"While my Hall bar gently heats":
Probing topological edge structures with thermal transport

Christian Spånslätt Rugarn

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Determination of topological edge quantum numbers of fractional quantum Hall phases by thermal conductance measurements,
S. K. Srivastav, R. Kumar, C. Spånslätt, K. Watanabe, T. Taniguchi, A. D. Mirlin, Y. Gefen, A. Das, Nat. Commun 13, 5185 (2022)

Thermal conductance and noise of Majorana modes along interfaced ν=5/2 fractional quantum Hall states,
M. Hein and C. Spånslätt, PRB 107, 245401 (2023)

Topology and geometry beyond perfect crystals, Nordita, June 13 2025

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 101031655 (TEAPOT).

Outline

Quantum Hall edge physics

- The quantum Hall edge and counter-propagating modes
- Charge conductance insufficient to probe topological order

Thermal conductance of FQH edges

- Tool to identify edge structures
- What is the thermal conductance and how to measure it?
- How to model?

Recent experiment and theory

- Experiment at fillings $1/3$, $2/3$, $2/5$, $3/5$ R. Kumar, S. K. Srivastav, C. Spånslätt et al.: Nat. Commun. 13, 1 (2022)
- Theoretical modelling of four-point thermal conductance M. Hein and C. Spånslätt: Phys. Rev. B 107 245401 (2023)

FQH liquids and their boundary

D. C. Tsui et al.: Phys. Rev. Lett. 48, 1559 (1982)
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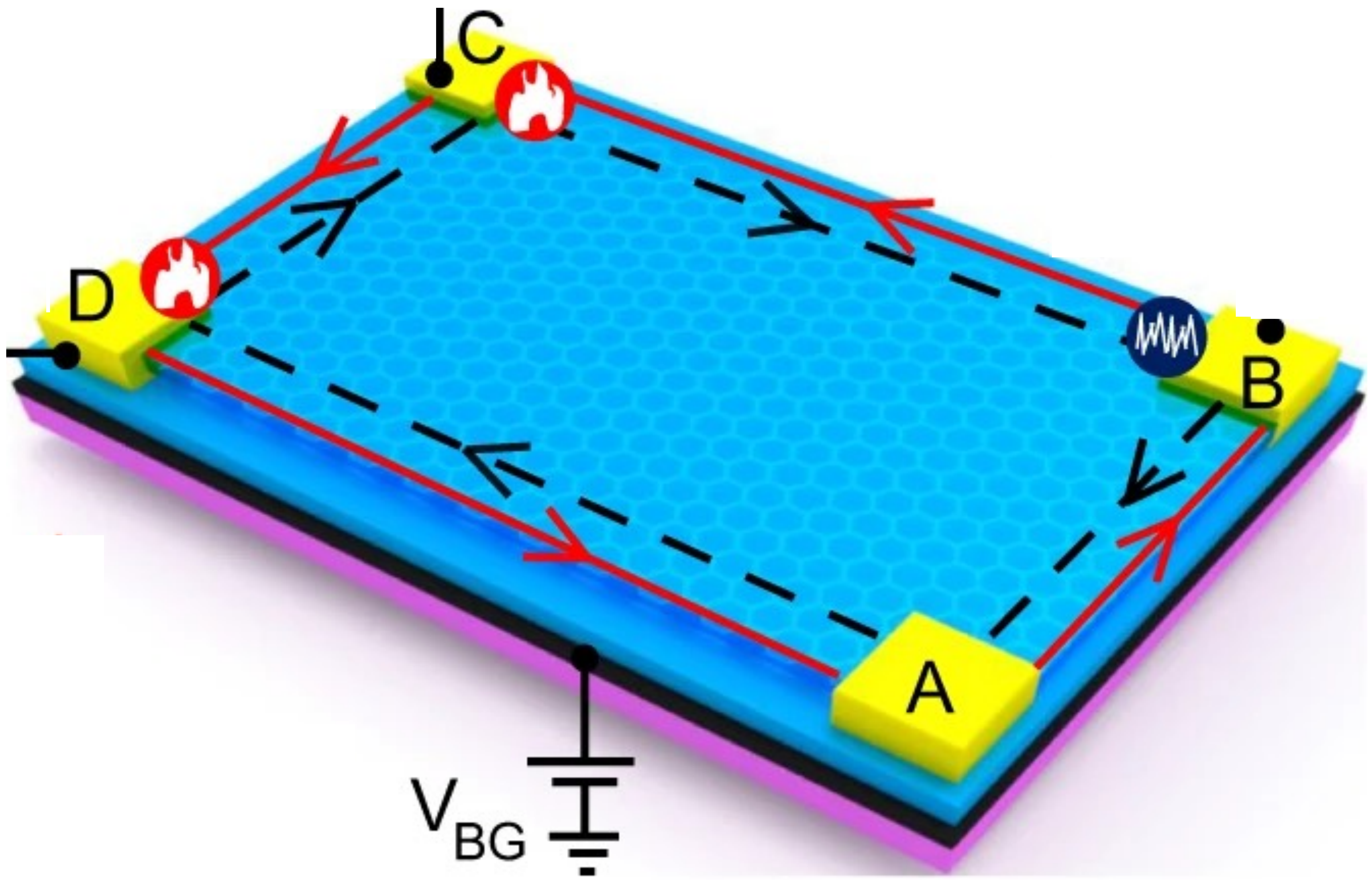

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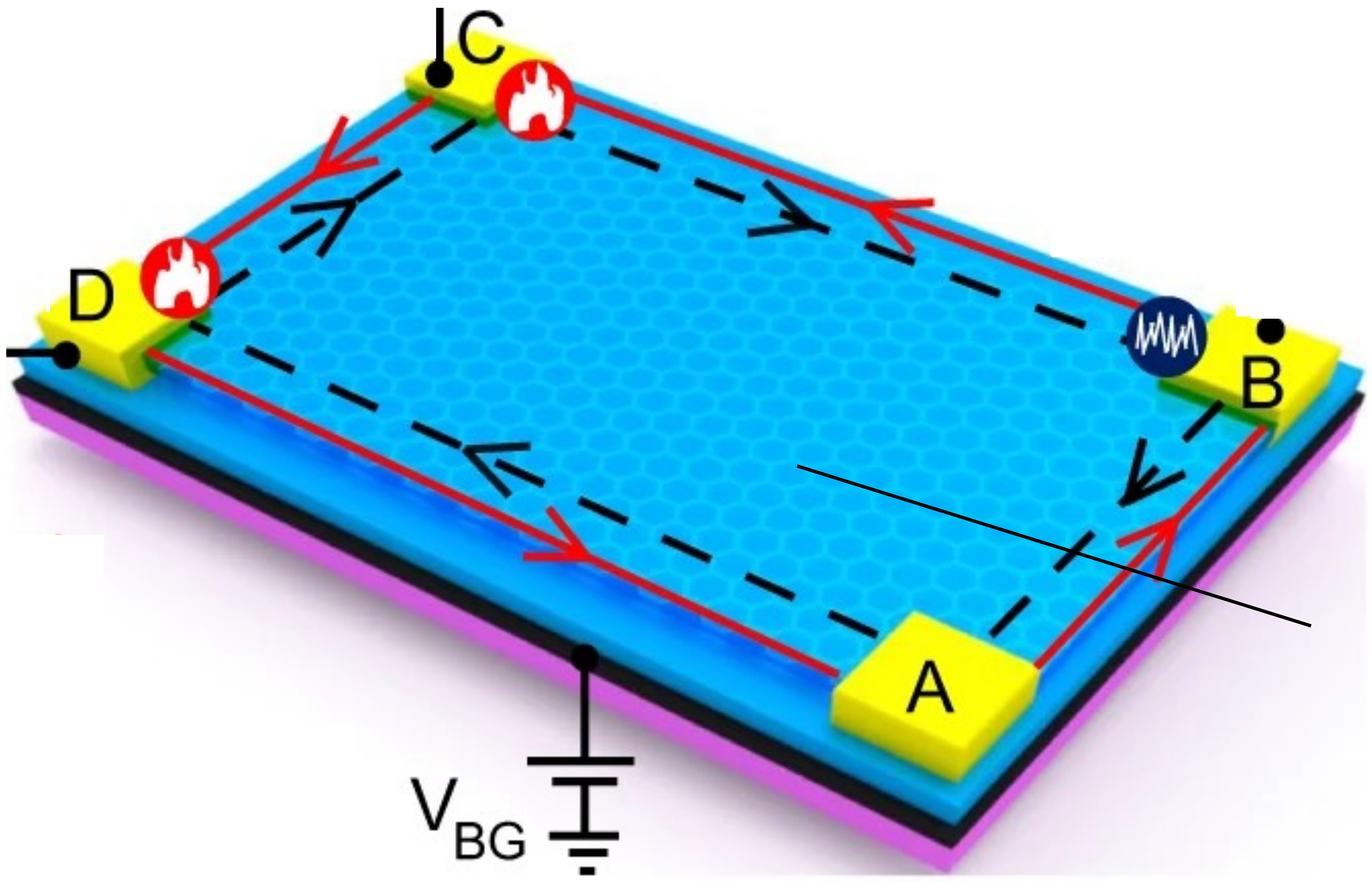

Charge insulating 2D bulk
Topological field theory, K-matrix
Quantum Statistics of excitations
Ground state degeneracy

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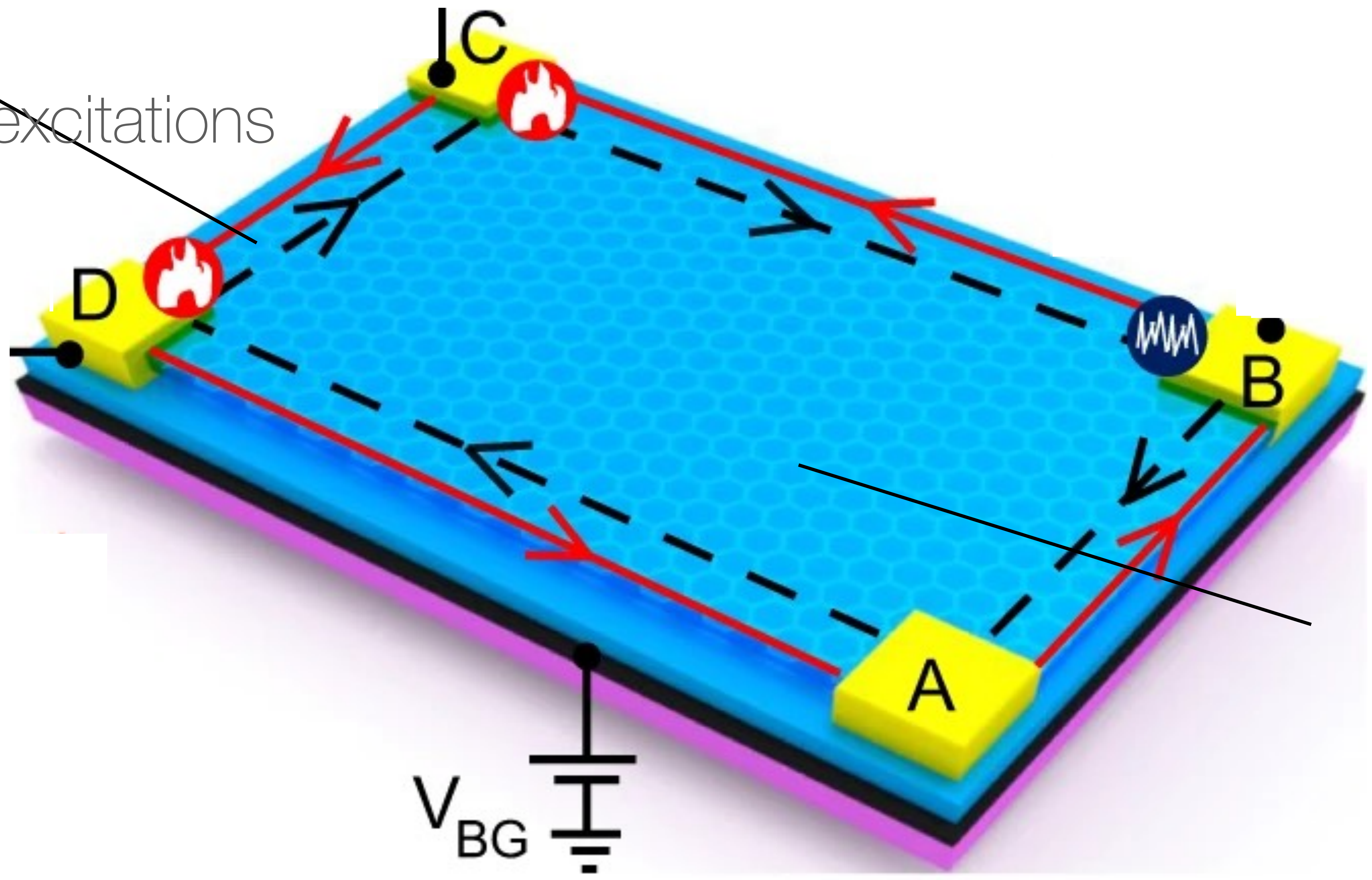
Conducting 1D edge
Chiral luttinger liquid
Conformal field theory
Wave guide of fractional excitations



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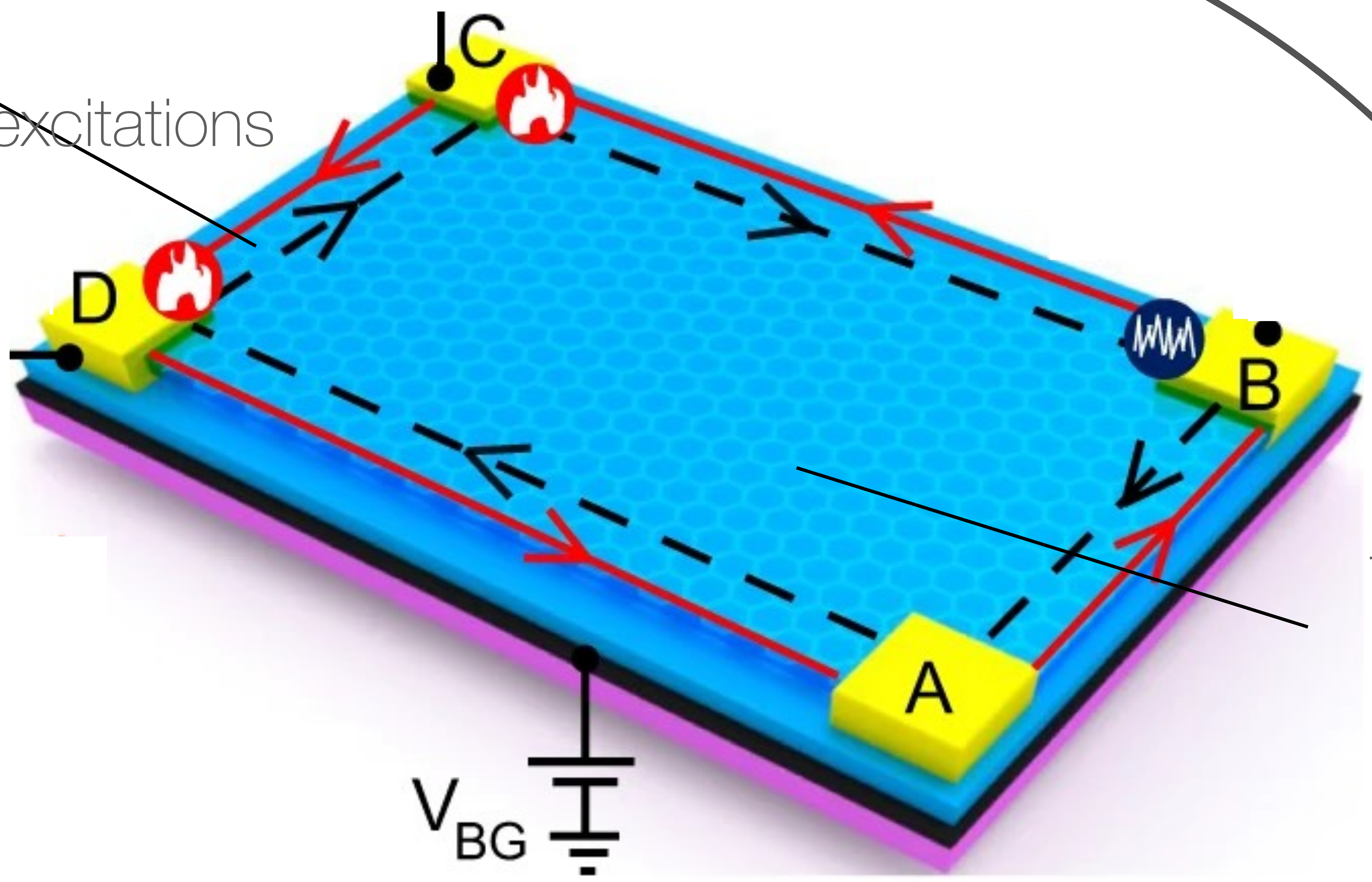
Bulk-Boundary
Correspondence



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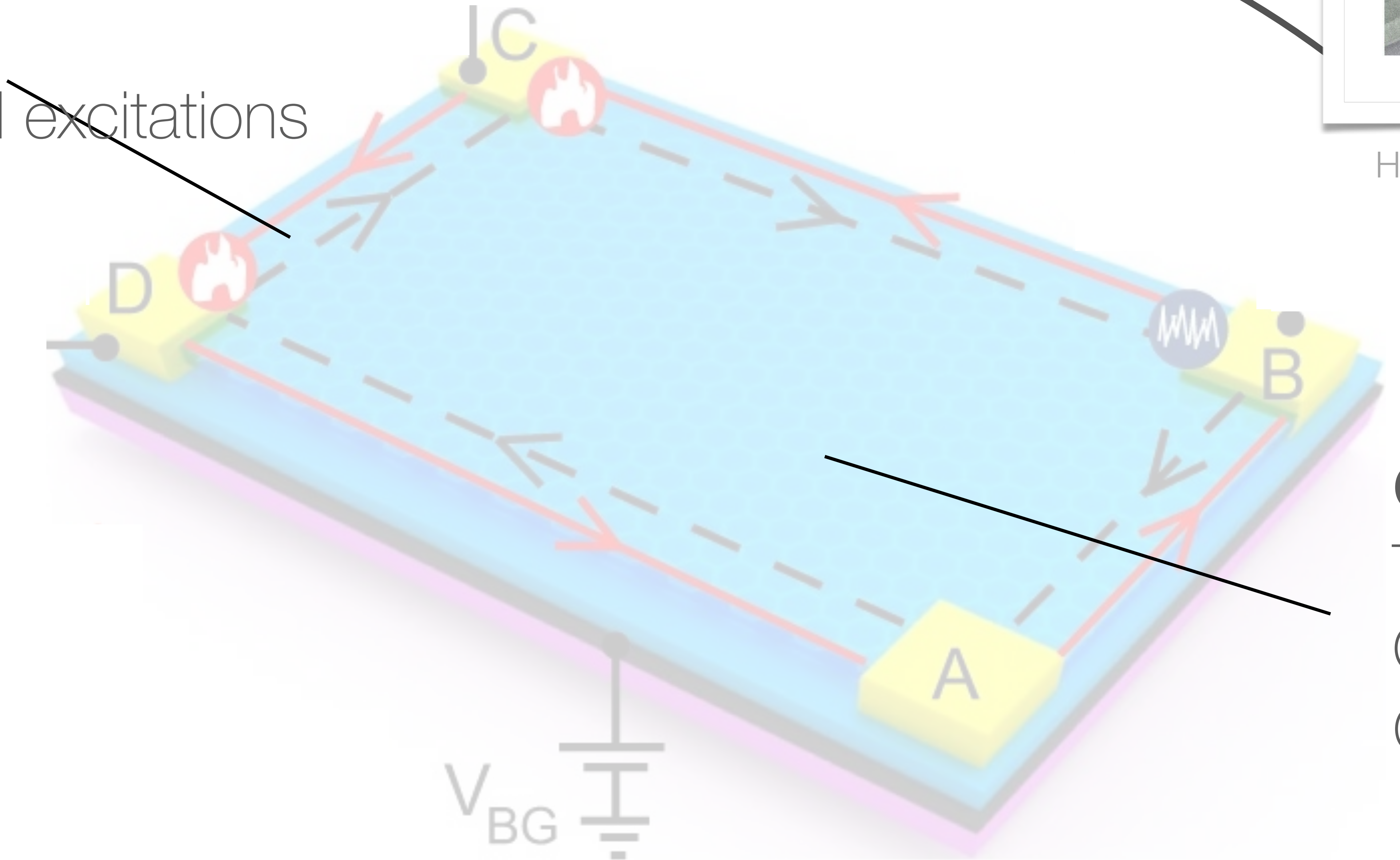

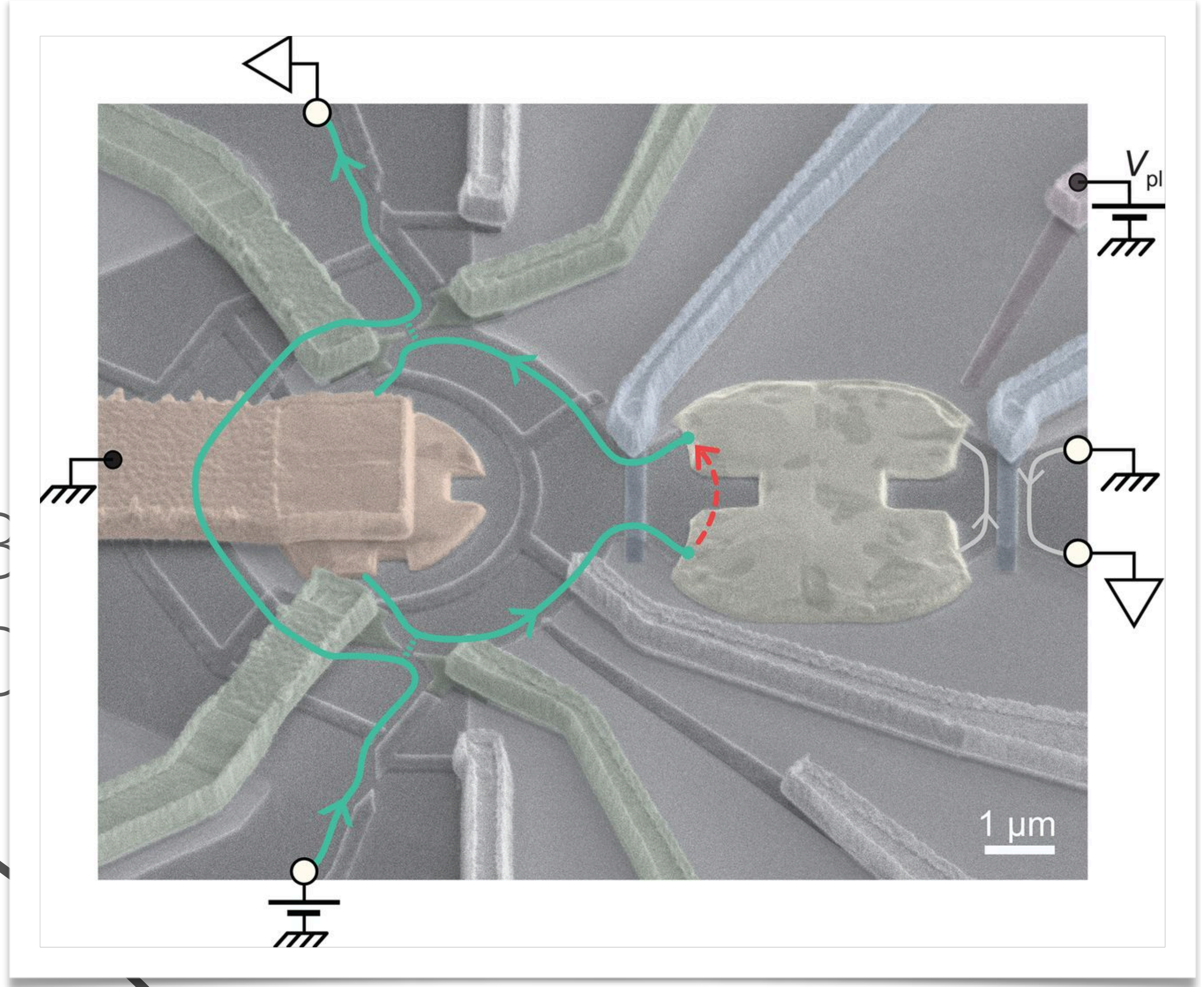

H. Duprez et al.: Science 366, 1243 (2019)

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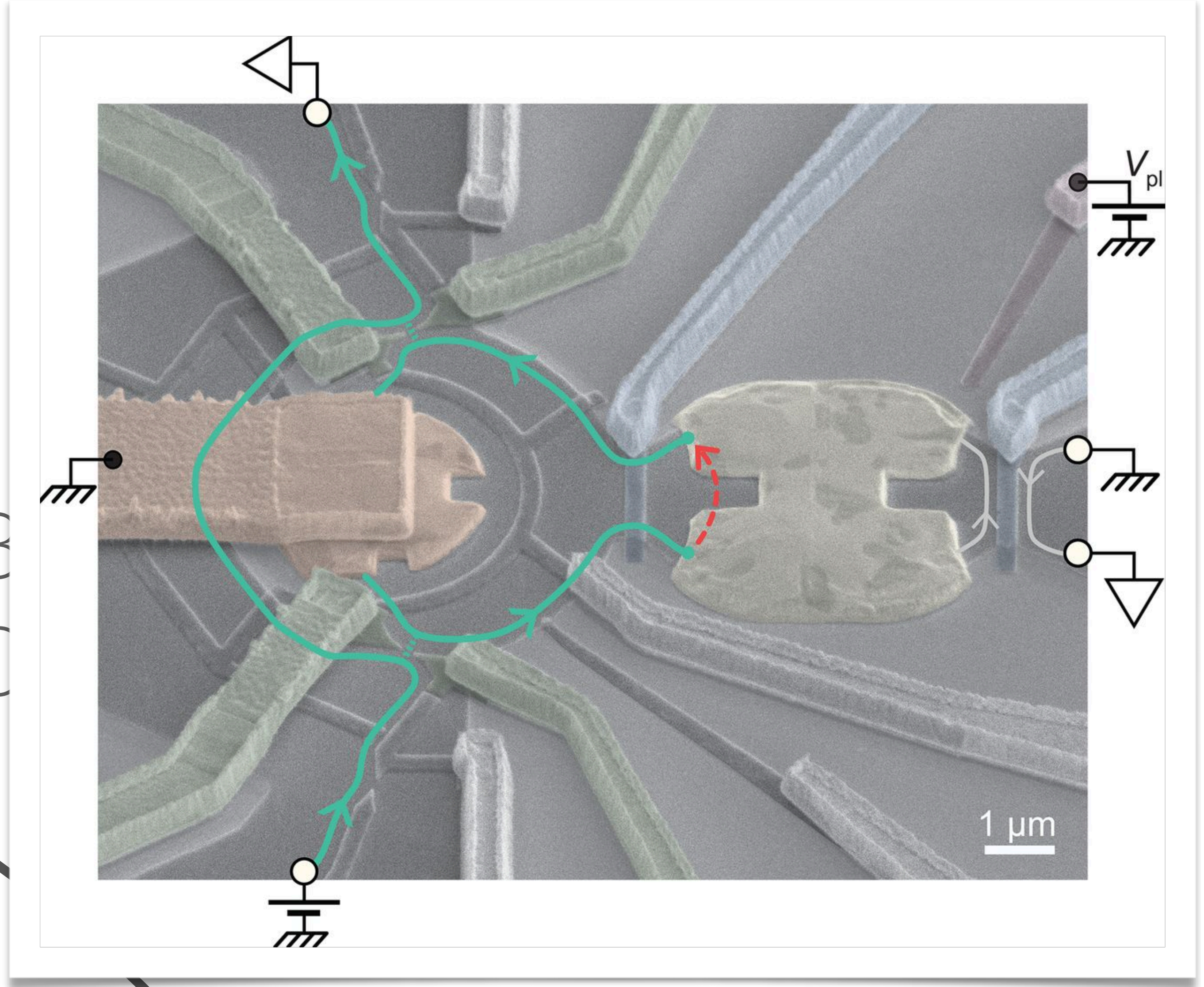

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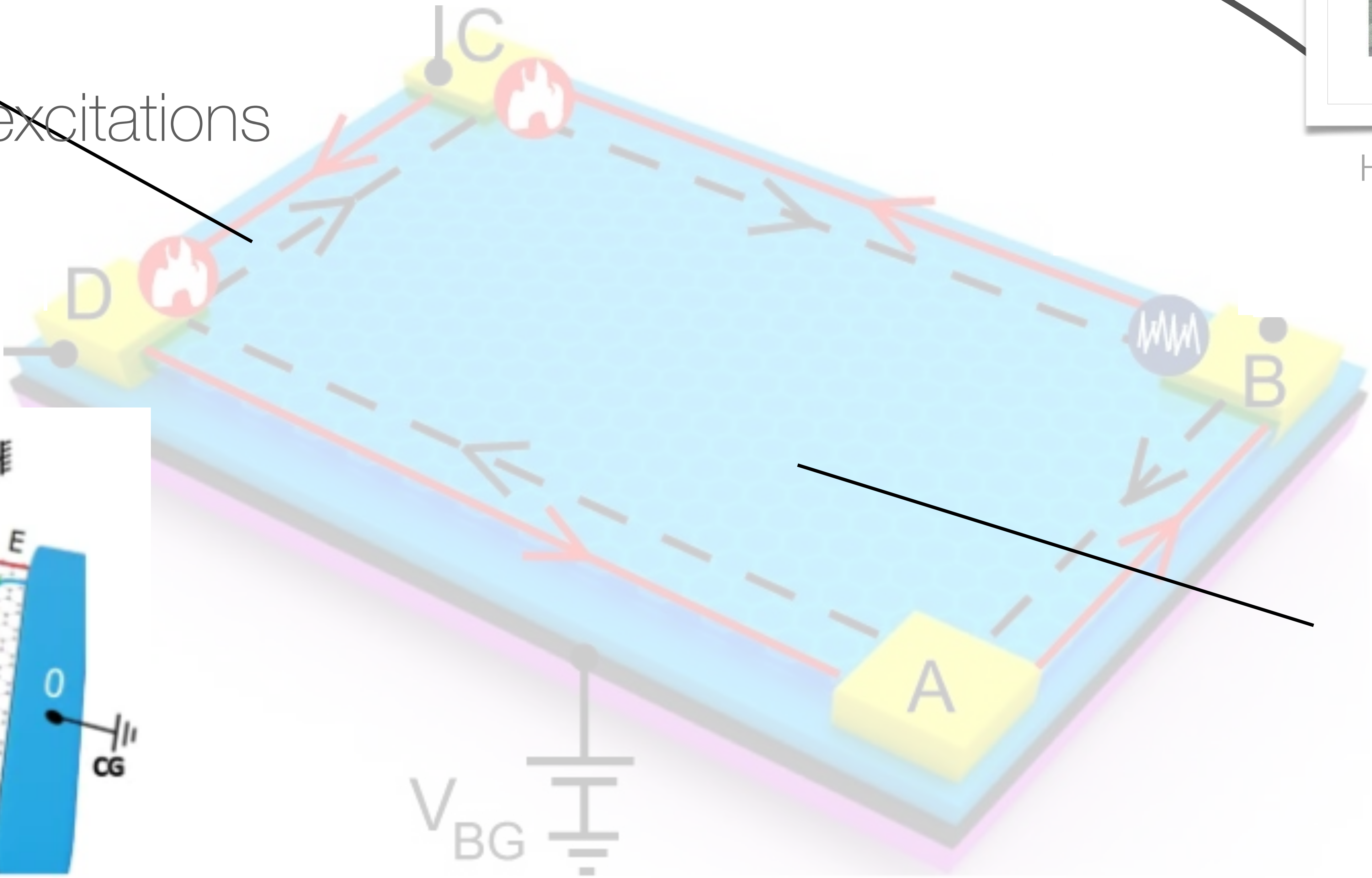

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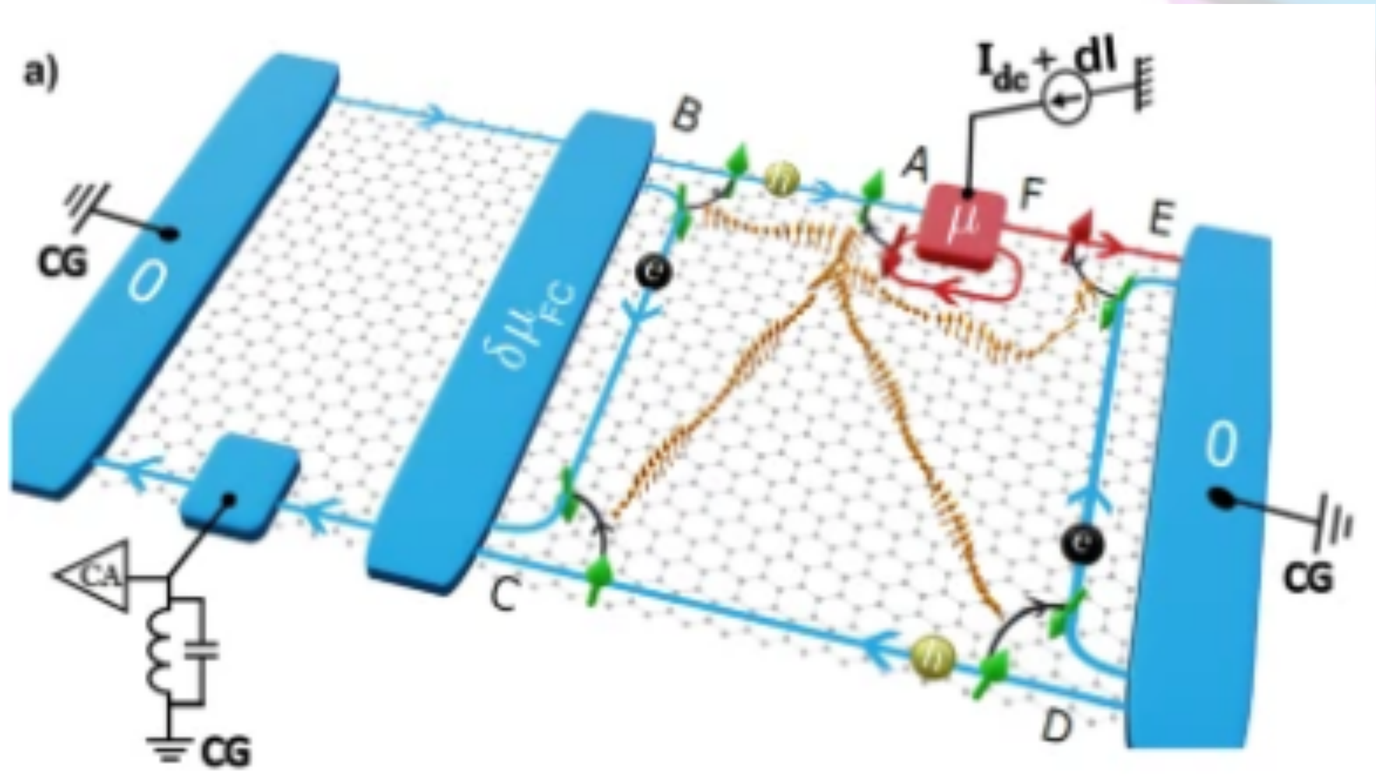

R. Kumar et al.: Nat. Commun. 15, 1 (2024)

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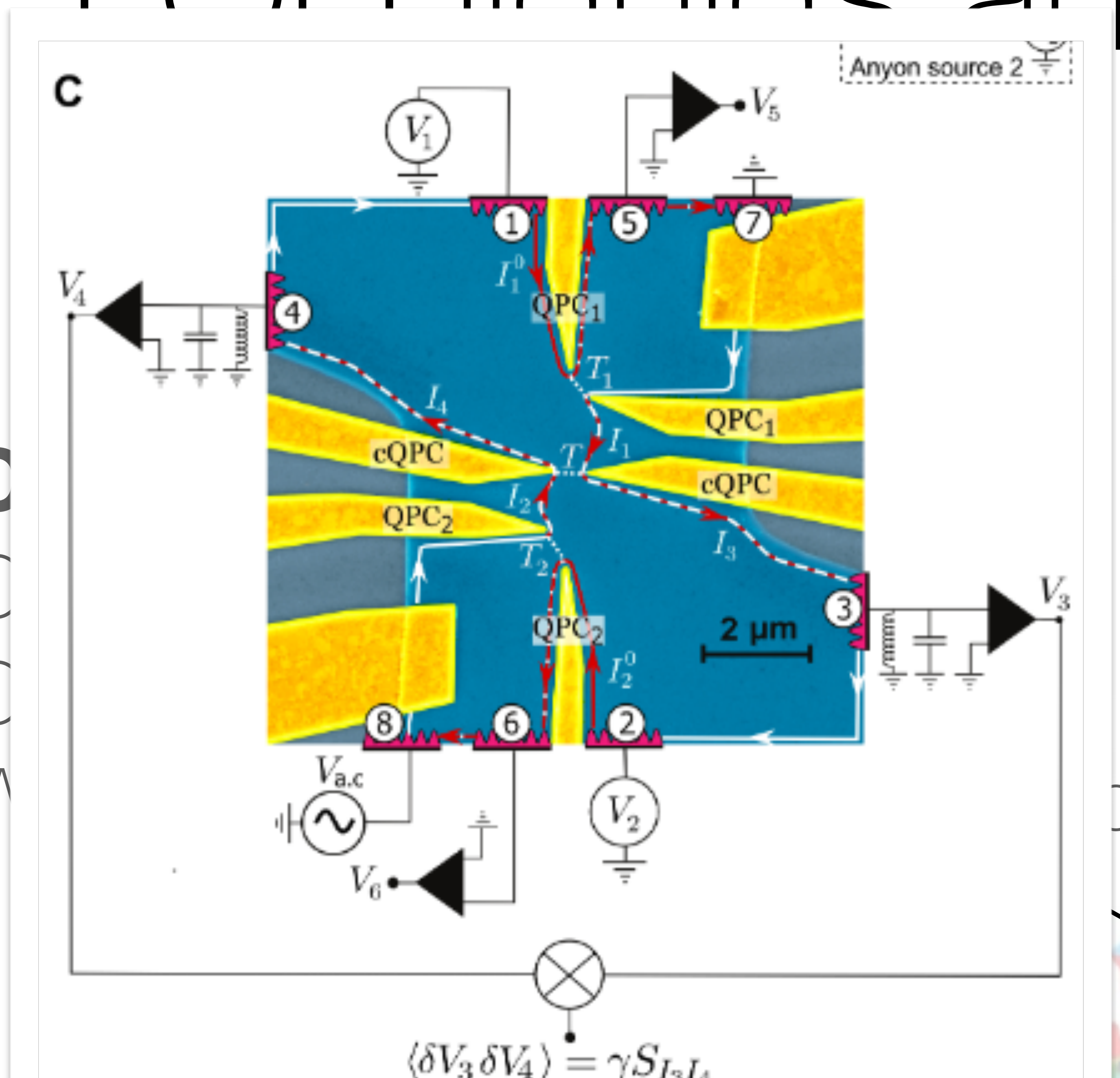
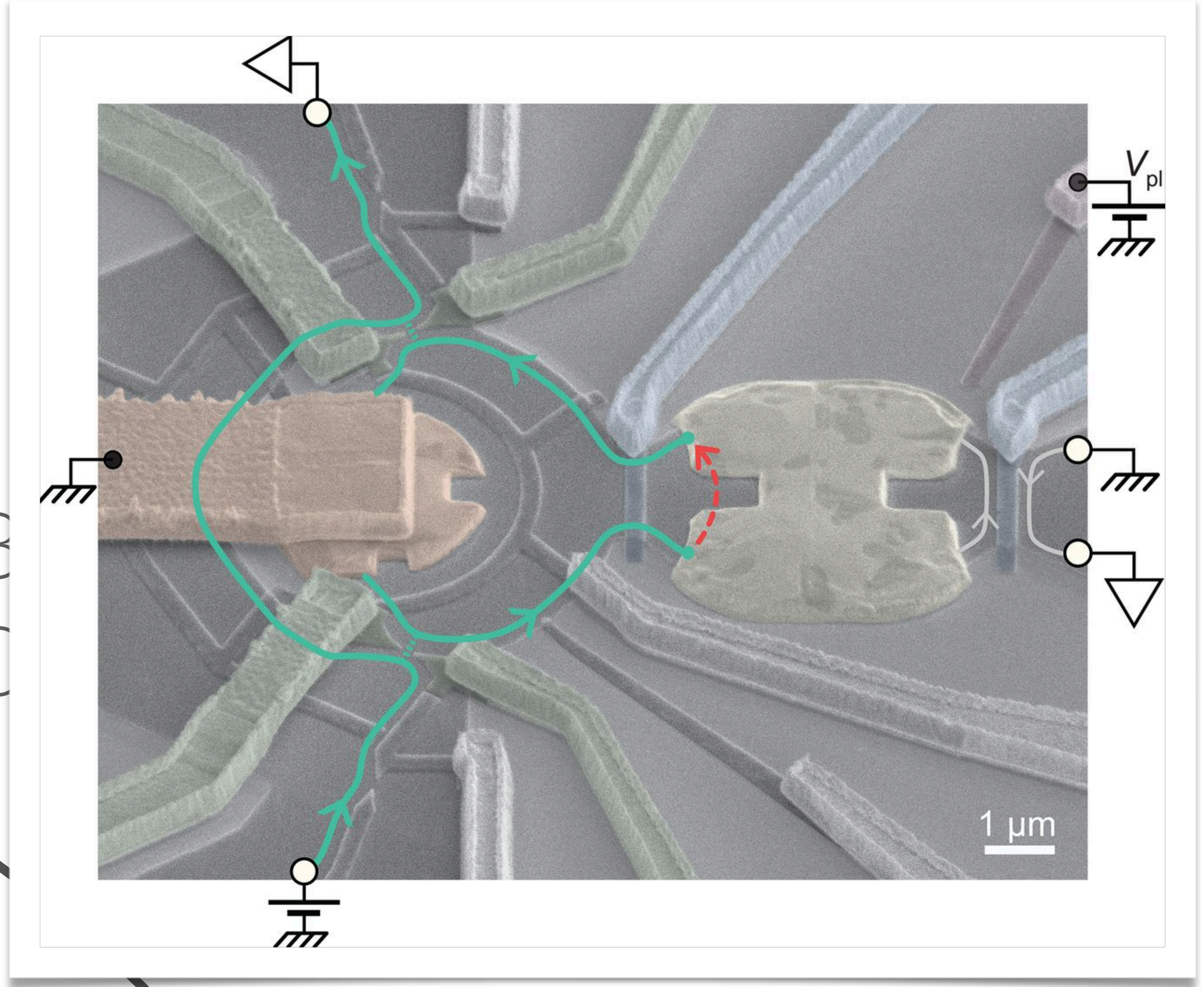


H. Bartolomei et al.: Science 368, 173 (2020)



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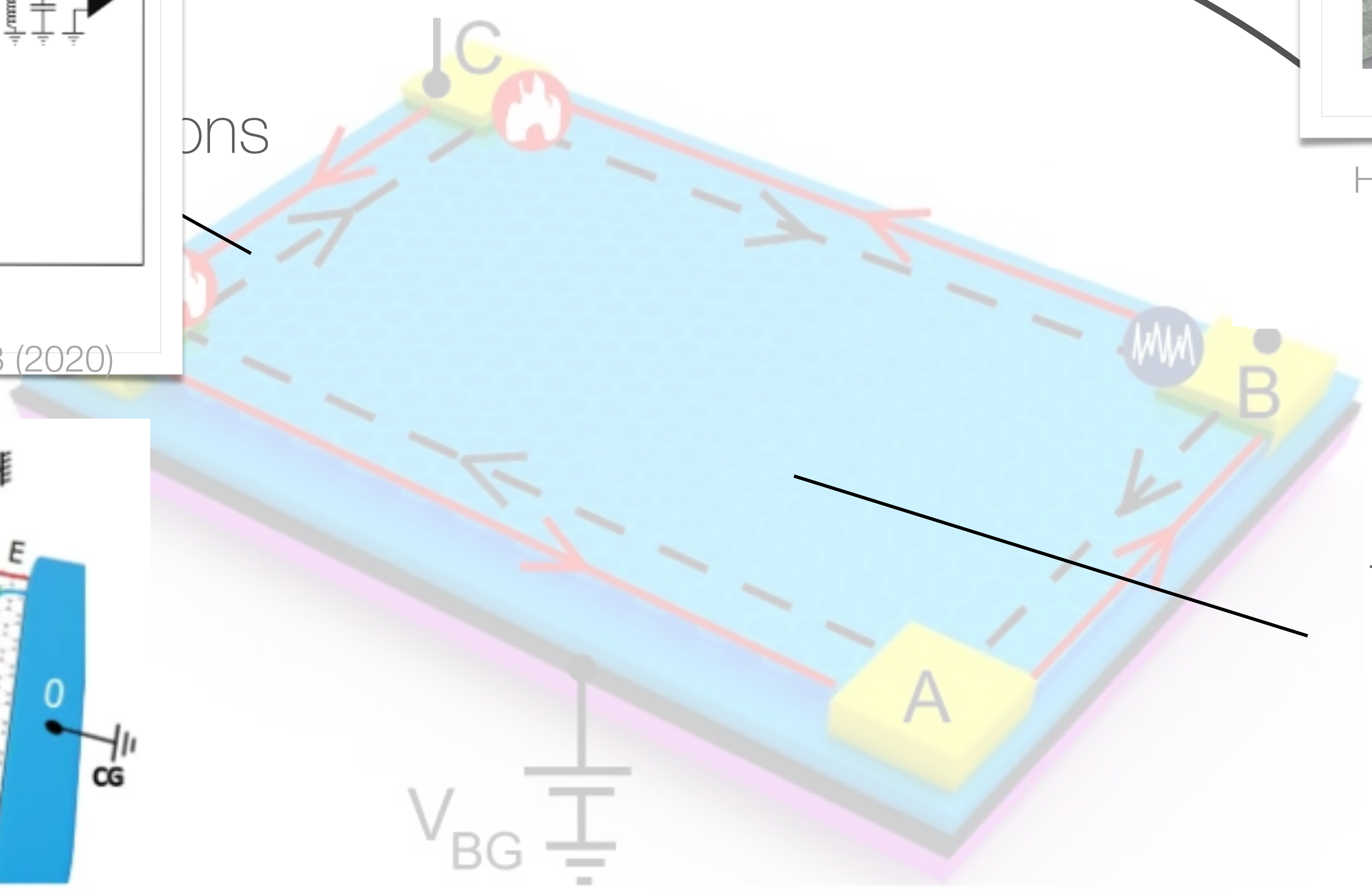

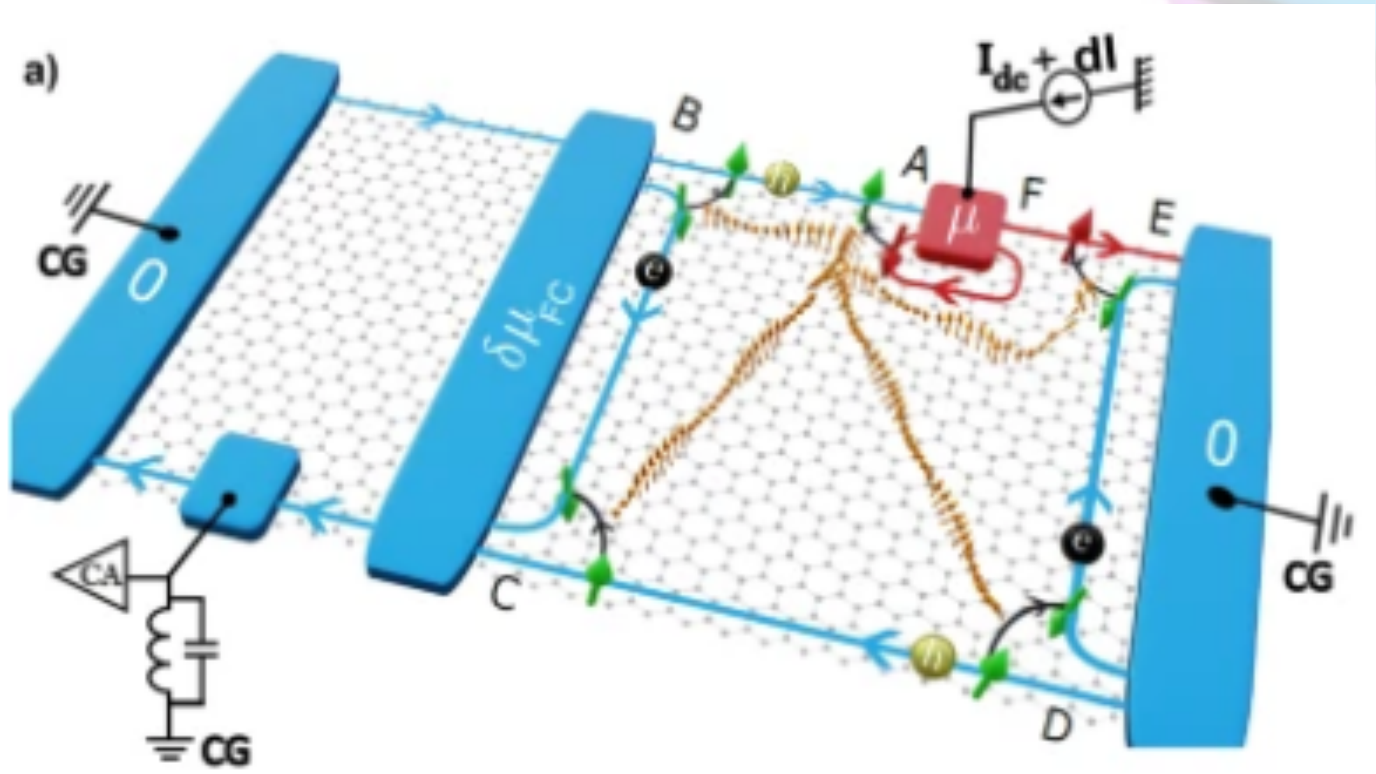

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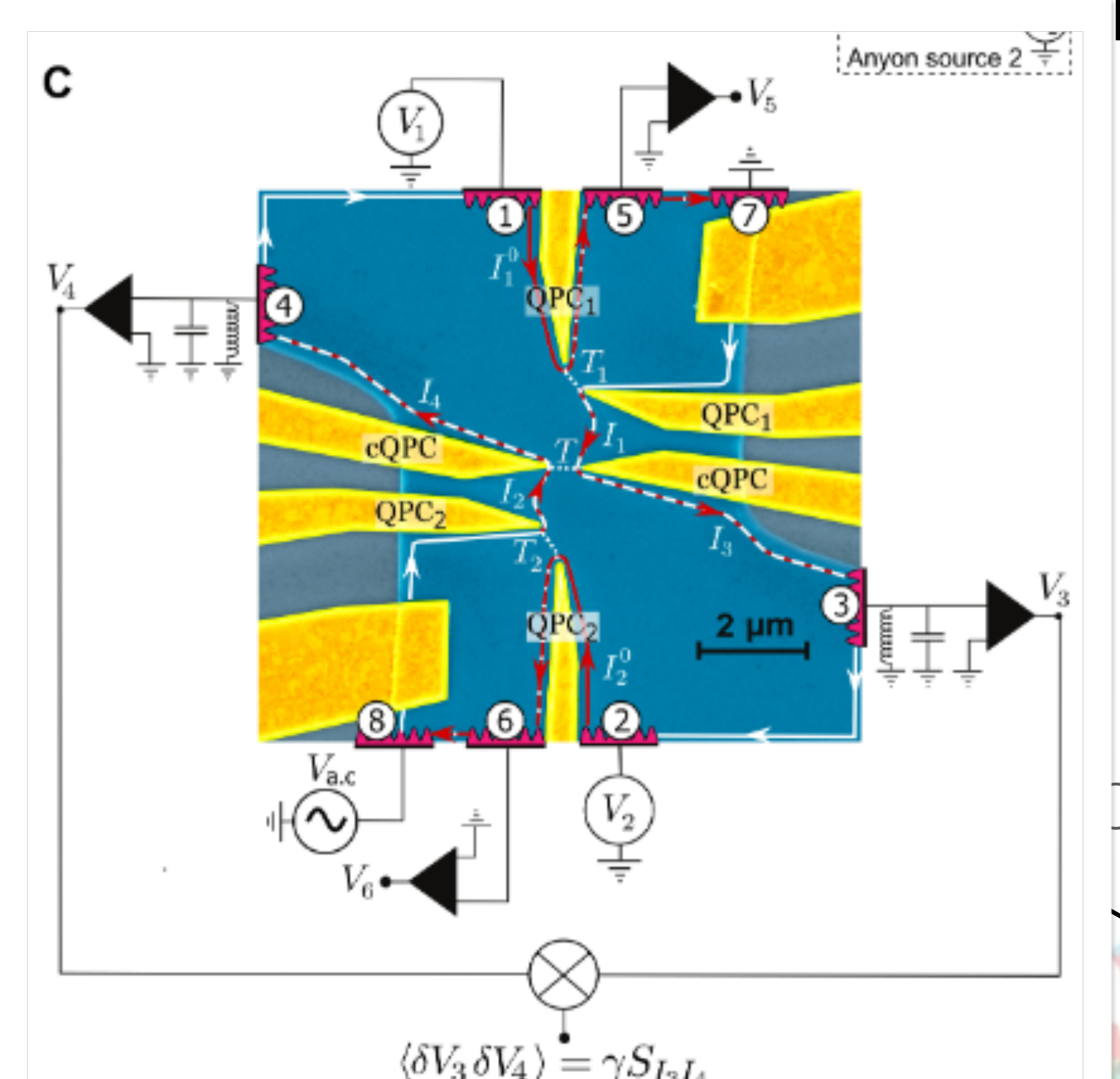

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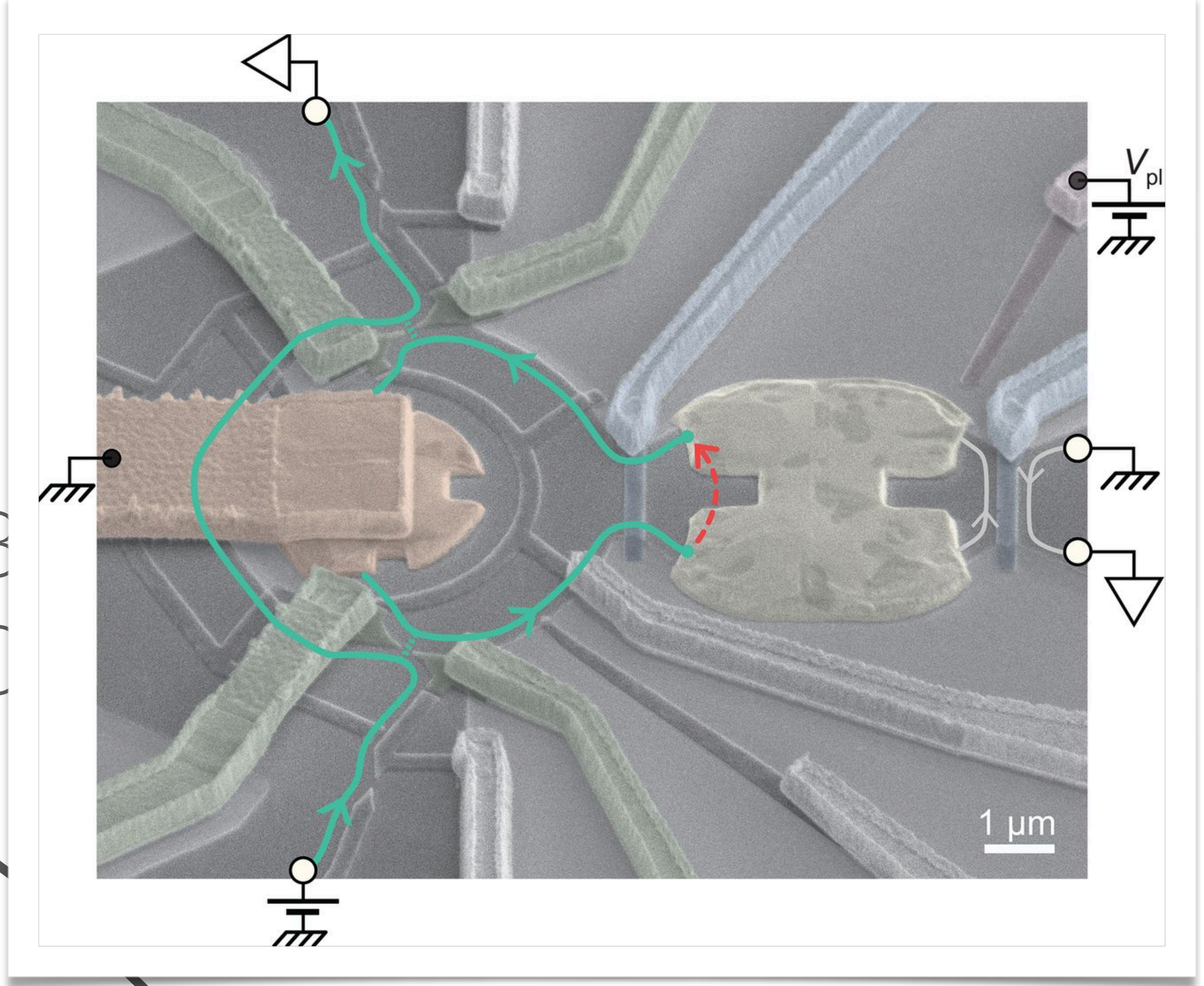

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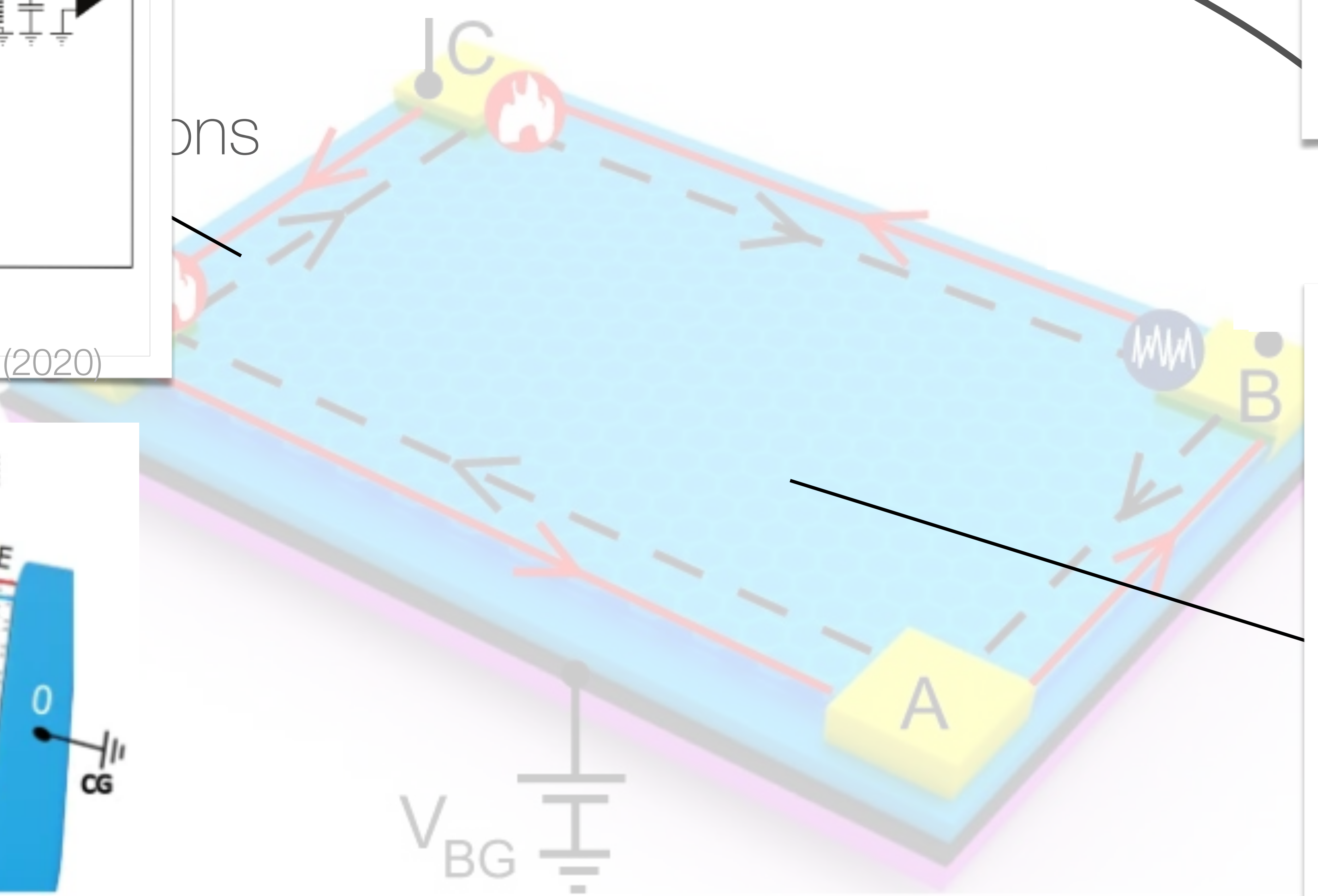

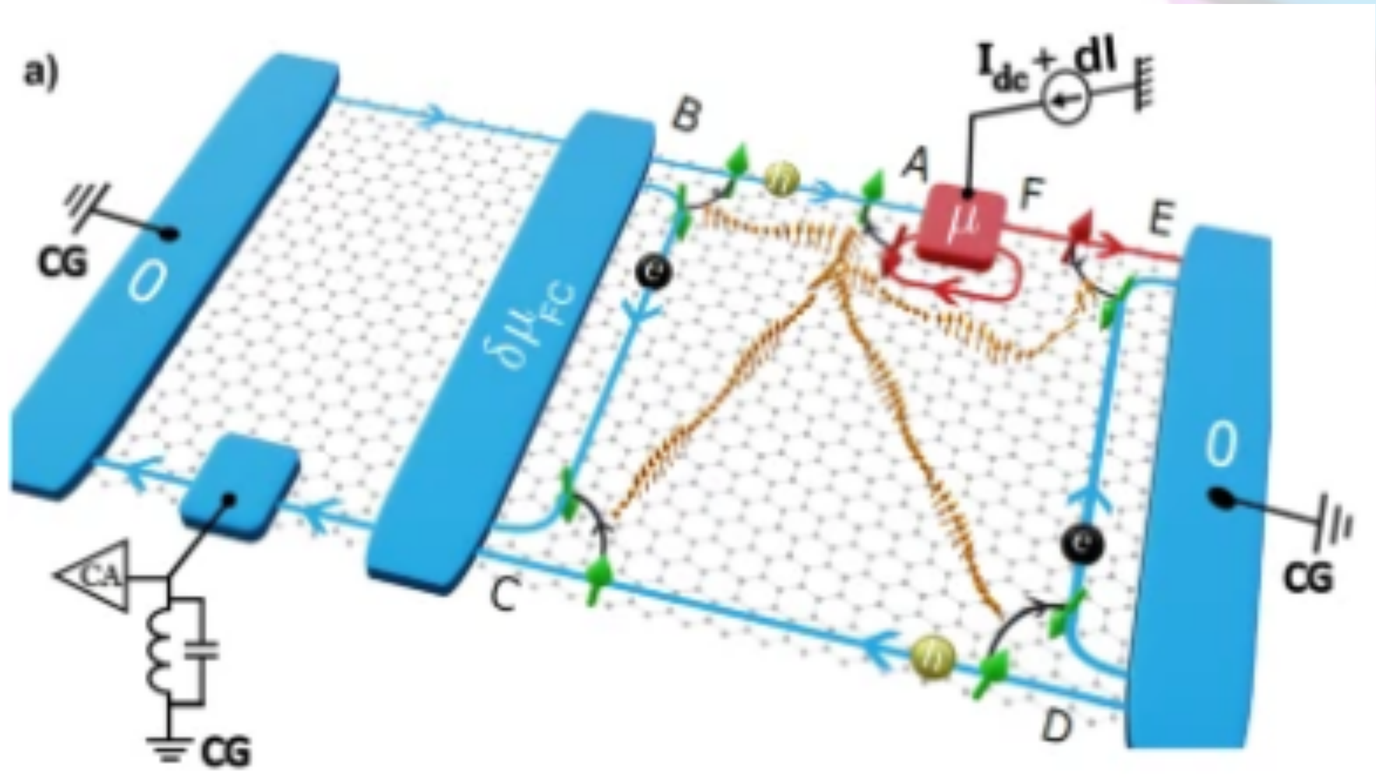

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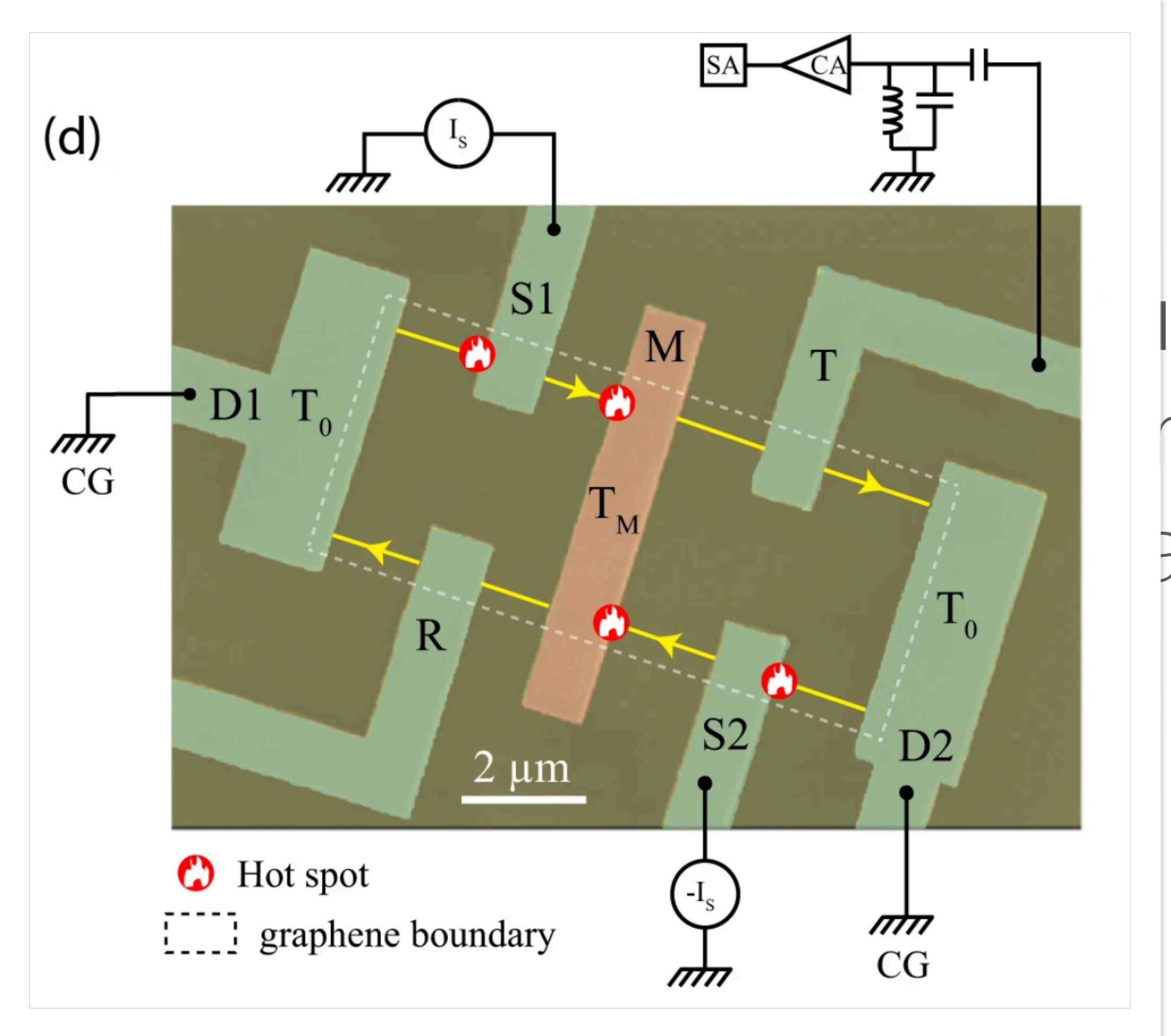
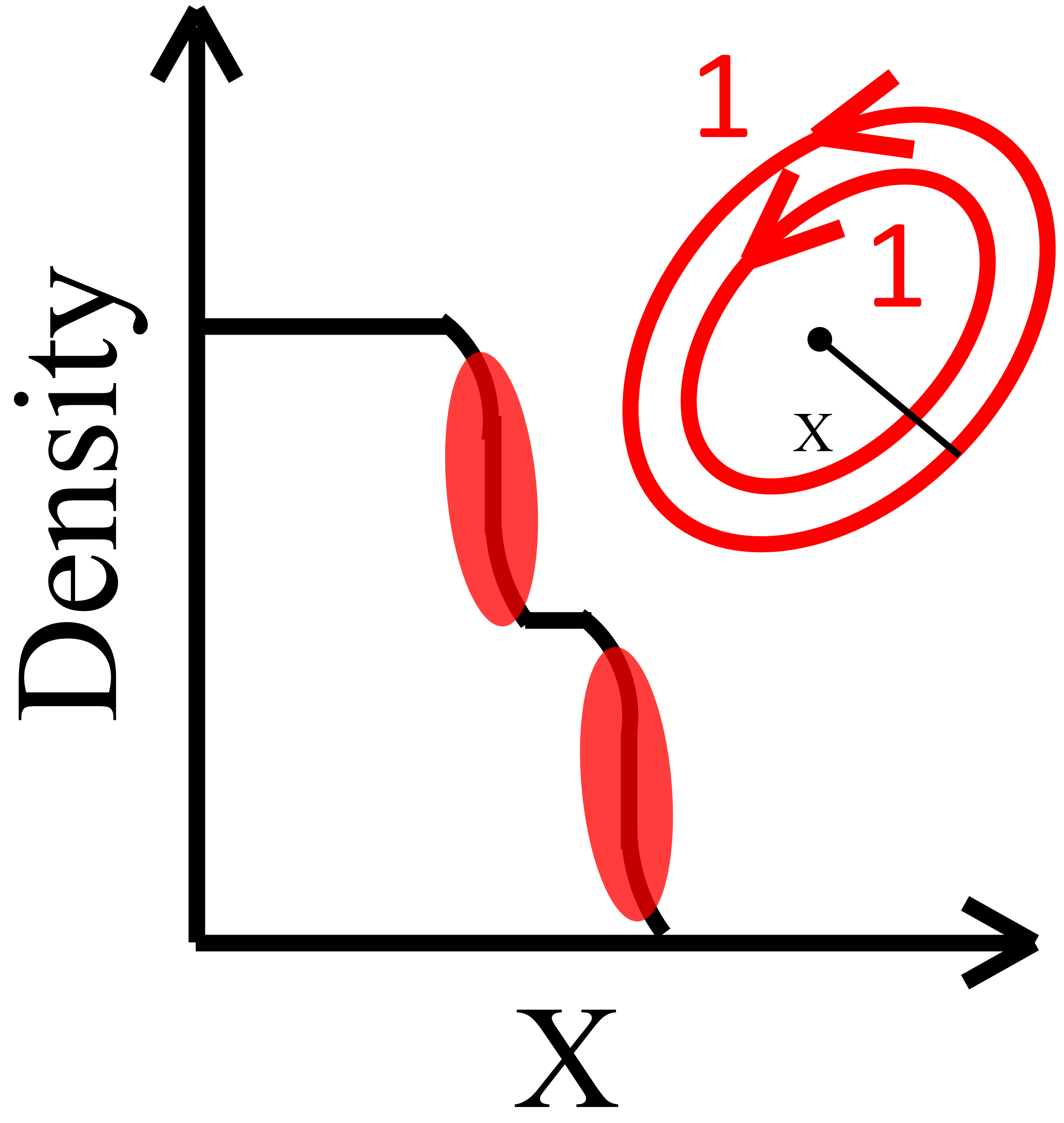
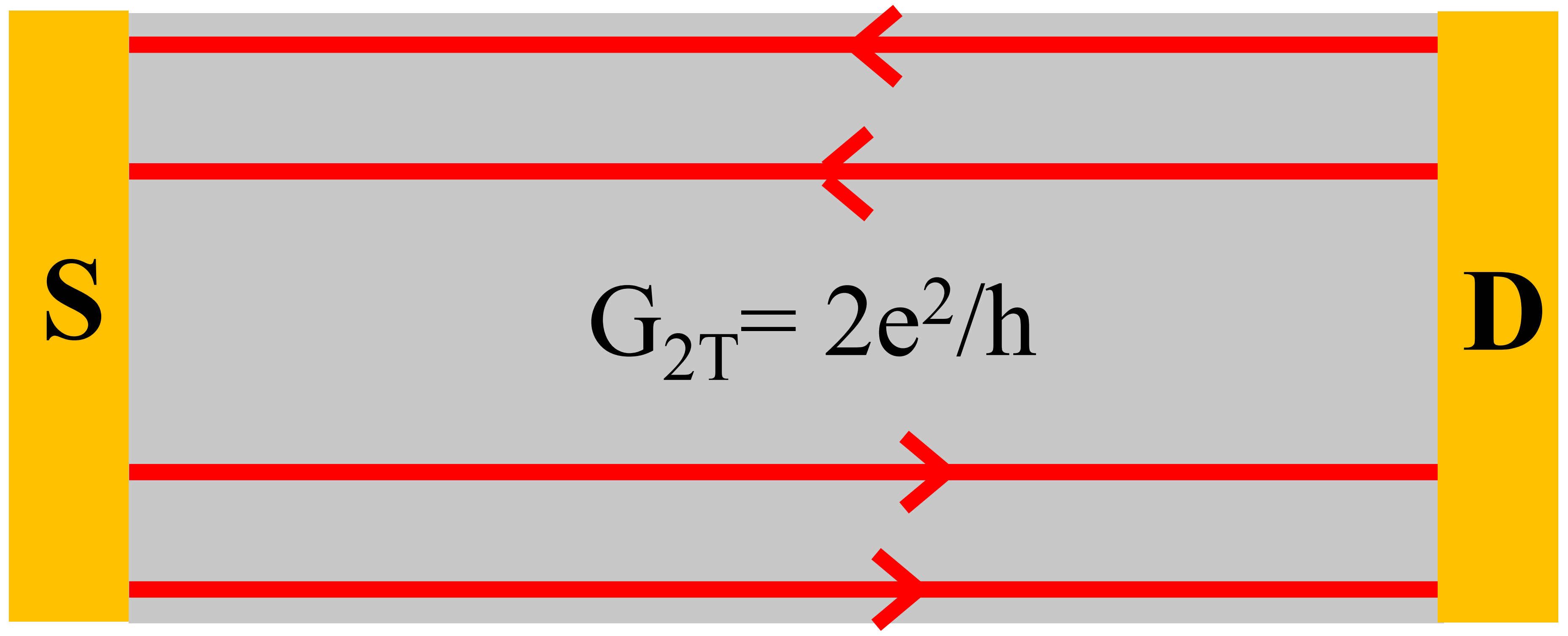


S. K. Srivastav et al.: Nat. Commun. 13, 1 (2022)

Co-propagating edge modes

Halperin, Phys. Rev. B. 25, 2185, 1982
M. Büttiker: Phys. Rev. B 38, 9375 (1988)
C. W. J. Beenakker: Phys. Rev. Lett. 64, 216 (1990)
X.-G. Wen: Int. J. Mod. Phys B 06, 1711 (1992)





$K = \mathrm{diag}(1,1)$

Counter-propagating edge modes

A. H. MacDonald: Phys. Rev. Lett. 64, 220 (1990)
C. L. Kane, M. P. A. Fisher, J. Polchinski: Phys. Rev. Lett. 72, 4129 (1994)
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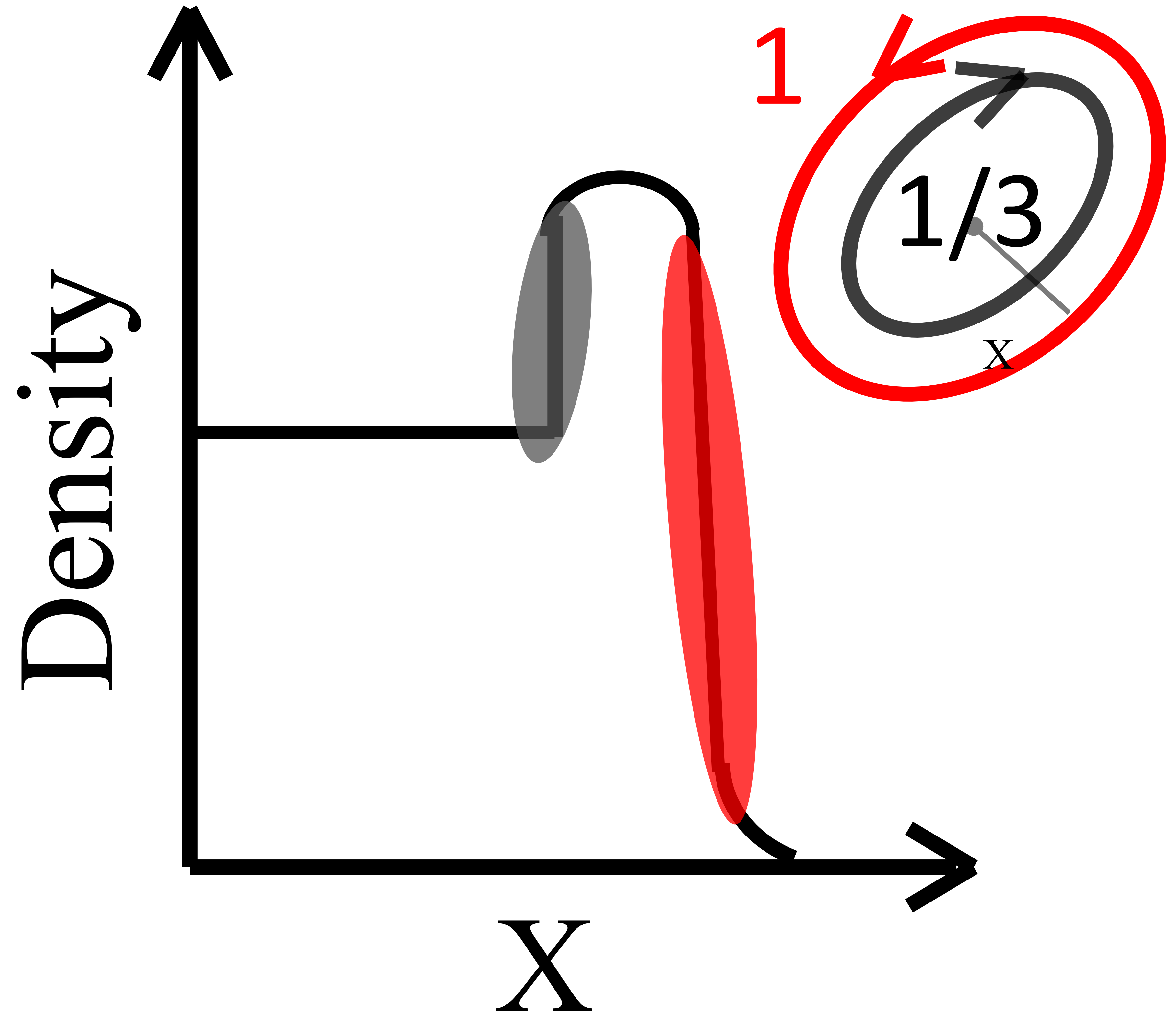
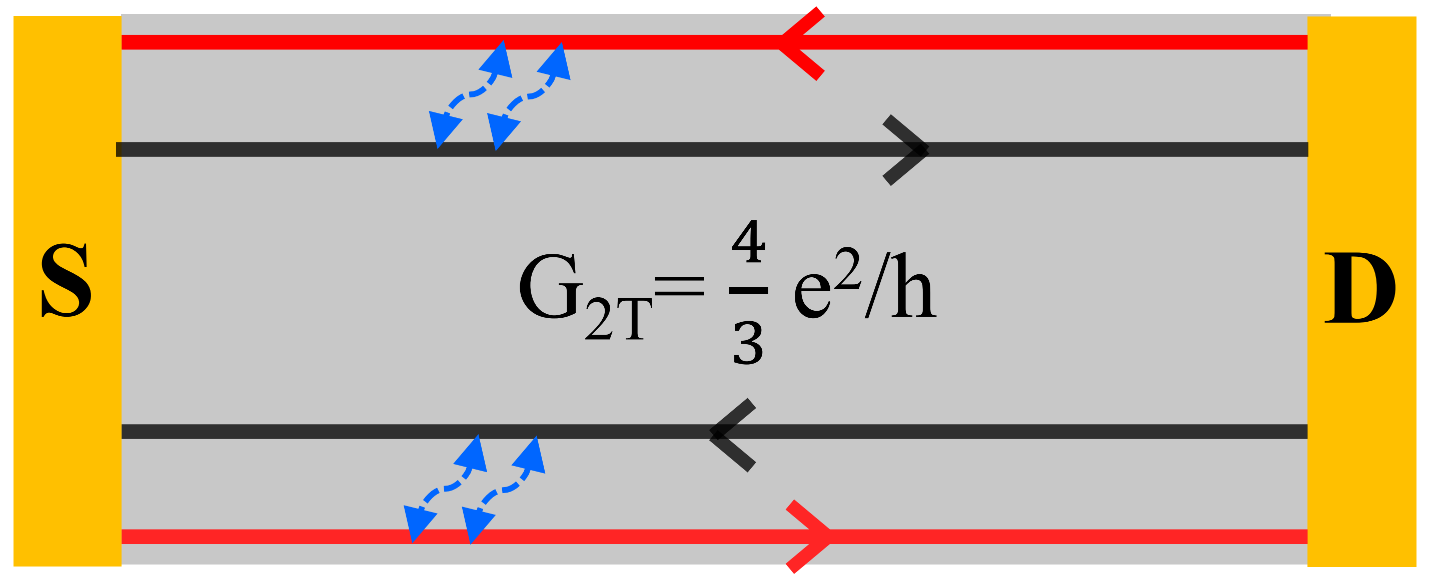


$K = \mathrm{diag}(1, -3)$



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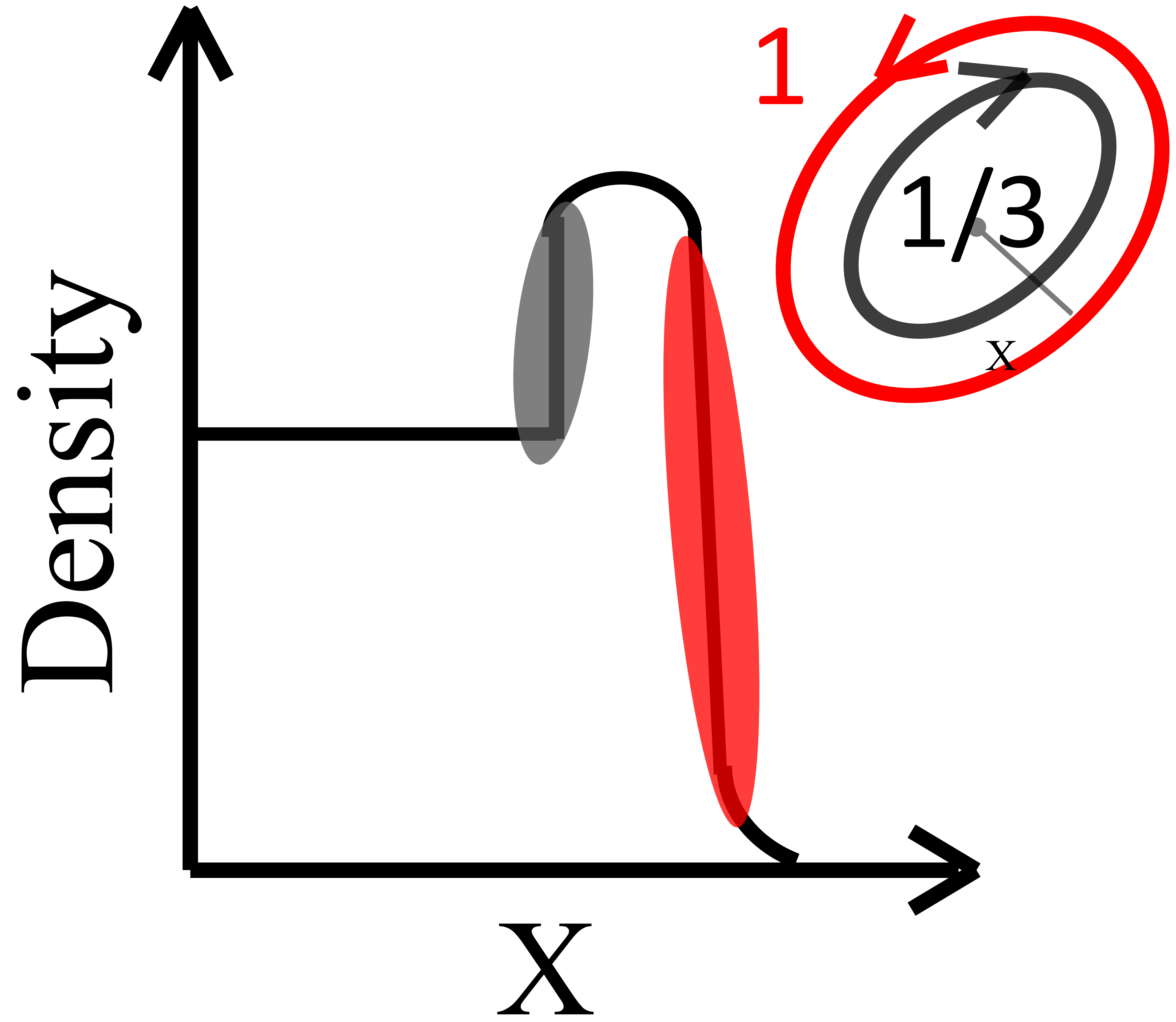

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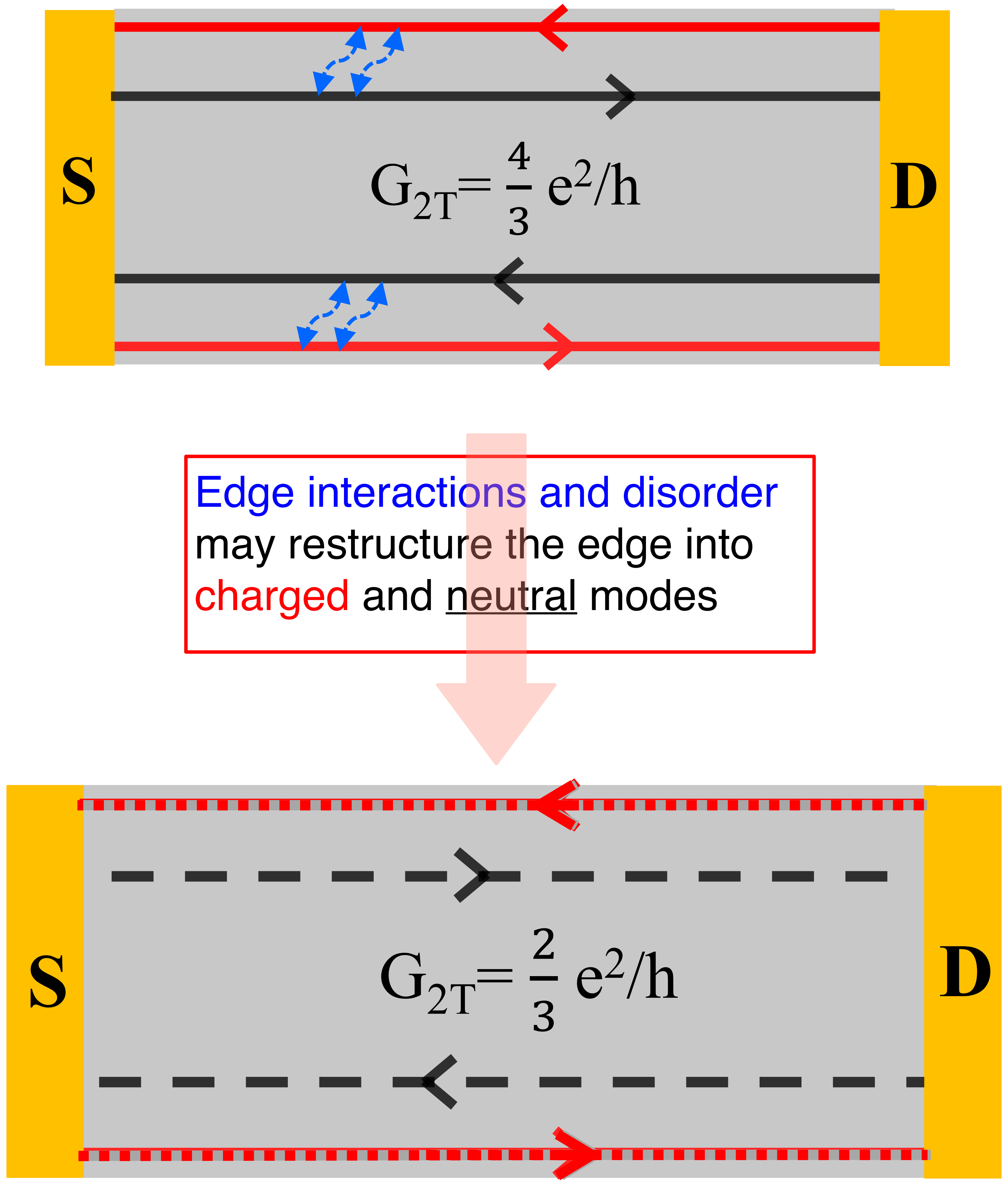

Abelian FQH edge structures

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$$\nu_Q = n_d - n_u$$

Downstream = direction according to B-field
Upstream = opposite to downstream

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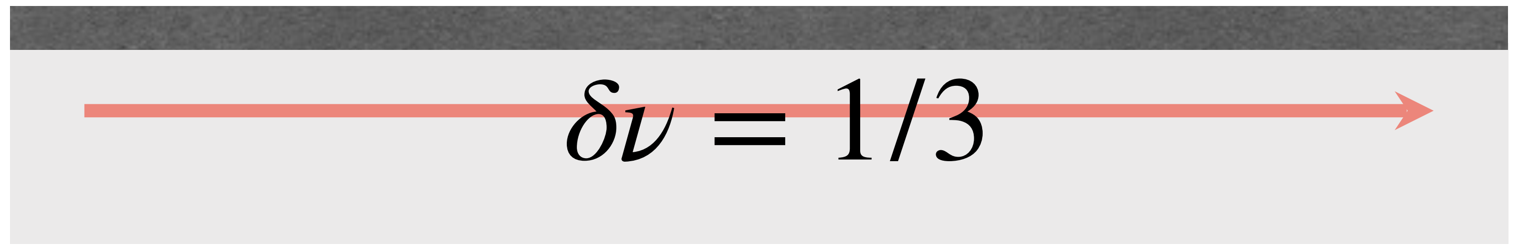

$\nu = 1/3 \Rightarrow \nu_Q = 1$

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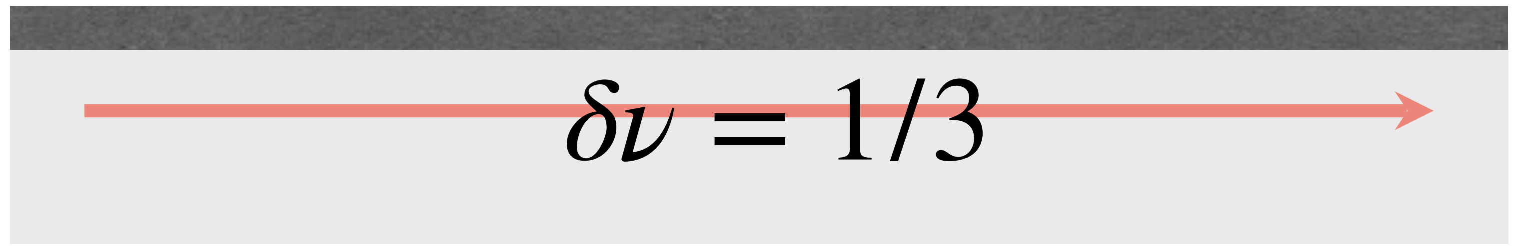

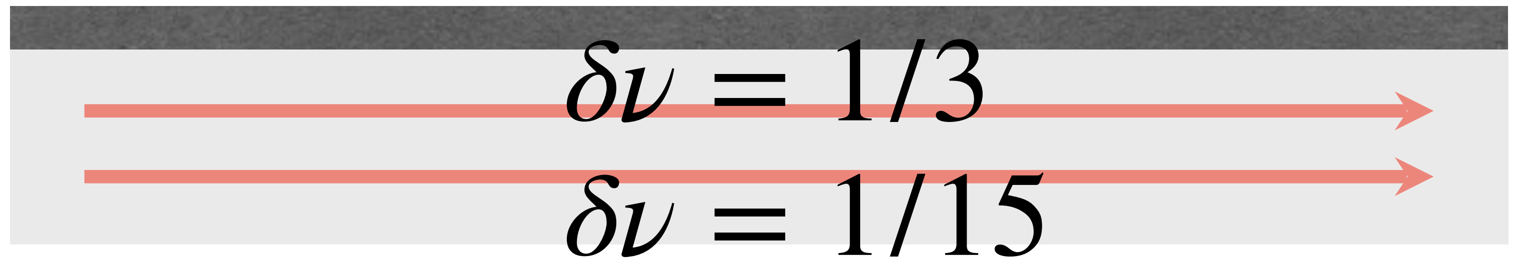

$$\nu = 1/3 \Rightarrow \nu_Q = 1$$

$$\nu = 2/5 \Rightarrow \nu_Q = 2$$

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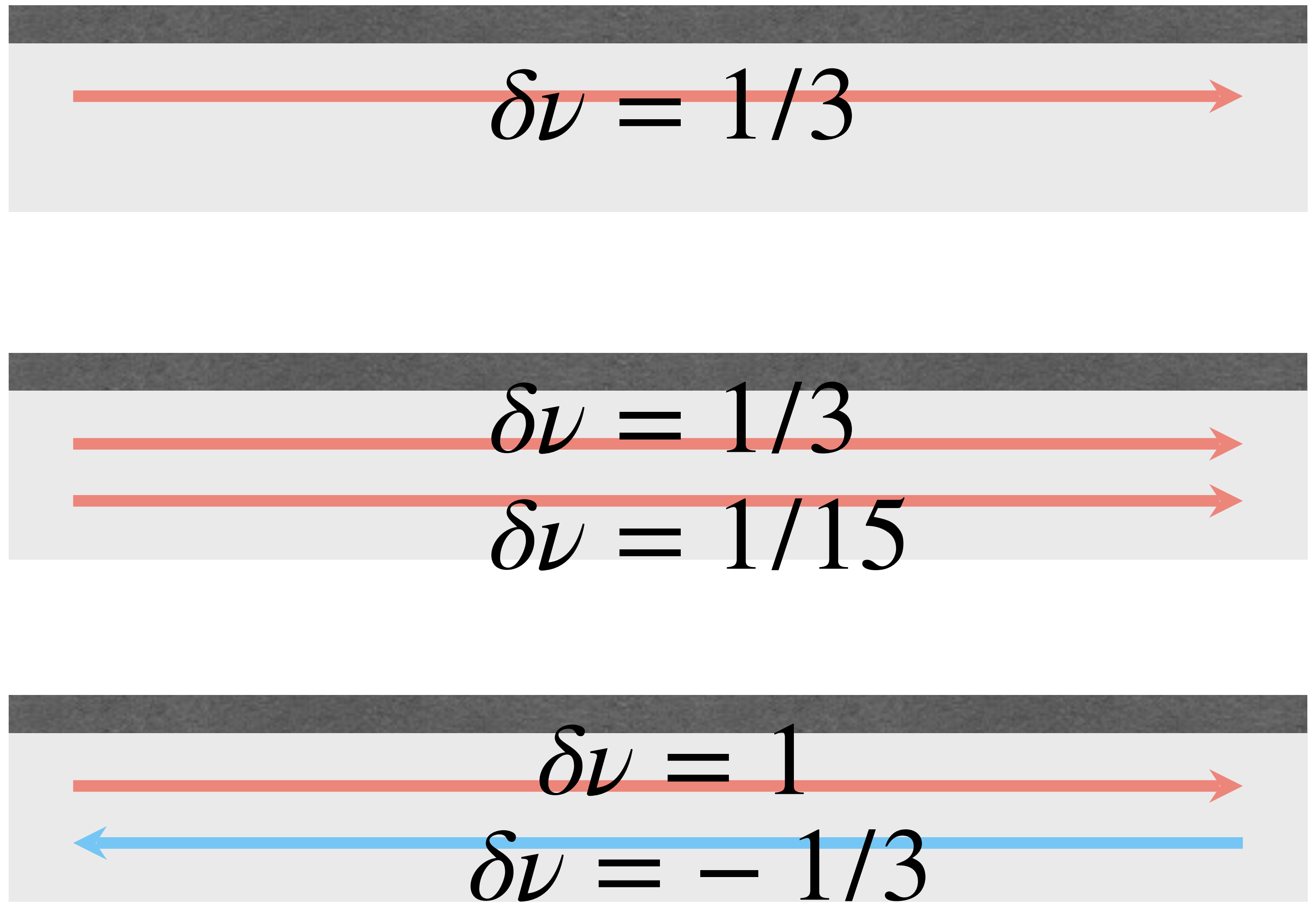

$\nu = 1/3 \Rightarrow \nu_Q = 1$

$\nu = 2/5 \Rightarrow \nu_Q = 2$

$\nu = 2/3 \Rightarrow \nu_Q = 0$

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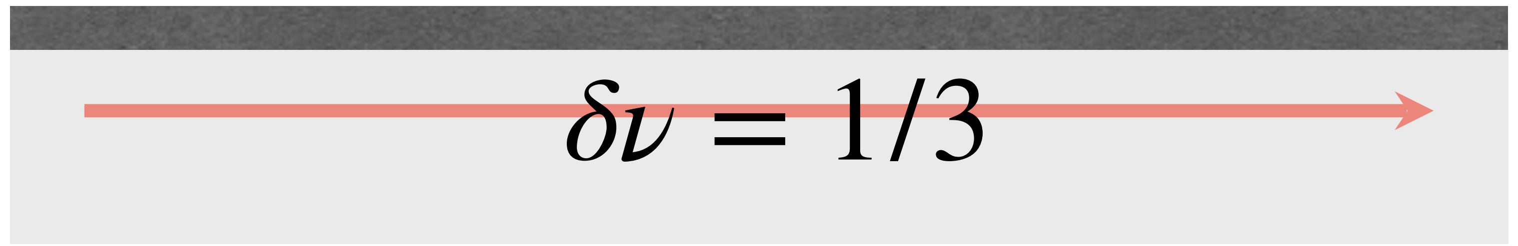

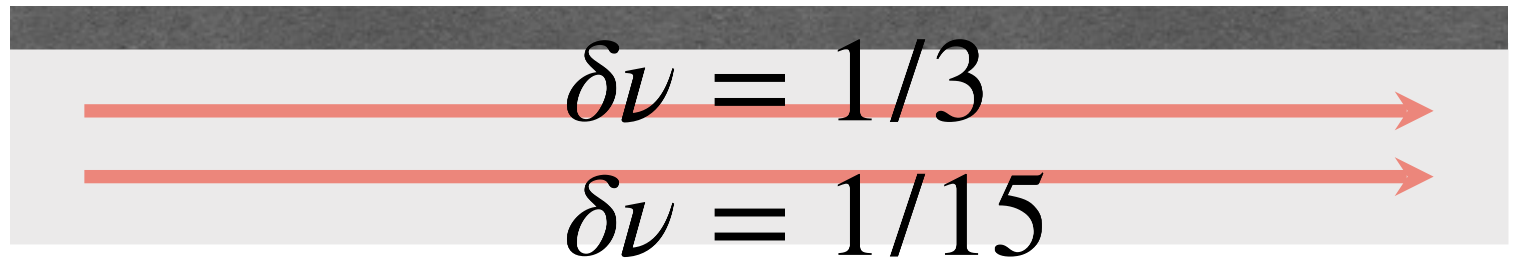

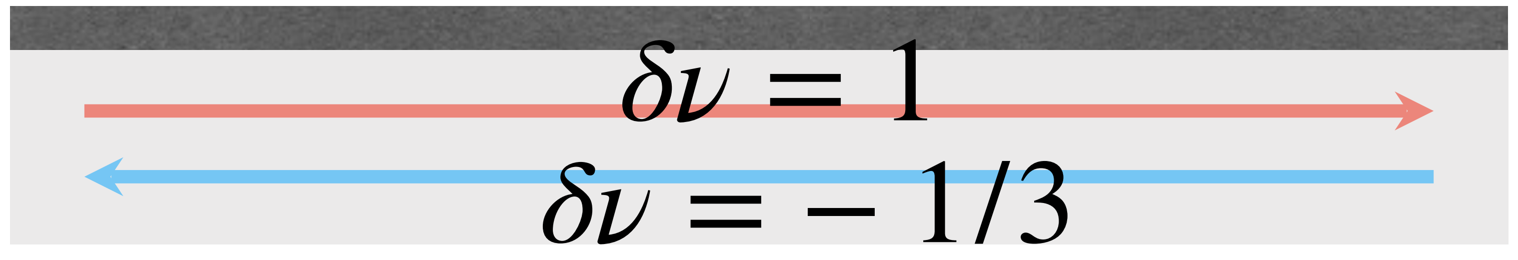

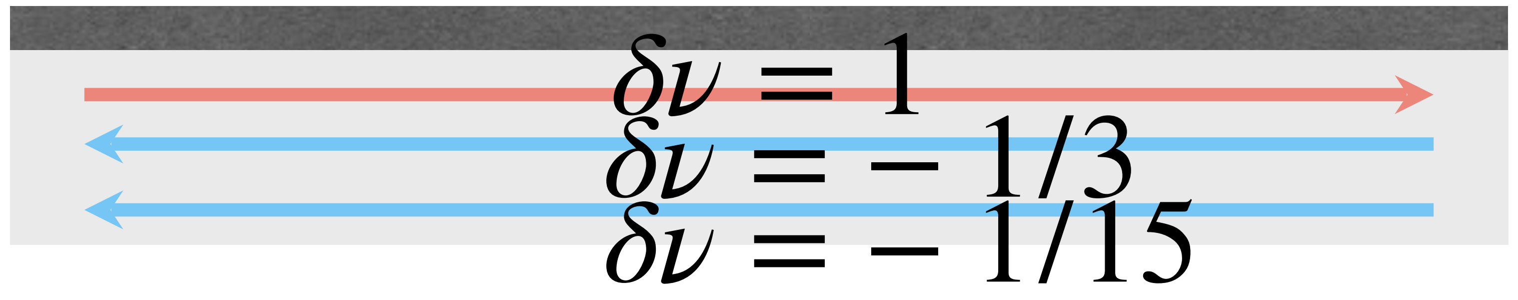

$$\nu = 1/3 \Rightarrow \nu_Q = 1$$

$$\nu = 2/5 \Rightarrow \nu_Q = 2$$

$$\nu = 2/3 \Rightarrow \nu_Q = 0$$

$$\nu = 3/5 \Rightarrow \nu_Q = -1$$

Non-abelian $\nu = 5/2$ FQH edge structures

G. Moore et al.: Nucl. Phys. B 360, 362 (1991); M. Levin et al.: Phys. Rev. Lett. 99, 236806 (2007)
S.-S. Lee et al.: Phys. Rev. Lett. 99, 236807 (2007); L. Fidkowski et al.: Phys. Rev. X 3, 041016 (2013)
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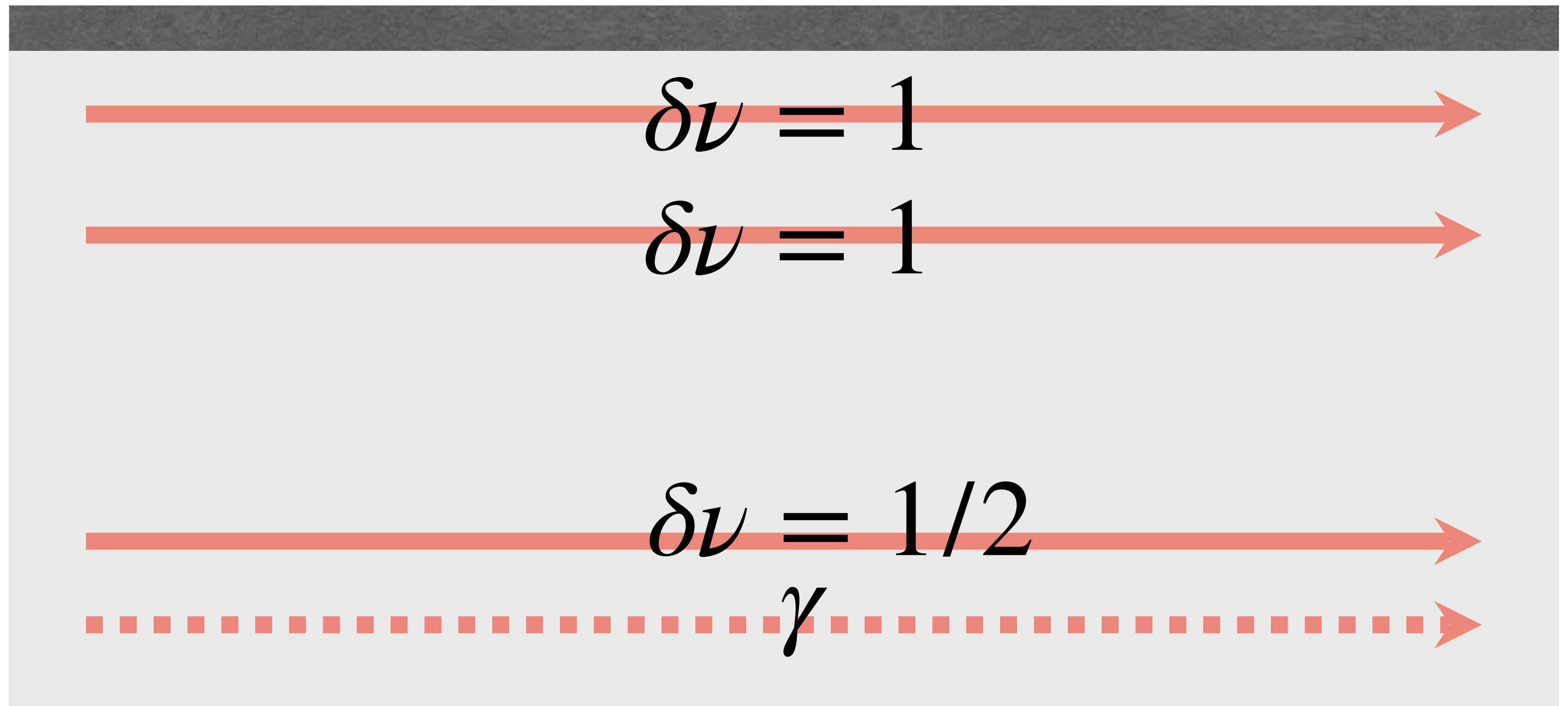
Pfaffian



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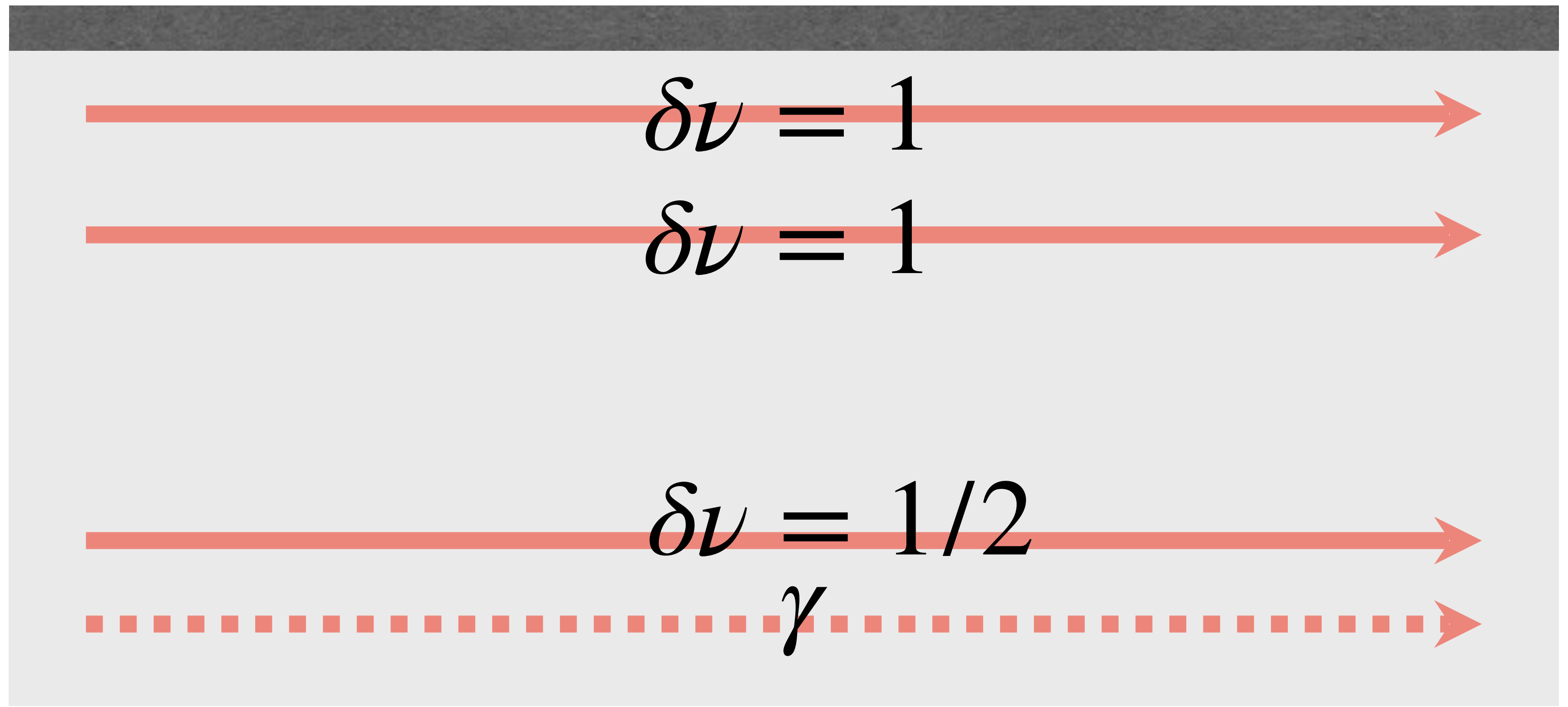
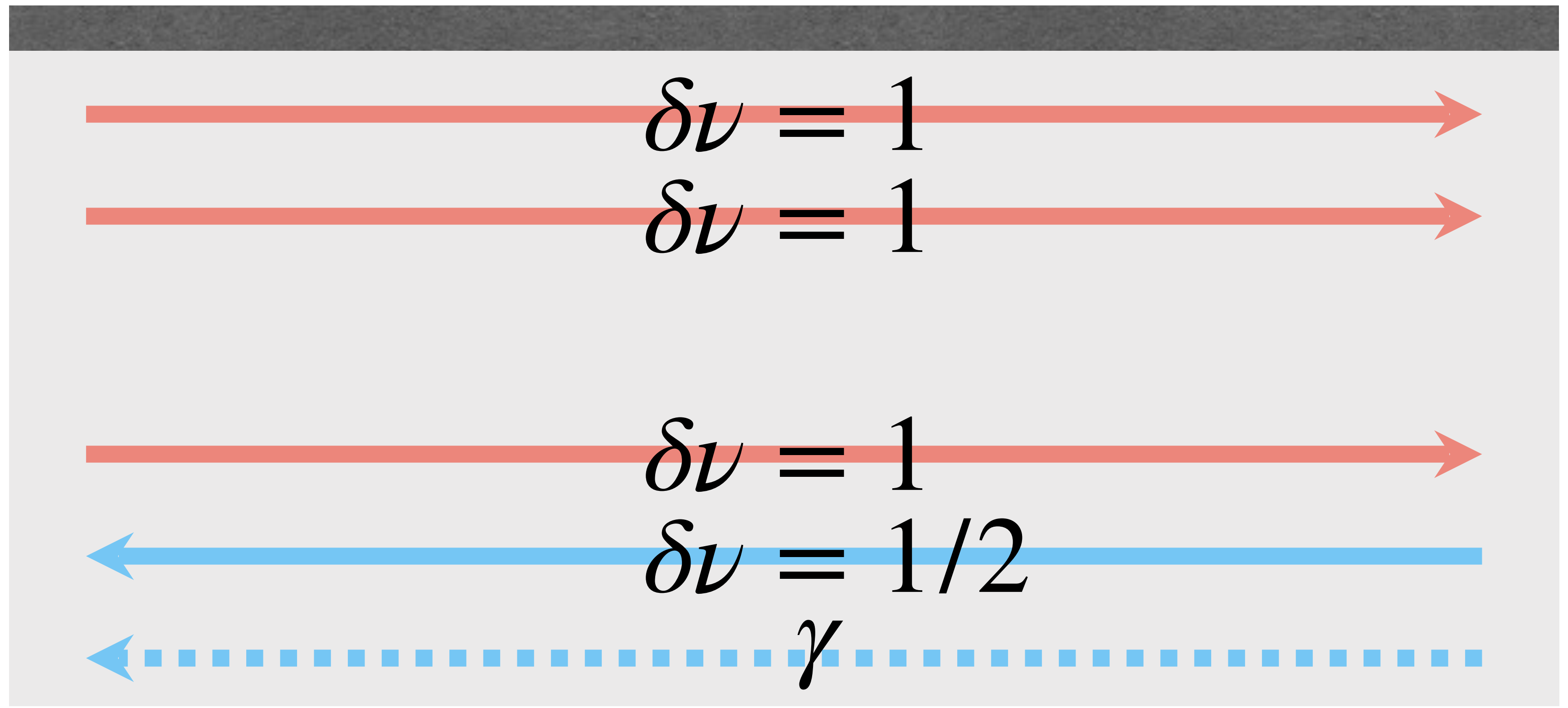


Anti-Pfaffian



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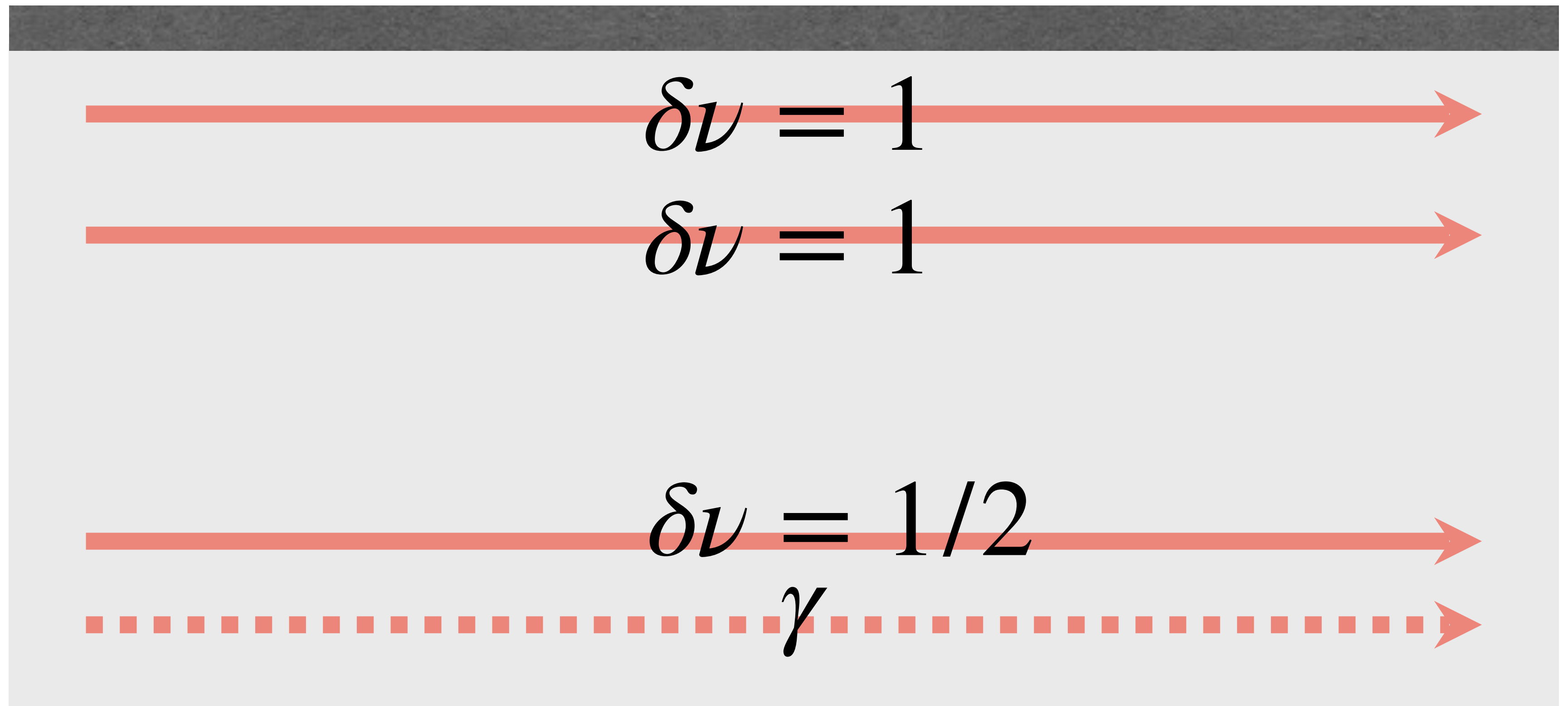

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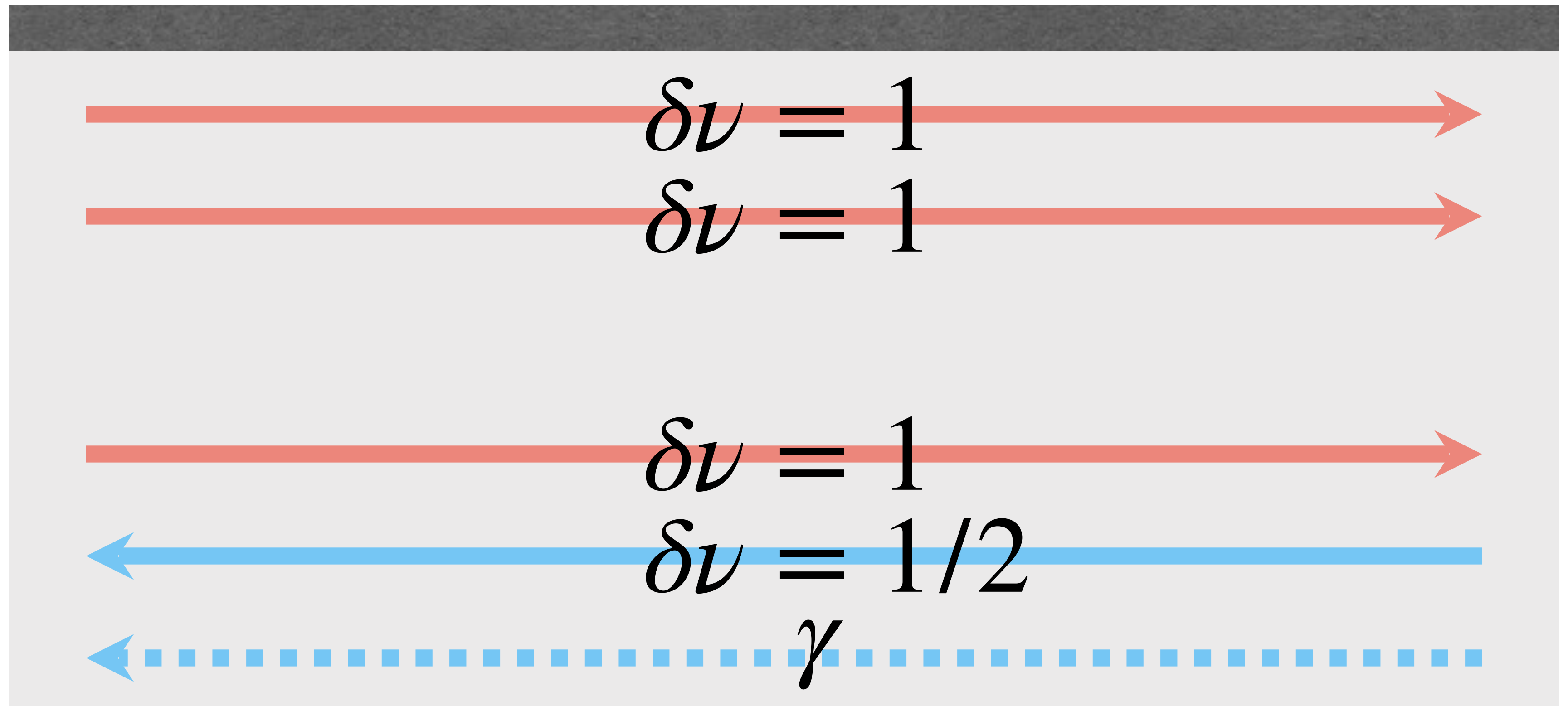
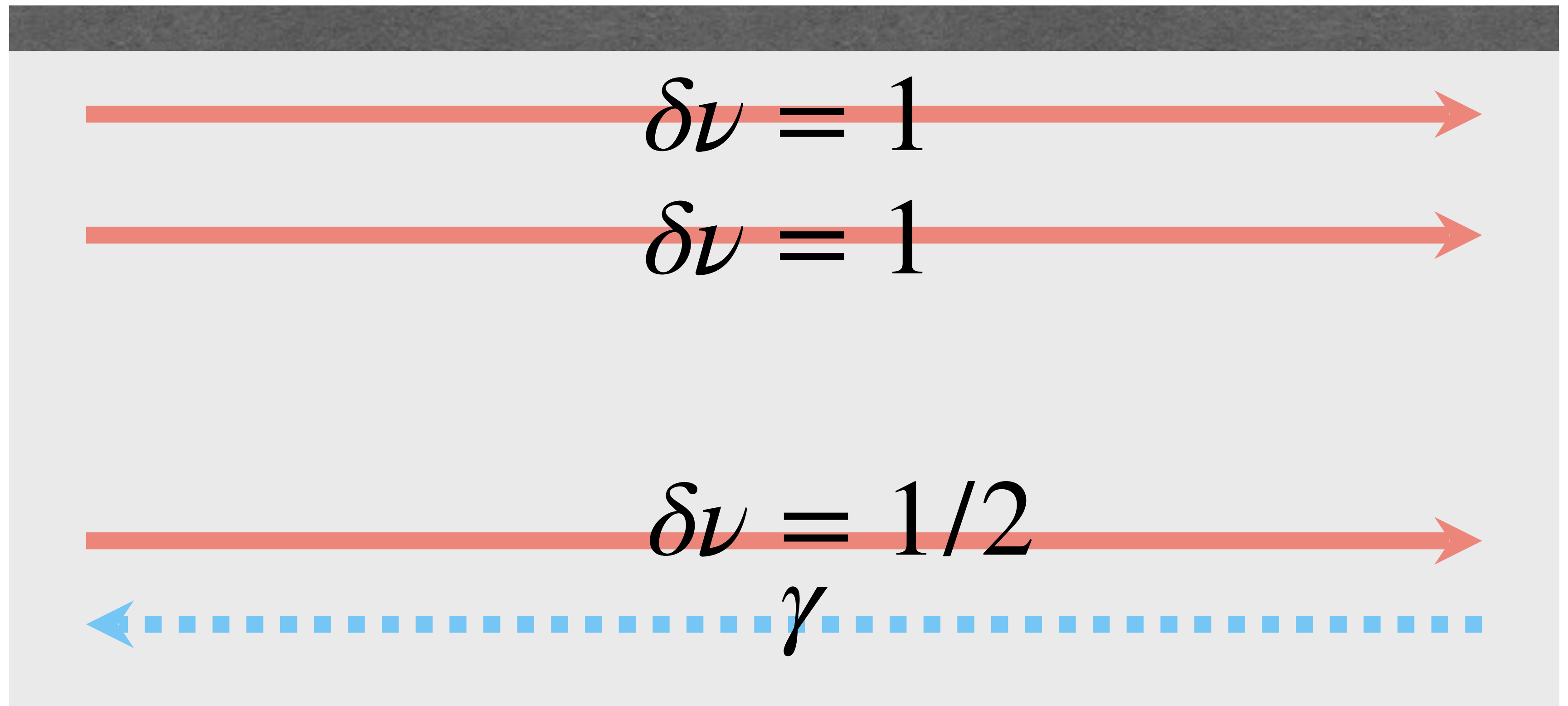


Particle-hole Pfaffian



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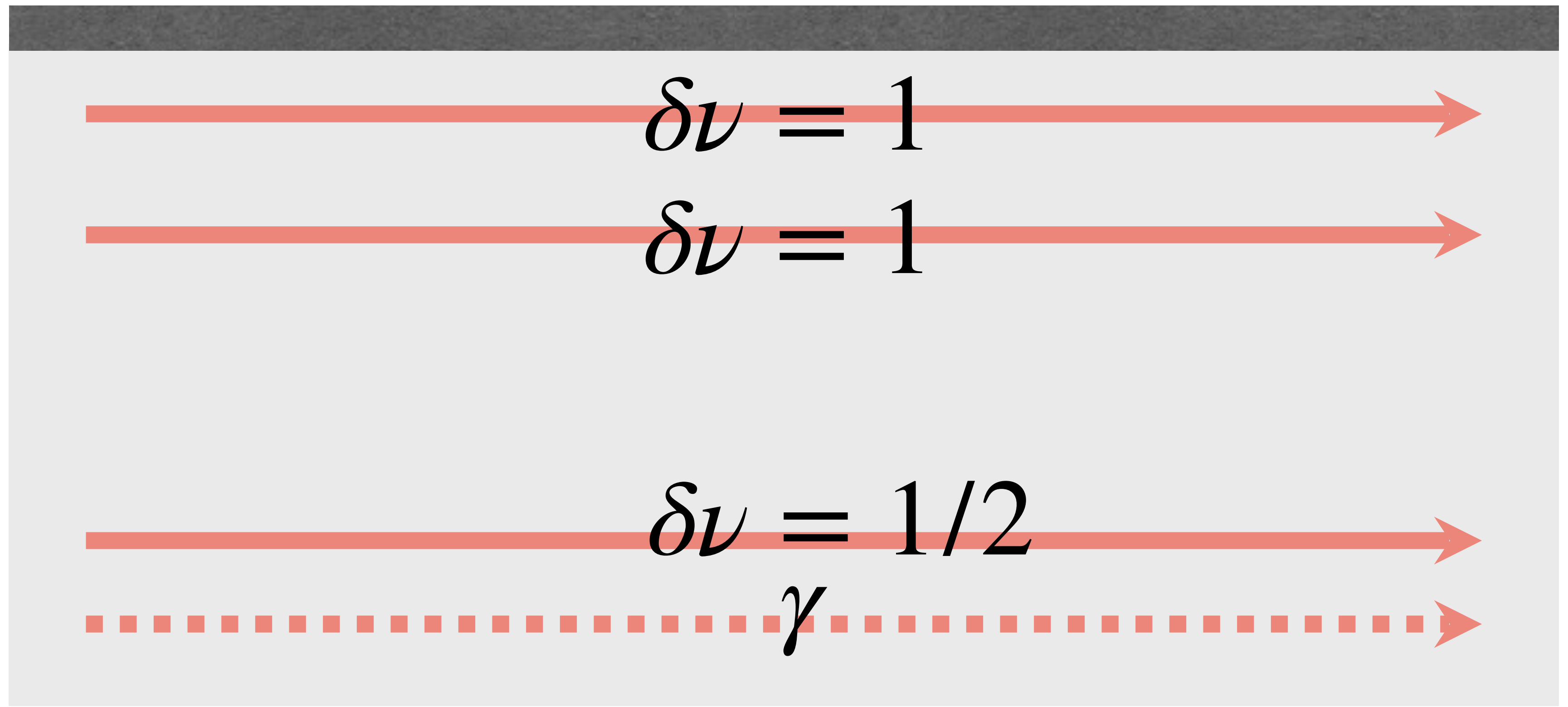

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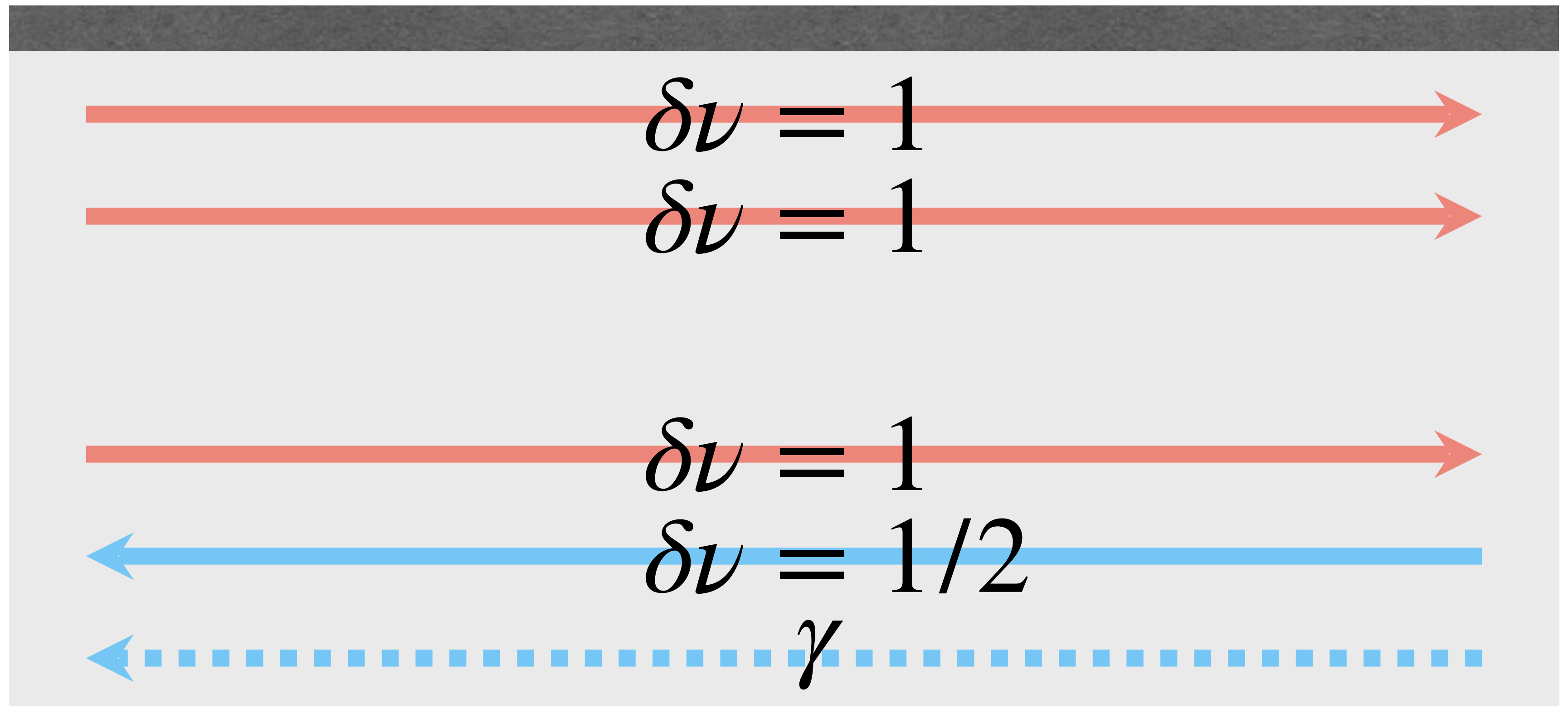

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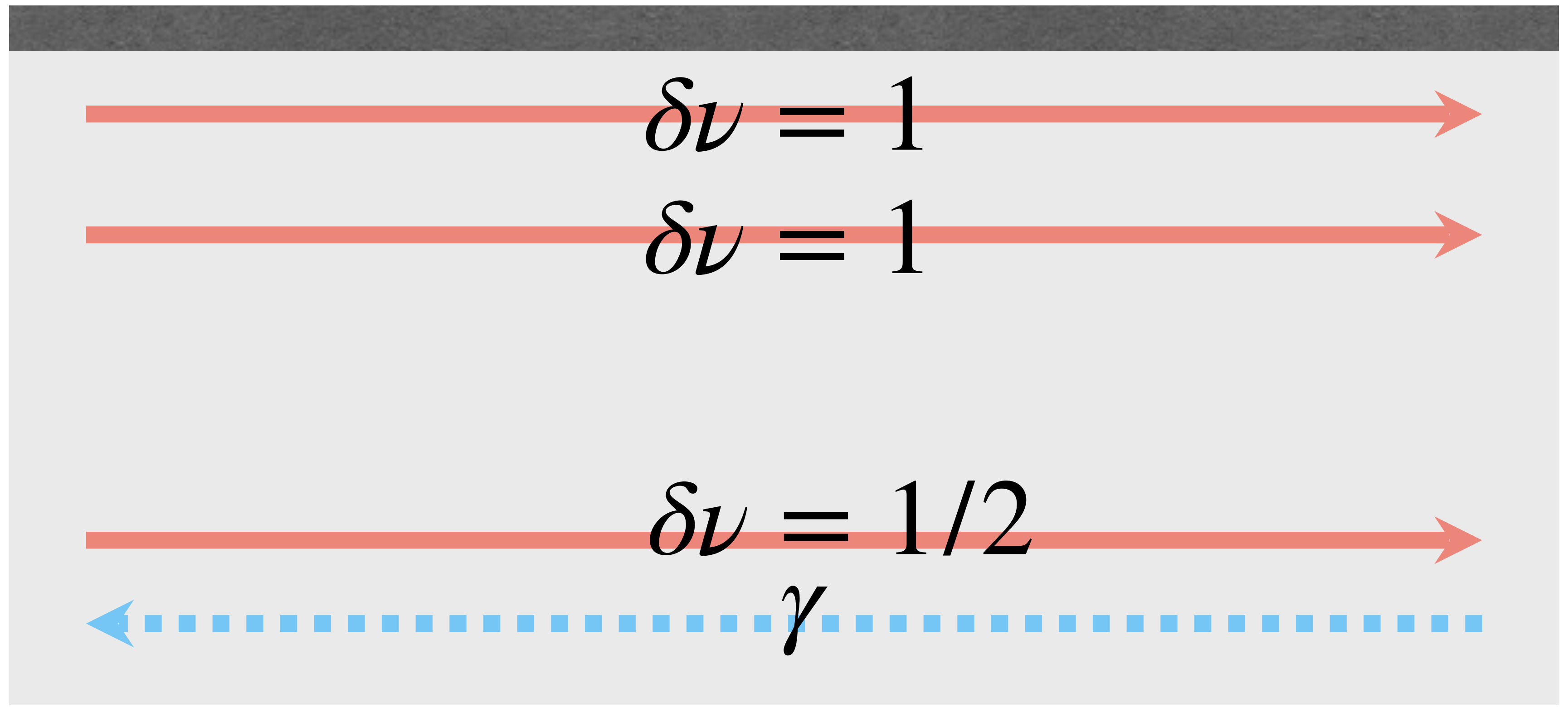
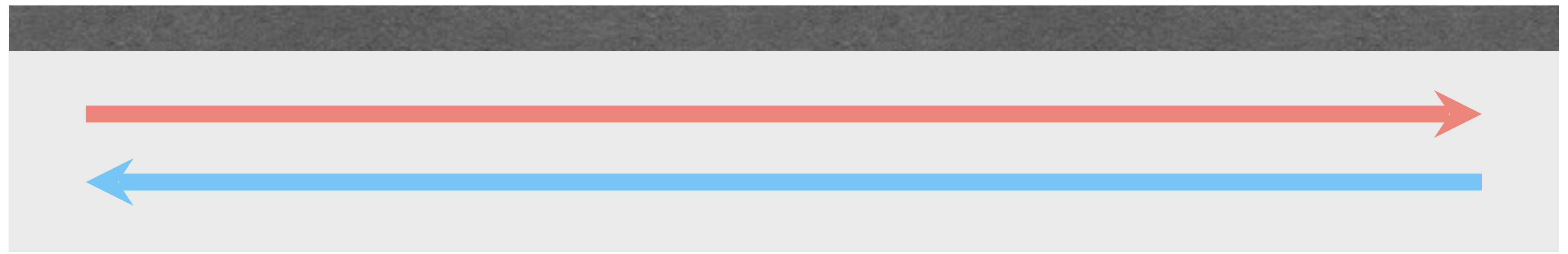


All states are compatible with $G = 5e^2/(2h)$

but differ in their edge structures!

$\nu = 1/3 + 1/3 = 2/3$

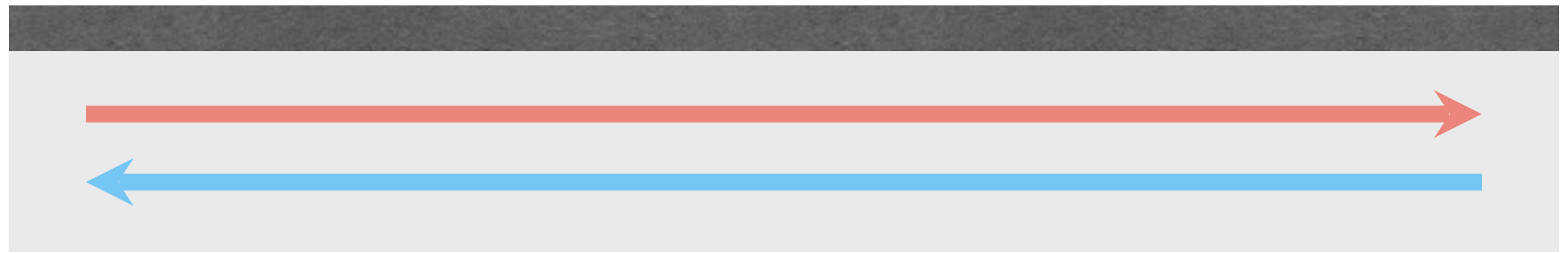
$\nu = 1 - 1/3 = 2/3$



How to distinguish?

$\nu = 1/3 + 1/3 = 2/3$

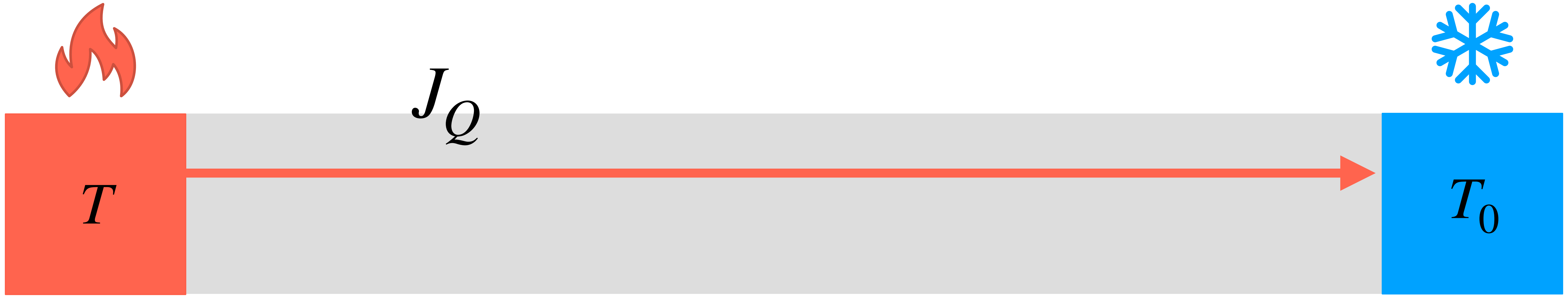
$\nu = 1 - 1/3 = 2/3$



How to distinguish?

Thermal conductance!

How to compute thermal conductances?



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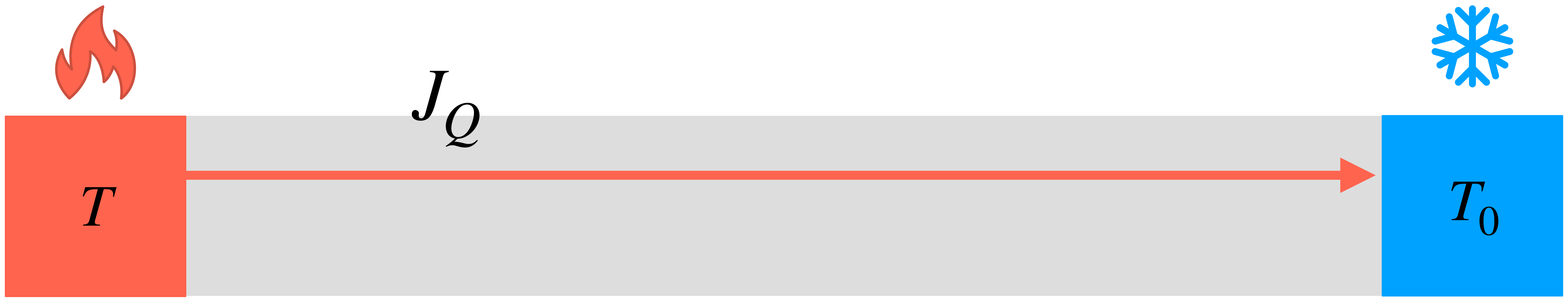

Bosons

$$J_Q = \int_0^\infty \frac{d\omega}{2\pi} \hbar\omega n_B(\hbar\omega) = \frac{\pi k_B^2 T^2}{12\hbar} = \frac{\pi^2 k_B^2 T^2}{6h} \equiv \frac{\kappa_0 T^2}{2}$$

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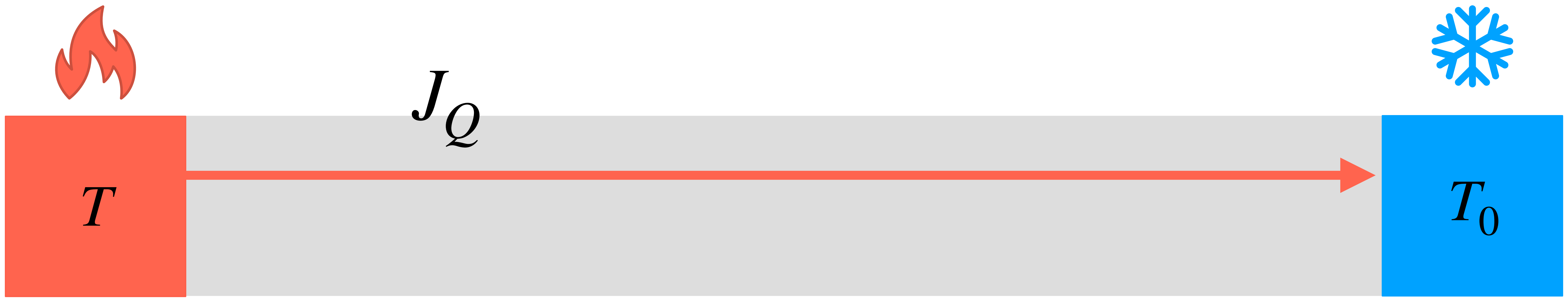

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$$\kappa_0 T = \frac{\pi^2 k_\mathrm{B}^2}{3h} T$$

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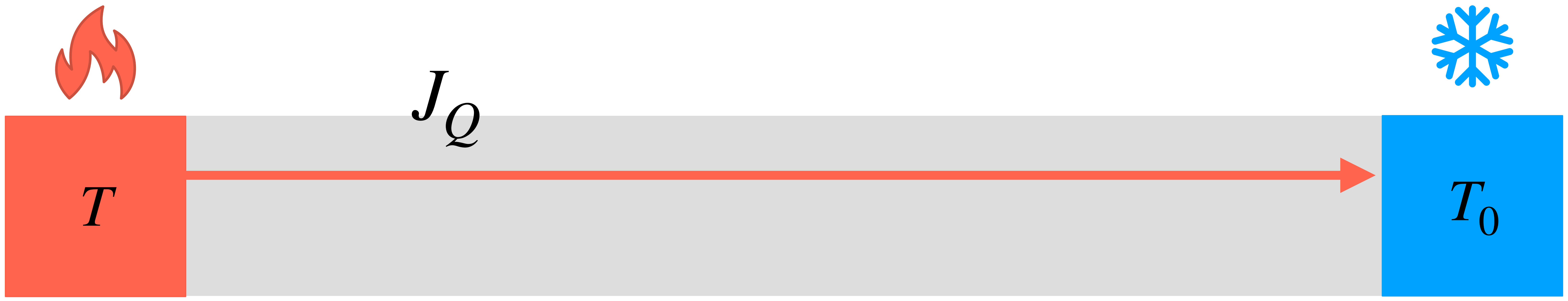

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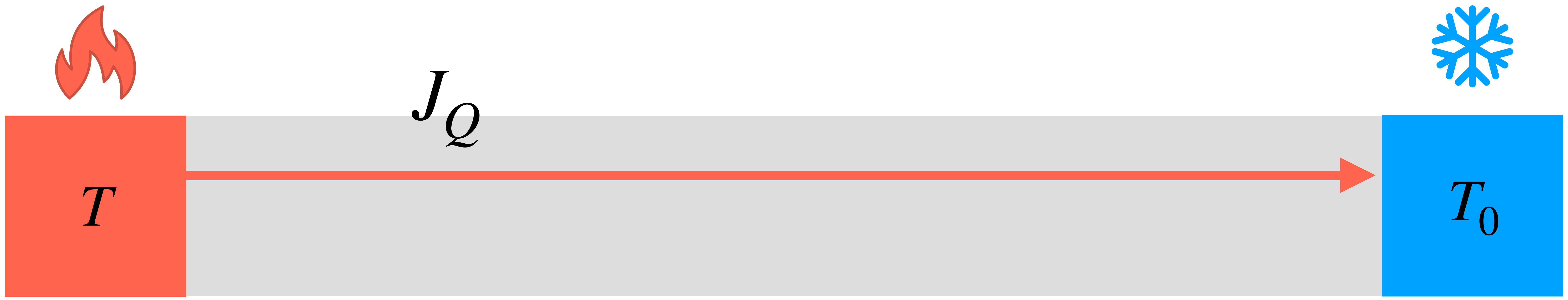

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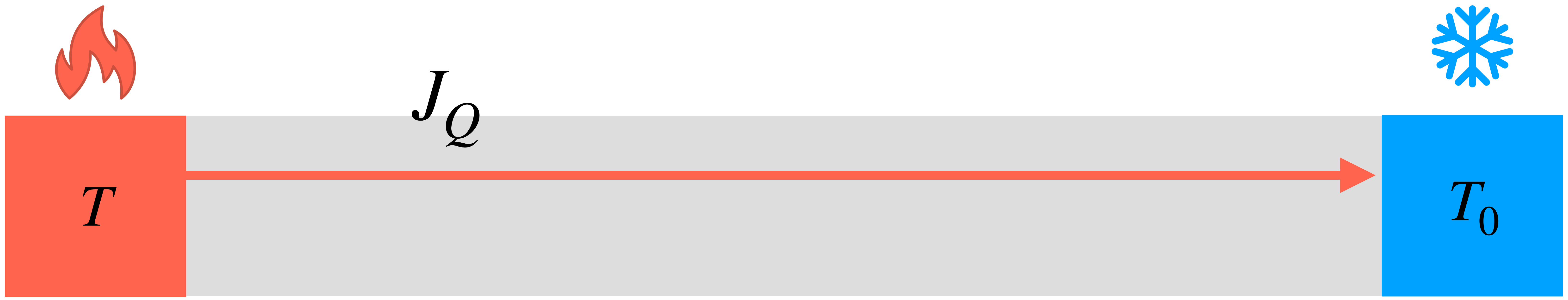
Generally: CFT

$$J_Q = \frac{v^2}{2\pi} \langle \mathcal{T} \rangle = c \frac{\kappa_0 T^2}{2}$$

A. Cappelli et al.: Nucl. Phys. B 636, 568 (2002)

How to compute thermal conductances?



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Abelian channel

$$c = 1$$

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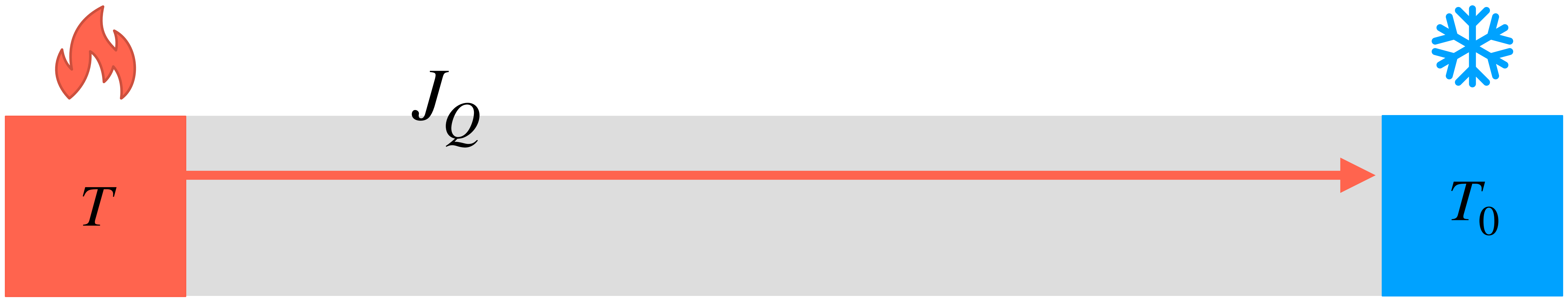
How to compute thermal conductances?



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A. Cappelli et al.: Nucl. Phys. B 636, 568 (2002)

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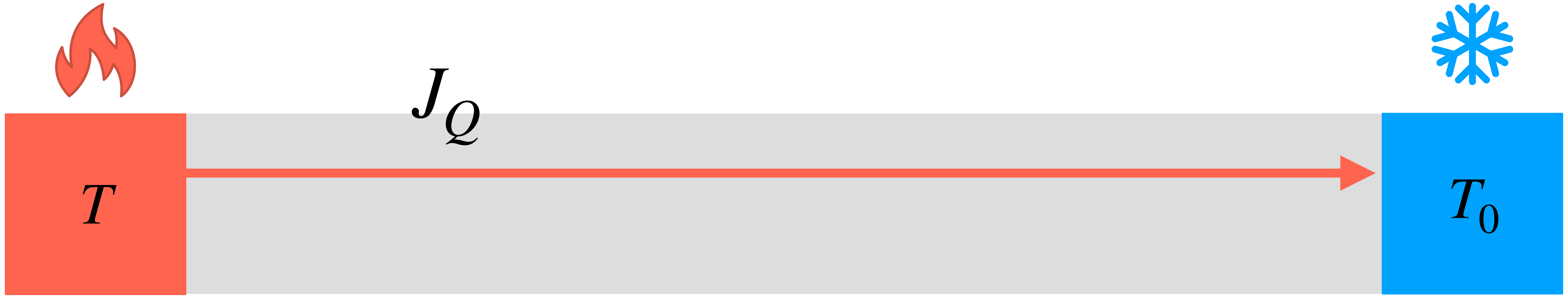
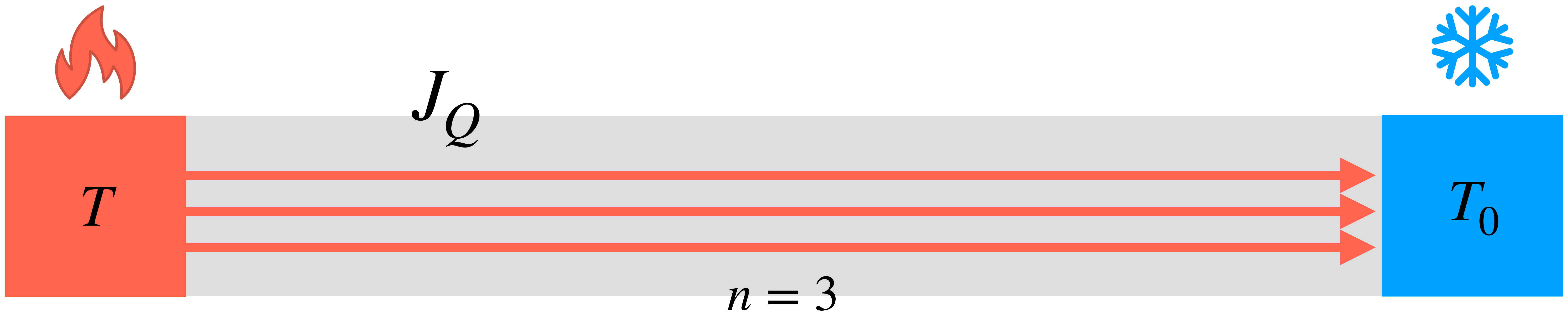


Thermal conductance of an edge channel

$$G_Q = \frac{dJ_Q}{dT} = c\kappa_0 T$$

How to compute thermal conductances?



Thermal conductance of n
co-propagating edge channels

$$G_Q = \frac{dJ_Q}{dT} = n\kappa_0 T$$

How to measure thermal conductances?

S. Jezouin et al.: Science 342, 601 (2013)

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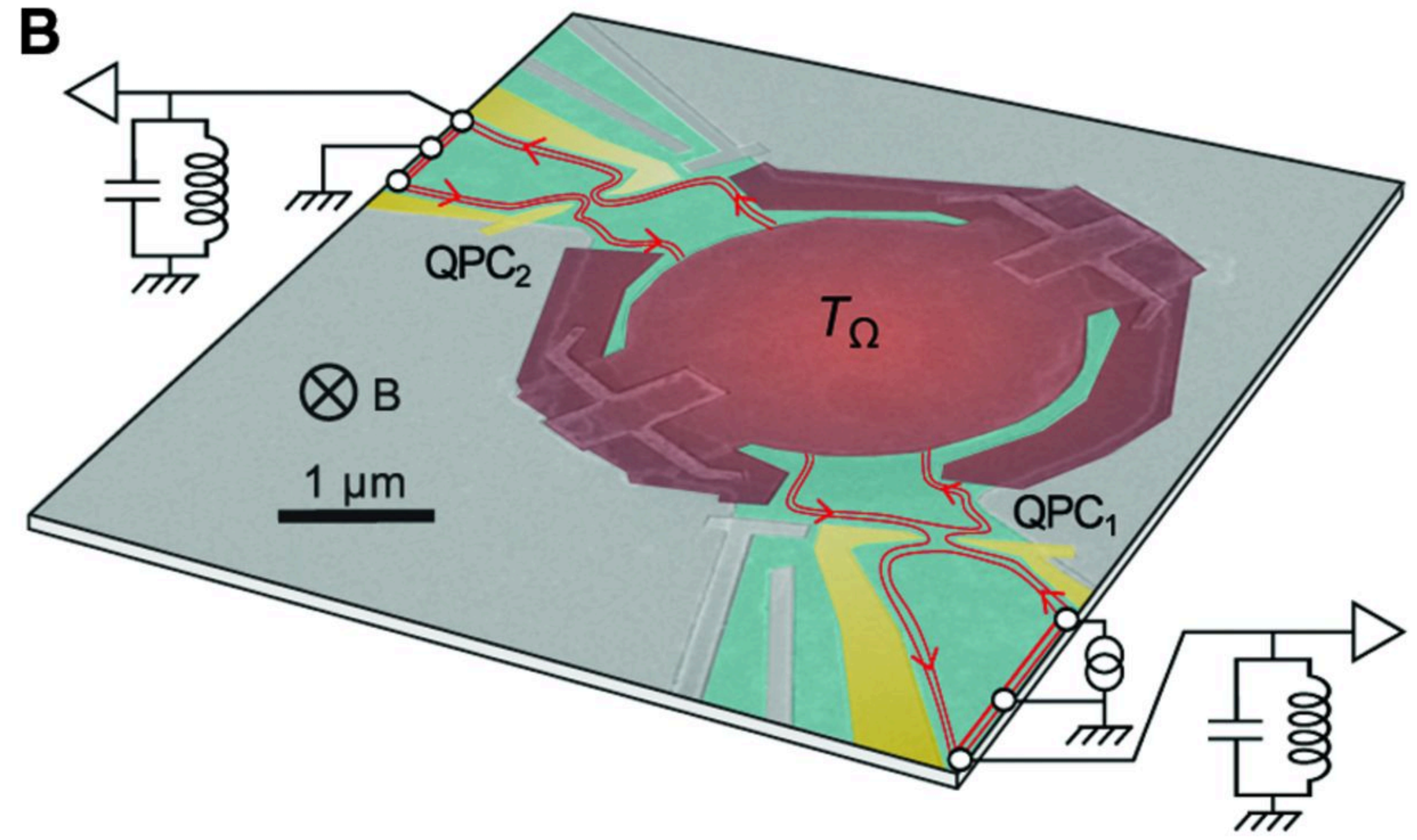

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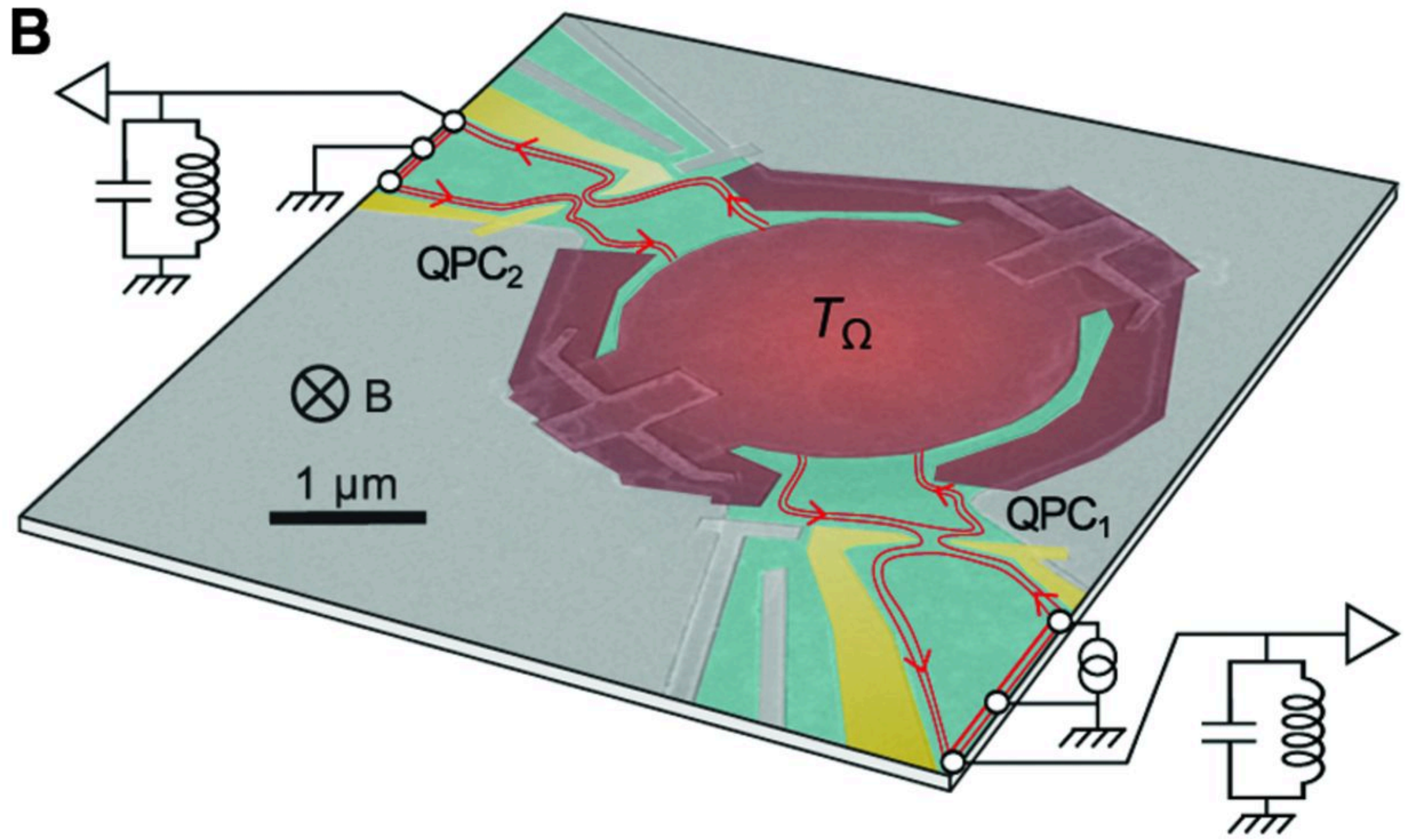

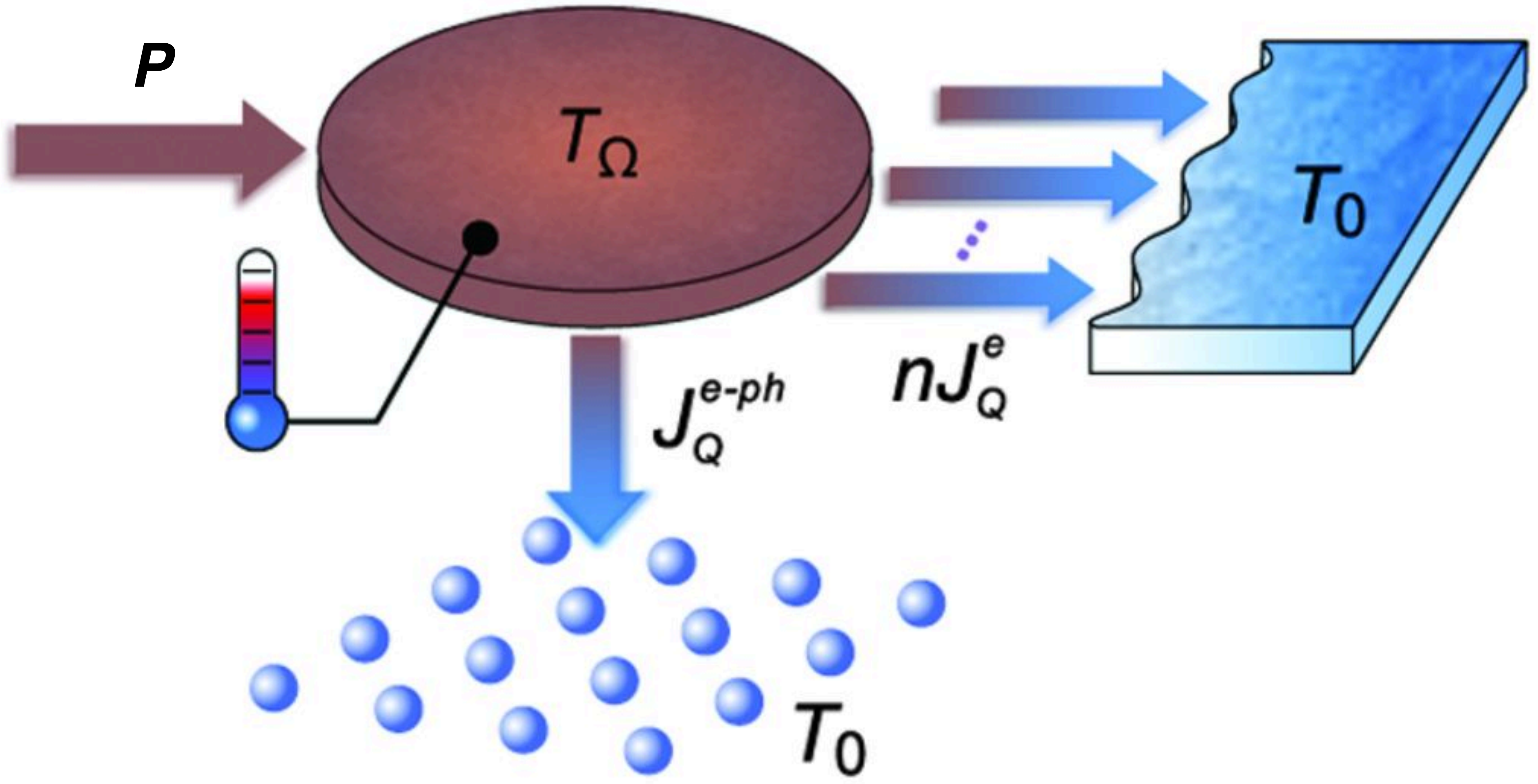

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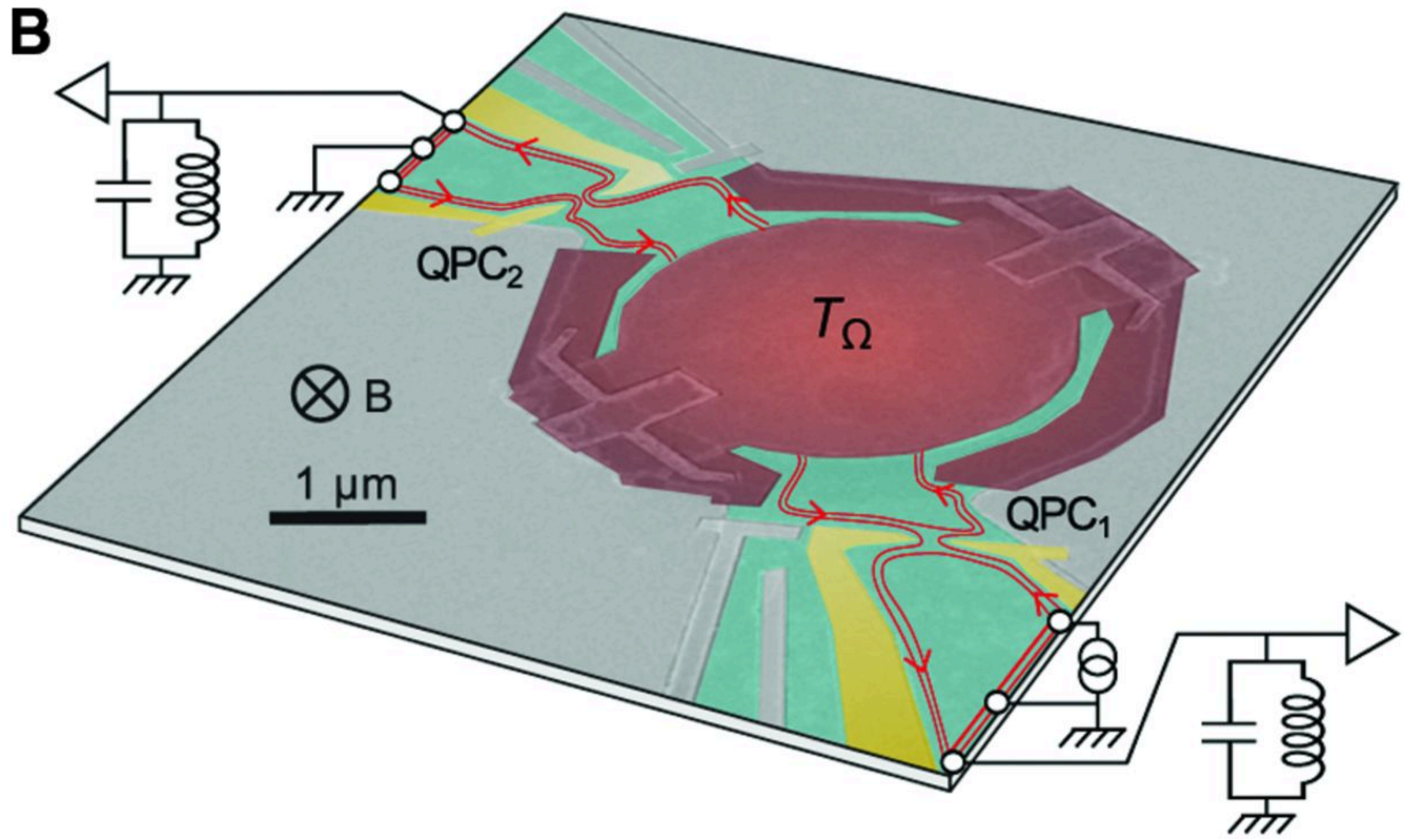
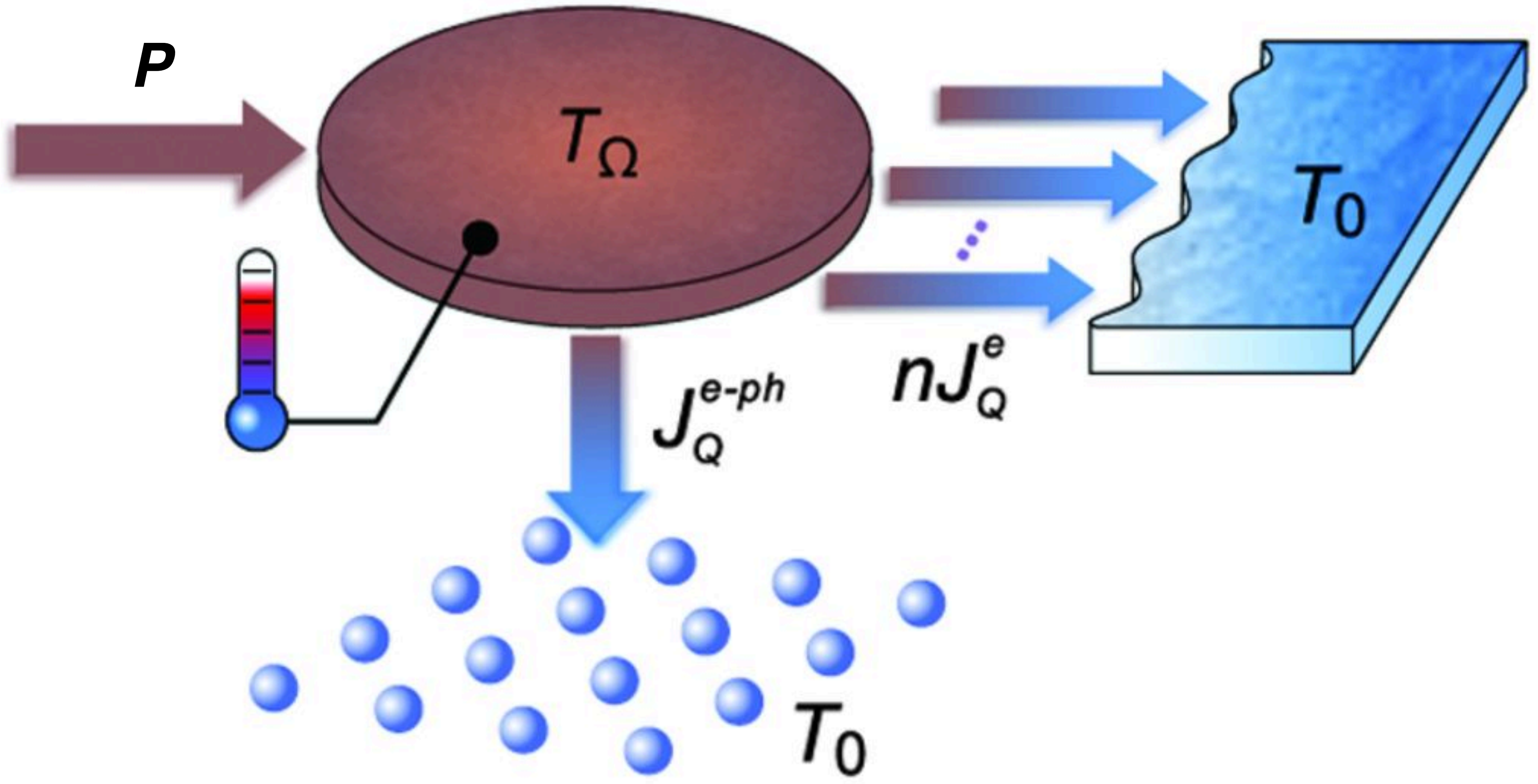




Power balance equation

$$P = J_Q = J_Q^{\mathrm{e-ph}} + nJ_Q^e \approx nJ_Q^e = \frac{n}{2}\kappa_0(T_\Omega^2 - T_0^2)$$

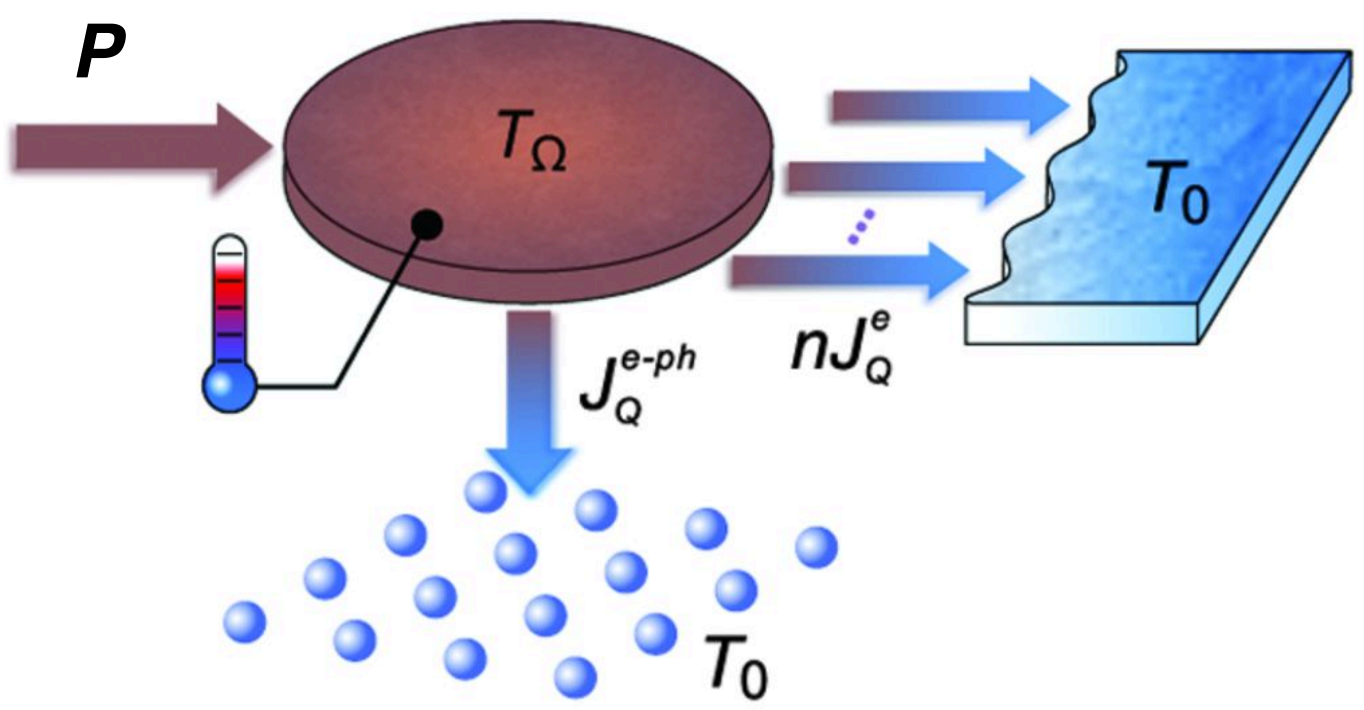
Extracting the thermal conductance

S. Jezouin et al.: Science 342, 601 (2013)

Extracting the thermal conductance

S. Jezouin et al.: Science 342, 601 (2013)

- P is known from injected current I_S



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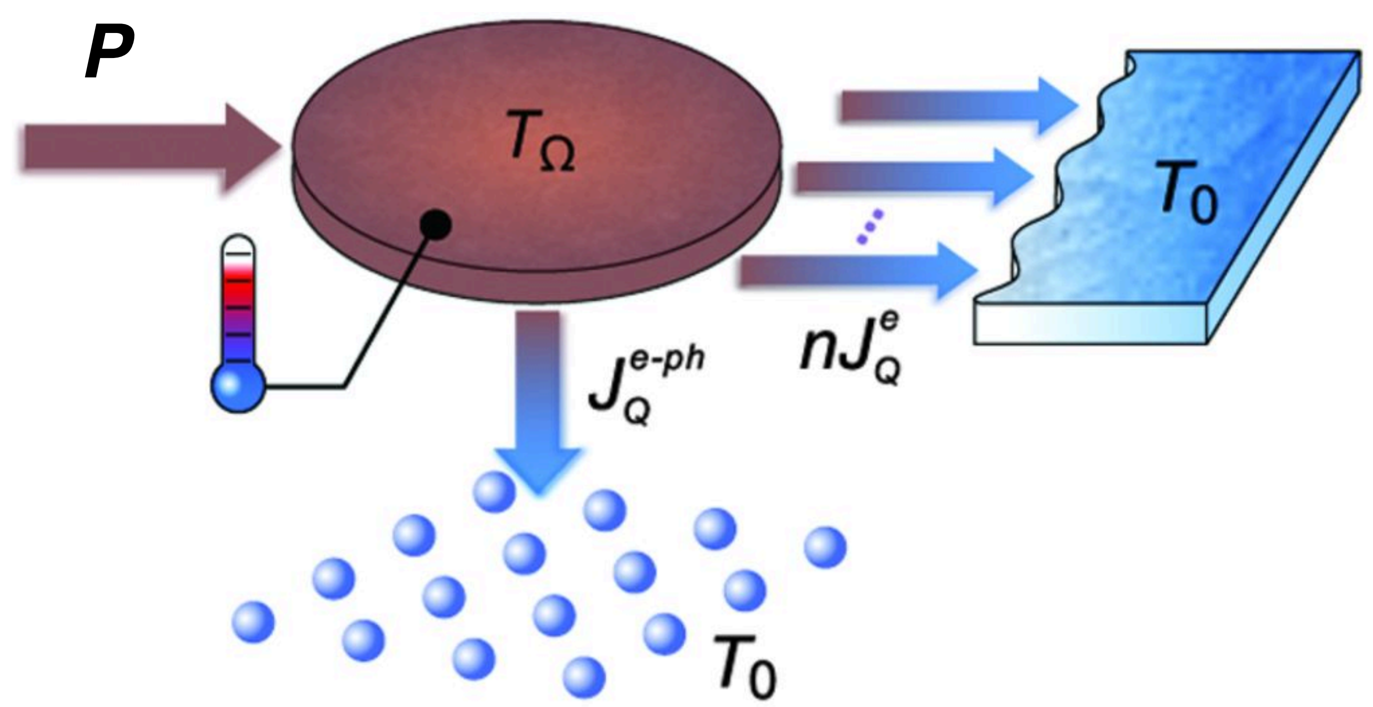
Extracting the thermal conductance

S. Jezouin et al.: Science 342, 601 (2013)

- P is known from injected current I_S
- Measure T_Ω, T_0 from thermal noise S_I



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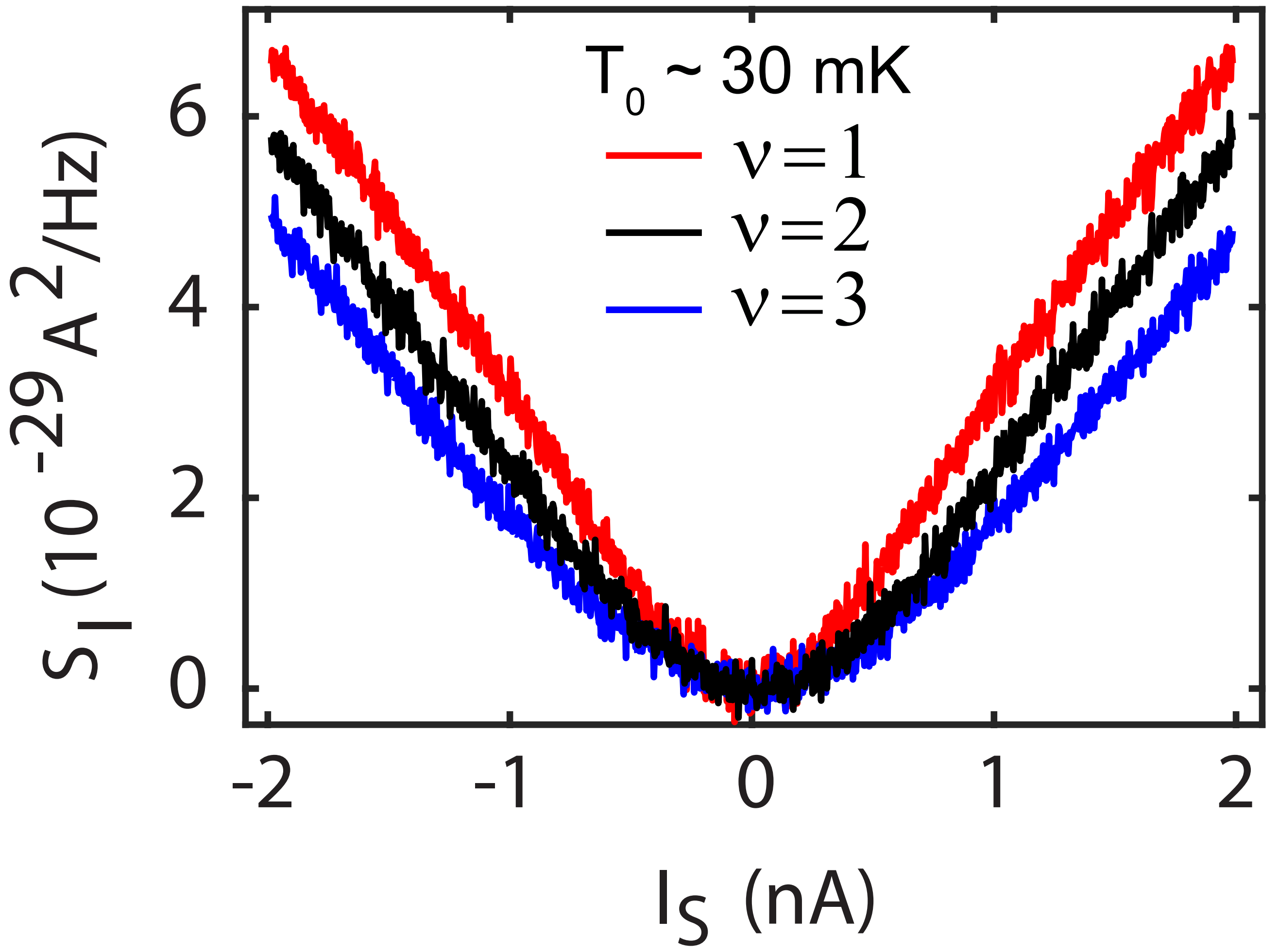
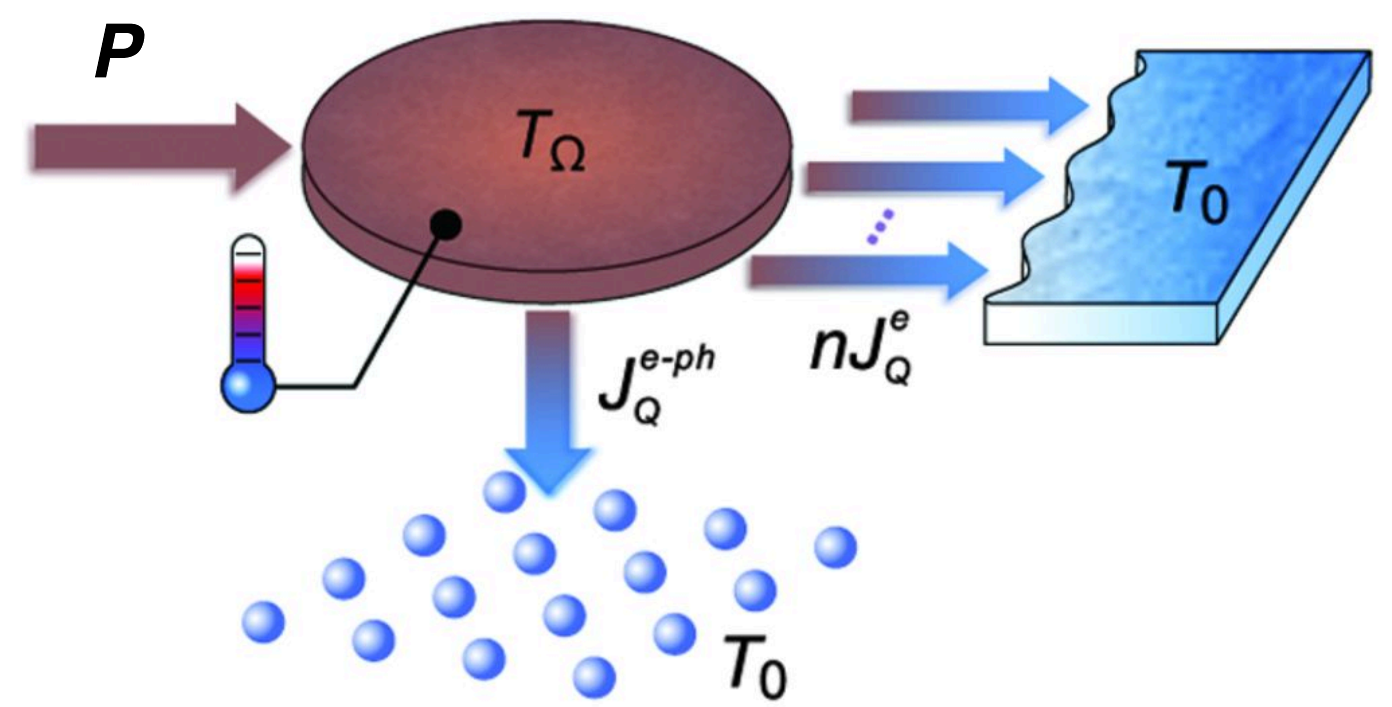


S. K. Srivastav, et al.:Phys. Rev. Lett. 126 216803 (2022)

Extracting the thermal conductance

S. Jezouin et al.: Science 342, 601 (2013)

- P is known from injected current I_S
- Measure T_Ω, T_0 from thermal noise S_I
- Plot P vs $T_\Omega^2 - T_0^2$



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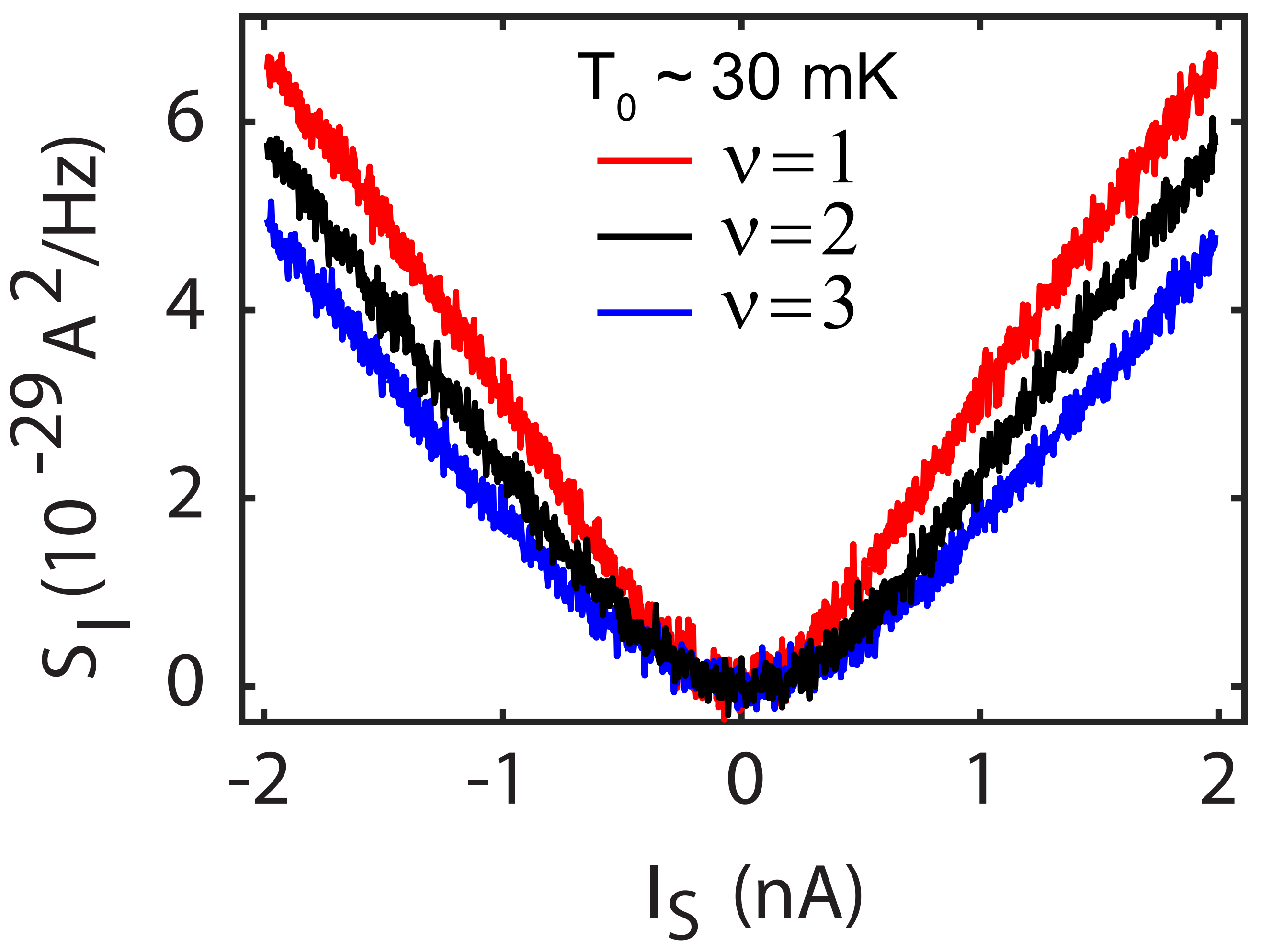

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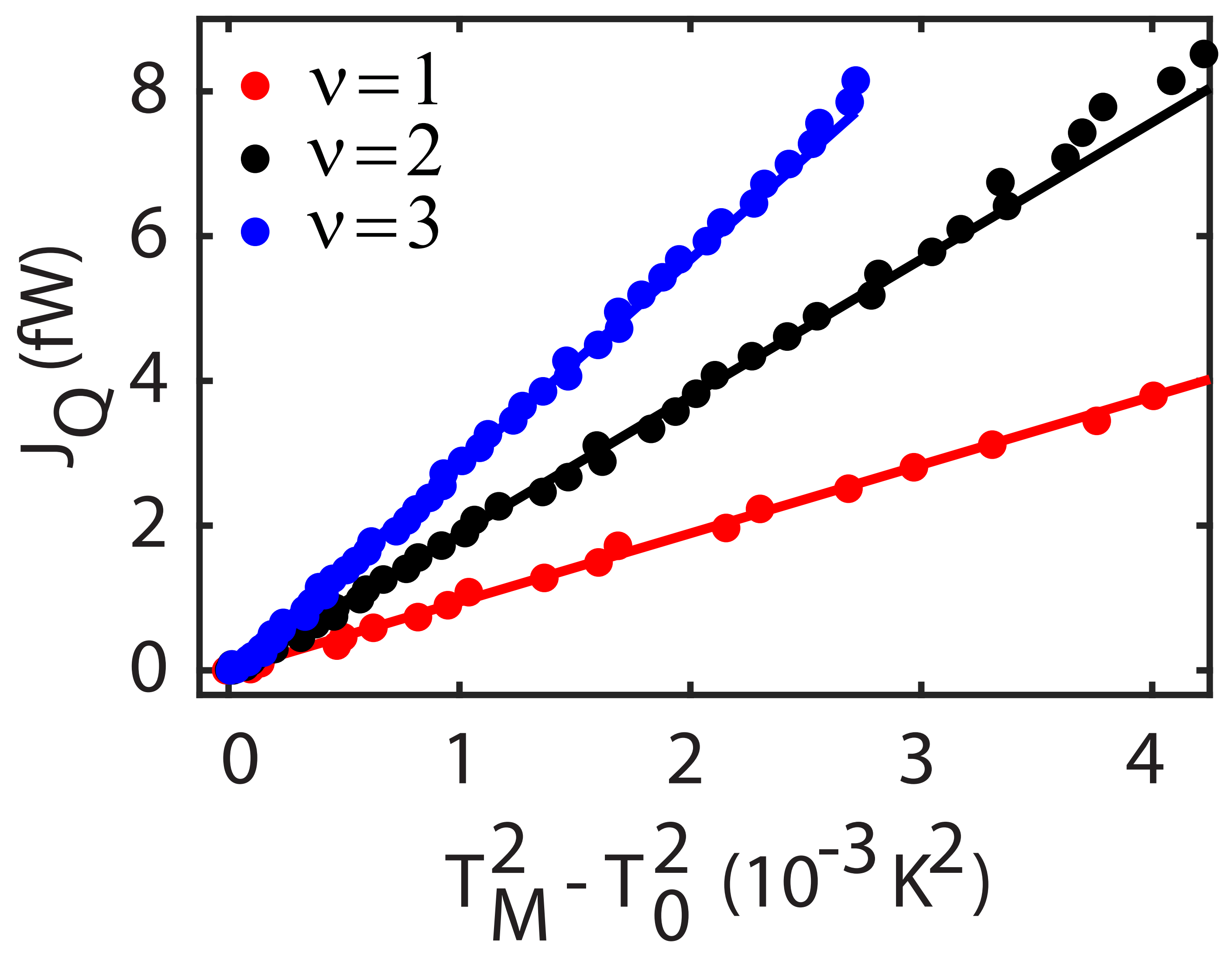

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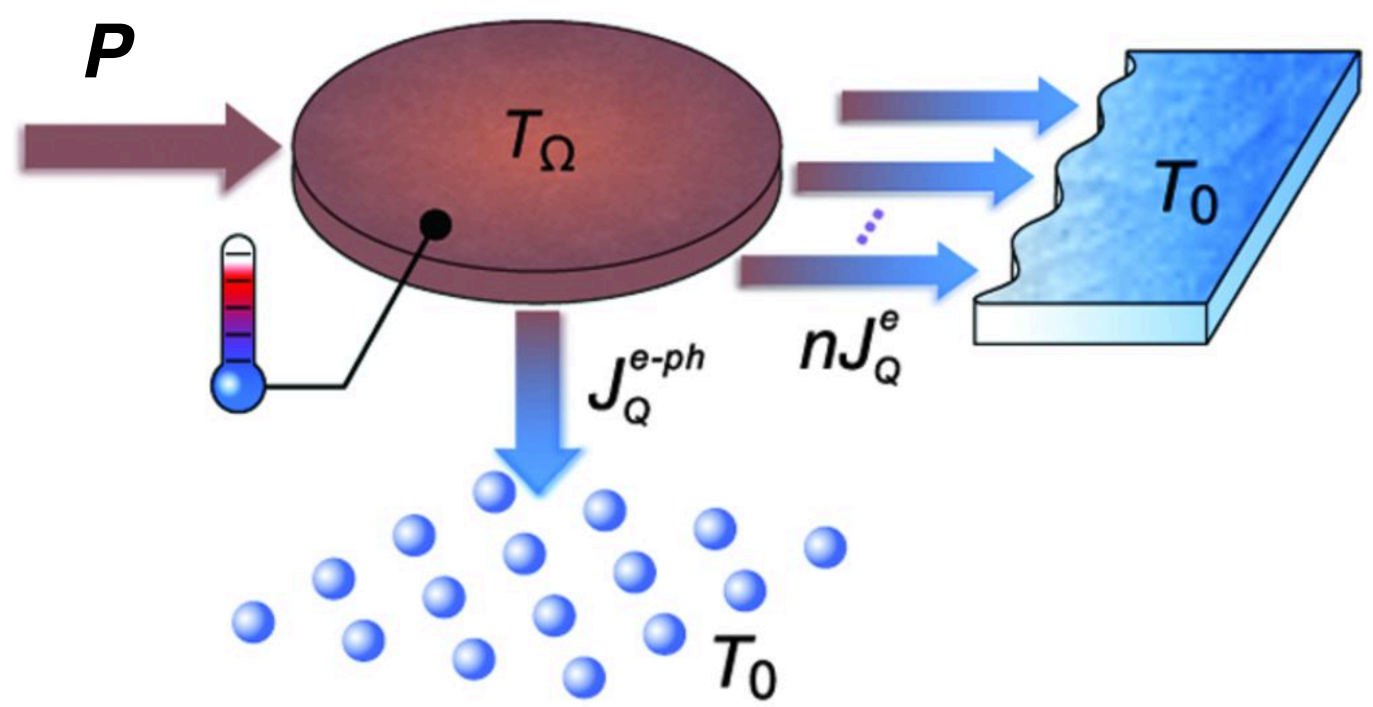
Extracting the thermal conductance

S. Jezouin et al.: Science 342, 601 (2013)

- P is known from injected current I_S
- Measure T_Ω, T_0 from thermal noise S_I
- Plot P vs $T_\Omega^2 - T_0^2$
- Slope of line gives $n \times \kappa_0/2$



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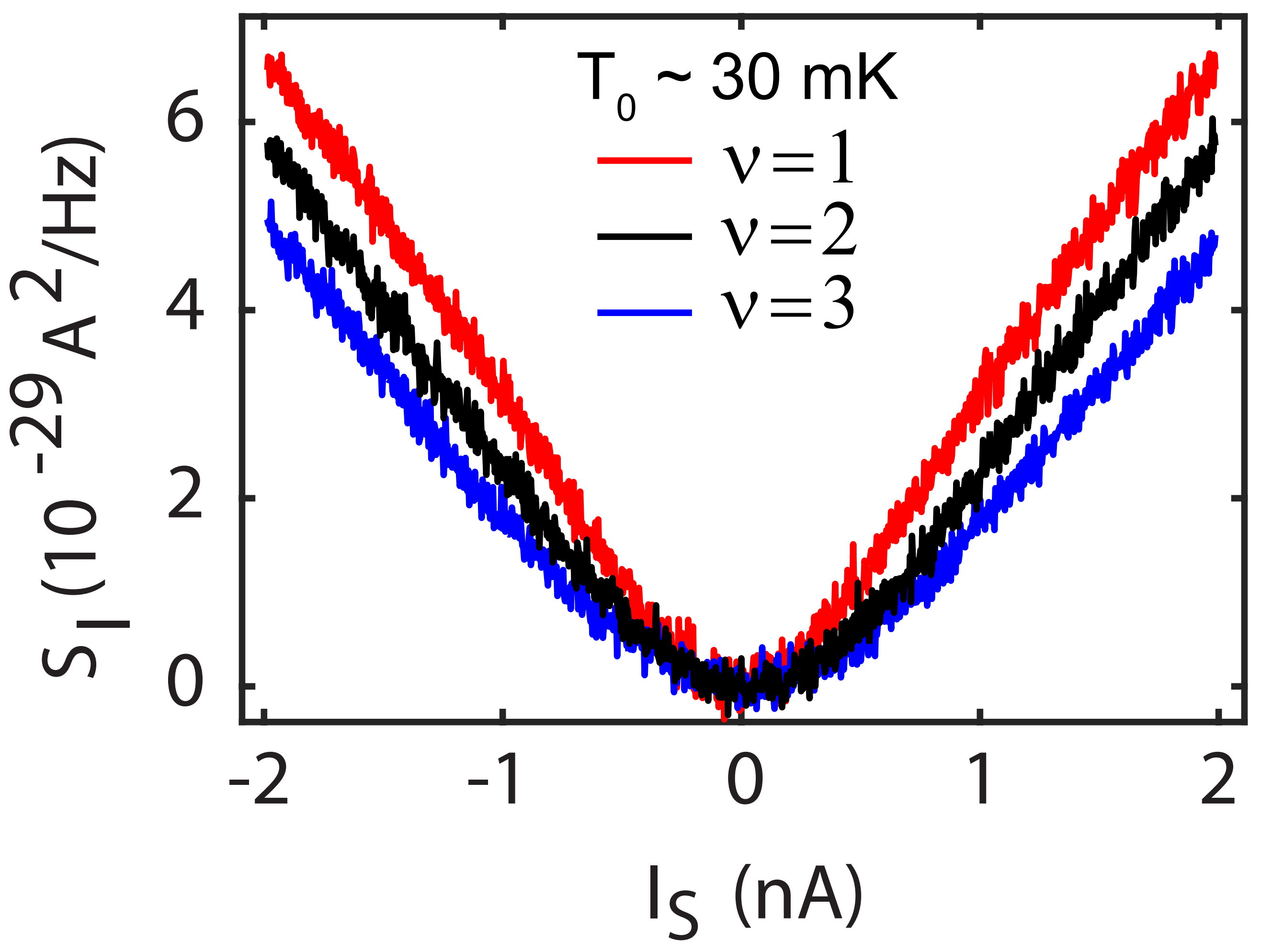

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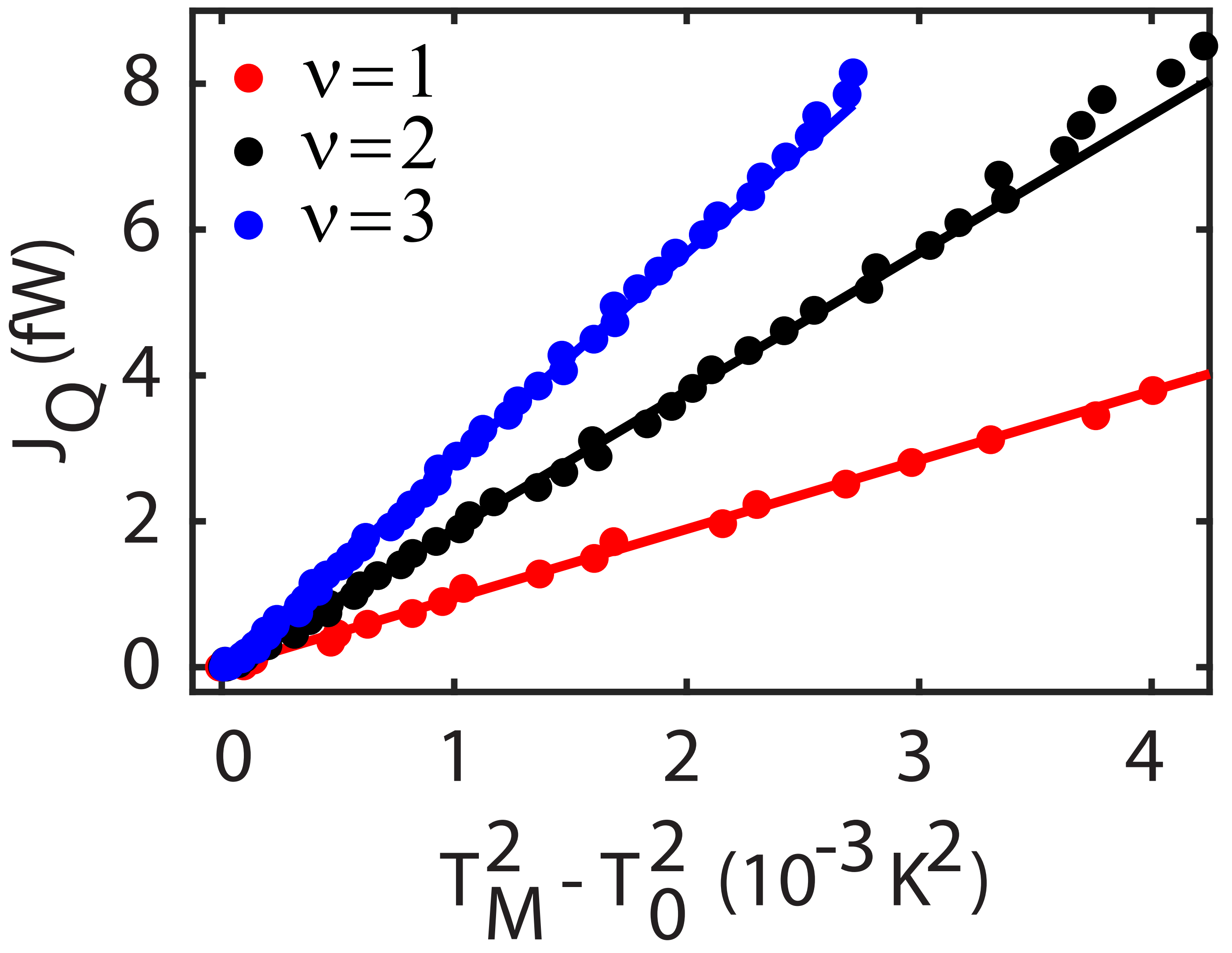

S. K. Srivastav, et al.:Phys. Rev. Lett. 126 216803 (2022)

Impressive recent progress

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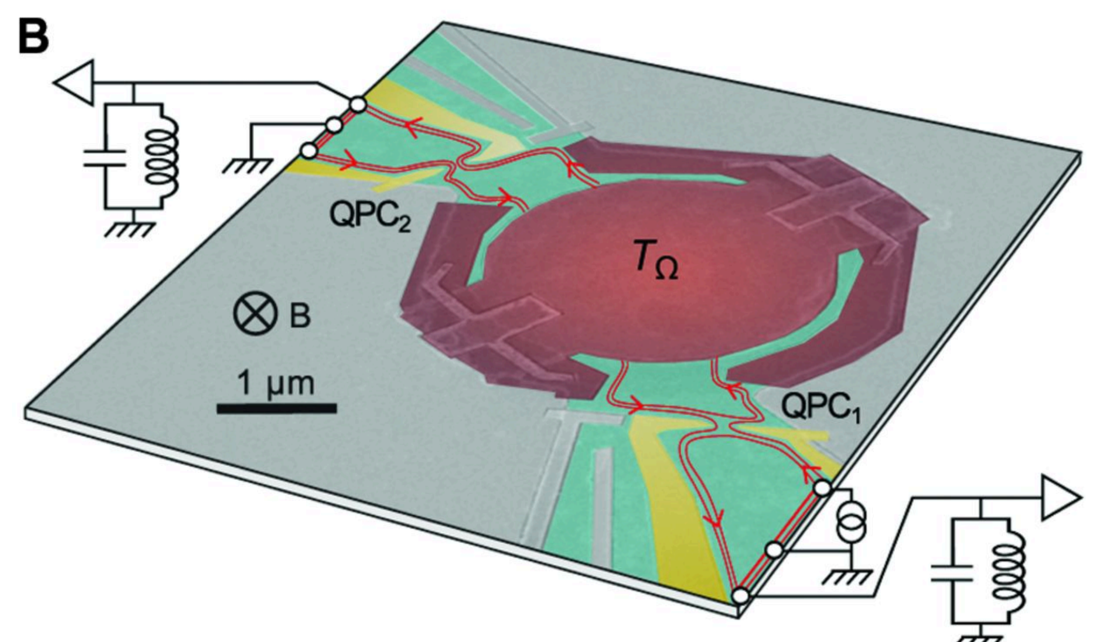
Integer states $c = 1$

S. Jezouin et al.: Science 342, 601 (2013)



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Abelian states: heat flow of anyons $c = 1$

M. Banerjee et al.: Nature 545, 75 (2017)
S. K. Srivastav et al.: Sci. Adv. 5, 7 (2019)
S. K. Srivastav, R. Kumar, C. Spånslätt et al.: Phys. Rev. Lett. 126, 216803 (2021)
R. A. Melcer, B. Dutta, C. Spånslätt et al.: Nat. Commun. 13, 1 (2022)
S. K. Srivastav, R. Kumar, C. Spånslätt et al.: Nat. Commun. 13, 5185 (2022)
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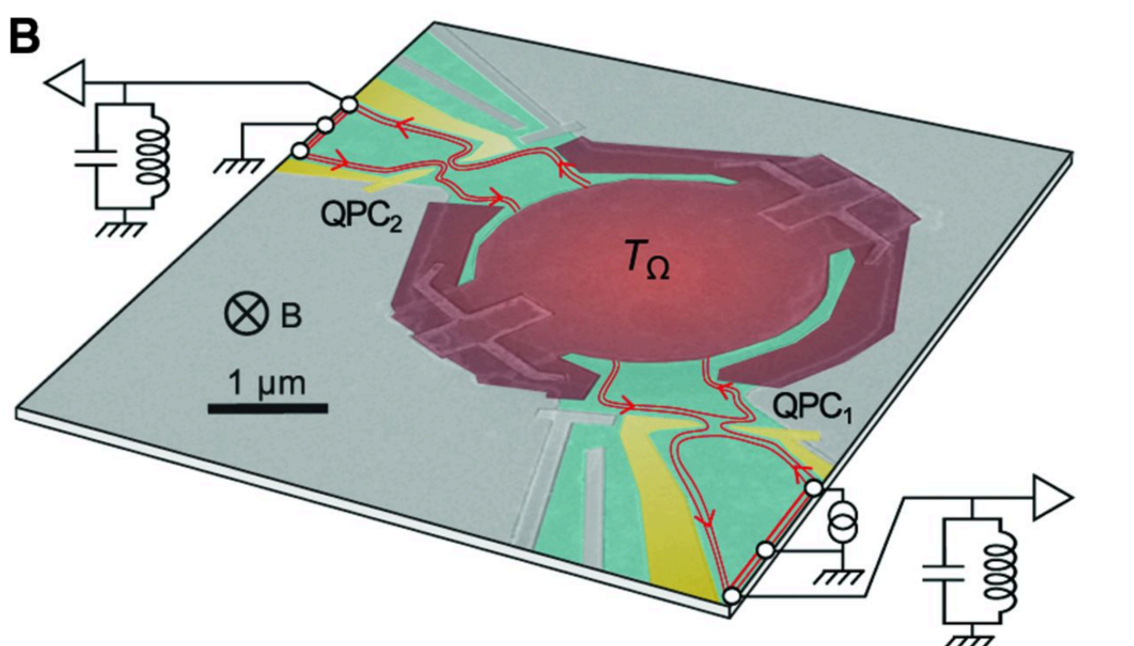

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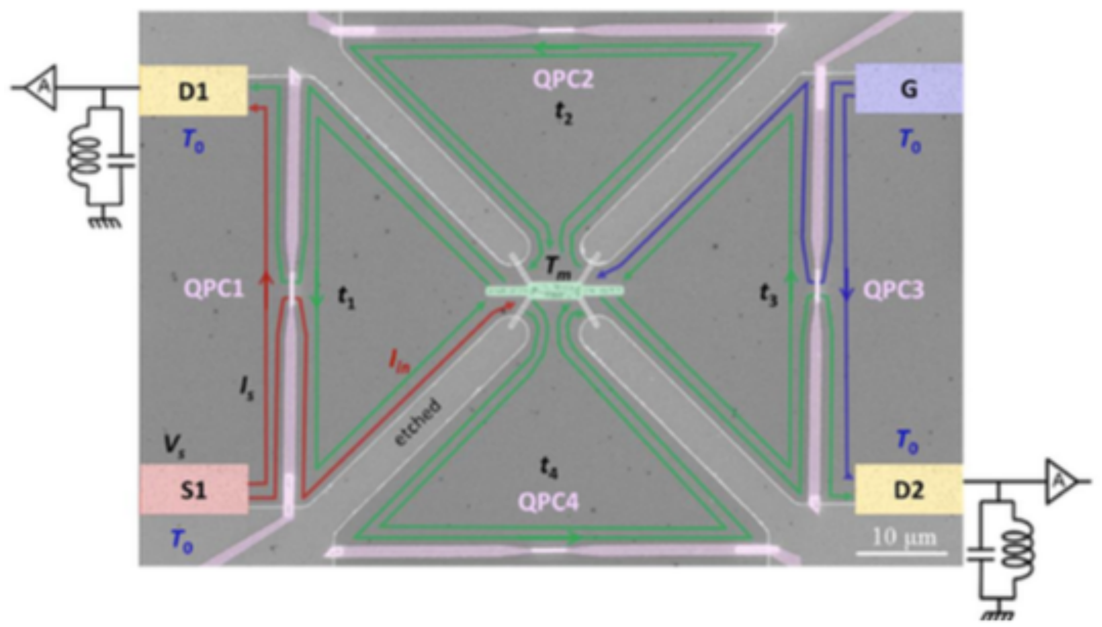

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Non-Abelian states: Majorana modes

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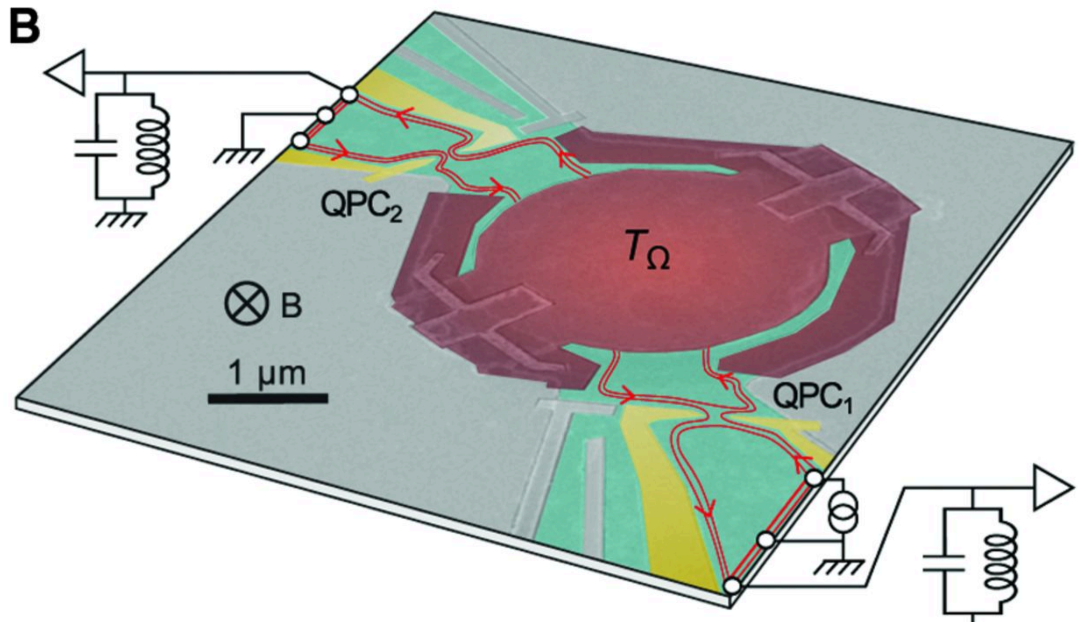
B. Dutta et al.: Science 377, 1198 (2022)
A. K. Paul et al.: Phys. Rev. Lett. 133, 076601 (2024)



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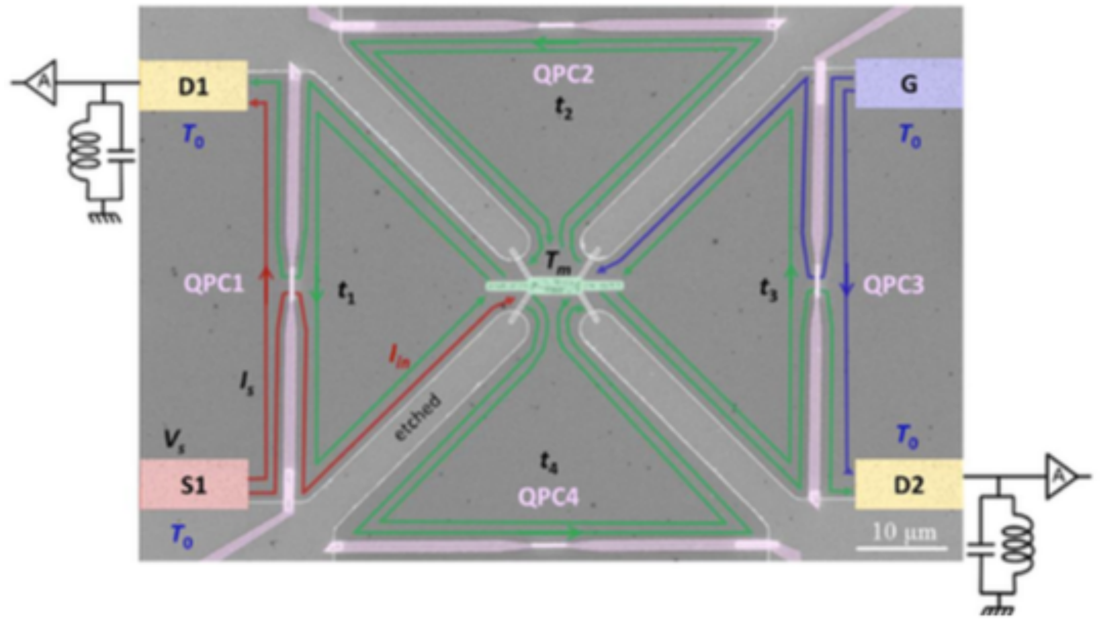

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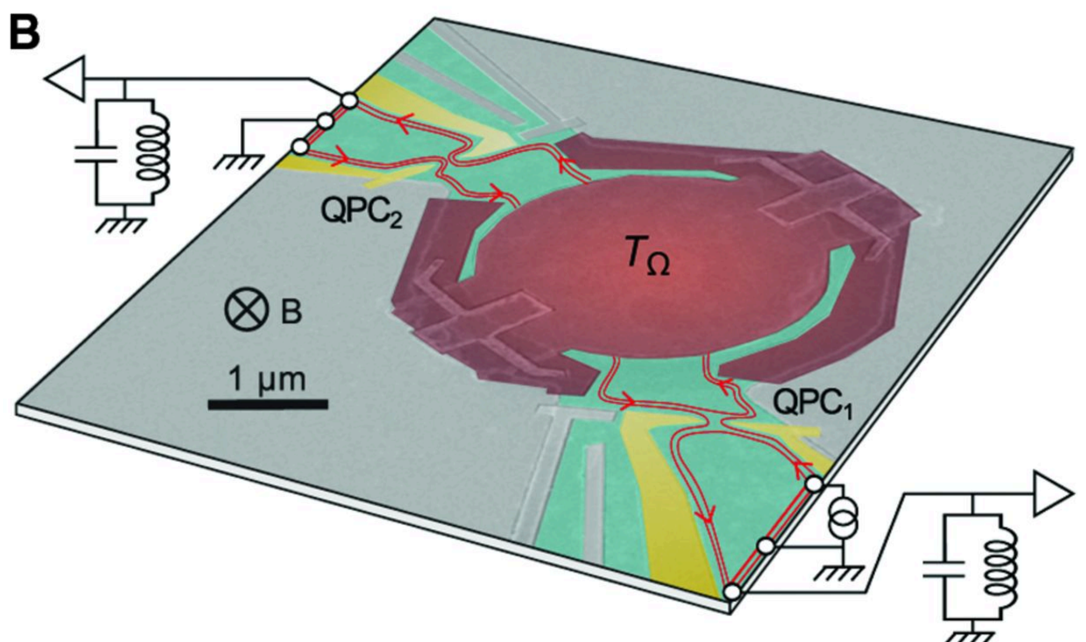
A. K. Paul et al.: Phys. Rev. Lett. 133, 076601 (2024)

Quantifying bulk contribution

R. A. Melcer et al.: Nature 625, 489 (2024)



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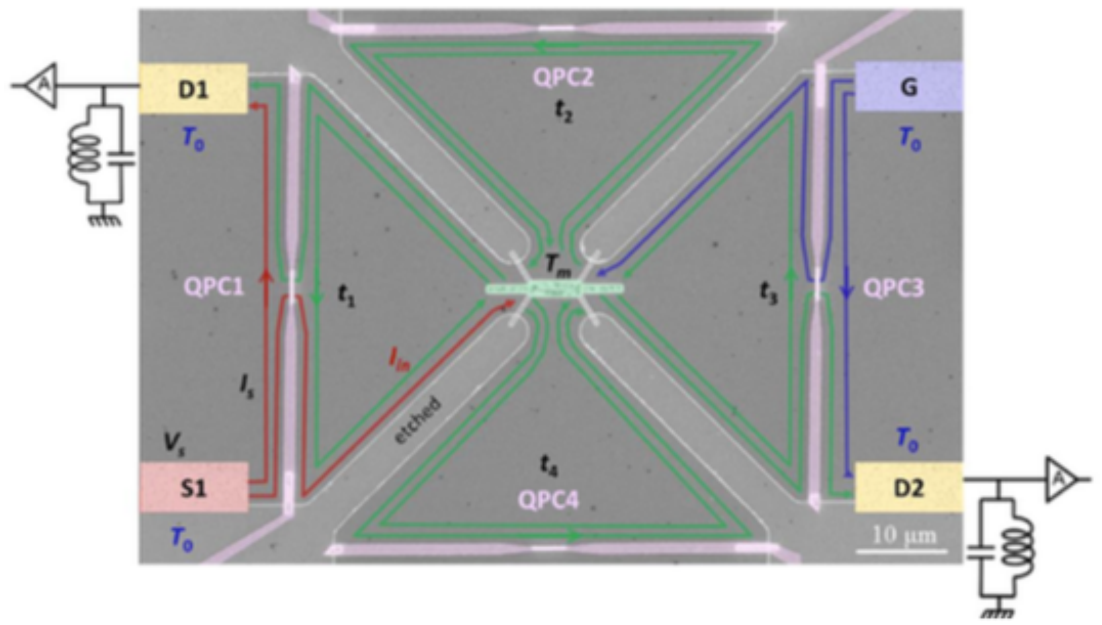

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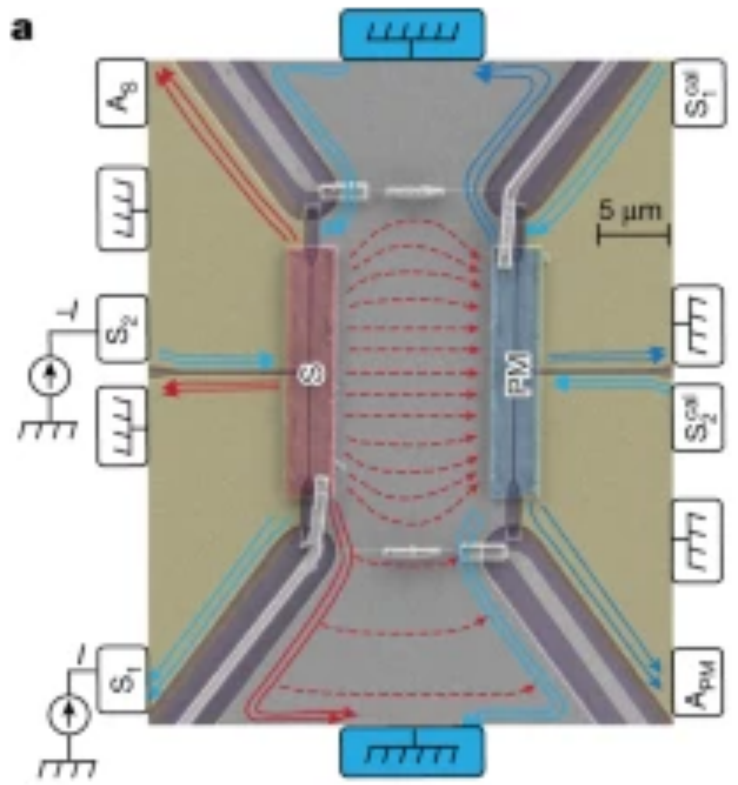
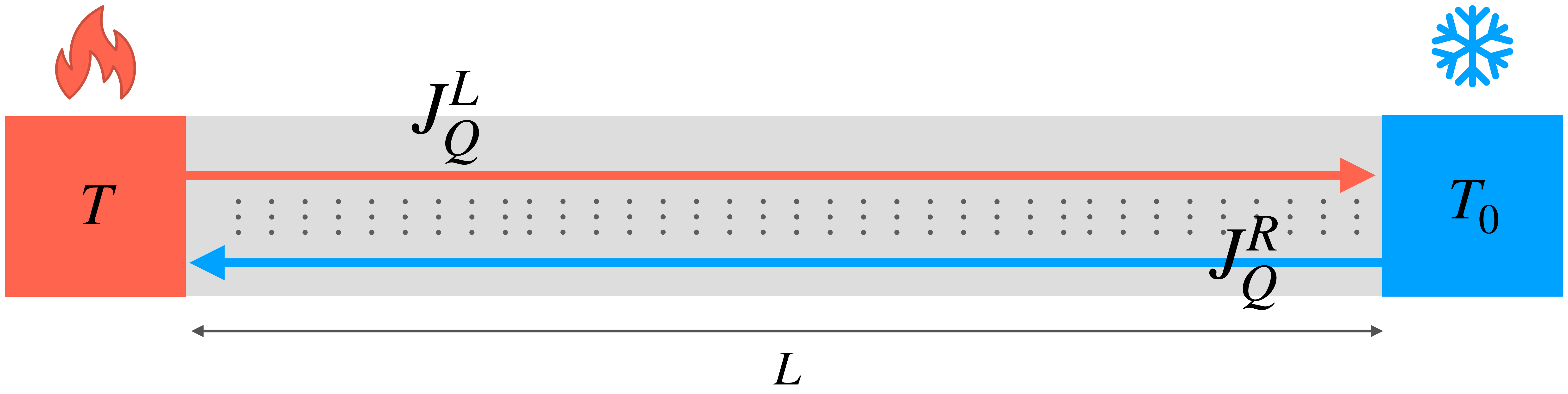


R. A. Melcer et al.: Nature 625, 489 (2024)

Theoretical model of equilibration

C. Nosiglia et al.: Phys. Rev. B 98, 115408 (2018); J. Park et al.: Phys. Rev. B 99, 161302 (2019)
C. Spånslätt et al.: Phys. Rev. Lett. 123, 137701 (2019) ; J. Park, C. Spånslätt et al.: Phys. Rev. Lett. 125, 157702 (2020)
C. Spånslätt et al.: Phys. Rev. B 104, 115416 (2021);M. Hein and C. Spånslätt: Phys. Rev. B 107 245401 (2023)



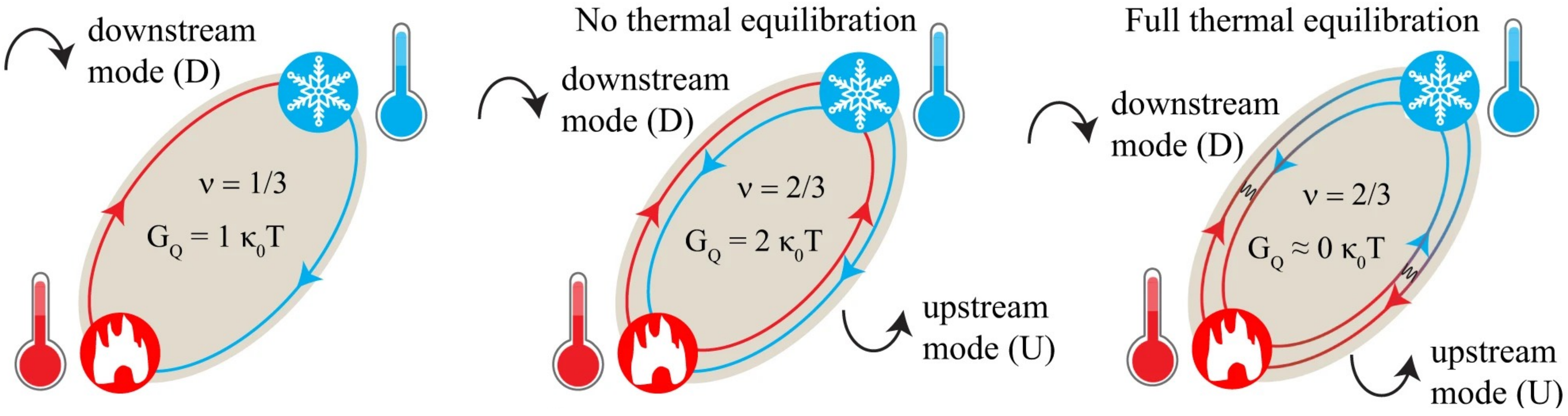
- Generic, incoherent circuit model for charge/heat currents, voltages, noise,…
- Characteristic thermal equilibration length $\ell_Q \sim a/g_Q \sim T^\alpha$, from LL physics
- Generally non-universal due to interactions, disorder

$$n_d + n_u \geq G^Q/(\kappa_0 T) \geq |n_d - n_u| = |\nu_Q|$$

Increasing thermal equilibration

$L \ll \ell_Q$ $\qquad$ $L \gg \ell_Q$

What to expect?



$$G^Q/(\kappa_0 T) = n_d$$

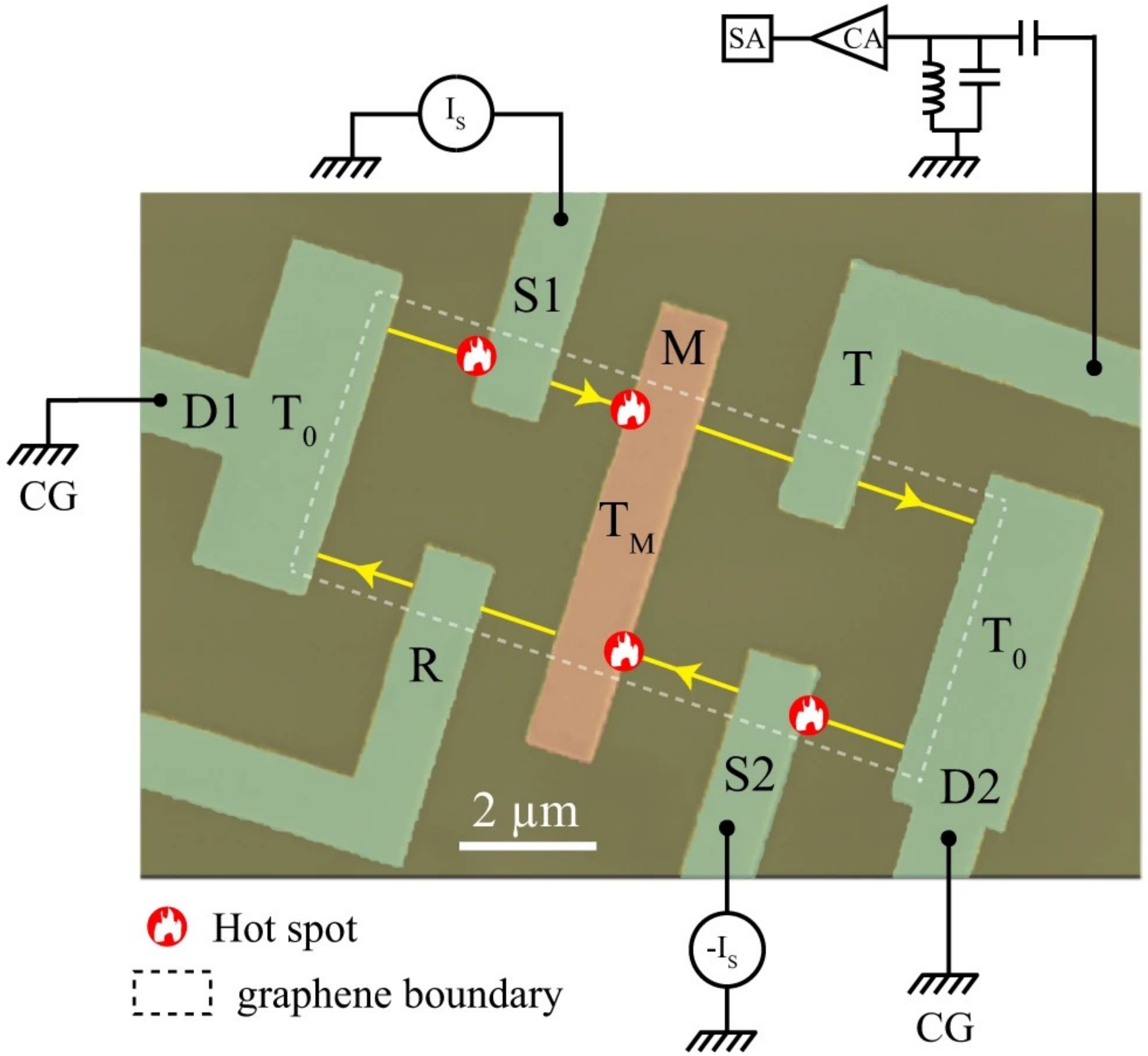
$$n_d + n_u \geq G^Q/(\kappa_0 T) \geq |n_d - n_u|$$

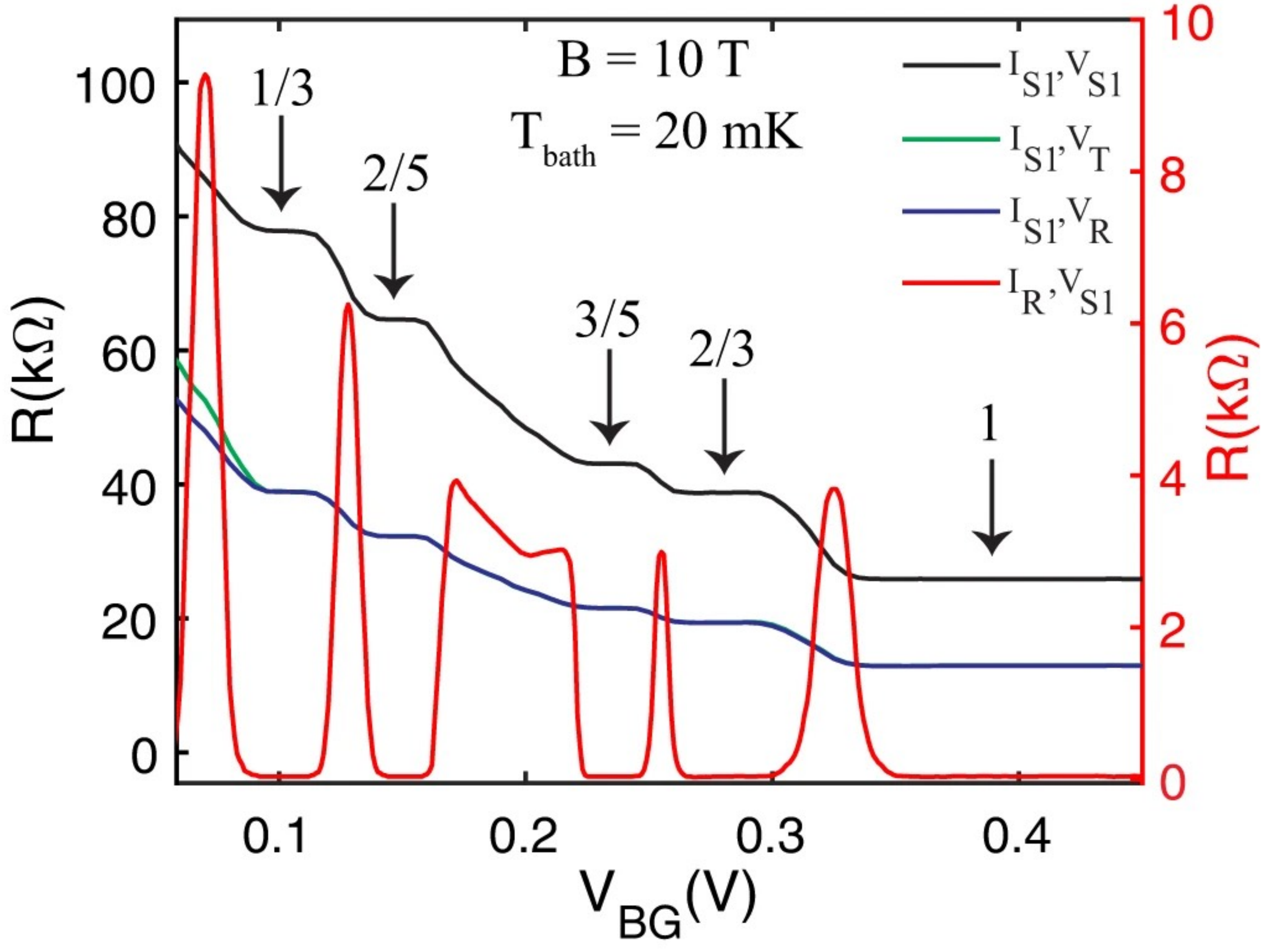
Increasing thermal equilibration

$L \ll \ell_Q$ $\qquad$ $L \gg \ell_Q$

Experiment in graphene

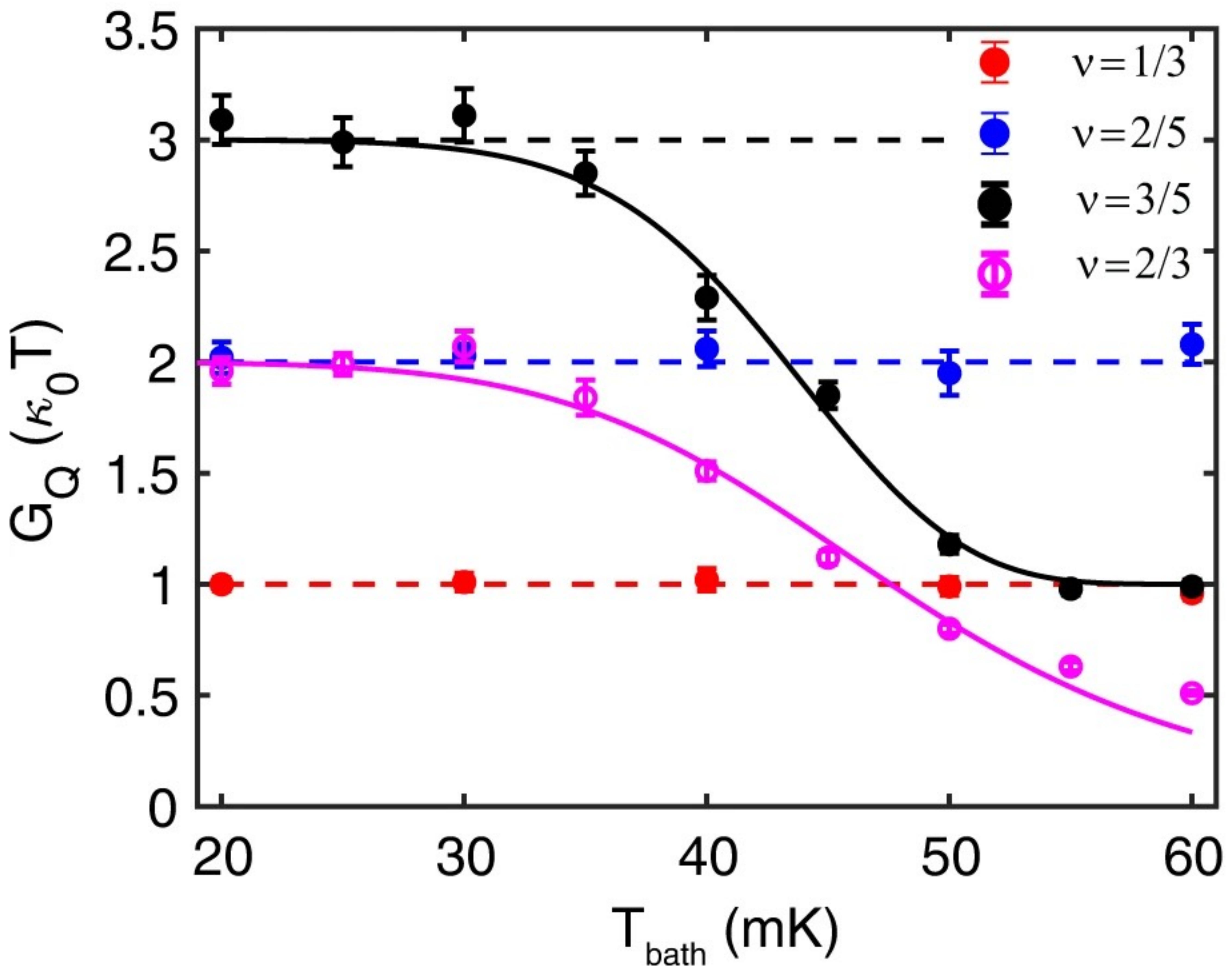
R. Kumar, S. K. Srivastav, C. Spånslätt et al.: Nat. Commun. 13, 1 (2022)

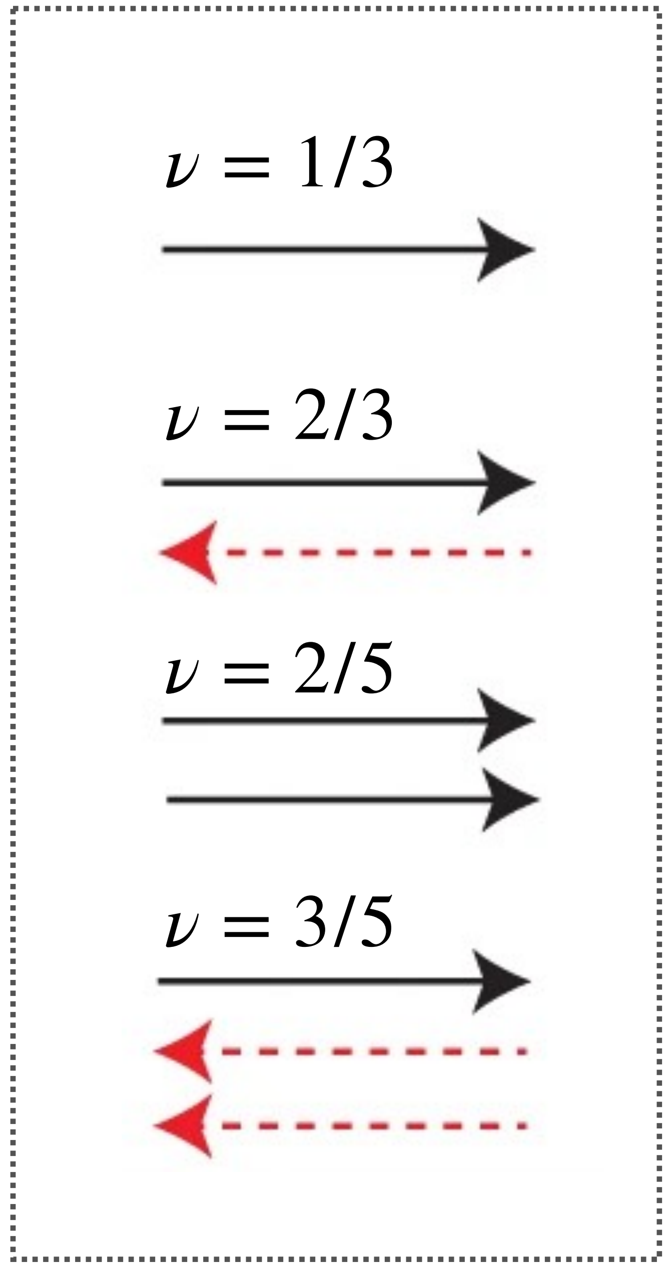




Full temperature crossover

R. Kumar, S. K. Srivastav, C. Spånslätt et al.: Nat. Commun. 13, 1 (2022)





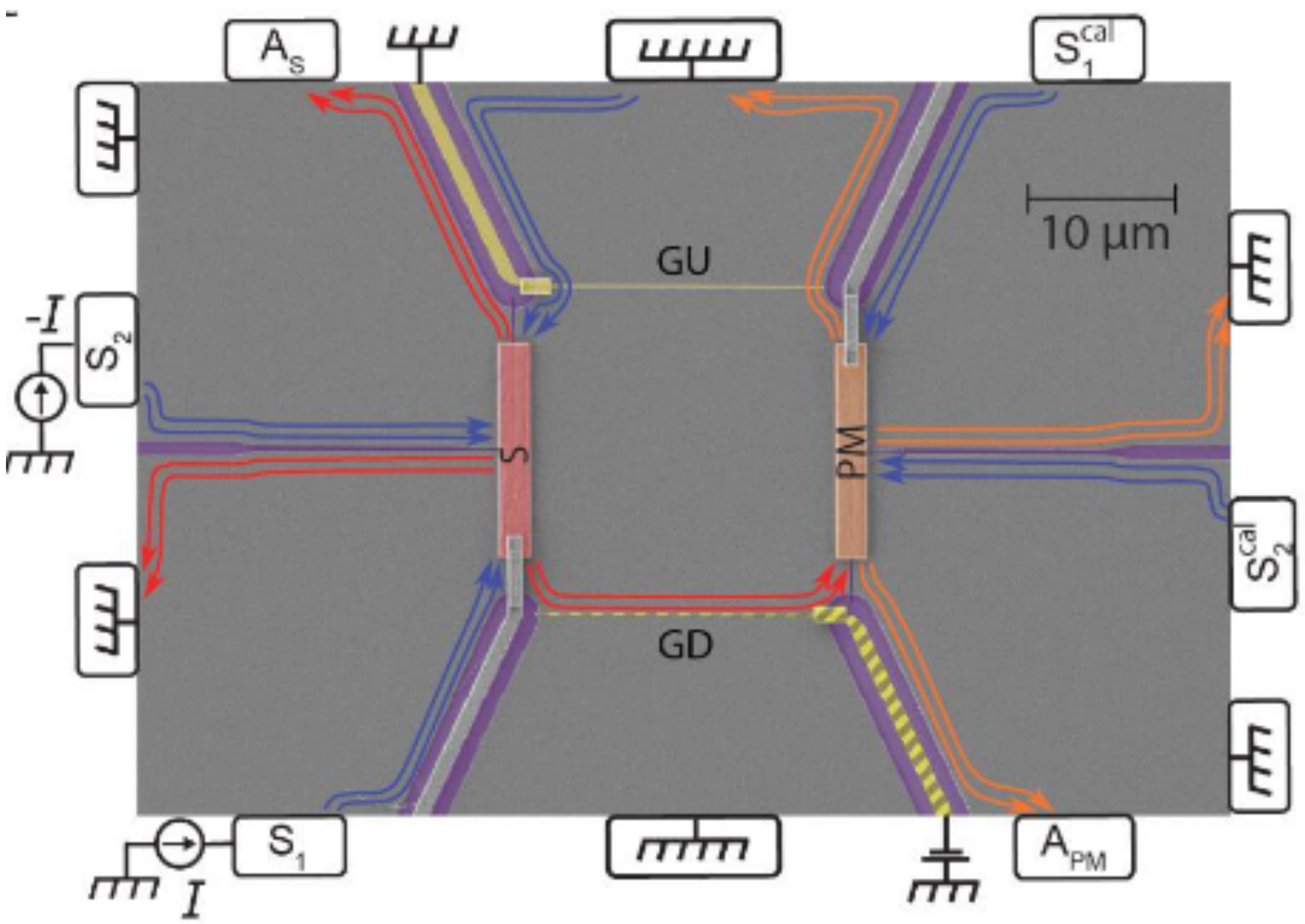
What if the full crossover cannot be accessed?

Four-point measurement

R. A. Melcer et al.: Nat. Phys. 19, 327 (2023)
R. A. Melcer et al.: Nature 625, 489 (2024)
A. K. Paul et al.: Phys. Rev. Lett. 133, 076601 (2024)

- Us and ds conductances were separated at $\nu = 2/3$: equilibration effects cancel
  $G_H^Q = G_{\text{ds}}^Q - G_{\text{us}}^Q \approx 0$

- Generalization to $\nu = 5/2$ could unambiguously determine the topological order as
  $G_H^Q = (c - \bar{c})\kappa_0 T$



Equilibration drops out!

R. A. Melcer et al.: Nat. Phys. 19, 327 (2023)
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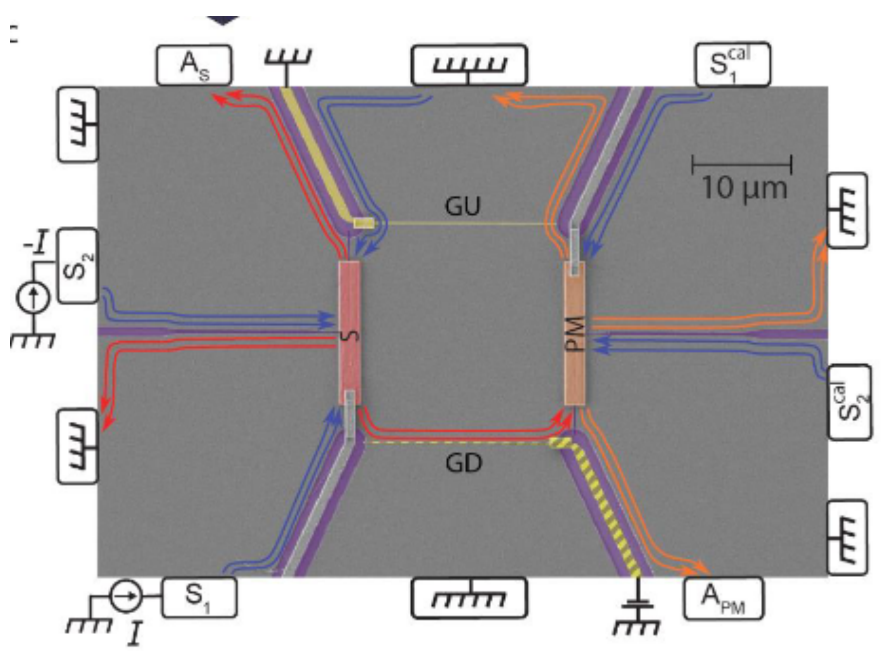

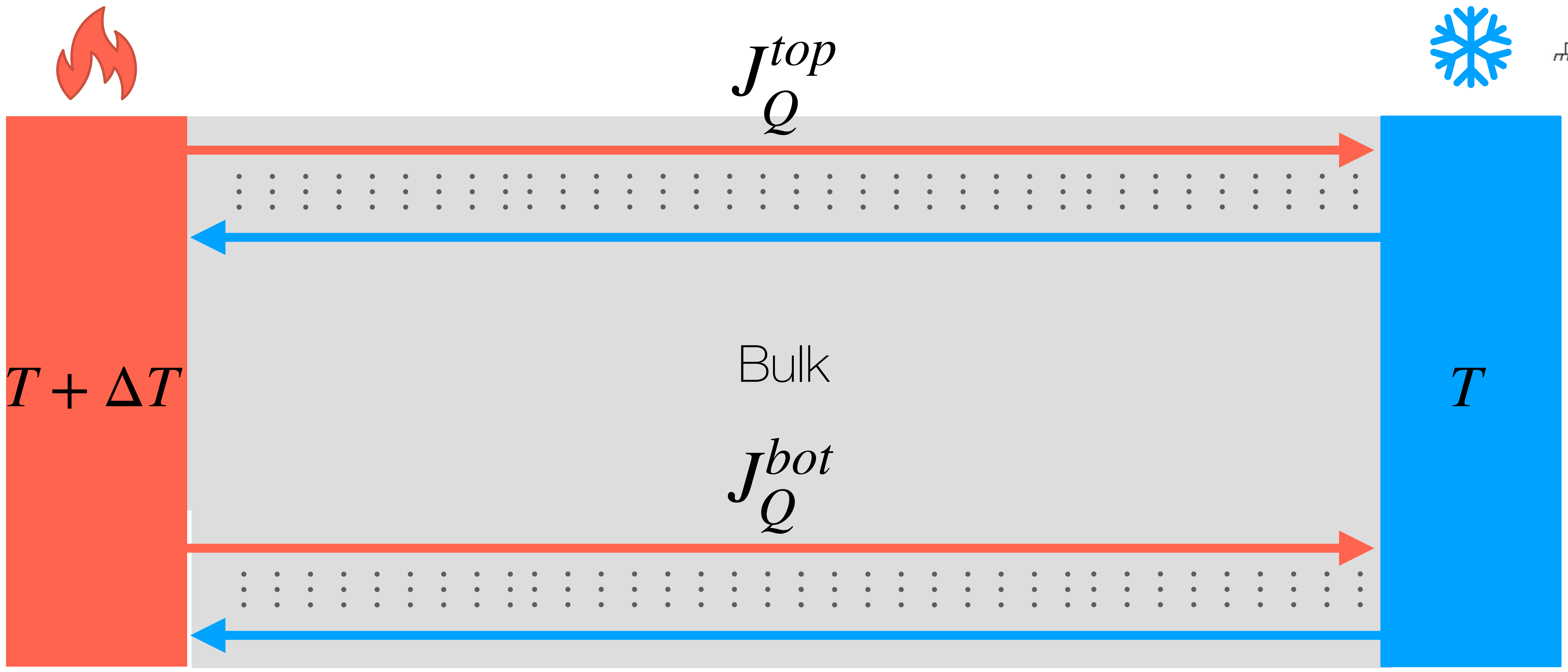

$$G_{\mathrm{H}}^{Q} \equiv \lim_{\Delta T \to 0} \left(\frac{d(J_Q^{\mathrm{top}} - J_Q^{\mathrm{bot}})}{d\Delta T} \right) = \kappa_0 T c + \text{equilibration} - \bar{c}\kappa_0 T - \text{equilibration} = (c - \bar{c})\kappa_0 T$$

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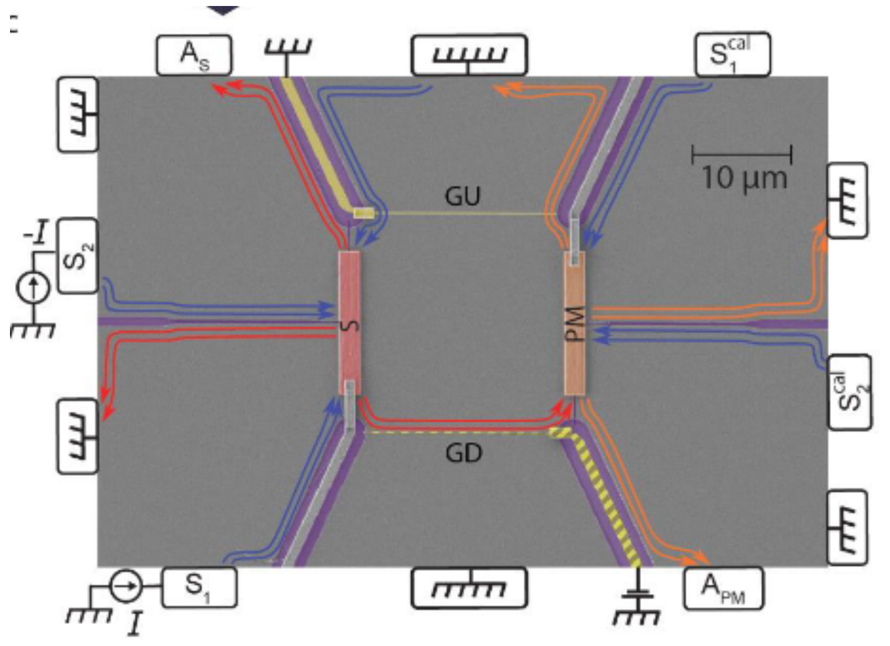

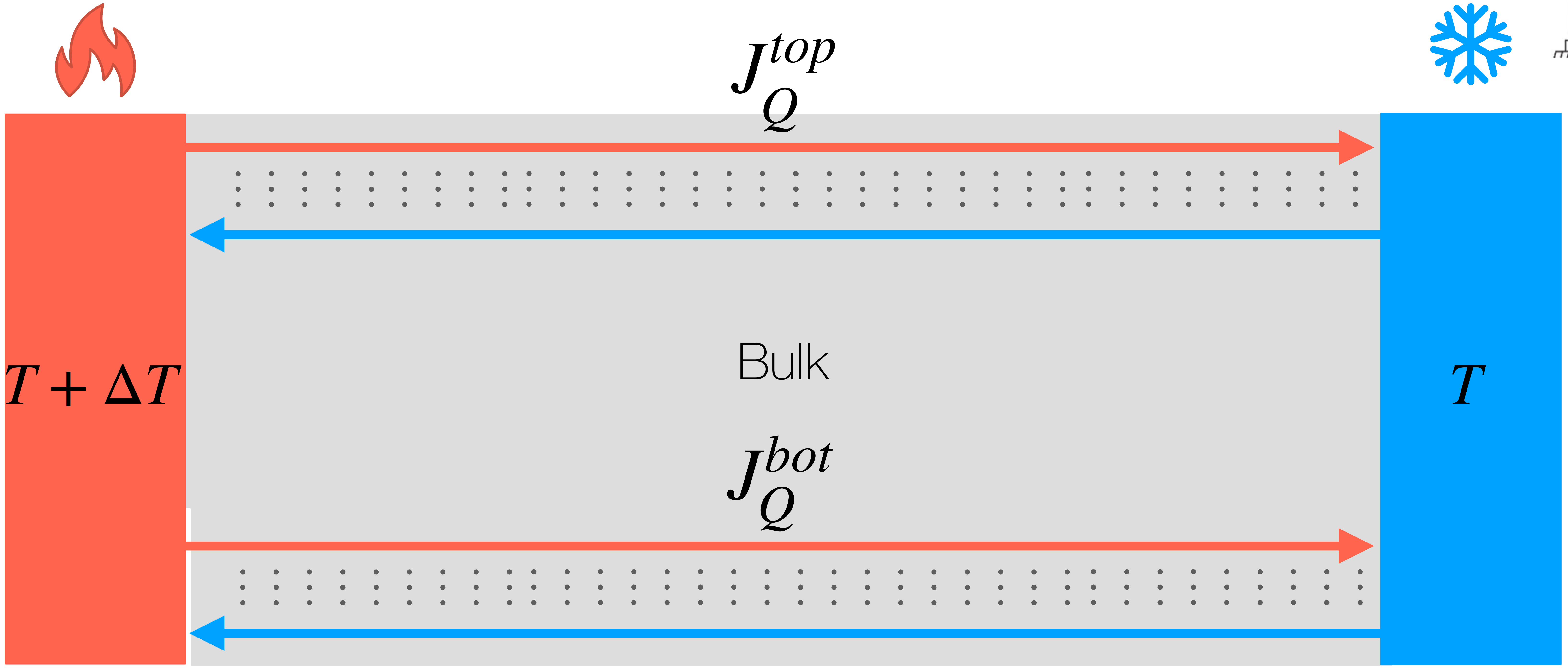

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Distinguishes non-Abelian candidates!

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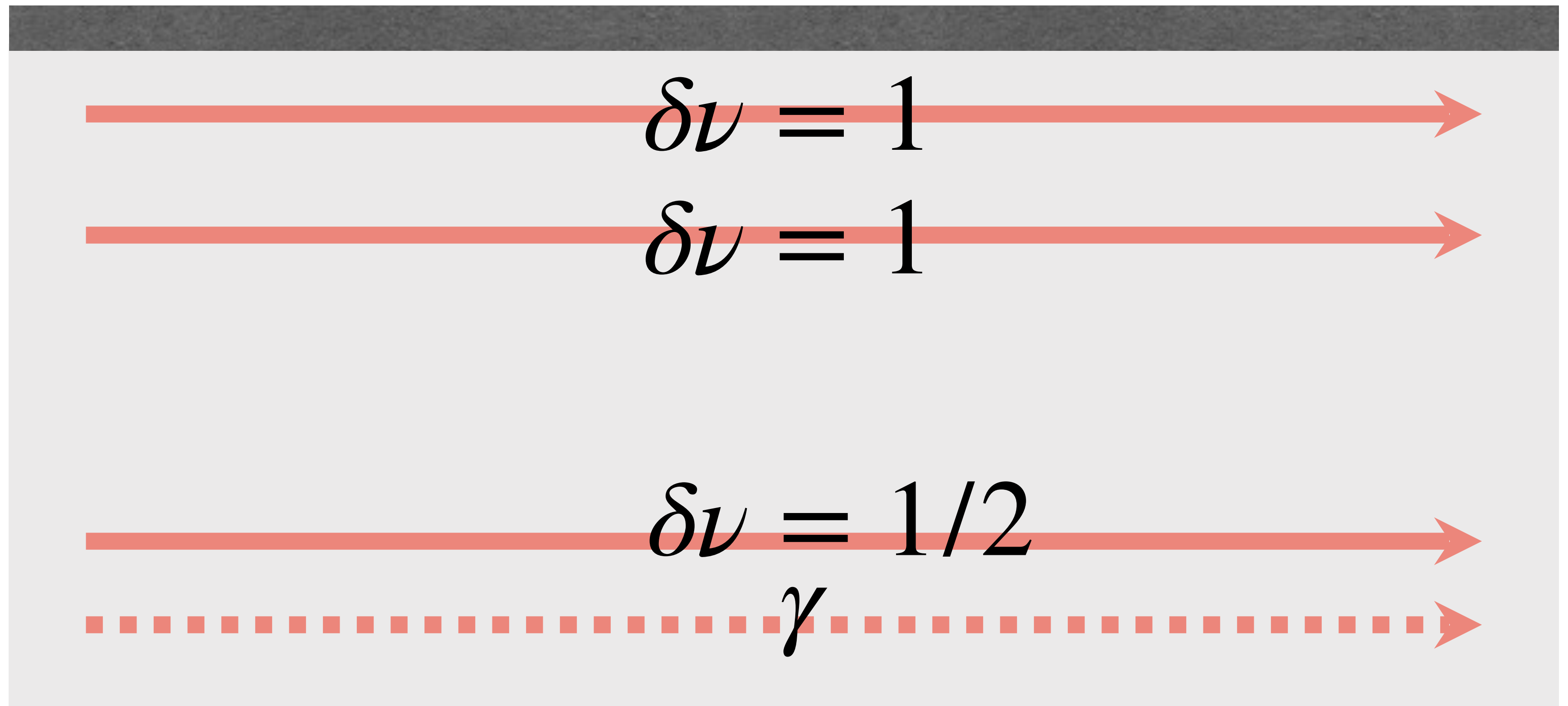
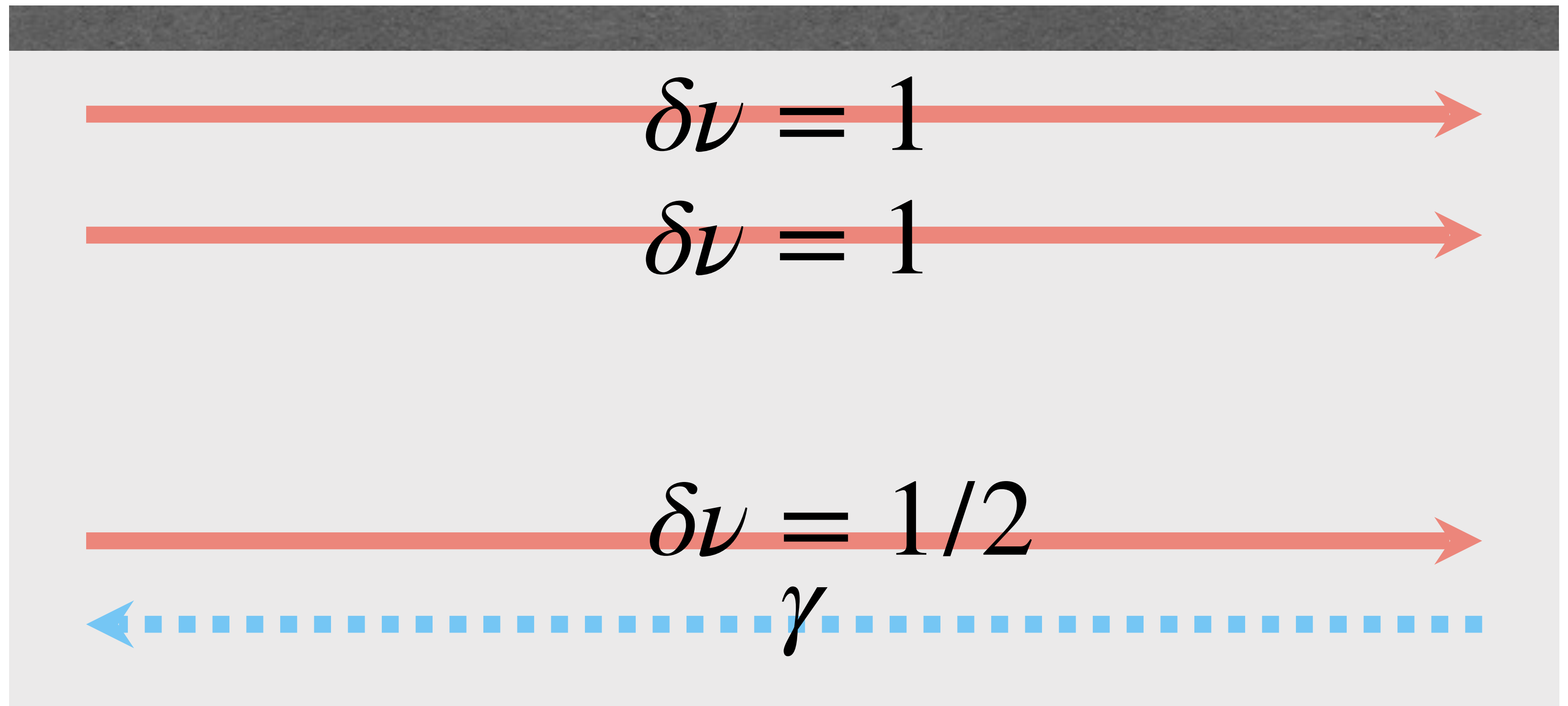
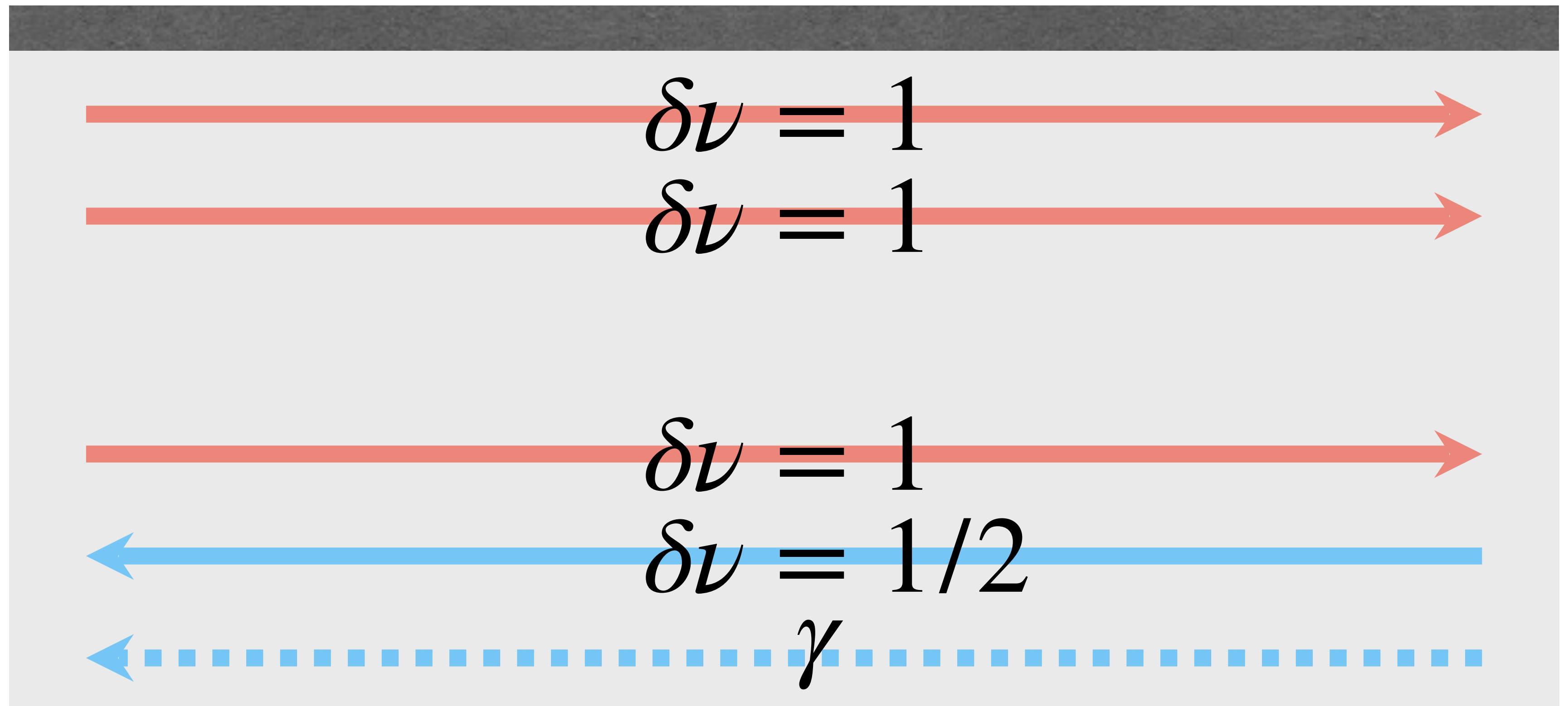
Pfaffian $G_{\mathrm{H}}^{Q}/(\kappa_0 T) = 7/2$



Particle-hole Pfaffian $G_{\mathrm{H}}^{Q}/(\kappa_0 T) = 5/2$



Anti-Pfaffian $G_{\mathrm{H}}^{Q}/(\kappa_0 T) = 3/2$



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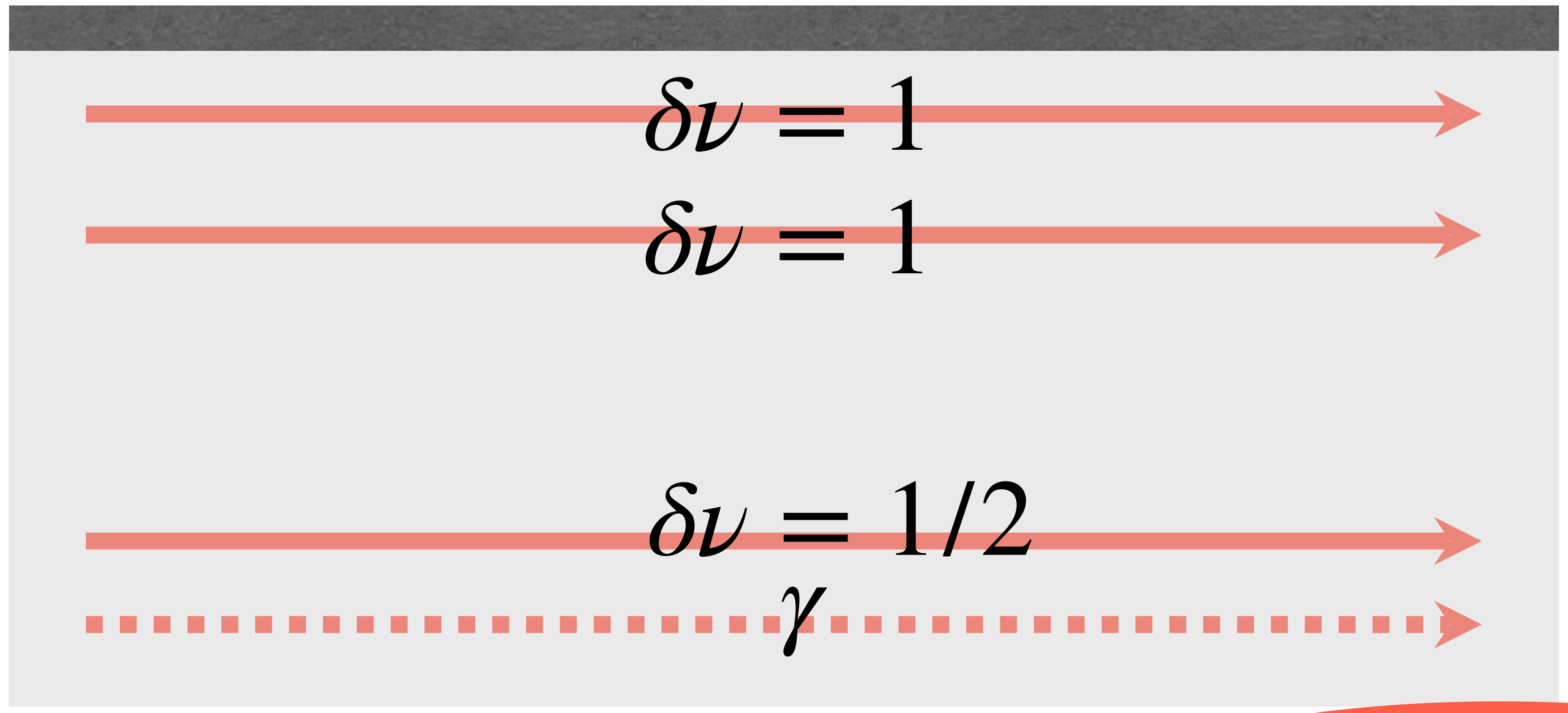

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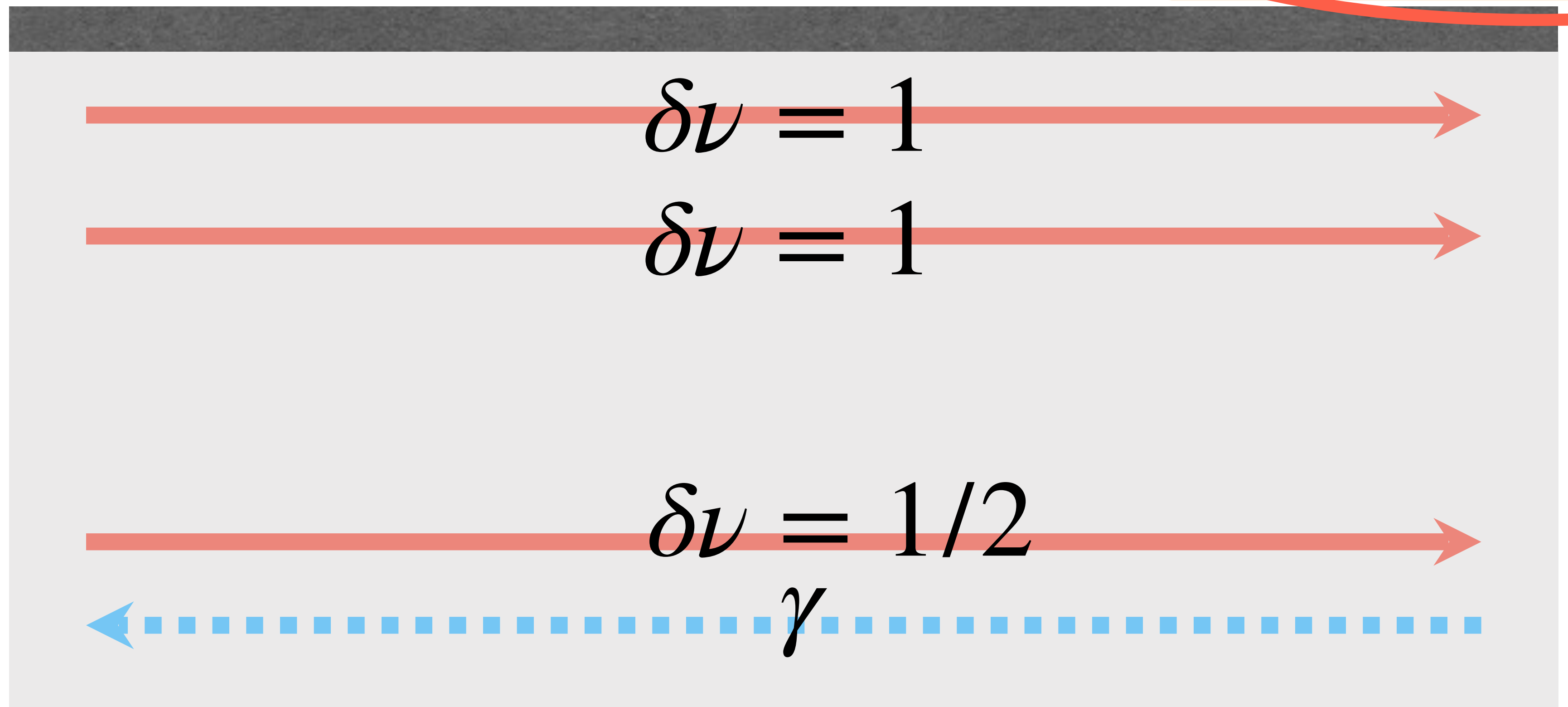

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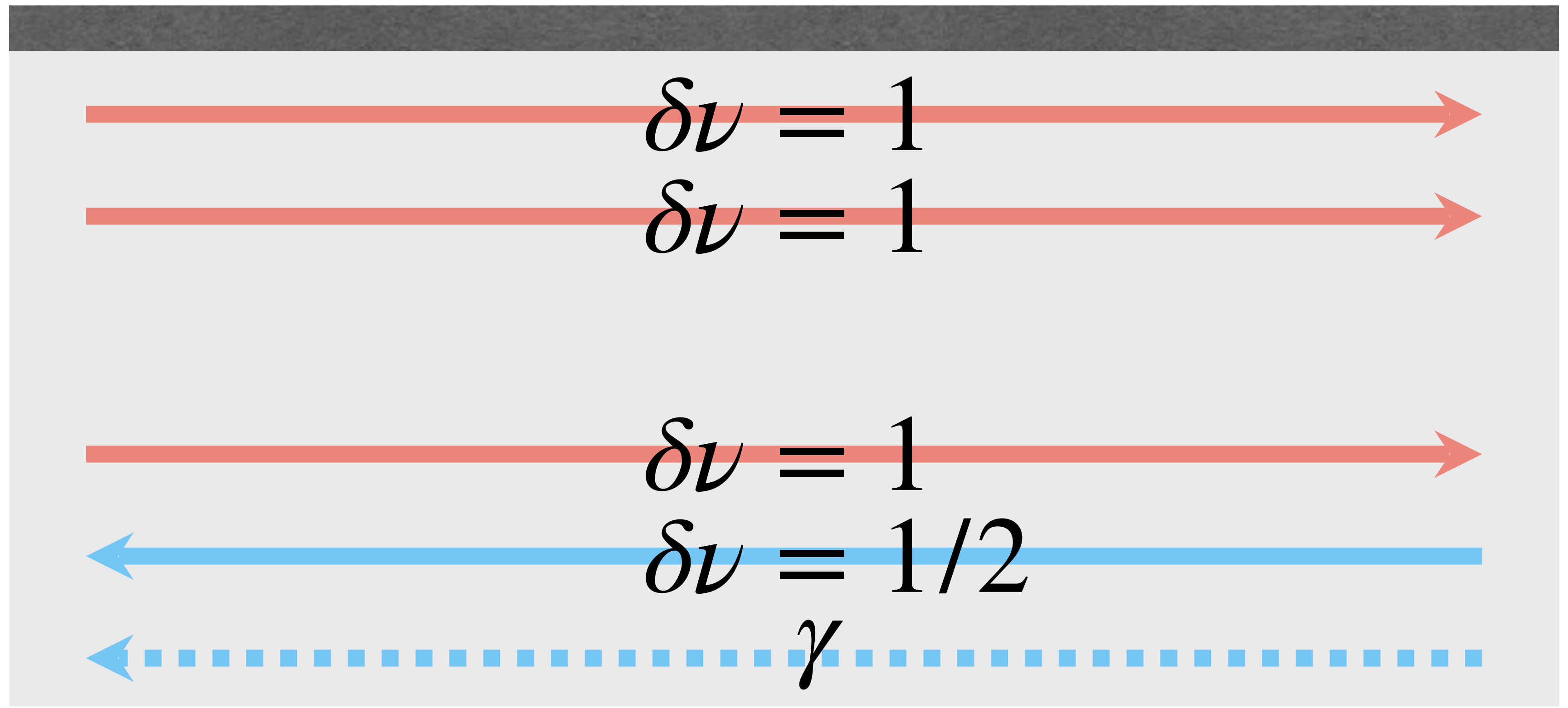

Puzzle: Not consistent with numerical work!

Disorder, Landau level mixing?

Main messages and outlook

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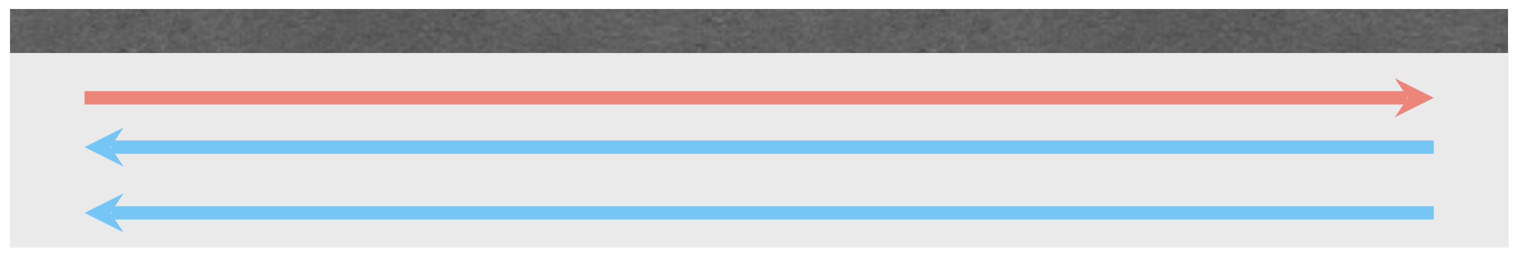
- The FQH edge is a fantastic platform for *strongly correlated and topological transport*
- Thermal conductance pin-points generic edge structures



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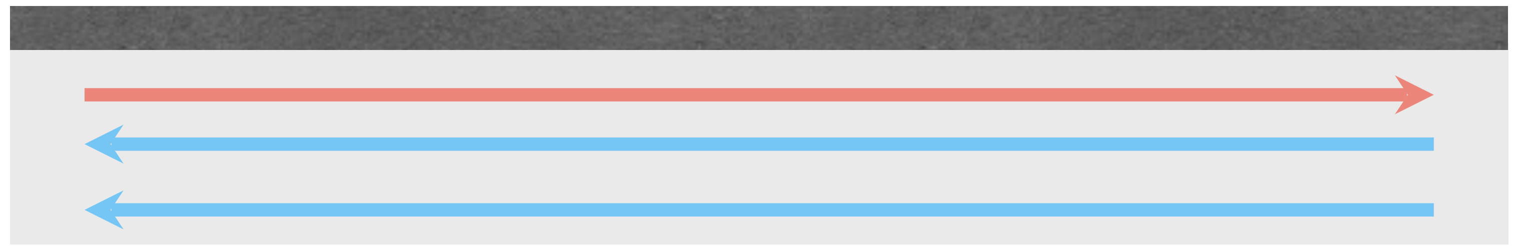

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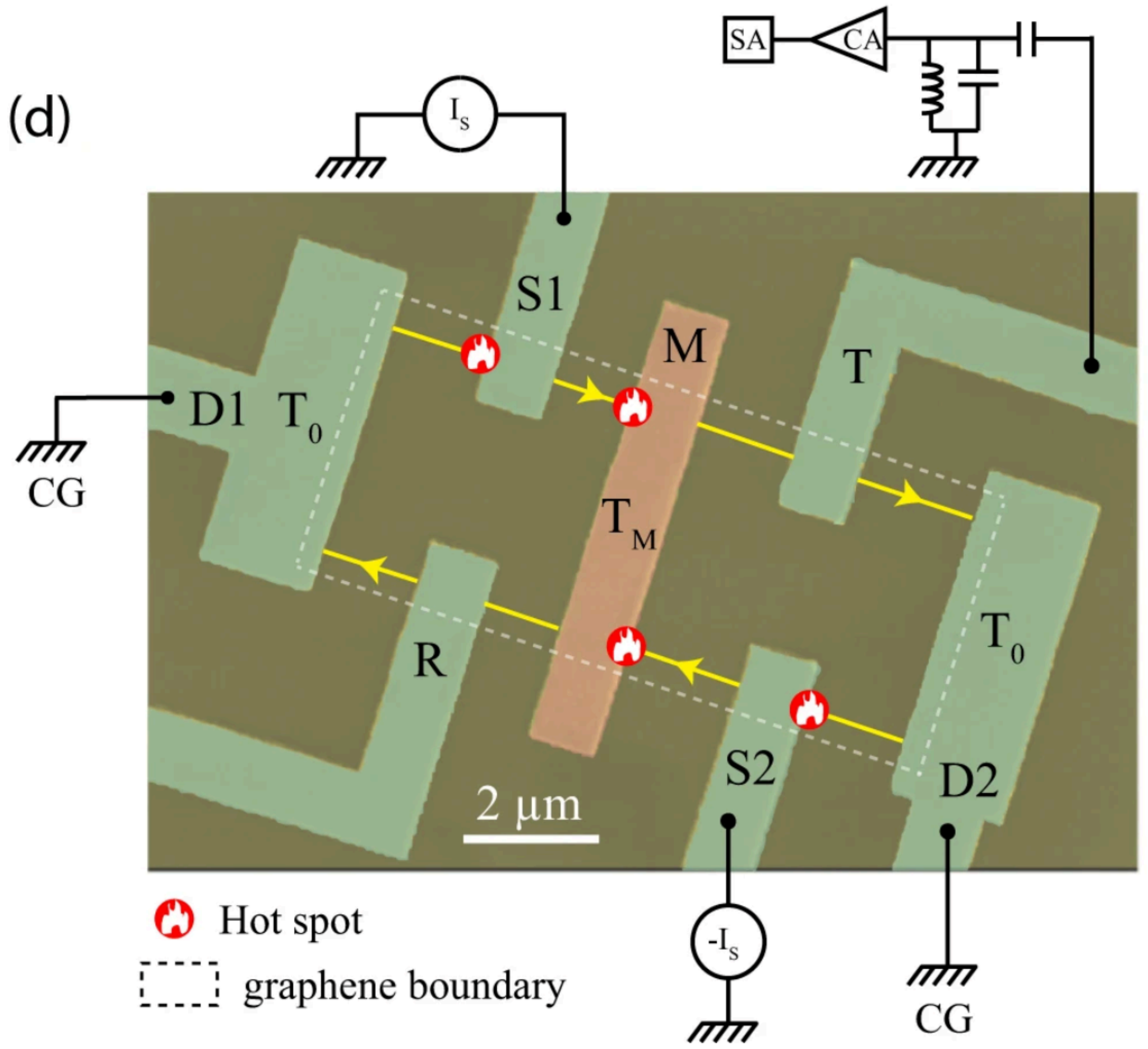

S. K. Srivastav et al.: Nat. Commun. 13, 1 (2022)

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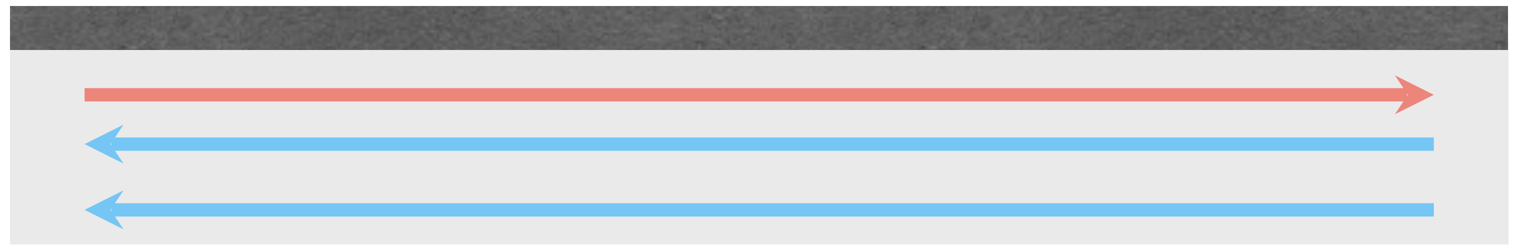

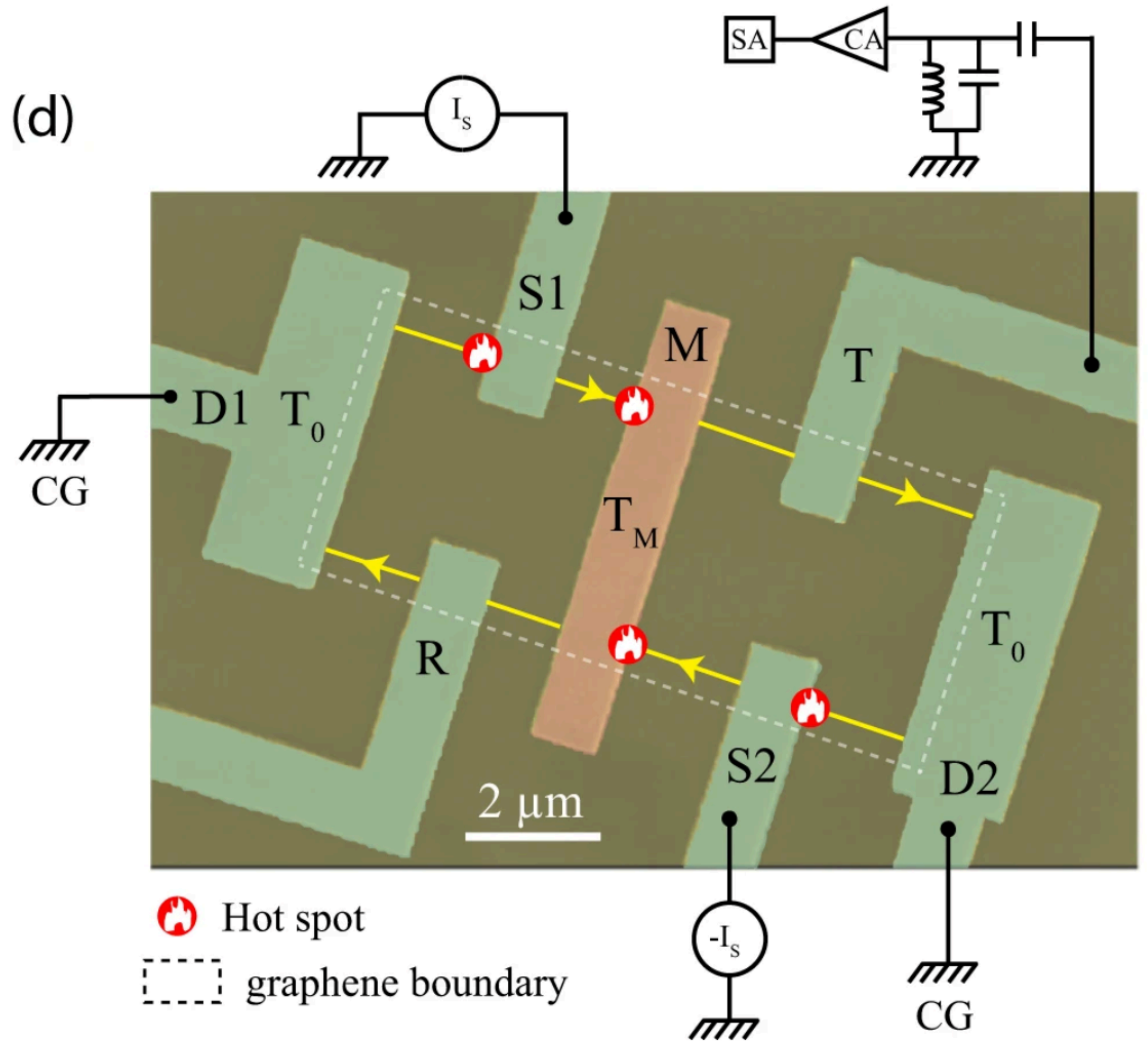

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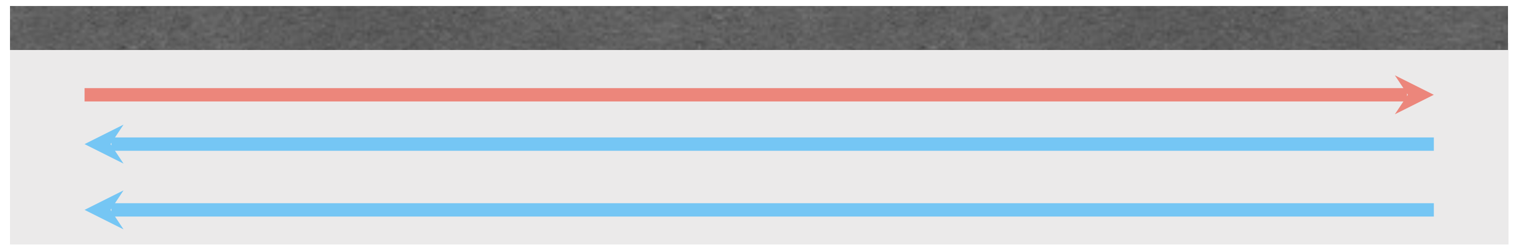

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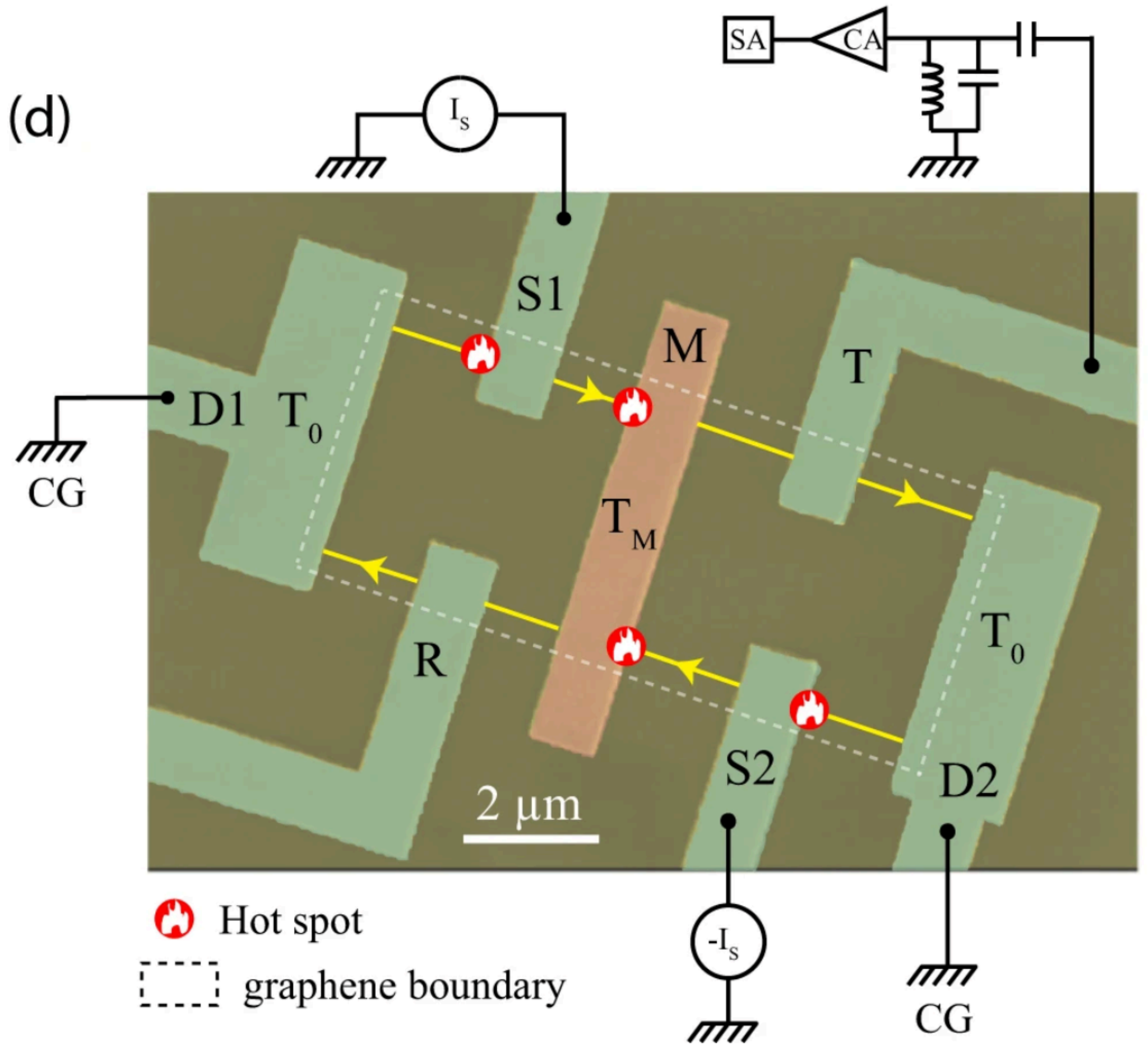

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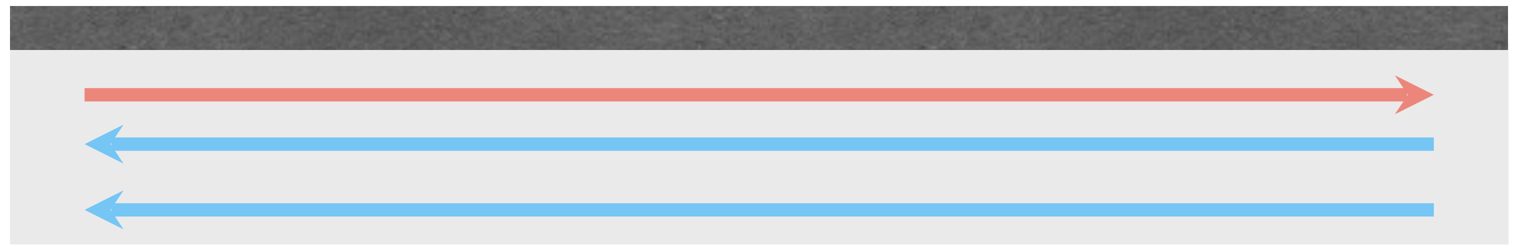

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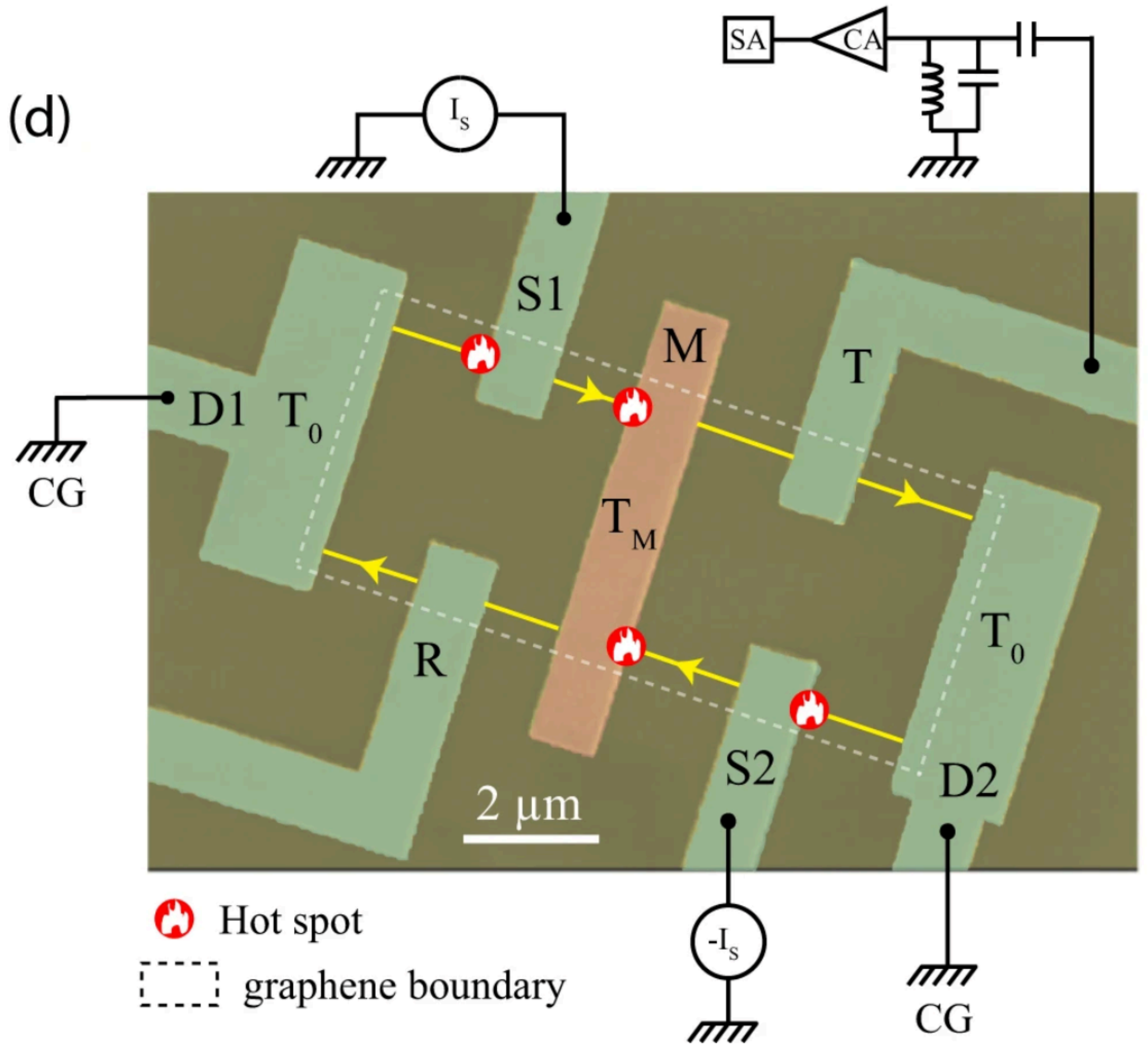

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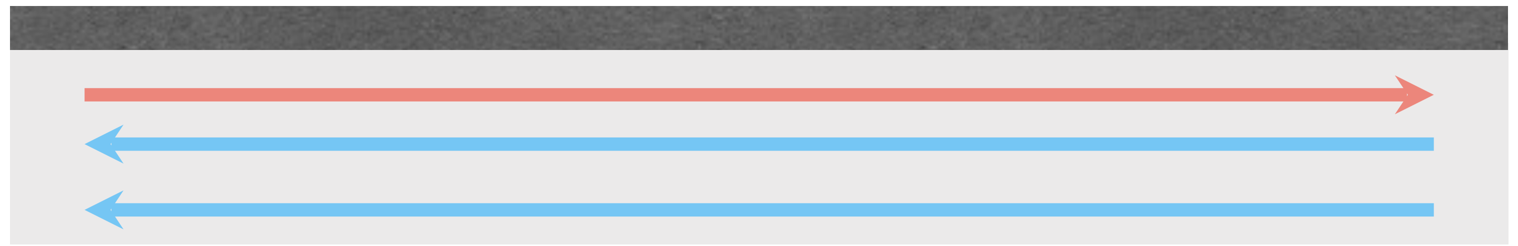
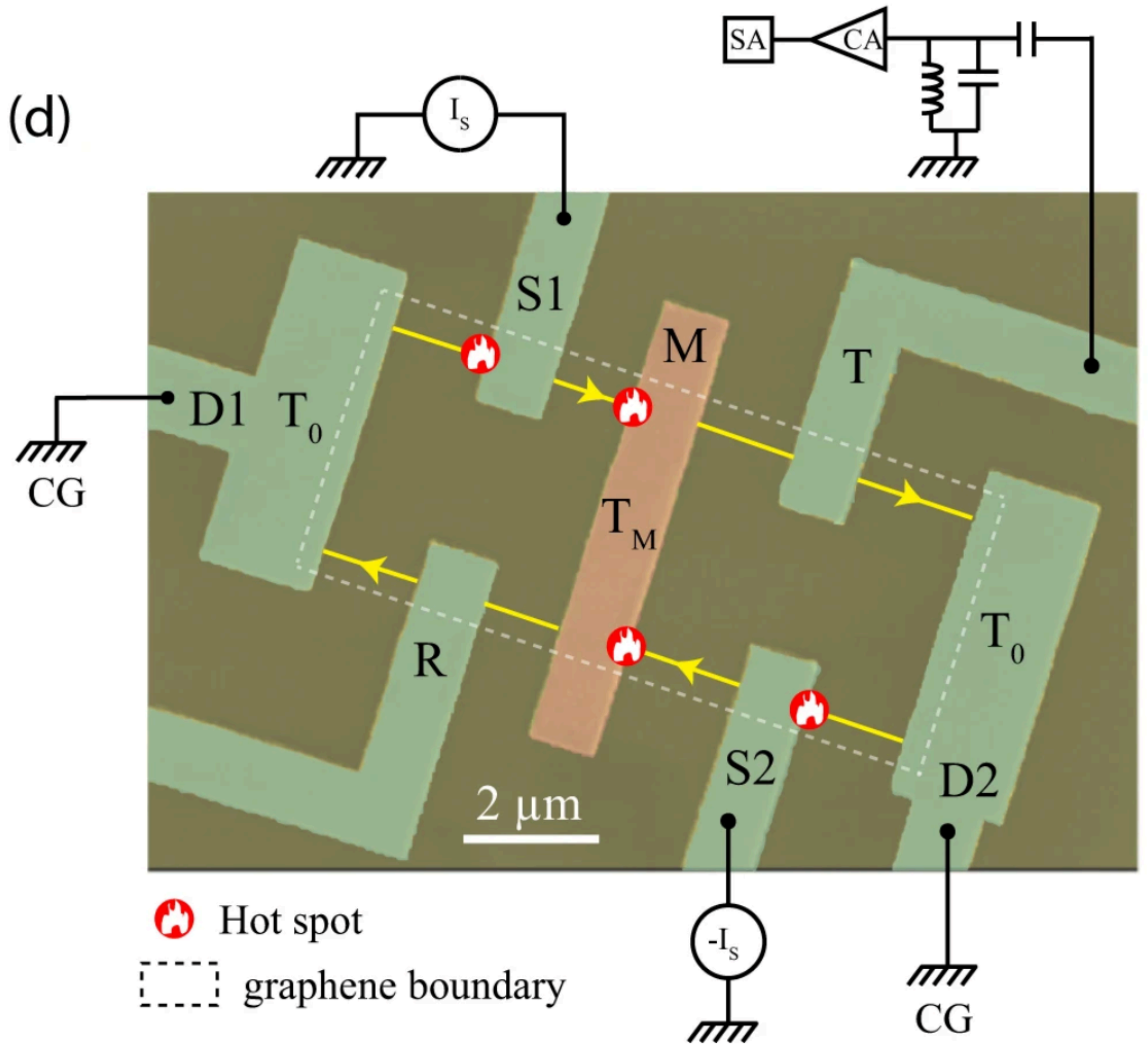
- Known edge structure: we can start classifying anyons
- Applications to fractional Chern insulators?





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Thank you!

Theory: Yuval Gefen, Alexander Mirlin, Jinhong Park, Michael Hein
Experiment: Saurabh Srivastav, Ravi Kumar, Anindya Das
Ron Melcer, Bivas Dutta,
Materials: Kenji Watanabe, Takashi Taniguchi, Vladimir Umansky

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MARIE CURIE ACTIONS

CHALMERS UNIVERSITY OF TECHNOLOGY | Area of Advance NANO

2D-TECH

KARLSTADS UNIVERSITET

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 101031655 (TEAPOT).

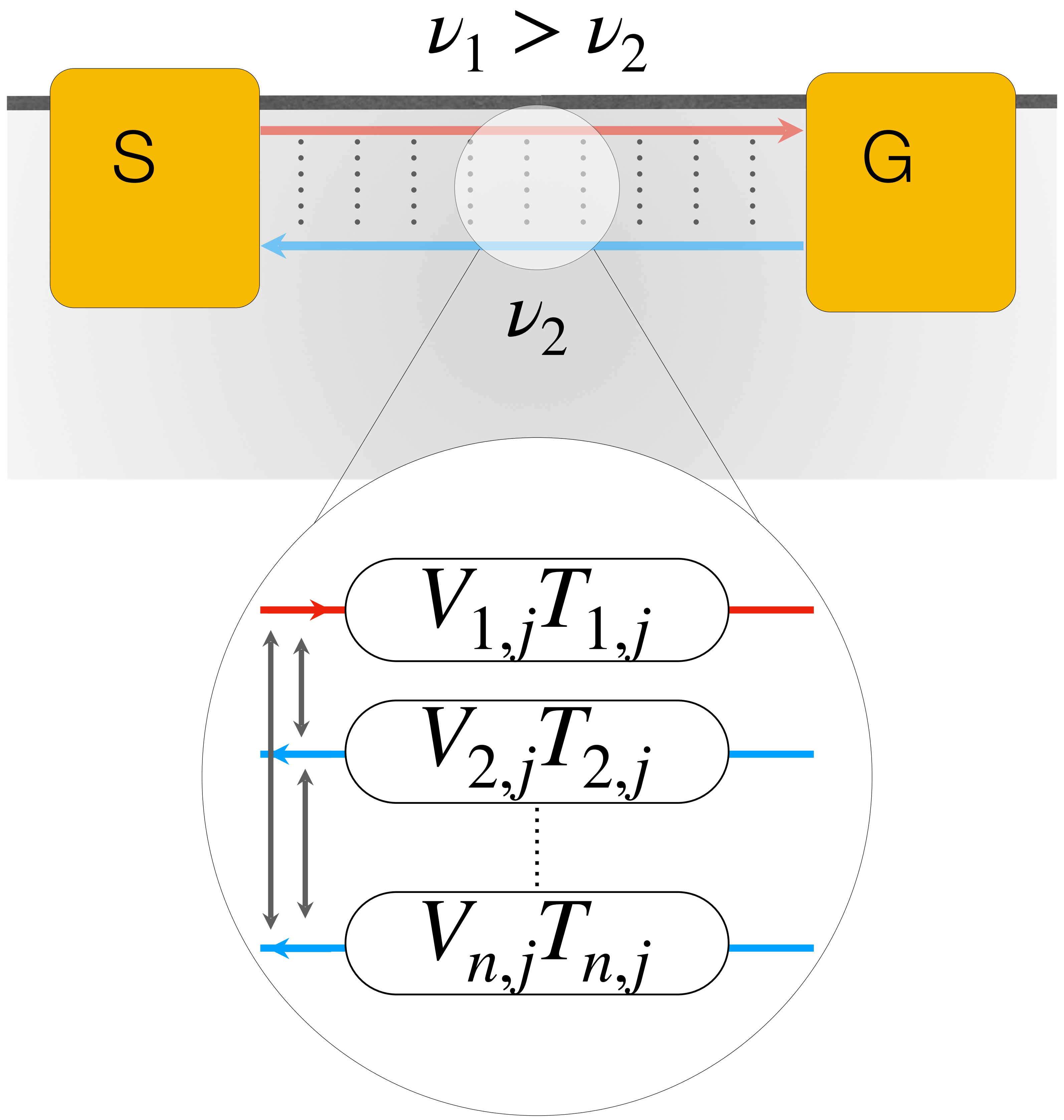
Bonus Material

Details of edge model

$$I_{n,j+1} = I_{n,j} - \sum_{m=1}^{N} I^{\tau}_{n,m,j},$$
$$J_{n,j+1} = J_{n,j} - \sum_{m=1}^{N} J^{\tau}_{n,m,j}.$$

$$I^{\tau}_{n,m,j} = g_{n,m}\frac{e^2}{h}\left(V_{n,j} - V_{m,j}\right)$$

$$J^{\tau}_{n,m,j} = g_{n,m}\frac{e^2}{2h}\left(V^2_{n,j} - V^2_{m,j}\right) + \gamma_{n,m}g_{n,m}\frac{k_B^2\pi^2}{6h}\left(T^2_{n,j} - T^2_{m,j}\right)$$



- Circuit type of model: charge currents, voltages, noise,…
- Characteristic thermal equilibration length $\ell_Q \sim a/g_Q \sim T^{\alpha}$
- Non universal: interactions, disorder strength, temperature

Four-point thermal conductance: proof

Appendix C: Proof of the universality of $G_{\rm H}^{Q}$ for equal thermal equilibration on top and bottom edges

Here, we prove Eq. (27), namely that the thermal Hall conductance (26) is universal as long as the two edges have the same degree of equilibration (even if it is poor).

We consider a generic edge structure with k downstream (ds) and $N-k$ upstream (us) modes, and write the heat transport equation (6) (with $\delta\vec{V}=0$, since we assume no voltage bias) as

$$\partial_x\vec{\theta}(x) = \mathcal{M}_T\vec{\theta}(x). \tag{C1}$$

Here, we defined $\vec{\theta}(x)=\vec{T}^2(x)$, and the matrix $\mathcal{M}_T$ (7) satisfies the heat current conservation law

$$\sum_j(\mathcal{M}_T)_{ij}=0, \quad \forall i. \tag{C2}$$

The general solution of Eq. (C1) for an edge segment with length $L>0$ can be written as

$$\vec{\theta}(L)=e^{L\mathcal{M}_T}\vec{\theta}(0), \tag{C3}$$

which we express in ds and us blockform as

$$\begin{pmatrix}\vec{\theta}_{\rm ds}(L)\\ \vec{\theta}_{\rm us}(L)\end{pmatrix} = \left(\begin{array}{c|c}\mathcal{A} & \mathcal{B}\\ \hline \mathcal{C} & \mathcal{D}\end{array}\right)\begin{pmatrix}\vec{\theta}_{\rm ds}(0)\\ \vec{\theta}_{\rm us}(0)\end{pmatrix}. \tag{C4}$$

Here, the block matrices $\mathcal{A}\in\mathbb{R}^{k\times k}$, $\mathcal{B}\in\mathbb{R}^{k\times(N-k)}$, $\mathcal{C}\in\mathbb{R}^{(N-k)\times k}$, and $\mathcal{D}\in\mathbb{R}^{(N-k)\times(N-k)}$. In terms of these matrices, the heat conservation law (C2) translates to

$$\sum_j\mathcal{A}_{ij}+\sum_j\mathcal{B}_{ij}=1, \quad \forall i, \tag{C5}$$

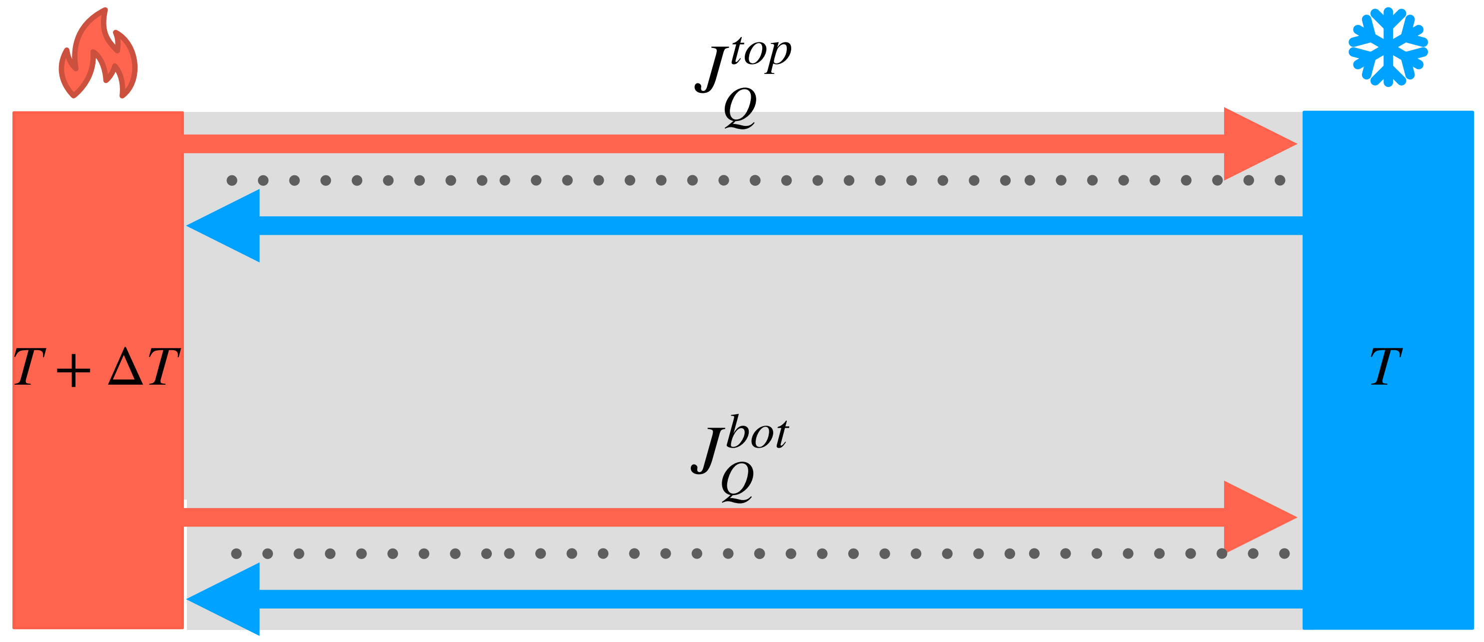
$$\sum_j\mathcal{C}_{ij}+\sum_j\mathcal{D}_{ij}=1, \quad \forall i. \tag{C6}$$

Next, we rearrange the terms in Eq. (C4) as

$$\begin{pmatrix}\vec{\theta}_{\rm ds}(L)\\ \vec{\theta}_{\rm us}(0)\end{pmatrix} = \left(\begin{array}{c|c}\mathcal{A}-\mathcal{B}\mathcal{D}^{-1}\mathcal{C} & \mathcal{B}\mathcal{D}^{-1}\\ \hline -\mathcal{D}^{-1}\mathcal{C} & \mathcal{D}^{-1}\end{array}\right)\begin{pmatrix}\vec{\theta}_{\rm ds}(0)\\ \vec{\theta}_{\rm us}(L)\end{pmatrix}. \tag{C7}$$

Let us now consider the top edge, for which the boundary conditions read $\vec{\theta}_{\rm ds}(0)=(\bar{T}+\Delta T)^2\times(1,\dots,1)_k^T$ and $\vec{\theta}_{\rm us}(L)=\bar{T}^2\times(1,\dots,1)_{N-k}^T$ (see Fig. 4). Plugging these quantitites into Eq. (C7), we write the top edge heat current (20) as

$$\begin{aligned}J_Q^{\rm top} &= \frac{\kappa_0}{2}\left(\sum_{i:\chi_i=+1}n_i\theta^i_{\rm ds}(L)-\bar{T}^2\sum_{i:\chi_i=-1}n_i\right)\\ &= \frac{\kappa_0}{2}\Bigg((\bar{T}+\Delta T)^2\sum_{i,j:\chi_i=+1}n_i(\mathcal{A}-\mathcal{B}\mathcal{D}^{-1}\mathcal{C})_{ij}\\ &\quad+\bar{T}^2\sum_{i,j:\chi_i=+1}n_i(\mathcal{B}\mathcal{D}^{-1})_{ij}-\bar{T}^2\sum_{i:\chi_i=-1}n_i\Bigg),\end{aligned} \tag{C8}$$



$$J_Q^{\rm top}\equiv\sum_{i:\chi_i=+1}J_i(L)-\sum_{i:\chi_i=-1}J_i(L), \tag{20}$$

$$J_Q^{\rm bot}\equiv\sum_{i:\chi_i=+1}J_i(0)-\sum_{i:\chi_i=-1}J_i(0), \tag{21}$$

we combine the two currents as

$$J_Q^{\rm top}-J_Q^{\rm bot}=\frac{\kappa_0}{2}\left((\bar{T}+\Delta T)^2)+\bar{T}^2\right)\times\left(\sum_{i,j:\chi_i=+1}n_i(\mathcal{A}-\mathcal{B}\mathcal{D}^{-1}\mathcal{C}+\mathcal{B}\mathcal{D}^{-1})_{ij}-\sum_{i:\chi_i=-1}n_i\right). \tag{C10}$$

Now, by using Eqs. (C5)-(C6), and identifying

$$\sum_{i:\chi_i=+1}n_i=c, \tag{C11}$$

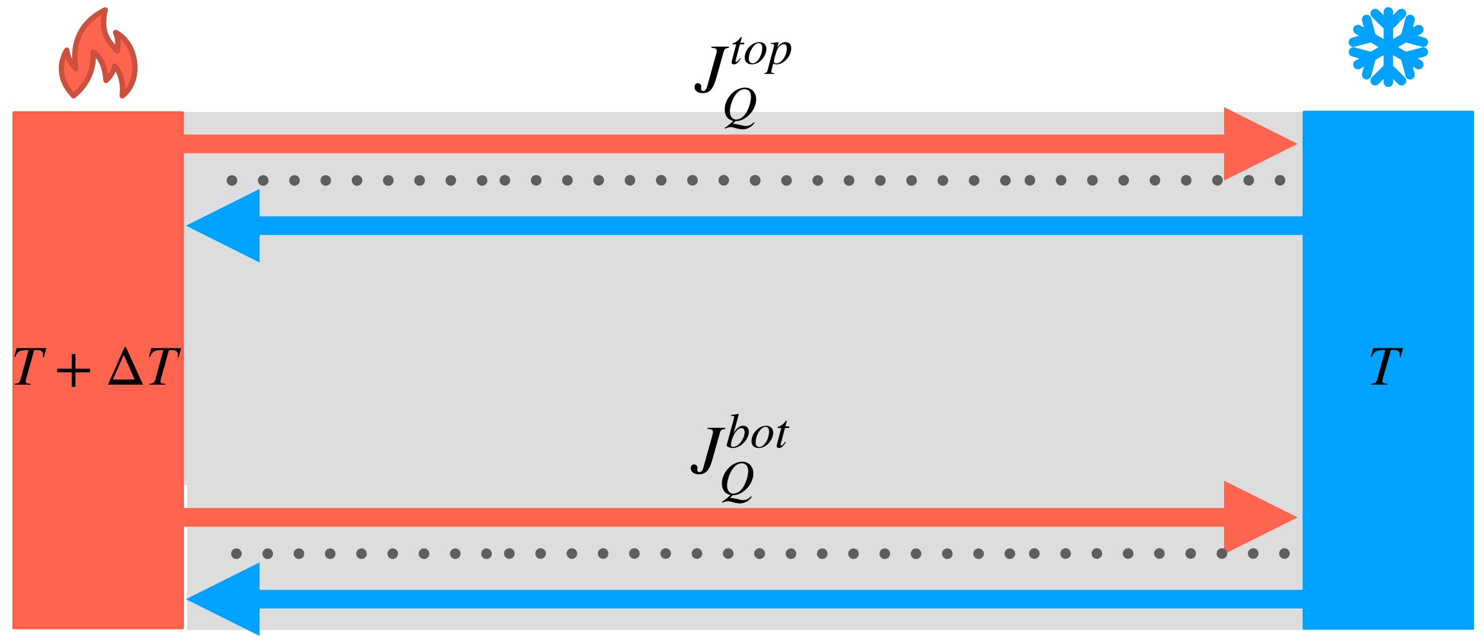
$$\sum_{i:\chi_i=-1}n_i=\bar{c}, \tag{C12}$$

the dependence on $\mathcal{A},\mathcal{B},\mathcal{C}$, and $\mathcal{D}$ cancels out, and we find that Eq. (C10) reduces to

$$J_Q^{\rm top}-J_Q^{\rm bot}=\frac{\kappa_0}{2}\left((\bar{T}+\Delta T)^2+\bar{T}^2\right)(c-\bar{c}). \tag{C13}$$

Finally, inserting this expression into the definition of G_H^Q, given in Eq. (26), gives our desired result (27). Our proof generalizes the theoretical analysis for $\nu=2/3$ in Ref. [37] to any edge structure.

Four-point thermal conductance: proof



where in the second line we used Eq. (C7). For the bottom edge, we have reversed boundary conditions $\vec{\theta}_{ds}(0) = \bar{T}^2 \times (1,\dots,1)_k^T$ and $\vec{\theta}_{us}(L) = (\bar{T}+\Delta T)^2 \times (1,\dots,1)_{N-k}^T$. The bottom edge heat current (21) then reads

$$\begin{aligned} J_Q^{\mathrm{bot}} &= -\frac{\kappa_0}{2}\left(\sum_{i:\chi_i=+1} n_i \theta^i_{\mathrm{ds}}(L) - \bar{T}^2 \sum_{i:\chi_i=-1} n_i\right) \\ &= \frac{\kappa_0}{2}\Big(\bar{T}^2 \sum_{i,j:\chi_i=+1} n_i(\mathcal{A}-\mathcal{B}\mathcal{D}^{-1}\mathcal{C})_{ij} \\ &+ (\bar{T}+\Delta T)^2 \sum_{i,j:\chi_i=+1} n_i(\mathcal{B}\mathcal{D}^{-1})_{ij} \\ &- (\bar{T}+\Delta T)^2 \sum_{i:\chi_i=-1} n_i\Big). \end{aligned} \tag{C9}$$

The crucial next step is the assumption that *the degrees of thermal equilibration on the top and bottom edges are identical.* This translates to identical block matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ for the two edges. Then, and only then, can

we combine the two currents as

$$\begin{aligned} J_Q^{\mathrm{top}} - J_Q^{\mathrm{bot}} &= \frac{\kappa_0}{2}\left((\bar{T}+\Delta T)^2) + \bar{T}^2\right) \\ &\times \left(\sum_{i,j:\chi_i=+1} n_i(\mathcal{A}-\mathcal{B}\mathcal{D}^{-1}\mathcal{C}+\mathcal{B}\mathcal{D}^{-1})_{ij} - \sum_{i:\chi_i=-1} n_i\right). \end{aligned} \tag{C10}$$

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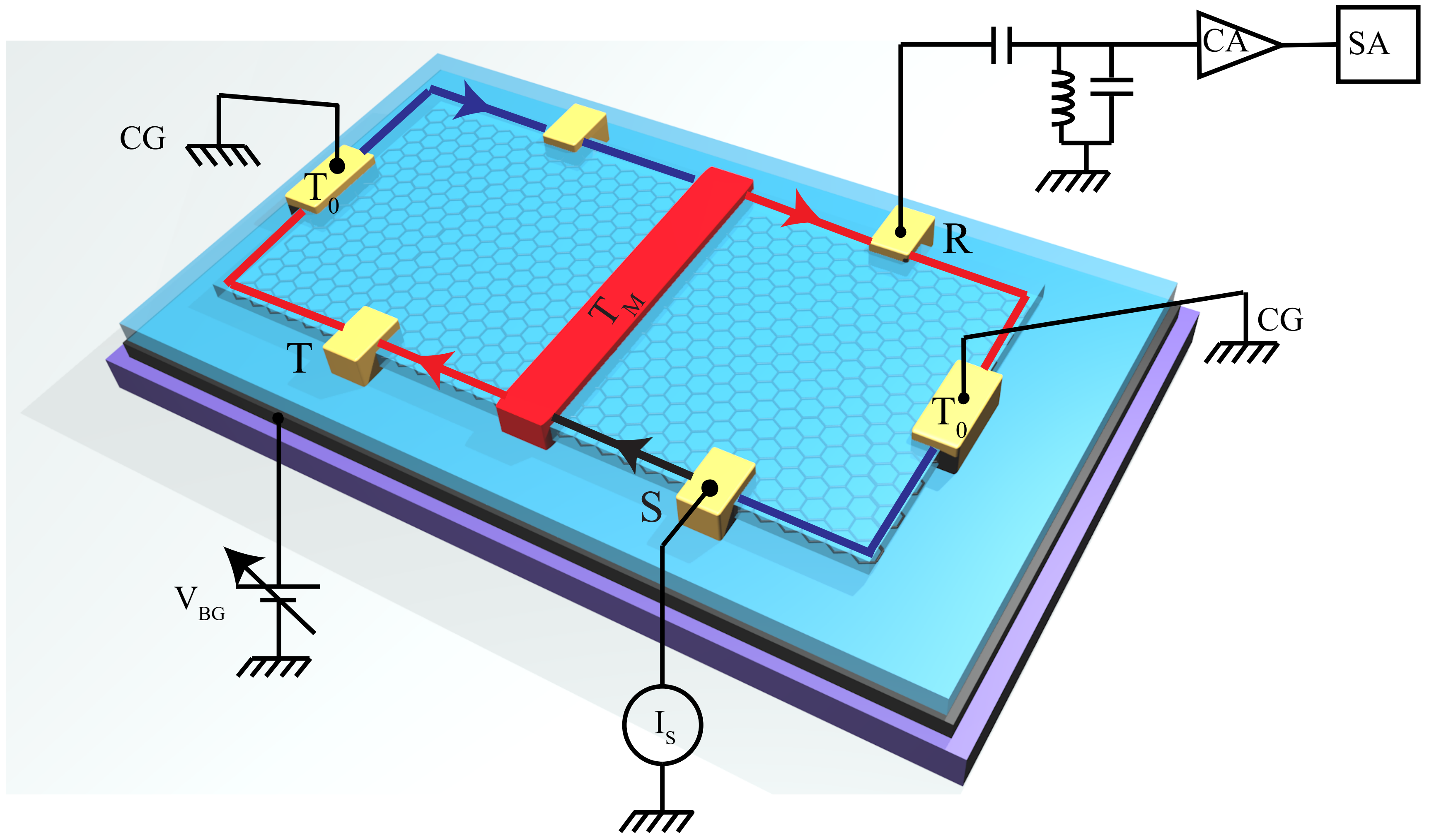
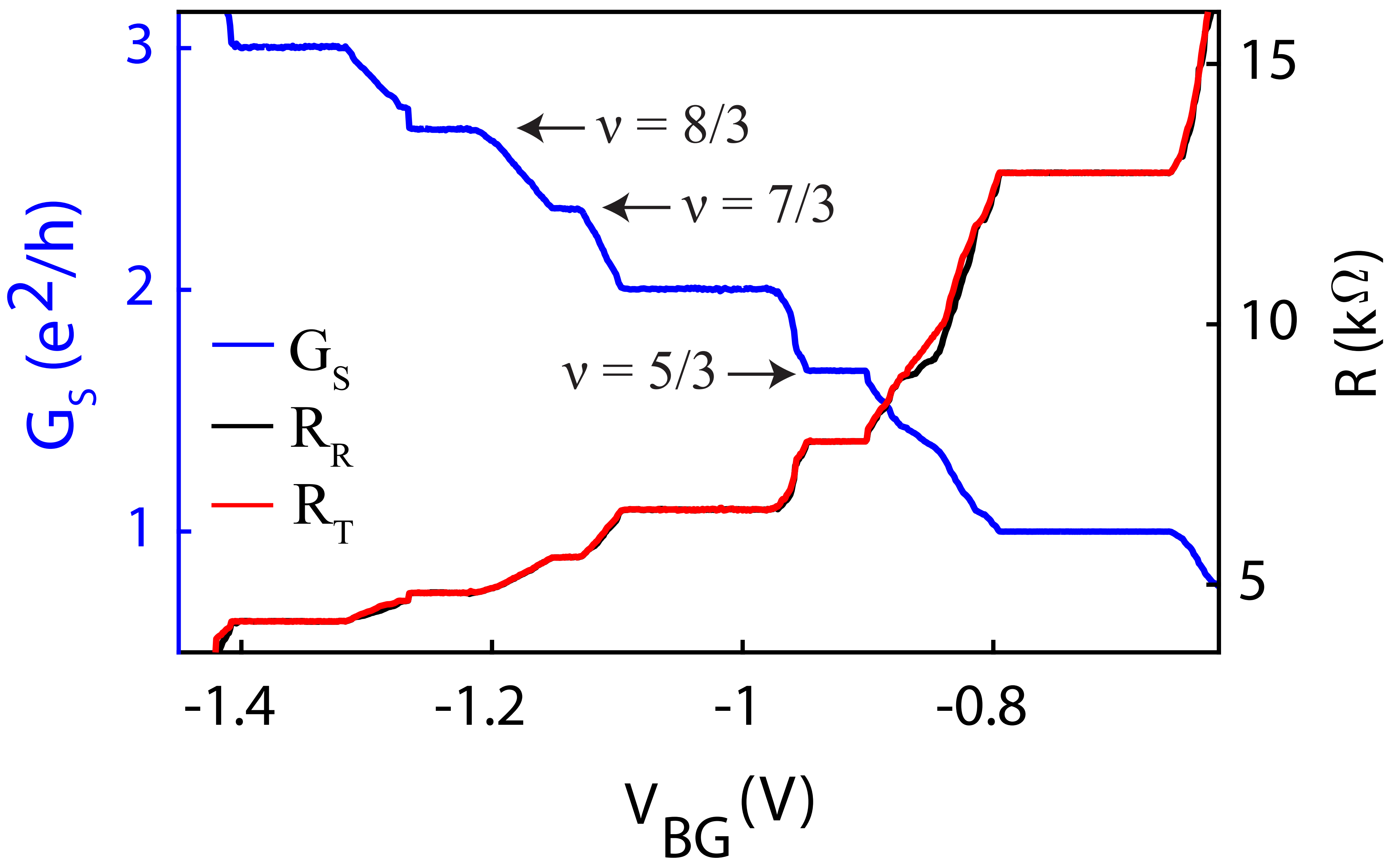
Complementary experiment at higher fillings

R. Kumar, S. K. Srivastav, C. Spånslätt et al.: Nat. Commun. 13, 1 (2022)

(b)





Edge structures similar to 1/3 or 2/3

- $\nu = 5/3 = 1 + 2/3 = 2 - 1/3$
- $\nu = 7/3 = 2 + 1/3$
- $\nu = 8/3 = 2 + 2/3 = 3 - 1/3$

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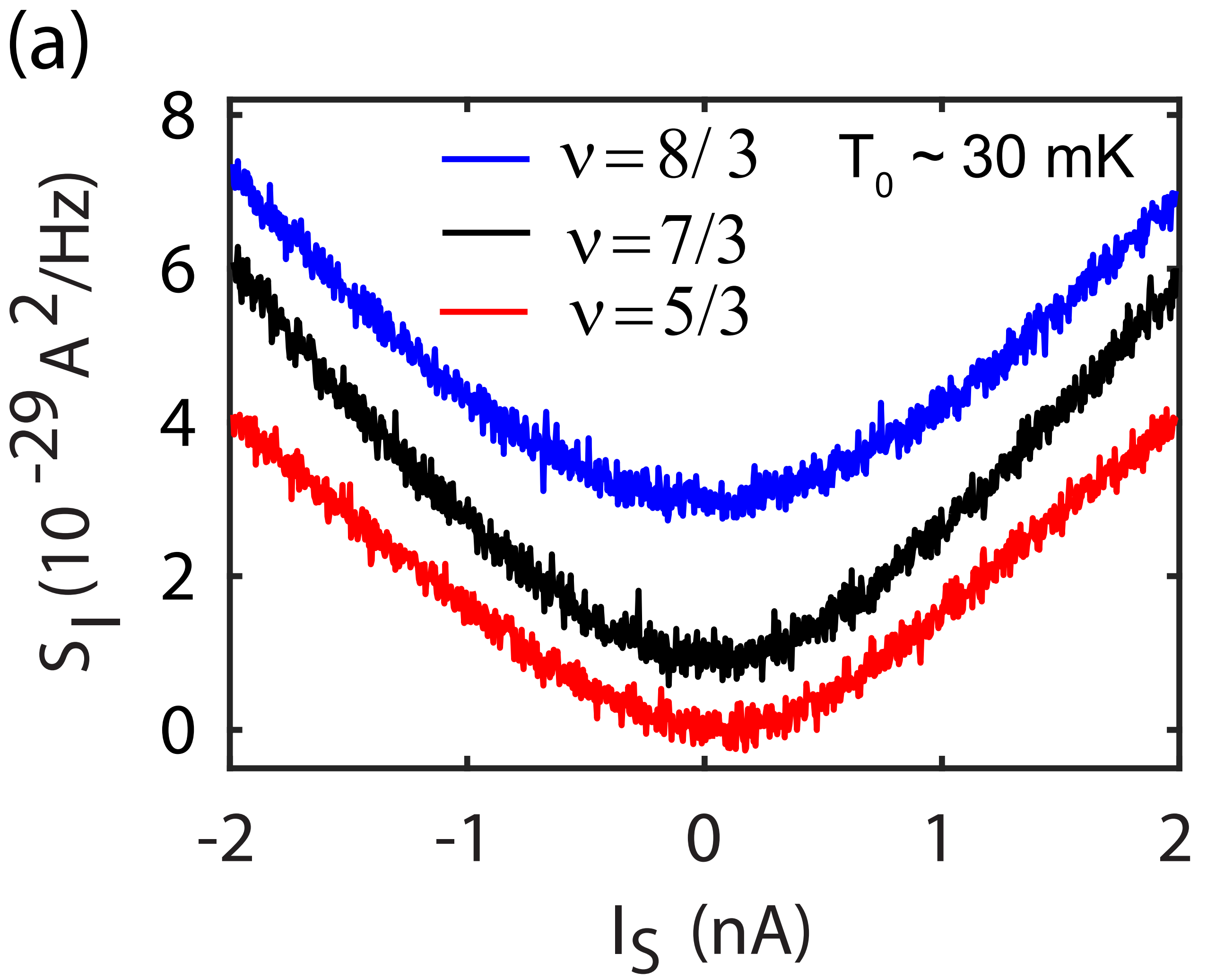
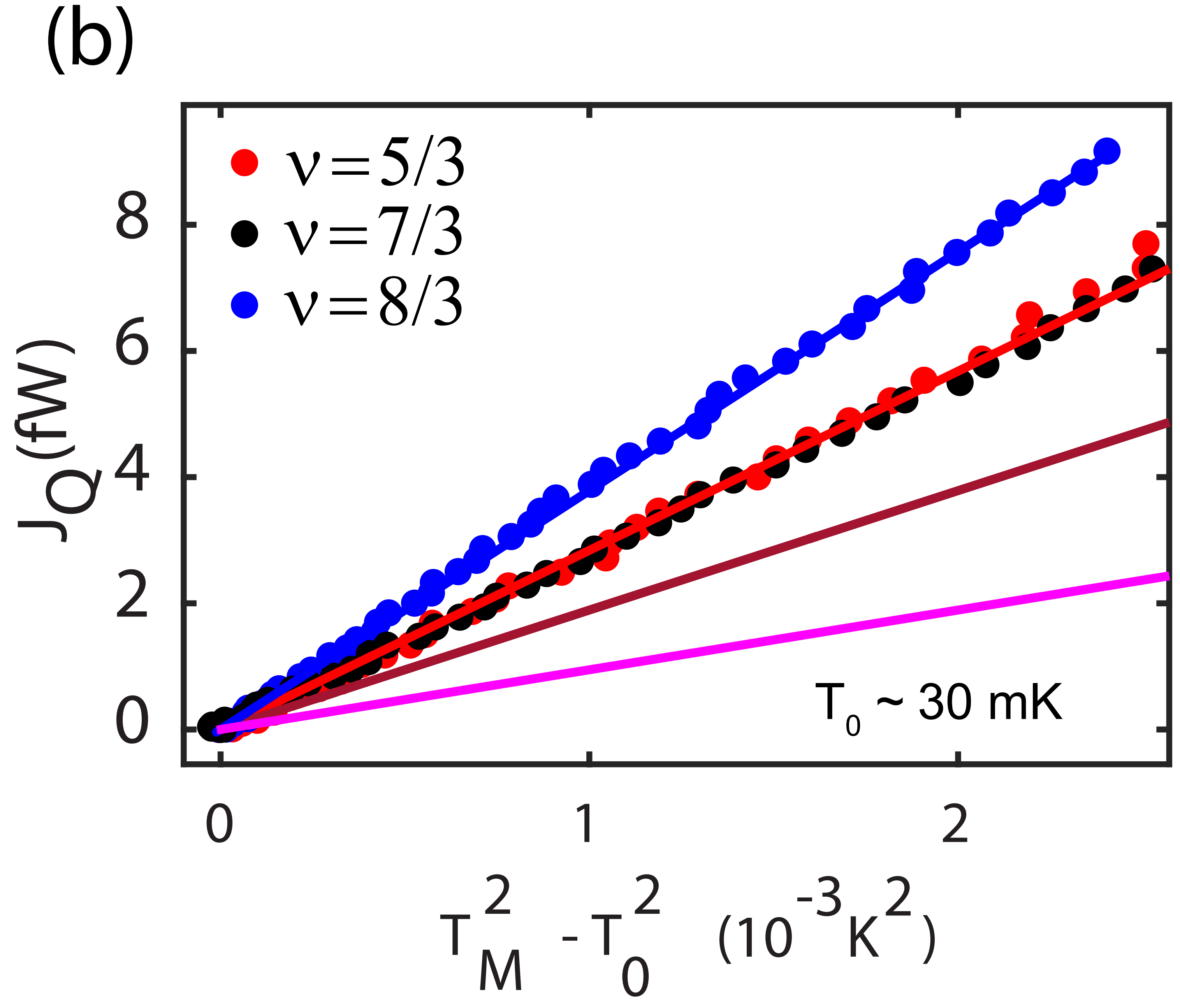


These are the values for non-equilibrated channels!

$$\frac{G_Q}{\kappa_0 T} = n_d + n_u$$

- $\nu = 5/3: \quad n_d + n_u = 2 + 1 = 3$
- $\nu = 7/3: \quad n_d + n_u = 2 + 1 = 3$
- $\nu = 8/3: \quad n_d + n_u = 3 + 1 = 4$



BUT, charge transport is quantized!

$$\frac{G}{e^2/h} = \nu$$

Big difference in charge and heat equilibration!

$$\ell_C \ll L \ll \ell_Q$$