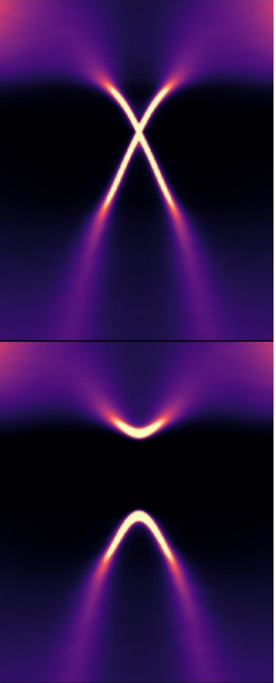
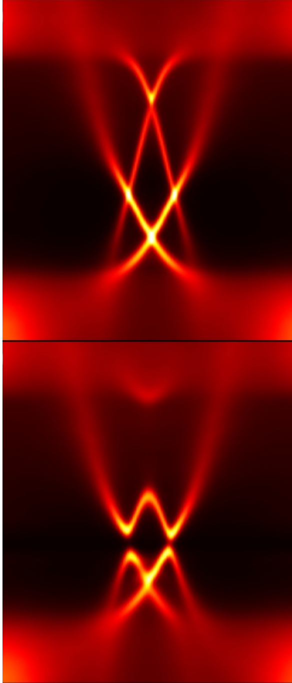


Isotropic 3D topological phases with broken time reversal symmetry

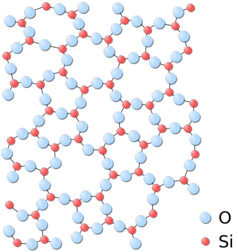
Hélène Spring, Anton R. Akhmerov,
Dániel Varjas

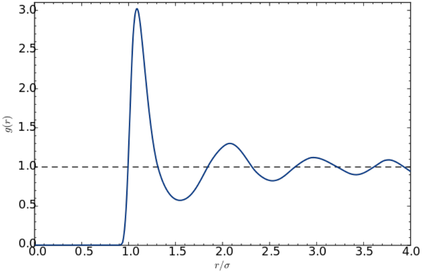
arXiv:2310.18400

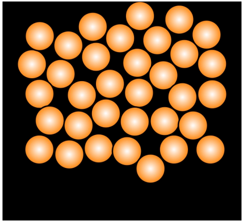


Amorphous materials

- Very common in technology
- Glass/liquid
- No long range order
- Short range correlations
- Uniform chemical environments





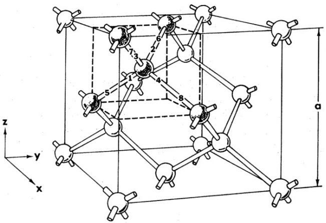


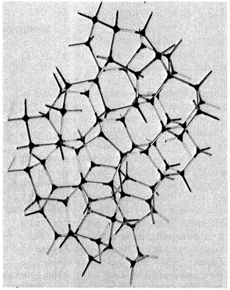
Weaire-Thorpe model of amorphous silicon

[Weaire, Thorpe: Phys. Rev. B **4** 2508 (1971)]

Explain band gap in amorphous silicon

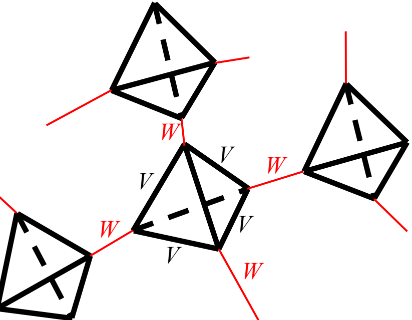
Tetrahedral bonding environment

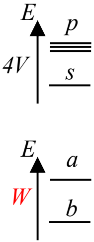




$V \,/\, W$

Replace every site with a tetrahedron





Resolvent method

no states where converges

$$\frac{1}{E - H_V - H_W} = \frac{1}{E - H_V} \sum_{n=0}^{\infty} \left(\frac{H_W}{E - H_V} \right)^n$$

$$\left| \frac{H_W}{E - H_V} \right| < 1 \qquad |H_W| < |E - H_V|$$

Weaire-Thorpe model of amorphous silicon

[Weaire, Thorpe: Phys. Rev. B **4** 2508 (1971)]

Explain band gap in amorphous silicon

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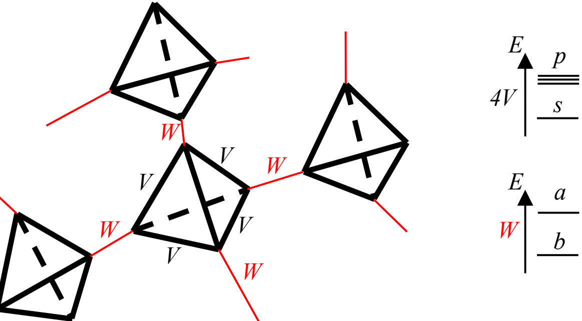
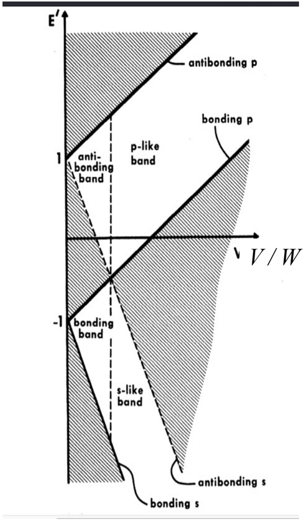
Replace every site with a tetrahedron





Analytical „band structure“

Previous results

Can spatial symmetries protect a topological insulator from disorder?

Previous results

Can spatial symmetries protect a topological insulator from disorder?

PRL **109,** 246605 (2012) PHYSICAL REVIEW LETTERS week ending 14 DECEMBER 2012

Topology, Delocalization via Average Symmetry and the Symplectic Anderson Transition

Liang Fu
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

C. L. Kane
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
(Received 16 August 2012; published 14 December 2012)

A field theory of the Anderson transition in two-dimensional disordered systems with spin-orbit interactions and time-reversal symmetry is developed, in which the proliferation of vortexlike topological defects is essential for localization. The sign of vortex fugacity determines the Z_2 topological class of the localized phase. There are two distinct fixed points with the same critical exponents, corresponding to transitions from a metal to an insulator and a topological insulator, respectively. The critical conductivity and correlation length exponent of these transitions are computed in an $N = 1 - \epsilon$ expansion in the number of replicas, where for small ϵ the critical points are perturbatively connected to the Kosterlitz-Thouless critical point. Delocalized states, which arise at the surface of weak topological insulators and topological crystalline insulators, occur because vortex proliferation is forbidden due to the presence of symmetries that are violated by disorder, but are restored by disorder averaging.

PHYSICAL REVIEW B **89**, 155424 (2014)

Statistical topological insulators

I. C. Fulga,[1] B. van Heck,[1] J. M. Edge,[1] and A. R. Akhmerov[1,2]
[1]*Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands*
[2]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
(Received 26 December 2012; revised manuscript received 3 April 2014; published 21 April 2014)

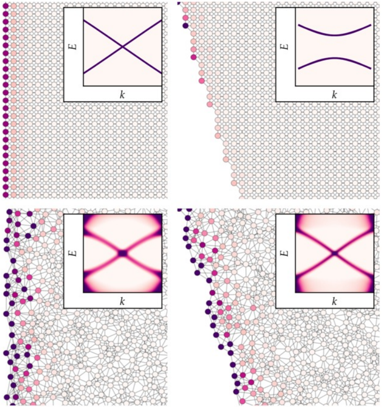
We define a class of insulators with gapless surface states protected from localization due to the statistical properties of a disordered ensemble, namely, due to the ensemble's invariance under a certain symmetry. We show that these insulators are topological, and are protected by a $\mathbb{Z}_2$ invariant. Finally, we prove that every topological insulator gives rise to an infinite number of classes of statistical topological insulators in higher dimensions. Our conclusions are confirmed by numerical simulations.

Yes! If the symmetries are preserved on average.

Can we get this in amorphous systems?

Previous results

Can spatial symmetries protect a topological insulator from disorder?



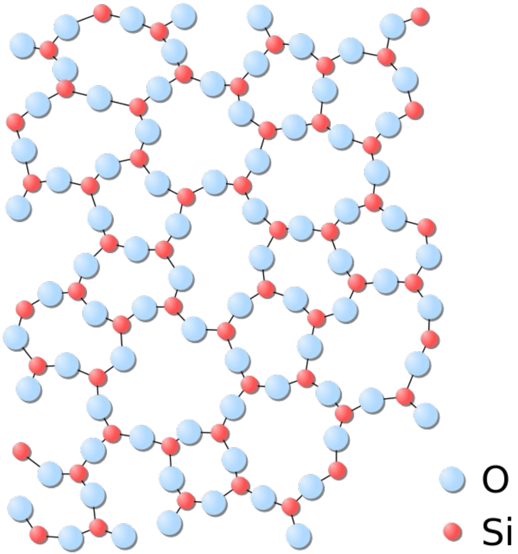
In isotropic amorphous systems all boundaries are protected

Amorphous topological phases in 2D require particle-hole or chiral symmetry ⇒ search in 3D.

Spring, Akhmerov, **DV,** SciPost Phys. 11, 022 (2021)

What is “spatial symmetry” in amorphous systems?

- Not “no symmetry”
- Bulk is “homogeneous” and “isotropic”



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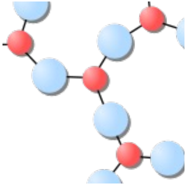
- Hamiltonian generated by a symmetric **local** rule

  Prodan & Kohn, PNAS 2005

$$H[\mathbf{r}] = H[\mathrm{env}_{\mathbf{r}}]$$



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Prodan & Kohn, PNAS 2005

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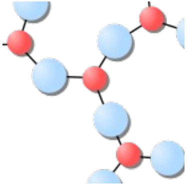

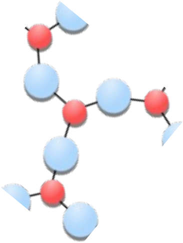

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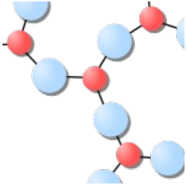
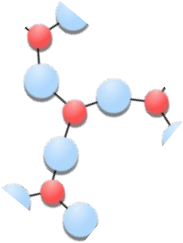




- Topological phases defined through deformations of the rule

DV, A. Lau, K. Pöyhönen, A. R. Akhmerov, D. I. Pikulin, I. C. Fulga, Phys. Rev. Lett. **123**, 196401 (2019)

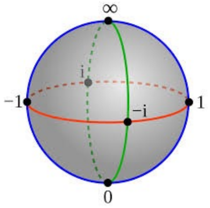
Effective continuum Hamiltonian

„fake“ k-space Hamiltonian

$$G = \lim_{\eta \to 0} (H + i\eta)^{-1}$$

$$G_{\text{eff}}(k)_{n,m} = \langle k, n | \, G \, | k, m \rangle$$

$$H_{\text{eff}} = G_{\text{eff}}^{-1}$$



gap closing in $H_{\text{eff}} \rightarrow$ gap closing in H

Obeys average symmetry

$$H_{\text{eff}}(\mathbf{k}) = U_g H_{\text{eff}}(R_g \mathbf{k}) U_g^\dagger$$

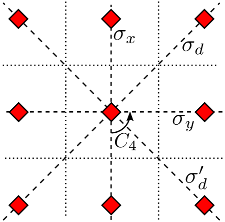
Allows use of symmetry indicators at $\mathbf{k} = 0$ and ∞

DV et.al., Phys. Rev. Lett. 123, 196401 (2019)

Lessnich et.al., Phys. Rev. B 104, 085116 (2021)

Symmetry indicators

Atomic insulators: $H(\mathbf{k}) = H_{\text{onsite}} = \text{const.}$

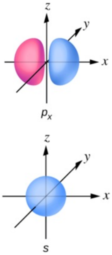


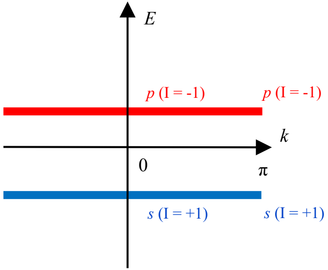
Atomic orbitals transform under symmetries

(representation content)

Bloch states at high symmetry momenta

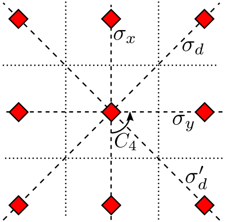
Atomic insulator: all the same





Symmetry indicators

Atomic insulators: $H(\mathbf{k}) = H_{\text{onsite}} = \text{const.}$



Atomic orbitals transform under symmetries

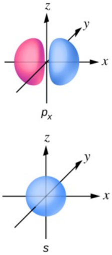
(representation content)

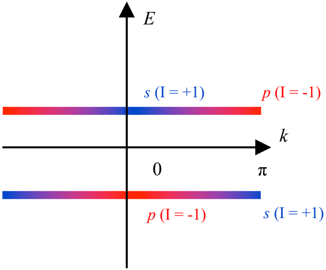
Bloch states at high symmetry momenta

Atomic insulator: all the same

Topological: band inversion

Generalized to all space groups





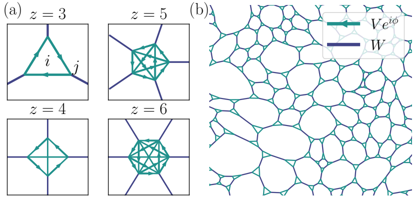
Fu, Kane PRB **76**, 045302 (2007)
Teo, Hughes PRL **111**, 047006 (2013)
Bradlyn *et al.* Nature **547** 298 (2017)
Po *et al.* Nature Communications **8**, 50 (2017)

2D Weaire-Thorpe class models

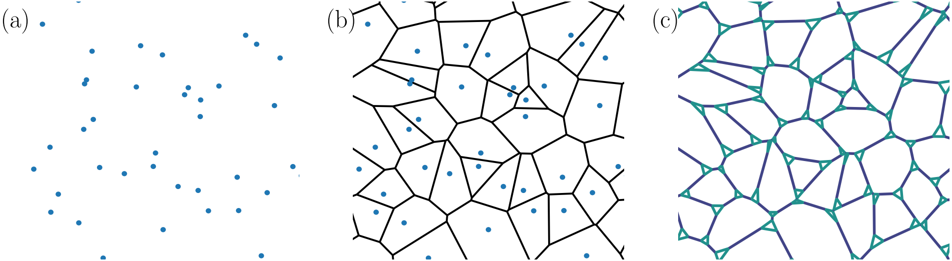
Q. Marsal, **DV**, A. G. Grushin, PNAS 202007384 (2020)

Fixed coordination z

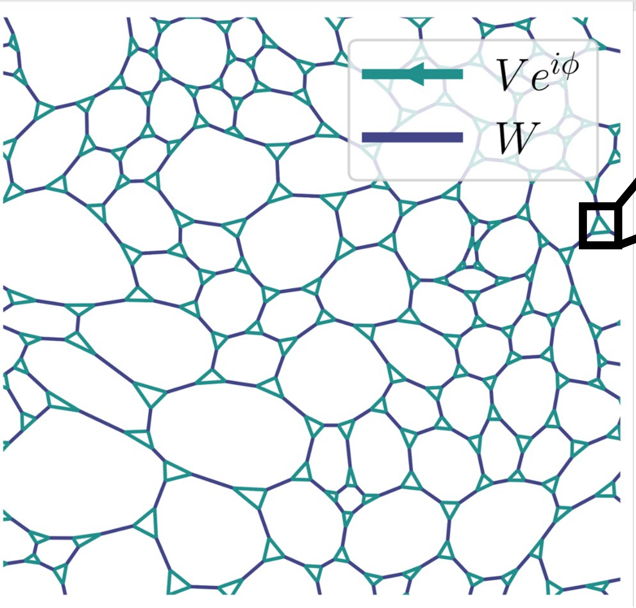
Allow time-reversal breaking (magnetic fluxes)

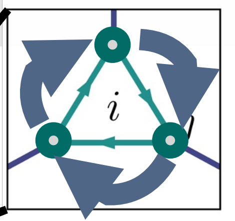


Voronization to get 3-fold coordination

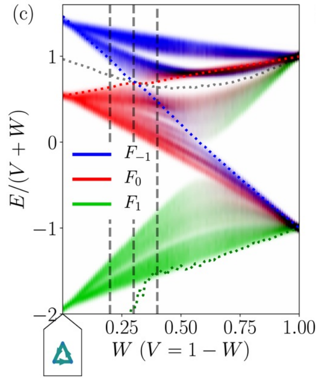


2D Weaire-Thorpe class models





C3 rotational symmetry



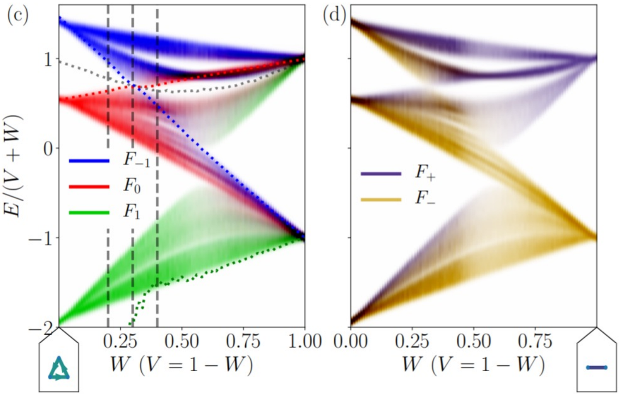
2D Weaire-Thorpe class models

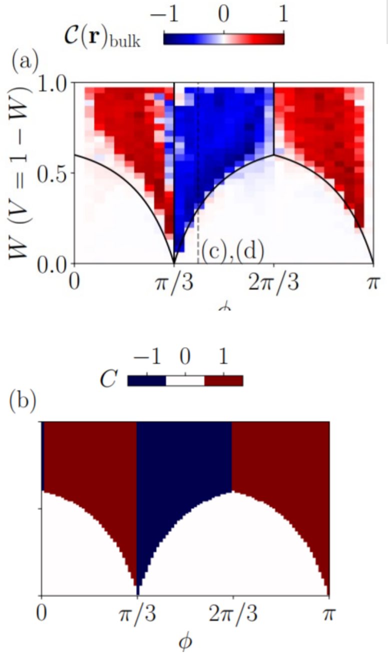
Chern insulator phases

“Band inversions“

3-fold rotation, bonding/antibonding character

Analytical phase diagram





3D continuum model with TR breaking

Key idea: Isotropy is a strong constraint → Minimal ***k.p*** model

$$H = (\mu + ak^2)\,\sigma_0\tau_z + (b\tau_x + c\tau_y)\,\boldsymbol{\sigma}\cdot\boldsymbol{k}$$

3D continuum model with TR breaking

Key idea: Isotropy is a strong constraint → Minimal ***k.p*** model

Make phase *k*-dependent to remove time-reversal.

$$H = (\mu + ak^2)\, \sigma_0\tau_z + (b\tau_x + ck^2\tau_y)\, \boldsymbol{\sigma}\cdot\boldsymbol{k}$$

- ✓ Has inversion, full rotation, (all mirrors,) and nothing else.
- ✓ Topological invariant: parity of inversion eigenvalues (here: sign μ).
- ✓ Surface: a disordered critical Dirac cone.

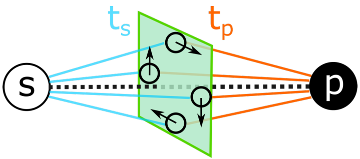
How to make it in tight-binding?

Tight-binding model

We need *scalar* time-reversal symmetry breaking.

- ✗ $\boldsymbol{M}$ (or $\boldsymbol{B}$): axial vector, breaks rotation.
- ✗ $\boldsymbol{P}$ (or $\boldsymbol{E}$): time-reversal symmetric, breaks rotation
- ✗ $\boldsymbol{P}\cdot\boldsymbol{M}$: pseudoscalar, breaks inversion.
- ✓ $\boldsymbol{P}\cdot\nabla\times\boldsymbol{M} \sim \boldsymbol{P}\cdot\boldsymbol{I}$: just right! Ferrotoroidal order

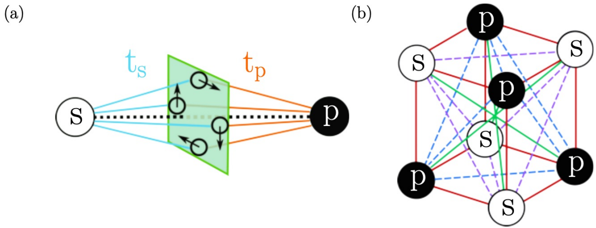
Possible probe: $\boldsymbol{P}\sim\boldsymbol{S}$ (Poynting vector)



(Analyzed with pymablock)

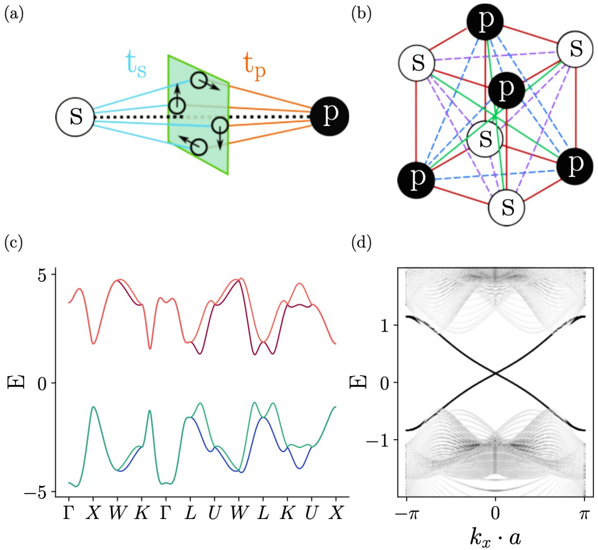
[Araya Day et al. https://pymablock.readthedocs.io/]

Crystalline phase



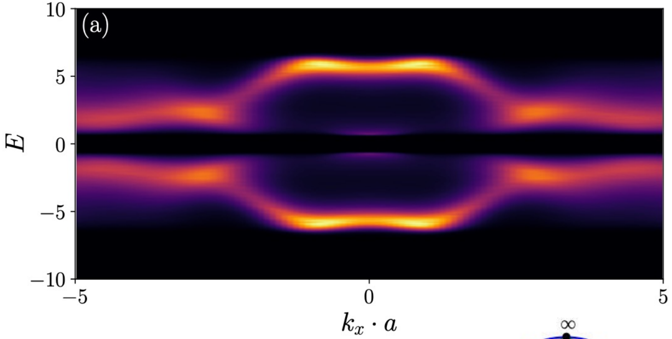
Crystalline phase

- Spin splitting only at low symmetry directions.



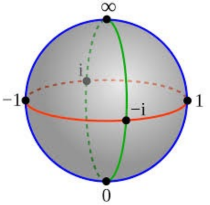
Amorphous phase

✓ Bulk gap



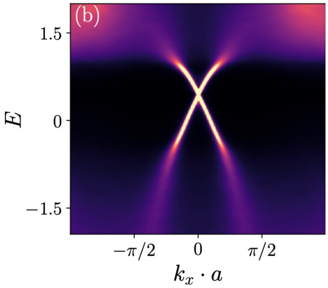
$$A(\mathbf{k}, l, E) = \langle \mathbf{k}, l | \delta(H - E) | \mathbf{k}, l \rangle$$

$$\langle \mathbf{r} | \mathbf{k}, l \rangle = \frac{1}{\sqrt{N}} \exp(i\mathbf{k}\mathbf{r}_c) \exp(i\phi_{\mathbf{r}} l)$$



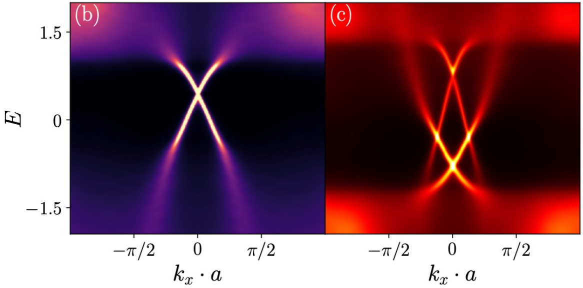
Amorphous phase

✓ Bulk gap
✓ Gapless surface



Amorphous phase

- ✓ Bulk gap
- ✓ Gapless surface
- ✓ Also with 2 Dirac cones (?)
- ✓ Amorphous mirror-Chern insulator



$$G_{\mathrm{eff}}(k)_{n,m} = \langle k, n | G | k, m \rangle$$

$$C_M = \frac{1}{2}(C_+ - C_-), \quad C_\pm = \oiint \mathcal{F}_\pm(\boldsymbol{k}) d^2\boldsymbol{k}$$

$$\nu_I = \frac{1}{2}\left[\iota_-(\boldsymbol{\infty}) - \iota_-(\boldsymbol{0})\right]$$

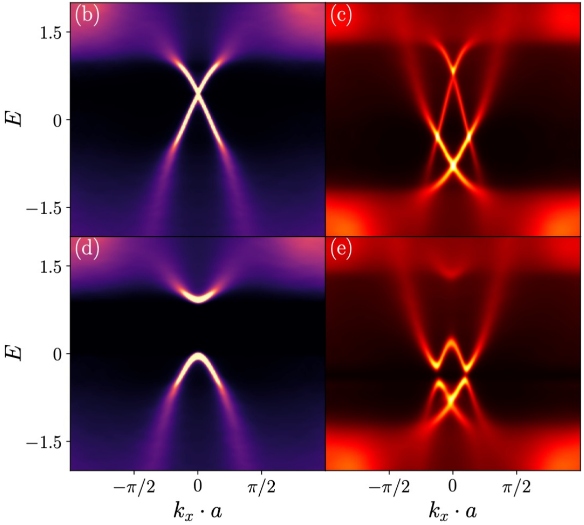
$$\iota_-(\boldsymbol{k}) = \mu_{-1}\left(\langle n(\boldsymbol{k}) | \mathcal{I} | m(\boldsymbol{k}) \rangle\right)$$

DV et al. PRL **123**, 196401(2019)

Marsal, **DV**, Grushin PNAS **117** 30260 (2020)

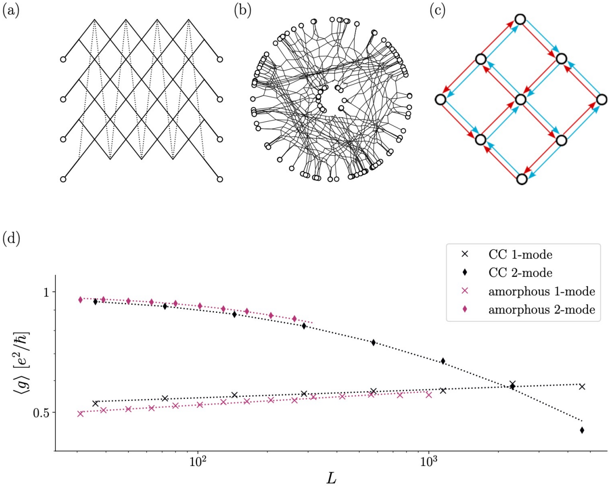
Amorphous phase

- ✓ Bulk gap
- ✓ Gapless surface
- ✓ Also with 2 Dirac cones (?)
- ✓ Amorphous mirror-Chern insulator
- ✓ Gapped by breaking reflection



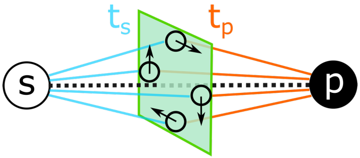
Amorphous phase

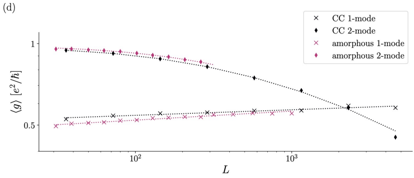
- ✓ Bulk gap
- ✓ Gapless surface
- ✓ Also with 2 Dirac cones (?)
- ✓ Amorphous mirror-Chern insulator
- ✓ Gapped by breaking reflection
- ✓ 2 Dirac cones localize, 1 is protected



Conclusions

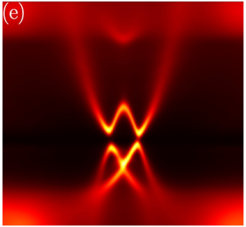
- Average spatial symmetries provide topological protection
- Symmetry indicator invariant
- Making such a phase requires a scalar time-reversal symmetry breaking.
- Any surface of this phase is protected.
- Odd phases are critical, even localize.





Further questions:

- Is the $C_M = 2$ phase's surface gappable?
- Generalization with on-site symmetries?



Hélène Spring, Anton R. Akhmerov, **Dániel Varjas**

Isotropic 3D topological phases with broken time reversal symmetry

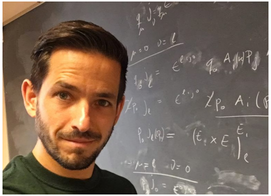
arXiv:2310.18400

Collaborators and funding

Anton R. Akhmerov

Hélène Spring

Quentin Marsal



Adolfo G. Grushin

MŰEGYETEM 1782

AZ NKFI ALAPBÓL MEGVALÓSULÓ PROJEKT

TU Delft

Vetenskapsrådet Swedish Research Council

DFG Deutsche Forschungsgemeinschaft

Quantum Tinkerer

Stockholms universitet

Knut and Alice Wallenberg Foundation

mpipks

IFW

NWO Netherlands Organisation for Scientific Research