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30/10/24

5d Conformal Matter SCFTs across dimensions

Based on past and upcoming work in collaboration with:

Michele Del Zotto, Mario De Marco, Michele Graffeo, Julius Grimminger

JHEP 05 (2024) 306, arXiv 2411……

Context: the landscape of QFTs

Motivation: chart the landscape of **QFTs** (incredibly hard problem!)

"Cheating":

- Focus on **supersymmetric QFTs**
- Fixed points of RG flows, i.e. **superconformal field theories (SCFTs)**

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"Cheating":

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- The last two decades have produced a deluge of results in several dimensions and with varying amount of supercharges, mostly employing the toolbox provided by **string theory/M-theory/F-theory**

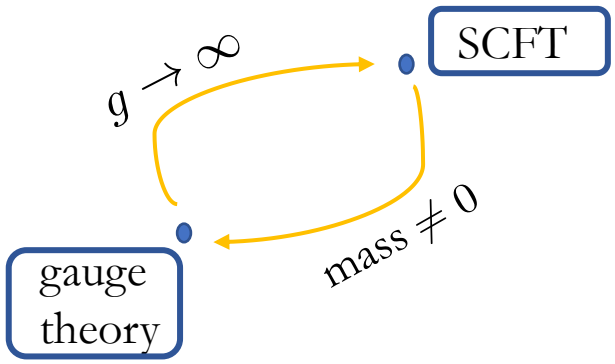
8 supercharges is the sweet spot: non-trivial dynamics, but sufficiently constrained to admit a (partial) classification

- In this talk: 5d $\mathcal{N} = 1$ SCFTs,
  and relations to 4d $\mathcal{N} = 2$ and 3d $\mathcal{N} = 4$

5d $\mathcal{N} = 1$ SCFTs

- Very difficult to treat from a gauge-theoretic perspective:

In 5d: $\left[\frac{1}{g^2}\right] = \text{mass}$



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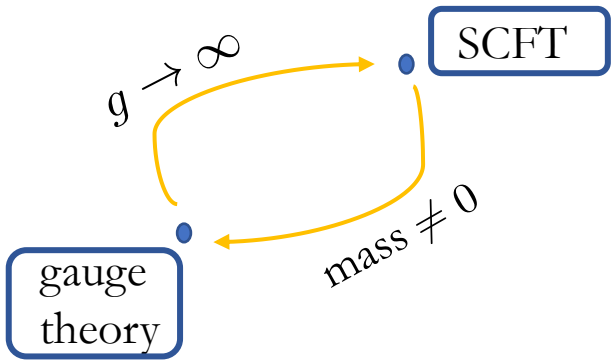
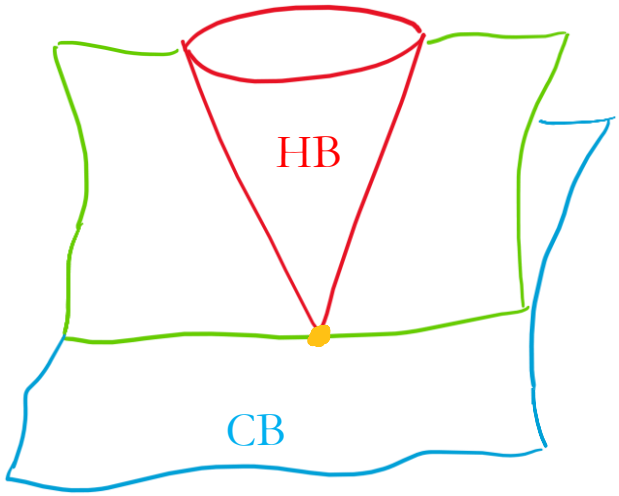


Since the mid-ninenties, thriving effort in the study of the properties of 5d SCFTs.

Interesting aspects to investigate:

- **Moduli spaces** of vacua
- **RG flows**
- **Symmetries** (gauge, flavor, generalized symmetries…)



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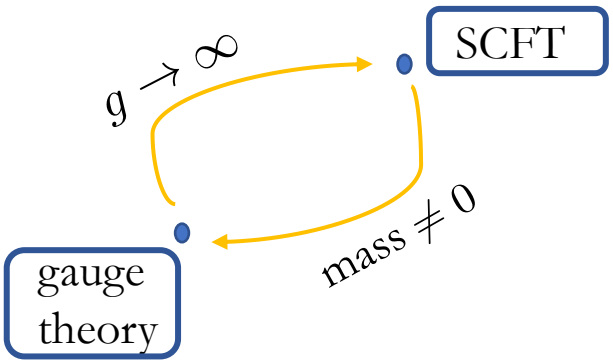

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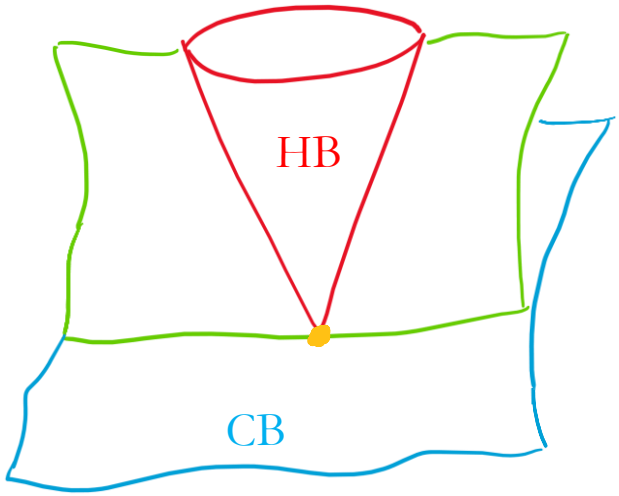

In recent years, huge investment in investigating 5d SCFTs:

- (p,q) 5-brane webs
- String dualities
- M-theory compactification on CY threefolds **(geometric engineering)**

Geometric engineering of 5d SCFTs

M-theory geometric engineering: study 5d SCFTs using M-theoretic setups involving only **geometry**

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$11d$ **M-theory**

dimensional reduction on
singular
non-compact
CY 3-fold

5d SCFT (8 supercharges)

Flat spacetime $\mathbb{R}^{4,1}$

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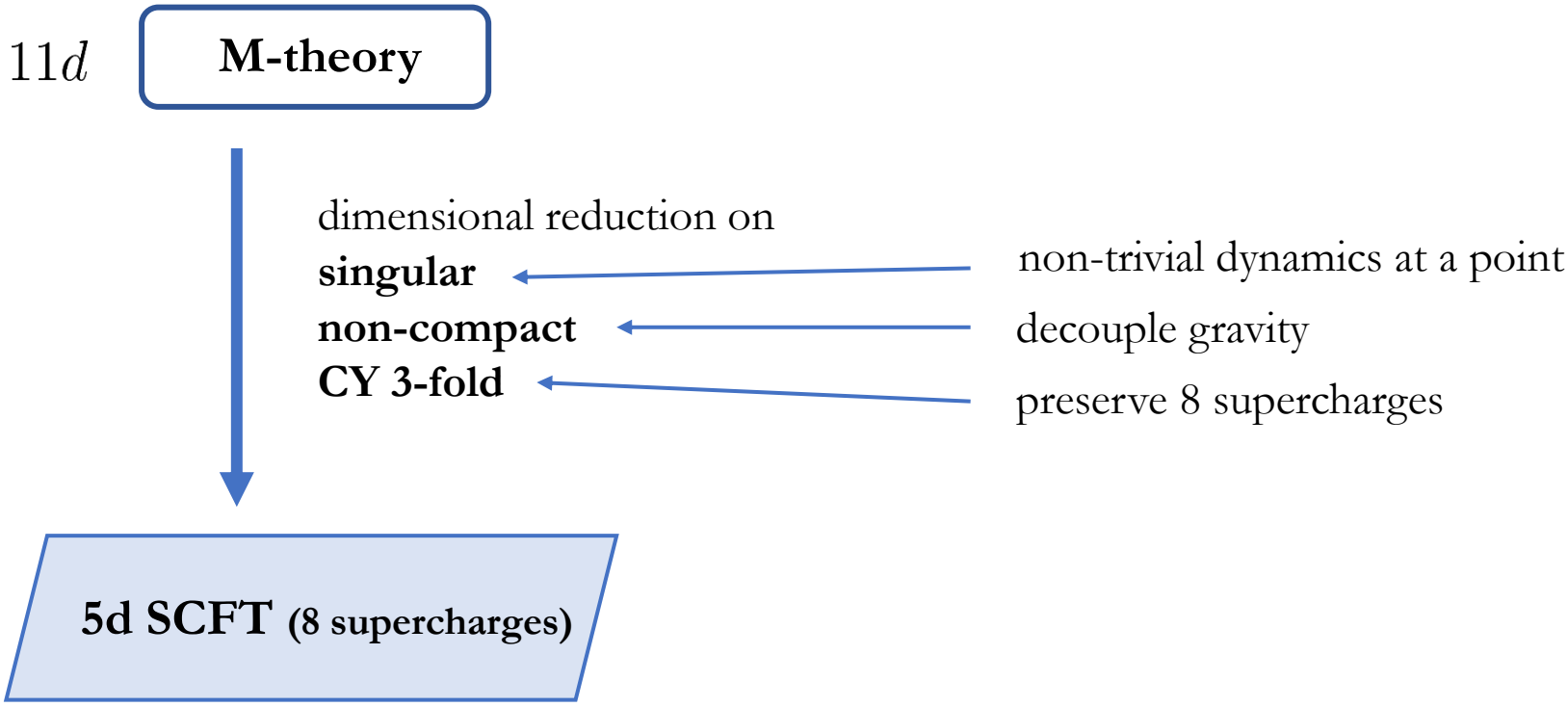

M-theory/5d SCFTs dictionary

5d SCFT | **DICTIONARY** | **CY threefold**

- Coulomb branch parameters (gauge symmetry)
- Mass parameters (flavor symmetry)
- Higgs branch parameters

- Compact divisors
- Non-compact divisors (dual to complex compact curves)
- Complex deformations ⟷ M2-brane states (GV invariants)

Physics ⟷ **Geometry**

Atomic classification of 5d SCFTs

- **Motivating question:** find analogue of 6d conformal matter (bifundamental matter with non-trivial gauge dynamics) in 5d setting

$\mathfrak{g} \times \mathfrak{g}$ flavor group

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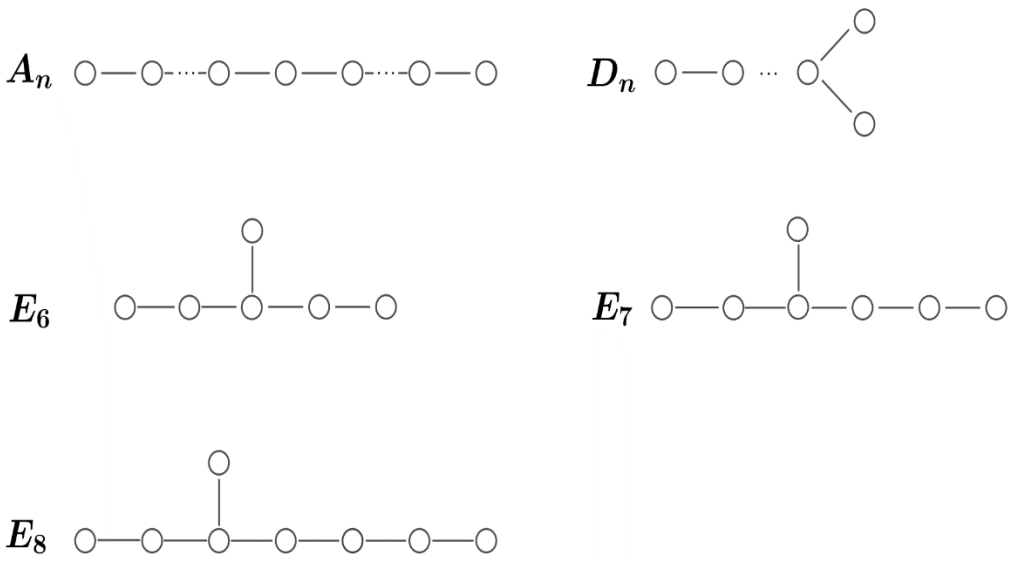
- **Geometric answer:**

5d conformal matter $(\mathfrak{g}, \mathfrak{g})$, $\mathfrak{g} \in ADE$
can be constructed from M-theory geometric engineering

$$\begin{cases} \boldsymbol{A_n}: & x^2 + y^2 + z^{n+1} = 0 \\ \boldsymbol{D_n}: & x^2 + zy^2 + z^{n-1} = 0 \\ \boldsymbol{E_6}: & x^2 + y^3 + z^4 = 0 \\ \boldsymbol{E_7}: & x^2 + y^3 + yz^3 = 0 \\ \boldsymbol{E_8}: & x^2 + y^3 + z^5 = 0 \end{cases}$$

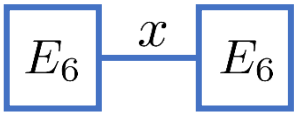


Atoms of 5d CM SCFTs

- **Example:**

5d conformal matter (E_6, E_6) can be built from M-theory geometric engineering on:

"Atom" of (E_6, E_6) conformal matter →



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base change — E_6 singularity

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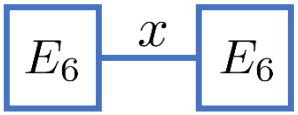

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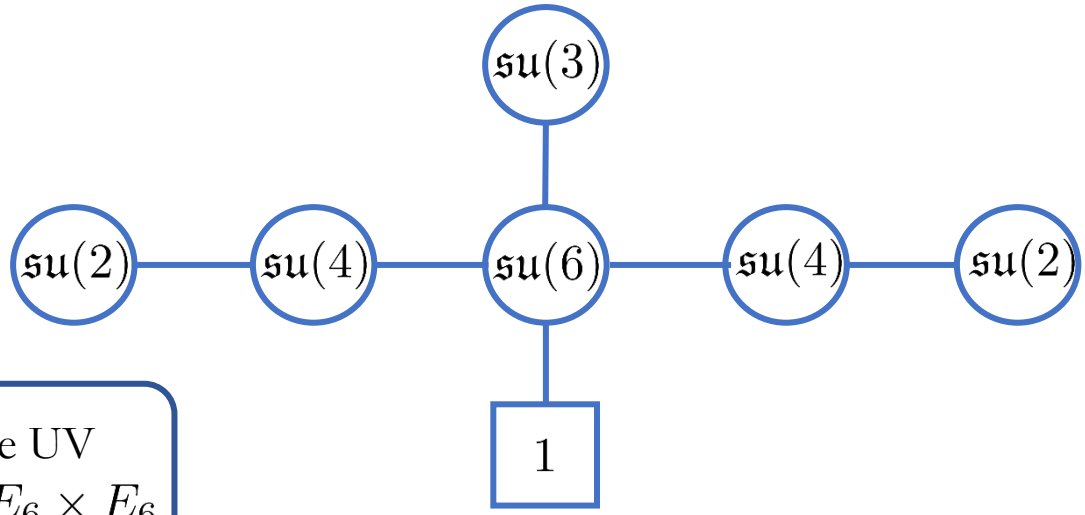
- Resolution: **low-energy quiver gauge theory phase**



Field theory arguments prove that the UV phase has at least **flavor symmetry** $E_6 \times E_6$

Atomic (partial) classification of 5d CM SCFTs

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5d conformal matter (E_6, E_6) can be built from M-theory geometric engineering on:

"Atom" of (E_6, E_6) conformal matter → $E_6 \;\; x \;\; E_6$ **CY3**: $\begin{cases} x^2 + y^3 + z^4 = 0 \\ x = uv \end{cases}$

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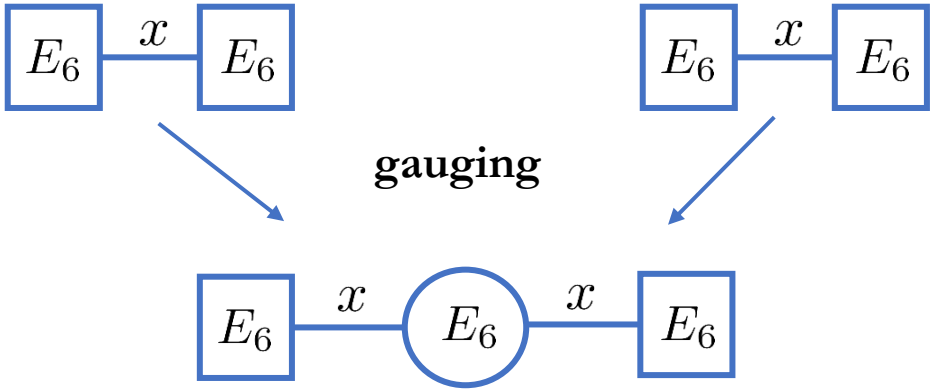
All the "atoms" have at least flavor symmetry $E_6 \times E_6$

Molecules of 5d CM SCFTs

- **5d conformal matter atoms** can be used as fundamental blocks to construct more complicated SCFTs of conformal matter type



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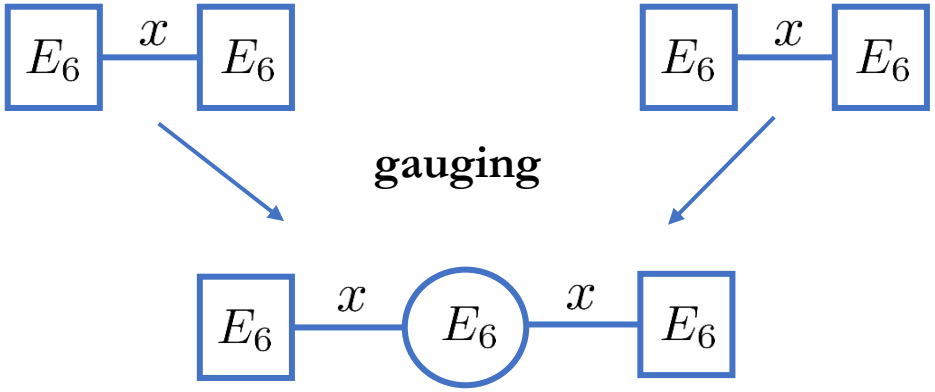

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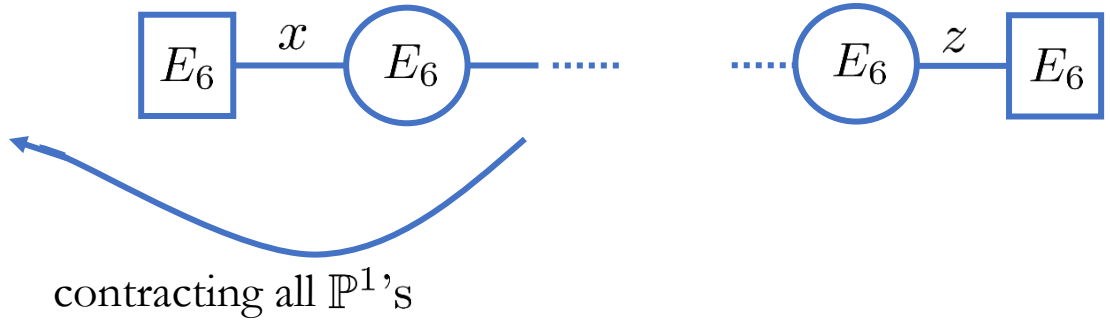
- **Most general molecule:**

CY3: $\begin{cases} x^2 + y^3 + z^4 = 0 \\ x^{n_1} y^{n_2} z^{n_3} = uv \end{cases}$



Circle reduction of 5d CM atoms

We wish to investigate the 4d $\mathcal{N} = 2$ theories obtained reducing 5d CM **atoms** on a circle

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- Non-trivially, the resulting 4d $\mathcal{N}=2$ theory is a **SCFT**

- It is a **class-S** fixture:
  6d $\mathcal{N}=(2,0)$ theory of type $\mathfrak{g} \in ADE$ on a sphere with **three regular punctures**
- Checks:

  matching **CB dimension**
  matching **HB dimension**
  matching **flavor symmetry**

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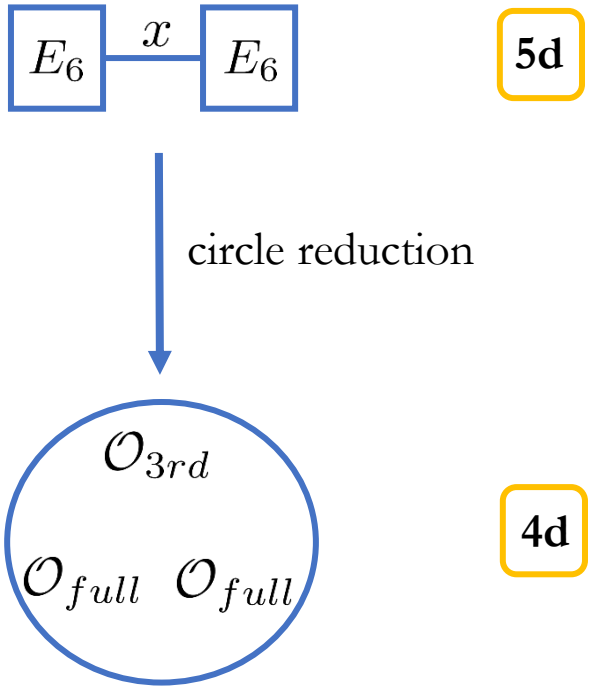

- punctures $\mathcal{O}_{full} \times \mathcal{O}_{full} \times \mathcal{O}_{3rd}$
- flavor sym $\mathfrak{g} \times \mathfrak{g} \times \mathfrak{g}_{rest}$

Circle reduction of 5d CM atoms

- The 4d $\mathcal{N}=2$ SCFT descendants can be identified for all 5d CM atoms:

Atom	$\mathcal{O}_{3rd}$	${\mathcal{O}_{3rd}}^L$	$\mathrm{rank}\,\mathrm{CB}_{\mathcal{T}_{4d}}$	flavor sym
$X^{(1)}_{A_{2j+1}}$	$[2^{j+1}]$	$[(j+1)^2]$	j^2	$A_{2j+1}\times A_{2j+1}\times \mathfrak{u}(1)$
$X^{(1)}_{A_{2j}}$	$[2^j,1]$	$[j+1,j]$	$j(j-1)$	$A_{2j}\times A_{2j}\times \mathfrak{u}(1)$
$X^{(1)}_{D_{2j+2}}$	$[3^2,2^{2j-2},1^2]$	$[(2j+1)^2,1^2]$	$j(2j+3)$	$D_{2j+2}\times D_{2j+2}\times \mathfrak{u}(1)^2$
$X^{(1)}_{D_{2j+3}}$	$[3^2,2^{2j-2},1^4]$	$[2j+3,2j+1,1^2]$	$j(2j+5)+1$	$D_{2j+3}\times D_{2j+3}\times \mathfrak{u}(1)$
$X^{(2)}_{D_{2j+2}}$	$[2^{2j},1^4]$	$[2j+3,2j+1]$	$2j^2+j-2$	$D_{2j+2}\times D_{2j+2}$
$X^{(2)}_{D_{2j+3}}$	$[2^{2j+2},1^2]$	$[(2j+3)^2]$	$j(2j+3)$	$D_{2j+3}\times D_{2j+3}\times \mathfrak{u}(1)$
$X^{(3)}_{D_j}$	$[3,1^{2j-3}]$	$[2j-3,1^3]$	$j-2$	$D_j\times D_j\times \mathfrak{su}(2)$
$X^{(1)}_{E_6}$	A_2	$E_6(a_3)$	15	$E_6\times E_6$
$X^{(2)}_{E_6}$	$2A_1$	D_5	10	$E_6\times E_6\times \mathfrak{u}(1)$
$X^{(3)}_{E_6}$	A_1	$E_6(a_1)$	5	$E_6\times E_6$
$X^{(1)}_{E_7}$	A_2+A_1	$E_6(a_1)$	31	$E_7\times E_7\times \mathfrak{u}(1)$
$X^{(2)}_{E_7}$	$(3A_1)''$	E_6	20	$E_7\times E_7\times \mathfrak{su}(2)$
$X^{(3)}_{E_7}$	A_1	$E_7(a_1)$	10	$E_7\times E_7$
$X^{(1)}_{E_8}$	A_2+A_1	$E_8(a_4)$	60	$E_8\times E_8$
$X^{(2)}_{E_8}$	$2A_1$	$E_8(a_2)$	38	$E_8\times E_8$
$X^{(3)}_{E_8}$	A_1	$E_8(a_1)$	21	$E_8\times E_8$

5d CM SCFTs across dimensions

Web of dualities relating 5d $\mathcal{N}=1$ CM theories with 4d $\mathcal{N}=2$ and 3d $\mathcal{N}=4$

5d

5d CM theory
M-theory on CY3

dual description

(p,q) 5-brane web description
(A and D cases)

4d

3d

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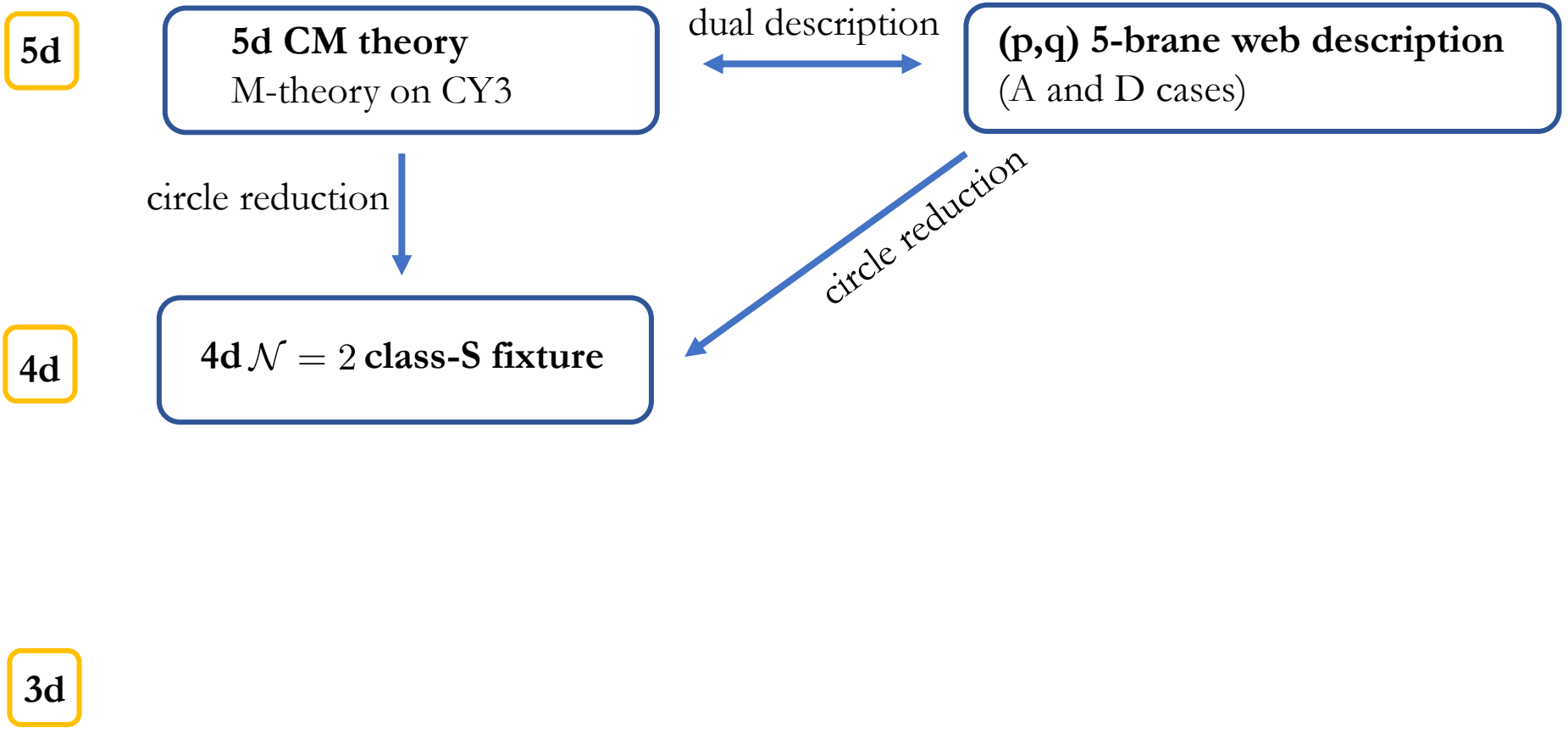

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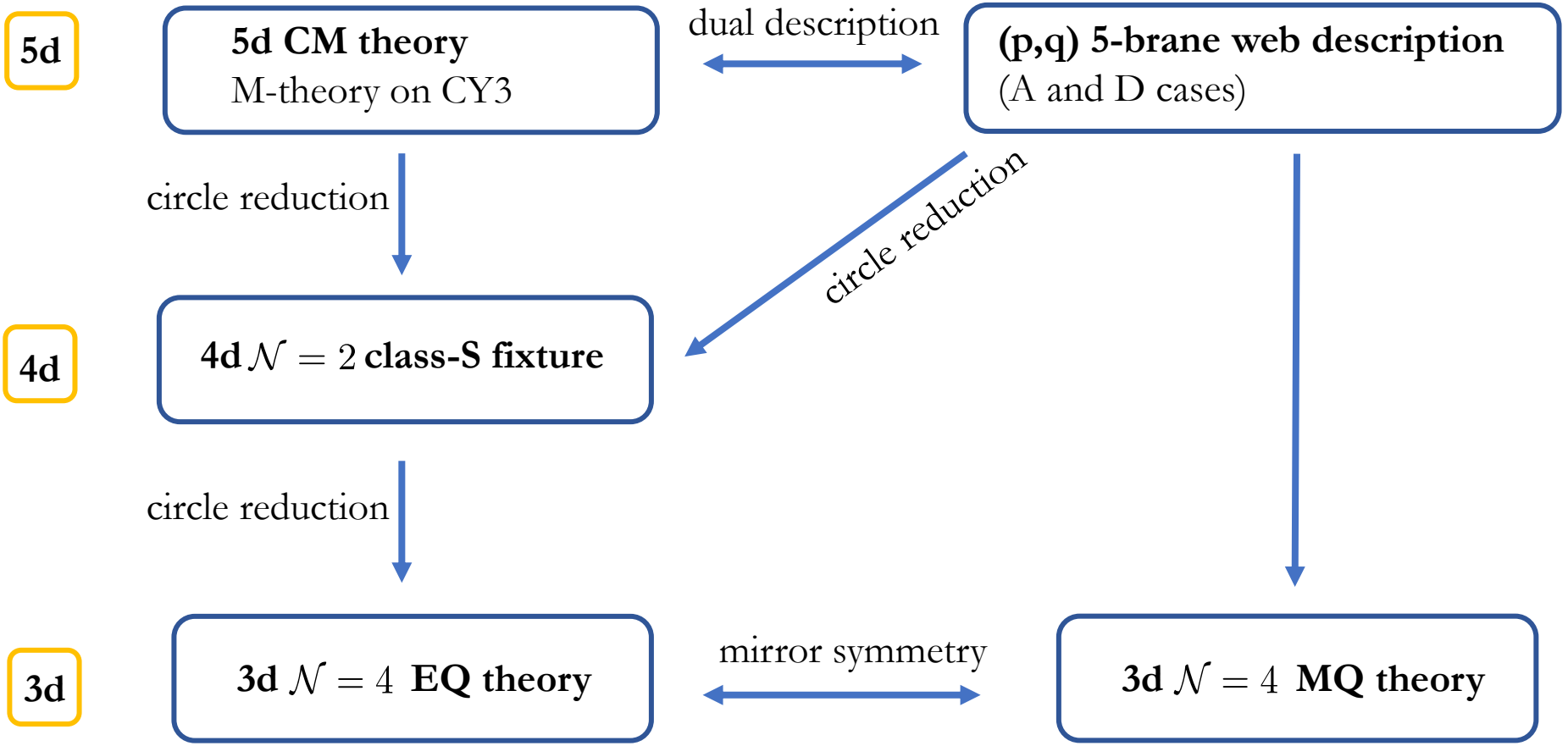

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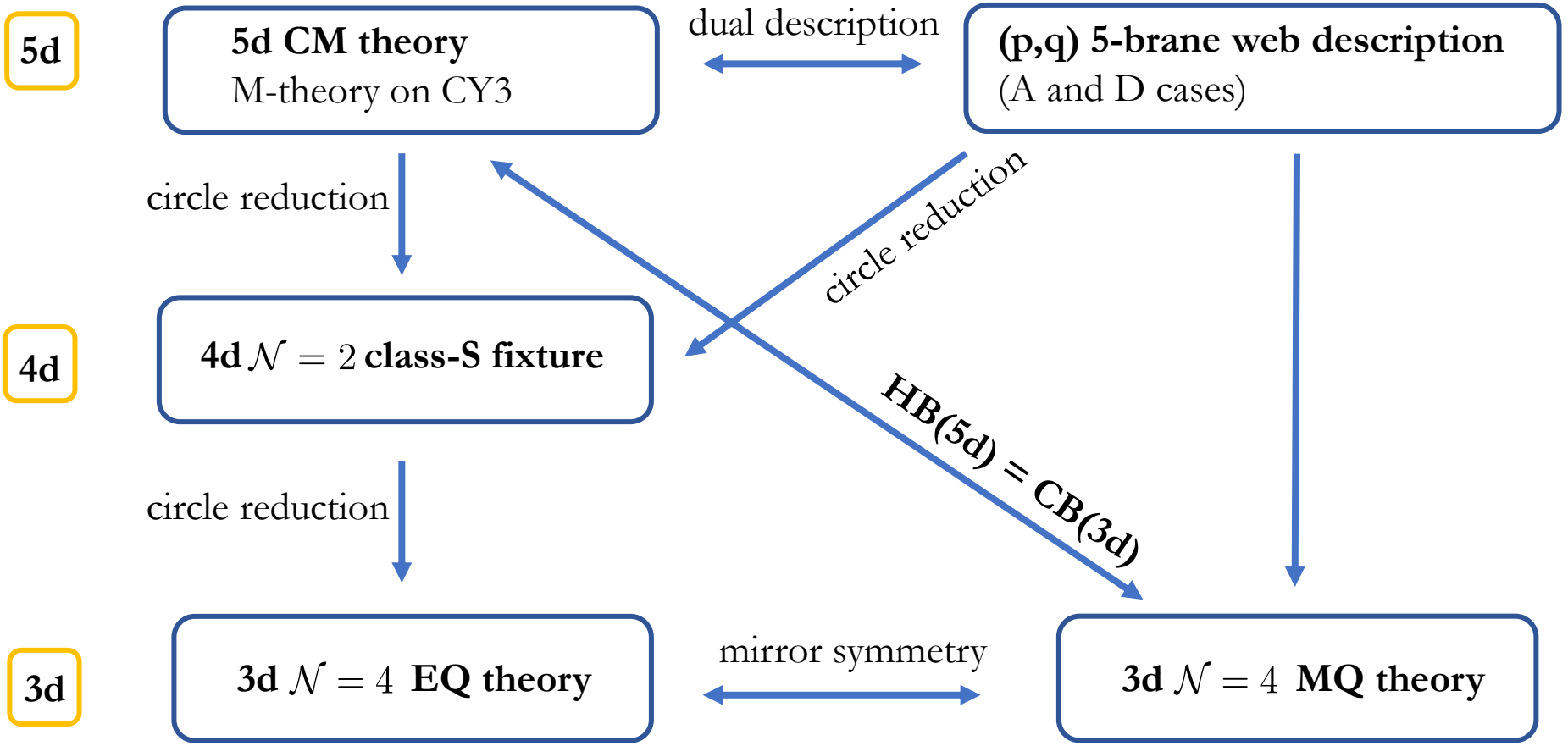
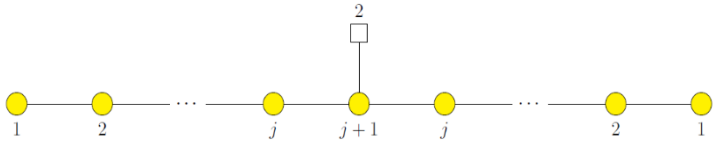


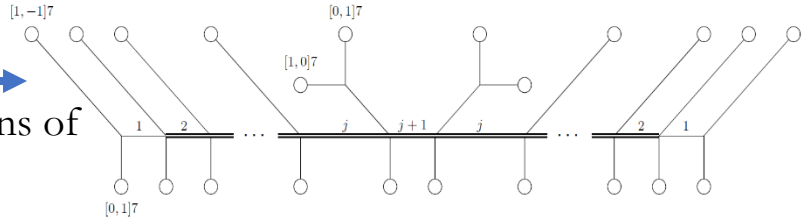
5d CM SCFTs across dimensions (example)

5d conformal matter (A_{2j+1}, A_{2j+1})

5d



dual descriptions of IR phase



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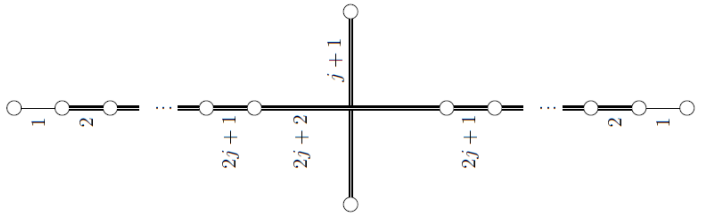
dual descriptions at UV point



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3d

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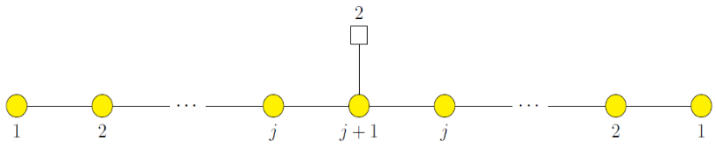

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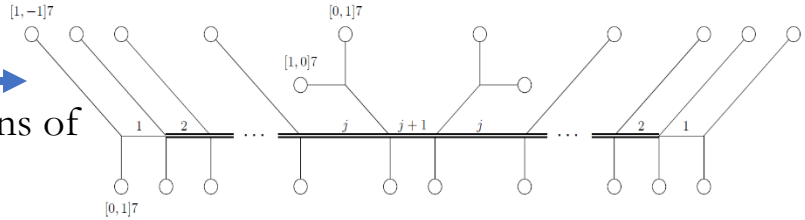

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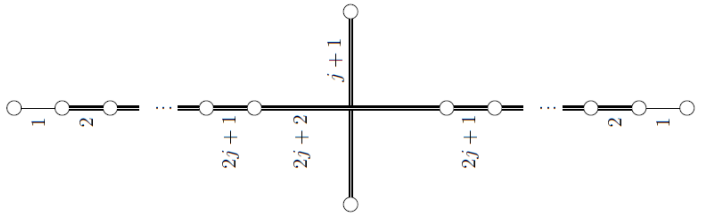

circle reduction

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6d $\mathcal{N} = (2,0)$ A_{2j+1} on: $[2^{j+1}]$ $[2j+2]$ $[2j+2]$

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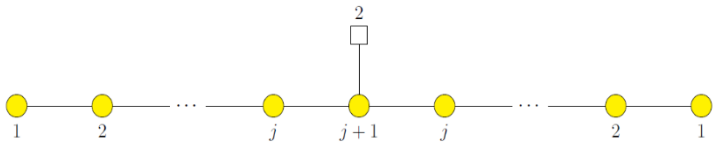

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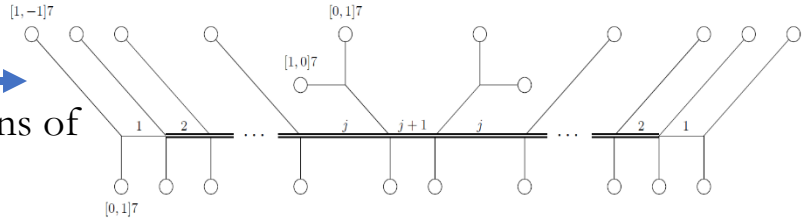

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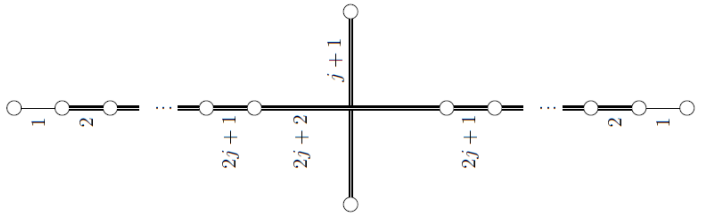

circle reduction

extract MQ

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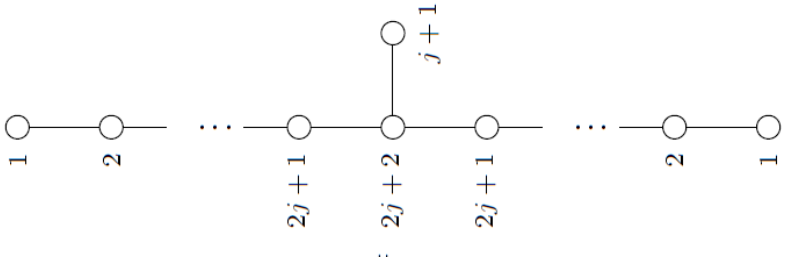

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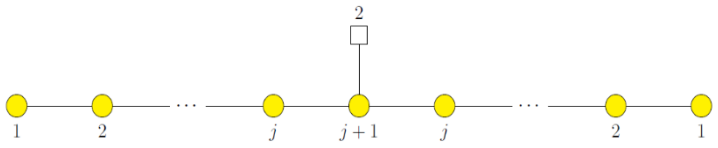

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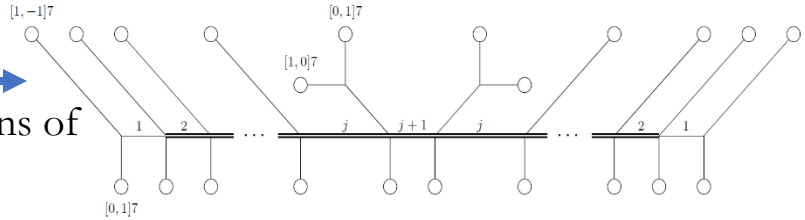

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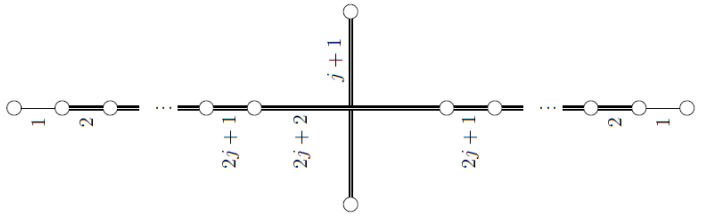

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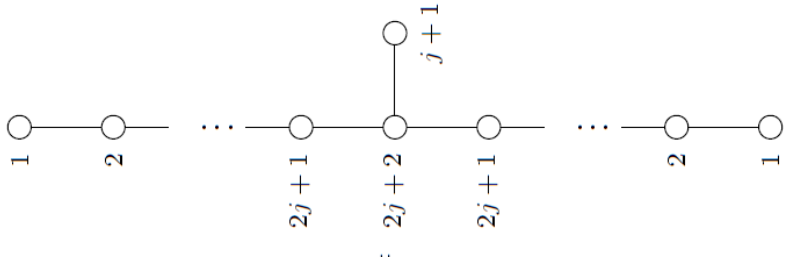
3d

5d HB data at UV point:

- Flavor sym: $\mathfrak{su}(2j+2) \oplus \mathfrak{su}(2j+2) \oplus \mathfrak{u}(1)$
- HB dimension: $4j^2 + 9j + 4$



Conclusions

Introduced new class of **5d SCFTs** with $\mathfrak{g} \times \mathfrak{g}$ flavor symmetry, $\mathfrak{g} \in ADE$:

- they can be constructed as gaugings of a finite set of **fundamental blocks** (atoms)

Exhibited their **circle reduction** and relation to **brane webs** and **magnetic quivers**:

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Future prospects:

- What about 5d SCFTs with $\mathfrak{g} \times \mathfrak{g} \times \mathfrak{g}$ (trinions) or $\mathfrak{g} \times \mathfrak{g} \times \mathfrak{g} \times \mathfrak{g}$ (tetraons) flavor symmetry?
- Have we exhausted all 5d SCFTs with $\mathfrak{g} \times \mathfrak{g}$ flavor symmetry? (answer: no, see e.g. T-branes)

Thank you for your attention!

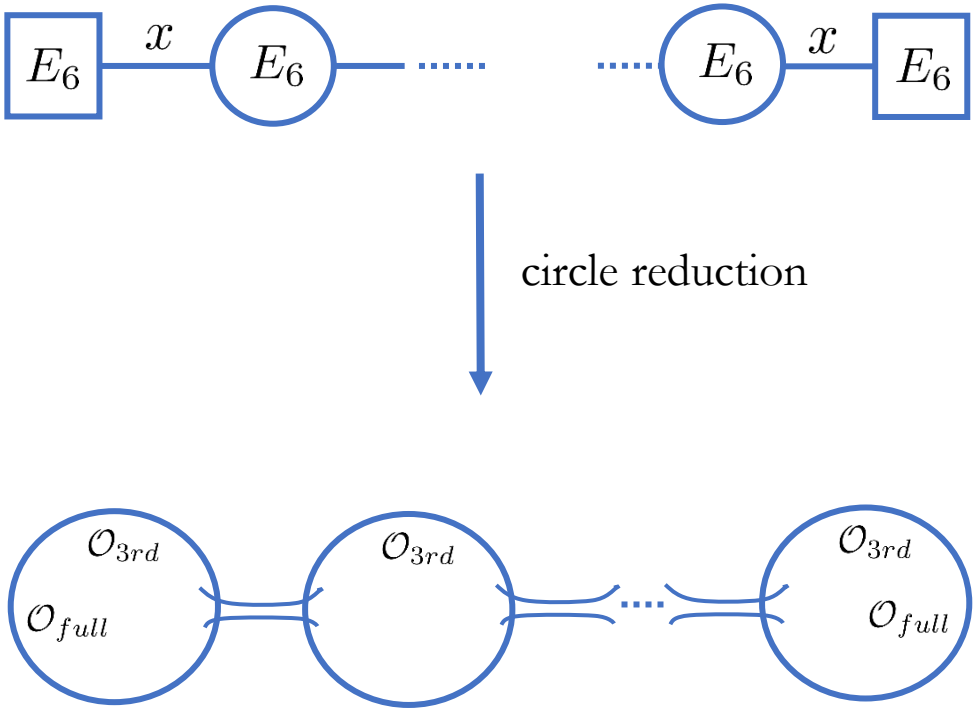
Circle reduction of 5d CM molecules

What about the 4d $\mathcal{N}=2$ theories obtained reducing 5d CM **molecules** on a circle?

- Molecules admit a 4d $\mathcal{N}=2$ **class-S** description which is **not a SCFT**

- It corresponds to a low-energy quiver gauge theory phase of the 5d SCFT engineered by:

CY3: $\begin{cases} x^2+y^3+z^4=0 \\ x^{n_1}=uv \end{cases}$



- punctures $\mathcal{O}_{full} \times \mathcal{O}_{full} \times (\mathcal{O}_{3rd})^{n_1}$
- flavor sym $\mathfrak{g} \times \mathfrak{g} \times (\mathfrak{g}_{rest})^{n_1}$