

The dilaton double copy

and why we want it

Ingrid A. Vazquez-Holm

John Joseph Carrasco (2010.13435, 2108.06798)
Asaad Elkhidir, Donal O'Connell, Matteo Sergola (2303.06211)
Henrik Johansson (coming soon)

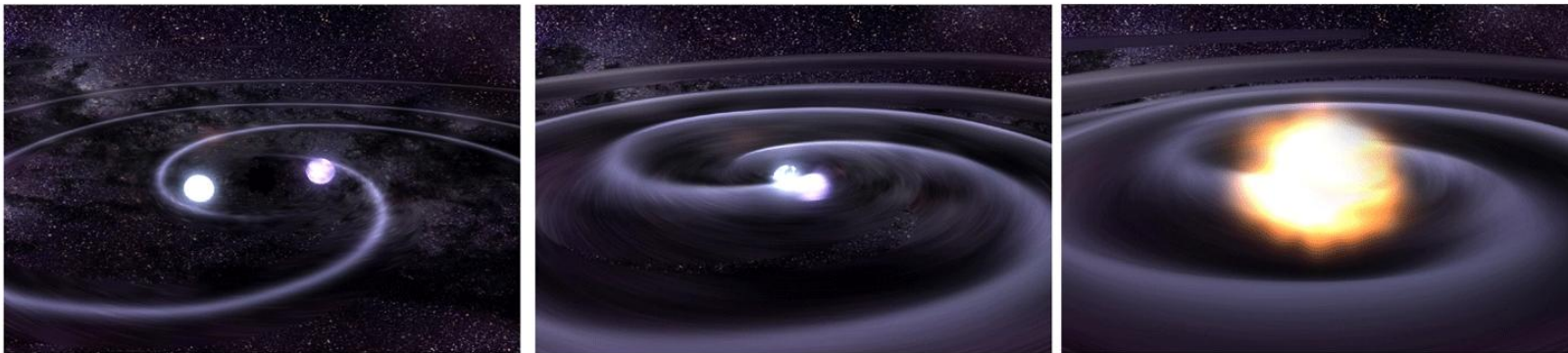


UPPSALA
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NORDITA
The Nordic Institute for Theoretical Physics

**33rd Nordic Network Meeting on “Strings,
Fields and Branes”,
Nordita,
31 October 2024**

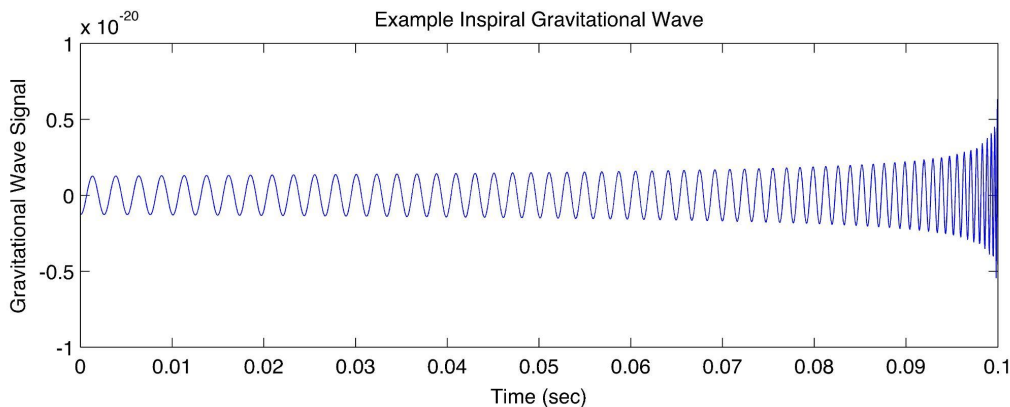


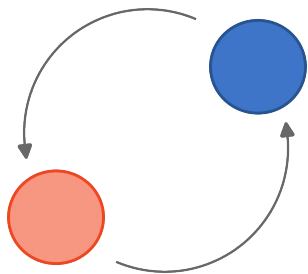
Elkhidir, O'Connell, Sergola,
IVH

Herderschee, Roiban, Teng

Brandhuber, Brown, Chen
De Angelis, Gowdy,
Travaglini

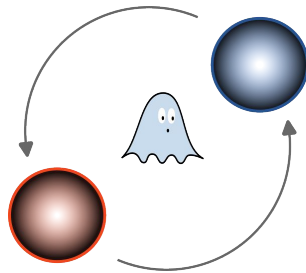
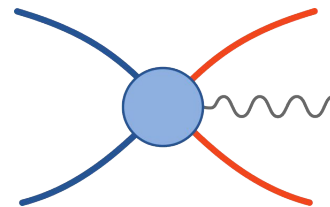
Georgoudis, Heissenberg,
IVH





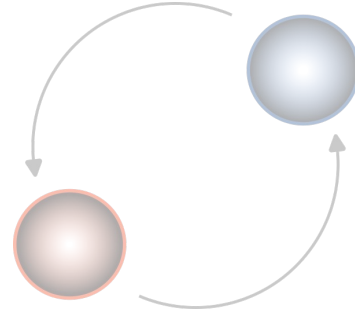
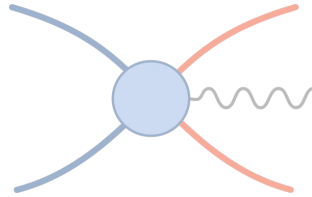
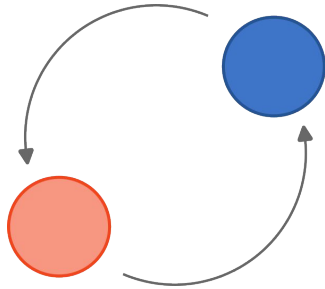
Express **radiation waveforms** in terms of scattering amplitudes

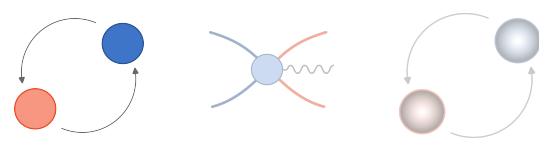
Amplitudes are bootstrapped using the color-kinematics duality



Gravity amplitudes are given using **dilaton double copy**

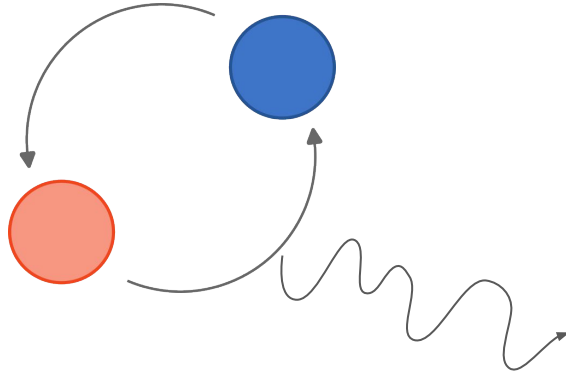
I. Radiation waveform





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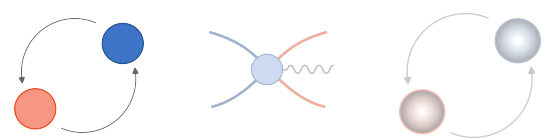
$$\langle F_{\mu\nu} \rangle \sim$$



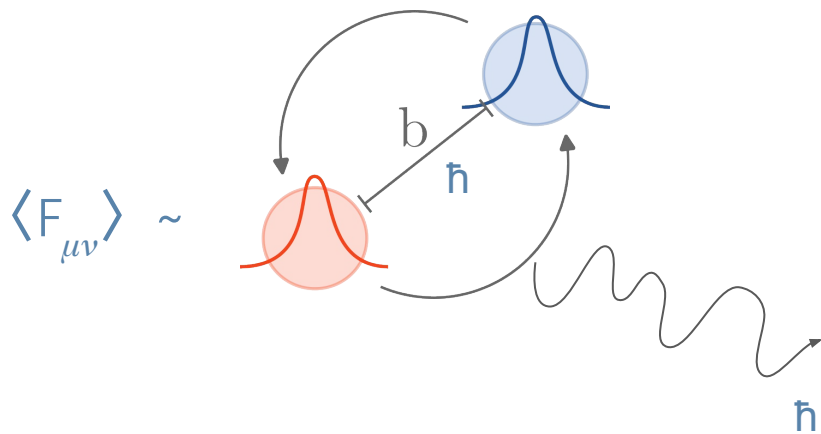
Radiation waveforms from the expectation value of the **field strength/Riemann tensor**

Define an initial state and the expectation in the far future, restore \hbar

Waveform from cuts of **amplitudes**



I. Radiation waveform



Radiation waveforms from the expectation value of the **field strength/Riemann tensor**

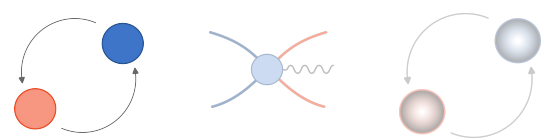
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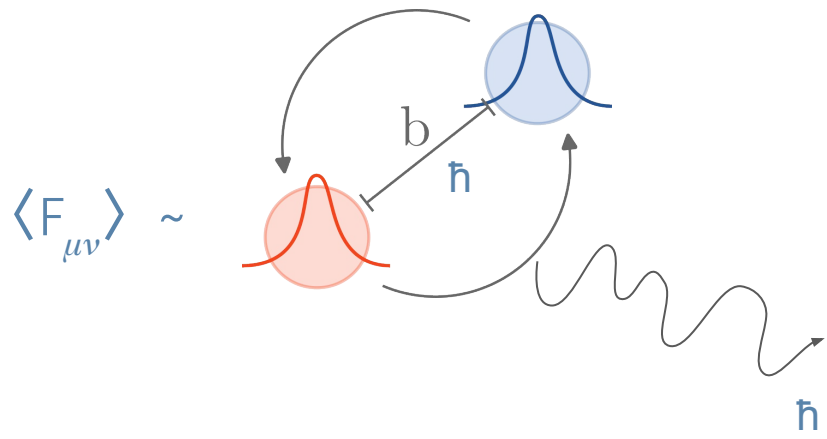
$$|\psi\rangle_{in} = \int d\tilde{k} \text{ (waveform) } e^{ib \cdot p_1/\hbar} |p_1 p_2\rangle_{in}$$

$$\langle \psi | S^\dagger F_{\mu\nu}(x) S | \psi \rangle$$

$$\langle p'_1 p'_2 | a_\eta(k) \text{Re } T + \frac{i}{2} ([a_\eta(k), T^\dagger] T - T^\dagger [a_\eta(k), T]) | p_1, p_2 \rangle$$



I. Radiation waveform



Radiation waveforms from the expectation value of the **field strength/Riemann tensor**

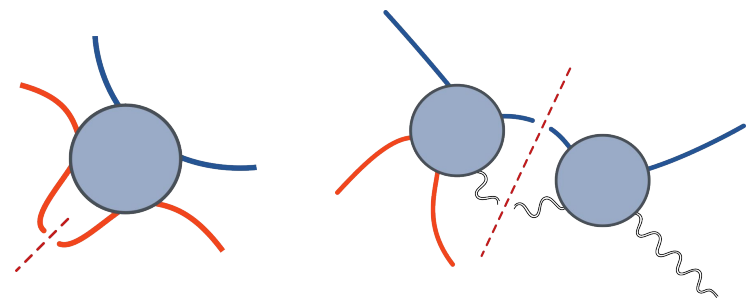
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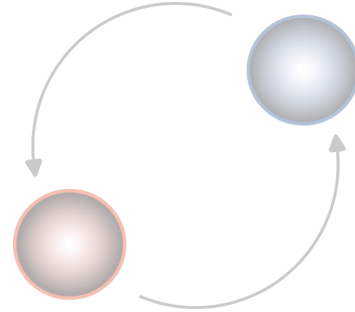
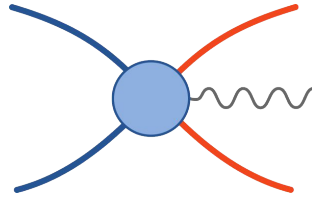
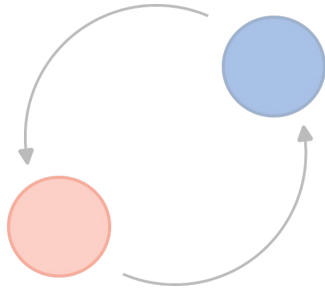
$$\langle \psi | S^\dagger F_{\mu\nu}(x) S | \psi \rangle$$

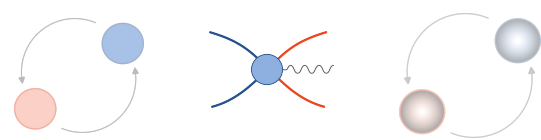
$$\langle p'_1 p'_2 | a_\eta(k) \text{Re } T + \frac{i}{2} ([a_\eta(k), T^\dagger] T - T^\dagger [a_\eta(k), T]) | p_1, p_2 \rangle$$

Waveform from cuts of **amplitudes**



II. Amplitudes





II. Amplitudes

We bootstrap **gauge and gravity amplitudes** using **color-kinematics duality** and unitarity cuts

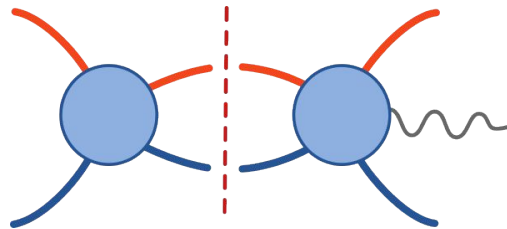
Bern, Carrasco, Johansson

$$f^{abl} f^{lcd} = f^{bcl} f^{lda} + f^{cal} f^{lbd}$$

$$\begin{array}{c} b \\ \text{wavy} \\ a \end{array}
 \begin{array}{c} c \\ \text{wavy} \\ d \end{array}
 =
 \begin{array}{c} c \\ \text{wavy} \\ b \end{array}
 \begin{array}{c} d \\ \text{wavy} \\ a \end{array}
 +
 \begin{array}{c} a \\ \text{wavy} \\ c \end{array}
 \begin{array}{c} b \\ \text{wavy} \\ d \end{array}$$

$$n(a, b, c, d) = n(b, c, d, a) + n(c, a, b, d)$$

Gives us amplitudes we can
double copy



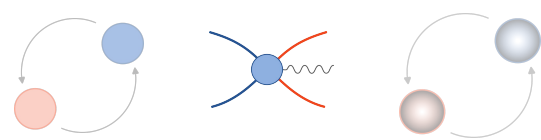
$$\frac{C_i n_i}{d_i}$$



$$\frac{n_i n_j}{d_i}$$

Gauge theory

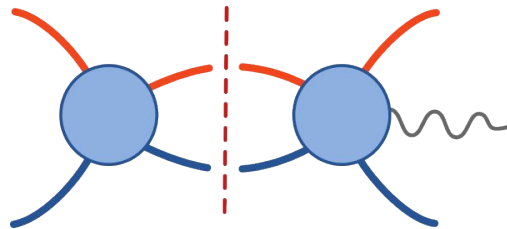
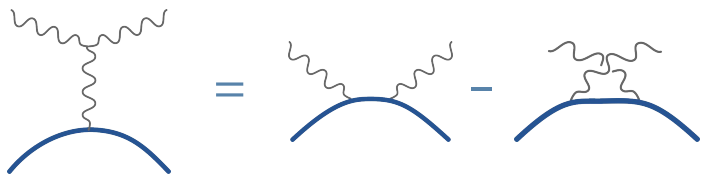
Gravity



II. Amplitudes

We bootstrap **gauge and gravity amplitudes** using **color-kinematics duality** and unitarity cuts

Bern, Carrasco, Johansson

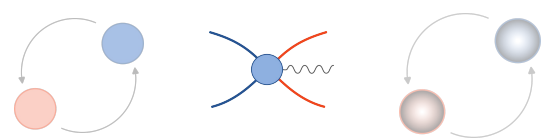


Gives us amplitudes we can
double copy

$$\frac{C_i n_i}{d_i} \rightarrow \frac{n_i n_j}{d_i}$$

Gauge theory

Gravity

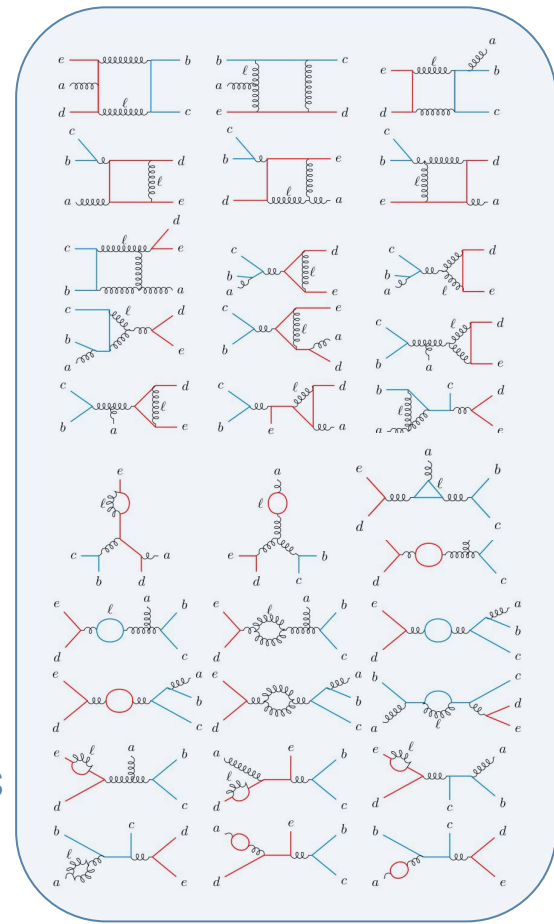


II. Amplitudes

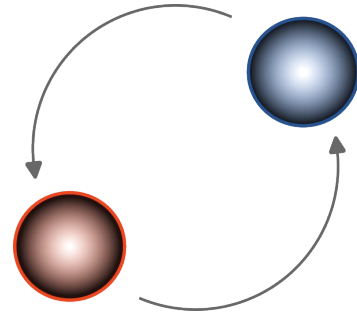
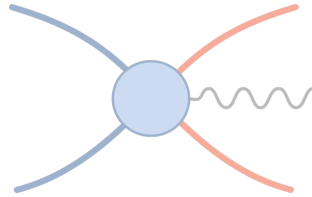
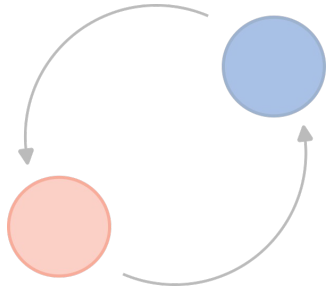
The 5-point one-loop amplitude has 116 graphs

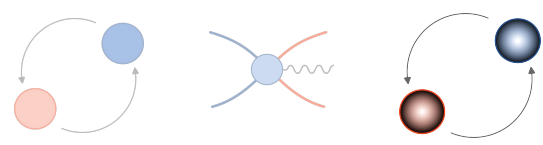
$$\begin{aligned}
 & \text{Diagram with external momenta } p'_1, p_1, p_2, p'_2 \text{ and internal momentum } k \\
 &= \int d^4\ell \sum_{g=1}^{116} \frac{n_g c_g}{d_g} \\
 &= \text{Sum of 116 Feynman diagrams (represented by a grid of 12 diagrams)} + \dots
 \end{aligned}$$

33 topologies

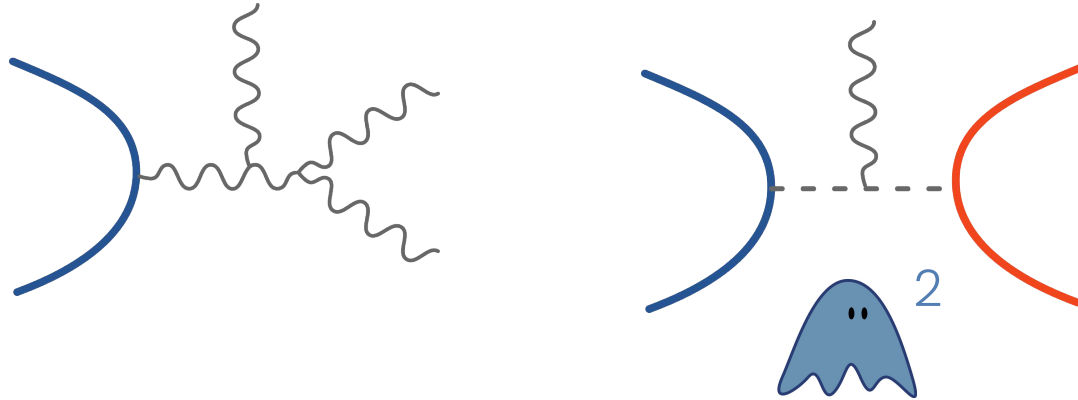


III. Dilaton double copy

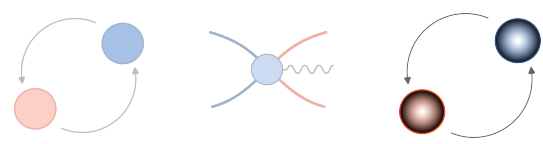




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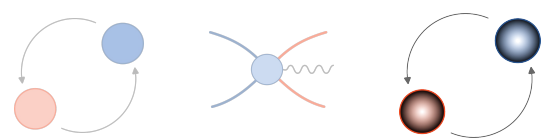
The double copy gives extra massless states



III. Dilaton double copy

$$n \left[\text{Diagram 1} \right] = n \left[\text{Diagram 2} \right]^2 - n \left[\text{Diagram 3} \right]^2$$

The equation shows a relationship between three diagrams. The first diagram is a blue semi-circular arc on the left and a red semi-circular arc on the right, connected by a wavy line. The second diagram is identical to the first but with a square superscript. The third diagram is identical to the first but with a dashed wavy line connecting the two arcs and a square superscript.



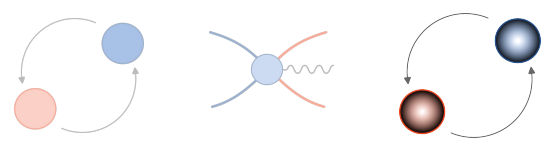
III. Dilaton double copy

$$n \left[\text{Diagram 1} \right] = n \left[\text{Diagram 2} \right]^2 - n \left[\text{Diagram 3} \right]^2$$

$$n \left[\text{Diagram 4} \right] \stackrel{\mathcal{N}=0}{=} \left(k_b \cdot k_d \right)^2 - \left(\left(k_b \cdot k_d \right)^2 - \frac{m_1^2 m_2^2}{(D_s - 2)} \right)$$

Einstein-Hilbert

$$\sum_s \mathcal{M}_3^{\text{trees}}(a, b, q^s) \mathcal{M}_3^{\text{trees}}(-q^{\bar{s}}, c, d)$$

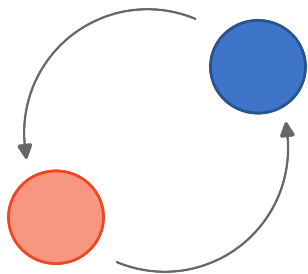


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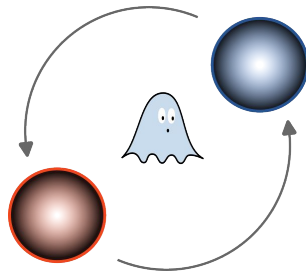
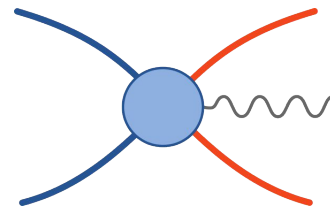
The equation shows a relationship between three diagrams. The first diagram is a blue semi-circular loop connected to a red semi-circular loop by a wavy line. The second diagram is identical to the first but with a square superscript. The third diagram is identical to the first but with a dashed line connecting the two loops and a square superscript.

Dilaton double copy up to six-point tree-level, and four-point one-loop*



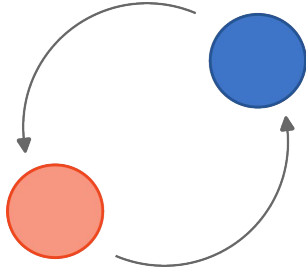
Express **radiation waveforms** in terms of scattering amplitudes

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What now?



Double copy dilatons at higher points?

Consider higher loops - waveform? Interesting physics?
Color-kinematics?

Spin?

