Matrix Theory Reloaded:
A BPS Road to Holography

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Vetenskapsrådet

based on work:

2410.03591, 2311.10564 (PRL) (Blair,Lahnsteiner,NO,Yan)

& to appear **24xx.yyyyy** (Harmark,Lahnsteiner,NO) → talk Troels Harmark tommow.

also: 2107.006542 (JHEP) (Bidussi,Harmark,Hartong,NO,Oling)

& earlier papers with Harmark,Hartong, and Menculini/Oling/Yan

Introduction/Motivation

- D-branes & their BPS nature underly major advances in ST
  - AdS/CFT correspondence (dual descriptions of D-branes)
  - matrix theory proposal (large N limit of D-brane wv. theory)

→ decoupling limits probe rich physics
  - simplification by removing part of spectrum
  - access to non-perturbative regimes

revisit decoupling limits in light of recent advances

- intruiging relations to non-Lorentzian corners of string theory
  e.g. NRST as a simpler corner of (relativistic) ST

  Gomis,Ooguri(2000),Danielsson,Guijosa,Kruczenski(2000)
  Harmark,Hartong,NO(2017)/Bergshoeff,Gomis,Yan(2018) /& many papers since

- new boostrap techniques/study of amplitudes in BFSS

  Han,Hartnoll,Kruthoff(2020)/Dorey,Mouland,Zhao(2022)/Mouland(2023),….
  Tropper,Wang(2023)/ Herderschee, Maldacena (2023)/Komatsu et al (2024)…

Questions

1. What are the guiding principles for mapping out self-consistent decoupling limits in ST ?

   Guided by a BPS road..

2. In the context of holography, what is the role of the ten-dimensional non-Lorentzian (NL) geometry coupled to matrix theory on the D-branes ?

   To Holography …

3. How is the ten-dimensional bulk geometry generated intrinsically ?

   And back via TTbar

Matrix Theory: A BPS perspective

Matrix theory arises from BPS decoupling limit of D-brane wv.

→ relies on fact that tension=charge & fine tuning RR gauge field

$$S_{\mathrm{D0}} = -\frac{1}{g_s\sqrt{\alpha'}}\int \mathrm{d}\tau\,\sqrt{-\dot{X}^{\mu}\dot{X}_{\mu}} + \frac{1}{\sqrt{\alpha'}}\int C^{(1)}\,, \qquad \mu = 0\,,\cdots,9\,.$$

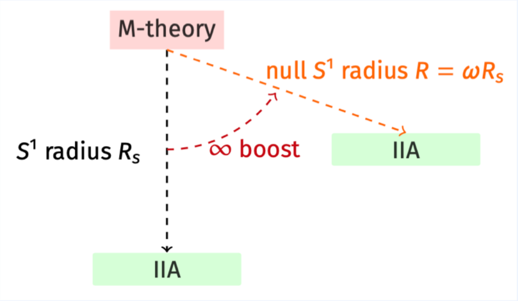
take limit: $g_s \to \omega^{-3/2}\,g_s\,, \quad X^0 \to \sqrt{\omega}\,X^0\,, \quad X^i \to \frac{X^i}{\sqrt{\omega}}\,, \quad C^{(1)} \to \omega^2\,g_s^{-1}\,\mathrm{d}X^0\,,$ $\quad \omega \to \infty.$

gives NR action: $$S = \frac{1}{g_s\sqrt{\alpha'}}\int \mathrm{d}\tau\left(\tfrac{1}{2}\,\dot{X}^i\,\dot{X}^i + 2\,\psi^{\intercal}\,\dot{\psi}\right),$$

non-abelian generalization is BFSS:

$$S_{\mathrm{BFSS}} = \frac{1}{R}\int \mathrm{d}\tau\,\mathrm{tr}\left[\tfrac{1}{2}\dot{X}^i\,\dot{X}^i + \tfrac{1}{4}[X^i\,,X^j]\,[X_i\,,X_j] + 2\Big(\psi^{\intercal}\,\dot{\psi} - \psi^{\intercal}\,\gamma^i\,[\psi\,,X^i]\Big)\right],$$

- BFSS (1996) conjecture: N,R to infinity = M-theory on flat spacetime
- Susskind (1997) fixed N: DLCQ of M-theory
- Seiberg/Sen (1997):
  light like circle as an infinite boost of spacelike circle



Intermezzo: Non-Lorentzian geometry

- start with flat relativistic metric

$$ds^2 = \omega\, dx^A\, dx^B\, \eta_{AB} + \omega^{-1}\, dx^{A'}\, dx^{A'}\,,$$

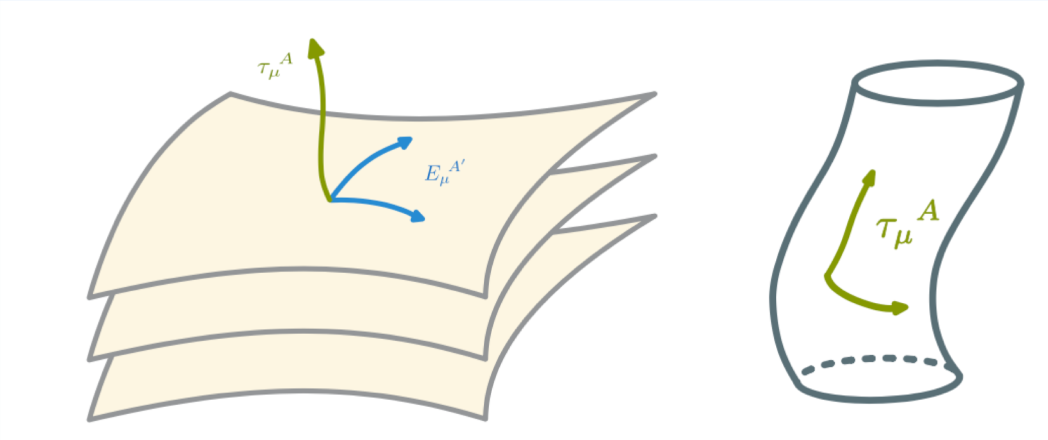
A =0..p longitudinal directions
A'= p+1,.. 9-p transverse directions

$\omega \to \infty$. geometry becomes non-Lorentzian: p-brane Newton-Cartan (NC) geometry

→ local SO(1,p) and SO(9-p) geometry
& p-brane boost symmetry

$$\delta_G x^A = 0\,, \qquad \delta_G x^{A'} = \Lambda^{A'}{}_A\, x^A\,,$$

- generalize to curved bgrs. $dx^A \to \tau_\mu{}^A\, dx^\mu$ $\quad dx^{A'} \to E_\mu{}^{A'}\, dx^\mu$



$$\tau_{\mu\nu} = \tau_\mu{}^A\, \tau_\nu{}^B\, \eta_{AB}\,, \qquad E_{\mu\nu} = E_\mu{}^{A'}\, E_\nu{}^{A'}\,,$$

M0T/MpT in curved spacetime

curved space of M0T

$$G_{\mu\nu} = \omega\,\tau_{\mu\nu} + \omega^{-1}\,E_{\mu\nu}\,, \qquad \Phi = \varphi - \tfrac{3}{2}\ln\omega\,, \qquad B^{(2)} = b^{(2)}\,,$$

$$C^{(1)} = \omega^2\,e^{-\varphi}\,\tau^0 + c^{(1)}\,, \qquad C^{(q)} = c^{(q)} \quad \text{if } q \neq 1\,.$$

$$\omega \to \infty.$$

Matrix theory of D0-branes (M0T) couples to a non-Lorentzian 10D geometry:

M0T target space = Newton-Cartan geometry + other string fields

$$S^{\text{M0T}}_{\text{D0}} = \frac{1}{2\sqrt{\alpha'}}\int \mathrm{d}\tau\, e^{-\varphi}\,\frac{\dot{X}^\mu\,\dot{X}^\nu\,E_{\mu\nu}}{\dot{X}^\mu\,\tau_\mu{}^0} + \frac{1}{\sqrt{\alpha'}}\int c^{(1)}\,.$$

- as opposed to earlier work: now applied to the full type II string theory containing all possible extended objects and in general backgrounds:

via T-duality → MpT = near-BPS decoupling limit of Dp-branes

MpT target space = p-brane NC geometry + other string fields

AdS/CFT

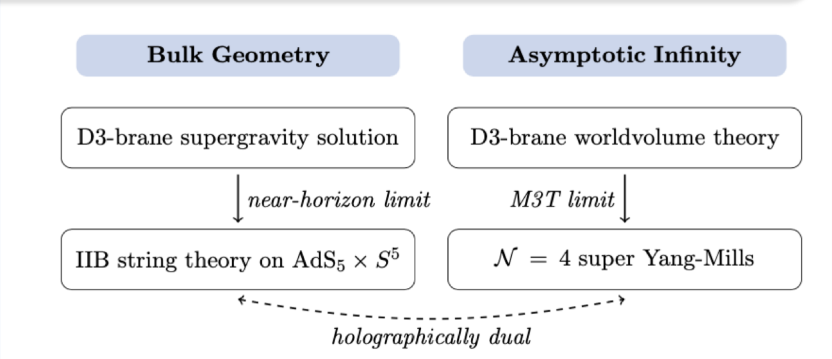
- BPS decoupling limit at asymptotic infinity
  → N=4 SYM coupled to non-Lorentzian M3T geometry
- **same** asymptotic BPS decoupling limit, applied to the D3-brane geometry generates the near-horizon limit !

$$(X^0, \cdots, X^p) = \omega^{\frac{1}{2}} (t, x^1, \cdots, x^p),$$

$$(X^{p+1}, \cdots, X^9) = \omega^{-\frac{1}{2}} (x^{p+1}, \cdots, x^9),$$

$$G_s = \omega^{\frac{p-3}{2}} g_s$$

$$C^{(p+1)} = \omega^2 g_s^{-1} \, \mathrm{d}t \wedge \cdots \wedge \mathrm{d}x^p .$$



$$\mathrm{d}s^2 = \Omega(r)\, \mathrm{d}x^A \, \mathrm{d}x^B \, \eta_{AB} + \frac{\mathrm{d}r^2 + r^2 \, \mathrm{d}\Omega^2_{8-p}}{\Omega(r)}, \qquad \Omega(r) = \left(\frac{r}{\ell}\right)^{\frac{7-p}{2}},$$

- applies to general Dp-brane solutions (IMSY)

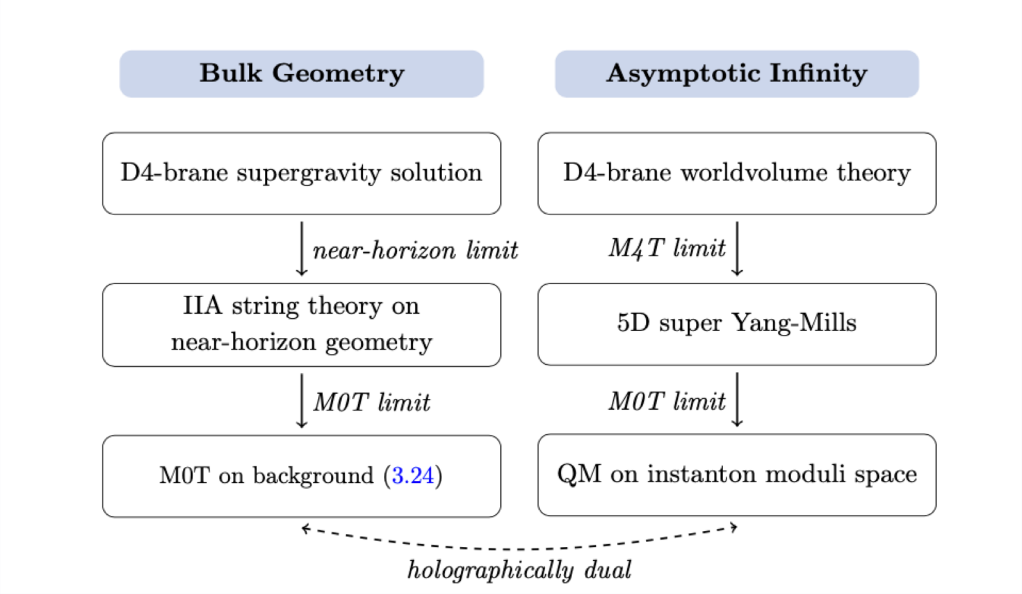
- near-horizon geometries themselves asymptotically approach an MpT limit
  - yields back NL geometry seen by SYM at asymptotic infinity
- geometrically explains relationship between matrix theory & AdS/CFT decoupling

Landscape of holographic duals

- can use this perspective to generate new examples of holographic bulk geometries
  → can do asymptotic MpT limit and apply to bulk Dq-brane with (p not q)

involves double BPS decoupling limit zooming in on intersecting background bound states

Ex.



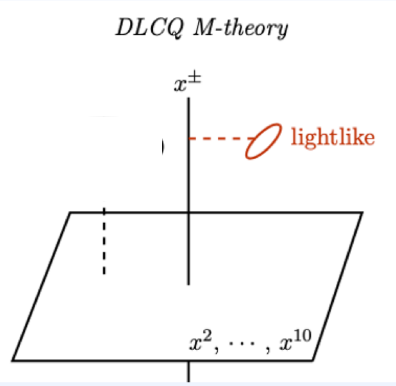
many more duals can be generated: involving NL geometries in the bulk !

see also: Lambert,Smith(2024)/Fontanella,Nieto-Garcia(2024)

Holography as DLCQ^n/DLCQ^m correspondence

can unify the BPS deocupling limit using M-theory and U-dualities

- M0T BPS decoupling limit
  = DLCQ in M-theory



- single BPS decoupling limits lie in U-duality orbit corresponding to ½ BPS
- double BPS decoupling limits lie in U-duality orbit corresponding to ¼ BPS

Blair,Lahnsteiner,NO,Yan (2023) Gomis,Yan (2023)
Dijkgraaf,de Boer,Harmark,NO (unpublished)

→ holographic dulaities unified as DLCQ^n/DLCQ^m with m > n

- AdS_5/CFT_4 = DLCQ^0/DLCQ^1 : DLCQ at asympt. inf = near-horizon

- AdS_3/CFT_2 = DLCQ^0/DLCQ^2

- novel holographic dualities with NL geometry in bulk: DLCQ^1/DLCQ^2

Generating near-horizon bulk geometry

- intrinsic perspective on relation between asymptotic infinity and bulk geometry

$$ds^2 = \left(\frac{r}{\ell}\right)^2 \mathrm{d}x^A\,\mathrm{d}x^B\,\eta_{AB} + \left(\frac{\ell}{r}\right)^2 \left(\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_5^2\right),$$

- geometrical version of M3T decoupling limit wit $(r/\ell)^2$ playing role of ω.
(can be promoted to bgr. valued fn. due to emergent dilation symmetry)

- reverse logic: can we "nvert" the BPS decoupling limit ?

Yes: related to TTbar deformation

$$\frac{\partial \mathcal{L}(t)}{\partial t} \sim \det T_{\alpha\beta}(t)\,,$$

in fact: NRST can be TTbar deformed back to relativistic string theory Blair (2020)

$$S_{\mathrm{F1}}^{\mathrm{NRST}} = -\frac{1}{4\pi\alpha'} \int \mathrm{d}^2\sigma\,\partial_\alpha X^{A'}\,\partial^\alpha X^{A'}, \qquad A' = 2\,,\cdots,9\,.$$

$$\mathcal{L}(t) = \frac{1}{t}\left[1 - \sqrt{1 + t\,\partial_\alpha X^{A'}\partial^\alpha X^{A'} - t^2\,\det\left(\partial_\alpha X^{A'}\partial_\beta X^{A'}\right)}\right].$$

→ expect (by U-duality): same for Dp-branes

New p-brane TTbar deformations

- deformations that induce a flow from SYM to DBI action (abelian)

- universal result for (p+1)-dimensional free scalar field theory to DNG

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{t}} = \frac{\sqrt{-\det \tau}}{2\,\boldsymbol{t}^2}\left\{\mathrm{tr}\left(\mathbb{1} - \boldsymbol{t}\,\mathcal{T}\right) - (p-1)\left[\det\left(\mathbb{1} - \boldsymbol{t}\,\mathcal{T}\right)\right]^{\frac{1}{p-1}} - 2\right\}.$$

$$T_{\alpha\beta}(\boldsymbol{t}) = -\frac{2}{\sqrt{-\det \tau}}\frac{\partial \mathcal{L}(\boldsymbol{t})}{\partial \tau^{\alpha\beta}}$$

- deforming gauge theories (including YM fields)

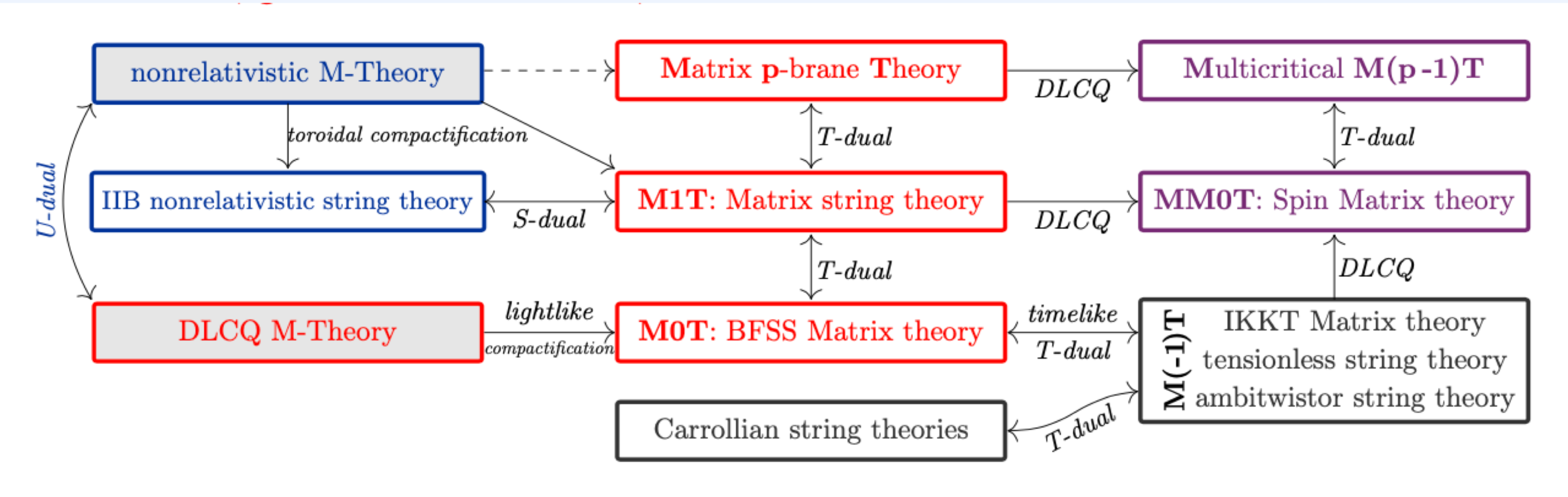
$$p=1: \quad \frac{\partial \mathcal{L}}{\partial \boldsymbol{t}} = -\frac{1}{2\,\boldsymbol{t}}\left(\mathrm{tr}\,\mathcal{T} + \sqrt{-\det F}\sqrt{-\mathrm{tr}\,\mathcal{T} + \boldsymbol{t}\det\mathcal{T}}\right)$$

$$p=2: \quad \frac{\partial \mathcal{L}}{\partial \boldsymbol{t}} = \frac{1}{4}\left[\mathrm{tr}(\mathcal{T}^2) - \left(\mathrm{tr}\,\mathcal{T}\right)^2\right] + \frac{\boldsymbol{t}}{2}\det\mathcal{T}\,.$$

& p=3 up to linear order in t

related formulae in Morone,Negro,Tateo(2024)/Tsolakidis(2024)

Web of decoupling limits



Blair,Lahnsteiner,NO,Yan (2023, 2024)), Gomis,Yan (2023)

obtained from insights using

- non-relativistic string theory on curved spactimes
- applying string theory U-dualities (including light-like compactifications)
- anatomy of BPS mass formulae in decoupling limits

Dijkgraaf,de Boer,Harmark,NO (unpublished)

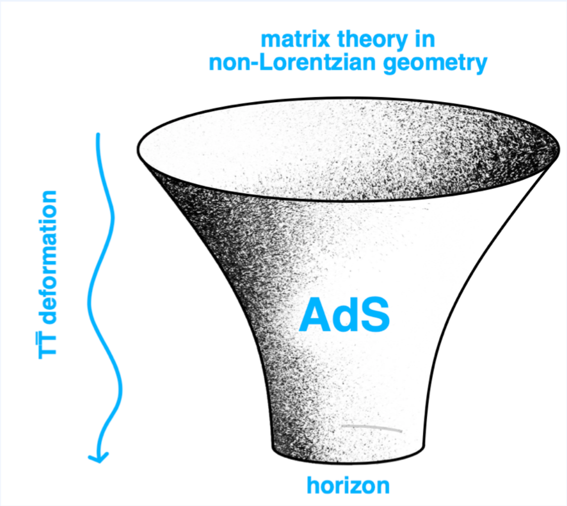
each node is its `own' decoupled theory including actions for:

- fundamental (light) degrees of freedom
  coupling to an appropriate (non-Lorentzian) target spacetime
- other (heavy) probe objects in the theory

Outlook

- correspondence between SUGRA and matrix QM: revisit

- elements of the AdS/CFT correspondence: deeper insight into relationships between brane configs and possible decoupling limits



-non-Lorentzian holography: properties of solutions, EOMs

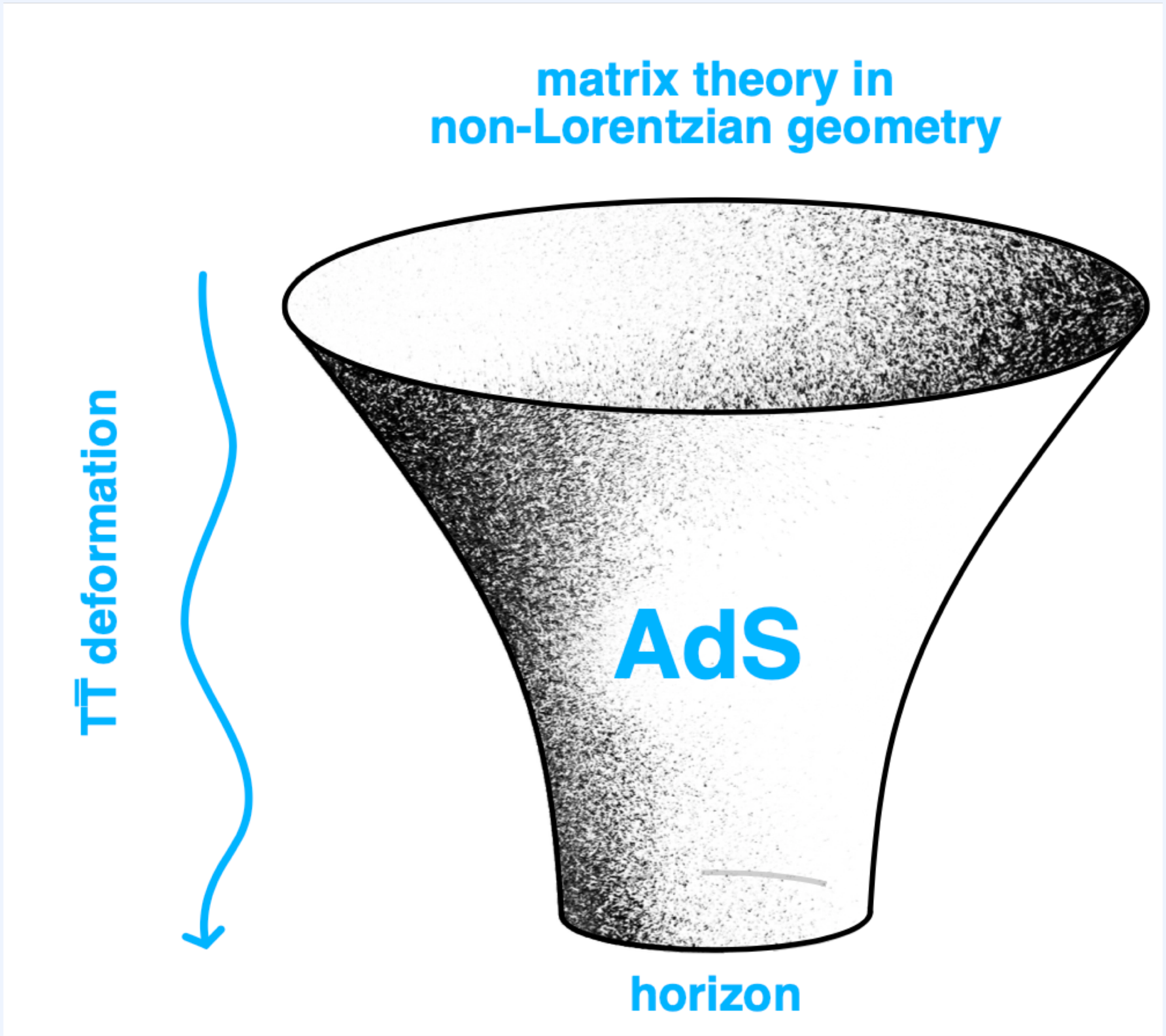
- dual description in terms of NRST (see talk Troels Hamark)

- new p-brane generalisations of TTbar deformation: generalizations

- algebraic aspects of matrix p-brane theory

- extensions: tensionless, Carrollian, heterotic (cf. the web)

The end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