Studying Inflation with Numerical Relativity

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Numerical Simulations of the Early Universe

Nordita, Stockholm

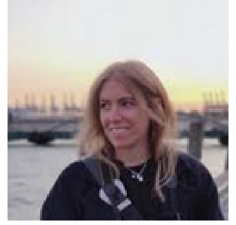
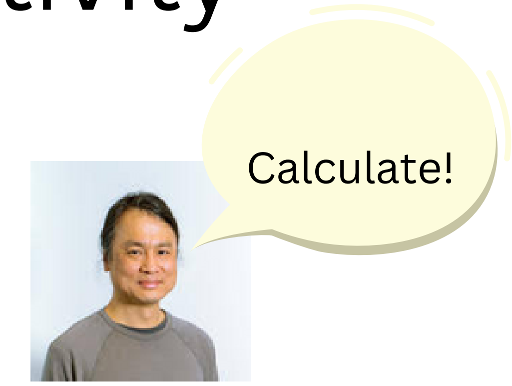
28/07/2025

Outlook

- Why Inflation?
- The Initial Condition problem-When inflation can inflate?-
- Testing Inflation with Numerical Relativity





J. Aurrekoetxea, K. Clough, M. Elley, R. Flauger, N. Righi, E. Lim

1608.04408: Clough, DiNunno, Fischer, Flauger, Lim, Paban
1712.07352: Clough, Flauger, Lim
1910.12547: Aurrekoetxea, Clough, Flauger, Lim
2405.03490: Aurrekoetxea, Clough, Elley, Flauger, **Giannadakis** and Lim
2508.xxxxx: Elley, Flauger, **Giannadakis**,Lim

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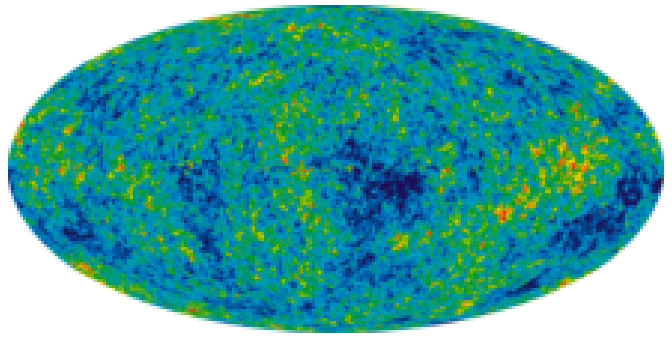
Our Universe

FRLW expansion



Big Bang
Initial Conditions?

Some Dynamics?

Observations

CMB T=2.7K

Homogeneous

Isotropic

Flat

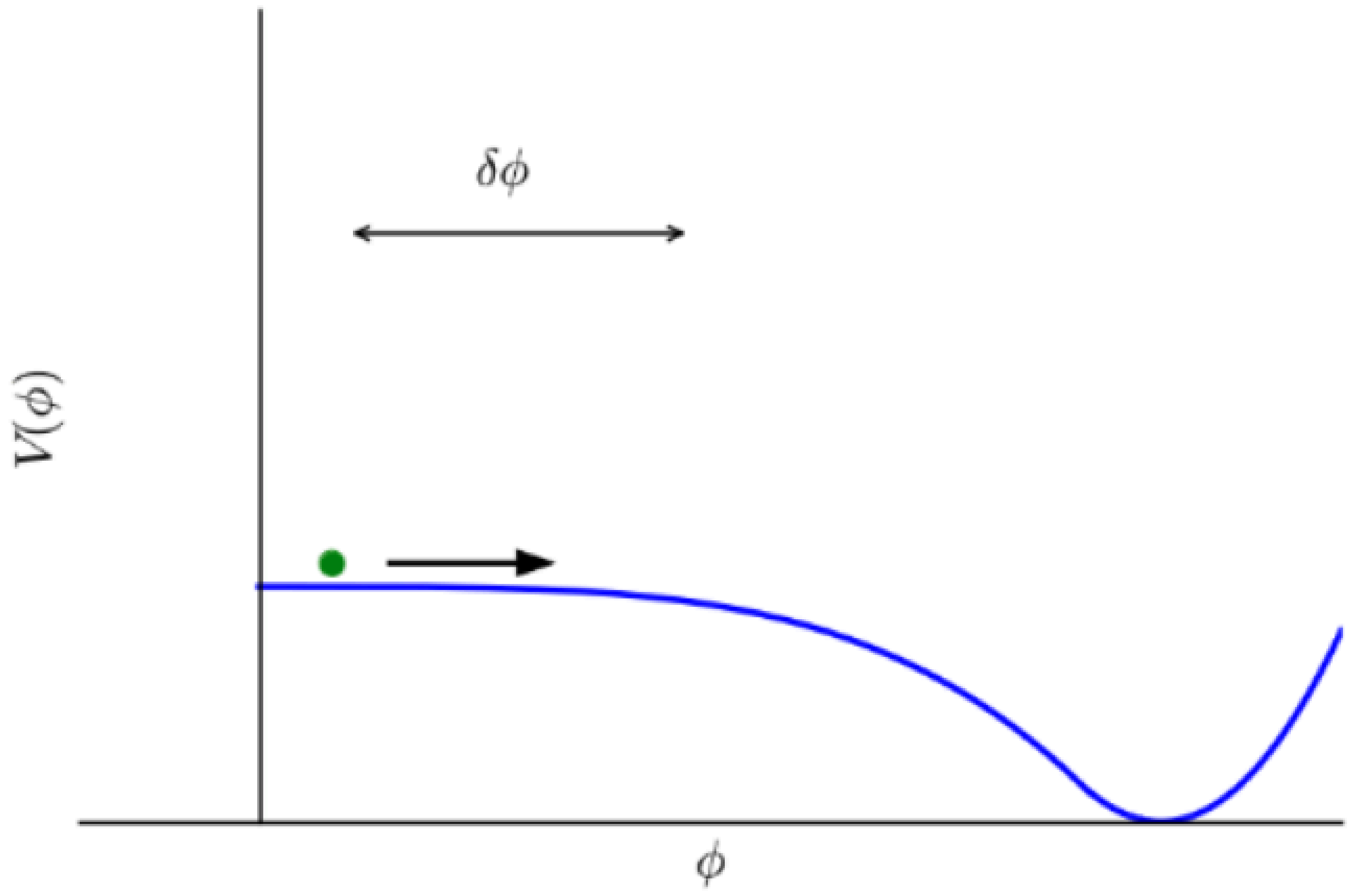
No relics

Universe today result of:

Initial Conditions?
Some Dynamics? → Inflation
Both?

The paradigm: Single Scalar Field Inflation

Once upon a time there was a spatially homogeneous scalar field $\phi(t)$ slowly rolling



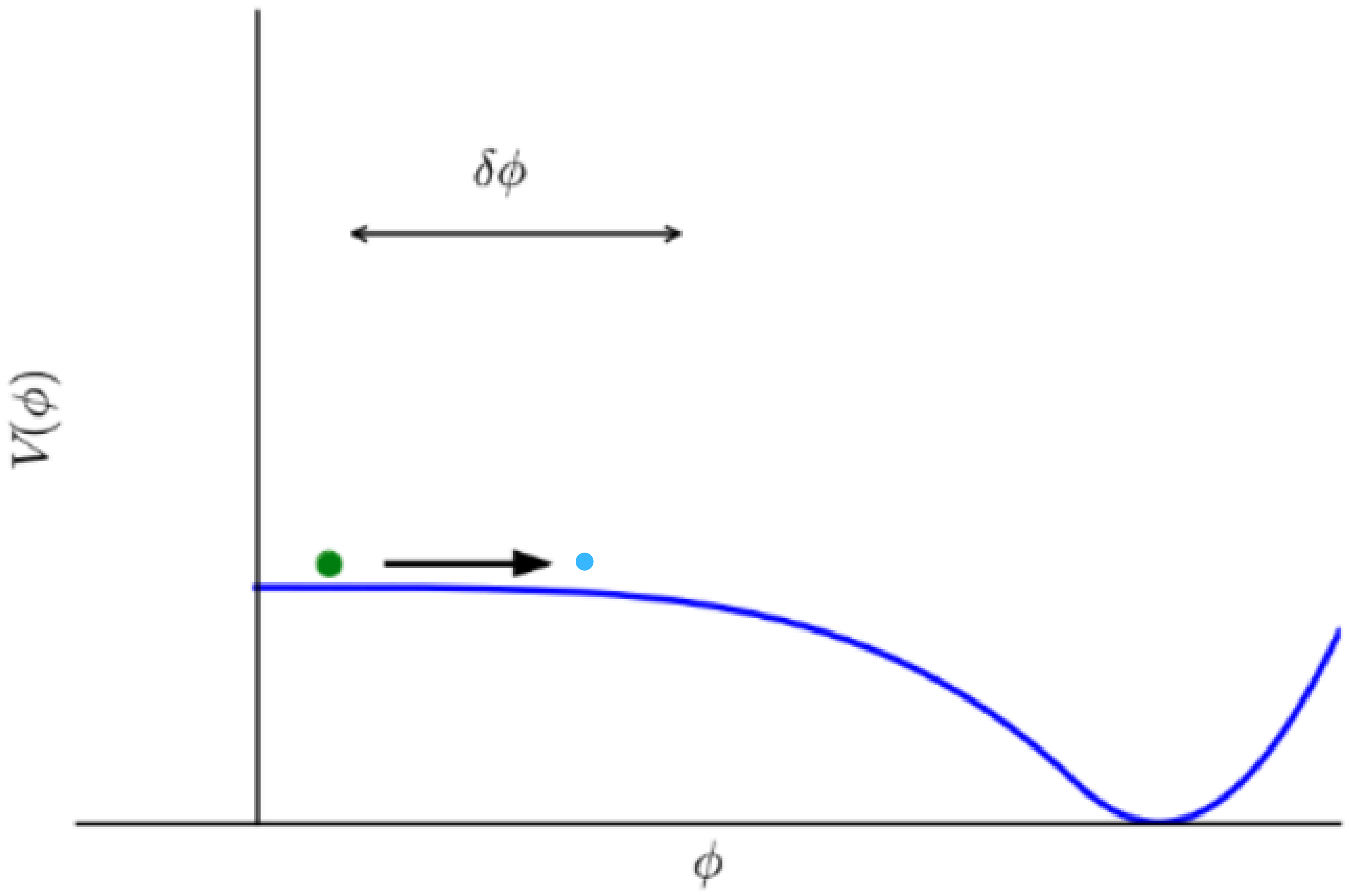
$$\frac{\rho}{p} = \frac{\cancel{\frac{1}{2}\dot{\phi}^2} + \cancel{\frac{1}{2a^2}(\nabla\phi)^2} + V(\phi)}{\cancel{\frac{1}{2}\dot{\phi}^2} + \cancel{\frac{1}{2a^2}(\nabla\phi)^2} - V(\phi)} \approx -1$$

This works!

The paradigm revisited

Once upon a time there was a spatially **inhomogeneous** scalar field $\phi(t,x,y,z)$ rolling

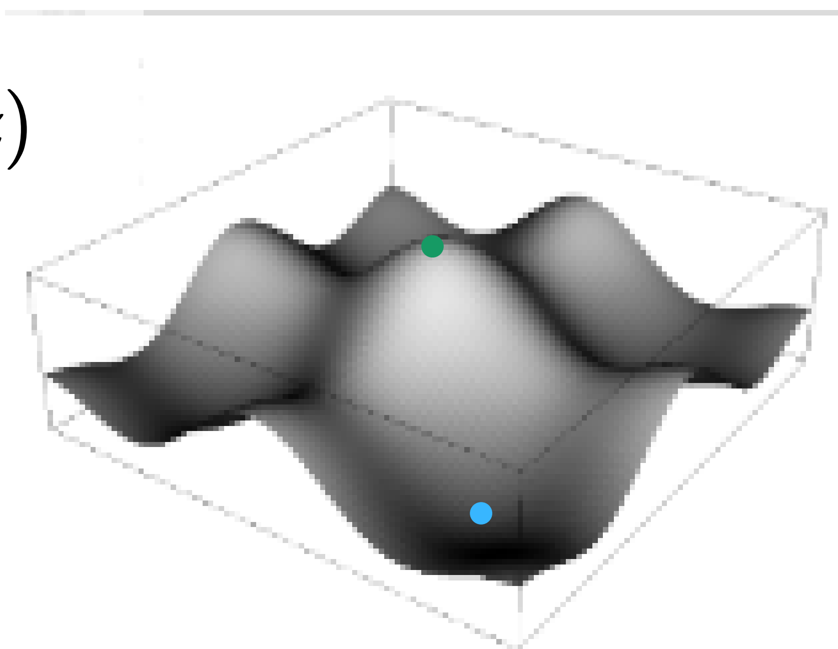
$$\frac{\rho}{p} \sim \frac{\frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi)}{\frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}(\nabla\phi)^2 - V(\phi)}$$



$g_{\mu\nu}(t,x,y,z)$

$\phi(t,x,y,z)$

$\dot{\phi}(t,x,y,z)$



The paradigm revisited

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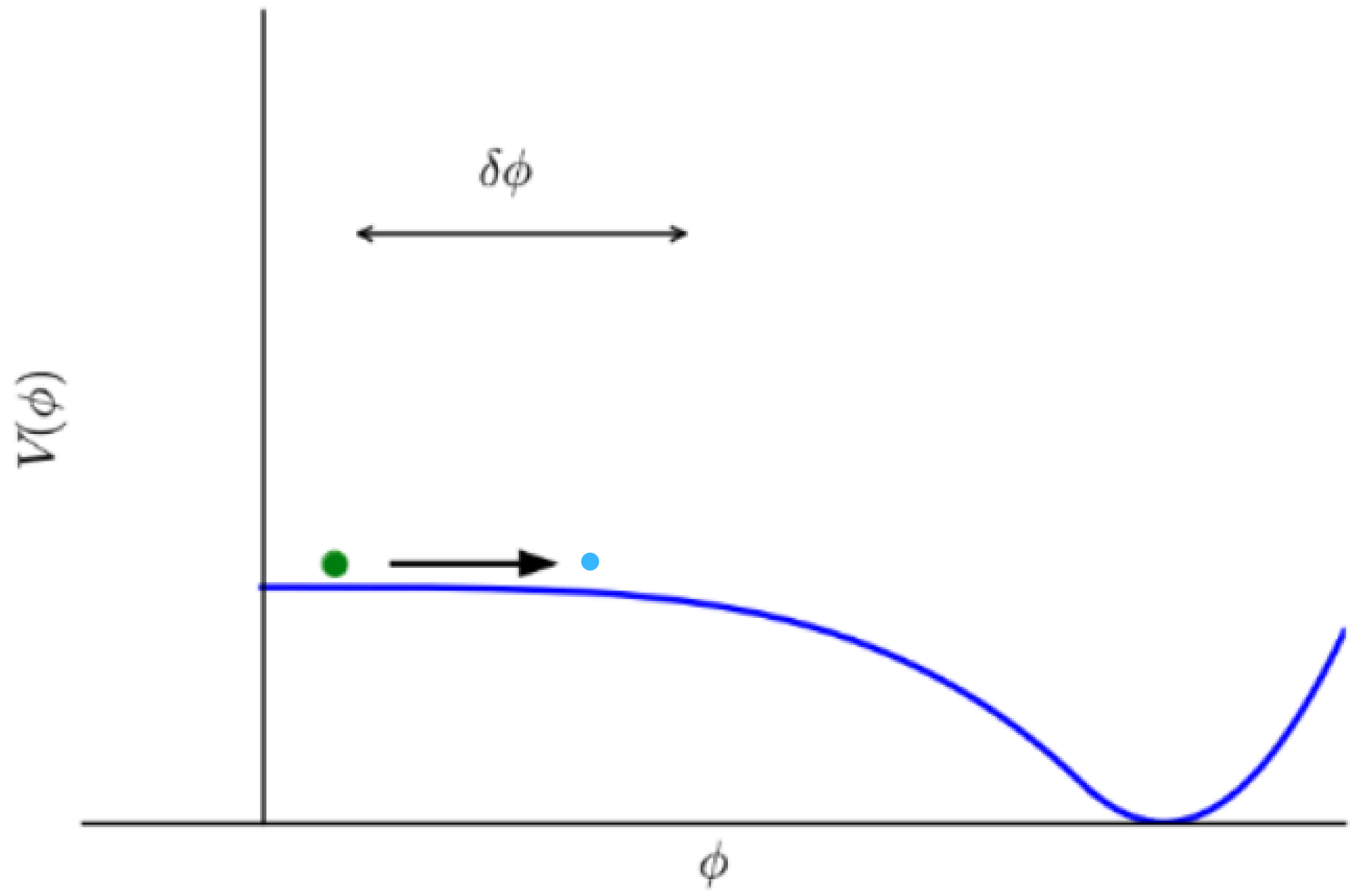
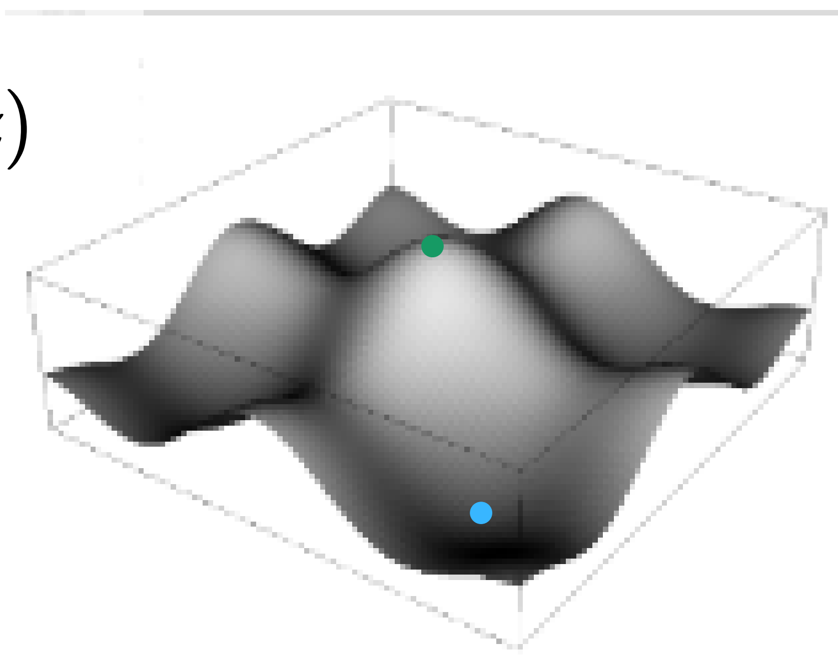


$$\frac{\rho}{p} \sim \frac{\frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi)}{\frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}(\nabla\phi)^2 - V(\phi)}$$

$g_{\mu\nu}(t, x, y, z)$

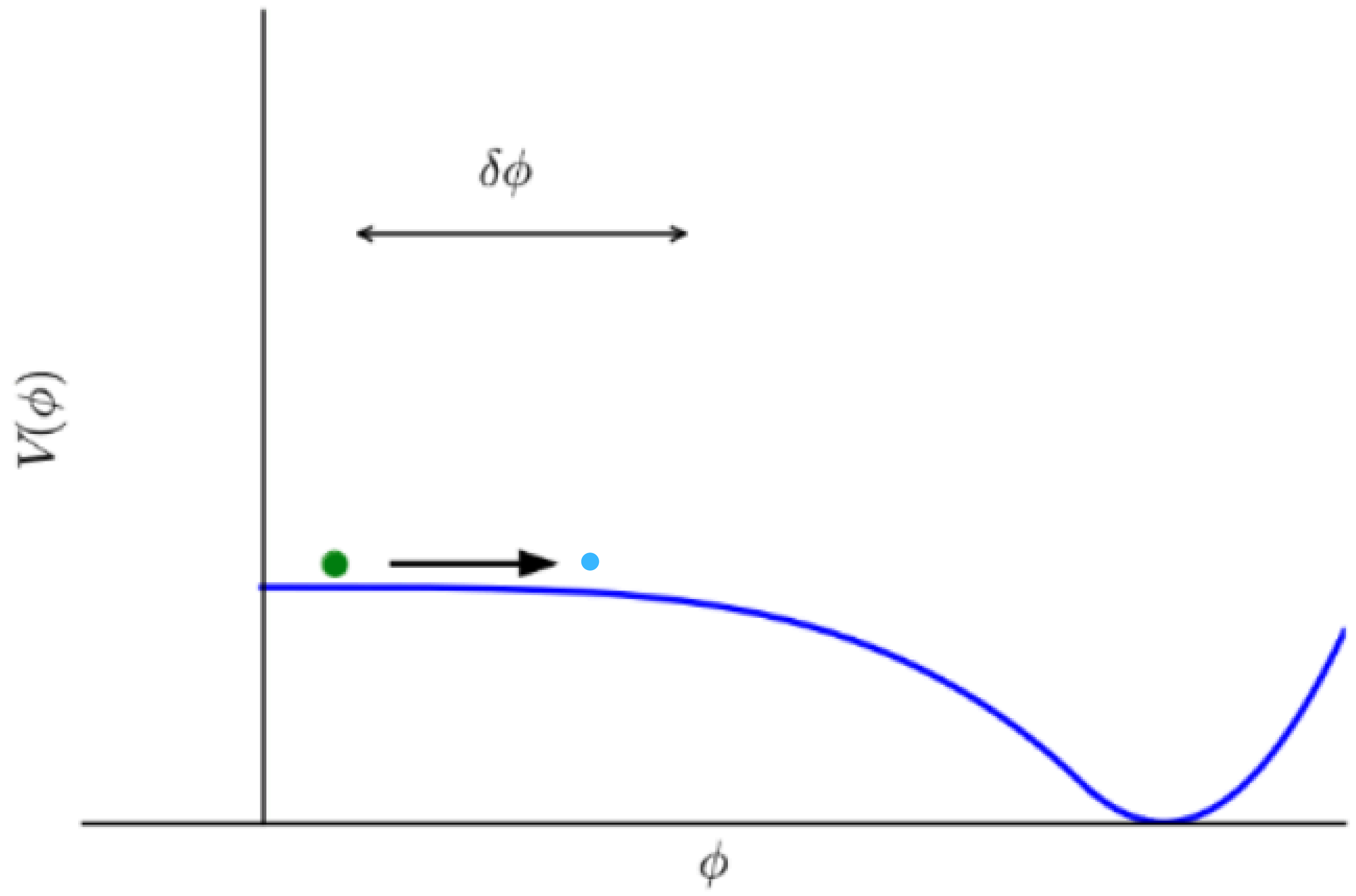
$\phi(t, x, y, z)$

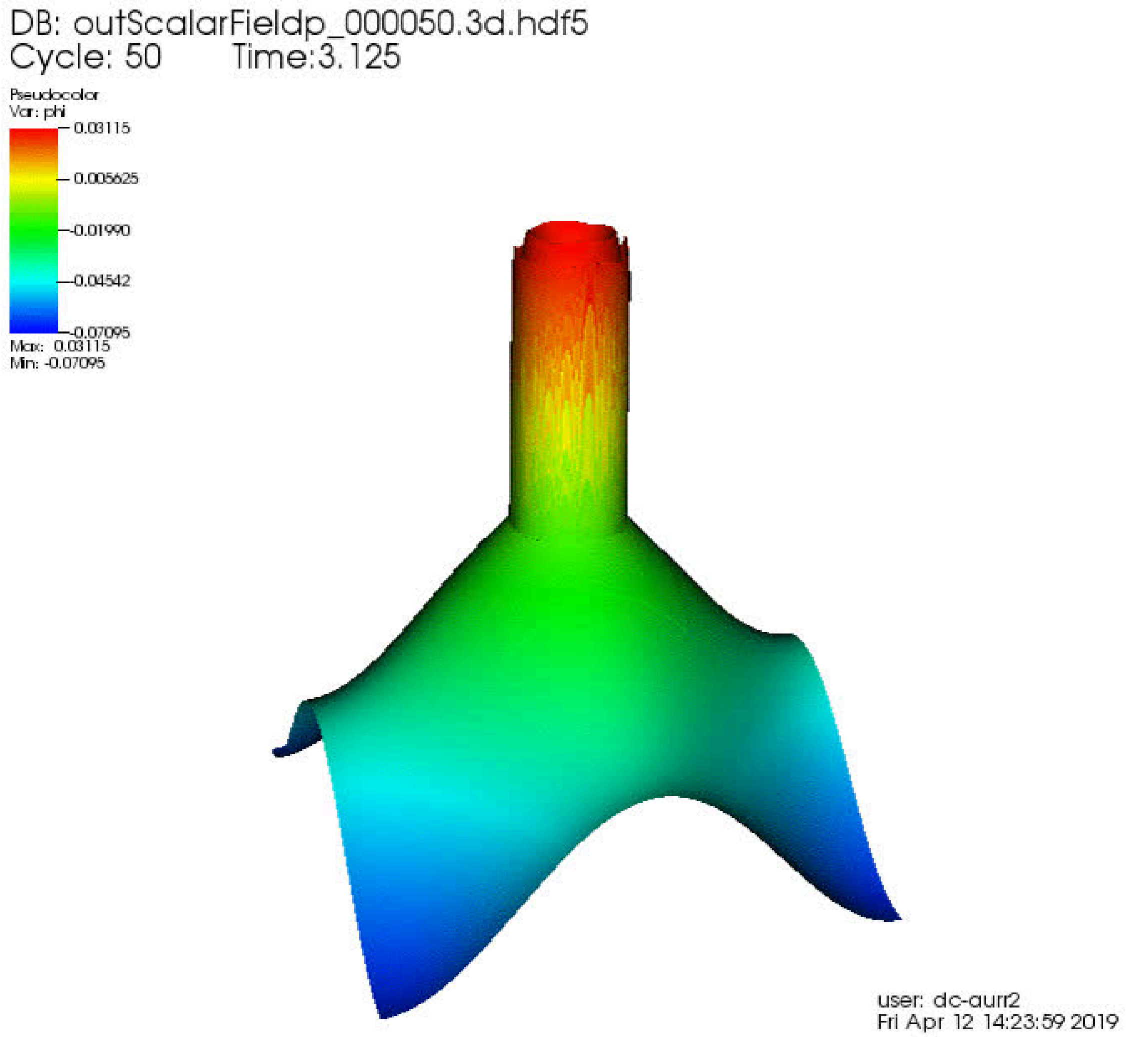
$\dot{\phi}(t, x, y, z)$



The paradigm revisited

Once upon a time there was a spatially **inhomogeneous** scalar field $\phi(t, x, y, z)$ rolling





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The paradigm revisited

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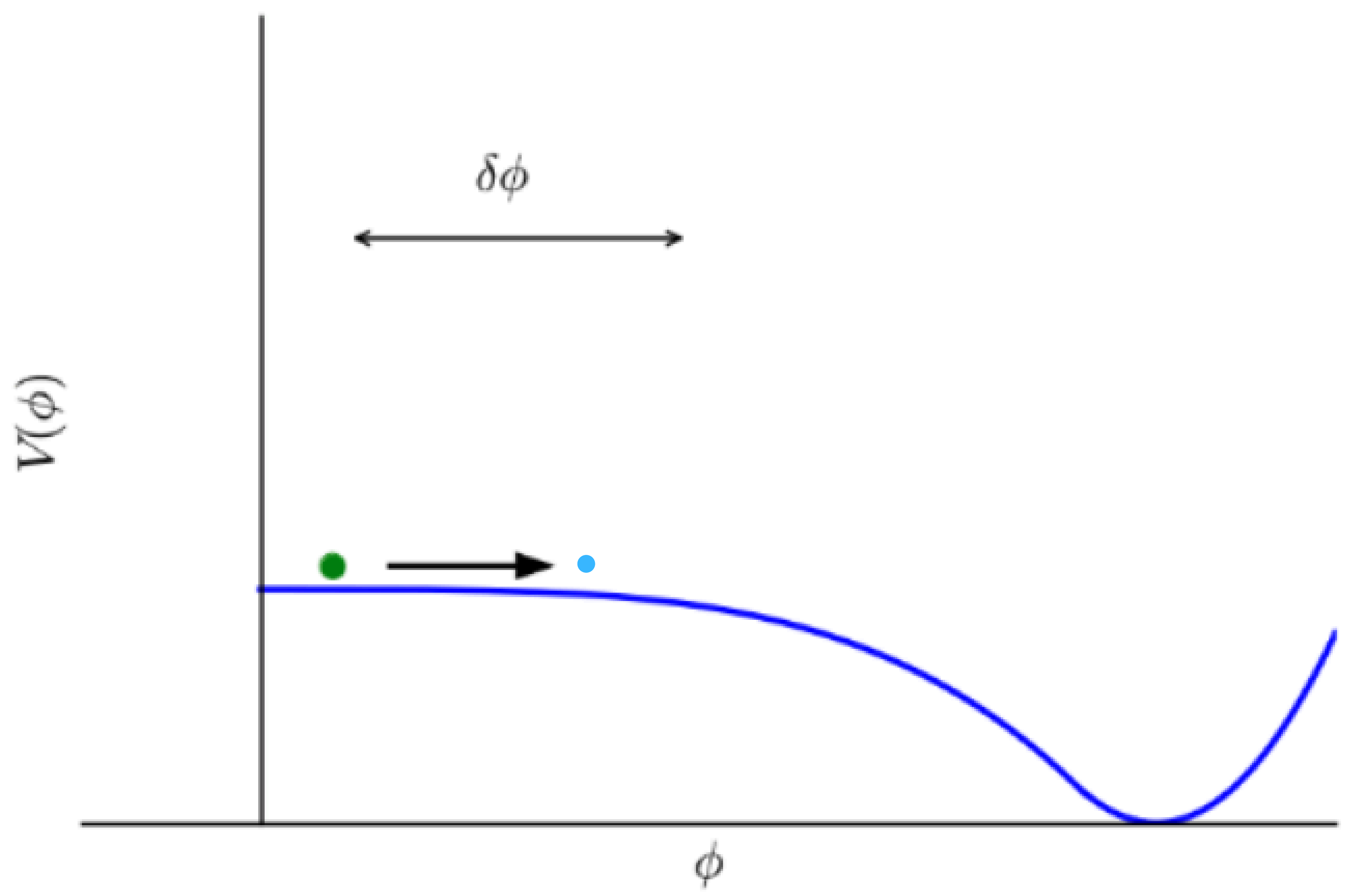
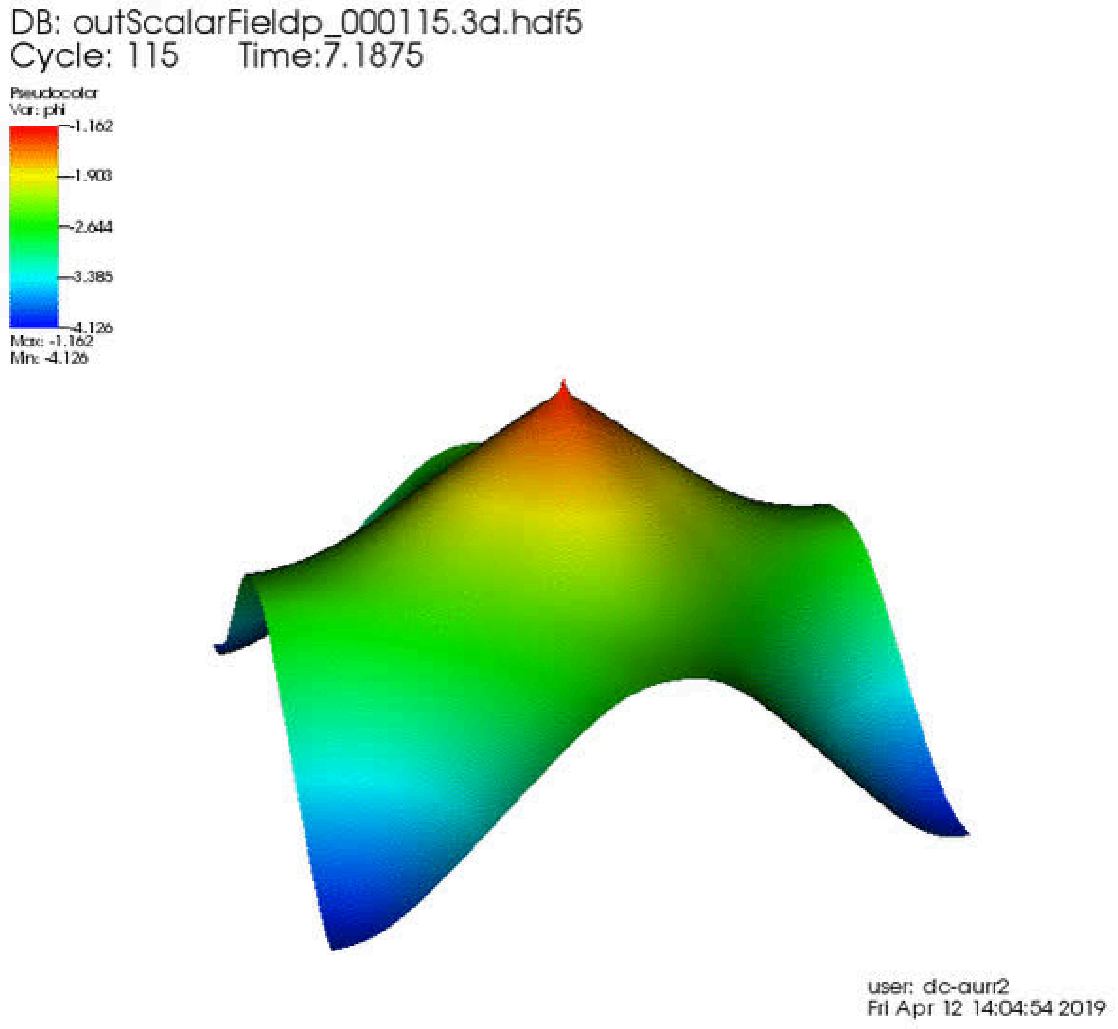
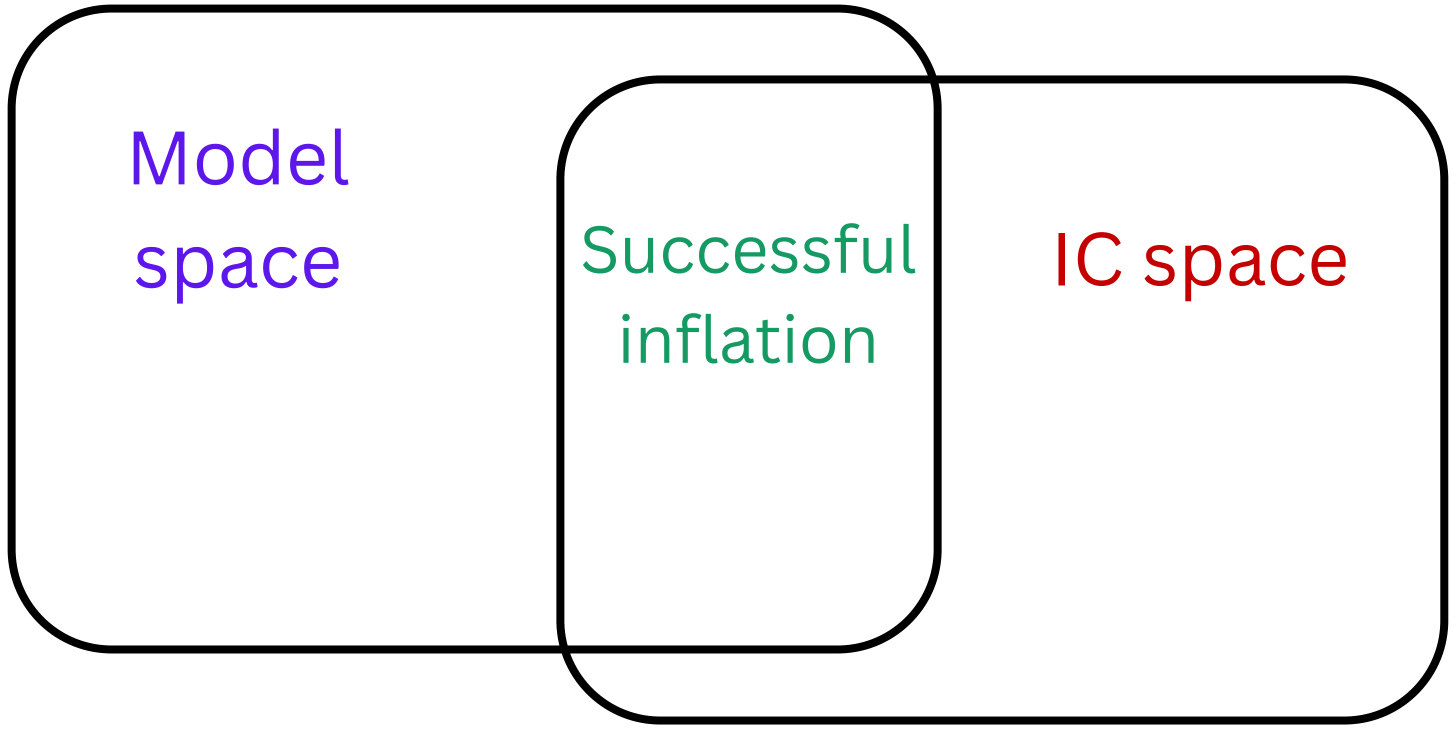




The phase space of a succesful inflation

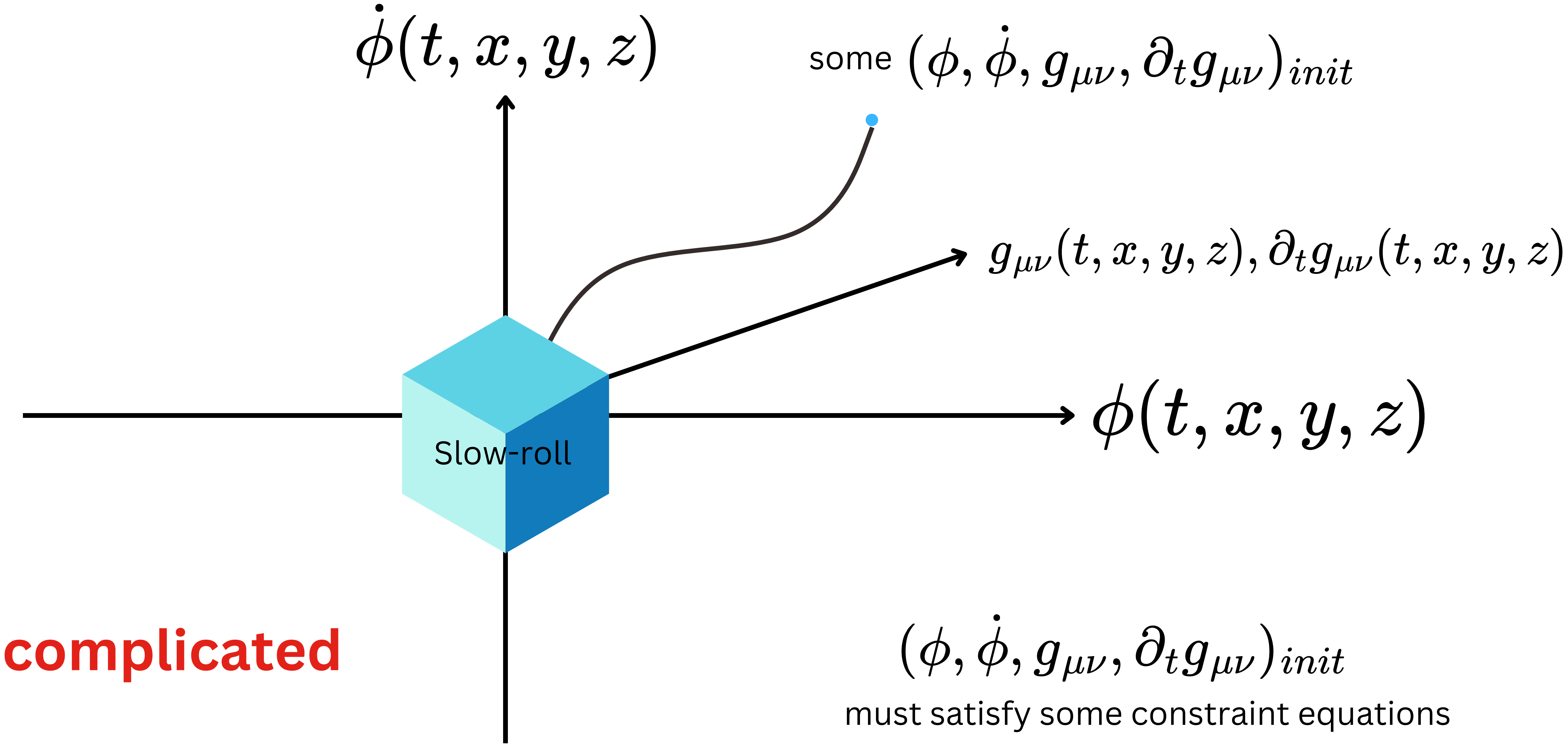
Model space: The different inflationary models (single field, multifield etc)

IC space: dynamical variables initial configuration



General Recipe: Take an inflation model and check how it works for different initial conditions

Single Field Inflation actual IC phase space



Very complicated

$(\phi, \dot{\phi}, g_{\mu\nu}, \partial_t g_{\mu\nu})_{init}$

must satisfy some constraint equations

Testing Inflation

Step 1: Pick an inflation model

Step 2: Choose initial conditions

Step 3: Check if it gives N>60 efolds

An inflationary model is **robust** to inhomogeneous initial conditions
if its phase space has an attractor behavior and
if N>60 at least at some patch of Hubble size

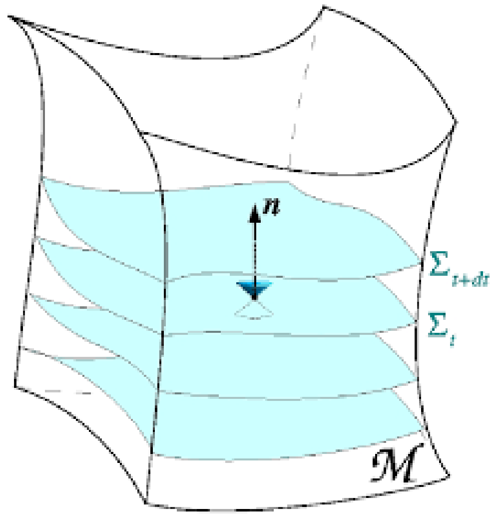
Can inflation be successful if it starts with inhomogeneous initial conditions?

↓

Numerical Relativity

3+1 Decomposition

Foliate the spacetime into 3D slices evolving in a time coordinate



Gravity sector:

3d metric γ_{ij}

Extrinsic curvature $K_{ij} \sim \partial_t \gamma_{ij}$

$$K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij} K$$

traceless: tensor modes

trace: expansion/contraction

Matter sector: $\phi(\vec{x}), \dot{\phi}(\vec{x})$

$K = -3H$ for FLRW

For IC specify matter , solve constraints for geometry, fix gauge dofs

How to simulate inhomogeneous inflation

- Take a model with potential $V(\phi)$
- Specify initial field configuration with harmonic perturbations in 3 directions and put maximum **only one** perturbation mode per species

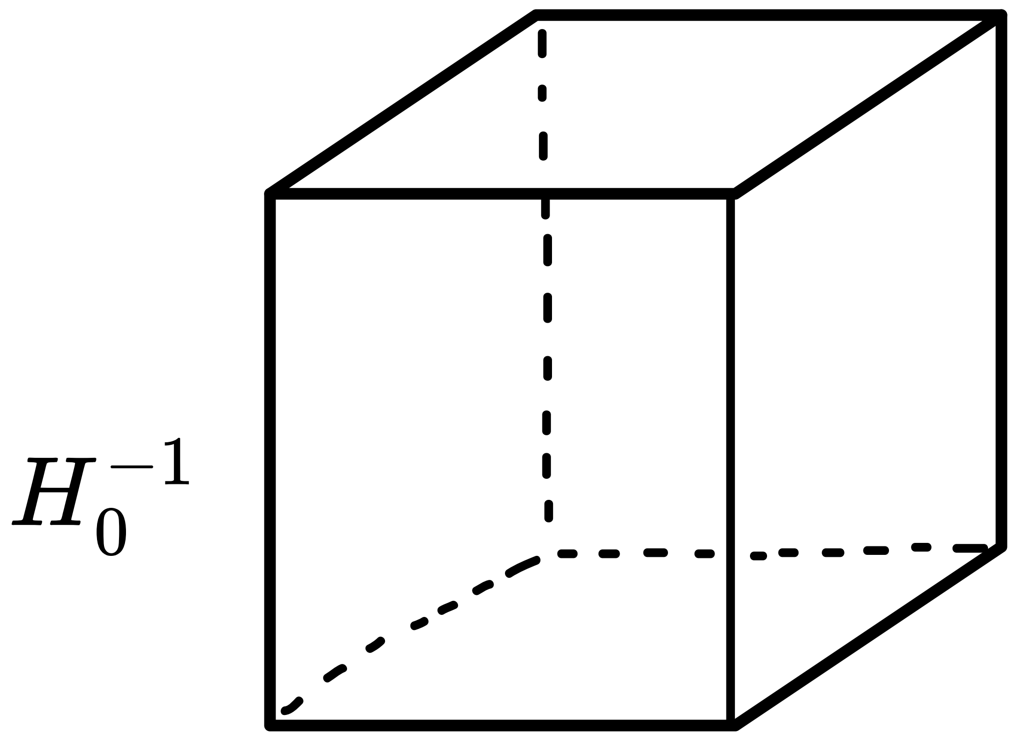
"Scalar" inhomogeneities: $\phi_{init}(\vec{x}) = \phi_0 + \Delta\phi(\vec{x})$, $\dot{\phi}_{init}(\vec{x}) = 0$ $\Rightarrow$ ρ non trivial

"Momentum" inhomogeneities: $\phi_{init}(\vec{x}) = \phi_0 + \Delta\phi(\vec{x})$, $\dot{\phi}_{init}(\vec{x}) = \dot{\phi}_0 + \Delta\Pi(\vec{x})$ $\Rightarrow$ S_i also non trivial

"Tensor" inhomogeneities: A_{ij}^{TT} Vacuum GW perturbations

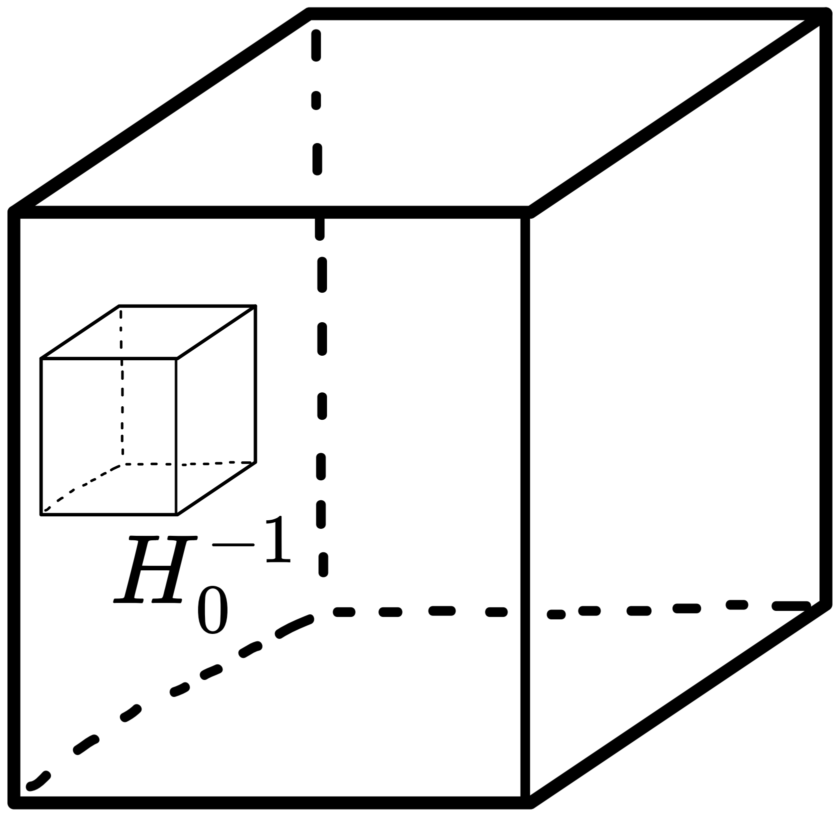
- Choose simulation domain size $L = H_0^{-1} = \left(\sqrt{\frac{8\pi G}{3}V(\phi_0)}\right)^{-1}$
- Solve constraints for the rest of geometric dofs

Simulate and count no. efolds locally in an patch of H_0^{-1} where the potential dominates and the scalar field has homogenised



$$N = \ln\left(\frac{a_{end}(t,\vec{x})}{\cancelto{1}{a_{init}}}\right)$$

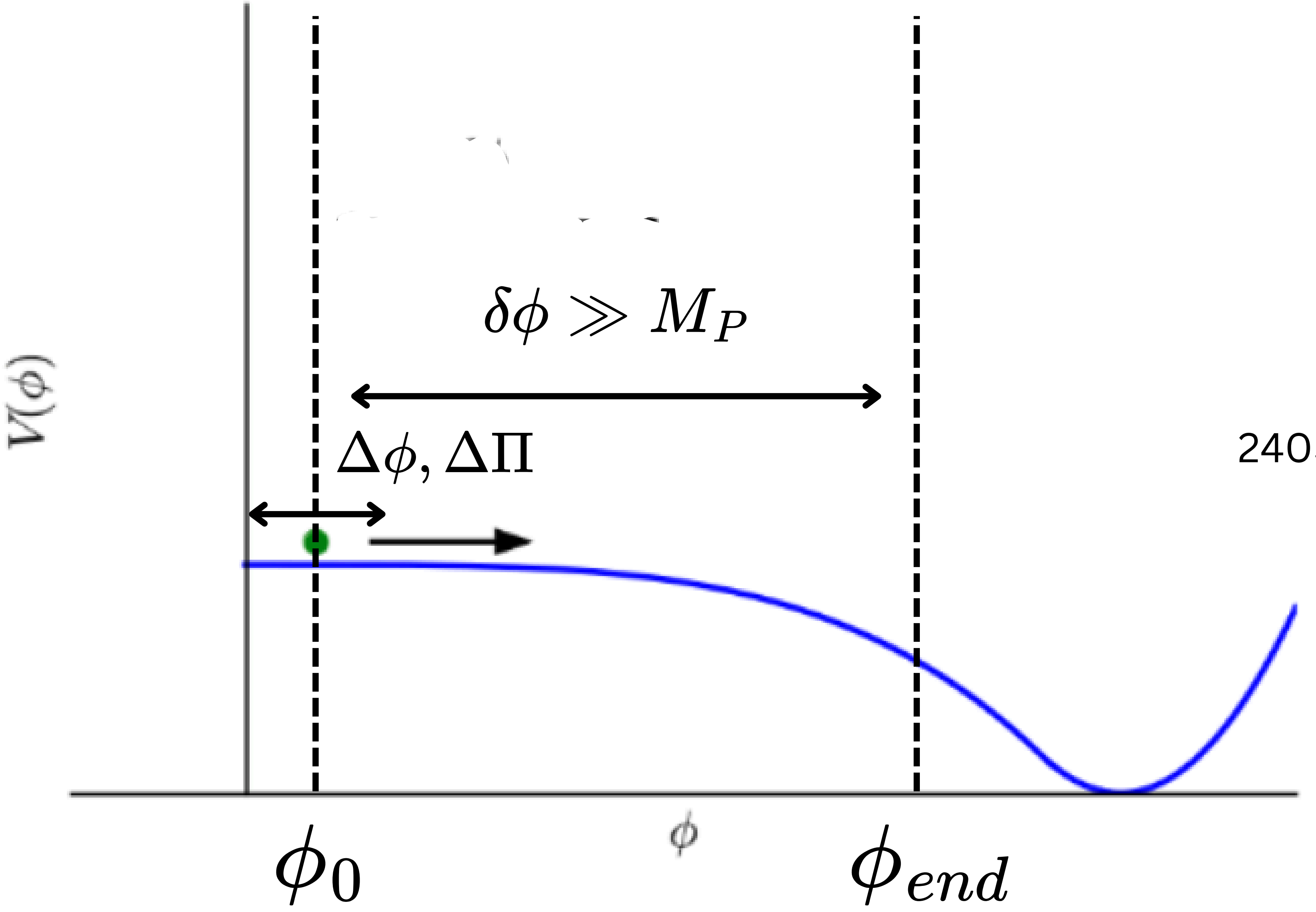
Any Lessons??



Lesson 1:

Models with **Superplanckian** characteristic scale $\delta\phi \gg M_P$ are very robust to all types of inhomogeneities

$$\rho_{inhom} \gg V(\phi_0)$$



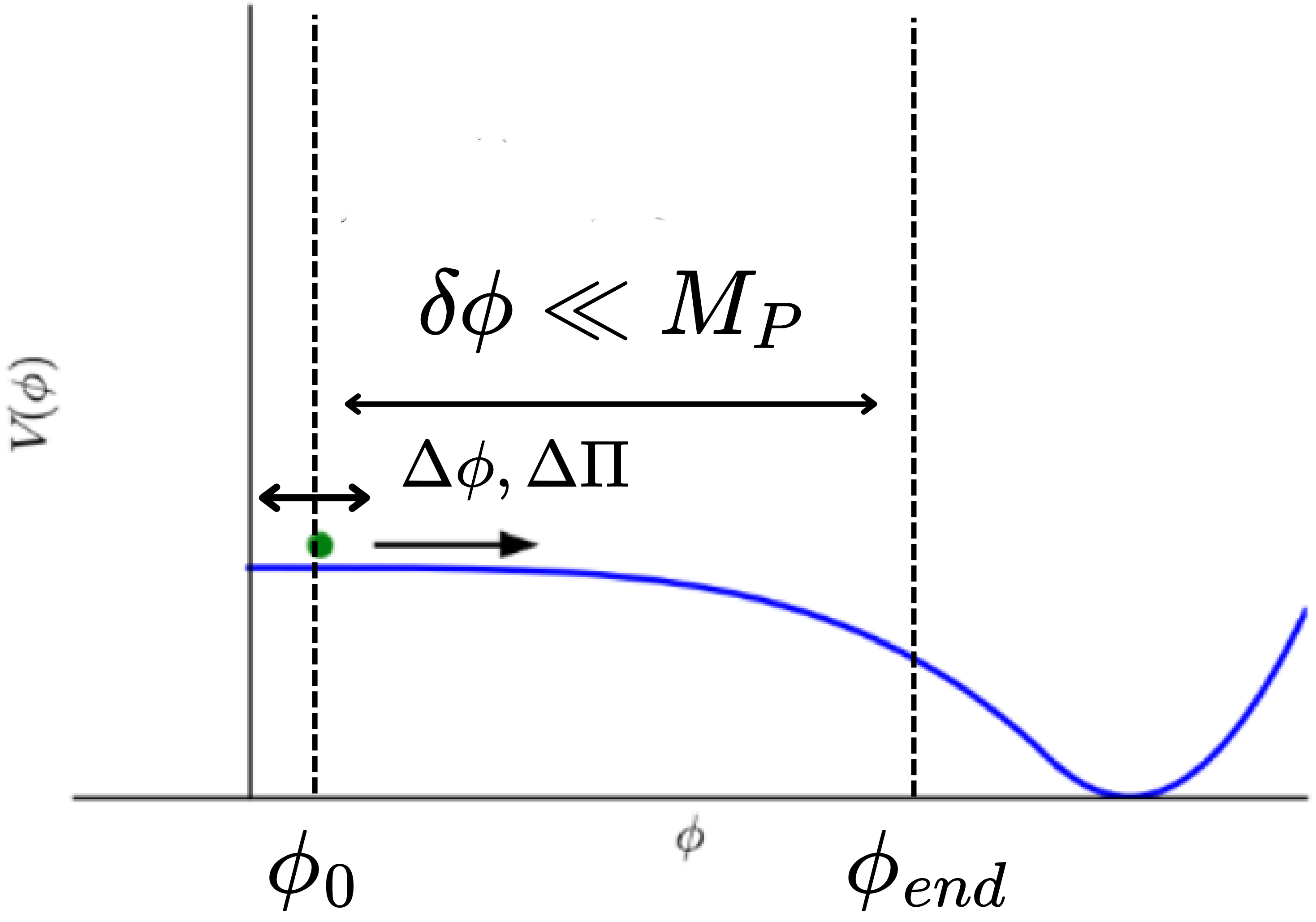
1608.04408: Clough, DiNunno, Fischer, Flauger, Lim, Paban

2405.03490: Aurrekoetxea, Clough, Elley, Flauger, **Giannadakis** and Lim

Lesson 2:

Models with **Subplanckian** characteristic scale $\delta\phi \ll M_P$ are NOT robust to inhomogeneities, inflation is pushed off the slow roll region

$$\rho_{inhom} \sim 10^{-4} V(\phi_0)$$



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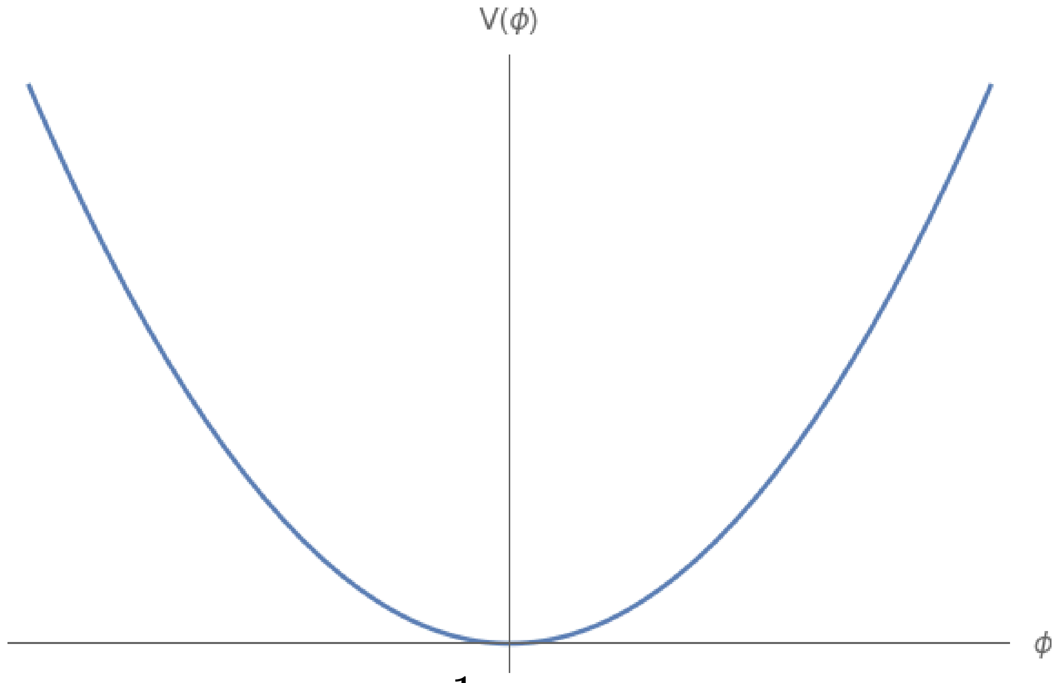
Lesson 3: Model space:
The shape of the potential is important:

Convex models: Robust to inhomogeneities

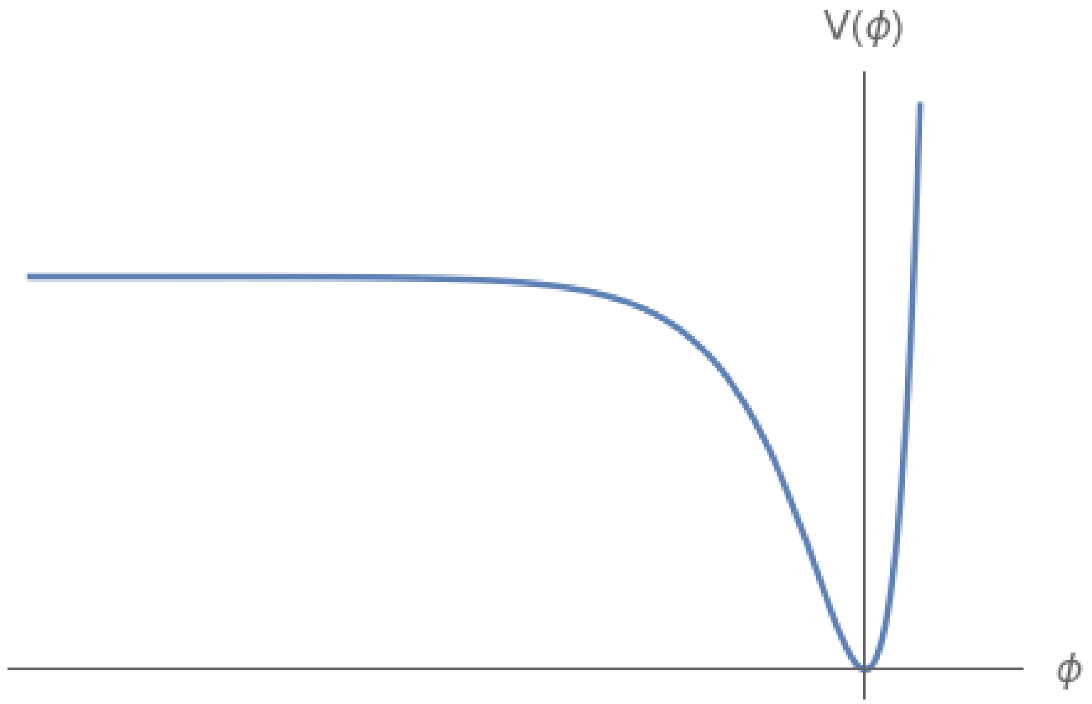
$$V''(\phi_0) > 0$$



$\frac{1}{2}m^2\phi^2$

Concave models: Not Robust to inhomogeneities

$$V''(\phi_0) < 0$$

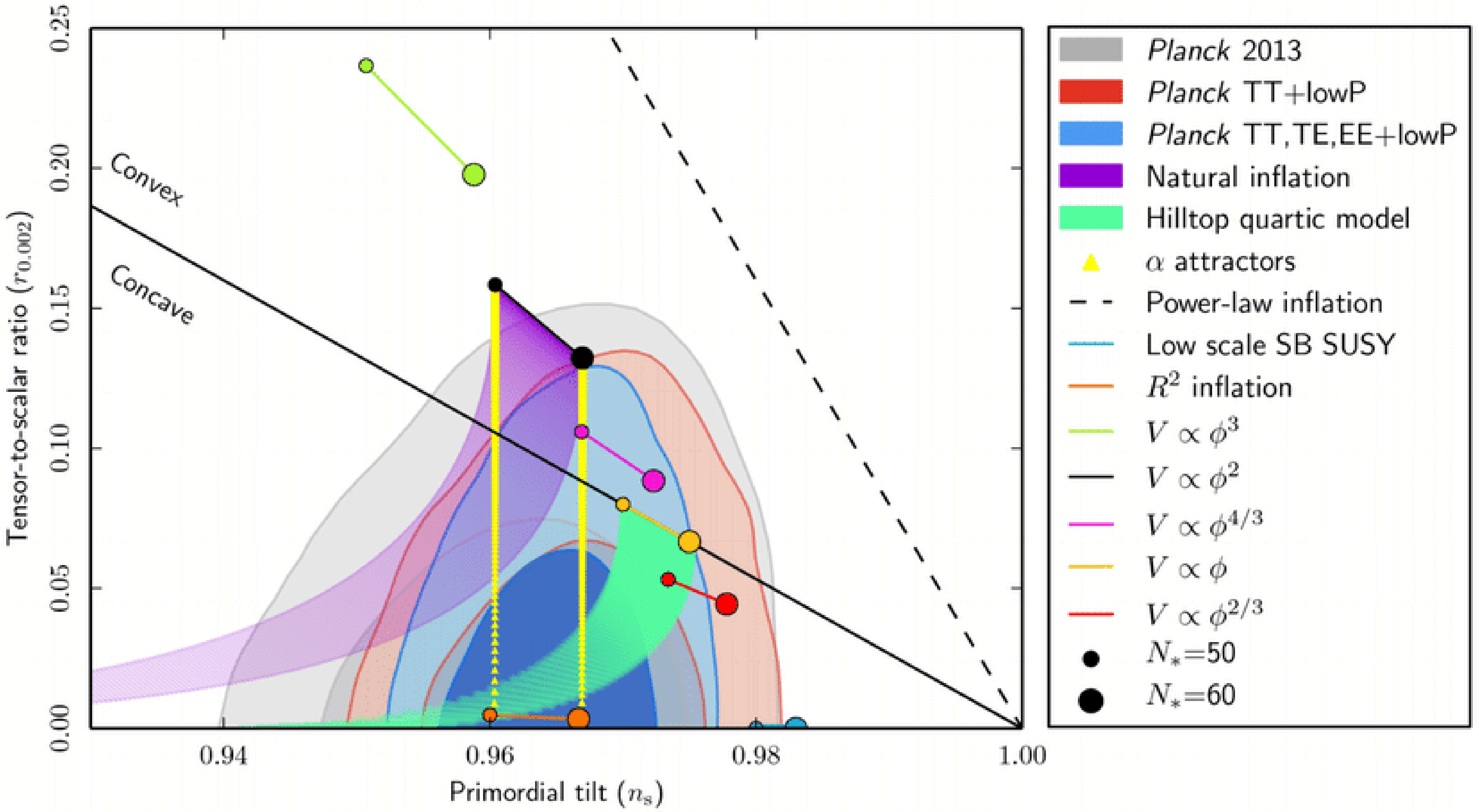


eg. of α-attractor

Lesson 3: Model space: The shape of the potential is important:

Convex models: Robust to inhomogeneities

Concave models: Not Robust to inhomogeneities



Lesson 4: most dangerous wavelength

$$\lambda = H_0^{-1}$$

Deep Subhorizon wavelengths: they homogenise and requires larger inhomogeneities to fail

Superhorizon wavelengths: give patches where inflation might fail very quickly and others where lasts N>>60

Near Horizon wavelengths $\lambda = H_0^{-1}$ **:** Inflation might fail everywhere

1608.04408: Clough, DiNunno, Fischer, Flauger, Lim, Paban

2405.03490: Aurrekoetxea, Clough, Elley, Flauger, **Giannadakis** and Lim

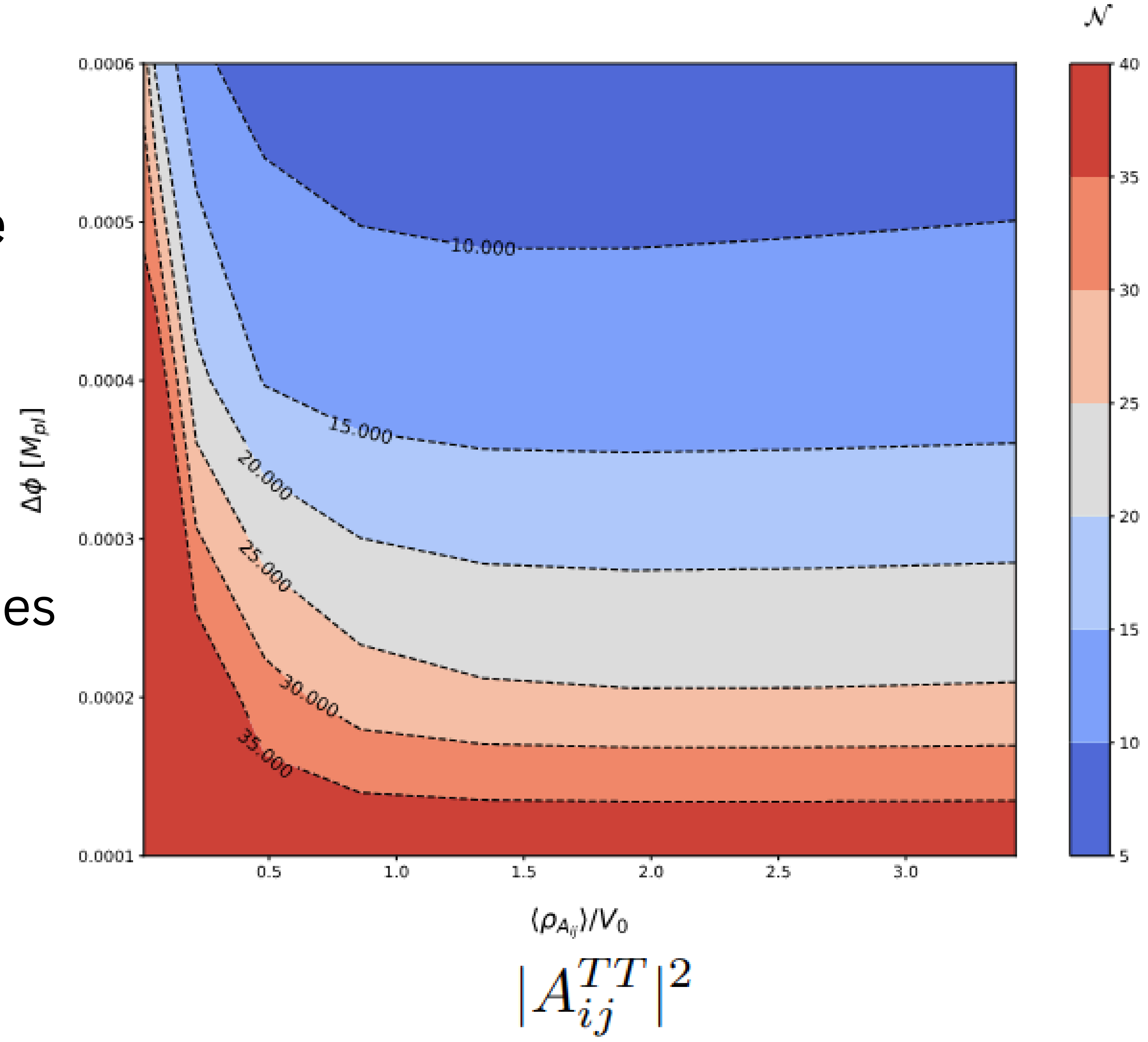
250x.xxxxx: Elley, Flauger, **Giannadakis**,Lim

Lesson 5: Tensor modes are not dangerous

Scalar inhomogeneity amplittude vs tensor mode amplitude

1712.07352: Clough, Flauger, Lim

N gets saturated with tensor modes amplitude



Lesson 6: Kinetic inhomogeneities make inflation less robust

$$N_\Pi = 0$$

$$\dot\phi_0 \neq 0$$

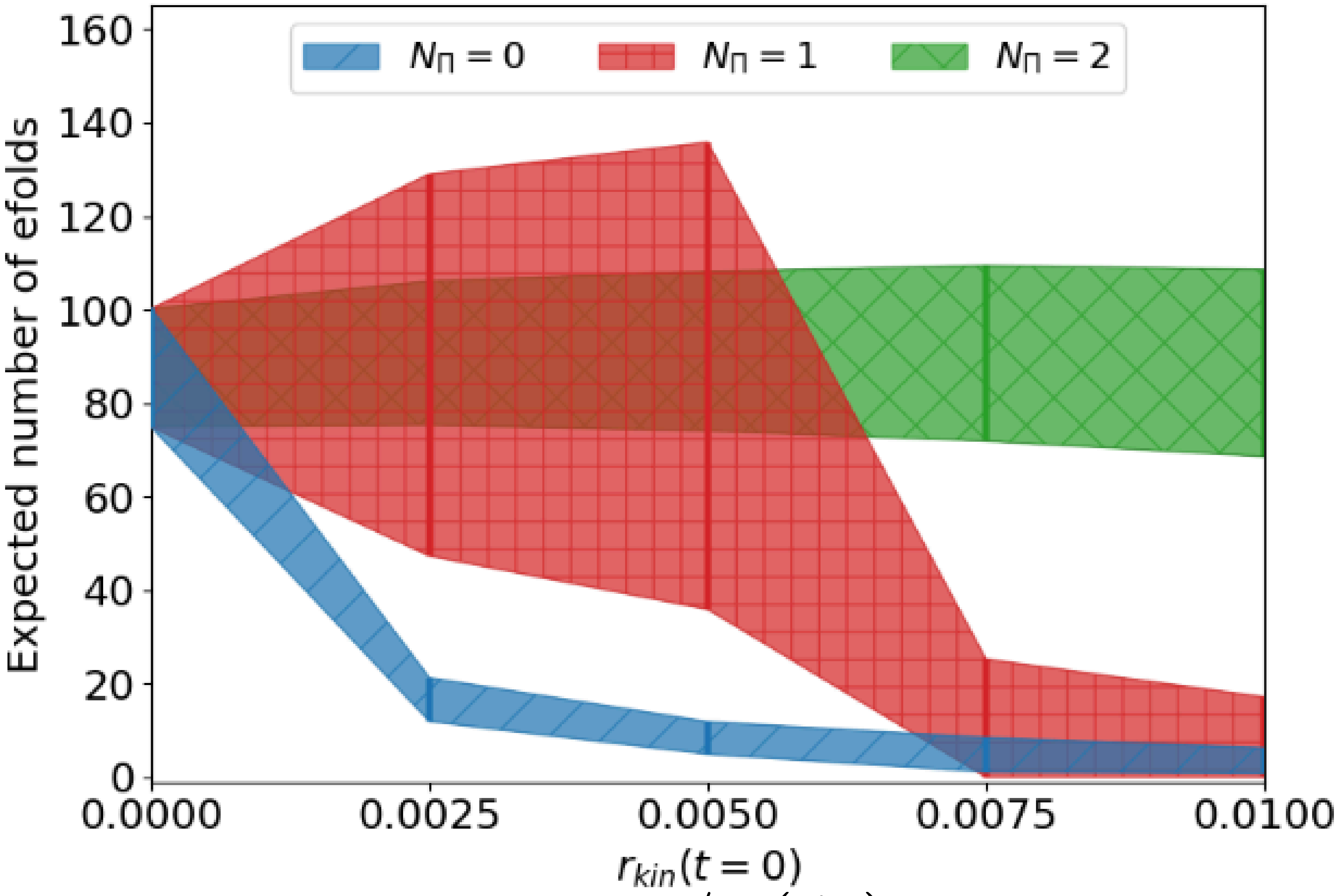
homogenous boost
most dangenrous

$$N_\Pi = 1$$

$$\dot\phi_0 + \Delta\Pi(\vec{x}) \neq 0$$

$$\lambda = H_0^{-1}$$

phase dependece



$$\rho_{kin\,0}/V(\phi_0)$$

2405.03490:
Aurrekoetxea, Clough, Elley, Flauger, **Giannadakis** and Lim

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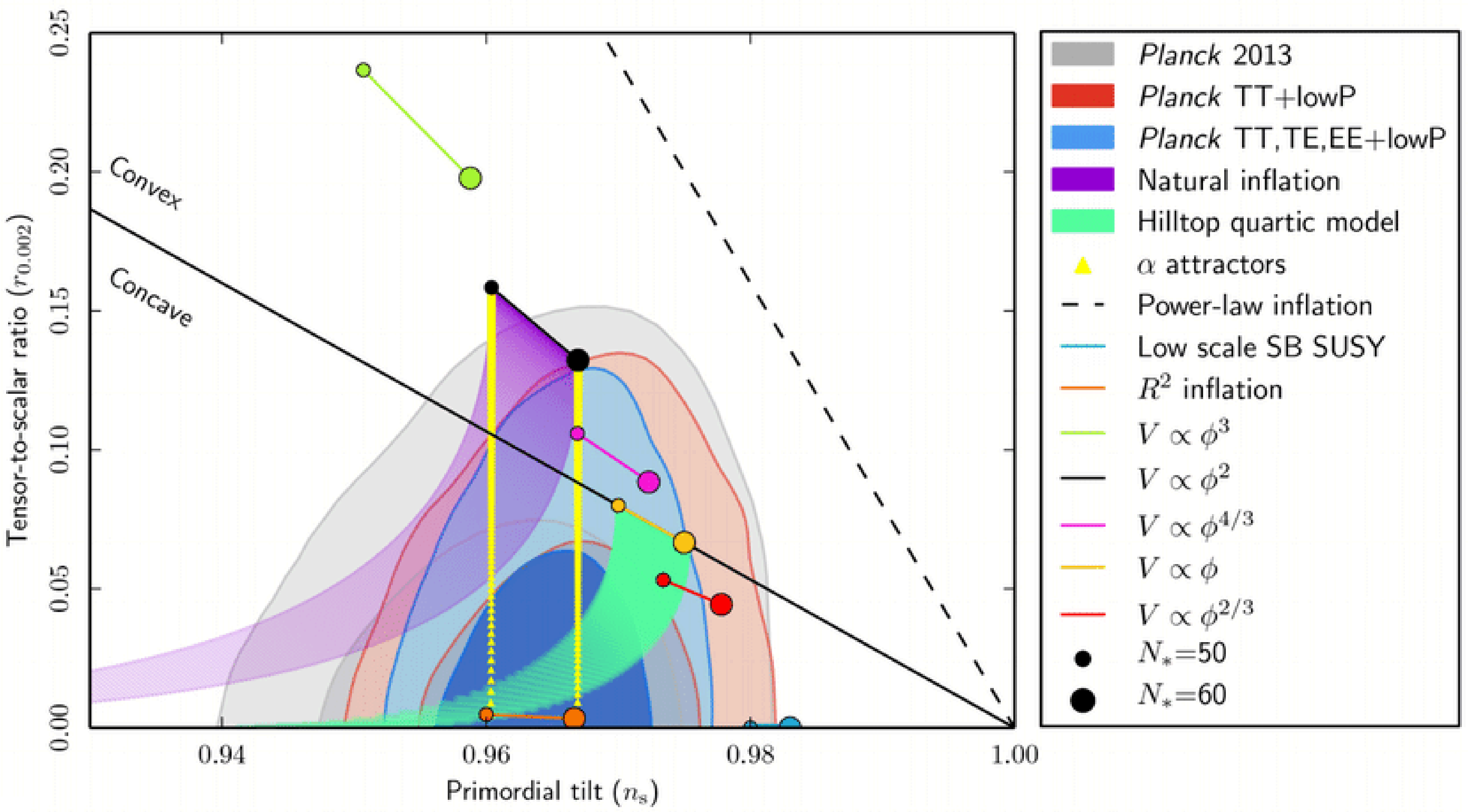
Constraining α-attractors

WORK IN PROGRESS



2508.xxxxx: Elley, Flauger, **Giannadakis**, Lim

Scalar Power Spectrum: $\mathcal{P}_s = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$

Tensor Power Spectrum: $\mathcal{P}_s = A_t \left(\frac{k}{k_*}\right)^{n_t}$

$$r = \frac{\mathcal{A}_t}{\mathcal{A}_s}$$

Upper bound from observations:

$$r \leq 0.036$$

What about lower bound?

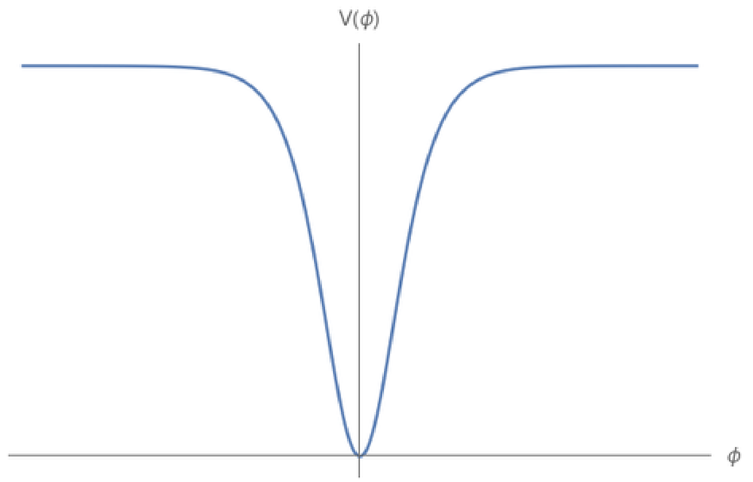
Lesson 8: Constraining r

Assume IC (dummy equipartition): $V(\phi_0) \sim \langle V \rangle \sim \langle \rho_{grad} \rangle \, (\sim \langle \rho_{kin} \rangle$ ✗$)$

Check for different μ small scale modelsand see where iflation starts to happen

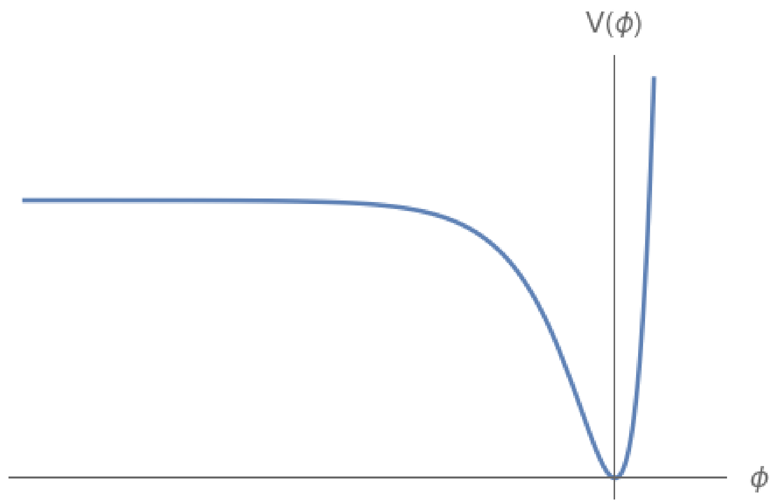
T-type:

$$V(\phi) = \Lambda^4 \tanh^2(\frac{\phi}{\mu})$$



E-type:

$$V(\phi) = \Lambda^4 (1 - e^{\frac{\phi}{\mu}})^2$$



Is there a critical μ for robustness? Possible low constraint for r:

$$\mu \leftrightarrow \delta\phi \Rightarrow r \approx 16\epsilon(\phi_0, \mu)$$

Lesson 8: Constraining r

T-model results:

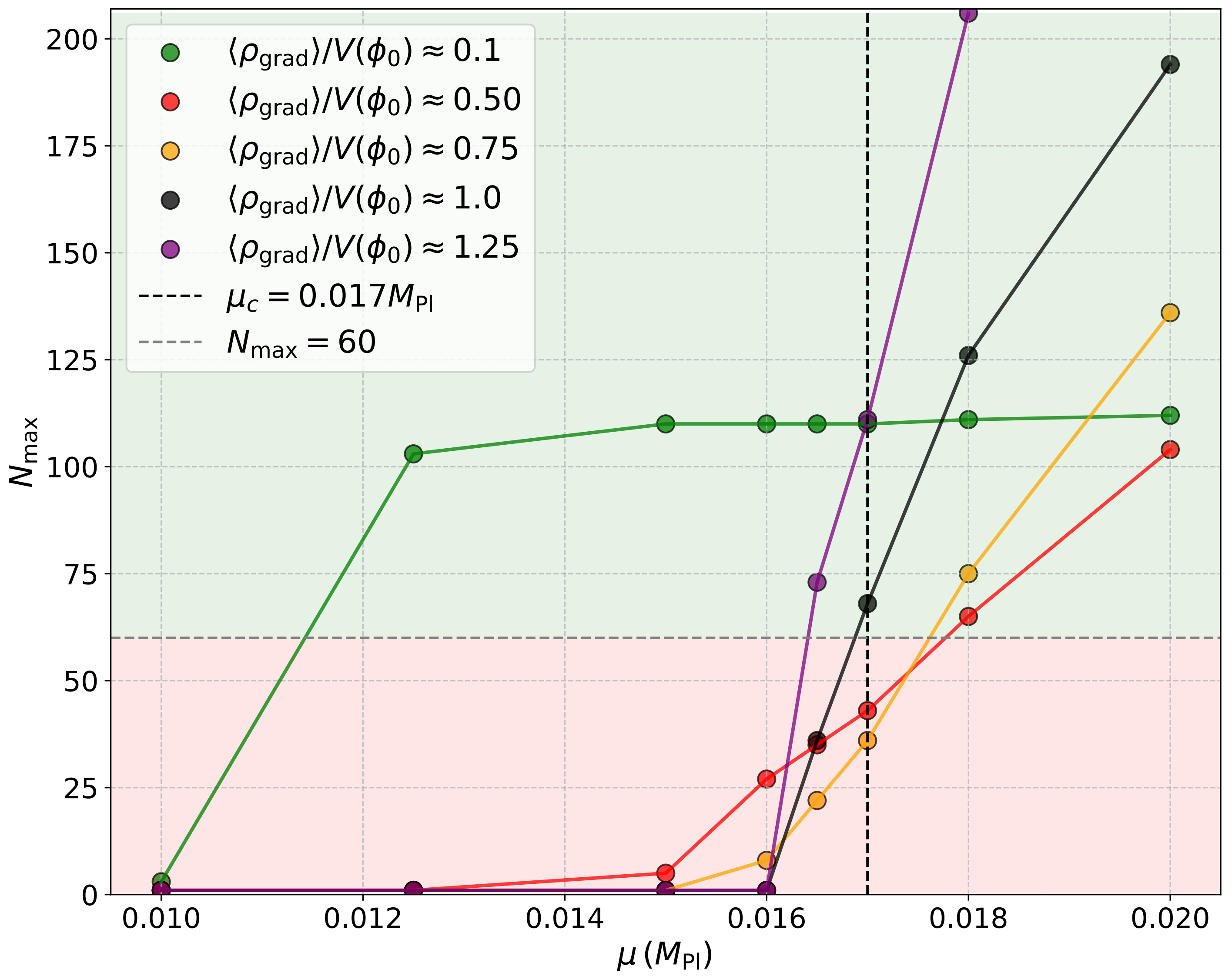
$$\mu_c = 0.017 M_P \Rightarrow r_c = 1.45 \times 10^{-6}$$

E-model results:

$$\mu_c = 0.017 M_P \Rightarrow r_c = 3.63 \times 10^{-7}$$

Warning: no such IC for $\mu < 0.017 M_P$

(scalar field starts exploring the exponential wall)



max no. efolds vs μ

Take Home

- Our Universe is unique and inflation “should” give a mechanism to obtain this “uniqueness” semi-independently of IC
- Zeroth order message: small scale dangerous, large scale safe
- Kinetic inhomogeneities dangerous especially homogeneous boosts
- Model dependence: Convex models > Concave models
- Possible low bound for tensor-to-scalar ratio for α-attractors

¡Thank you!