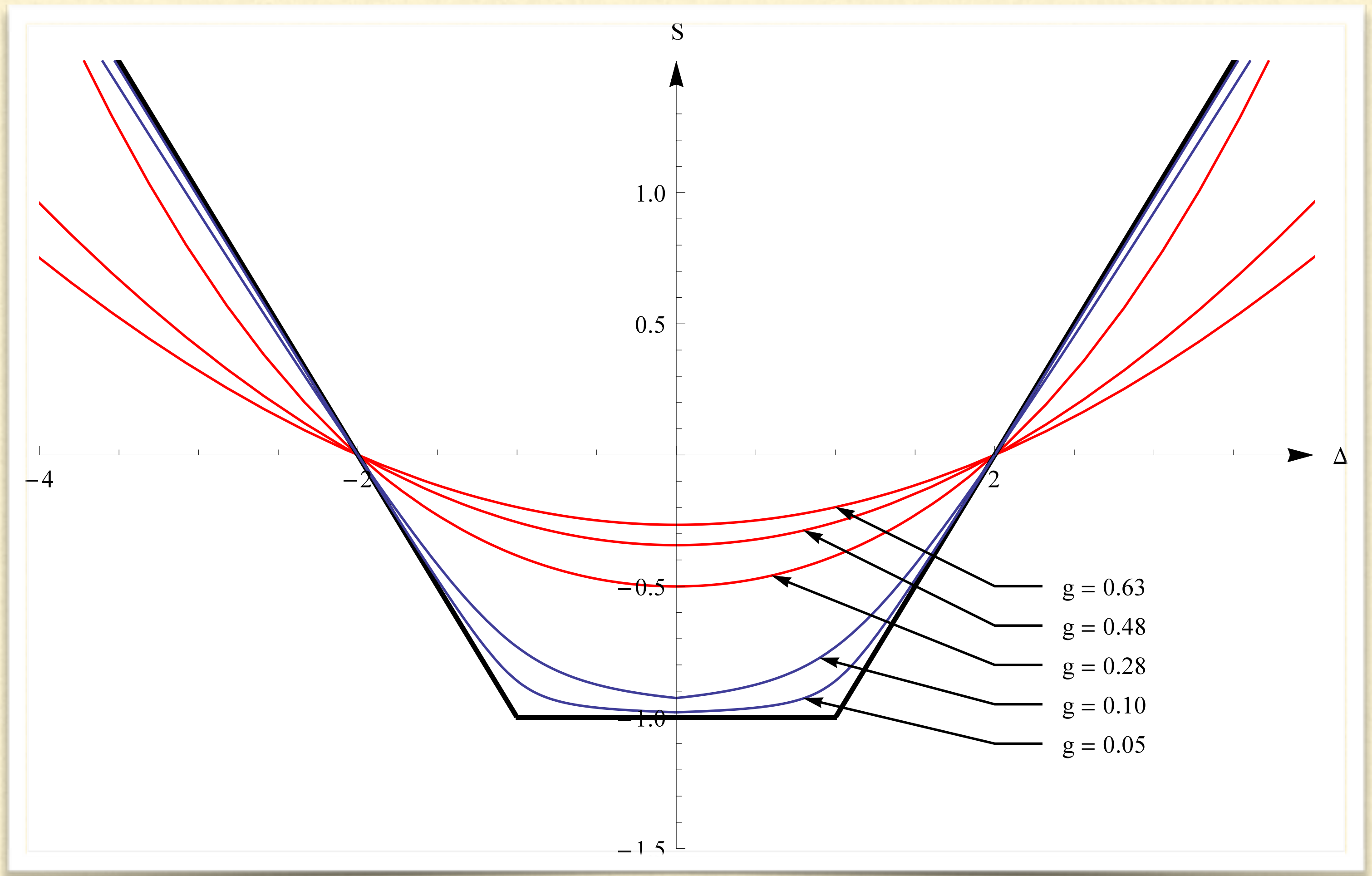
QQ-relations

Asymptotic for large u: $\mathbf{P}_a(u) \sim A_a u^{-\tilde{M}_a}, \; \mathbf{Q}_i(u) \sim B_i u^{\hat{M}_i - 1}$

Analytic properties:
$$\tilde{\mathbf{Q}}^i(u) = M^{ij}(u)\mathbf{Q}_j(-u)\,,$$
$$\tilde{\mathbf{Q}}_i(u) = -\left(M^{-1}\right)_{ji}(u)\mathbf{Q}^j(-u)\,.$$

$$M = \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 & 0 \\ \ell_2 & 0 & 0 & 0 \\ \ell_3 & 0 & \ell_4 & \ell_5 \\ 0 & 0 & \ell_5 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & \ell_6 & 0 \\ 0 & 0 & 0 & 0 \\ \ell_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{2\pi u} + \begin{pmatrix} 0 & 0 & \ell_7 & 0 \\ 0 & 0 & 0 & 0 \\ \ell_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{-2\pi u}\,.$$

Pomeron in $\mathcal{N} = 4$ SYM



[Gromov, Levkovich-Maslyuk, Sizov, Valatka '14]
[Alfimov, Gromov, Kazakov '15]

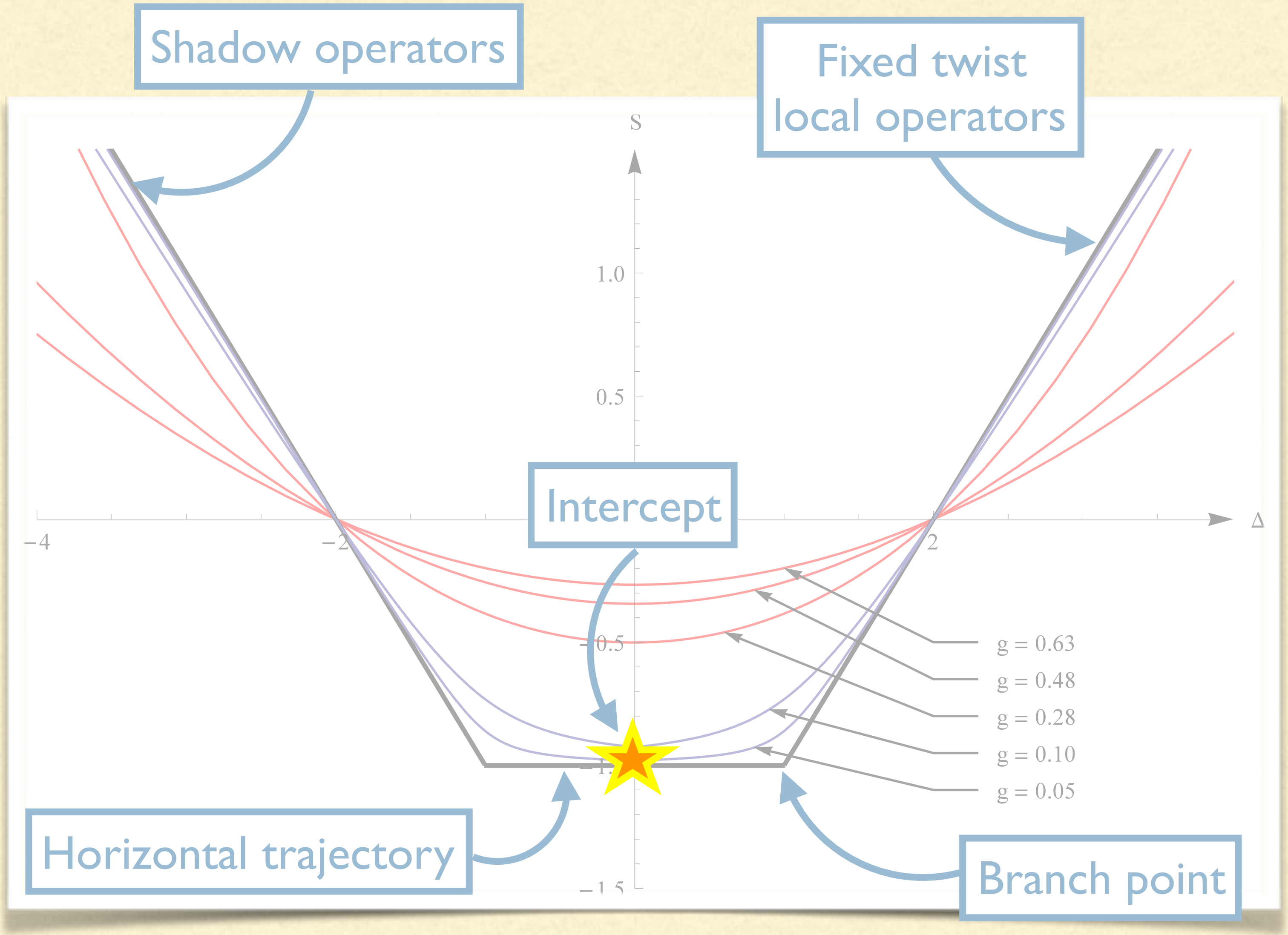
$$\mathrm{Tr}(ZD^S Z) + \mathrm{perm}\,.$$

$$(2,0,0\,|\,2 + S + \gamma, S, 0)$$

$$\alpha(\Delta) = -1 + 4\left(2\psi(1) - \psi\left(\frac{1-\Delta}{2}\right) - \psi\left(\frac{1+\Delta}{2}\right)\right) g^2 + \mathcal{O}(g^4)$$

$$\alpha_{\mathbb{P}}(0) = \alpha(0) + 2$$

Pomeron in $\mathcal{N} = 4$ SYM



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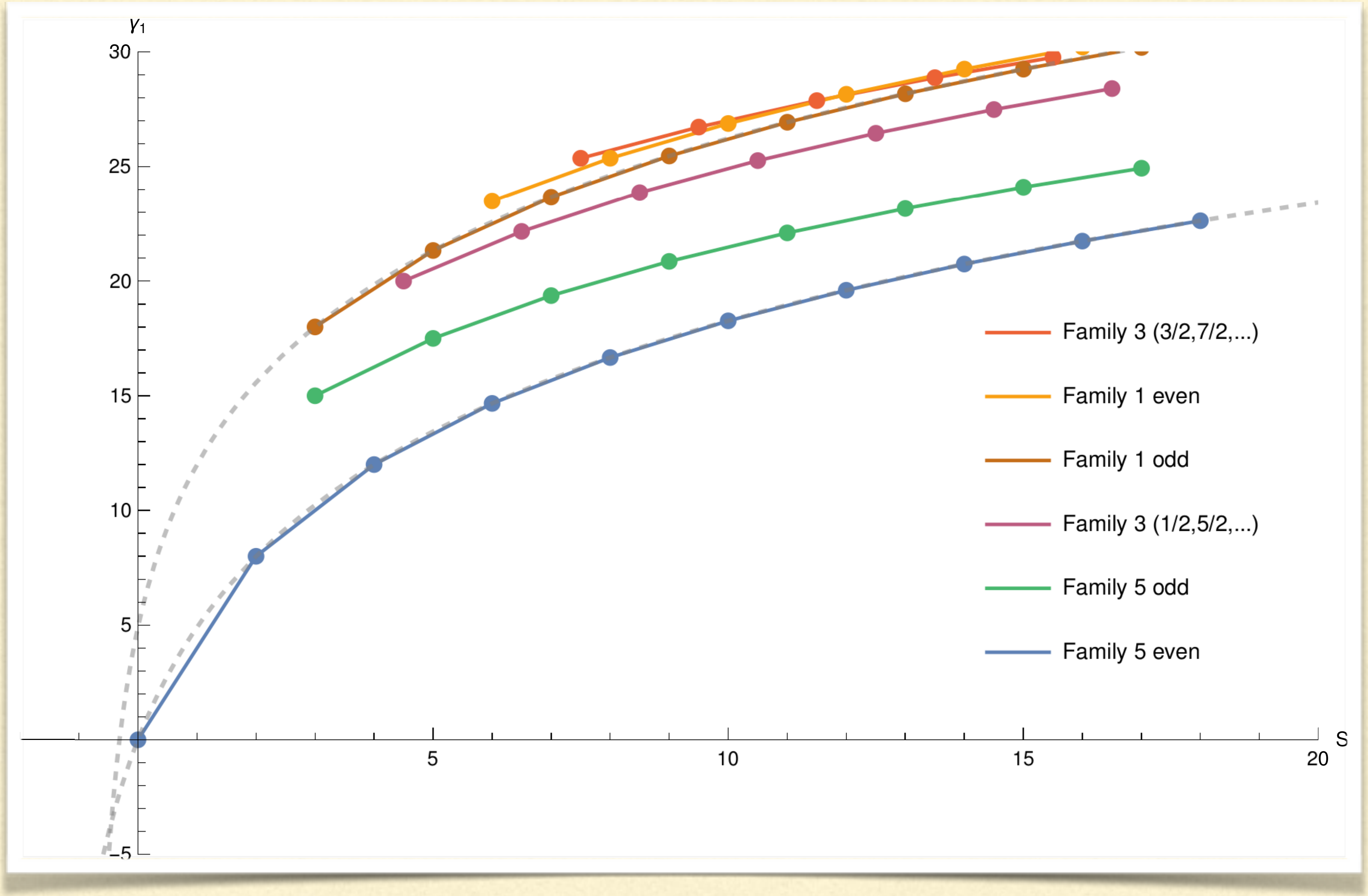
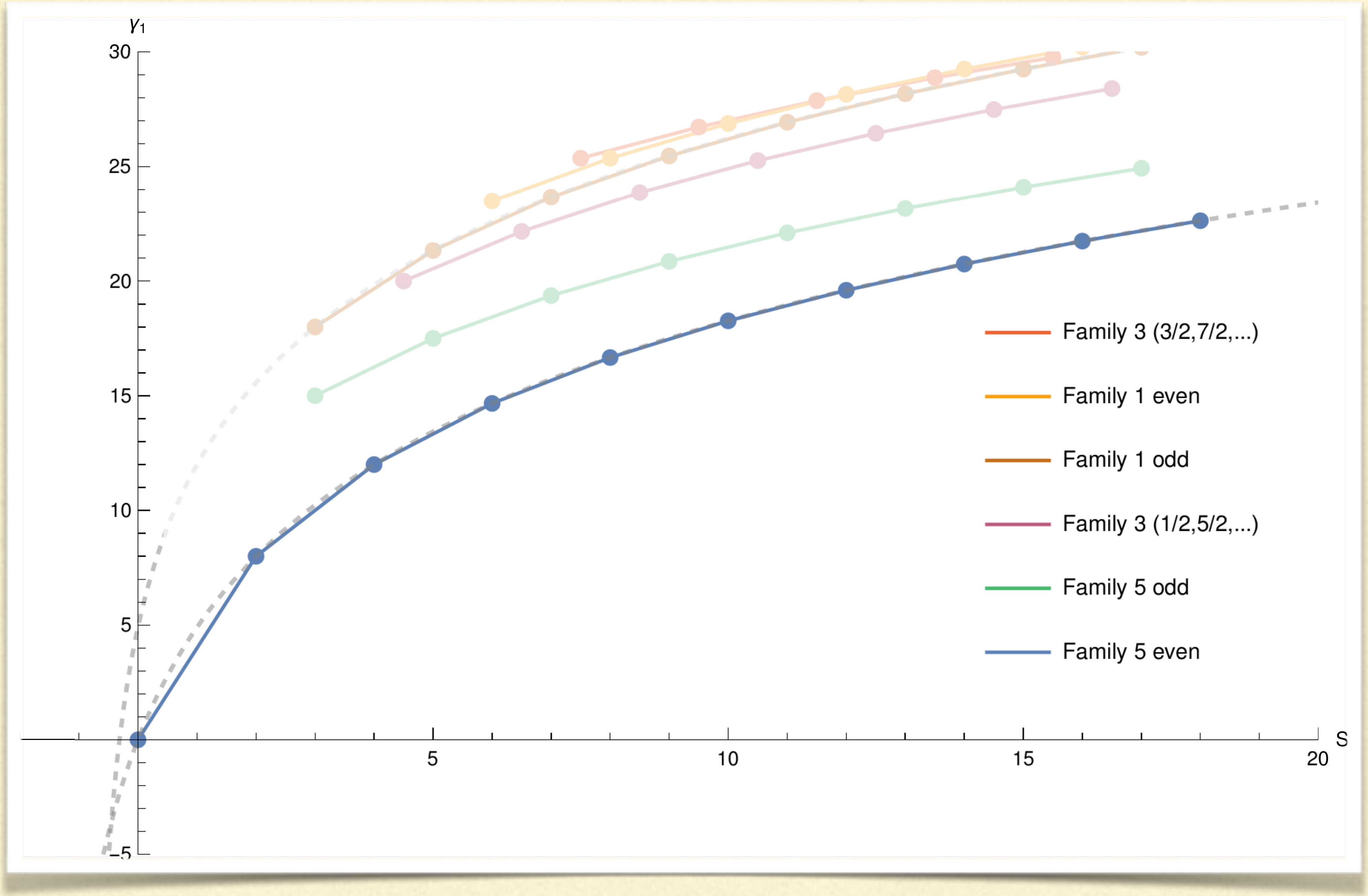
Twist-3 states $\mathcal{N} = 4$ SYM



Recently studied also in: [Homrich, Simmons-Duffin, Vieira '22]

Twist-3 states $\mathcal{N} = 4$ SYM



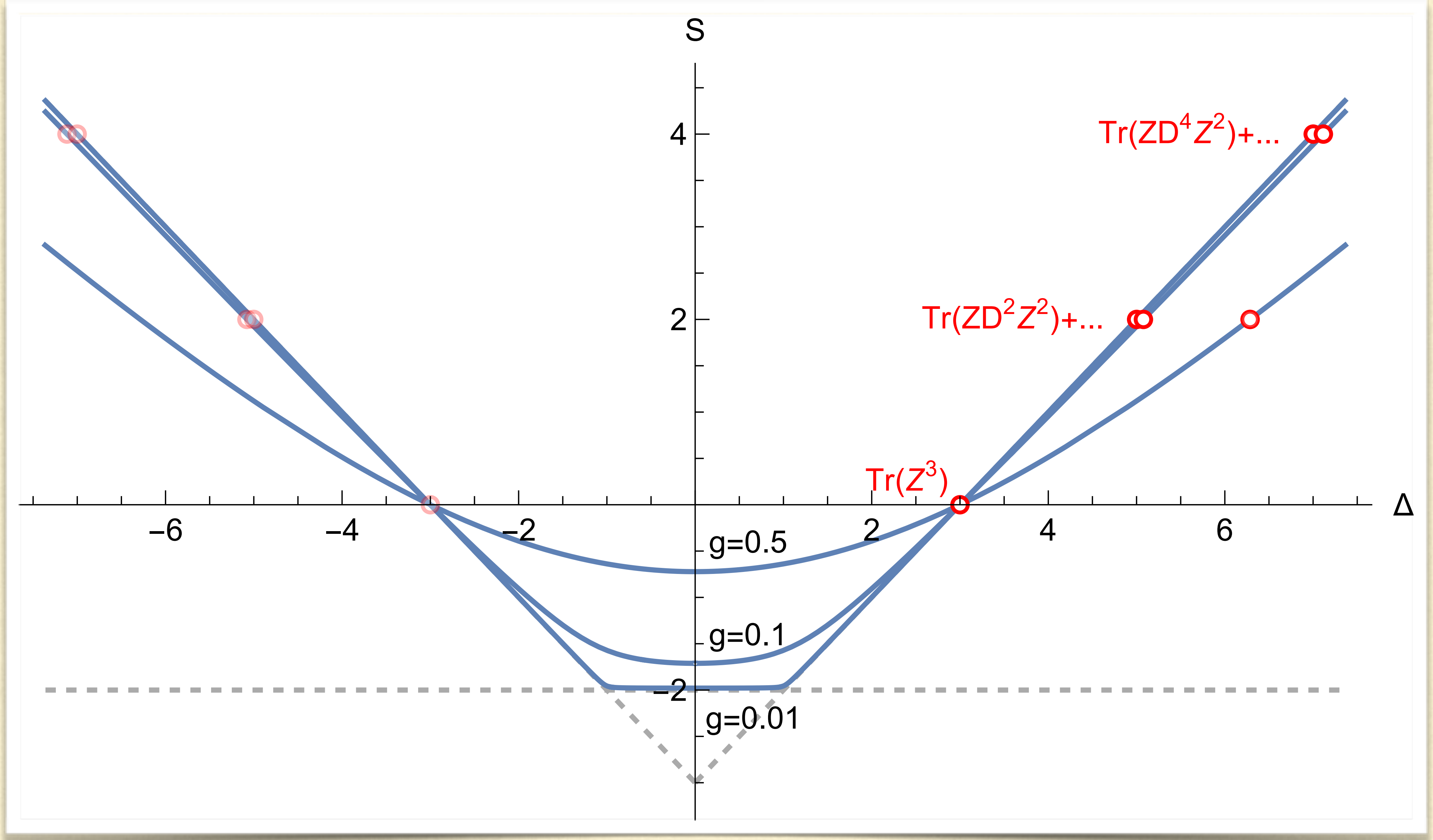
$$\mathcal{O}_S = \mathrm{Tr}(D^S ZZZ) + \text{perm}\,.$$

$$(3,0,0\,|\,3 + S + \gamma, S, 0)$$

parity singlet

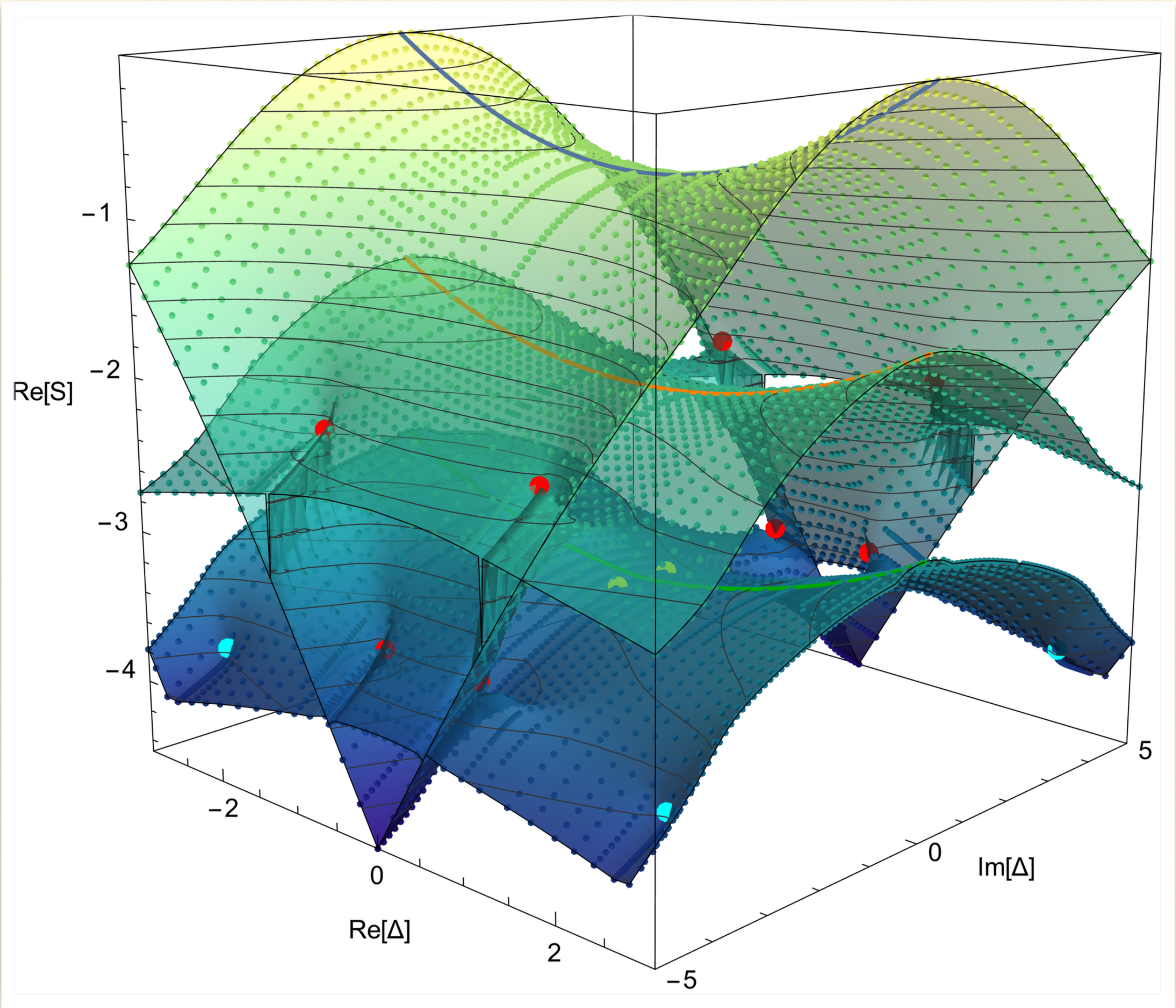
Leading trajectory

[Klabbers, Preti, **IMSZ** '24]



Riemann surface, $g = 1/2$

[Klabbers, Preti, **IMSZ** '24]



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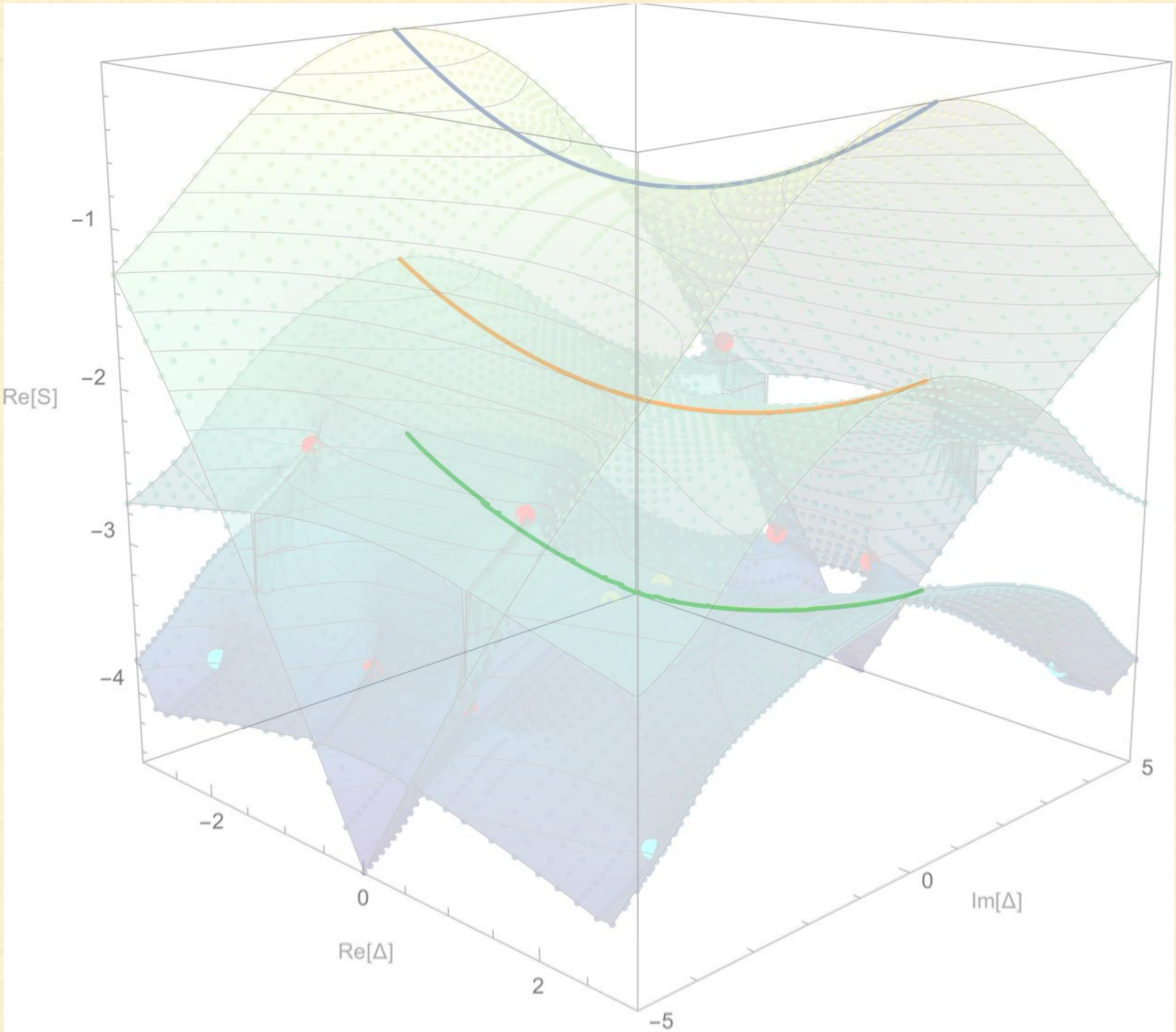

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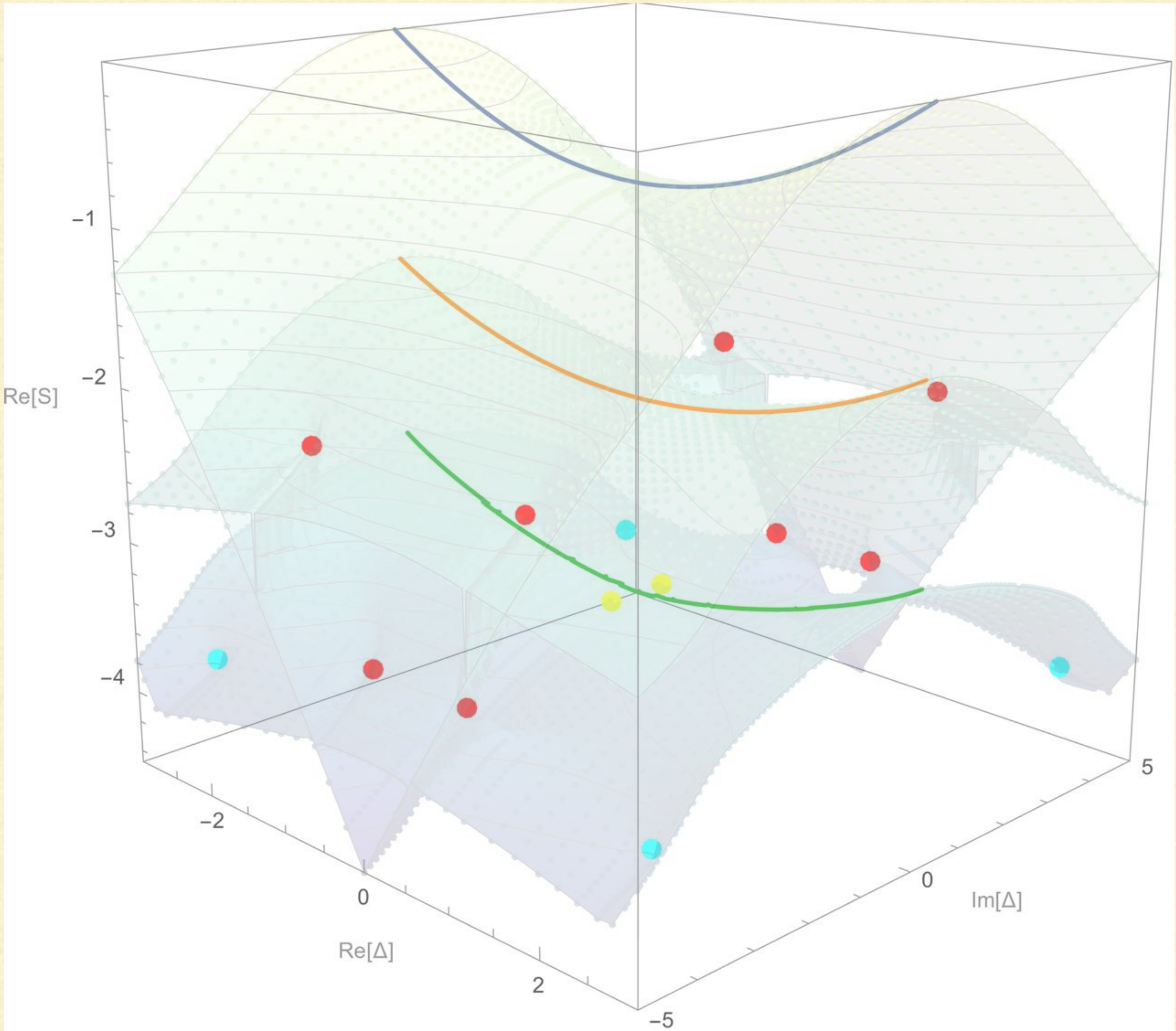

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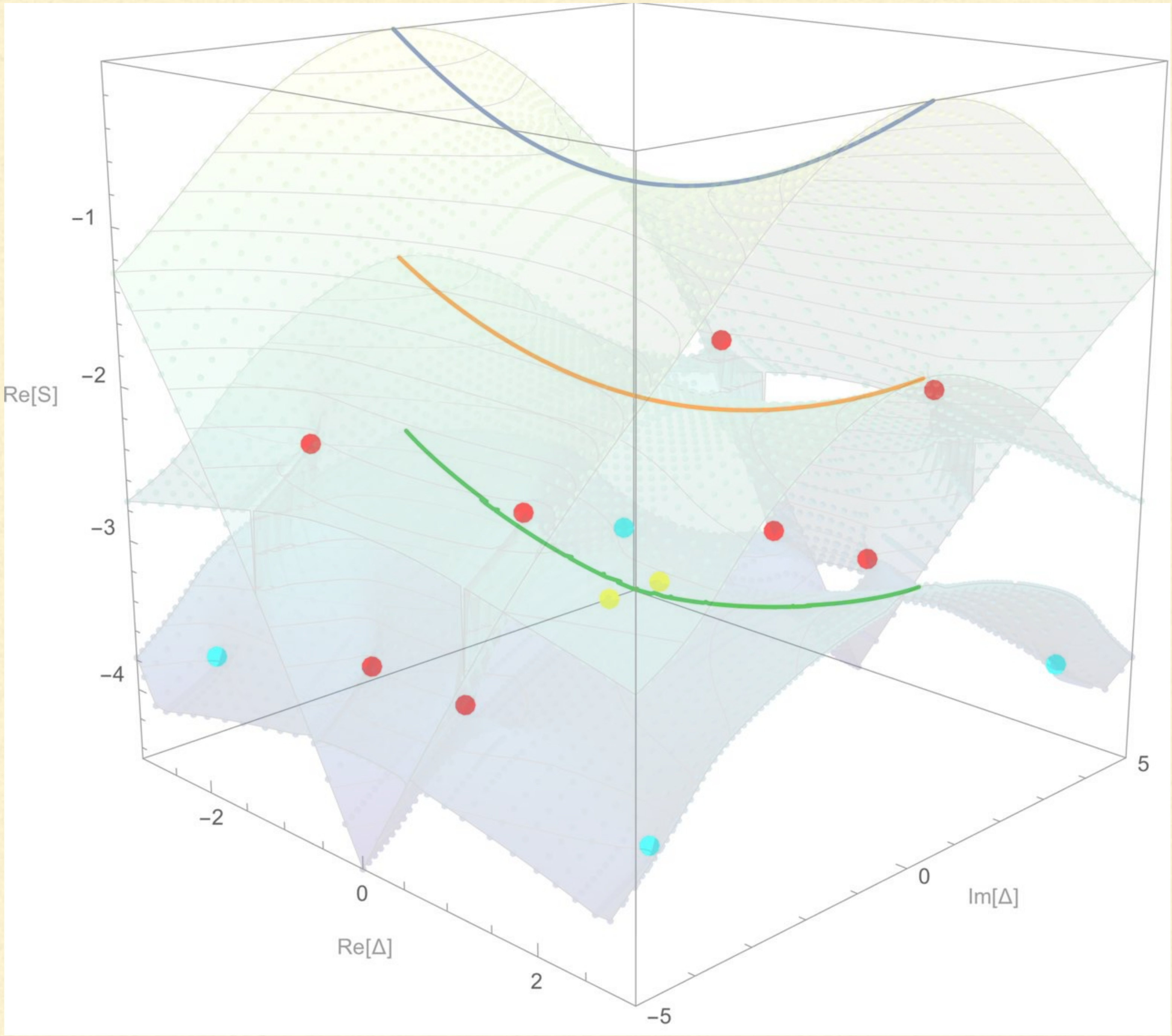
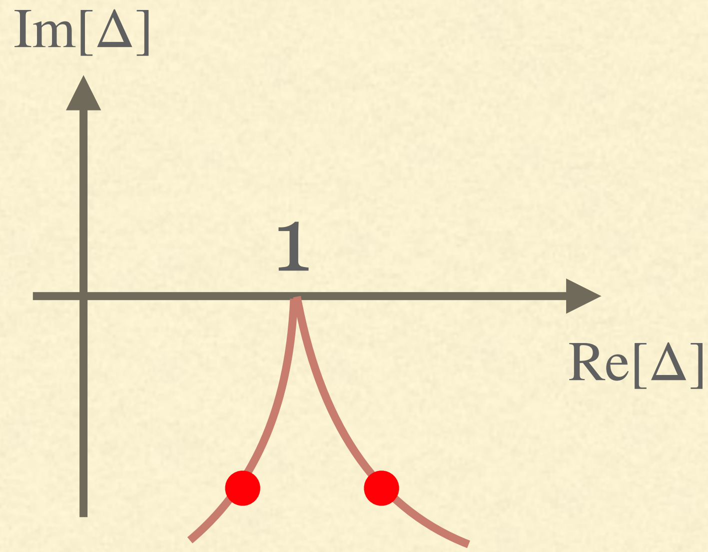
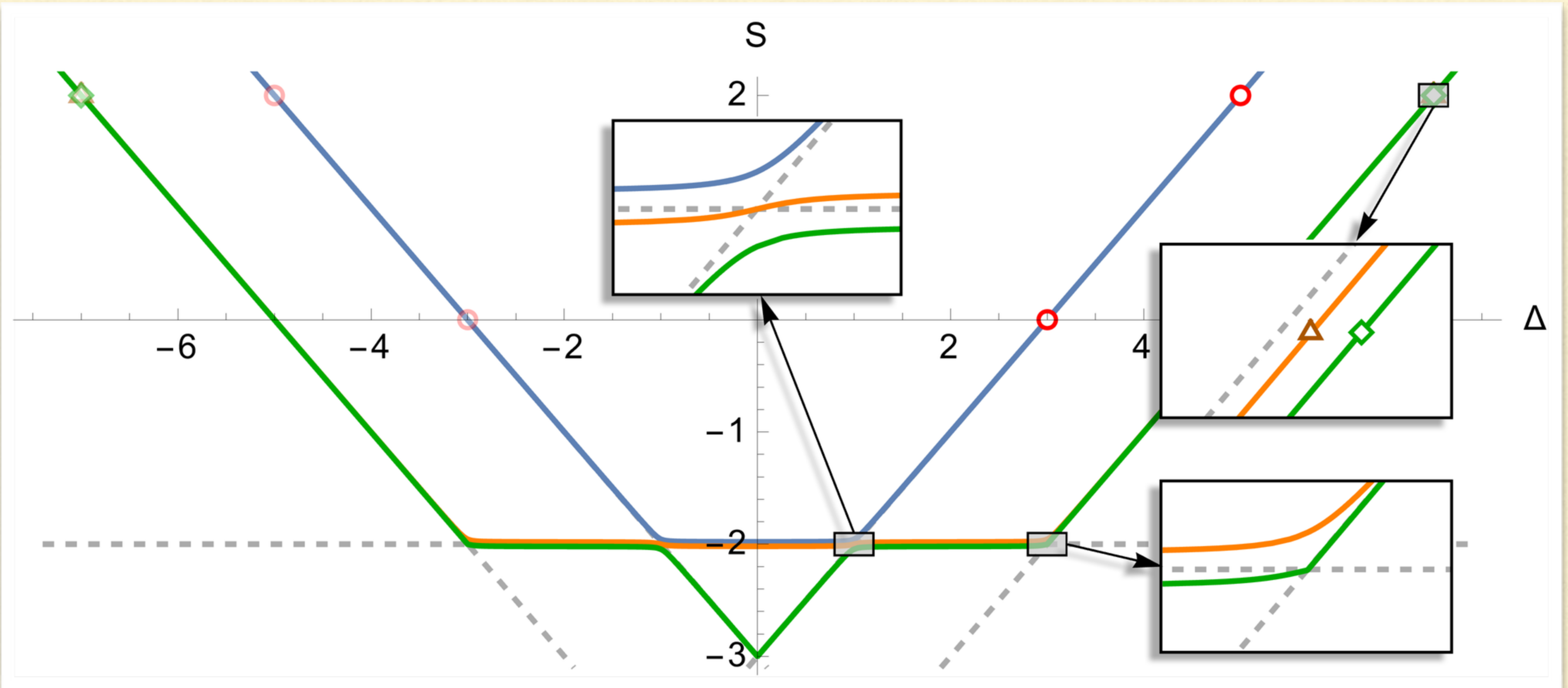




Trajectories at weak coupling, $g = 1/100$

[Klabbers, Preti, **IMSZ** '24]



Degenerate horizontal trajectories

Resolution of the degeneracy

[Klabbers, Preti, **IMSZ** '24]

Perturbative solution to QSC [Alfimov, Gromov, Kazakov '15]

$$S = -2 + \sum_{i=1} I_i(\Delta) g^{i}$$

[Marboe, Volin '18]

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↓

technical derivation…

↓

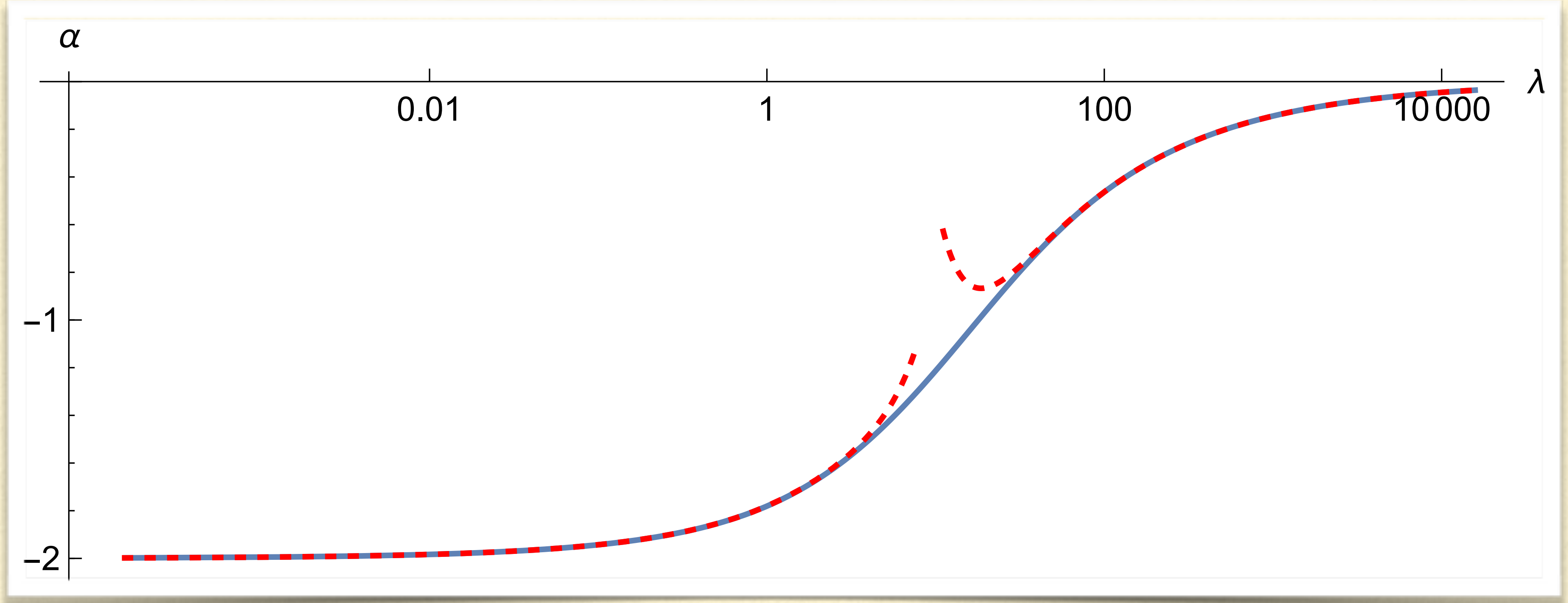
$$I_1 = \pm 2$$

Linear g dependence
is present (!)

Intercept

[Klabbers, Preti, **IMSZ** '24]

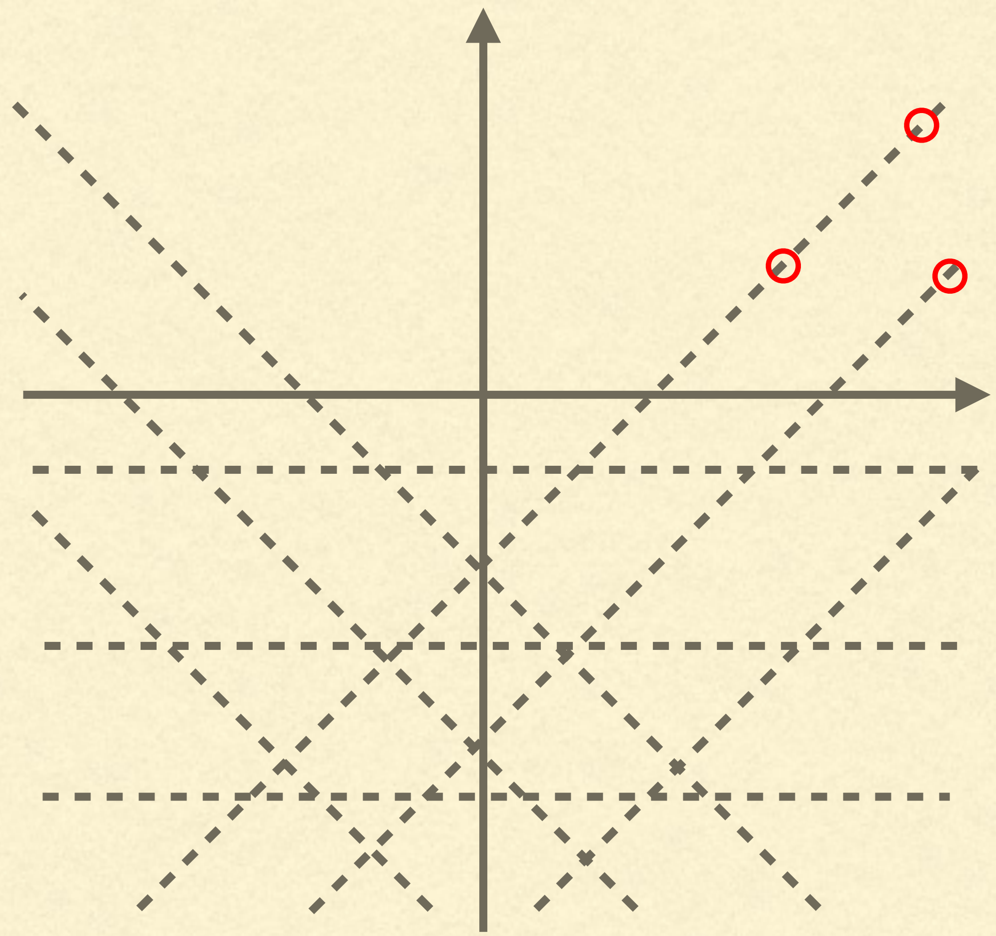
$$\alpha(0) = -2 + 2g + 16\log 2\, g^2 - \frac{2\pi^2}{3} g^3$$
$$-204.77377158292661 g^4 + 136.29333638813 g^5$$
$$+4733.39078974 g^6 - 6116.79585 g^7 + \ldots,$$



[Brower, Costa, Djuric, Raben, Tan '15] [Gromov, Levkovich-Maslyuk, Sizov, Valatka '14]

General properties

States with fixed $(J_1, J_2, J_3 \,|\, S_2)$
+ discrete symmetries
form the surface



2nd order branch points,
but extra degeneracies
at weak coupling

Linear g dependence → Perturbative inversion of $\Delta(S)$ more complex

→ $g \mapsto -g$ connection between trajectories

Summary

Analytic continuation for the twist-3 $\mathcal{O}_S$ trajectory

Extended the QSC numerics and explored
the Riemann surface and its connectivity

Found explicit degeneracy of horizontal trajectories

Resolved the degeneracy analytically

→ linear g dependence

Outlook

Analytic continuation of the other trajectories

Understand the degeneracies in an operatorial way
(light-ray operators)

Finding the $\mathcal{N}=4$ Odderon intercept for all coupling

Thank you for your attention!

Resolution of the degeneracy

[Klabbers, Preti, **IMSZ** '24]

Perturbative solution to QSC [Alfimov, Gromov, Kazakov '15]

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$\mathbf{P}_a$ at $\mathcal{O}(g)$

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$\mathbf{P}_a$ at $\mathcal{O}(g)$ → Diff. eq. for $\mathbf{Q}_i$

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[Marboe, Volin '18]

$\mathbf{P}_a$ at $\mathcal{O}(g)$ → Diff. eq. for $\mathbf{Q}_i$ → factorisation

$$\frac{2(u+i)+igI_1}{(u+i)^{3/2}}\mathbf{Q}_i(u+i) + \frac{2(u-i)-igI_1}{(u-i)^{3/2}}\mathbf{Q}_i(u-i) - \frac{1+8u^2-\Delta^2+2gI_1}{2u^{5/2}}\mathbf{Q}_i(u) = 0$$

Resolution of the degeneracy

[Klabbers, Preti, **IMSZ** '24]

→ Mellin transformation

[Faddeev, Korchemsky '95]
[Korchemsky '95]

Related to hypergeometric differential equation

$$\alpha_1 = \frac{1-\Delta+gI_1}{2}, \quad \alpha_2 = \frac{1+\Delta+gI_1}{2}, \quad \beta_1 = 1+g\frac{I_1}{2}.$$

Solutions:

$$\sim \sum_n \frac{(\alpha_1)_n(\alpha_2)_n}{(1)_n(1_n)} z^n \left[\mathscr{A} + \mathscr{B}\log(z) + \mathscr{C}\log(z)^2\right]$$

With $\mathscr{A}, \mathscr{B}, \mathscr{C}$ combinations of ψ_0, ψ_1

Resolution of the degeneracy

[Klabbers, Preti, **IMSZ** '24]

→ Solution for $\mathbf{Q}_1$ and $\mathbf{Q}_3$

$$\mathbf{Q}_{1,3} \sim \sqrt{u}$$

I_1 is still free

Resolution of the degeneracy

[Klabbers, Preti, **IMSZ** '24]

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→ New regularity condition: $\mathbf{Q}_i \sqrt{x}^{-1} + \tilde{\mathbf{Q}}_i \sqrt{x}$

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- $I_1^2 = 4$

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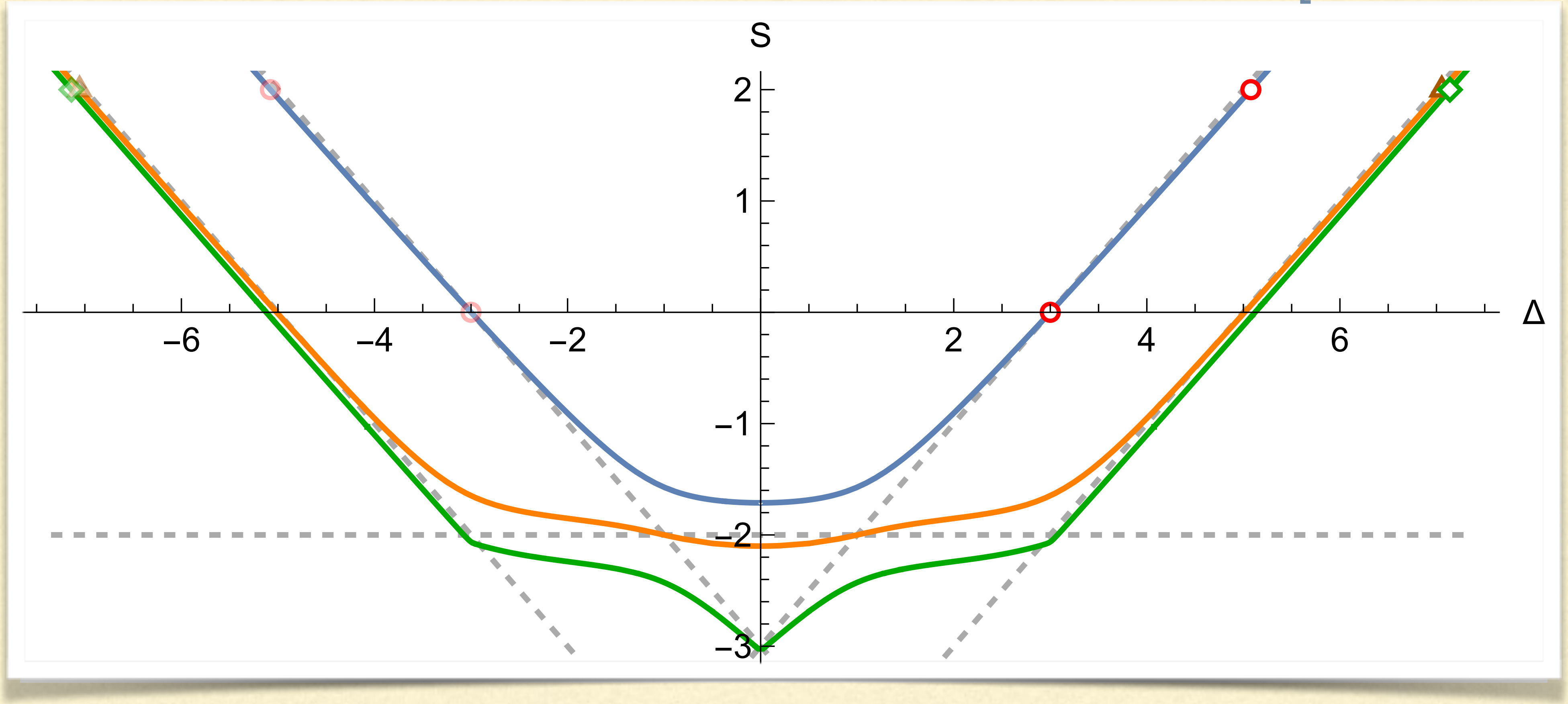
→ $I_1^2 = 4$ → $I_1 = \pm 2$

Linear g dependence is present (!)

Identifying the extra trajectories

[Klabbers, Preti, **IMSZ** '24]



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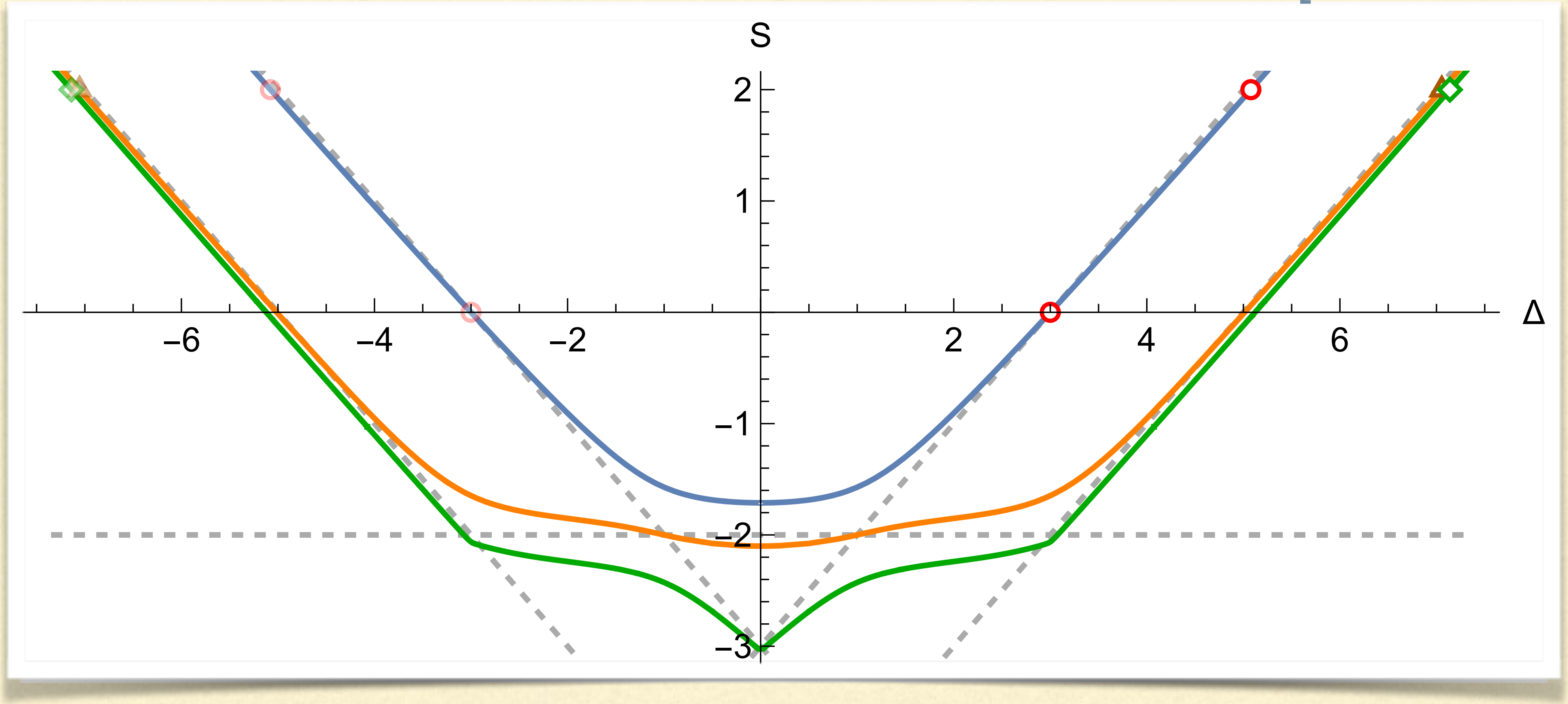
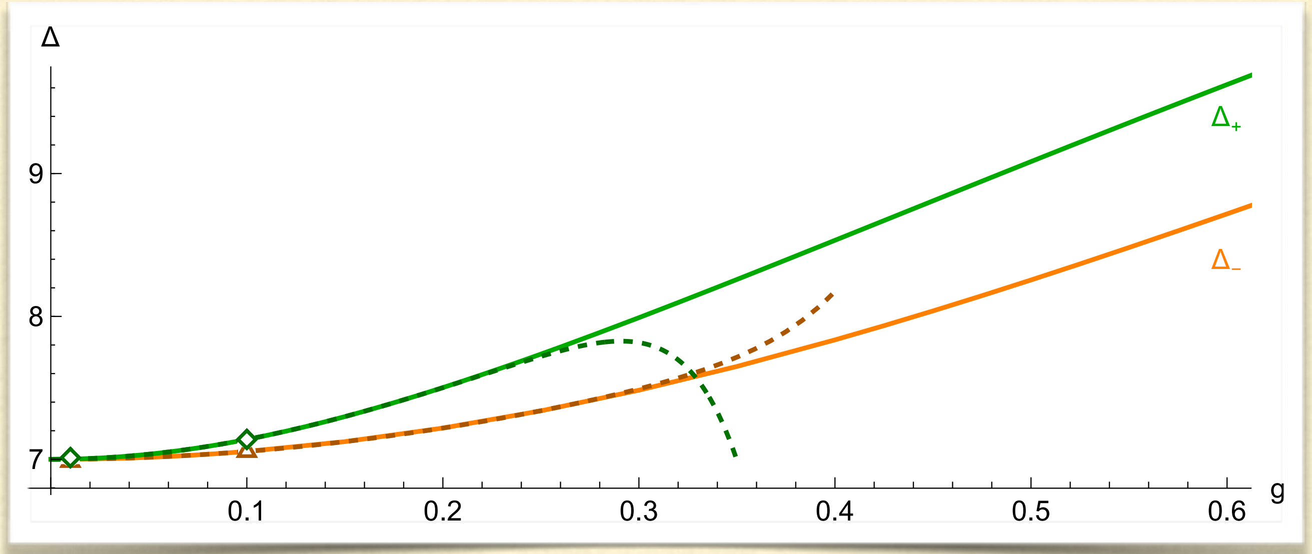
[Klabbers, Preti, **IMSZ** '24]





$$(3,0,0\,|\,5+S+\gamma, S,0)$$

Identi … s

Preti, **IMSZ** '24]

$$\mathcal{O}\_=1.45\mathrm{Tr}(\mathcal{D}\Psi ZZZ\bar\Psi)+0.553\mathrm{Tr}(\mathcal{D}\Psi ZZ\bar\Psi Z)+0.553\mathrm{Tr}(\mathcal{D}\Psi Z\bar\Psi ZZ)+1.45\mathrm{Tr}(\mathcal{D}\Psi\bar\Psi ZZZ)-0.106\mathrm{Tr}(\mathcal{D}Z\Psi ZZ\bar\Psi)-0.106\mathrm{Tr}(\mathcal{D}Z\Psi\bar\Psi ZZ)+0.106\mathrm{Tr}(\mathcal{D}Z\Psi ZZ\bar\Psi)+0.106\mathrm{Tr}(\mathcal{D}Z\Psi\bar\Psi ZZ)-$$

$$1.00\mathrm{Tr}(\mathcal{D}ZZ\Psi Z\bar\Psi)-1.00\mathrm{Tr}(\mathcal{D}ZZ\Psi\bar\Psi Z)+1.00\mathrm{Tr}(\mathcal{D}ZZ\Psi Z\bar\Psi)+1.00\mathrm{Tr}(\mathcal{D}ZZ\Psi\bar\Psi Z)-1.79\mathrm{Tr}(\mathcal{D}ZZZ\Psi\bar\Psi)+1.79\mathrm{Tr}(\mathcal{D}ZZZ\Psi\bar\Psi)-0.894\mathrm{Tr}(\mathcal{D}ZZZX\mathcal{D}\bar X)+0.894\mathrm{Tr}(\mathcal{D}ZZZY\mathcal{D}\bar Y)+0.894\mathrm{Tr}(\mathcal{D}ZZZ\bar Y\mathcal{D}Y)-$$

$$0.894\mathrm{Tr}(\mathcal{D}ZZZ\bar X\mathcal{D}X)-3.58\mathrm{Tr}(\mathcal{D}ZZZ\mathcal{D}Z\bar Z)-0.0528\mathrm{Tr}(\mathcal{D}ZZZ\mathcal{D}X\bar X)+0.0528\mathrm{Tr}(\mathcal{D}ZZZ\mathcal{D}Y\bar Y)+0.0528\mathrm{Tr}(\mathcal{D}ZZZ\mathcal{D}\bar YY)-0.0528\mathrm{Tr}(\mathcal{D}ZZZ\mathcal{D}\bar XX)-0.106\mathrm{Tr}(\mathcal{D}ZZZ\bar\Psi\Psi)+0.106\mathrm{Tr}(\mathcal{D}ZZZ\bar\Psi\Psi)-$$

$$0.500\mathrm{Tr}(\mathcal{D}ZZXZ\mathcal{D}\bar X)+1.84\mathrm{Tr}(\mathcal{D}ZZX\mathcal{D}Z\bar X)-0.500\mathrm{Tr}(\mathcal{D}ZZX\mathcal{D}\bar XZ)+0.500\mathrm{Tr}(\mathcal{D}ZZYZ\mathcal{D}\bar Y)-1.84\mathrm{Tr}(\mathcal{D}ZZY\mathcal{D}Z\bar Y)+0.500\mathrm{Tr}(\mathcal{D}ZZY\mathcal{D}\bar YZ)+0.500\mathrm{Tr}(\mathcal{D}ZZ\bar YZ\mathcal{D}Y)-1.84\mathrm{Tr}(\mathcal{D}ZZ\bar Y\mathcal{D}ZY)+$$

$$0.500\mathrm{Tr}(\mathcal{D}ZZ\bar Y\mathcal{D}YZ)-0.500\mathrm{Tr}(\mathcal{D}ZZ\bar XZ\mathcal{D}X)+1.84\mathrm{Tr}(\mathcal{D}ZZ\bar X\mathcal{D}ZX)-0.500\mathrm{Tr}(\mathcal{D}ZZ\bar X\mathcal{D}XZ)-2.00\mathrm{Tr}(\mathcal{D}ZZ\mathcal{D}ZZ\bar Z)+0.658\mathrm{Tr}(\mathcal{D}ZZ\mathcal{D}ZX\bar X)-0.658\mathrm{Tr}(\mathcal{D}ZZ\mathcal{D}ZY\bar Y)-0.658\mathrm{Tr}(\mathcal{D}ZZ\mathcal{D}Z\bar YY)+$$

$$0.658\mathrm{Tr}(\mathcal{D}ZZ\mathcal{D}Z\bar XX)-2.00\mathrm{Tr}(\mathcal{D}ZZ\mathcal{D}\bar ZZZ)-0.500\mathrm{Tr}(\mathcal{D}ZZ\mathcal{D}X\bar XZ)+0.500\mathrm{Tr}(\mathcal{D}ZZ\mathcal{D}Y\bar YZ)+0.500\mathrm{Tr}(\mathcal{D}ZZ\mathcal{D}\bar YYZ)-0.500\mathrm{Tr}(\mathcal{D}ZZ\mathcal{D}\bar XXZ)-1.00\mathrm{Tr}(\mathcal{D}ZZ\bar\Psi\Psi Z)+1.00\mathrm{Tr}(\mathcal{D}ZZ\bar\Psi\Psi Z)-$$

$$0.0528\mathrm{Tr}(\mathcal{D}ZXZZ\mathcal{D}\bar X)+1.84\mathrm{Tr}(\mathcal{D}ZXZ\mathcal{D}Z\bar X)+1.84\mathrm{Tr}(\mathcal{D}ZX\mathcal{D}Z\bar XZ)-0.0528\mathrm{Tr}(\mathcal{D}ZX\mathcal{D}\bar XZZ)+0.0528\mathrm{Tr}(\mathcal{D}ZYZZ\mathcal{D}\bar Y)-1.84\mathrm{Tr}(\mathcal{D}ZYZ\mathcal{D}Z\bar Y)-1.84\mathrm{Tr}(\mathcal{D}ZY\mathcal{D}Z\bar YZ)+0.0528\mathrm{Tr}(\mathcal{D}ZY\mathcal{D}\bar YZZ)+$$

$$0.0528\mathrm{Tr}(\mathcal{D}Z\bar YZZ\mathcal{D}Y)+0.0528\mathrm{Tr}(\mathcal{D}Z\bar Y\mathcal{D}YZZ)-0.0528\mathrm{Tr}(\mathcal{D}Z\bar XZZ\mathcal{D}X)-0.0528\mathrm{Tr}(\mathcal{D}Z\bar X\mathcal{D}XZZ)-0.211\mathrm{Tr}(\mathcal{D}Z\mathcal{D}ZZZ\bar Z)+0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}ZZX\bar X)-0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}ZZY\bar Y)-$$

$$0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}ZZ\bar YY)+0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}ZZ\bar XX)+0.658\mathrm{Tr}(\mathcal{D}Z\mathcal{D}ZXZ\bar X)+0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}ZX\bar XZ)-0.658\mathrm{Tr}(\mathcal{D}Z\mathcal{D}ZYZ\bar Y)-0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}ZY\bar YZ)-0.658\mathrm{Tr}(\mathcal{D}Z\mathcal{D}Z\bar YZY)-0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}Z\bar YYZ)+$$

$$0.658\mathrm{Tr}(\mathcal{D}Z\mathcal{D}Z\bar XZX)+0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}Z\bar XXZ)-0.211\mathrm{Tr}(\mathcal{D}Z\mathcal{D}\bar ZZZZ)-0.0528\mathrm{Tr}(\mathcal{D}Z\mathcal{D}XZZ\bar X)-0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}XZ\bar XZ)-0.894\mathrm{Tr}(\mathcal{D}Z\mathcal{D}X\bar XZZ)+0.0528\mathrm{Tr}(\mathcal{D}Z\mathcal{D}YZZ\bar Y)+0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}YZ\bar YZ)+$$

$$0.894\mathrm{Tr}(\mathcal{D}Z\mathcal{D}Y\bar YZZ)+0.0528\mathrm{Tr}(\mathcal{D}Z\mathcal{D}\bar YZZY)+0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}\bar YZYZ)+0.894\mathrm{Tr}(\mathcal{D}Z\mathcal{D}\bar YYZZ)-0.0528\mathrm{Tr}(\mathcal{D}Z\mathcal{D}\bar XZZX)-0.500\mathrm{Tr}(\mathcal{D}Z\mathcal{D}\bar XZXZ)-0.894\mathrm{Tr}(\mathcal{D}Z\mathcal{D}\bar XXZZ)-1.79\mathrm{Tr}(\mathcal{D}Z\bar\Psi\Psi ZZ)-$$

$$1.00\mathrm{Tr}(\mathcal{D}Z\bar\Psi Z\Psi Z)-0.106\mathrm{Tr}(\mathcal{D}Z\bar\Psi ZZ\Psi)+1.79\mathrm{Tr}(\mathcal{D}Z\Psi\bar\Psi ZZ)+1.00\mathrm{Tr}(\mathcal{D}Z\Psi Z\bar\Psi Z)+0.106\mathrm{Tr}(\mathcal{D}Z\Psi ZZ\bar\Psi)+1.45\mathrm{Tr}(\mathcal{D}XZZZ\mathcal{D}\bar X)+0.553\mathrm{Tr}(\mathcal{D}XZZ\mathcal{D}\bar XZ)+0.553\mathrm{Tr}(\mathcal{D}XZ\mathcal{D}\bar XZZ)+$$

$$1.45\mathrm{Tr}(\mathcal{D}X\mathcal{D}\bar XZZZ)-1.45\mathrm{Tr}(\mathcal{D}YZZZ\mathcal{D}\bar Y)-0.553\mathrm{Tr}(\mathcal{D}YZZ\mathcal{D}\bar YZ)-0.553\mathrm{Tr}(\mathcal{D}YZ\mathcal{D}\bar YZZ)-1.45\mathrm{Tr}(\mathcal{D}Y\mathcal{D}\bar YZZZ)-0.106\mathrm{Tr}(\bar\Psi\mathcal{F}ZZ\Psi)-1.00\mathrm{Tr}(\bar\Psi\mathcal{F}Z\Psi Z)-1.79\mathrm{Tr}(\bar\Psi\mathcal{F}\Psi ZZ)-0.500\mathrm{Tr}(\bar\Psi\Psi\Psi Z\Psi)-$$

$$0.658\mathrm{Tr}(\bar\Psi\Psi\Psi\Psi Z)-0.658\mathrm{Tr}(\bar\Psi\Psi Z\Psi\Psi)+0.0528\mathrm{Tr}(\bar\Psi\Psi ZZ\mathcal{D}\bar X)+1.00\mathrm{Tr}(\bar\Psi\Psi Z\bar X\mathcal{D}Z)+1.00\mathrm{Tr}(\bar\Psi\Psi Z\mathcal{D}Z\bar X)+1.84\mathrm{Tr}(\bar\Psi\Psi\Psi Z\Psi)+0.842\mathrm{Tr}(\bar\Psi\Psi\bar XZ\mathcal{D}Z)+1.00\mathrm{Tr}(\bar\Psi\Psi\bar X\mathcal{D}ZZ)+0.842\mathrm{Tr}(\bar\Psi\Psi\mathcal{D}ZZ\bar X)+$$

$$1.00\mathrm{Tr}(\bar\Psi\Psi\mathcal{D}Z\bar XZ)+0.0528\mathrm{Tr}(\bar\Psi\Psi\mathcal{D}\bar XZZ)+1.84\mathrm{Tr}(\bar\Psi\Psi\Psi\Psi Z)+1.84\mathrm{Tr}(\bar\Psi\Psi\Psi Z\Psi)+0.500\mathrm{Tr}(\bar\Psi\Psi\Psi Z\Psi)+0.658\mathrm{Tr}(\bar\Psi\Psi\Psi\Psi Z)+0.658\mathrm{Tr}(\bar\Psi\Psi Z\Psi\Psi)-0.0528\mathrm{Tr}(\bar\Psi\Psi ZZ\mathcal{D}Y)-1.00\mathrm{Tr}(\bar\Psi\Psi ZY\mathcal{D}Z)-$$

$$1.00\mathrm{Tr}(\bar\Psi\Psi Z\mathcal{D}ZY)-1.84\mathrm{Tr}(\bar\Psi\Psi Z\Psi\Psi)-0.842\mathrm{Tr}(\bar\Psi\Psi YZ\mathcal{D}Z)-1.00\mathrm{Tr}(\bar\Psi\Psi Y\mathcal{D}ZZ)-0.842\mathrm{Tr}(\bar\Psi\Psi\mathcal{D}ZZY)-1.00\mathrm{Tr}(\bar\Psi\Psi\mathcal{D}ZYZ)-0.0528\mathrm{Tr}(\bar\Psi\Psi\mathcal{D}YZZ)-1.84\mathrm{Tr}(\bar\Psi\Psi\Psi\Psi Z)-1.84\mathrm{Tr}(\bar\Psi\Psi\Psi Z\Psi)+$$

$$0.106\mathrm{Tr}(\bar\Psi\Psi ZZ\mathcal{D}Z)+1.00\mathrm{Tr}(\bar\Psi\Psi Z\mathcal{D}ZZ)+1.79\mathrm{Tr}(\bar\Psi\Psi\mathcal{D}ZZZ)-1.00\mathrm{Tr}(\bar\Psi Z\mathcal{F}\Psi Z)-0.500\mathrm{Tr}(\bar\Psi Z\Psi\Psi\Psi)+0.500\mathrm{Tr}(\bar\Psi Z\Psi Z\mathcal{D}\bar X)+0.842\mathrm{Tr}(\bar\Psi Z\Psi\bar X\mathcal{D}Z)+0.842\mathrm{Tr}(\bar\Psi Z\Psi\mathcal{D}Z\bar X)+0.500\mathrm{Tr}(\bar\Psi Z\Psi\mathcal{D}\bar XZ)+$$

$$1.84\mathrm{Tr}(\bar\Psi Z\Psi\Psi\Psi)+0.500\mathrm{Tr}(\bar\Psi Z\Psi\Psi\Psi)-0.500\mathrm{Tr}(\bar\Psi Z\Psi Z\mathcal{D}Y)-0.842\mathrm{Tr}(\bar\Psi Z\Psi Y\mathcal{D}Z)-0.842\mathrm{Tr}(\bar\Psi Z\Psi\mathcal{D}ZY)-0.500\mathrm{Tr}(\bar\Psi Z\Psi\mathcal{D}YZ)-1.84\mathrm{Tr}(\bar\Psi Z\Psi\Psi\Psi)+1.00\mathrm{Tr}(\bar\Psi Z\Psi\mathcal{D}ZZ)-0.106\mathrm{Tr}(\bar\Psi ZZ\mathcal{F}\Psi)+$$

$$0.894\mathrm{Tr}(\bar\Psi ZZ\Psi\mathcal{D}\bar X)-0.894\mathrm{Tr}(\bar\Psi ZZ\Psi\mathcal{D}Y)+0.106\mathrm{Tr}(\bar\Psi ZZ\Psi\mathcal{D}Z)+0.894\mathrm{Tr}(\bar\Psi ZZY\mathcal{D}\Psi)-0.894\mathrm{Tr}(\bar\Psi ZZ\bar X\mathcal{D}\Psi)-0.0528\mathrm{Tr}(\bar\Psi ZZ\mathcal{D}\Psi\bar X)+0.0528\mathrm{Tr}(\bar\Psi ZZ\mathcal{D}\Psi Y)+1.79\mathrm{Tr}(\bar\Psi ZZ\mathcal{D}Z\Psi)-$$

$$0.0528\mathrm{Tr}(\bar\Psi ZZ\mathcal{D}Y\Psi)+0.0528\mathrm{Tr}(\bar\Psi ZZ\mathcal{D}\bar X\Psi)-1.79\mathrm{Tr}(\bar\Psi ZZ\bar\Psi\mathcal{F})+0.342\mathrm{Tr}(\bar\Psi ZY\Psi\mathcal{D}Z)+0.500\mathrm{Tr}(\bar\Psi ZYZ\mathcal{D}\Psi)+0.500\mathrm{Tr}(\bar\Psi ZY\mathcal{D}\Psi Z)-1.00\mathrm{Tr}(\bar\Psi ZY\mathcal{D}Z\Psi)-0.342\mathrm{Tr}(\bar\Psi Z\bar X\Psi\mathcal{D}Z)-0.500\mathrm{Tr}(\bar\Psi Z\bar XZ\mathcal{D}\Psi)-$$

$$0.500\mathrm{Tr}(\bar\Psi Z\bar X\mathcal{D}\Psi Z)+1.00\mathrm{Tr}(\bar\Psi Z\bar X\mathcal{D}Z\Psi)-0.500\mathrm{Tr}(\bar\Psi Z\mathcal{D}\Psi\bar XZ)+0.500\mathrm{Tr}(\bar\Psi Z\mathcal{D}\Psi YZ)-0.342\mathrm{Tr}(\bar\Psi Z\mathcal{D}Z\Psi\bar X)+0.342\mathrm{Tr}(\bar\Psi Z\mathcal{D}Z\Psi Y)+1.00\mathrm{Tr}(\bar\Psi Z\mathcal{D}Z\Psi Z)+1.00\mathrm{Tr}(\bar\Psi Z\mathcal{D}ZZ\Psi)-1.00\mathrm{Tr}(\bar\Psi Z\mathcal{D}ZY\Psi)+$$

$$1.00\mathrm{Tr}(\bar\Psi Z\mathcal{D}Z\bar X\Psi)-0.500\mathrm{Tr}(\bar\Psi Z\mathcal{D}Y\Psi Z)+0.500\mathrm{Tr}(\bar\Psi Z\mathcal{D}\bar X\Psi Z)-1.00\mathrm{Tr}(\bar\Psi Z\bar\Psi\mathcal{F}Z)-0.658\mathrm{Tr}(\bar\Psi Z\bar\Psi\Psi\Psi)+0.658\mathrm{Tr}(\bar\Psi Z\bar\Psi\Psi\Psi)-1.00\mathrm{Tr}(\bar\Psi Z\bar\Psi Z\mathcal{F})+0.342\mathrm{Tr}(\bar\Psi Y\Psi Z\mathcal{D}Z)+0.342\mathrm{Tr}(\bar\Psi Y\Psi\mathcal{D}ZZ)+$$

$$0.342\mathrm{Tr}(\bar\Psi YZ\Psi\mathcal{D}Z)+0.0528\mathrm{Tr}(\bar\Psi YZZ\mathcal{D}\Psi)-0.842\mathrm{Tr}(\bar\Psi YZ\mathcal{D}Z\Psi)+0.0528\mathrm{Tr}(\bar\Psi Y\mathcal{D}\Psi ZZ)-0.842\mathrm{Tr}(\bar\Psi Y\mathcal{D}Z\Psi Z)-1.00\mathrm{Tr}(\bar\Psi Y\mathcal{D}ZZ\Psi)-0.342\mathrm{Tr}(\bar\Psi\bar X\Psi Z\mathcal{D}Z)-0.342\mathrm{Tr}(\bar\Psi\bar X\Psi\mathcal{D}ZZ)-$$

$$0.342\mathrm{Tr}(\bar\Psi\bar XZ\Psi\mathcal{D}Z)-0.0528\mathrm{Tr}(\bar\Psi\bar XZZ\mathcal{D}\Psi)+0.842\mathrm{Tr}(\bar\Psi\bar XZ\mathcal{D}Z\Psi)-0.0528\mathrm{Tr}(\bar\Psi\bar X\mathcal{D}\Psi ZZ)+0.842\mathrm{Tr}(\bar\Psi\bar X\mathcal{D}Z\Psi Z)+1.00\mathrm{Tr}(\bar\Psi\bar X\mathcal{D}ZZ\Psi)-0.0528\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi ZZ\bar X)-0.500\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi Z\bar XZ)-$$

$$0.894\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi\bar XZZ)+0.0528\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi ZZY)+0.500\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi ZYZ)+0.894\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi YZZ)-0.342\mathrm{Tr}(\bar\Psi\mathcal{D}Z\Psi Z\bar X)-0.342\mathrm{Tr}(\bar\Psi\mathcal{D}Z\Psi\bar XZ)+0.342\mathrm{Tr}(\bar\Psi\mathcal{D}Z\Psi ZY)+0.342\mathrm{Tr}(\bar\Psi\mathcal{D}Z\Psi YZ)+$$

$$0.106\mathrm{Tr}(\bar\Psi\mathcal{D}Z\Psi ZZ)-0.342\mathrm{Tr}(\bar\Psi\mathcal{D}ZZ\Psi\bar X)+0.342\mathrm{Tr}(\bar\Psi\mathcal{D}ZZ\Psi Y)+0.106\mathrm{Tr}(\bar\Psi\mathcal{D}ZZZ\Psi)-0.842\mathrm{Tr}(\bar\Psi\mathcal{D}ZZY\Psi)+0.842\mathrm{Tr}(\bar\Psi\mathcal{D}ZZ\bar X\Psi)-0.842\mathrm{Tr}(\bar\Psi\mathcal{D}ZY\Psi Z)-1.00\mathrm{Tr}(\bar\Psi\mathcal{D}ZYZ\Psi)+$$

$$0.842\mathrm{Tr}(\bar\Psi\mathcal{D}Z\bar X\Psi Z)+1.00\mathrm{Tr}(\bar\Psi\mathcal{D}Z\bar XZ\Psi)-0.894\mathrm{Tr}(\bar\Psi\mathcal{D}Y\Psi ZZ)-0.500\mathrm{Tr}(\bar\Psi\mathcal{D}YZ\Psi Z)-0.0528\mathrm{Tr}(\bar\Psi\mathcal{D}YZZ\Psi)+0.894\mathrm{Tr}(\bar\Psi\mathcal{D}\bar X\Psi ZZ)+0.500\mathrm{Tr}(\bar\Psi\mathcal{D}\bar XZ\Psi Z)+0.0528\mathrm{Tr}(\bar\Psi\mathcal{D}\bar XZZ\Psi)-$$

$$0.106\mathrm{Tr}(\Psi\bar\Psi\mathcal{F}ZZ)-0.500\mathrm{Tr}(\Psi\bar\Psi\Psi\Psi Z)-0.658\mathrm{Tr}(\Psi\bar\Psi\Psi Z\Psi)+0.500\mathrm{Tr}(\Psi\bar\Psi\Psi\Psi Z)+0.658\mathrm{Tr}(\Psi\bar\Psi\Psi Z\Psi)-0.500\mathrm{Tr}(\Psi\bar\Psi Z\Psi\Psi)+0.500\mathrm{Tr}(\Psi\bar\Psi Z\Psi\Psi)-0.106\mathrm{Tr}(\Psi\bar\Psi ZZ\mathcal{F})-0.0528\mathrm{Tr}(\Psi\Psi ZZ\mathcal{D}\bar Y)-$$

$$1.00\mathrm{Tr}(\Psi\Psi Z\bar Y\mathcal{D}Z)-1.00\mathrm{Tr}(\Psi\Psi Z\mathcal{D}Z\bar Y)-0.842\mathrm{Tr}(\Psi\Psi\bar YZ\mathcal{D}Z)-1.00\mathrm{Tr}(\Psi\Psi\bar Y\mathcal{D}ZZ)-0.842\mathrm{Tr}(\Psi\Psi\mathcal{D}ZZ\bar Y)-1.00\mathrm{Tr}(\Psi\Psi\mathcal{D}Z\bar YZ)-0.0528\mathrm{Tr}(\Psi\Psi\mathcal{D}\bar YZZ)+0.0528\mathrm{Tr}(\Psi\Psi ZZ\mathcal{D}X)+1.00\mathrm{Tr}(\Psi\Psi ZX\mathcal{D}Z)+$$

$$1.00\mathrm{Tr}(\Psi\Psi Z\mathcal{D}ZX)+0.842\mathrm{Tr}(\Psi\Psi XZ\mathcal{D}Z)+1.00\mathrm{Tr}(\Psi\Psi X\mathcal{D}ZZ)+0.842\mathrm{Tr}(\Psi\Psi\mathcal{D}ZZX)+1.00\mathrm{Tr}(\Psi\Psi\mathcal{D}ZXZ)+0.0528\mathrm{Tr}(\Psi\Psi\mathcal{D}XZZ)-0.106\mathrm{Tr}(\Psi\Psi ZZ\mathcal{D}Z)-1.00\mathrm{Tr}(\Psi\Psi Z\mathcal{D}ZZ)-1.79\mathrm{Tr}(\Psi\Psi\mathcal{D}ZZZ)-$$

$$0.500\mathrm{Tr}(\Psi Z\Psi Z\mathcal{D}\bar Y)-0.842\mathrm{Tr}(\Psi Z\Psi\bar Y\mathcal{D}Z)-0.842\mathrm{Tr}(\Psi Z\Psi\mathcal{D}Z\bar Y)-0.500\mathrm{Tr}(\Psi Z\Psi\mathcal{D}\bar YZ)+0.500\mathrm{Tr}(\Psi Z\Psi Z\mathcal{D}X)+0.842\mathrm{Tr}(\Psi Z\Psi X\mathcal{D}Z)+0.842\mathrm{Tr}(\Psi Z\Psi\mathcal{D}ZX)+0.500\mathrm{Tr}(\Psi Z\Psi\mathcal{D}XZ)-$$

$$1.00\mathrm{Tr}(\Psi Z\Psi\mathcal{D}ZZ)-0.894\mathrm{Tr}(\Psi ZZ\Psi\mathcal{D}\bar Y)+0.894\mathrm{Tr}(\Psi ZZ\Psi\mathcal{D}X)-0.106\mathrm{Tr}(\Psi ZZ\Psi\mathcal{D}Z)-0.894\mathrm{Tr}(\Psi ZZX\mathcal{D}\Psi)+0.894\mathrm{Tr}(\Psi ZZ\bar Y\mathcal{D}\Psi)+0.0528\mathrm{Tr}(\Psi ZZ\mathcal{D}\Psi\bar Y)-0.0528\mathrm{Tr}(\Psi ZZ\mathcal{D}\Psi X)-$$

$$1.79\mathrm{Tr}(\Psi ZZ\mathcal{D}Z\Psi)+0.0528\mathrm{Tr}(\Psi ZZ\mathcal{D}X\Psi)-0.0528\mathrm{Tr}(\Psi ZZ\mathcal{D}\bar Y\Psi)-0.342\mathrm{Tr}(\Psi ZX\Psi\mathcal{D}Z)-0.500\mathrm{Tr}(\Psi ZXZ\mathcal{D}\Psi)-0.500\mathrm{Tr}(\Psi ZX\mathcal{D}\Psi Z)+1.00\mathrm{Tr}(\Psi ZX\mathcal{D}Z\Psi)+0.342\mathrm{Tr}(\Psi Z\bar Y\Psi\mathcal{D}Z)+$$

$$0.500\mathrm{Tr}(\Psi Z\bar YZ\mathcal{D}\Psi)+0.500\mathrm{Tr}(\Psi Z\bar Y\mathcal{D}\Psi Z)-1.00\mathrm{Tr}(\Psi Z\bar Y\mathcal{D}Z\Psi)+0.500\mathrm{Tr}(\Psi Z\mathcal{D}\Psi\bar YZ)-0.500\mathrm{Tr}(\Psi Z\mathcal{D}\Psi XZ)+0.342\mathrm{Tr}(\Psi Z\mathcal{D}Z\Psi\bar Y)-0.342\mathrm{Tr}(\Psi Z\mathcal{D}Z\Psi X)-1.00\mathrm{Tr}(\Psi Z\mathcal{D}Z\Psi Z)-1.00\mathrm{Tr}(\Psi Z\mathcal{D}ZZ\Psi)+$$

$$1.00\mathrm{Tr}(\Psi Z\mathcal{D}ZX\Psi)-1.00\mathrm{Tr}(\Psi Z\mathcal{D}Z\bar Y\Psi)+0.500\mathrm{Tr}(\Psi Z\mathcal{D}X\Psi Z)-0.500\mathrm{Tr}(\Psi Z\mathcal{D}\bar Y\Psi Z)-0.342\mathrm{Tr}(\Psi X\Psi Z\mathcal{D}Z)-0.342\mathrm{Tr}(\Psi X\Psi\mathcal{D}ZZ)-0.342\mathrm{Tr}(\Psi XZ\Psi\mathcal{D}Z)-0.0528\mathrm{Tr}(\Psi XZZ\mathcal{D}\Psi)+$$

$$0.842\mathrm{Tr}(\Psi XZ\mathcal{D}Z\Psi)-0.0528\mathrm{Tr}(\Psi X\mathcal{D}\Psi ZZ)+0.842\mathrm{Tr}(\Psi X\mathcal{D}Z\Psi Z)+1.00\mathrm{Tr}(\Psi X\mathcal{D}ZZ\Psi)+0.342\mathrm{Tr}(\Psi\bar Y\Psi Z\mathcal{D}Z)+0.342\mathrm{Tr}(\Psi\bar Y\Psi\mathcal{D}ZZ)+0.342\mathrm{Tr}(\Psi\bar YZ\Psi\mathcal{D}Z)+0.0528\mathrm{Tr}(\Psi\bar YZZ\mathcal{D}\Psi)-$$

$$0.842\mathrm{Tr}(\bar\Psi\bar YZ\mathcal{D}Z\Psi)+0.0528\mathrm{Tr}(\bar\Psi\bar Y\mathcal{D}\Psi ZZ)-0.842\mathrm{Tr}(\bar\Psi\bar Y\mathcal{D}Z\Psi Z)-1.00\mathrm{Tr}(\bar\Psi\bar Y\mathcal{D}ZZ\Psi)+0.0528\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi ZZ\bar Y)+0.500\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi Z\bar YZ)+0.894\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi\bar YZZ)-0.0528\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi ZZX)-$$

$$0.500\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi ZXZ)-0.894\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi XZZ)+0.342\mathrm{Tr}(\bar\Psi\mathcal{D}Z\Psi Z\bar Y)+0.342\mathrm{Tr}(\bar\Psi\mathcal{D}Z\Psi\bar YZ)-0.342\mathrm{Tr}(\bar\Psi\mathcal{D}Z\Psi ZX)-0.342\mathrm{Tr}(\bar\Psi\mathcal{D}Z\Psi XZ)-0.106\mathrm{Tr}(\bar\Psi\mathcal{D}Z\Psi ZZ)+0.342\mathrm{Tr}(\bar\Psi\mathcal{D}ZZ\Psi\bar Y)-$$

$$0.342\mathrm{Tr}(\bar\Psi\mathcal{D}ZZ\Psi X)-0.106\mathrm{Tr}(\bar\Psi\mathcal{D}ZZZ\Psi)+0.842\mathrm{Tr}(\bar\Psi\mathcal{D}ZZX\Psi)-0.842\mathrm{Tr}(\bar\Psi\mathcal{D}ZZ\bar Y\Psi)+0.842\mathrm{Tr}(\bar\Psi\mathcal{D}ZX\Psi Z)+1.00\mathrm{Tr}(\bar\Psi\mathcal{D}ZXZ\Psi)-0.842\mathrm{Tr}(\bar\Psi\mathcal{D}Z\bar Y\Psi Z)-1.00\mathrm{Tr}(\bar\Psi\mathcal{D}Z\bar YZ\Psi)+$$

$$0.894\mathrm{Tr}(\bar\Psi\mathcal{D}X\Psi ZZ)+0.500\mathrm{Tr}(\bar\Psi\mathcal{D}XZ\Psi Z)+0.0528\mathrm{Tr}(\bar\Psi\mathcal{D}XZZ\Psi)-0.894\mathrm{Tr}(\bar\Psi\mathcal{D}\bar Y\Psi ZZ)-0.500\mathrm{Tr}(\bar\Psi\mathcal{D}\bar YZ\Psi Z)-0.0528\mathrm{Tr}(\bar\Psi\mathcal{D}\bar YZZ\Psi)+1.45\mathrm{Tr}(\bar\Psi ZZZ\mathcal{D}\Psi)+0.553\mathrm{Tr}(\bar\Psi ZZ\mathcal{D}\Psi Z)+$$

$$0.553\mathrm{Tr}(\bar\Psi Z\mathcal{D}\Psi ZZ)+1.45\mathrm{Tr}(\bar\Psi\mathcal{D}\Psi ZZZ)+1.45\mathrm{Tr}(\mathcal{D}^2ZZZZ\bar Z)-0.0528\mathrm{Tr}(\mathcal{D}^2ZZZX\bar X)+0.0528\mathrm{Tr}(\mathcal{D}^2ZZZY\bar Y)+0.0528\mathrm{Tr}(\mathcal{D}^2ZZZ\bar YY)-0.0528\mathrm{Tr}(\mathcal{D}^2ZZZ\bar XX)+0.553\mathrm{Tr}(\mathcal{D}^2ZZZ\bar ZZ)-$$

$$0.500\mathrm{Tr}(\mathcal{D}^2ZZXZ\bar X)+0.500\mathrm{Tr}(\mathcal{D}^2ZZYZ\bar Y)+0.500\mathrm{Tr}(\mathcal{D}^2ZZ\bar YZY)-0.500\mathrm{Tr}(\mathcal{D}^2ZZ\bar XZX)+0.553\mathrm{Tr}(\mathcal{D}^2ZZ\bar ZZZ)-0.894\mathrm{Tr}(\mathcal{D}^2ZXZZ\bar X)-0.500\mathrm{Tr}(\mathcal{D}^2ZXZ\bar XZ)-0.0528\mathrm{Tr}(\mathcal{D}^2ZX\bar XZZ)+$$

$$0.894\mathrm{Tr}(\mathcal{D}^2ZYZZ\bar Y)+0.500\mathrm{Tr}(\mathcal{D}^2ZYZ\bar YZ)+0.0528\mathrm{Tr}(\mathcal{D}^2ZY\bar YZZ)+0.894\mathrm{Tr}(\mathcal{D}^2Z\bar YZZY)+0.500\mathrm{Tr}(\mathcal{D}^2Z\bar YZYZ)+0.0528\mathrm{Tr}(\mathcal{D}^2Z\bar YYZZ)-0.894\mathrm{Tr}(\mathcal{D}^2Z\bar XZZX)-0.500\mathrm{Tr}(\mathcal{D}^2Z\bar XZXZ)-$$

$$0.0528\mathrm{Tr}(\mathcal{D}^2Z\bar XXZZ)+1.45\mathrm{Tr}(\mathcal{D}^2Z\bar ZZZZ)-0.894\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi ZZ\bar X)-0.500\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi Z\bar XZ)-0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi\bar XZZ)+0.894\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi ZZY)+0.500\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi ZYZ)+0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi YZZ)-$$

$$1.45\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi ZZZ)-0.500\mathrm{Tr}(\mathcal{D}\bar\Psi Z\Psi Z\bar X)+0.500\mathrm{Tr}(\mathcal{D}\bar\Psi Z\Psi ZY)-0.553\mathrm{Tr}(\mathcal{D}\bar\Psi Z\Psi ZZ)-0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi ZZ\Psi\bar X)+0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi ZZ\Psi Y)-0.553\mathrm{Tr}(\mathcal{D}\bar\Psi ZZ\Psi Z)-1.45\mathrm{Tr}(\mathcal{D}\bar\Psi ZZZ\Psi)+$$

$$0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi ZZY\Psi)-0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi ZZ\bar X\Psi)+0.500\mathrm{Tr}(\mathcal{D}\bar\Psi ZYZ\Psi)-0.500\mathrm{Tr}(\mathcal{D}\bar\Psi Z\bar XZ\Psi)+0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi Y\Psi ZZ)+0.500\mathrm{Tr}(\mathcal{D}\bar\Psi YZ\Psi Z)+0.894\mathrm{Tr}(\mathcal{D}\bar\Psi YZZ\Psi)-0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi\bar X\Psi ZZ)-$$

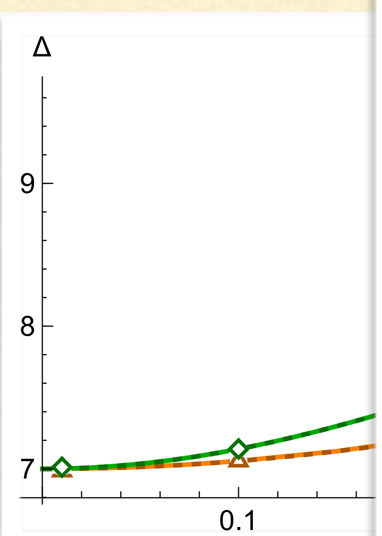
$$0.500\mathrm{Tr}(\mathcal{D}\bar\Psi\bar XZ\Psi Z)-0.894\mathrm{Tr}(\mathcal{D}\bar\Psi\bar XZZ\Psi)+0.894\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi ZZ\bar Y)+0.500\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi Z\bar YZ)+0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi\bar YZZ)-0.894\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi ZZX)-0.500\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi ZXZ)-0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi XZZ)+$$

$$1.45\mathrm{Tr}(\mathcal{D}\bar\Psi\Psi ZZZ)+0.500\mathrm{Tr}(\mathcal{D}\bar\Psi Z\Psi Z\bar Y)-0.500\mathrm{Tr}(\mathcal{D}\bar\Psi Z\Psi ZX)+0.553\mathrm{Tr}(\mathcal{D}\bar\Psi Z\Psi ZZ)+0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi ZZ\Psi\bar Y)-0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi ZZ\Psi X)+0.553\mathrm{Tr}(\mathcal{D}\bar\Psi ZZ\Psi Z)+1.45\mathrm{Tr}(\mathcal{D}\bar\Psi ZZZ\Psi)-$$

$$0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi ZZX\Psi)+0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi ZZ\bar Y\Psi)-0.500\mathrm{Tr}(\mathcal{D}\bar\Psi ZXZ\Psi)+0.500\mathrm{Tr}(\mathcal{D}\bar\Psi Z\bar YZ\Psi)-0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi X\Psi ZZ)-0.500\mathrm{Tr}(\mathcal{D}\bar\Psi XZ\Psi Z)-0.894\mathrm{Tr}(\mathcal{D}\bar\Psi XZZ\Psi)+0.0528\mathrm{Tr}(\mathcal{D}\bar\Psi\bar Y\Psi ZZ)+$$

$$0.500\mathrm{Tr}(\mathcal{D}\bar\Psi\bar YZ\Psi Z)+0.894\mathrm{Tr}(\mathcal{D}\bar\Psi\bar YZZ\Psi)+1.45\mathrm{Tr}(\bar{\mathcal{F}}\mathcal{F}ZZZ)+0.106\mathrm{Tr}(\bar{\mathcal{F}}\Psi\Psi ZZ)+1.00\mathrm{Tr}(\bar{\mathcal{F}}\Psi Z\Psi Z)+1.79\mathrm{Tr}(\bar{\mathcal{F}}\Psi ZZ\Psi)-0.106\mathrm{Tr}(\bar{\mathcal{F}}\Psi\Psi ZZ)-1.00\mathrm{Tr}(\bar{\mathcal{F}}\Psi Z\Psi Z)-1.79\mathrm{Tr}(\bar{\mathcal{F}}\Psi ZZ\Psi)+$$

$$0.553\mathrm{Tr}(\bar{\mathcal{F}}Z\bar{\mathcal{F}}ZZ)+1.00\mathrm{Tr}(\bar{\mathcal{F}}Z\Psi Z\Psi)-1.00\mathrm{Tr}(\bar{\mathcal{F}}Z\Psi Z\Psi)+0.553\mathrm{Tr}(\bar{\mathcal{F}}ZZ\bar{\mathcal{F}}Z)+0.106\mathrm{Tr}(\bar{\mathcal{F}}ZZ\Psi\Psi)-0.106\mathrm{Tr}(\bar{\mathcal{F}}ZZ\Psi\Psi)+1.45\mathrm{Tr}(\bar{\mathcal{F}}ZZZ\mathcal{F})$$



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