Gravitational solitons and non-relativistic string theory

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Based on: upcoming paper with J. Lahnsteiner and N. Obers

What is non-relativistic string theory (NRST)?

Klebanov & Maldacena '00; Gomis & Ooguri '00; Danielsson, Guijosa & Kruczenski '00;

Start very innocently with closed string theory in flat space:

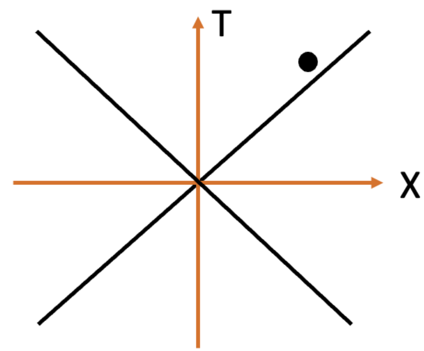
$$ds^2 = -dT^2 + dX^2 + \sum_{i=1}^{8} dr^i dr^i \ , \quad B = 0 \ , \quad g_s e^{\Phi} = g_s$$

X-direction compact with radius R_X and momentum: $p_X = \frac{w}{R_X}$

w = a positive integer

Now do coordinate transformation:

$$T = ct \ , \quad X = ct - \frac{1}{c}u$$



Large c: Close to edge of lightcone

R_X is small $R_u = cR_X$

$$\Rightarrow \quad ds^2 = -2dtdu + \frac{1}{c^2}du^2 + \sum_{i=1}^{8} dr^i dr^i \ , \quad B = 0 \ , \quad g_s e^{\Phi} = g_s$$

c → ∞ gives null reduction with finite radius R_u: DLCQ

T-duality along the u-direction:

$$ds^2 = c^2(-dt^2 + dv^2) + dr^i dr^i \ , \quad B = c^2 dt \wedge dv \ , \quad g_s e^\Phi = g_s c$$

$$v \equiv v + 2\pi R_v \ , \quad R_v = \frac{\alpha'}{R_u}$$

c → ∞ with R_v and α' fixed is non-relativistic string theory (NRST) limit

DLCQ/null-reduction is T-dual to NRST

Klebanov & Maldacena '00; Gomis & Ooguri '00; Danielsson, Guijosa & Kruczenski '00
Bergshoeff, Gomis, Yan '17; TH, Hartong, Obers '17
TH, Hartong, Menculini, Obers, Yan '18; TH, Hartong, Menculini, Obers, Oling '19

$g_s e^\Phi$ goes to infinity in NRST limit?

9D string coupling, related to adding handles: $\frac{g_s e^\Phi \sqrt{\alpha'}}{\sqrt{g_{vv}} R_v}$

Danielsson, Guijosa & Kruczenski '00

Sufficient criteria for this coupling being small:

Smallness of string coupling in T-dual frame $g_s = \frac{g_s e^\Phi}{\sqrt{g_{vv}}} \ll 1$

Large radius in string units $\frac{R_v}{\sqrt{\alpha'}} \gg 1$

Closed string mass formulas (bosonic part)

Original frame:

$$p_T^2 - \frac{w^2}{R_X^2} - \delta^{ij} p_i p_j = n^2 \frac{R_X^2}{(\alpha')^2} + \frac{2}{\alpha'}(N + \bar{N} - k)$$

Energy — Momentum — Transverse momenta — Winding — String excitations

After transformation:

$$\frac{1}{c^2} p_t^2 + 2p_t \frac{w}{R_u} - \delta^{ij} p_i p_j = \frac{1}{c^2}\left(\frac{nR_u}{\alpha'}\right)^2 + \frac{2}{\alpha'}(N + \tilde{N} - k)$$

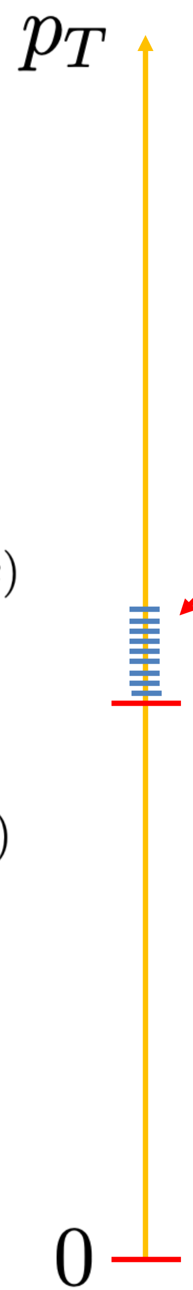
After T-duality:

$$\frac{1}{c^2} p_t^2 + 2p_t \frac{wR_v}{\alpha'} - \frac{1}{c^2}\frac{n^2}{R_v^2} = \delta^{ij} p_i p_j + \frac{2}{\alpha'}(N + \tilde{N} - k)$$

In the c → ∞ limit:

$$p_t = \frac{\alpha'}{2wR_v}\left[\delta^{ij} p_i p_j + \frac{2}{\alpha'}(N + \tilde{N} - k)\right]$$

Note: $R_v = \frac{\alpha'}{R_u}$ $\quad R_u = cR_X$



Limit zooms in on modes just above the w/R_X mode in original frame

$$\frac{w}{R_X} = c\frac{w}{R_u} = c\frac{wR_v}{\alpha'}$$

A near-BPS limit

The role of the F-string soliton

What if we include backreaction of the winding mode?

F-string soliton with charge = winding w

$$ds^2 = c^2 H^{-1}(-dt^2 + dv^2) + dr^2 + r^2 d\Omega_7^2$$

$$B = c^2 H^{-1} dt \wedge dv \ , \quad g_s e^{\Phi} = \frac{g_s c}{\sqrt{H}}$$

$$\frac{H}{c^2} = \frac{1}{c^2} + \frac{L^6}{r^6}$$

$$L^6 = 32\pi^2 w\, g_s^2 \alpha'^3$$

Coupling of F-string to NS-NS gravity sector set by

$$\text{Tension} \times \text{Gravitational coupling} = \frac{w}{2\pi\alpha'} 8\pi^6 (\alpha')^4 g_s^2 e^{2\Phi} \sim c^2$$

Goes to infinity for c → ∞

NRST limit = Near-horizon limit of F-string

$$ds^2 = \frac{r^6}{L^6}(-dt^2 + dv^2) + dr^2 + r^2 d\Omega_7^2 \ , \quad B = \frac{r^6}{L^6} dt \wedge dv \ , \quad g_s e^{\Phi} = g_s \frac{r^3}{L^3}$$

Ávila, Guijosa & Olmedo '23; TH, Lahnsteiner & Obers (upcoming)

Interpretation?

Ávila, Guijosa & Olmedo ’23:

- Since near-horizon F-string soliton geometry is relativistic → No NRST
- Second limit r/L → ∞?

Our interpretation:

T-dual frame: $$ds^2 = -2dtdu + \frac{H}{c^2}du^2 + dr^2 + r^2 d\Omega_7^2 \ , \quad B = 0 \ , \quad \Phi = 0$$

Limit: $$\frac{H}{c^2} \to \frac{L^6}{r^6} \quad \text{for} \quad c \to \infty$$

Gravity solution can be trusted when

1) Curvatures are small: $r \gg \sqrt{\alpha'}$
2) String coupling small: $g_s \ll 1$

Itzhaki, Maldacena, Sonnenschein & Yankielowicz '98

For $g_s \ll \frac{1}{\sqrt{w}} \Leftrightarrow L \ll \sqrt{\alpha'}$ ⇨ No backreaction, DLCQ of flat space

⇨ **T-dual frame is weakly coupled NRST**

For $g_s \gg \frac{1}{\sqrt{w}} \Leftrightarrow L \gg \sqrt{\alpha'}$ ⇨ Backreaction = Near-horizon string soliton

Conclusion:

- *NRST is a viable limit of relativistic ST for sufficiently weak string coupling*
- *Near-horizon string soliton is NRST at strong coupling*

(Even higher coupling: S-dual phases of IIA/B)

NS5-brane soliton in NRST

Are there backreacted brane solutions of type IIA/B string theory that survive NRST limit?

We can repeat DLCQ procedure with a NS5-brane solution

$$H = 1 + \frac{N\alpha'}{r^2}$$

$$ds^2 = -dT^2 + dX^2 + \sum_{m=1}^{4} dy_m^2 + H \sum_{i=1}^{4} dr_i^2 \ , \quad (dB)_{ijk} = \varepsilon_{ijk}{}^{l} H \partial_l H \ , \quad g_s e^{\Phi} = g_s H^{1/2}$$

$$T = c\,t \ , \quad X = c\,t - \frac{1}{c} u$$

$$ds^2 = -2dt\,du + \frac{1}{c^2} du^2 + \sum_{m=1}^{4} dy_m^2 + H \sum_{i=1}^{4} dr_i^2 \ , \quad (dB)_{ijk} = \varepsilon_{ijk}{}^{l} H \partial_l H \ , \quad g_s e^{\Phi} = g_s H^{1/2}$$

T-duality along u

$$ds^2 = c^2(-dt^2 + dv^2) + \sum_{m=1}^{4} dy_m^2 + H \sum_{i=1}^{4} dr_i^2 \ , \quad B_{tv} = c^2 \ , \quad (dB)_{ijk} = \varepsilon_{ijk}{}^{l} H \partial_l H \ , \quad g_s e^{\Phi} = g_s c\, H^{1/2}$$

Gives NRST geometry!

NRST geometry:

$$\tau_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dv^2\ , \quad E_{\mu\nu}dx^\mu dx^\nu = \sum_{m=1}^{4} dy_m^2 + H\sum_{i=1}^{4} dr_i^2$$

$$(dB)_{ijk} = \varepsilon_{ijk}{}^{l} H\partial_l H\ , \quad g_s e^\Phi = g_s c\, H^{1/2}\ , \quad H = 1 + \frac{N\alpha'}{r^2}$$

Shown in Bergshoeff, Lahnsteiner, Romano & Rosseel to be ½ BPS

Why does this survive the NRST limit?

$$\text{Tension} \times \text{Gravitational coupling} = \frac{N}{(2\pi)^5(\alpha')^3 g_s^2 c^2} 8\pi^6 (\alpha')^4 g_s^2 c^2 \sim c^0$$

NS5-brane is lighter than F-string in NRST limit!

Dp-brane soliton in NRST

Start with D(p+1)-brane, do DLCQ procedure:

$$ds^2 = H^{-\frac{1}{2}}\left(-dT^2 + dX^2 + \sum_{a=1}^{p}(dy^a)^2\right) + H^{\frac{1}{2}}\sum_{i=1}^{8-p} dr^i dr^i\ , \quad H = 1 + \frac{(2\pi\sqrt{\alpha'})^{6-p} g_s N}{(6-p)\Omega_{7-p} r^{6-p}}$$

$$A^{(p+1)} = (H^{-1}-1) dT \wedge dX \wedge dy^1 \wedge \cdots \wedge dy^p\ , \quad g_s e^{\Phi} = g_s H^{\frac{2-p}{4}}$$

$$T = c\,t\ , \quad X = c\,t - \frac{1}{c}u$$

$$ds^2 = H^{-\frac{1}{2}}\left(-2dt\,du + \frac{1}{c^2}du^2 + \sum_{a=1}^{p}(dy^a)^2\right) + H^{\frac{1}{2}}\sum_{i=1}^{8-p} dr^i dr^i\ , \quad H = 1 + \frac{(2\pi\sqrt{\alpha'})^{6-p} g_s N}{(6-p)\Omega_{7-p} r^{6-p}}$$

$$A^{(p+1)} = -(H^{-1}-1) dt \wedge du \wedge dy^1 \wedge \cdots \wedge dy^p\ , \quad g_s e^{\Phi} = g_s H^{\frac{2-p}{4}}$$

T-duality along u

$$ds^2 = H^{-\frac{1}{2}}\left(-c^2 dt^2 + \sum_{a=1}^{p}(dy^a)^2\right) + H^{\frac{1}{2}}\left(c^2 dv^2 + \sum_{i=1}^{8-p} dr^i dr^i\right)\ , \quad H = 1 + \frac{(2\pi\sqrt{\alpha'})^{6-p} g_s N}{(6-p)\Omega_{7-p} r^{6-p}}$$

$$B = c^2 dt \wedge dv\ , \quad A^{(p+1)} = -(H^{-1}-1) dt \wedge dy^1 \wedge \cdots \wedge dy^p\ , \quad g_s e^{\Phi} = g_s c\, H^{\frac{3-p}{4}}$$

Gives NRST geometry!

NRST geometry:

$$\tau_{\mu\nu}dx^\mu dx^\nu = -H^{-\frac{1}{2}}dt^2 + H^{\frac{1}{2}}dv^2 \ , \quad E_{\mu\nu}dx^\mu dx^\nu = H^{-\frac{1}{2}}\sum_{a=1}^{p}(dy^a)^2 + H^{\frac{1}{2}}\sum_{i=1}^{8-p}dr^i dr^i$$

$$A^{(p+1)} = -(H^{-1}-1)dt\wedge dy^1\wedge\cdots\wedge dy^p \ , \quad g_s e^{\Phi} = g_s H^{\frac{3-p}{4}}$$

$$H = 1 + \frac{(2\pi\sqrt{\alpha'})^{6-p}g_s N}{(6-p)\Omega_{7-p}r^{6-p}}$$

Dp-brane smeared along v-direction (transverse direction)

Agrees with open string POV — Gomis, Yan & Yu ‘20; Hartong & Have ‘24

Why did it survive limit?

$$\text{Tension (D}p) \times \text{Gravitational coupling (9D)} = \frac{N}{(2\pi)^p(\alpha')^{\frac{p+1}{2}}g_s c}\frac{8\pi^6(\alpha')^4 g_s^2 c^2}{cR_v} \sim c^0$$

Issue: does not obey foliation conditions in literature → Seemingly not SUSY?

Conclusions:

Results:

- Near-horizon limit of F-string soliton = NRST limit
- Weakly coupled NRST describes a corner of weakly coupled string theory, despite strong backreaction of string soliton
- Near-horizon F-string solution describes part of strong coupling phase of NRST
- Sourced gravitational solitons in NRST: NS5-brane & Transverse Dp-branes

Future directions:

- SUSY and foliation condition for transverse Dp-branes?
- Non-extremal branes → Seems to be excluded?
- Connection to Blair, Lahnsteiner, Obers & Yan '23 and '24