

– Higgs Branch RG-flows –  
Decay and Fission of Magnetic Quivers

**Marcus Sperling**

33rd Nordic Network Meeting on *Strings, Fields and Branes*

— October 30, 2024 —



# Motivation

**Supersymmetric / superconformal theories** expanded QFT landscape

- higher dim. QFTs
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How to explore?

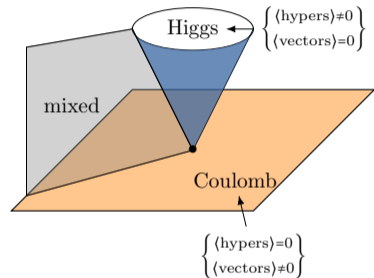
- Ideally: all local operators & correlation functions  $\longrightarrow$  too hard!
- Instead: **simple, robust (and computable) quantities**  
e.g. symmetries, **vacua**, partition functions, ...

# Problem

Despite extensive research on  $d=4,5,6$  theories w/ 8 SUSY,

**Higgs branches**  $\mathcal{H}$  not systematically understood

→ issue:  $\mathcal{H}$  often **beyond hyper-Kähler quotient**



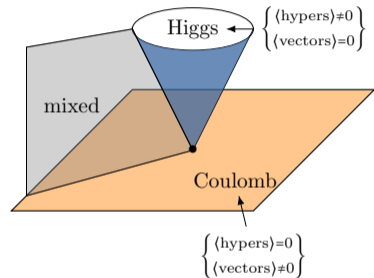
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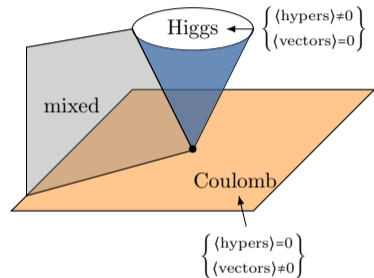
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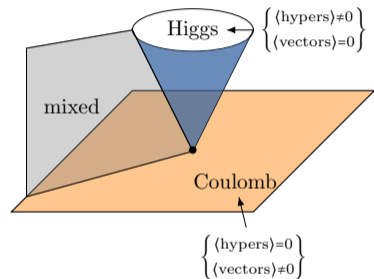
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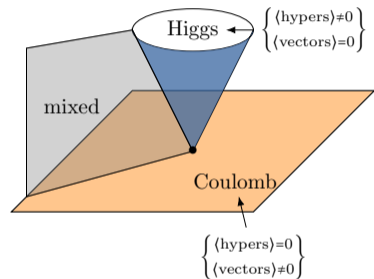
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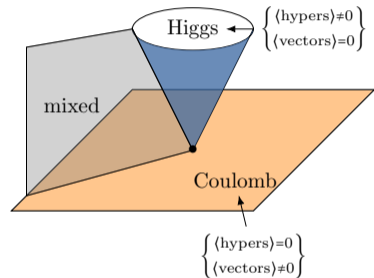
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Q: How to approach “quantum Higgs branches”?

→

**A: Magnetic quivers!**



## Magnetic quivers — the basics

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$$\text{M5} \times \rightarrow \left| \text{M9} \right.$$

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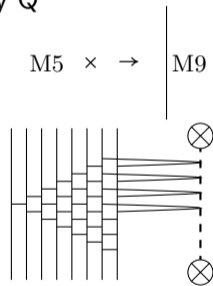
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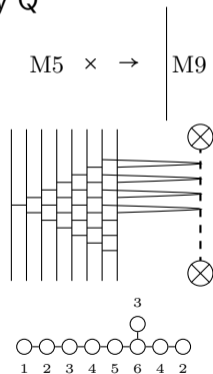
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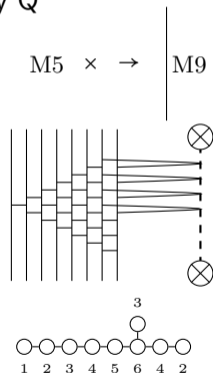
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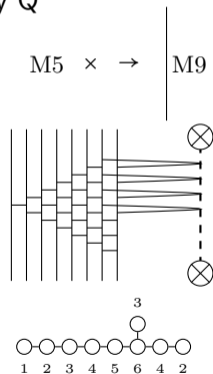
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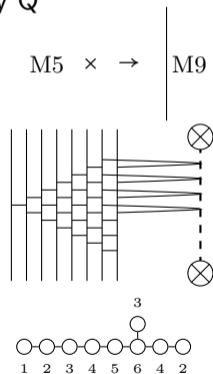
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Comp.  
data?

- ▶ Dimensions
- ▶ Operator spectrum
- ▶ Symmetries
- ▶ Stratification



[Cabrera, Hanany, MS '19]

# Magnetic quiver have been used in ...

## 6d $\mathcal{N} = (1, 0)$ SCFTs

- ▶ Type II [Cabrera, Hanany, MS '19](#); [MS, Zhong '21](#); [Hanany, MS '22](#); Bourget, Gimminger '22; Fazzi Giacomelli, Giri '23; ...
- ▶ F-theory inspired [Lawrie, Mansi '23](#); [Bao, Zhang '24](#)

## 6d LSTs

- ▶ Type II based [Del Zotto, Fazzi, Giri '23](#); [Lawrie, Mansi '23](#); [Mansi, MS '23](#), ...

## 5d $\mathcal{N} = 1$ SCFTs

- ▶ 5-branes [Cabrera, Hanany '18](#); [Bourget, Grimminger, Hanany, MS, Zhong '20](#); Akhond, Carta, Dwivedi, Hayashi, Kim, Yagi '20 & '22; [Akhond, Carta '21](#); ...
- ▶ M-theory on (toric) CY3 [Beest, Bourget, Eckhard, Schäfer-Nameki '20](#), [Closset, Schäfer-Nameki, Wang '20](#); ...

## 4d $\mathcal{N} = 2$ SCFTs

- ▶ Class  $\mathcal{S}$  [Benini, Tachikawa, Xie '15](#)
- ▶ Argyres Douglas [Bourget, Gimminger, Hanany, MS, Zhong '20](#); [Giacomelli, Mekareeya, Mininno '21](#); ....

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## 6d LSTs

- ▶ Type II base

**Magnetic Quivers** = **novel access** to quantum Higgs branch ✓  
→ geometry, duality checks, classifications, ...

## 5d $\mathcal{N} = 1$ SCFT

- ▶ 5-branes [Cabrera](#)

but missing **connection to RG-flows** ✗

'20 & '22; [Akhshor](#)

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[Bourget, MS, Zhong: *Phys.Rev.Lett.* 132 (2024) 22 & *Phys.Rev.D* 109 (2024) 12]

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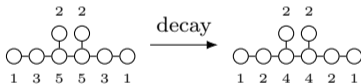
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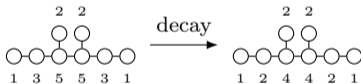
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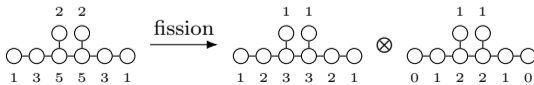
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- Fission:** shape remains, rank vector splits



# Higgs branch RG-flows via Decay & Fission

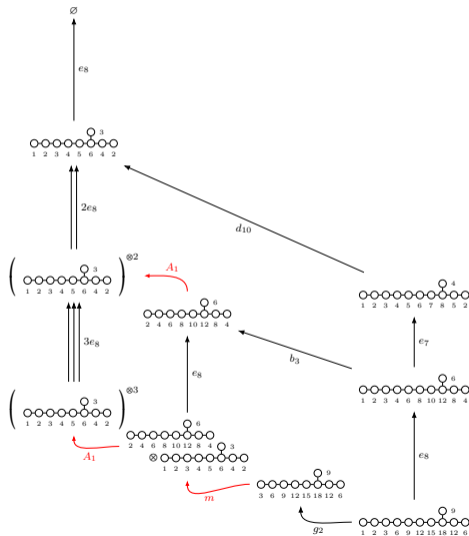
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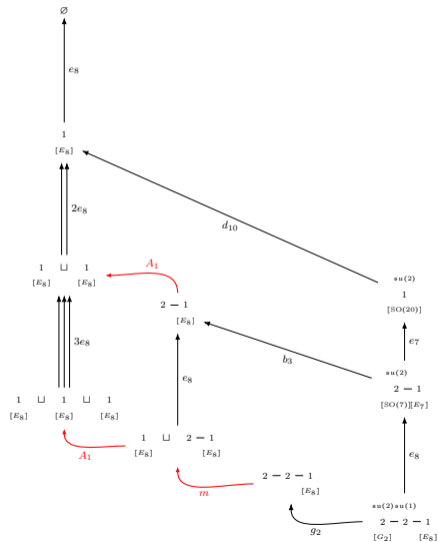
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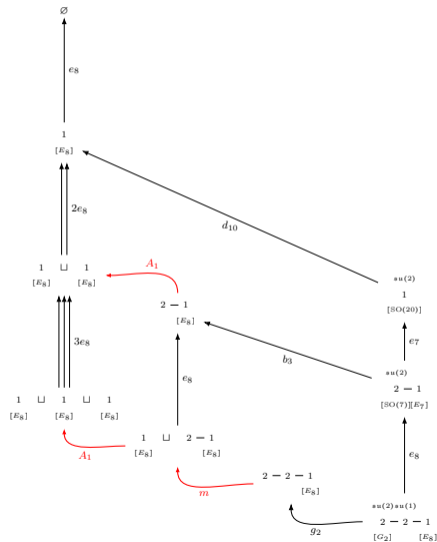
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Distinct Higgsing types

- **Decay** – a.k.a. nilpotent Higgsing
- **Fission** into products

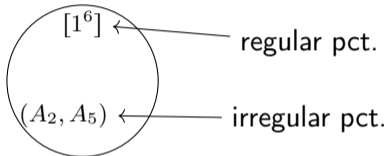






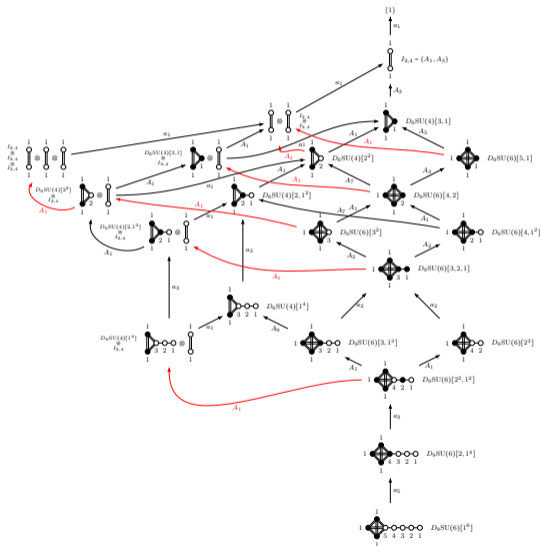
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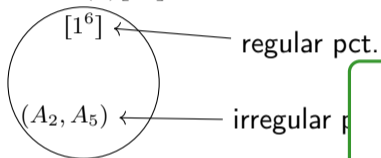
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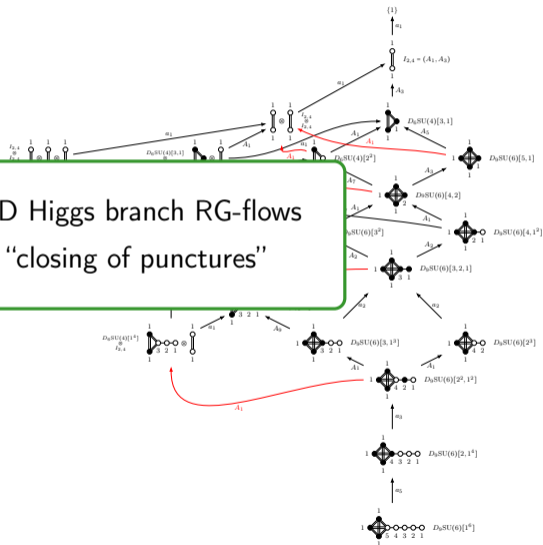
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4d class  $\mathcal{S}$  / AD Higgs branch RG-flows  
more than just “closing of punctures”

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- **Magnetic quivers** = window into geometry, symmetries, ...
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