A roundabout for waves

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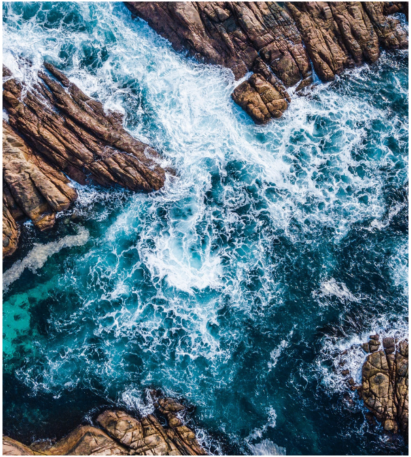
Antonio Manesco, Mert Bozkurt, and Valla Fatemi
SciPost Phys. 18, 098 (+source code)

Jun 2, 2025, Nordita workshop

TU Delft Delft University of Technology

Quantum Tinkerer

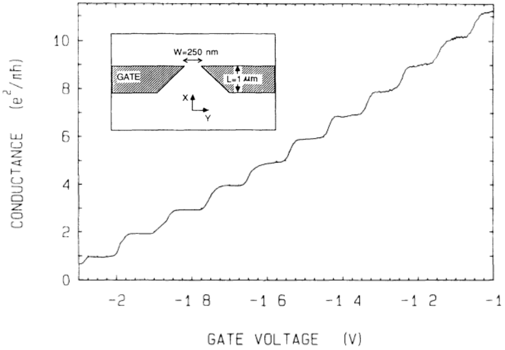
The problem



Background: protected transmission

Waves scatter, but sometimes transmission is protected.

- Quantum point contact: transmission protected by smooth potential

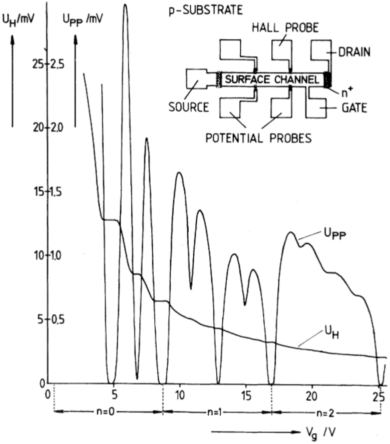


van Wees *et al.*, 1988

Background: protected transmission

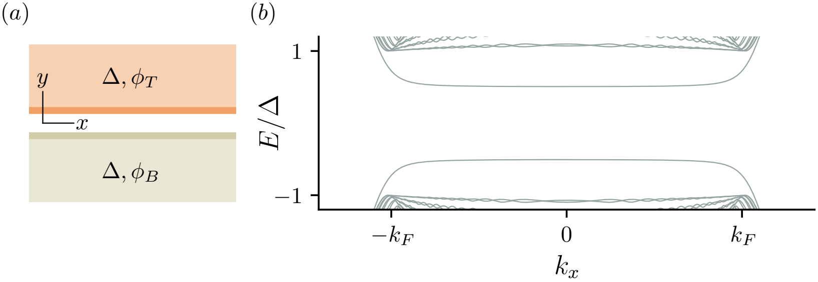
Waves scatter, but sometimes transmission is protected.

- Quantum point contact: transmission protected by smooth potential
- Quantum Hall effect: *chiral* transmission protected by topology



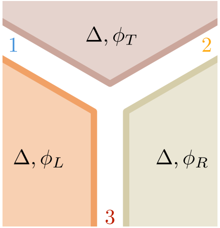
Klitzing *et al.*, 1980

The solution



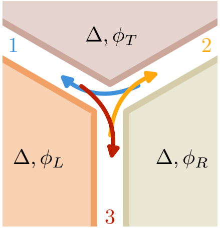
1. Take a short Josepshon junction

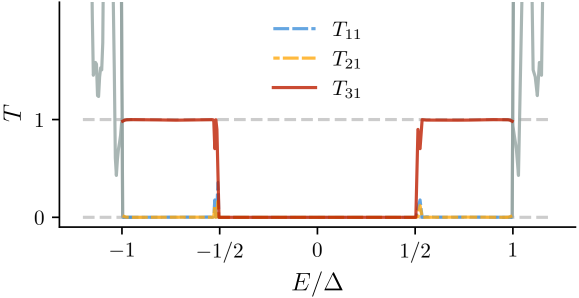
The solution



1. Take a short Josepshon junction
2. Make 3 meet

The solution





1. Take a short Josepshon junction
2. Make 3 meet
3. Get chiral and quantized transmission

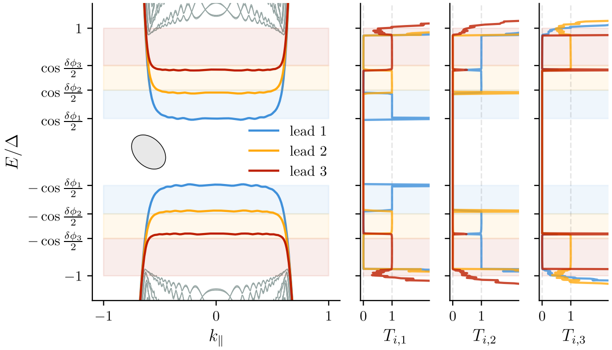
The end.
Questions?

Testing the limits

Is it really protected?

Works if we:

- Make phase differences unequal



Testing the limits

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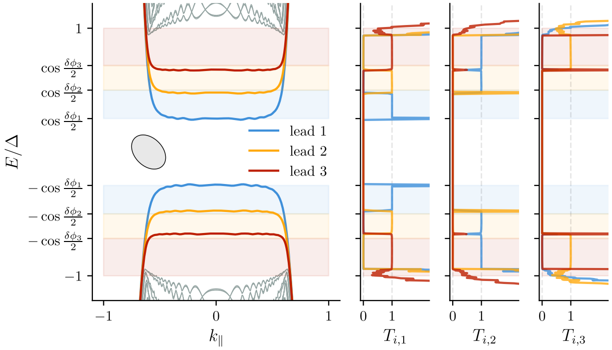
Works if we:

- Make phase differences unequal
- Make junction asymmetric



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Works if we:

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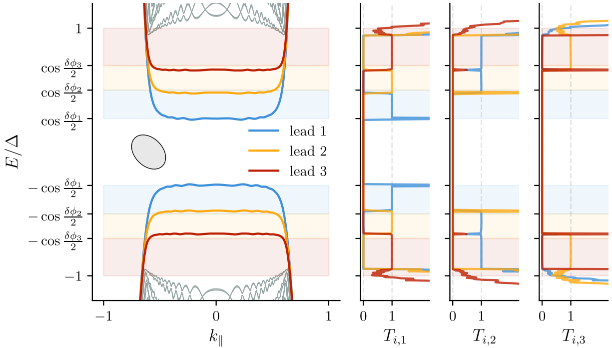
- Make dispersion anisotropic



Testing the limits

Is it really protected?

Works if we:

- Make phase differences unequal
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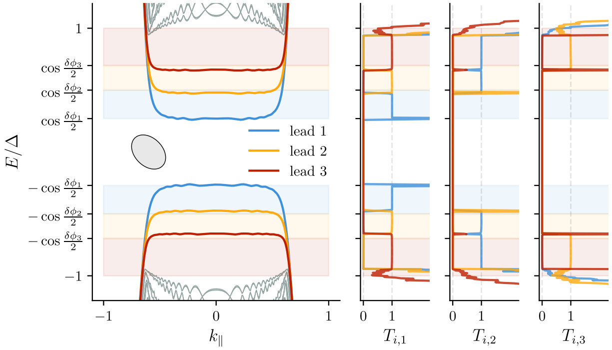
- Make dispersion anisotropic
- ¡Break particle-hole symmetry $H_{ee} \to 2 \times H_{ee}$!



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Breaks if we:

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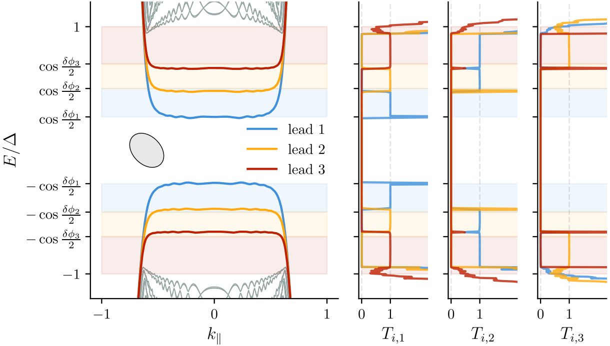

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Breaks if we:

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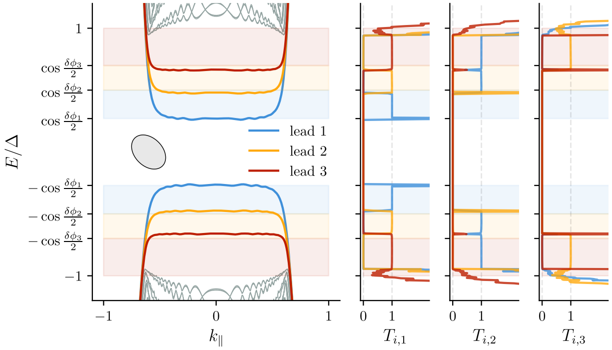
- Make $\mu \sim \Delta$



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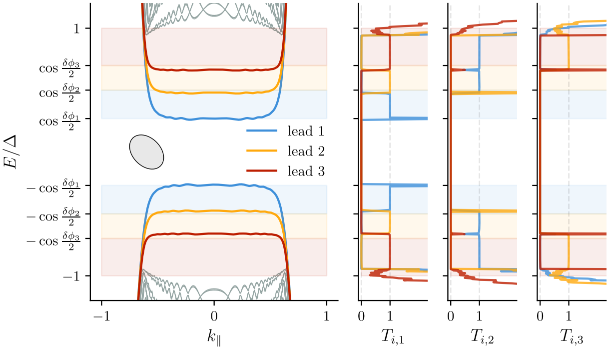
- Make junction asymmetric
- Make dispersion anisotropic
- ¡Break particle-hole symmetry $H_{ee} \to 2 \times H_{ee}$!

Breaks if we:

- Make $\mu \to \mu(r)$
- Make $\mu \sim \Delta$
- Introduce sharp $< 90^\circ$ turns



What is going on?

Foundation

Probing Fermi Sea Topology by Andreev State Transport

Pok Man Tam and Charles L. Kane
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

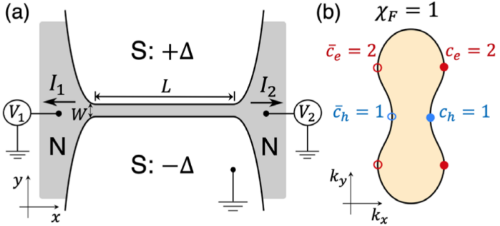
(Received 14 October 2022; accepted 16 January 2023; published 1 March 2023)

We show that the topology of the Fermi sea of a two-dimensional electron gas (2DEG) is reflected in the ballistic Landauer transport *along* a long and narrow Josephson π junction that proximitizes the 2DEG. The low-energy Andreev states bound to the junction are shown to exhibit a dispersion that is sensitive to the Euler characteristic of the Fermi sea (χ_F). We highlight two important relations: one connects the electron or hole nature of Andreev states to the convex or concave nature of Fermi surface critical points, and one relates these critical points to χ_F. We then argue that the transport of Andreev states leads to a quantized conductance that probes χ_F. An experiment is proposed to measure this effect, from which we predict an *I-V* characteristic that not only captures the topology of the Fermi sea in metals, but also resembles the rectification effect in diodes. Finally, we evaluate the feasibility of measuring this quantized response in graphene, InAs and HgTe 2DEGs.

DOI: 10.1103/PhysRevLett.130.096301

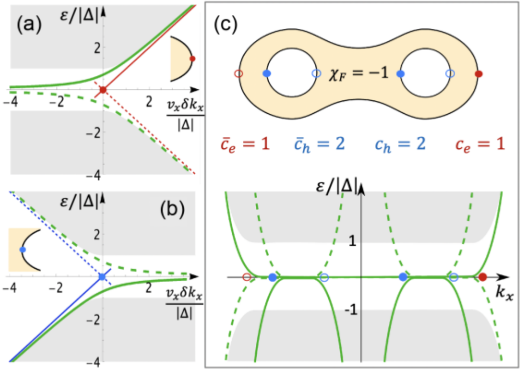
Andreev states in a π-junction

- Claim: nonlocal Andreev conductance in a π-junction counts Fermi surfaces



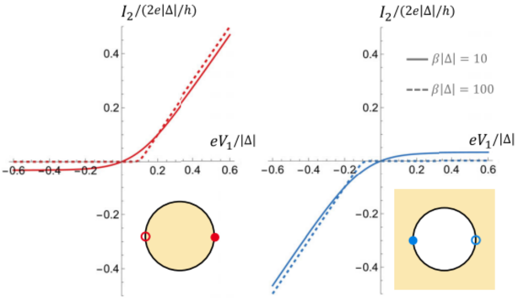
Andreev states in a π-junction

- Claim: nonlocal Andreev conductance in a π-junction counts Fermi surfaces
- Zero modes split near *critical points* and acquire dispersion

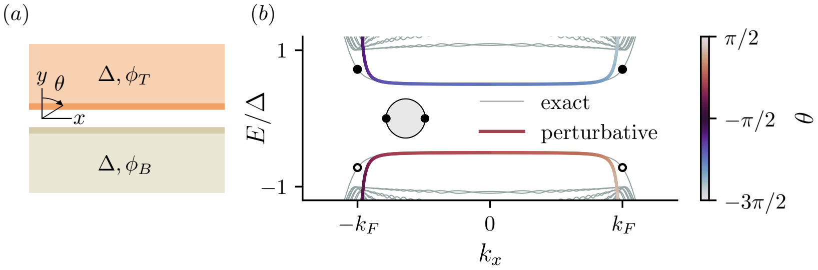


Andreev states in a π-junction

- Claim: nonlocal Andreev conductance in a π-junction counts Fermi surfaces
- Zero modes split near *critical points* and acquire dispersion
- Adiabaticity prevents backscattering and quantizes charge



Generalizing to $\phi \neq \pi$



Two states at $(k_x, \pm k_y)$ couple due to dispersion nonlinearity $\sim k_y^2$

$$H_\pm = \Delta \begin{pmatrix} \cos(\delta\phi/2) & \frac{\Delta}{2E_x}\sin^2(\delta\phi/2) \\ \frac{\Delta}{2E_x}\sin^2(\delta\phi/2) & -\cos(\delta\phi/2) \end{pmatrix}.$$

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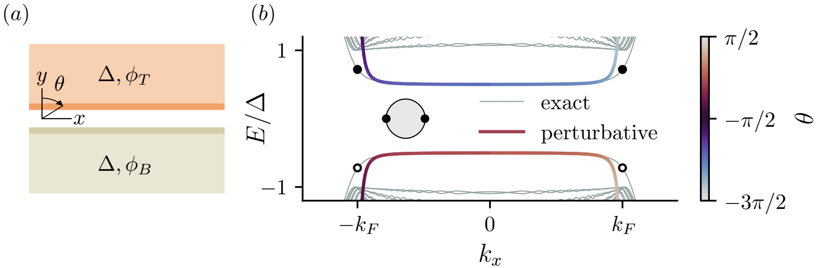

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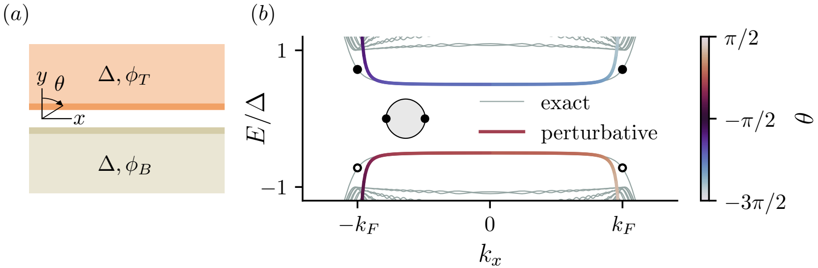

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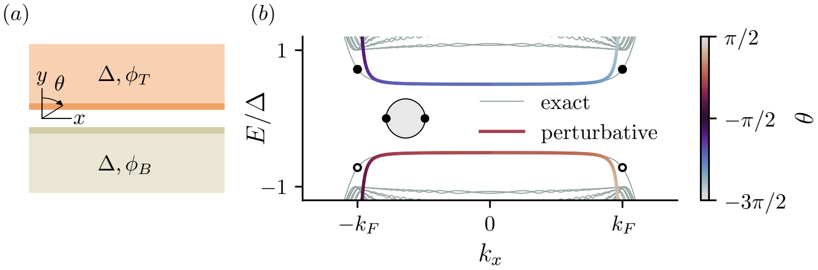

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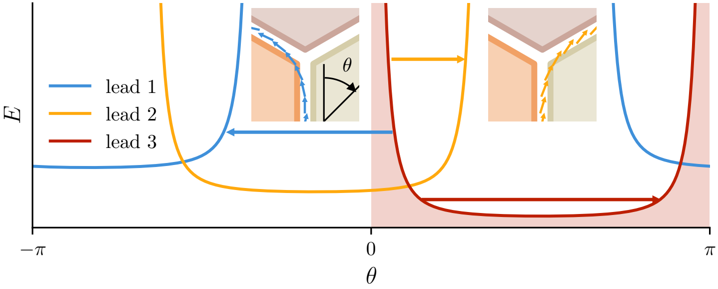
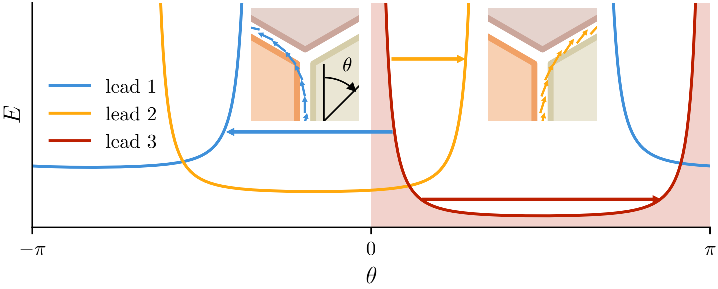
Dispersion forms an energy barrier for momenta!

Protection by energy barriers



- Momentum evolves adiabatically $\Rightarrow$ never reaches prohibited regions

Protection by energy barriers



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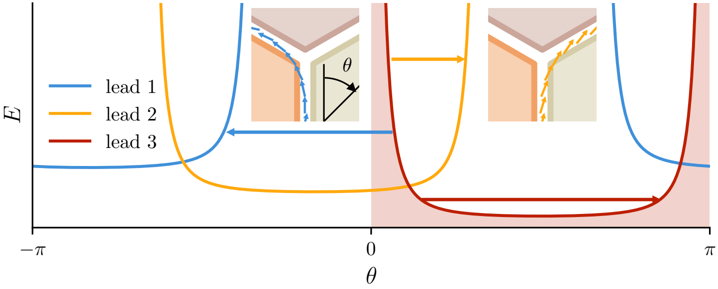
Protection by energy barriers



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- $\Rightarrow$ only chiral transmission is allowed!

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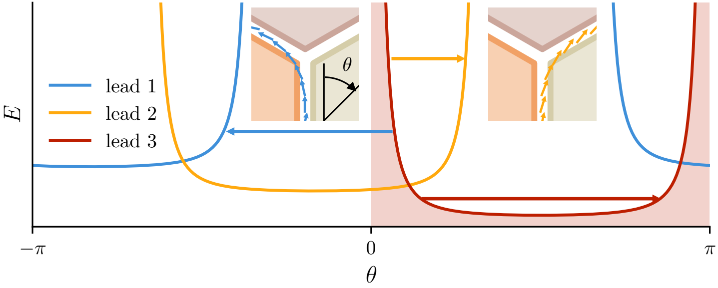

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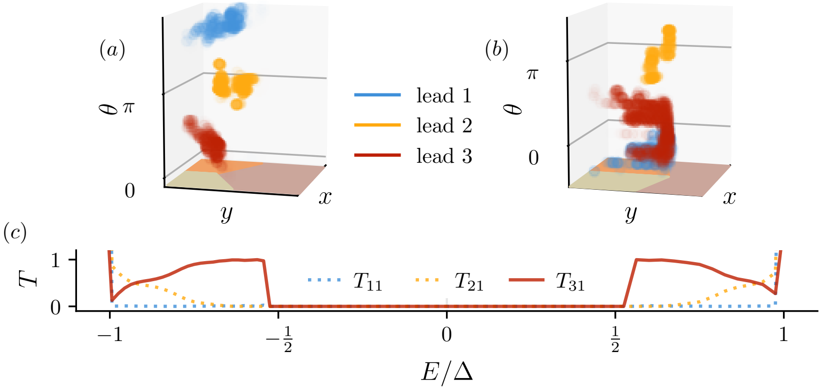
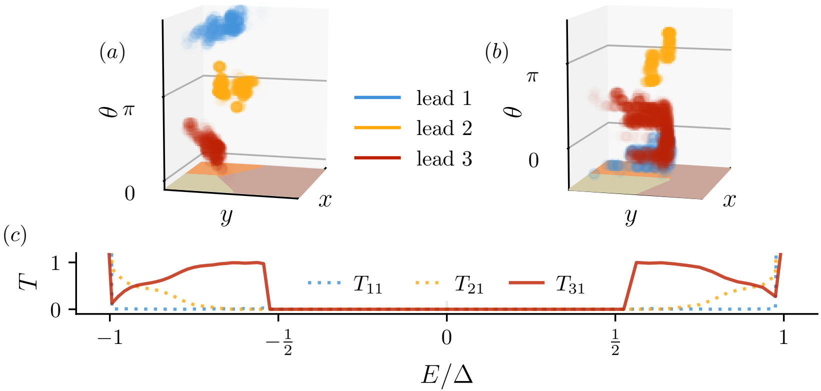
Can we break this pattern?

Sharp turns



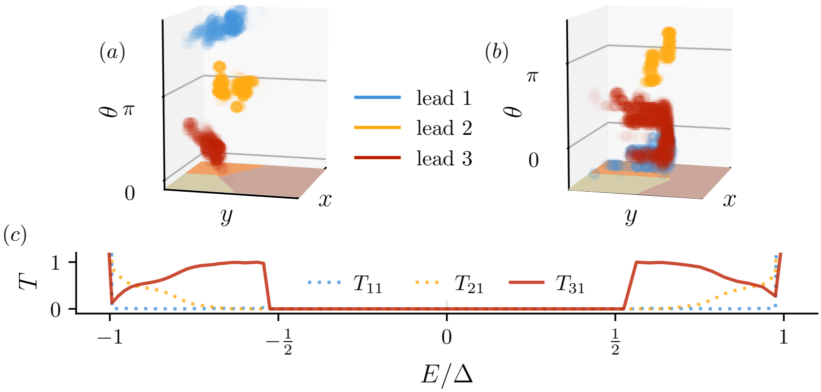
- When turns are $< 90^{\circ}$, transmission is not quantized

Sharp turns



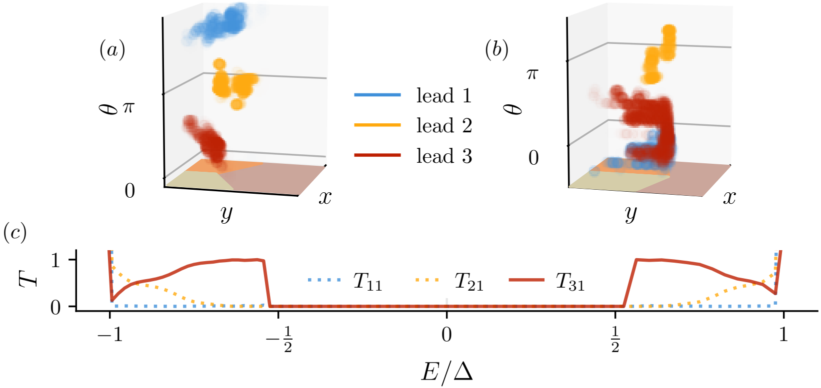
- When turns are $< 90^\circ$, transmission is not quantized
- The wave functions start overlapping in the phase space

Sharp turns



- When turns are $< 90^\circ$, transmission is not quantized
- The wave functions start overlapping in the phase space
- With smooth turns everything is protected

Sharp turns



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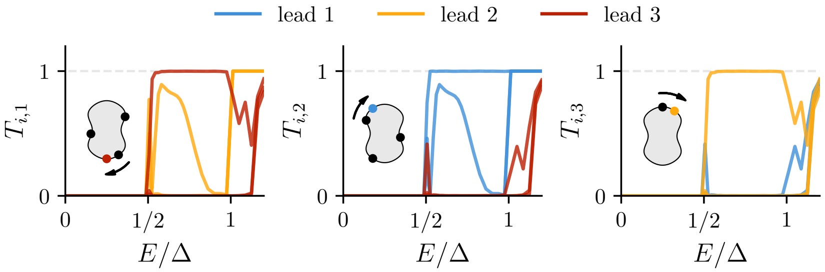
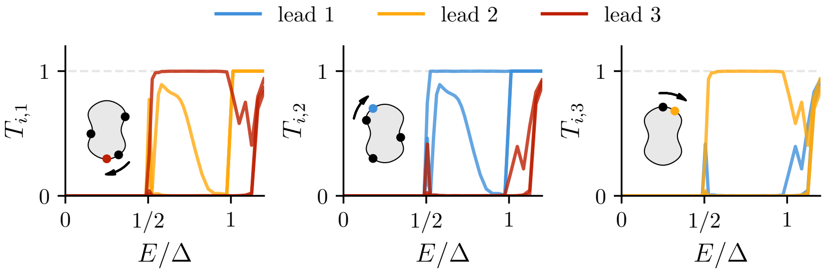
But what about more complex Fermi surfaces?

Non-convex Fermi surfaces



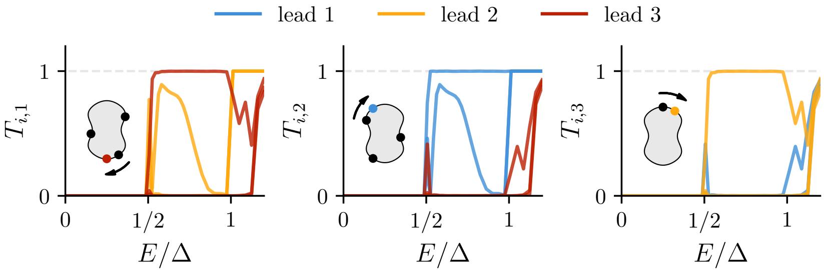
- Peanut-shaped Fermi surface

Non-convex Fermi surfaces



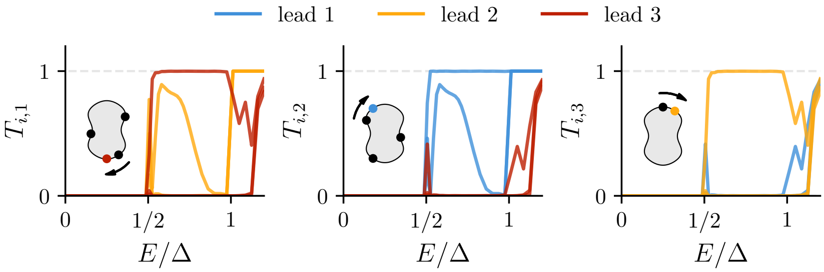
- Peanut-shaped Fermi surface
- Some transmission eigenvalues not quantized. . .

Non-convex Fermi surfaces



- Peanut-shaped Fermi surface
- Some transmission eigenvalues not quantized...
- However, at least one must stay quantized!

Non-convex Fermi surfaces



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- However, at least one must stay quantized!

$\Rightarrow$ chiral transport counts Fermi surfaces!

Electrical conductance

What about electric charge?

Electrical conductance

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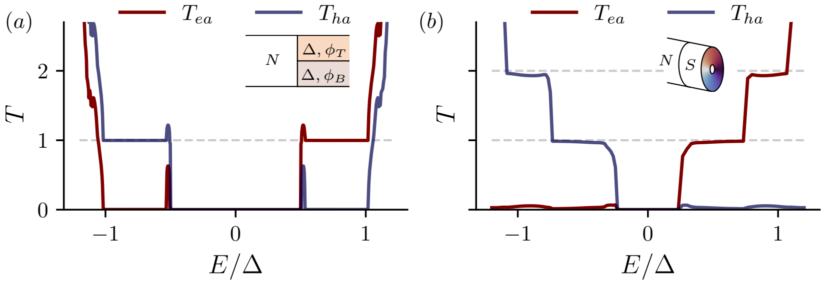
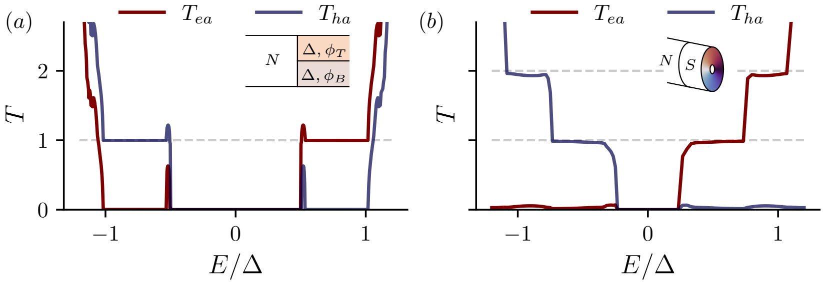
- Electrical conductance protected by adiabatically $\Rightarrow$ applies to any ϕ



Electrical conductance

What about electric charge?

- Electrical conductance protected by adiabatically $\Rightarrow$ applies to any ϕ
- Same protection applies to a vortex!



Outlook

- Momentum space separation protects chiral transmission
- Phenomenon generic to gapped multiband systems
- Observation in superconductors possible but hard
- Metamaterials an alternative plaform
- A lot of open question about the nature of protection and generality

The end.
Questions?

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