Fractionalized excitations in Quantum Spin Liquids and their Detection

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https://indico.fysik.su.se/event/8803/


QSL?

QSL?

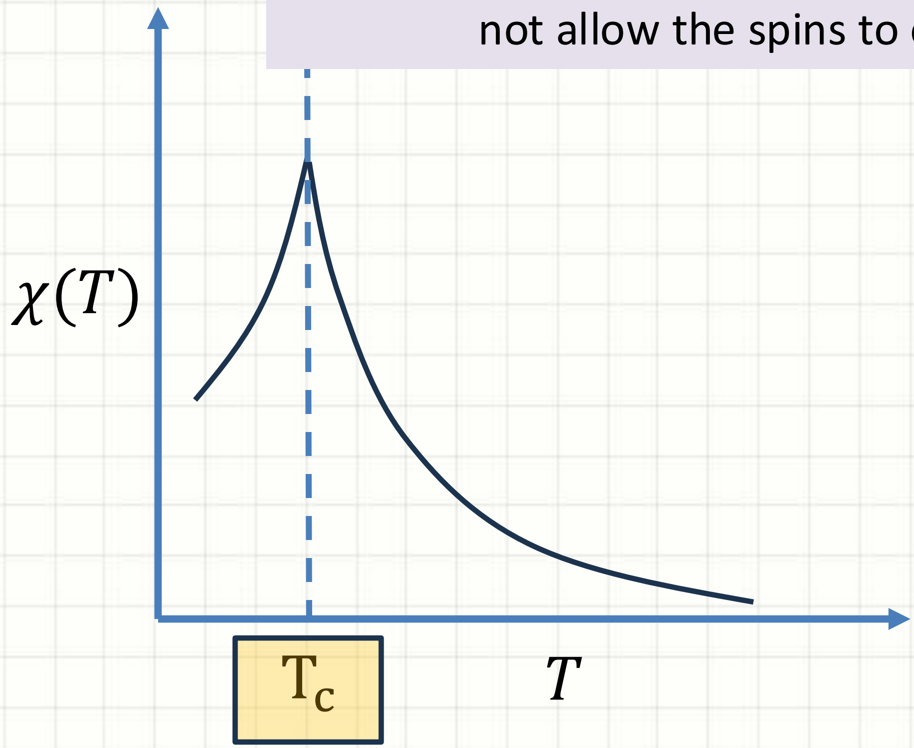
Q → quantum fluctuations

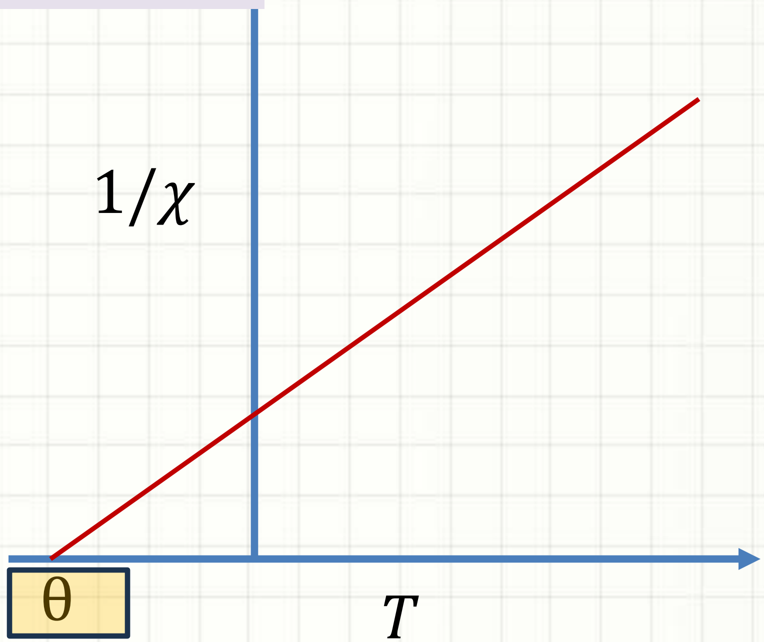
S → spin (or J) → strong correlations → Mott insulator

L → liquid → linear superposition of many configurations
(not solid, no disorder, not spin glass, not spin ice)

Very first signatures of a QSL: local moments exist; they interact strongly

But interaction and geometric frustration do not allow the spins to order





$$f = \theta/Tc$$

Ordered: $f \sim O(1)$
QSL: $f \to \infty$

Candidate QSL

Large f → very first signatures of a QSL

Quantum Matter

Landau paradigm

spontaneously broken symmetry
→ **local** order parameter
→ bosonic excitations

Topological Paradigm

“IQHE”
Topological Bands

“FQHE”
Topological Order

- Ground state degeneracy
- Long range entanglement
- Fractionalized Excitations

Important for storing information non-locally;
robust against decoherence

Topological Order

- ❖ Ground state of gapped many body system is degenerate
- ❖ Degeneracy depends on genus of Riemann surface
- ❖ Topological degeneracy cannot be lifted by any local perturbation (different from symmetry)

Fractionalized Excitations

Experimental significance of topological order:
(different from any symmetry breaking states)

1. Finite-energy quasiparticles of topological ordered systems carry fractional charges and fractional statistics

2. Topological ordered states can have gapless boundary excitations
→ topologically protected (against local perturbations)
→ perfect conducting boundary channels → device applications.

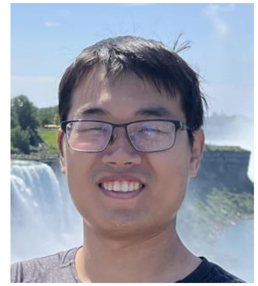
3. Emergent gauge theories

4. Building block for topological qubits → robust quantum computation

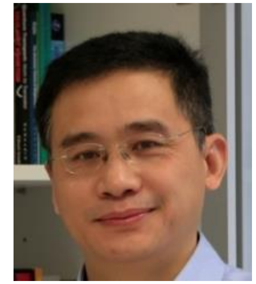
Shi Feng, OSU

Adhip
Agarwala
IIT Kanpur

Subhro
Bhattacharya
ICTS Bangalore

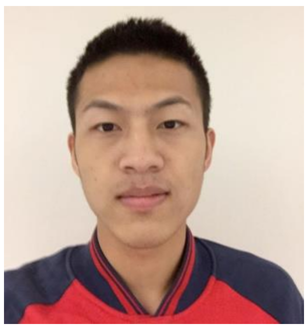


Penghao Zhu
OSU



Tao Xiang
IOP, Beijing

Kang Wang
IOP 王康



Xu Yang, OSU

Ryan Buechele, OSU

Exact Diagonalization
DMRG, iDMRG, iPEPS
Partons and mean field theory
Variational wavefunctions

Detection of anyon braiding through pump-probe spectroscopy, Xu Yang, R. Buechele, N. Trivedi, arXiv:2503.22792

Anyon dynamics in field-driven phases of the anisotropic Kitaev model,
S. Feng, A. Agarwala, S. Bhattacharjee, N. Trivedi, Phys. Rev. B 108, 035149 (2023)

Fractionalization Signatures in the Dynamics of Quantum Spin Liquids
Kang Wang, Shi Feng, Penghao Zhu, Runze Chi, Hai-Jun Liao, N. Trivedi, Tao Xiang, PRB 111, L100402, (2025)

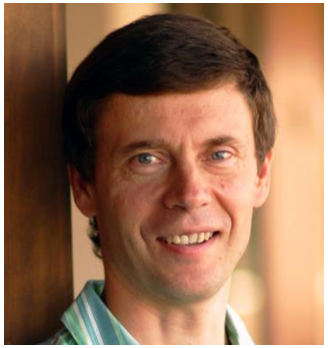
Emergent Majorana Metal in a Chiral Quantum Spin Liquid
Penghao Zhu, Shi Feng, Kang Wang, Tao Xiang, and N. Trivedi, Nature Communications 16, 2420 (2025)

Today's Focus:

1. Rich phase diagram of Kitaev model as a function of magnetic field
   -- new intermediate gapless QSL phase

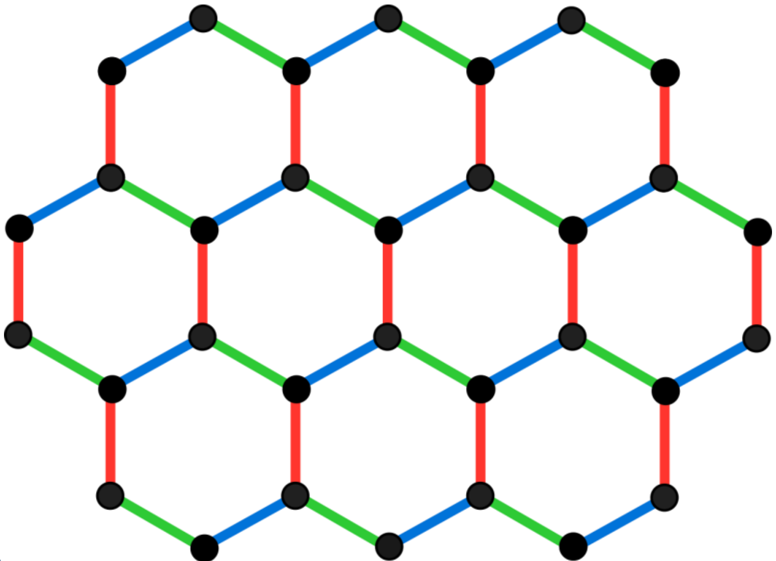
2. Proposal for how to see sharp signatures of fractionalized excitations in experiments

Kitaev Model: bond-dependent interactions between qubits



A. Kitaev, Annals of Physics **321**, 2-111 (2006)

$$\mathcal{H} = K\left[\sum_{\langle ij\rangle\in x}\sigma_i^x\sigma_j^x + \sum_{\langle ij\rangle\in y}\sigma_i^y\sigma_j^y + \sum_{\langle ij\rangle\in z}\sigma_i^z\sigma_j^z\right]$$



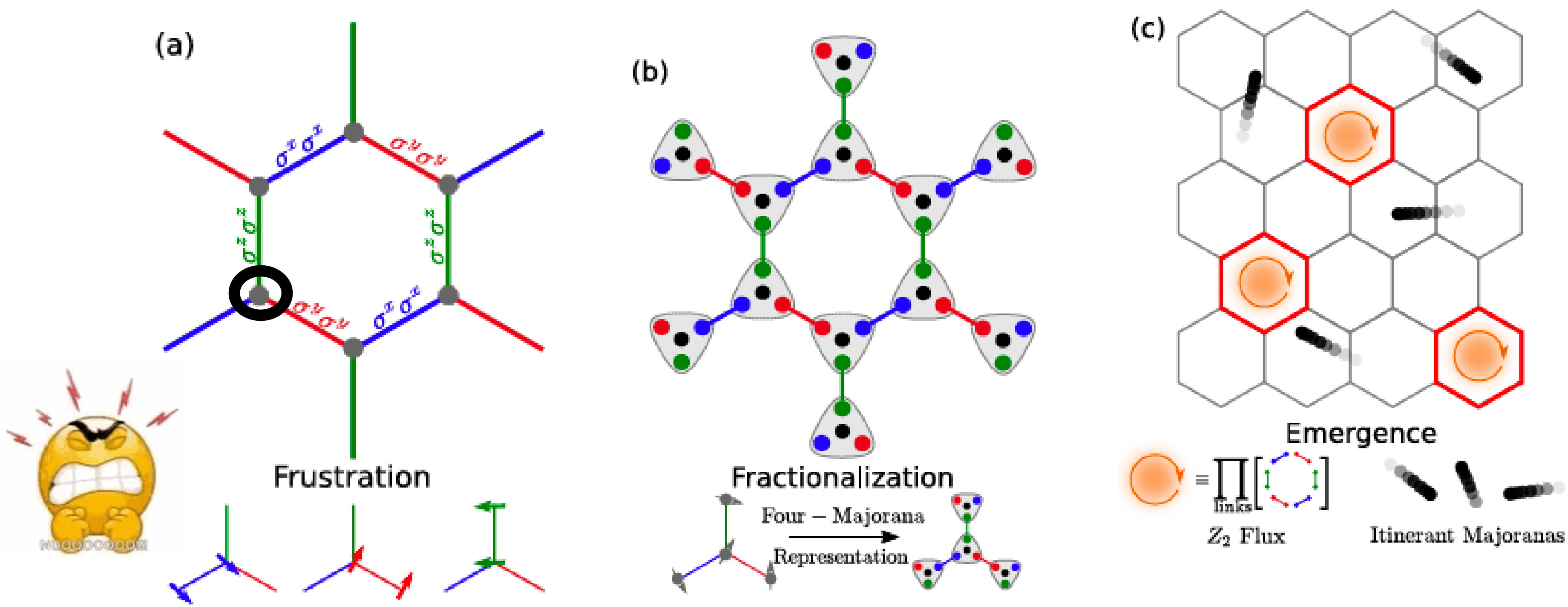
Exact solution

No magnetic long range order

Topological order

→ Quantun spin liquid with **long range entanglement**

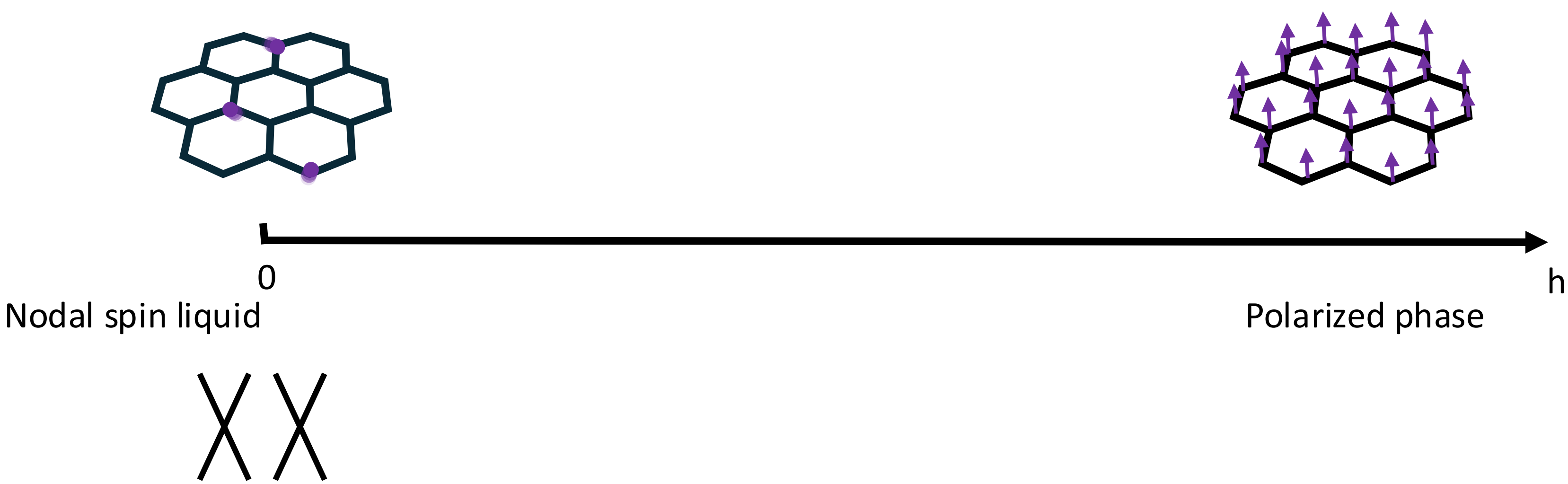
Frustration Fractionalization Emergence



Emergent Gauge Theory
Fractionalized Matter

Kitaev honeycomb model + [111] field

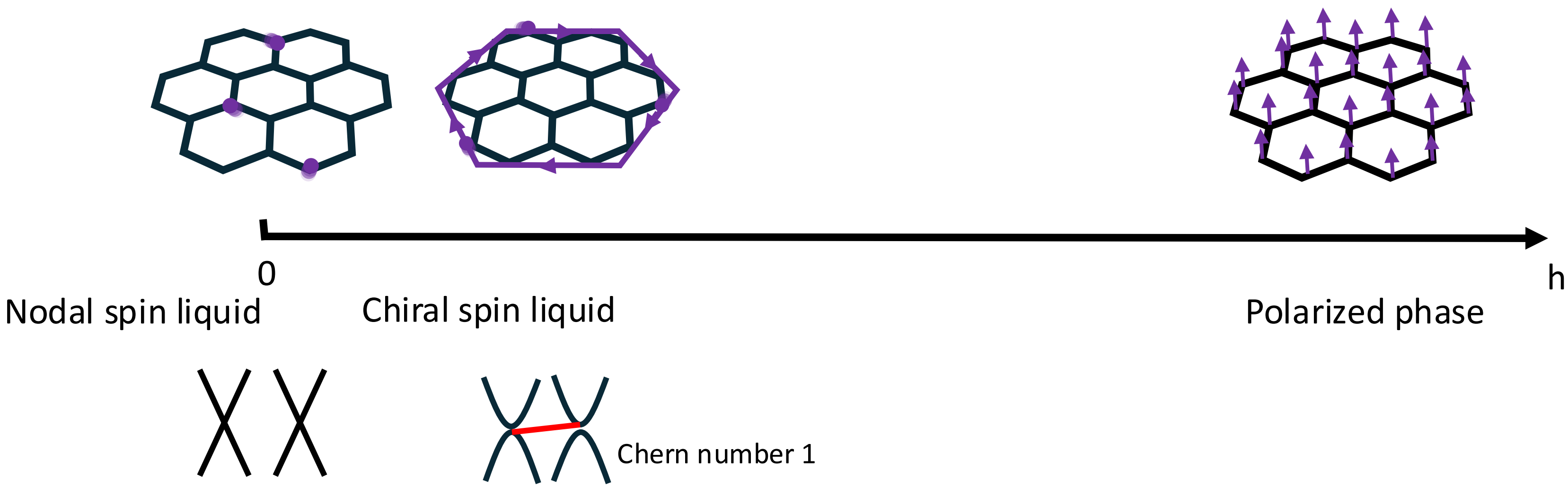
$$H = K \sum_{<jk>_\alpha} \sigma_j^\alpha \sigma_k^\alpha - h \sum_{j,\alpha} \sigma_j^\alpha$$



Kitaev honeycomb model + [111] field

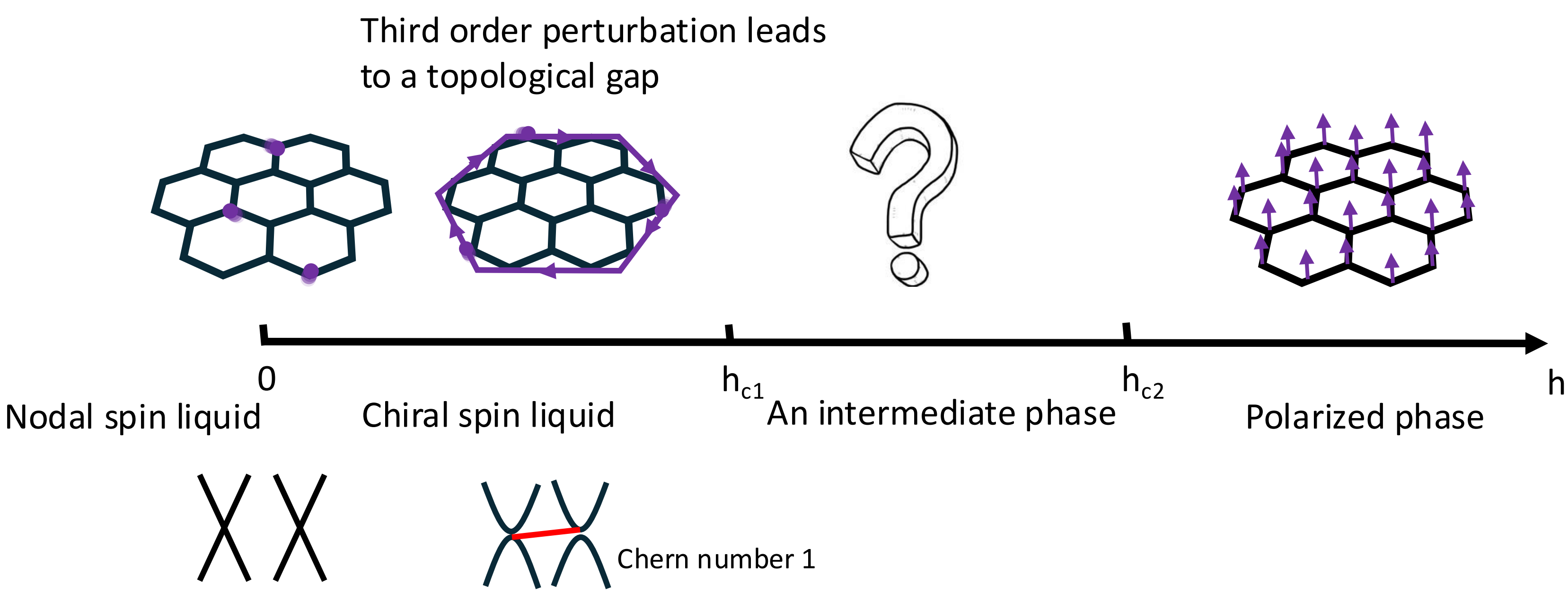
$$H = K \sum_{<jk>_\alpha} \sigma_j^\alpha \sigma_k^\alpha - h \sum_{j,\alpha} \sigma_j^\alpha$$

Third order perturbation leads
to a topological gap



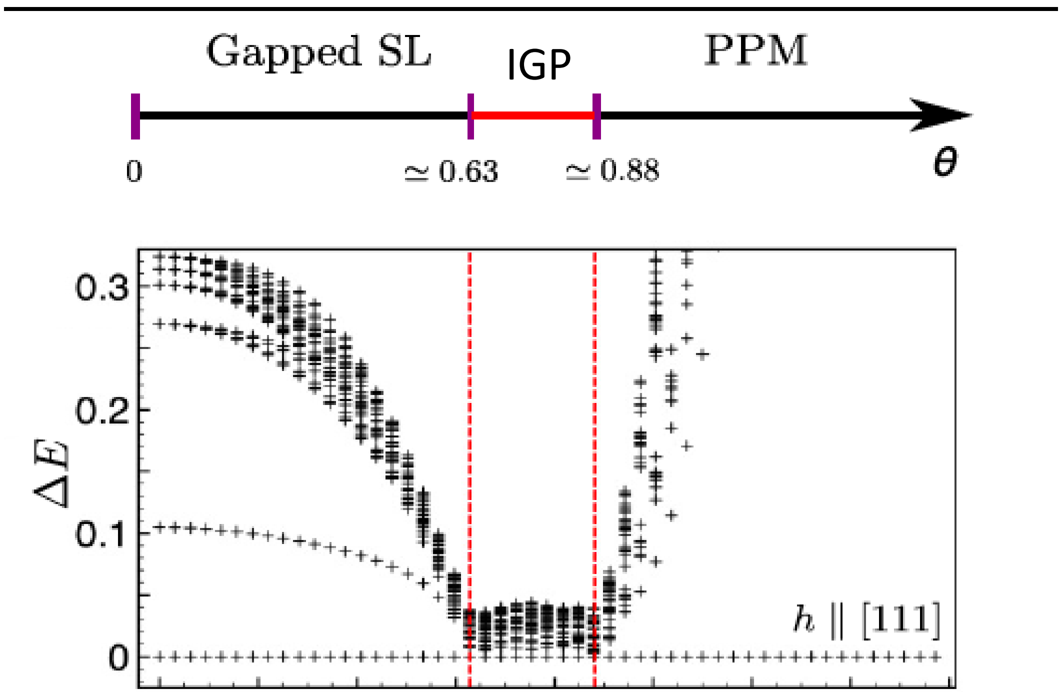
Kitaev honeycomb model + [111] field

$$H = K \sum_{<jk>_\alpha} \sigma_j^\alpha \sigma_k^\alpha - h \sum_{j,\alpha} \sigma_j^\alpha$$



Intermediate gapless phase

$h||[111]$

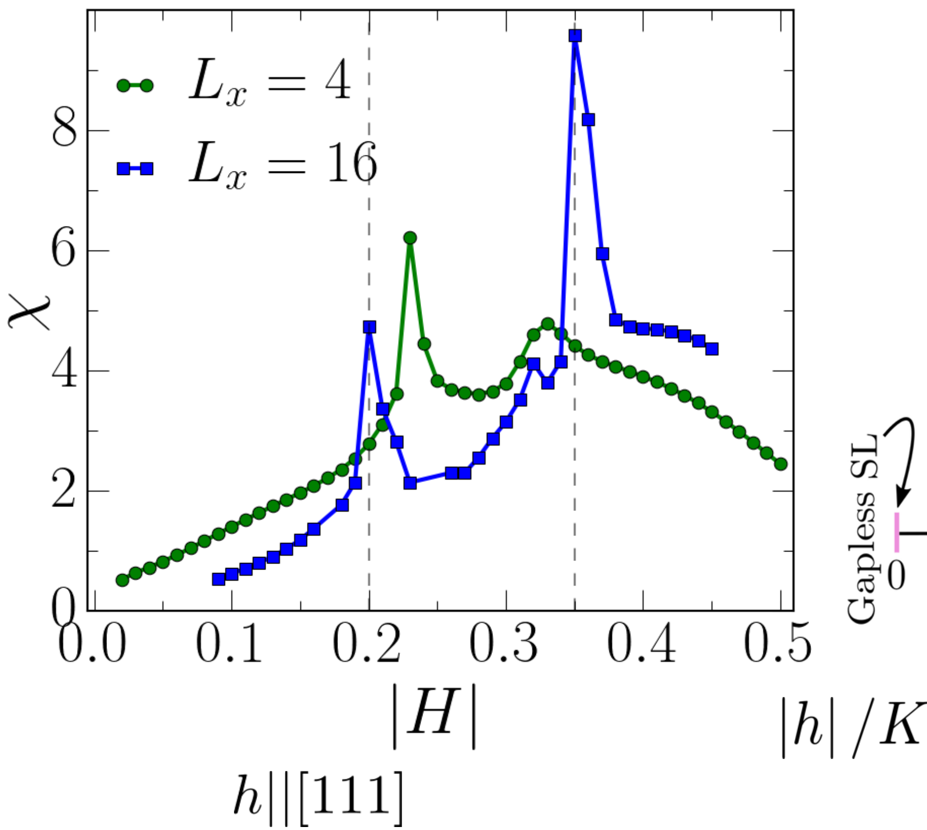


$$\theta \approx h/K$$

Exact diagonalization
24 sites

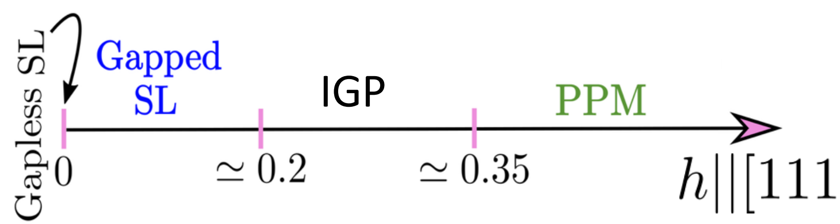
Ronquillo, Vengal, Trivedi, PRB 99, 140413 (2019)

Evidence for Two Transitions



$$H = H_K + h \sum S_i^{\alpha}$$

$$\chi_h = \frac{\partial^2 E_{gs}(K_z, h)}{\partial h^2}$$



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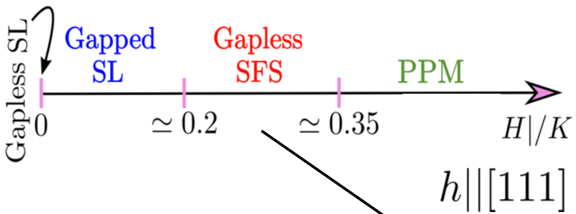
160 sites DMRG

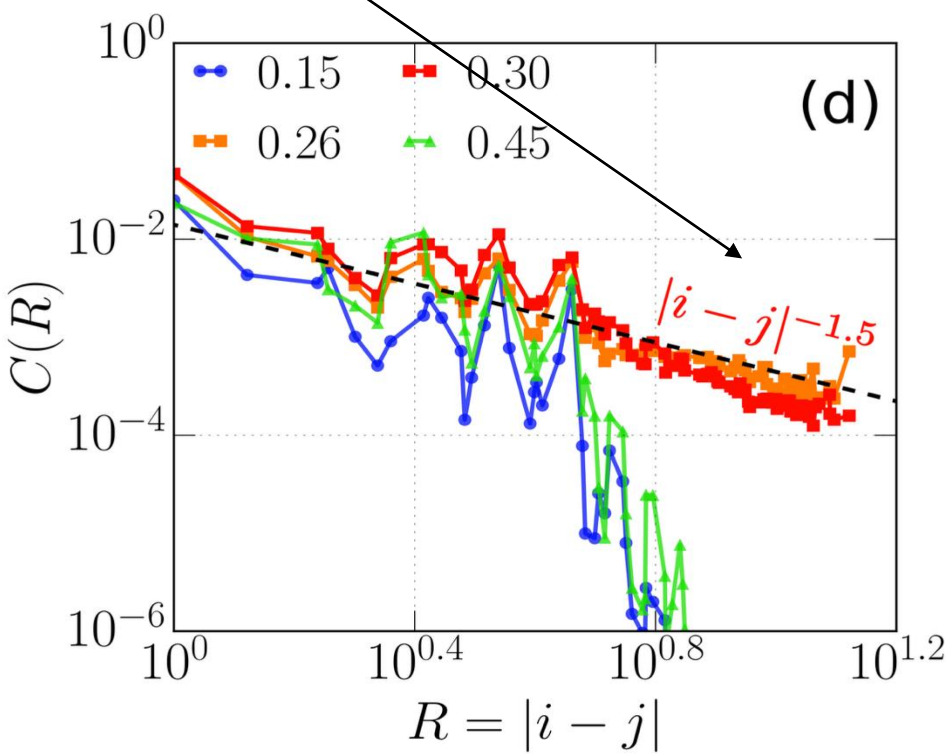
Patel & Trivedi, PNAS 116,12199 (2019)

Spin-Spin Correlations in Intermediate phase

IGP



$$C(R) = \frac{1}{N_R} \sum_{R=|i-j|} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle - \langle \mathbf{S}_i \rangle \cdot \langle \mathbf{S}_j \rangle$$

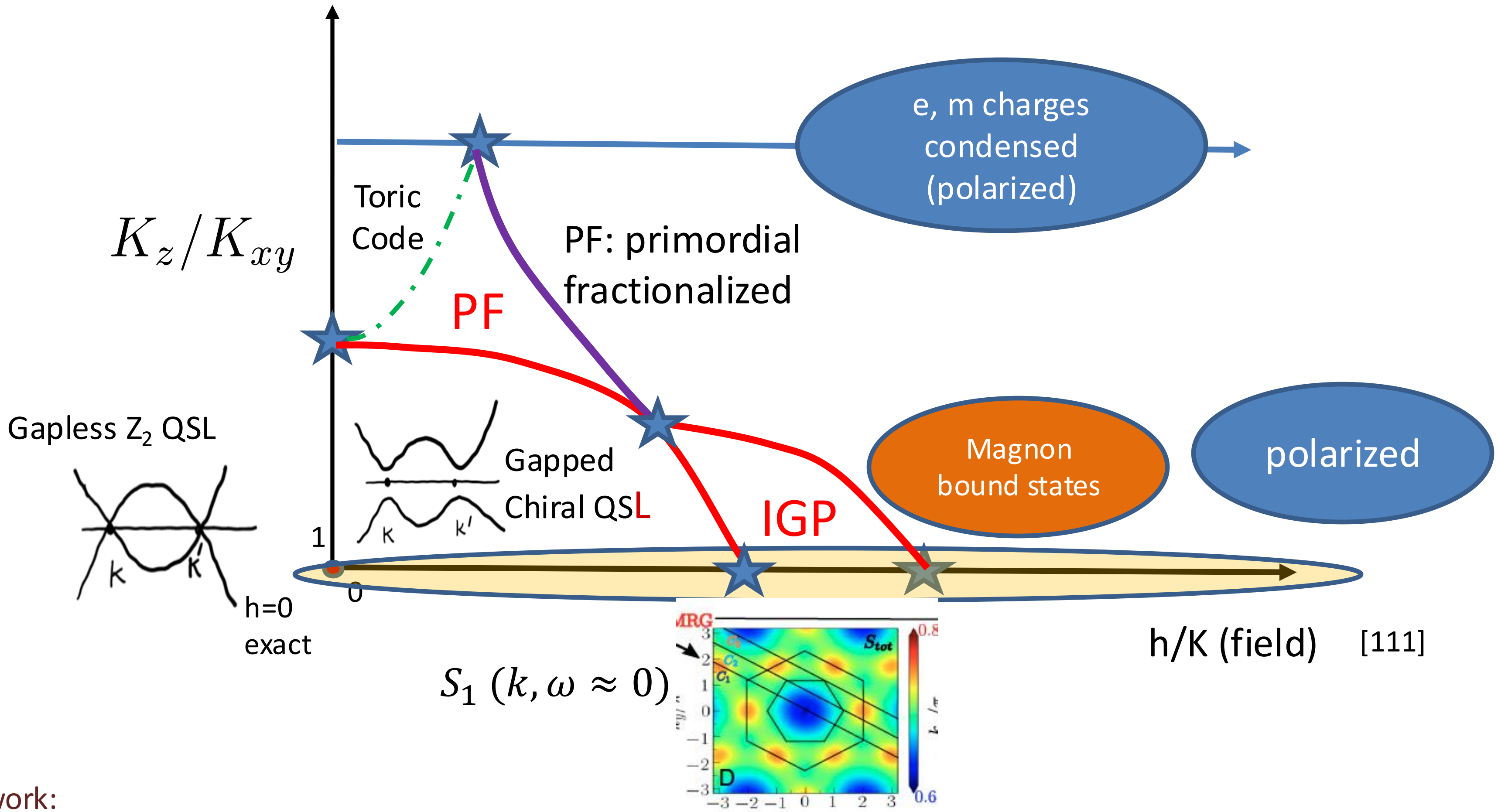


Distinct power law decay of real- space spin-spin correlations!

5

DMRG: 160 sites

Rich Phase Diagram



Related work:
Z. Zhu, et al., Phys. Rev. B 97, 241110 (2018)
M. Gohlke, et al., Phys. Rev. B 98, 014418 (2018)
C. Hickey and S. Trebst, Nat. Comm. 10, 530 (2019)
H.C. Jiang et al. arXiv 1809.08247
Y. Motome and J. Nasu, J. Phys. Soc. Jpn. 89, 012002 (2020)

Ronquillo, Vengal, Trivedi, PRB 99, 140413 (2019)
Patel & Trivedi, PNAS 116,12199 (2019)
Pradhan, Patel, Trivedi, PRB 101, 180401 (2020)
Feng, Agarwala, Bhattacharjee, Trivedi, PRB 108, 035149 (2023)

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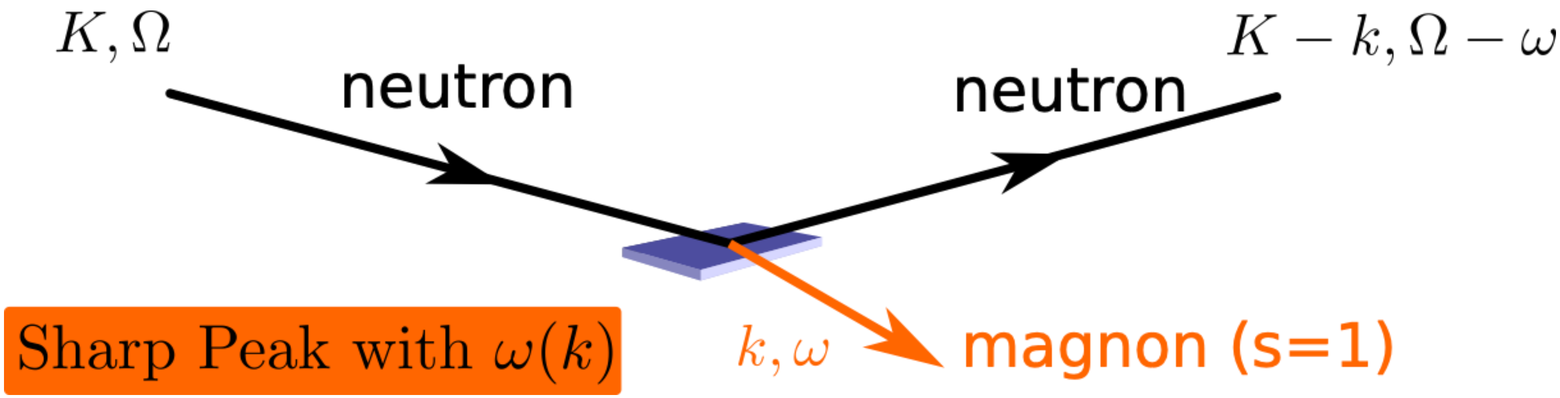
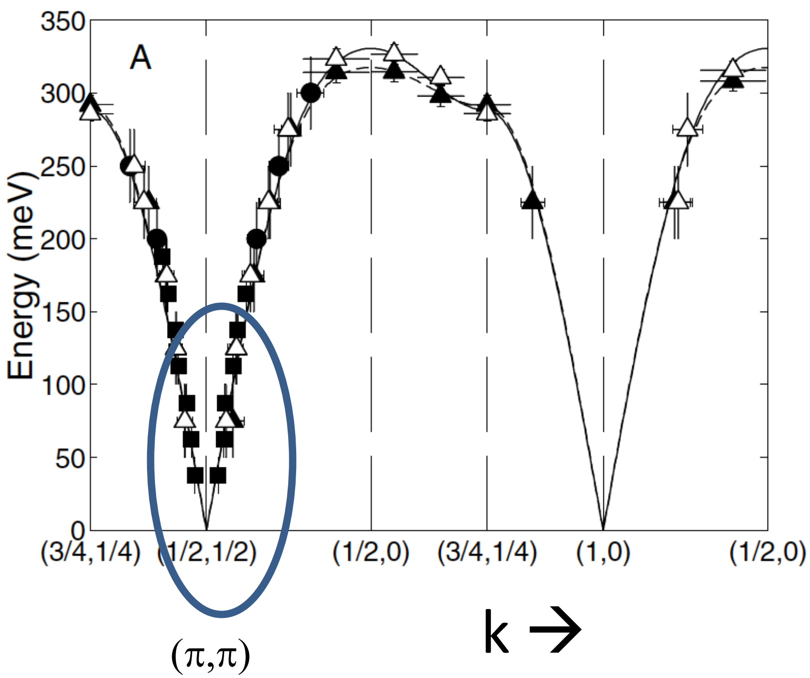
   -- new intermediate gapless QSL phase

2. Proposal for how to see sharp signatures of fractionalized excitations in experiments

Inelastic neutron scattering

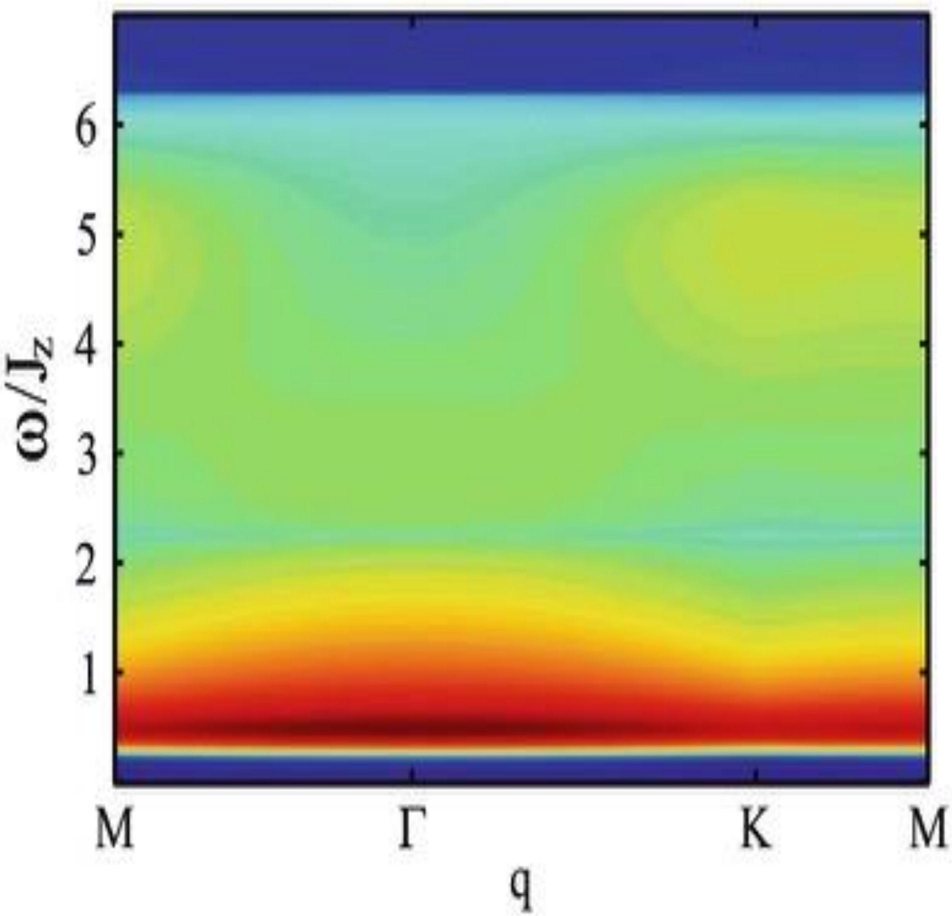


La_2CuO_4: Mott Insulator Antiferromagnet



Long range antiferromagnetic order
Sharp peaks in inelastic neutron scattering structure factor S(**q**,ω)
→ Magnon modes with dispersion E(**k**)

Coldea, Hayden, Aeppli, Perring, Frost, Mason, Cheong, Fisk, PRL 86, 5377 (2001)



QSL Kitaev Model:

- Local moments exist,
- they interact strongly,
- BUT there are no well-defined modes

Theory: Knolle, Kovrizhin, Chalker, Moessner PRL 112, 207203 (2014)

α-$RuCl_3$: Mott Insulator Quantum Spin Liquid candidate

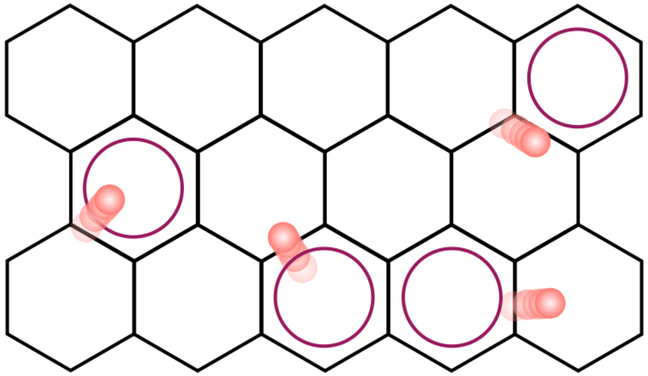
$$|\theta_{CW}| = 40K$$

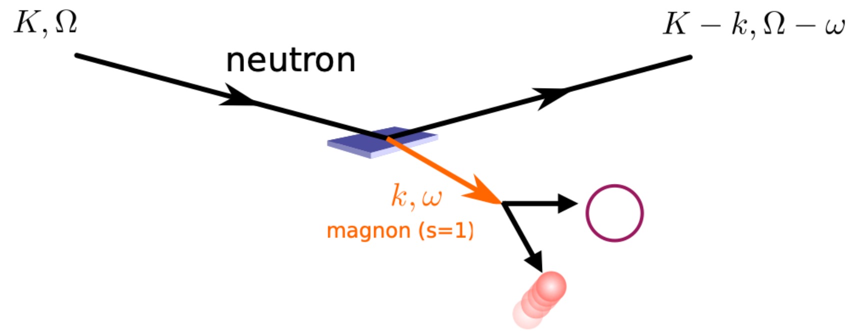
$$T_c = 7K$$

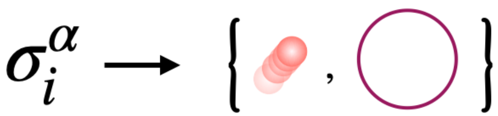
$$f \approx 6$$

Experiment: Banerjee, Yan, Knolle, Bridges, Stone, Lumsden, Mandrus, Tennant, Moessner, and Nagler, Science 356, 1055 (2017)

Fractionalization







$$S(\mathbf{k}, \omega) \sim \mathrm{F.T.}[\sigma_{\mathbf{k}}(t)\sigma_{-\mathbf{k}}(0)]$$

Local spin excitation fractionalizes into majorana + Z_2 fluxes

Inelastic neutron scattering mixes fluxes and majorana fermions

Prediction for a sharp signature in a QSL

$$\mathcal{O}1 = \sigma_j^+$$

$$S1(q,t) = \langle\ \sigma_i^-(t)\sigma_j^+(0)\ \rangle$$

FT(i-j)

$$\mathcal{O}2 = \sigma_j^+\sigma_{j+z}^+$$

$$S2(q,t) = \langle\ \sigma_i^-\sigma_{i+z}^-(t)\ \sigma_j^+\sigma_{j+z}^+(0)\ \rangle$$

FT(i-j)

Excitation	Kitaev	Heisenberg
1 spin flip	Broad High Energy	Sharp Strong
2 spin flip	Sharp; directional Low energy	Broad Weak

Feng, Agarwala, Bhattacharjee, Trivedi, PRB 108, 035149 (2023)

Dynamical spin-spin correlations

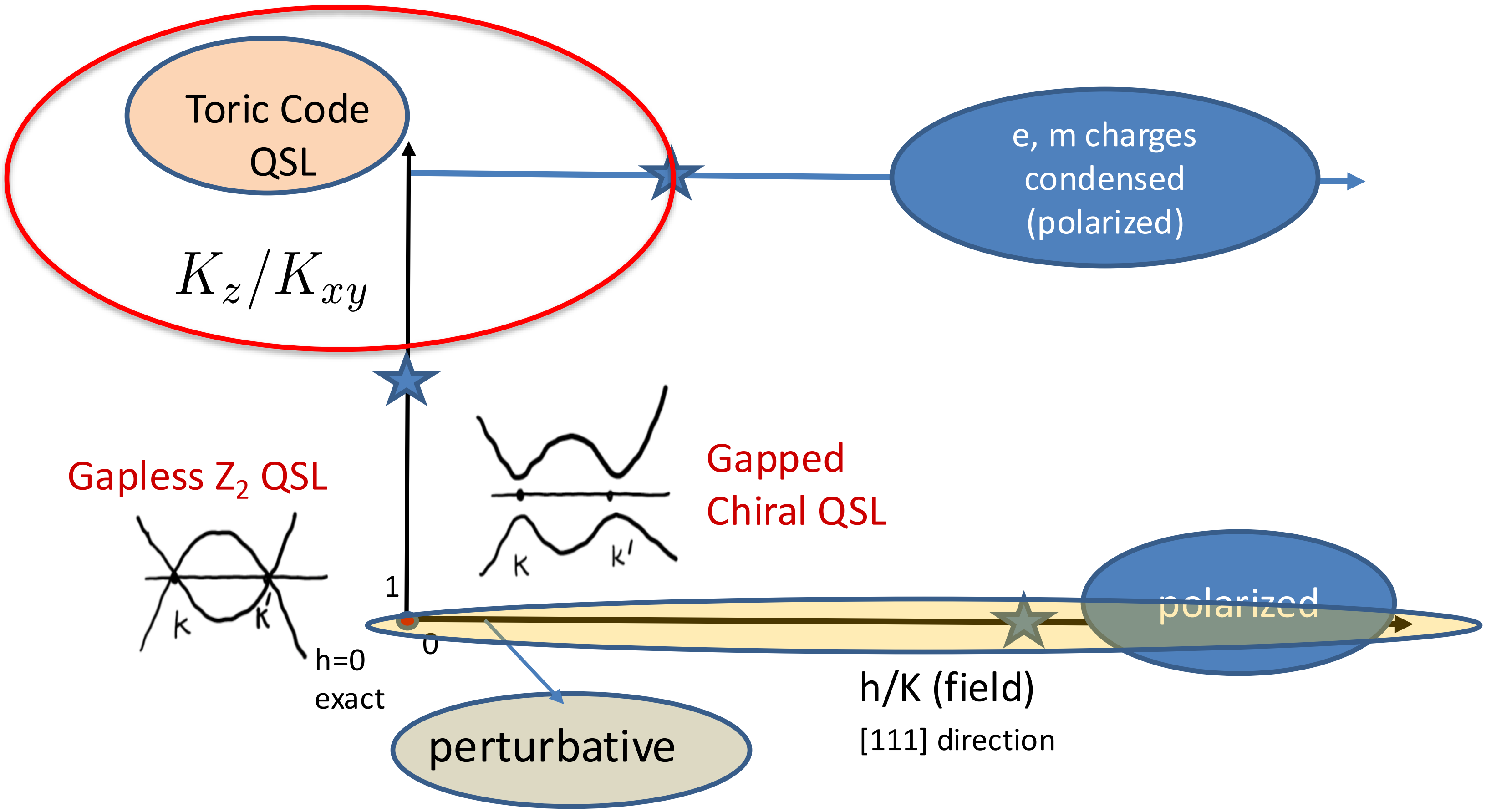
$$S_1^\alpha(\mathbf{k},\omega) = \sum_{m\neq 0} \langle 0|\sigma_{\mathbf{k}}^\alpha|m\rangle \ \langle m|\sigma_{-\mathbf{k}}^\alpha|0\rangle \ \delta(\omega - E_m + E_0)$$

Dynamical dimer-dimer correlations

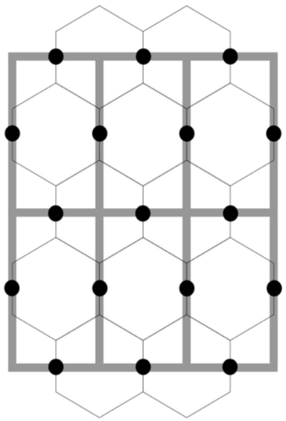
$$S_2^\alpha(\mathbf{k},\omega) = \sum_{m\neq 0} \langle 0|\mathcal{D}_{\mathbf{k}}^\alpha|m\rangle \ \langle m|\mathcal{D}_{-\mathbf{k}}^\alpha|0\rangle \ \delta(\omega - E_m + E_0)$$

$$\mathcal{D}_j^\alpha \equiv \sigma_j^\alpha \sigma_{j+z}^\alpha$$

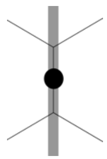
Anisotropy + Field → sharp signature of anyons in the flux sector



Toric Code in large K_z limit

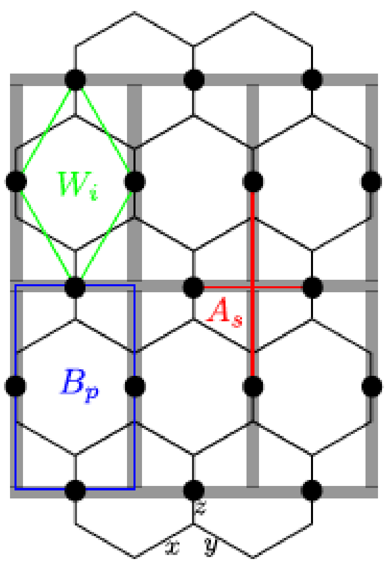


$$H = K \sum_{\text{x bond}} S_i^x S_j^x + K \sum_{\text{y bond}} S_i^y S_j^y + K_z \sum_{\text{z bond}} S_i^z S_j^z$$

 $= \quad |\uparrow\downarrow\rangle \text{ or } |\downarrow\uparrow\rangle \longrightarrow \tau$

$\tau = (\sigma_A^z - \sigma_B^z)/2$

1 spin flip→ high energy sector
2 spin flips → returns to the low energy sector



Fourth order perturbation theory:

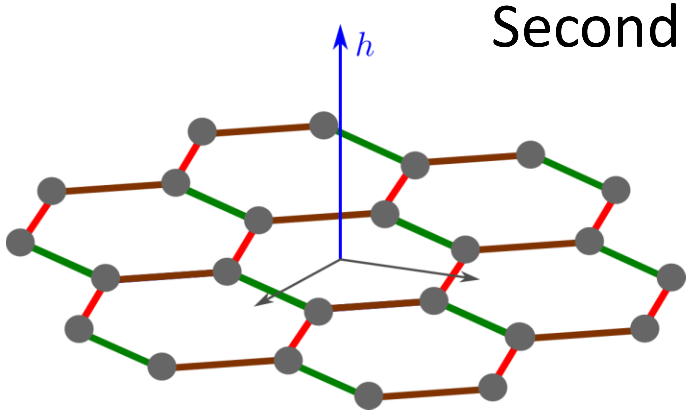
$$H_{\text{eff}} \sim -\sum_s A_s - \sum_p B_p$$

$$A_s = \prod_{i \in s} \tau_i^x \qquad B_p = \prod_{i \in p} \tau_i^z$$

Excitations of gauge field:
e, m (bosons)
e x m (fermions)

Magnetic-field induces anyon dynamics

$$H = K \sum_{\text{x bond}} S_i^x S_j^x + K \sum_{\text{y bond}} S_i^y S_j^y + K_z \sum_{\text{z bond}} S_i^z S_j^z - \vec{h} \cdot \sum_i \vec{S}_i$$



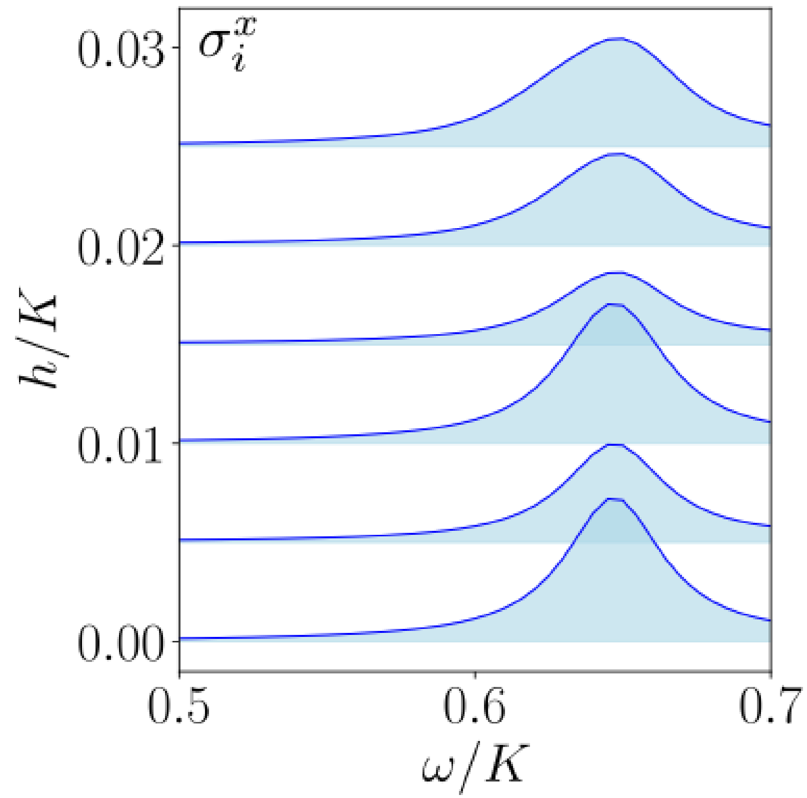
Second order perturbation theory:

$$H_{\text{eff}} \sim -\sum_s A_s - \sum_p B_p + \sum_i \frac{h^2}{K_z} \tau_i^y$$

q-integrated 2 Spin Flip Spectrum

$$\sum_k S_2(k,\omega)$$

Heisenberg

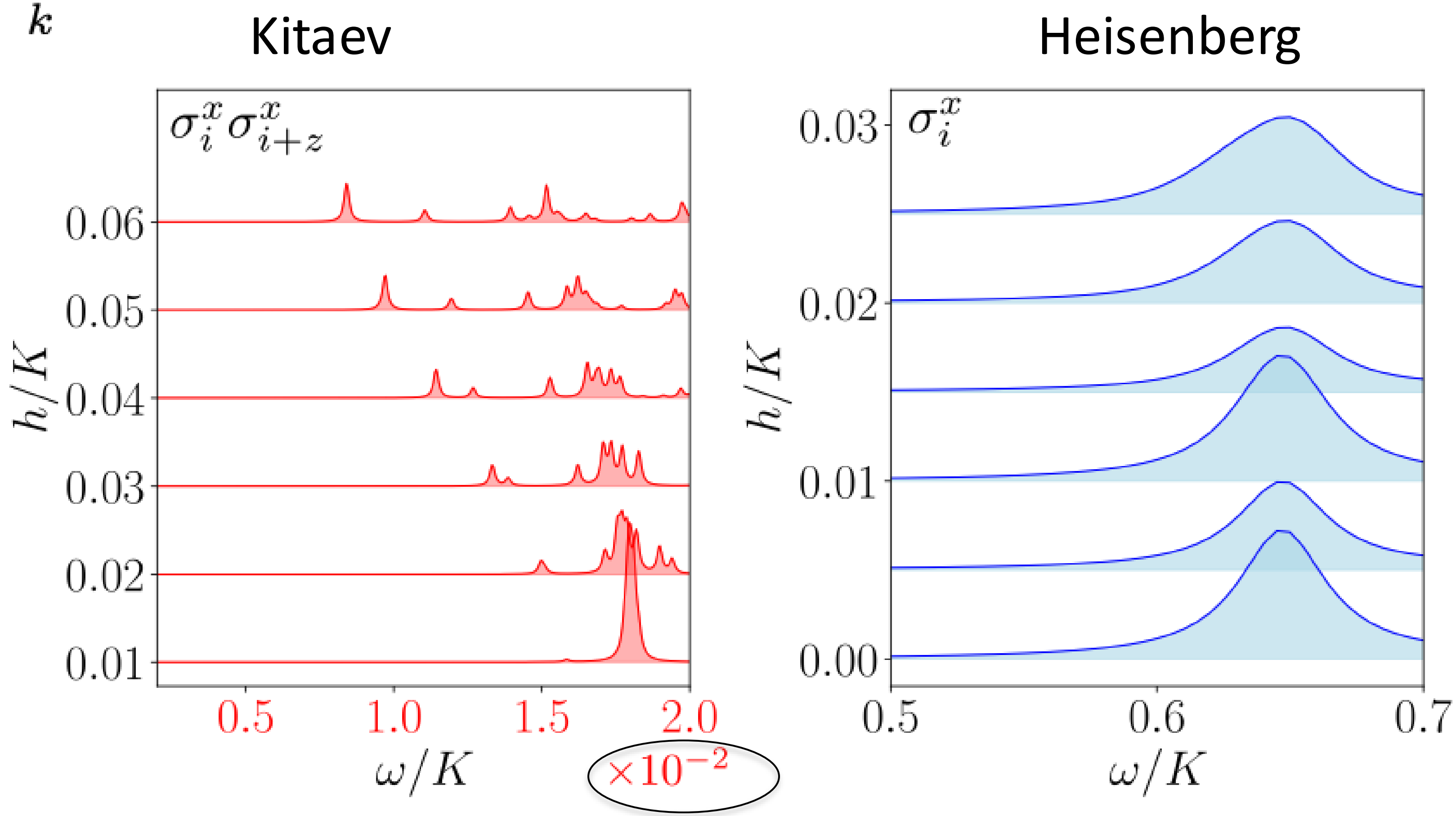


$$S_2^\alpha(\mathbf{k},\omega) = \sum_{m\neq 0} \langle 0|\mathcal{D}^\alpha_{\mathbf{k}}|m\rangle \ \langle m|\mathcal{D}^\alpha_{-\mathbf{k}}|0\rangle \ \delta(\omega - E_m + E_0)$$

$$\mathcal{D}_j^\alpha \equiv \sigma_j^\alpha \sigma_{j+z}^\alpha$$

q-integrated 2 Spin Flip Spectrum

$$\sum_k S_2(k,\omega)$$



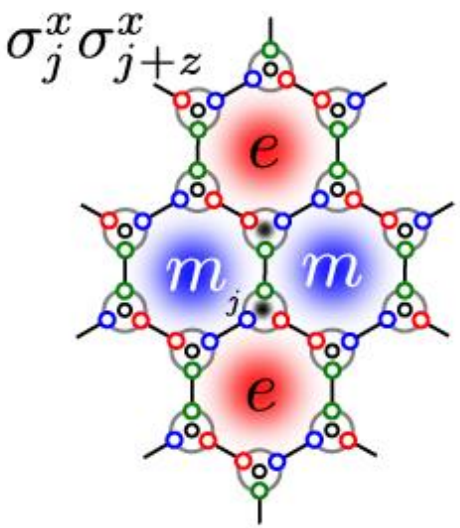
$$S_2^\alpha(\mathbf{k},\omega) = \sum_{m\neq 0} \langle 0|\mathcal{D}_{\mathbf{k}}^\alpha|m\rangle \; \langle m|\mathcal{D}_{-\mathbf{k}}^\alpha|0\rangle \; \delta(\omega - E_m + E_0)$$

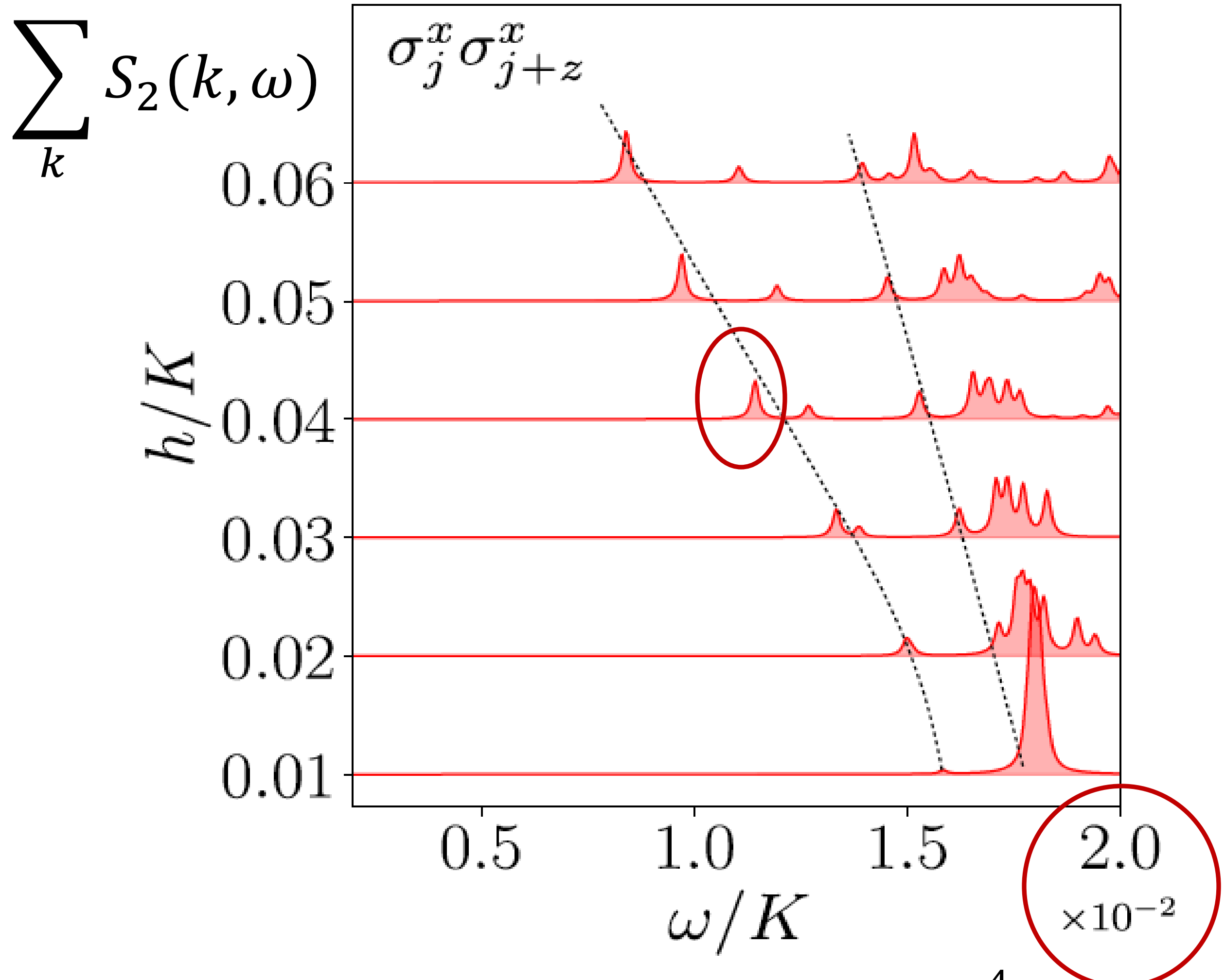
$$\mathcal{D}_j^\alpha \equiv \sigma_j^\alpha \sigma_{j+z}^\alpha$$

Two-spin flip spectral function:

$$S_2^\alpha(\mathbf{k},\omega) = \sum_{m\neq 0} \langle 0|\mathcal{D}_{\mathbf{k}}^\alpha|m\rangle\ \langle m|\mathcal{D}_{-\mathbf{k}}^\alpha|0\rangle\ \delta(\omega - E_m + E_0)$$

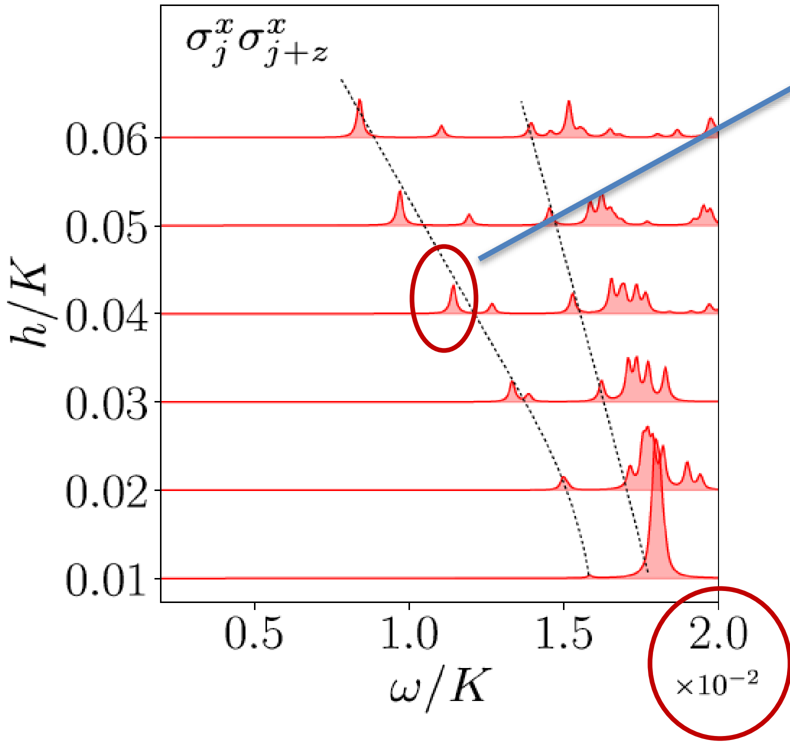
$$\mathcal{D}_j^\alpha \equiv \sigma_j^\alpha \sigma_{j+z}^\alpha$$



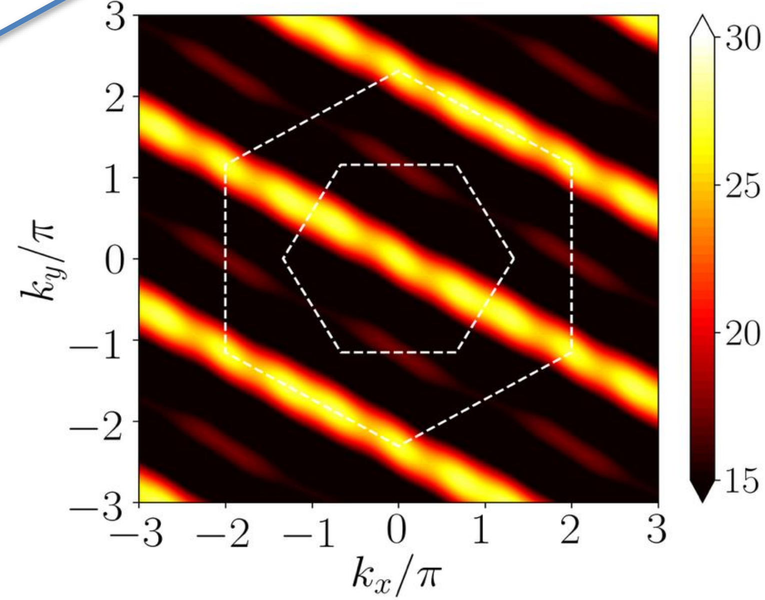


$$J_{TC} = \frac{K^4}{16Kz^3}$$

Constrained anyon dispersion



$\omega = \omega_{peak}$



$$\varepsilon(\mathbf{k}) = 4J_{\mathrm{TC}} - \frac{4h^2}{K_z}\cos\left(\frac{\sqrt{3}}{2}k_x + \frac{3}{2}k_y\right)$$

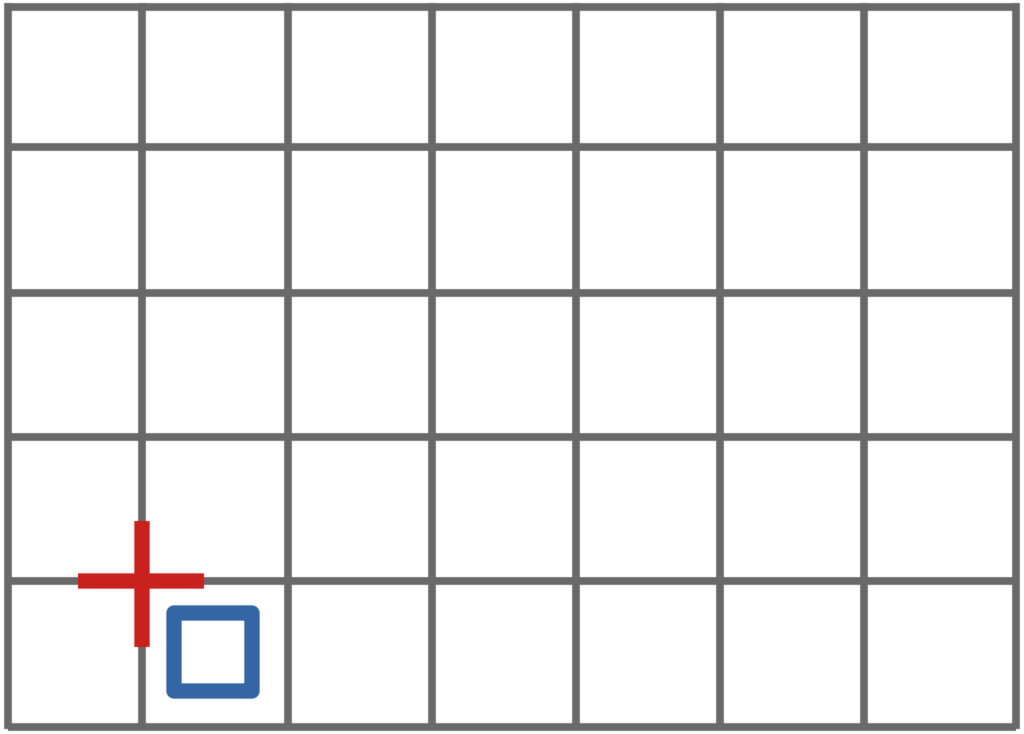
$$\mathbf{v}(\mathbf{k}) = \partial_{\mathbf{k}}\varepsilon_x(\mathbf{k}) = \frac{4h^2}{K_z}\mathbf{d}_1 \sin(\mathbf{d}_1 \cdot \mathbf{k})$$

Velocity in $\mathbf{d}_1 = (\frac{\sqrt{3}}{2}, \frac{3}{2})$ direction

DMRG 160 sites

Magnetic-field-induced anyon dynamics

$$H_{\text{eff}} \sim -\sum_s A_s - \sum_p B_p + \sum_i \frac{h^2}{K_z}\tau_i^y$$



$$A_s = \prod_{i\in s}\tau_i^x$$

$$B_p = \prod_{i\in p}\tau_i^z$$

$$\tau_i^y \sim \tau_i^z\tau_i^x$$

$$[\tau_i^x, A_s] = 0$$

$$[\tau_i^z, B_p] = 0$$

Magnetic-field-induced anyon dynamics

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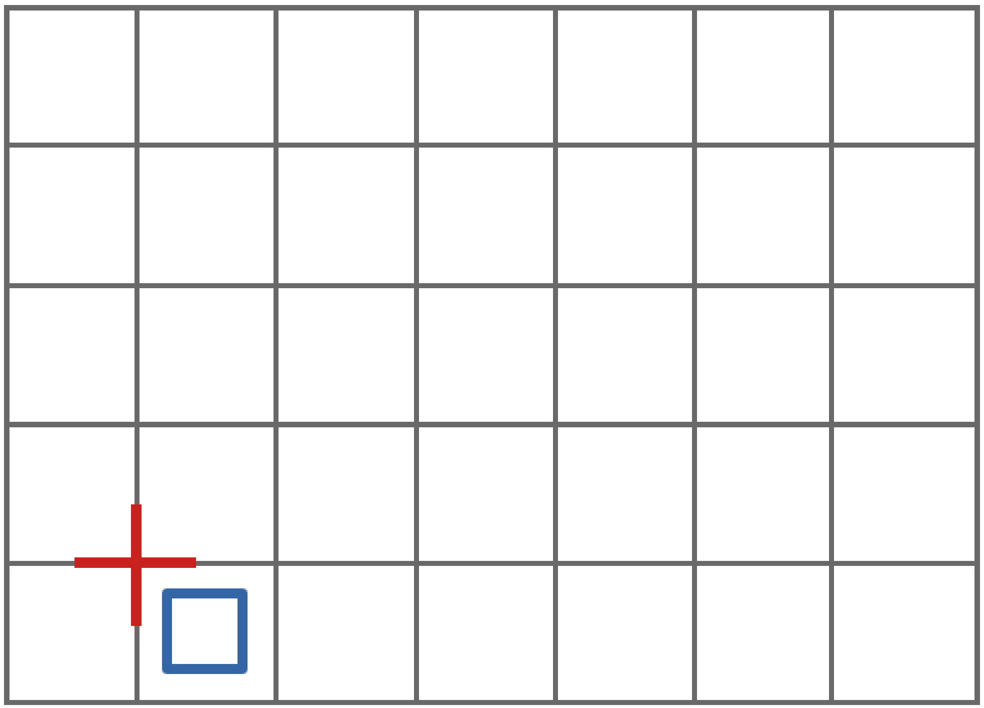

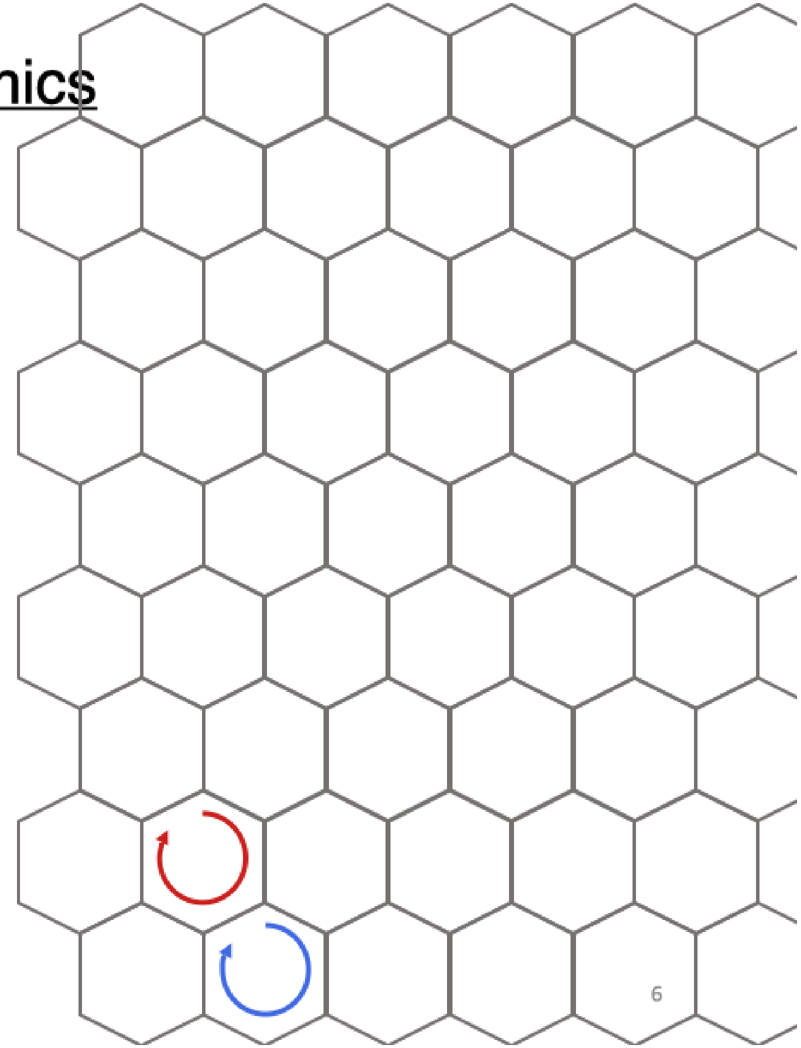

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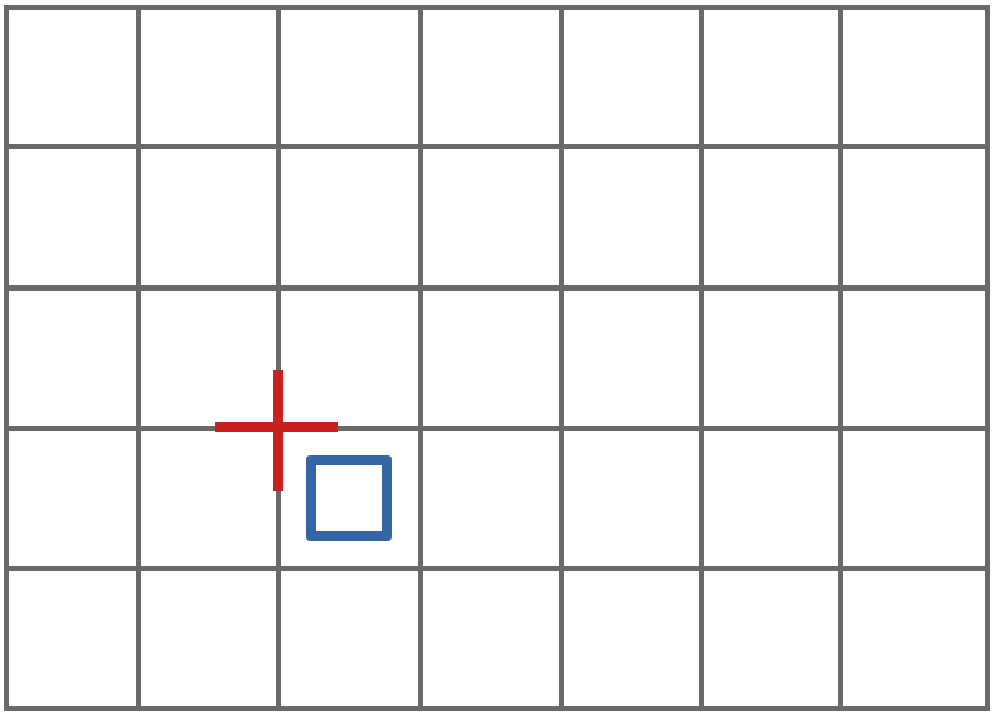
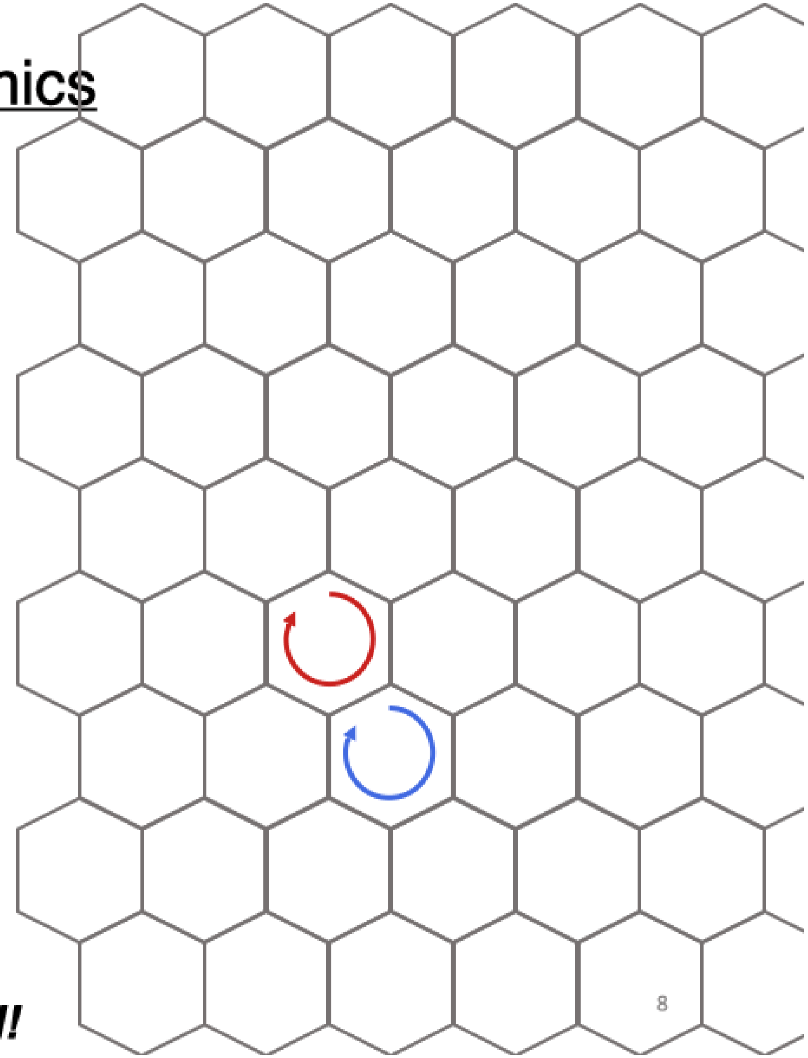
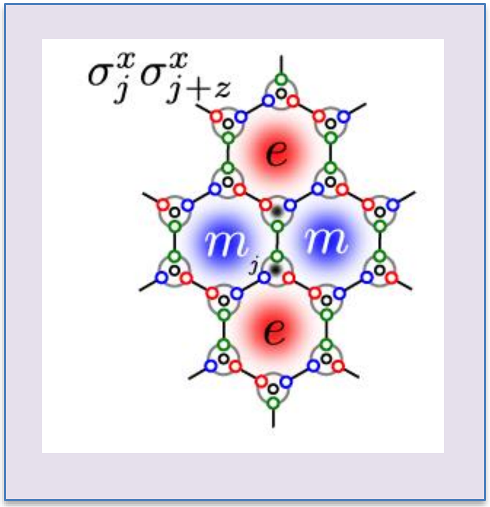
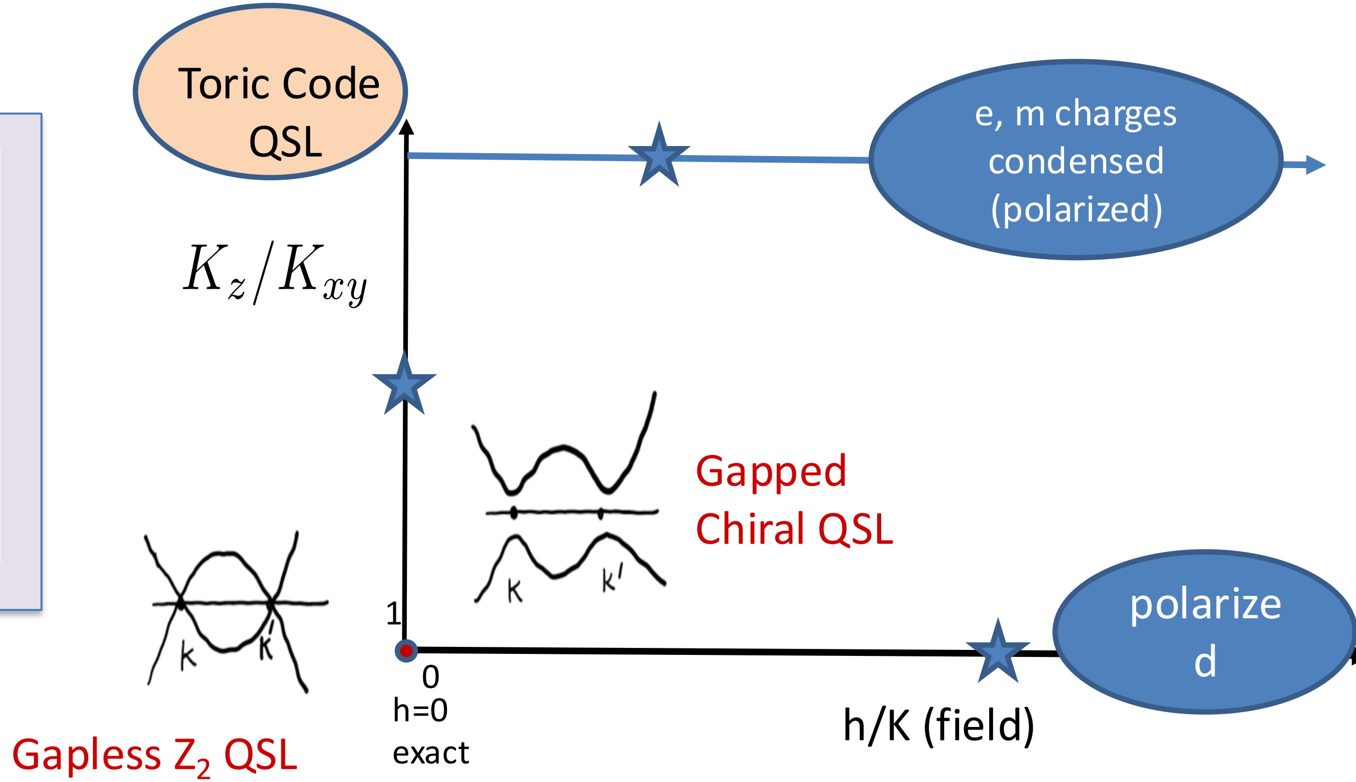


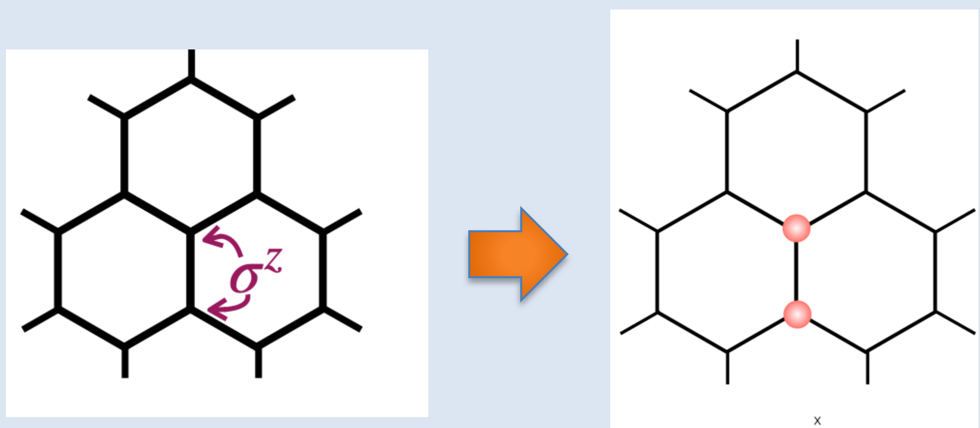


1D transport of composite anyon in a 2D model!

2-spin flip response







Only majorana response

How can we see the ***statistics*** of the fractionalized anyons?

Dynamics of Toric Code

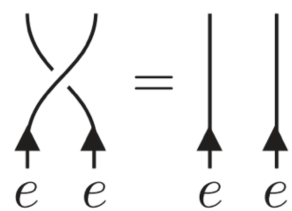
$$H_0 = -J_A \sum_v \hat{A}_v - J_B \sum_p \hat{B}_p$$

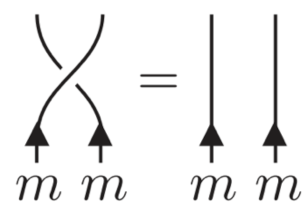
$$H = H_0 - h_X \sum_j \hat{X}_j - h_Z \sum_j \hat{Z}_j$$

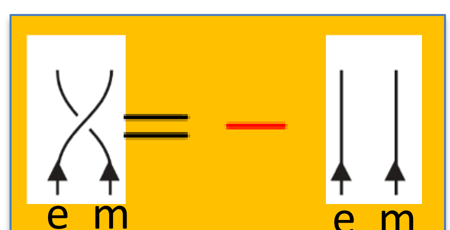
$$\hat{A}_v = \prod_{j \in v} \hat{X}_e \qquad \hat{B}_p = \prod_{j \in p} \hat{Z}_p$$

Z_2 Abelian anyons:

- e $(A_s = -1)$ boson
- m $(B_p = -1)$ boson
- $\epsilon = e \times m$ fermion







Linear response

t=0 : PROBE

$$M^\alpha \equiv \sum_{j=1}^{N} \sigma_j^\alpha \;\; (\alpha \in \{x, y, z\})$$

t : MEASURE

$$M^\beta(t)$$

$$\chi^{(1)}_{\alpha,\beta}(t) = -\frac{i}{N}\langle 0|[M^\beta(t), M^\alpha(0)]|0\rangle$$

Toric Code

$$\chi^{(1)}_{zz}(t) \sim \frac{b}{t} \times \sin(\Delta t)$$

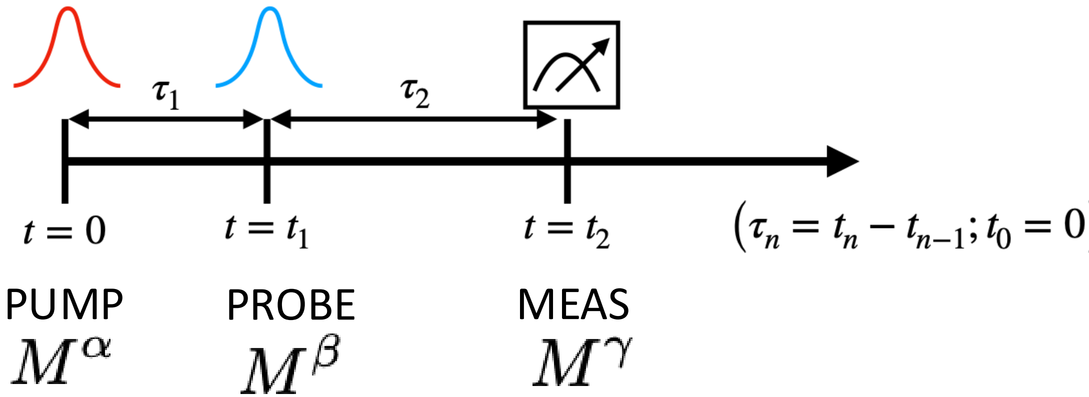
Decreasing probability to find the two anyons at the same location to be annihilated together at time t

each Mz can either excite a pair of e anyons with a gap Δ = 4J

Fava, Gopalakrishnan, Vasseur, Essler, Parameswaran, PRL 131, 256505 (2023)
McGinley, Fava, Parameswaran, PRL 132, 066702 (2024)

NON-Linear response

Wan, Armitage PRL 122, 257401 (2019);
Wonjune Choi, Ki Hoon Lee, Yong Baek Kim, 124, 117205 (2020)
S. Mukamel, Principles of Nonlinear Optical Spectroscopy (Oxford 1995).

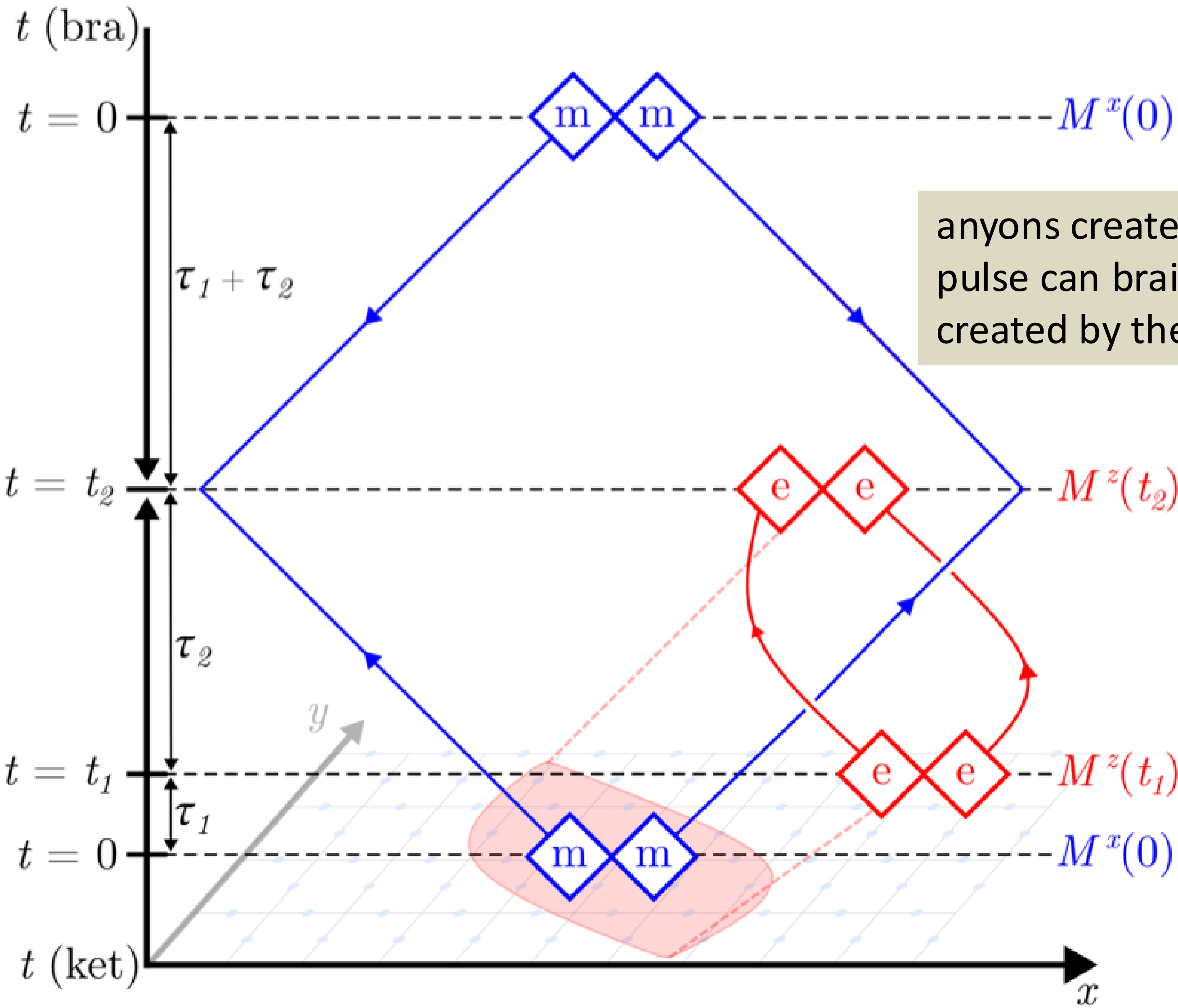


$$\chi^{(1)}_{\alpha,\beta}(t) = -\frac{i}{N}\langle 0 | [M^\alpha(0), M^\beta(t)] | 0 \rangle$$

$$\boxed{\begin{aligned}\Xi^{PP}_{\alpha,\beta,\gamma}(t_1, t_2) &= -\frac{i}{N}\Big(\underbrace{\langle 0 | e^{i\mu M^\alpha}[M^\beta(t_1), M^\gamma(t_2)]e^{-i\mu M^\alpha} | 0 \rangle}_{\chi^{(1)}\text{in pumped state}} - \underbrace{\langle 0 | [M^\beta(t_1), M^\gamma(t_2)] | 0 \rangle}_{\chi^{(1)}\text{in g.s.}}\Big) \\ &= \mu \chi^{(2)}_{\alpha,\beta,\gamma}(t_1, t_2) + \frac{\mu^2}{2}\chi^{(3)}_{\alpha,\beta,\gamma}(t_1, t_2) + \mathcal{O}(\mu^3)\end{aligned}}$$

Braiding

$$\chi^{(3)}(t_1, t_2) \sim \langle 0 | M^x(0) M^z(t_2) M^z(t_1) M^x(0) | 0 \rangle$$



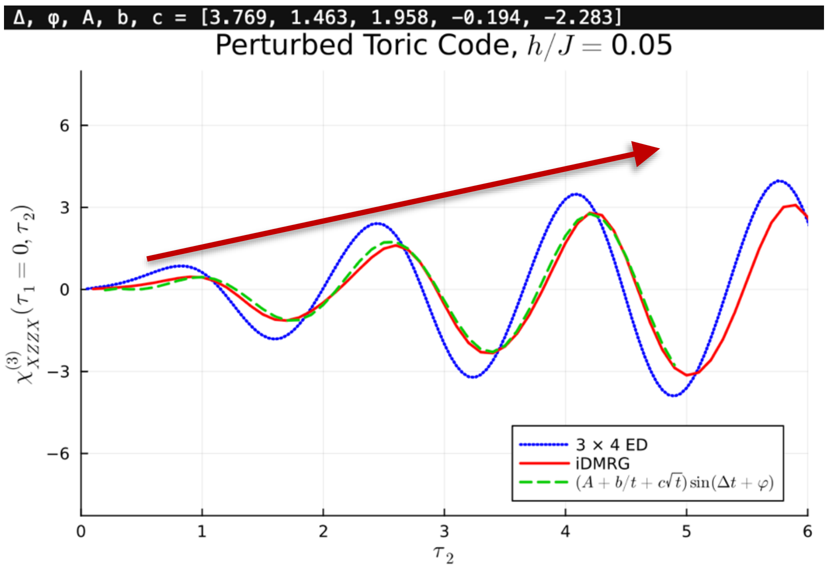
anyons created by the first pulse can braid with those created by the second pulse

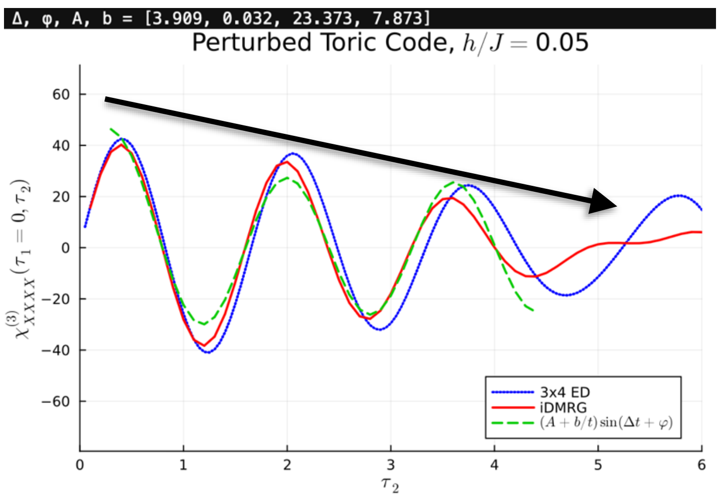
Braiding

iDMRG simulations
+ ED (test)

- XZZ response creates e-m-e-m excitations that braid
- does not quickly decay
- contribution from divergent piece of response

- XXX response creates e-e-e-e excitations; no braiding
- Decays
- essentially proportional to linear response



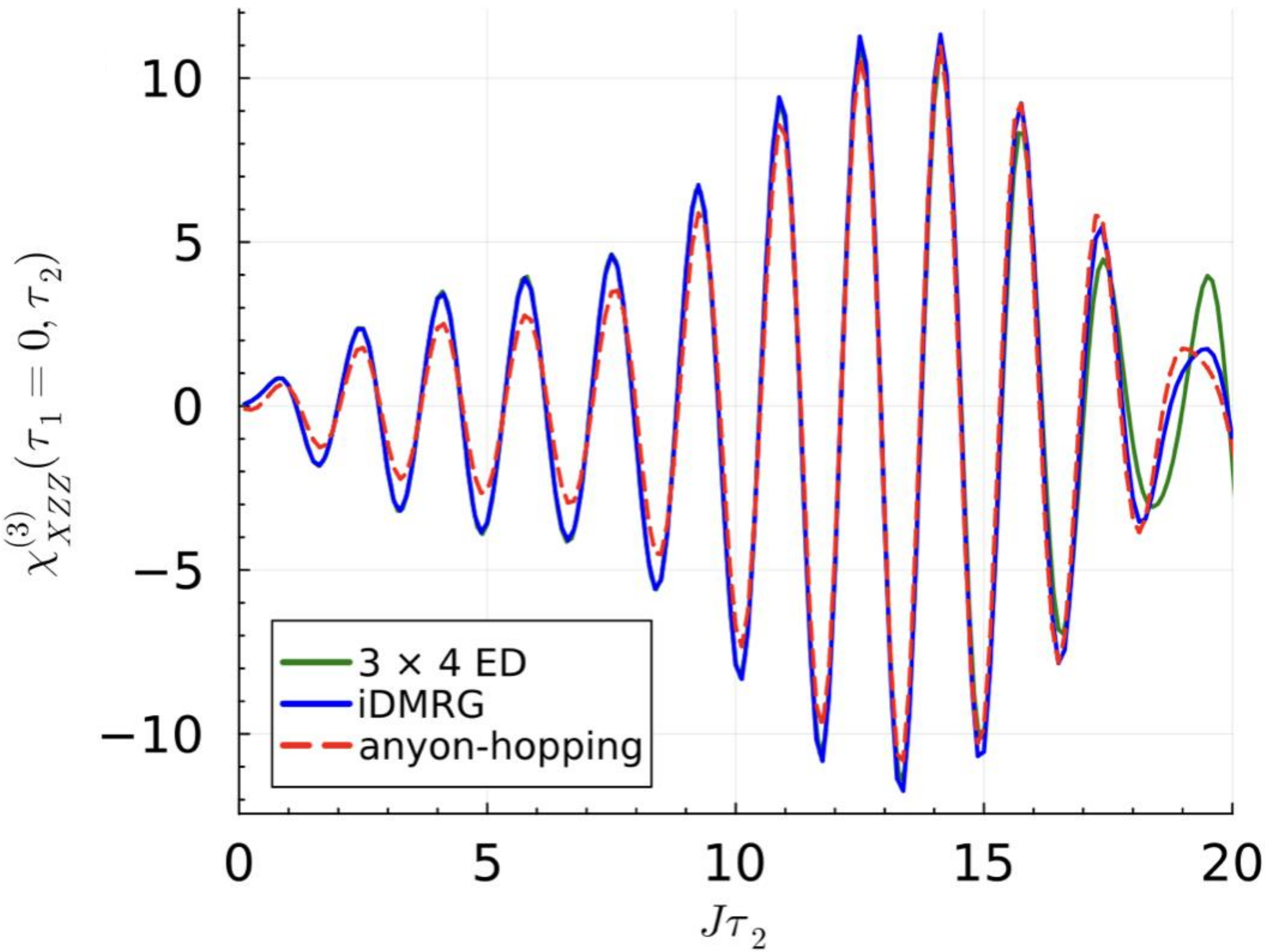


$$\chi^{(3)}_{XZZ}(\tau_1 = 0, \tau_2) = \left[a + \frac{b}{\tau_2} + c\tau_2^{\alpha}\right] \times \sin(\Delta\tau_2)$$

$$\chi^{(3)}_{XXX}(\tau_1 = 0, \tau_2) = \left[a + \frac{b}{\tau_2}\right] \times \sin(\Delta\tau_2)$$

divergent behavior from braiding
(due to increased probability
of braiding at long times)

$h = 0.05J$



$t \sim 1/h$

Outlook for QSLs

- Need more quantum materials
- Synthetic platforms for model QSLs
- New types of probes of fractionalized excitations and their statistics → two spin flips
- Braiding signatures in non-linear susceptibility to distinguish abelian and non-abelian signatures

Detection of anyon braiding through pump-probe spectroscopy, Xu Yang, R. Buechele, N. Trivedi, **arXiv:2503.22792**

Anyon dynamics in field-driven phases of the anisotropic Kitaev model,
S. Feng, A. Agarwala, S. Bhattacharjee, N. Trivedi, PRB 108, 035149 (2023)

Fractionalization Signatures in the Dynamics of Quantum Spin Liquids
Kang Wang, Shi Feng, Penghao Zhu, Runze Chi, Hai-Jun Liao, N. Trivedi, and Tao Xiang, PRB 111, L100402, (2025)

Emergent Majorana Metal in a Chiral Quantum Spin Liquid
Penghao Zhu, Shi Feng, Kang Wang, Tao Xiang, and N. Trivedi, Nature Comm. 16, 2420 (2025)

- **Band limit of Kitaev magnets promising for new design principle for flat bands -- kinetic frustration due to interference of pathways**

Orbital frustration and topological flat bands
Wenjuan Zhang , Z. Addison, N. Trivedi
PRB 104, 235202 (2021)
Flat bands arising from spin-orbit assisted orbital frustration
Z. Addison and N. Trivedi PRB 106, 235144 (2022)