August 8th, 2025

Gravitational effects on sound waves:

-a perturbative approach for large bubbles in cosmological FOPTs-

Numerical Simulations of Early Universe sources of GWs @ NORDITA

Jun'ya Kume (University of Padova)

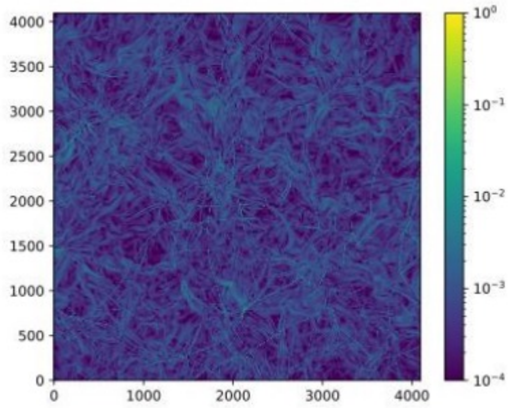
Based on JCAP 02 (2025) 057 (arXiv:2408.10770)

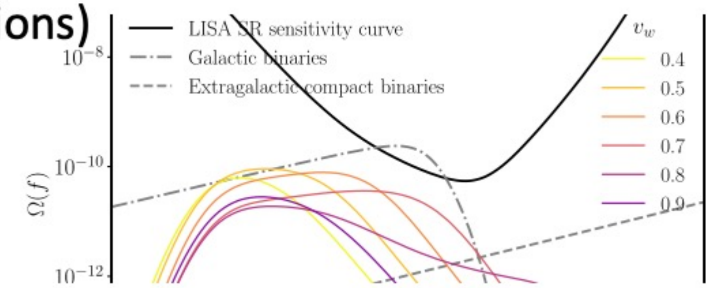
with Ryusuke Jinno (U. Kobe)

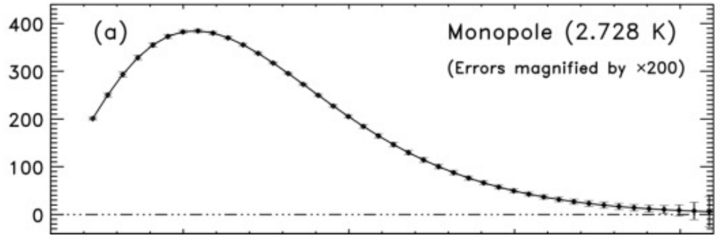
- “Wishlist” from Mark’s talk

Can we make GWs as good as the CMB?

- More simulation work needed:
  - Convergence at peak wavenumbers (longer runs)
  - More (v_w, α_n) - is $\tilde{\Omega}_{gw}^{\infty} = 0.017$ for both runs a fluke?
  - More realistic equations of state
    - e.g. constant c_s model Giese et al 2021
  - Temperature-dependent nucleation & friction (deflagrations)
  - Strongly supercooled transitions Lewicki, Vaskonen 2022
  - Hubble-sized bubbles: gravitational effects
- Models to incorporate simulation developments
  - Reheating effects in deflagrations
  - Kinetic energy decay
  - Large bubble regime







Outline

- Need for large bubble SGWB templates!
- Higgsless scheme for Minkowski background
- Accommodating gravity: "cosmological hydrodynamics"
- Summary & Outlook

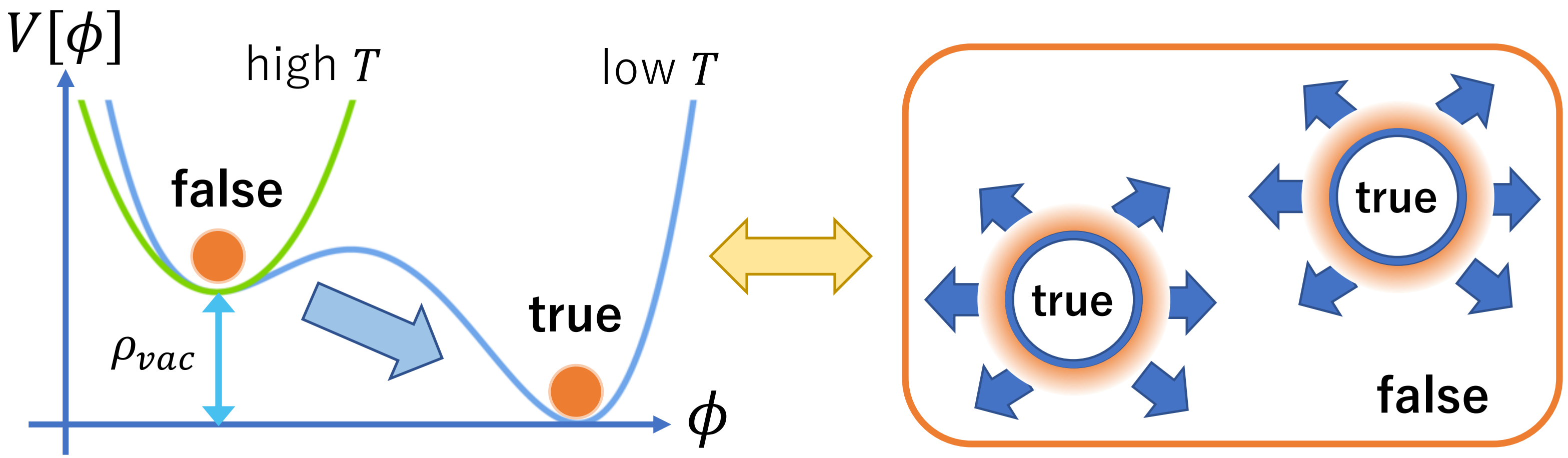
Need for large bubble SGWB templates!

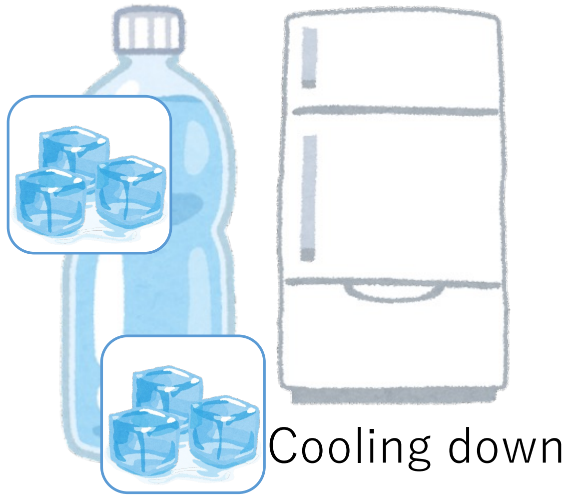
- GW production in FOPT

microscopic: quantum tunneling of a "Higgs" field ϕ

→ "bubble" nucleation in real space.

various realization in **BSM**





Need for large bubble SGWB templates!

- GW production in FOPT

microscopic: quantum tunneling of a "Higgs" field ϕ ← various realization in **BSM**

→ "bubble" nucleation in real space.

macroscopic: bubbles stir plasma

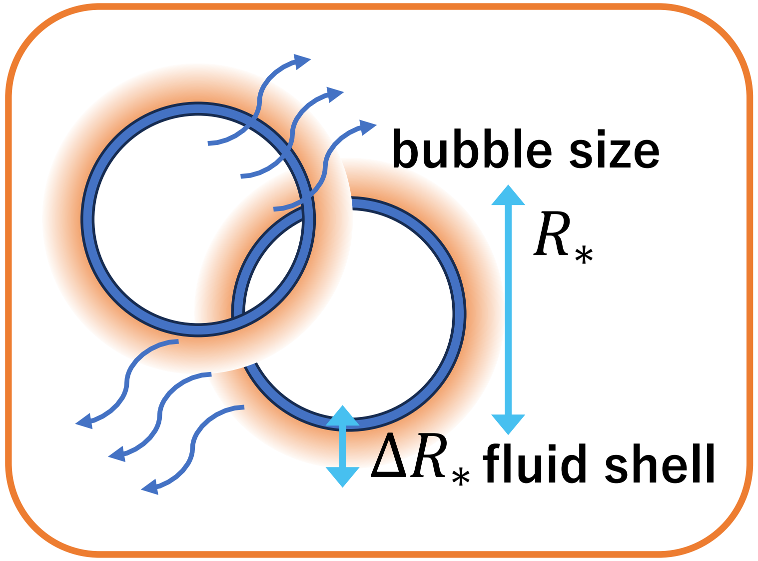
→ Bubble & fluid dynamics source **SGWB!**

α: strength of PT v_w: wall velocity

β: $\simeq$ (PT duration)$^{-1}$ T_*: nucleation temperature

collision of walls, **sound waves**, turbulence

→ extensively investigated both analytically & numerically

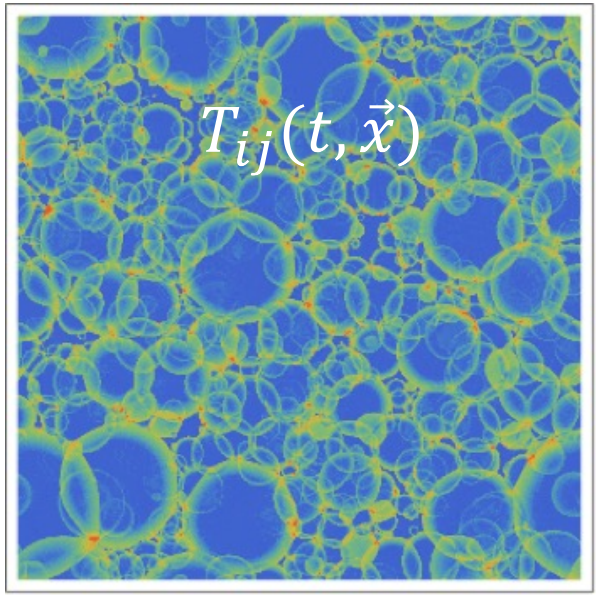


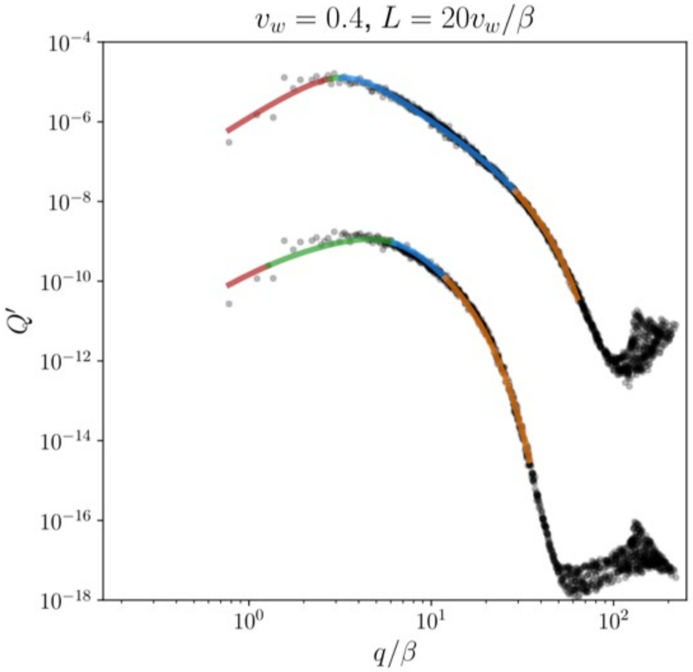
- SGWB from sound waves

3D hydrodynamic simulations → extracting SGWB spectrum

(Hindmarsh+ 2013, 2015, 2017, 2019, Jinno+ 2022, 2023, Caprini+ 2024, ...)

(Jinno+ 2022)





- vs analytic models?

e.g. sound shell model (Hindmarsh 2016)

- parametric dependence?

→ SGWB templates

e.g. CosmoGW (Roper-Pol & Stomberg)

- strong transition, non-linearity, ...

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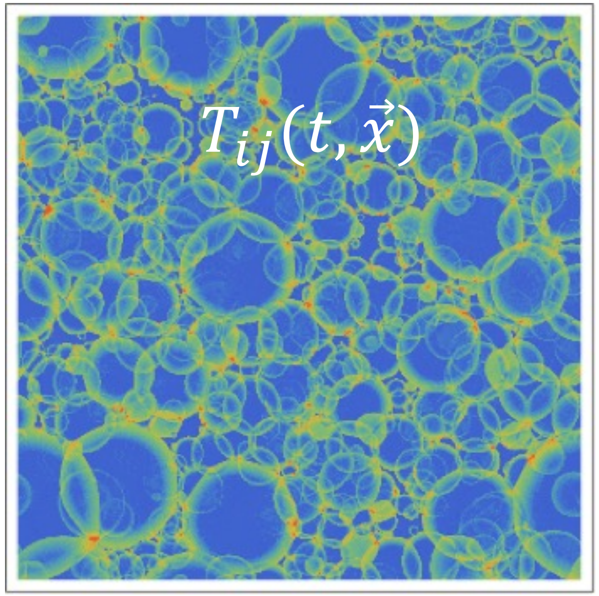

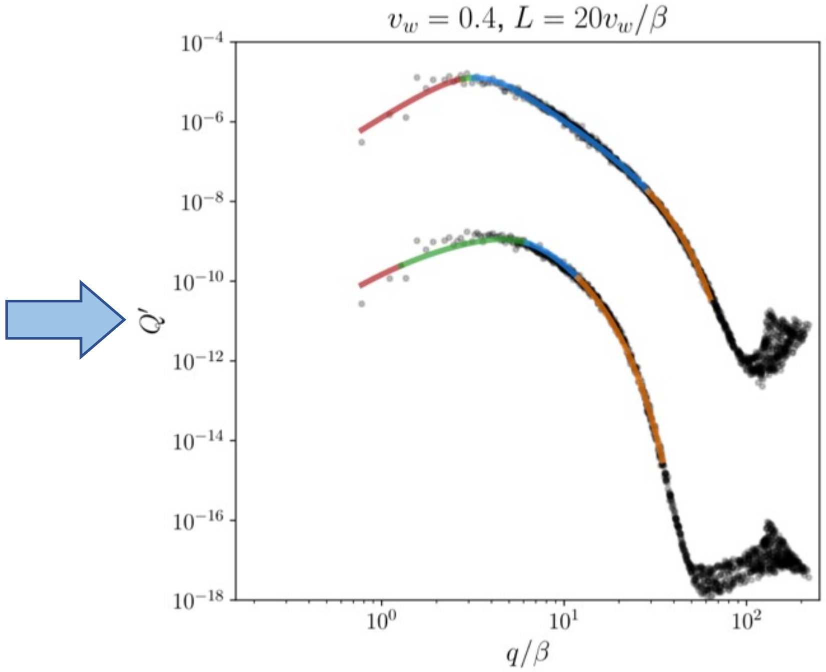

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e.g. CosmoGW (Roper-Pol & Stomberg)

- strong transition, non-linearity, ...

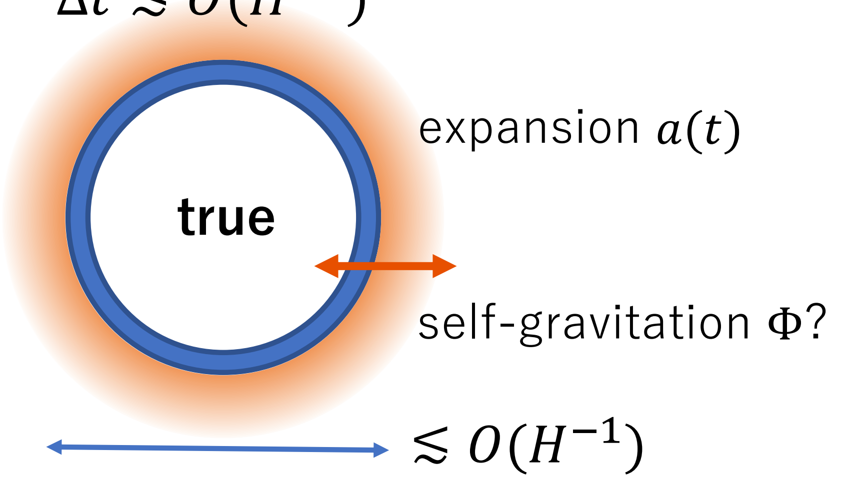
※ fast transition ↔ flat background

- Is this enough...? 🤔

Testable by near-future observation

→ **bubbles of Hubble size**

$\Delta t \lesssim O(H^{-1})$



(NANOGrav collaboration 2022)

$$\Omega_s(f) = \mathcal{D}\,\tilde{\Omega}_s \Upsilon(\tau_{\rm sw}) \left(\frac{\kappa_s\,\alpha_*}{1+\alpha_*}\right)^2 (H_* R_*)\, \mathcal{S}(f/f_s)$$

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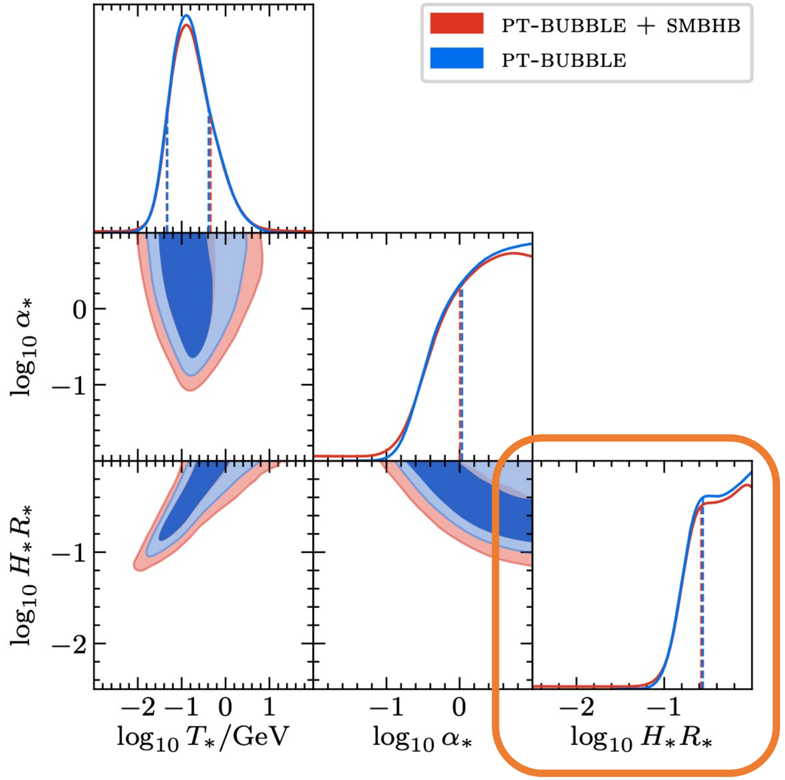

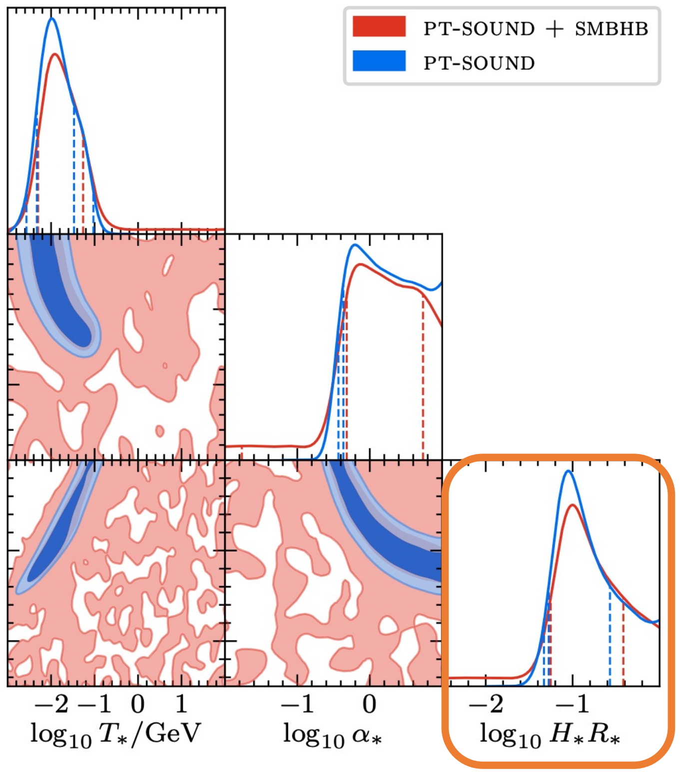

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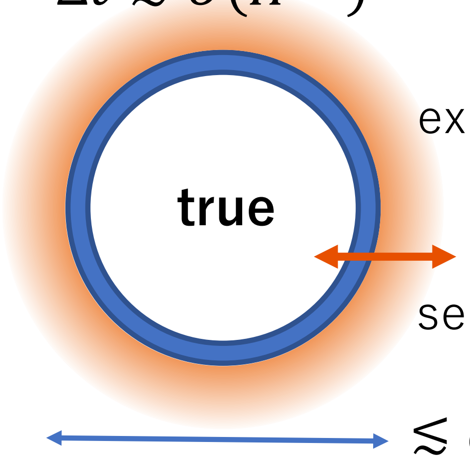

expansion $a(t)$

self-gravitation Φ?

$\lesssim O(H^{-1})$

General Relativistic treatment!

- Cai & Wang 2022
- Giombi & Hindmarsh 2023, Giombi+ 2025
- Jinno & Kume 2024
  → fluid profile around a bubble

(NANOGrav collaboration 2022)

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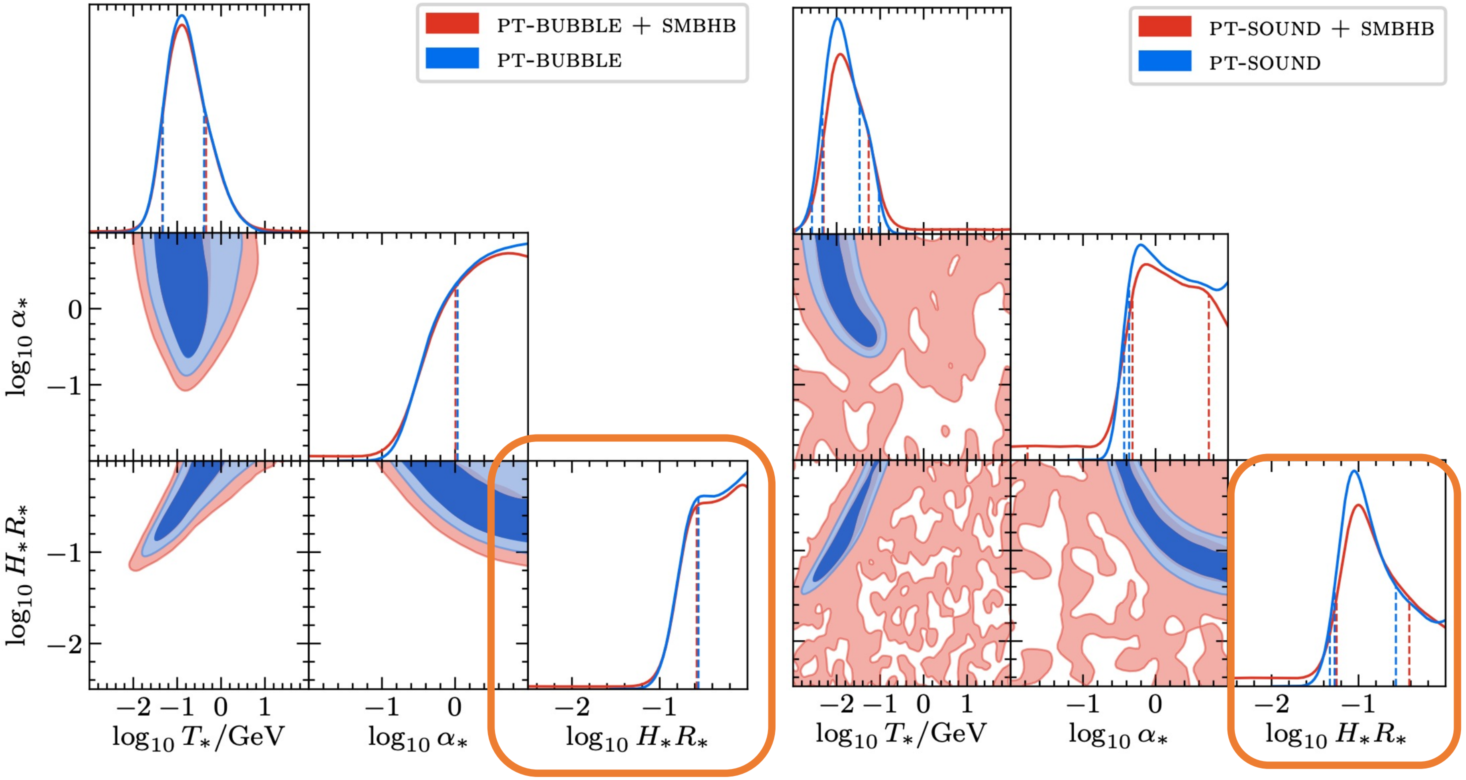

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Higgs-less scheme in Minkowski background

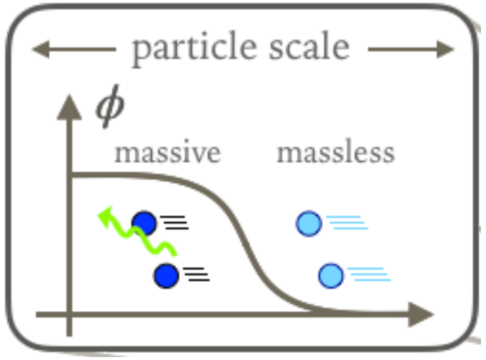
- How do we simulate the system?

scalar ϕ + hydrodynamics

$$\nabla_\mu T^{\mu\nu}_{\phi} = \delta^\nu$$

$$\nabla_\mu T^{\mu\nu}_{\text{fluid}} = -\delta^\nu$$

$$\delta^\nu = \eta u^\mu \partial_\mu \phi \partial^\nu \phi$$



$V(\phi, T)$ & η ↔ wall velocity v_w

(Hindmarsh+ 2013, 2014, …)

computationally demanding… ⌛

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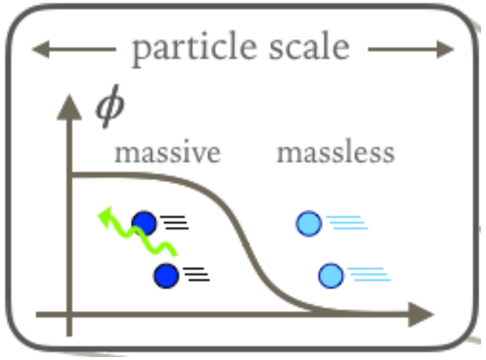

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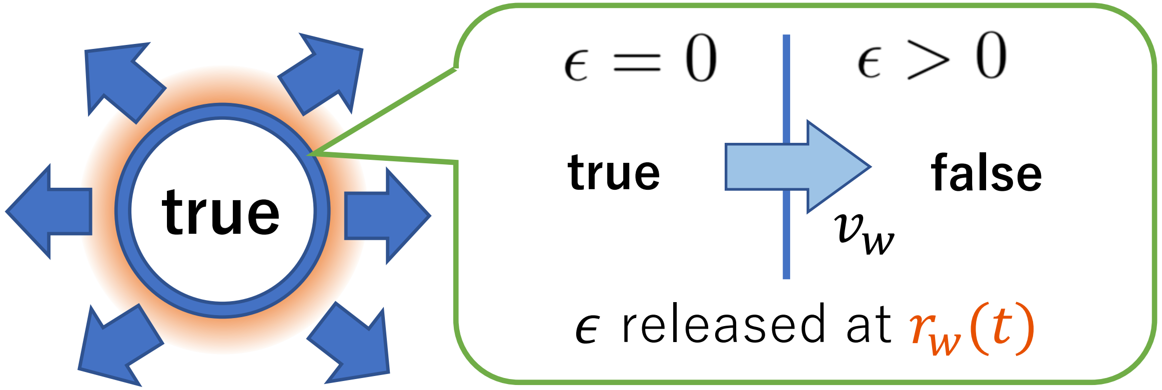
From a macroscopic point of view:



$$\nabla_\mu T^{\mu\nu}_{\text{f+vac}} = 0 \quad \text{with} \quad \rho_{\text{vac}} = \epsilon\, \Theta(r - r_w(t))$$

→ Hydrodynamics w/ **propagating boundary**

(Jinno+ 2020, 2022, Talks by Henrique & Isak!)

- Fluid dynamics around a single bubble

Bag EoS: $\rho(t,r) = aT^4(t,r) + \epsilon\,\Theta(r - r_w(t))$
$p(t,r) = (a/3)T^4(t,r) - \epsilon\,\Theta(r - r_w(t))$

EoM: $\nabla_\mu T^{\mu\nu}_{\mathrm{f+vac}} = 0$

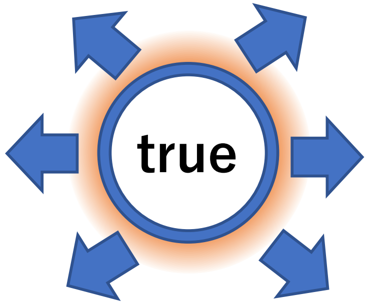
$T^{\mu\nu} = wu^\mu u^\nu + pg^{\mu\nu} \quad w = \rho + p$

$u^\mu = \gamma(1, v^i) \quad \gamma = 1/\sqrt{1 - v^2}$ ※ $c = 1$

$$\partial_t\left(w\gamma^2 - \frac{w}{4}\right) + \partial_r(w\gamma^2 v - v_w \epsilon\,\Theta(r - r_w(t))) + \frac{2}{r}(w\gamma^2 v) = 0,$$

$$\partial_t(w\gamma^2 v) + \partial_r\left(w\gamma^2 v^2 + \frac{w}{4} - \epsilon\,\Theta(r - r_w(t))\right) + \frac{2}{r}(w\gamma^2 v^2) = 0.$$

spherical symmetry → (t,r)



Bubble wall:

$v_w = const.$

$\alpha \equiv \epsilon/aT_\infty^4$

Minkowski background

※ discontinuity in ∂_r part

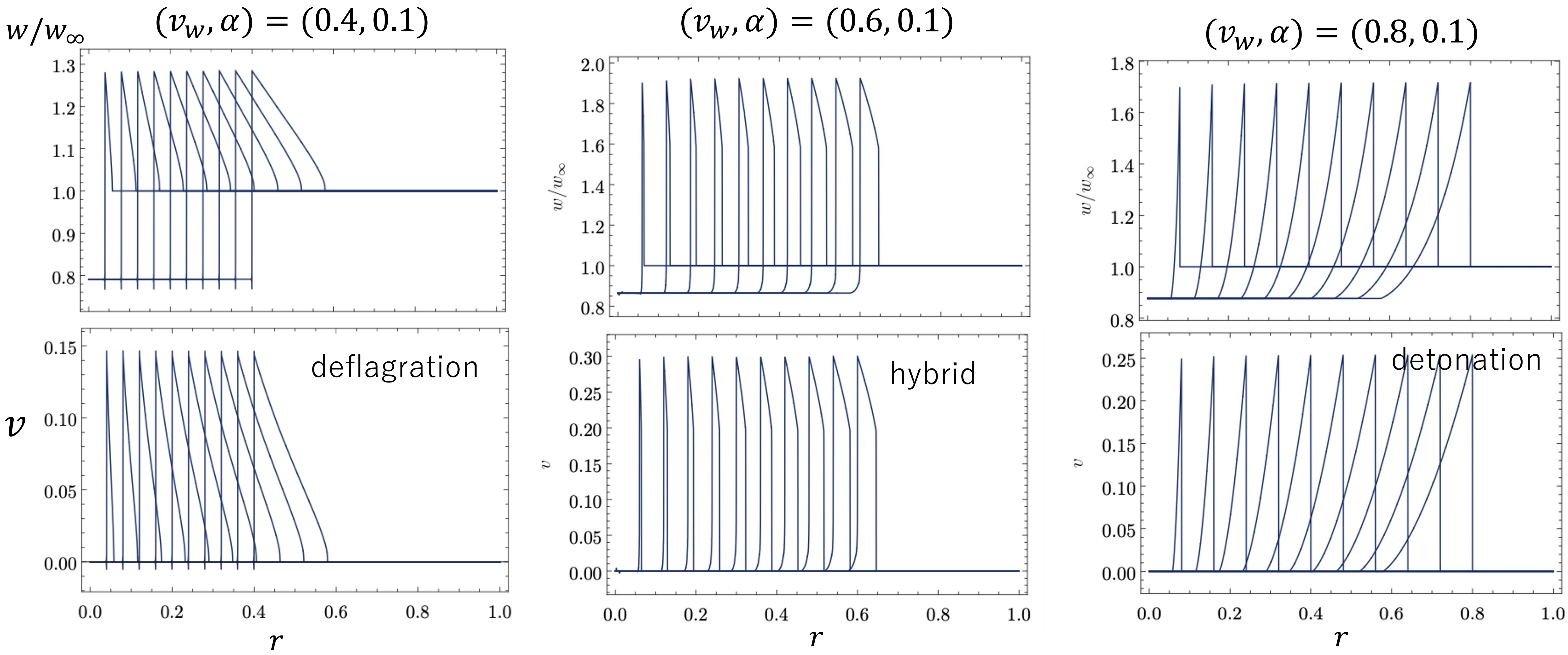
→ shock capturing by **Kurganov-Tadmor** scheme

(see Henrique's slides!)

Evaluate $v(t,r)$ & $w(t,r) = (4/3)aT^4(t,r)$ for the input (v_w, α).

(See e.g. Espinosa+ 2010 for analytical understanding)

$\Delta r = 10^{-4},\ \Delta t = 0.5\Delta r,\ n_r = 10^4$ RK4 solves $T^t \equiv w(\gamma^2 - 1)$ & $T^r \equiv w\gamma^2 v \to (w, v)$



→ self-similar evolution, consistent with the analytic prediction

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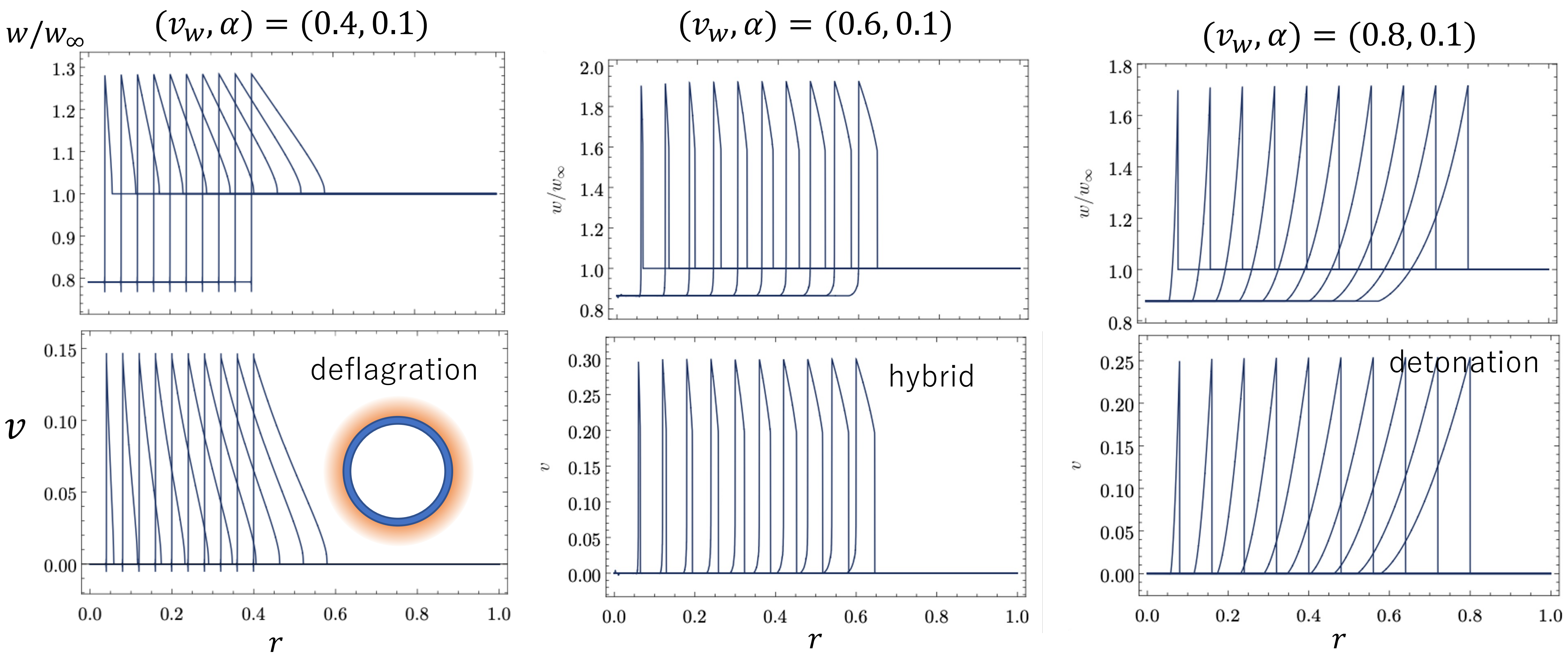

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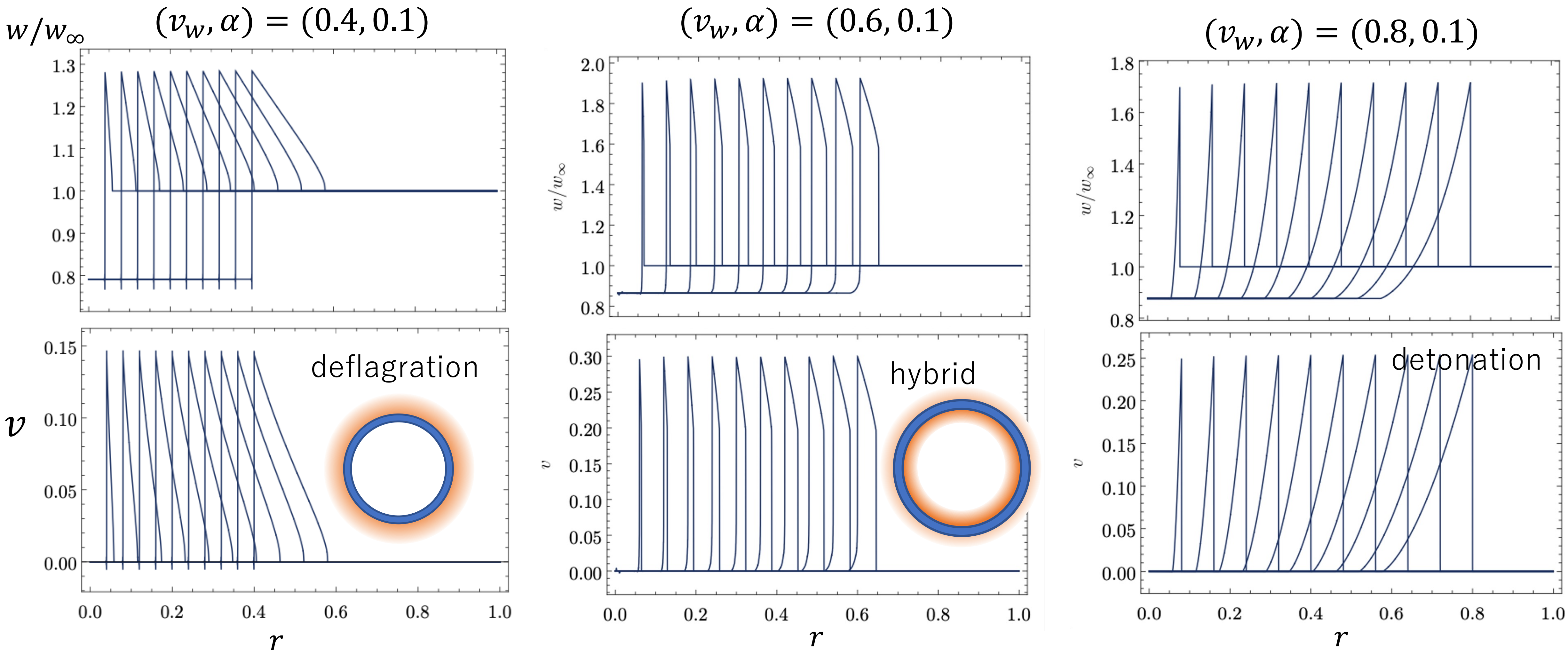

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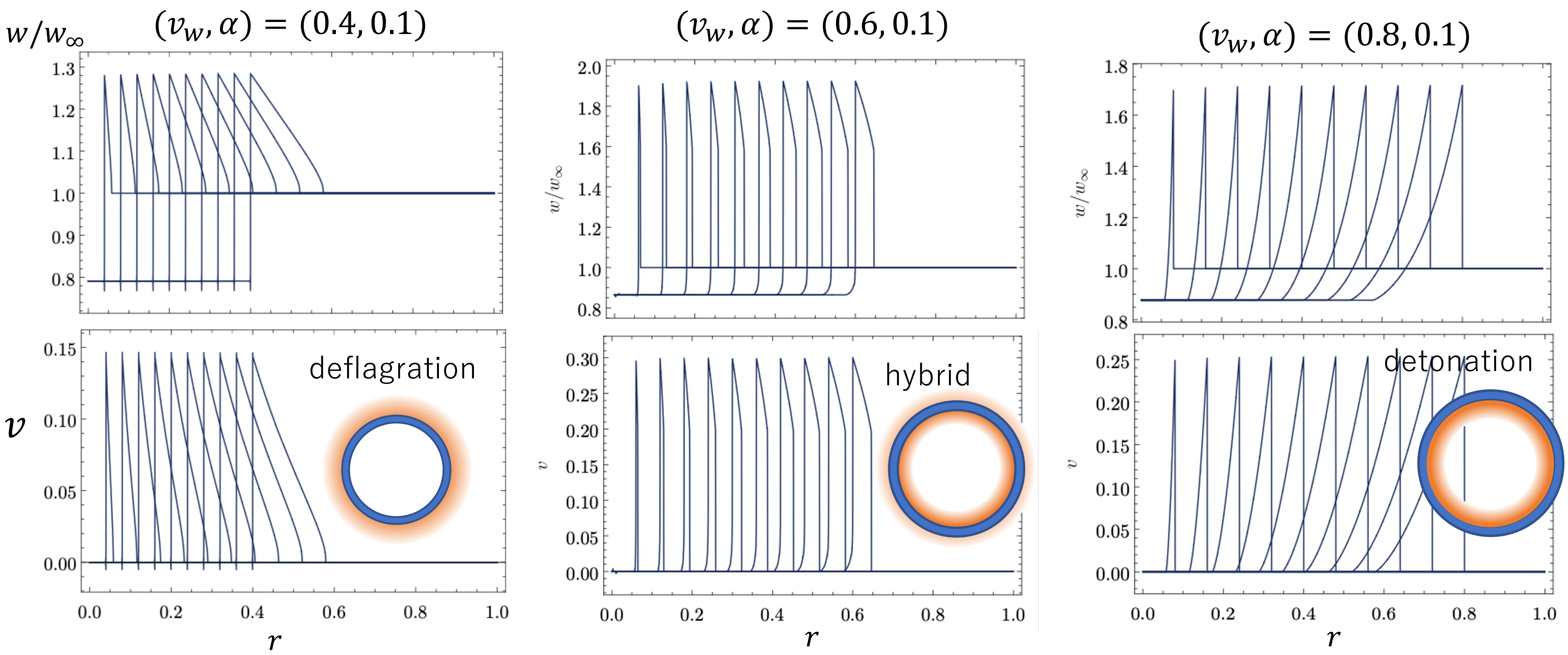

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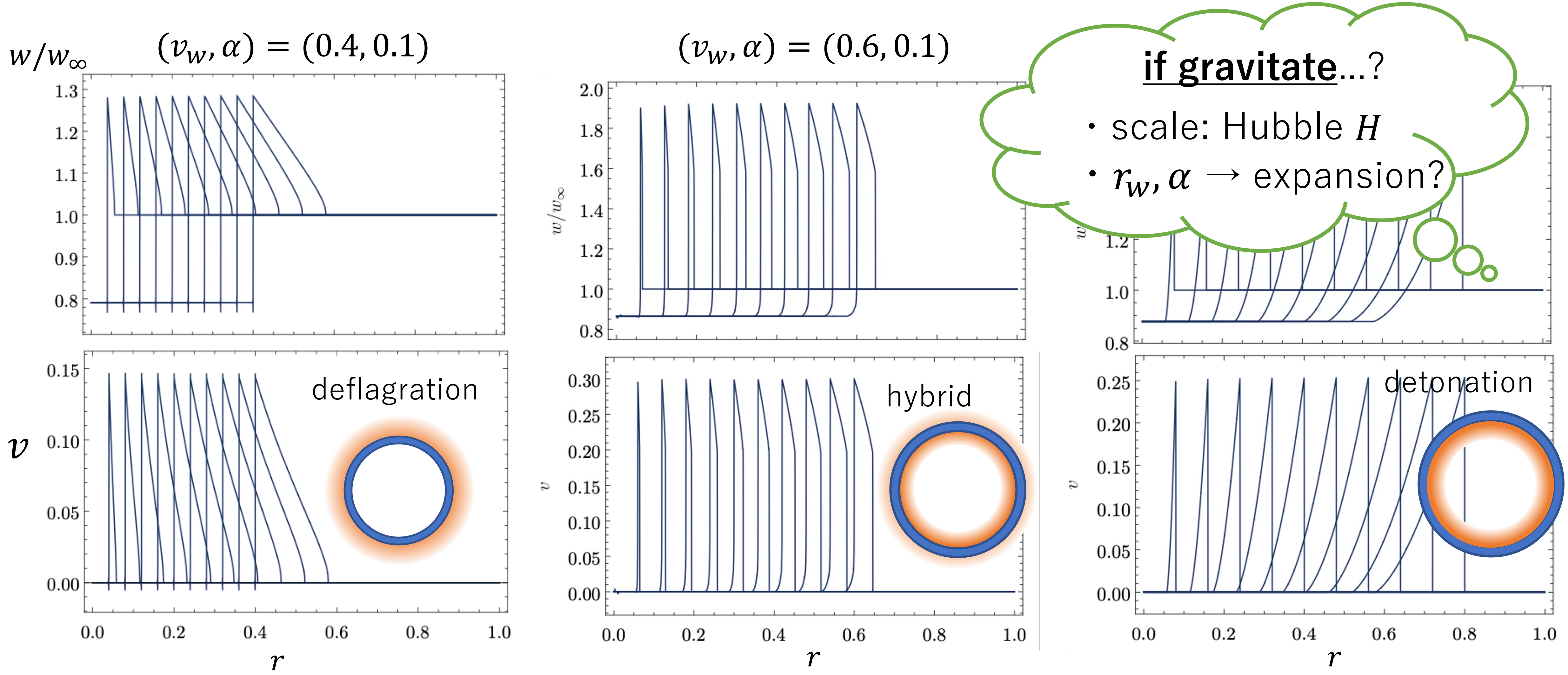

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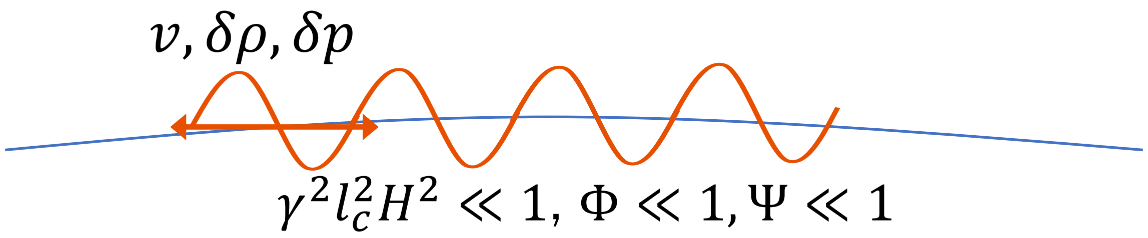
Accommodating gravity: "cosmological hydrodynamics"

- Peculiar fluid motion around FLRW spacetime (Noh+ 2018)

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$
$$\nabla_\mu T^{\mu\nu} = 0$$

$$ds^2 = -(1+2\Phi)dt^2 - 2\chi_i dt dx^i + (1-2\Psi)a(t)^2\delta_{ij}dx^i dx^j$$

$$T^{\mu\nu} = (w_b + \delta w)u^\mu u^\nu + (p_b + \delta p)g^{\mu\nu} \quad u^\mu = \gamma(1, \vec{v}/a)$$



b.g. equations:

$$3M_{\rm Pl}^2 H^2 = \rho_b \quad \dot{\rho}_b + 3Hw_b = 0$$
$$\ddot{a}/a = -(w_b + 2p_b)/6M_{\rm Pl}^2$$

→ EoMs for perturbations

- zero spatial shear & uniform expansion ($\kappa = 0$) → no gauge redundancy
- up to linear order in metric pert. → self-gravitation described by Φ

- <u>fluid quantities</u>

$v, \quad \rho = \rho_b + \delta\rho, \quad p = p_b + \delta p,$

$w = w_b + \delta w = (\rho_b + p_b) + (\delta\rho + \delta p)$

→ gauge invariant quantities (e.g. $v_{\rm GI} = v + C\,\kappa$)

defined in the frame where <u>the expansion is uniform</u>

※ $v_{\rm GI}$ is same for zero-shear gauge ($\chi = 0$)

- <u>Einstein equations</u> ※ $\rho_b(t), p_b(t) \to a(t)$

$$\frac{\Delta}{a^2}\Phi = \frac{1}{2M_{\rm Pl}^2}(\delta\rho + 3\delta p) + \frac{1}{2M_{\rm Pl}^2}w\gamma^2 v^2 \qquad \frac{\Delta}{a^2}\Psi = \frac{1}{2M_{\rm Pl}^2}\delta\rho + \frac{1}{2M_{\rm Pl}^2}w\gamma^2 v^2$$

(※constraint equation for χ & $\partial_r\chi/a \ll v \to \gamma^2 l_c^2 H^2 \ll 1$)

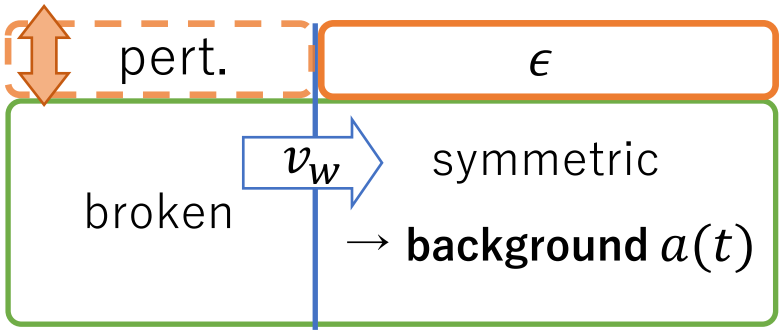
- <u>Conservation equations</u>

$$\partial_t(w\gamma^2 - p) + \frac{1}{a}\left(\partial_r + \frac{2}{r}\right)(w\gamma^2 v) + (3 + v^2)Hw\gamma^2 + w\gamma^2 v\frac{1}{a}\partial_r\Phi = 0,$$

$$\partial_t(w\gamma^2 v) + \frac{1}{a}\left\{\partial_r(w\gamma^2 v^2 + p) + \frac{2}{r}(w\gamma^2 v^2)\right\} + 4Hw\gamma^2 v + w\gamma^2\frac{1}{a}\partial_r\Phi = 0,$$

For FOPT?

- Application to cosmological FOPT (Jinno & JK 2024)



※broken phase as b.g. → similar results

i) How to decompose the system?

$$\rho_b(t) = (3/4)w_b(t) + \epsilon$$
$$p_b(t) = (1/4)w_b(t) - \epsilon$$
$$w_b(t) = w_{b,0}\, a^{-4}(t)$$

⇒

$$\delta\rho = (3/4)\delta w - \epsilon\,\Theta(r_w(t) - r)$$
$$\delta p = (1/4)\delta w + \epsilon\,\Theta(r_w(t) - r)$$

Solving $(v, \delta w, \Phi)$ around b.g.

ii) Effect of b.g. expansion?

$$H^2 = H_0^2 \left[\frac{1}{1+\alpha_0} \left(\frac{a}{a_0} \right)^{-4} + \frac{\alpha_0}{1+\alpha_0} \right] \Rightarrow \frac{a(t)}{a_0} = \alpha_0^{-\frac{1}{4}} \sqrt{\sinh\left(2H_0 t \sqrt{\frac{\alpha_0}{1+\alpha_0}} + \operatorname{arctanh}\left(\sqrt{\frac{\alpha_0}{1+\alpha_0}} \right) \right)}$$

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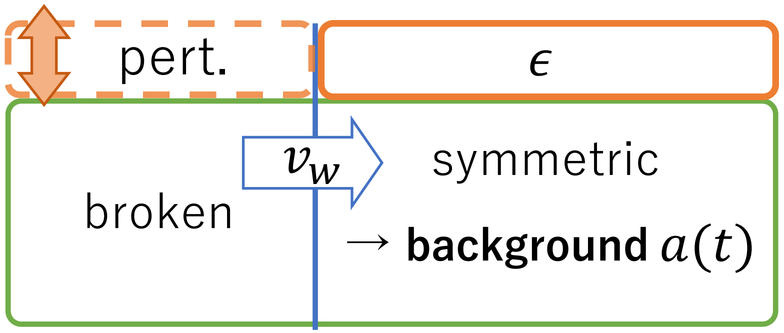

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- $\alpha(t) \equiv 4\epsilon/3w_b(t) = \alpha_0(a/a_0)^4$: increases as radiation dilutes away
- wall position $a(t)\dfrac{dr_w}{dt} = v_w$ ⇒ $r_w(t) = \int dt\, v_w/a(t) = \int d\eta\, v_w$
- inputs in our system: (v_w, α_0) ※ H_0, or $w_{b,0}/M_{\rm Pl}^2$ defines the box-size/time scale

・Higgs-less scheme + cosmological hydro.

$$\partial_t \tilde{T}^t + \frac{1}{a(t)}\left[\left(\partial_r + \frac{2}{r}\right)(\tilde{w}\gamma^2 v) - \partial_r\left(\frac{3}{4}\alpha(t) v_w(t)\Theta(r - r_w(t))\right) + \tilde{w}\gamma^2 v\,\partial_r\Phi\right] = 0,$$

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fluid gravitation

broken phase w.r.t background

Code variables:

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$$\tilde{T}^r \equiv \tilde{w}\gamma^2 v$$

$$\tilde{w}(\tilde{T}^t, \tilde{T}^r)\quad v(\tilde{T}^t, \tilde{T}^r)$$

$$\tilde{X} = X/w_b$$

・Regime of validity?

$\Phi \ll 1$? → "Hole" inside the bubble grows over time

→ smaller v_w or α_0 to simulate longer

$\gamma^2 l_c^2 H^2 \ll 1$ earlier time: $l_c \sim$ (fluid shell) $\ll H^{-1}$

late time: Φ induces large scale profile & $\alpha(t)\nearrow \to \gamma(v)\nearrow$

・Higgs-less scheme + cosmological hydro.

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→ i)Safer prediction for $t \leq H_0^{-1}$

Effects mostly come from $a(t)$

ii)Late time solution until $\Phi \sim \mathcal{O}(0.1)$

- Early time solution $(0 \le t \le H_0^{-1})$

$v_w = const.$ & weak PT: $(\alpha_0 = 10^{-2})$

grid: $\Delta r = \Delta t = 5 \times 10^{-5} H_0^{-1},\ n_t = n_r = 2 \times 10^4$

$\rightarrow \quad r_{\max} = H_0^{-1}$ (horizon at initial)

b.g. quantities evolve as

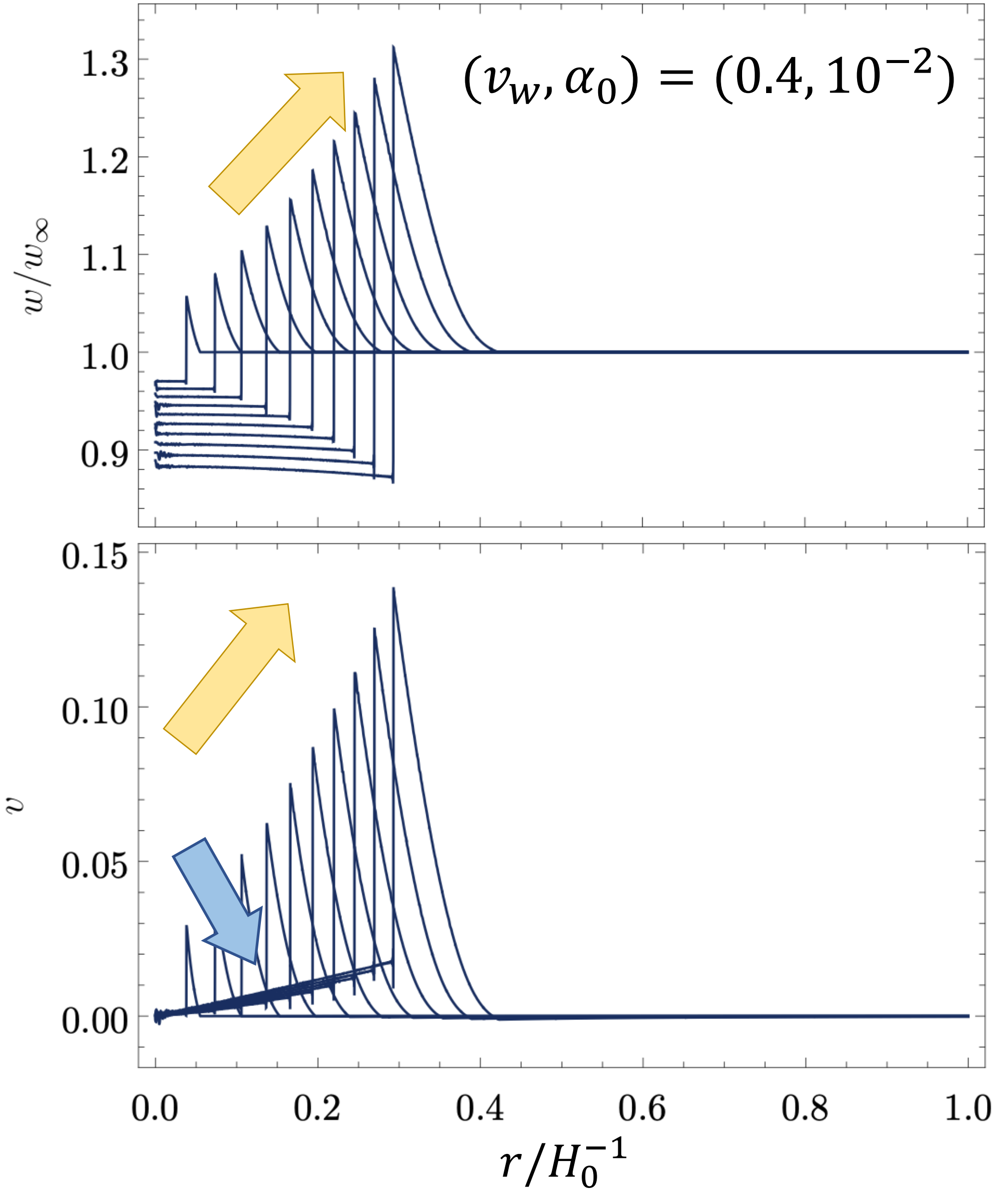
$H(t = H_0^{-1}) \sim 3H_0^{-1}$

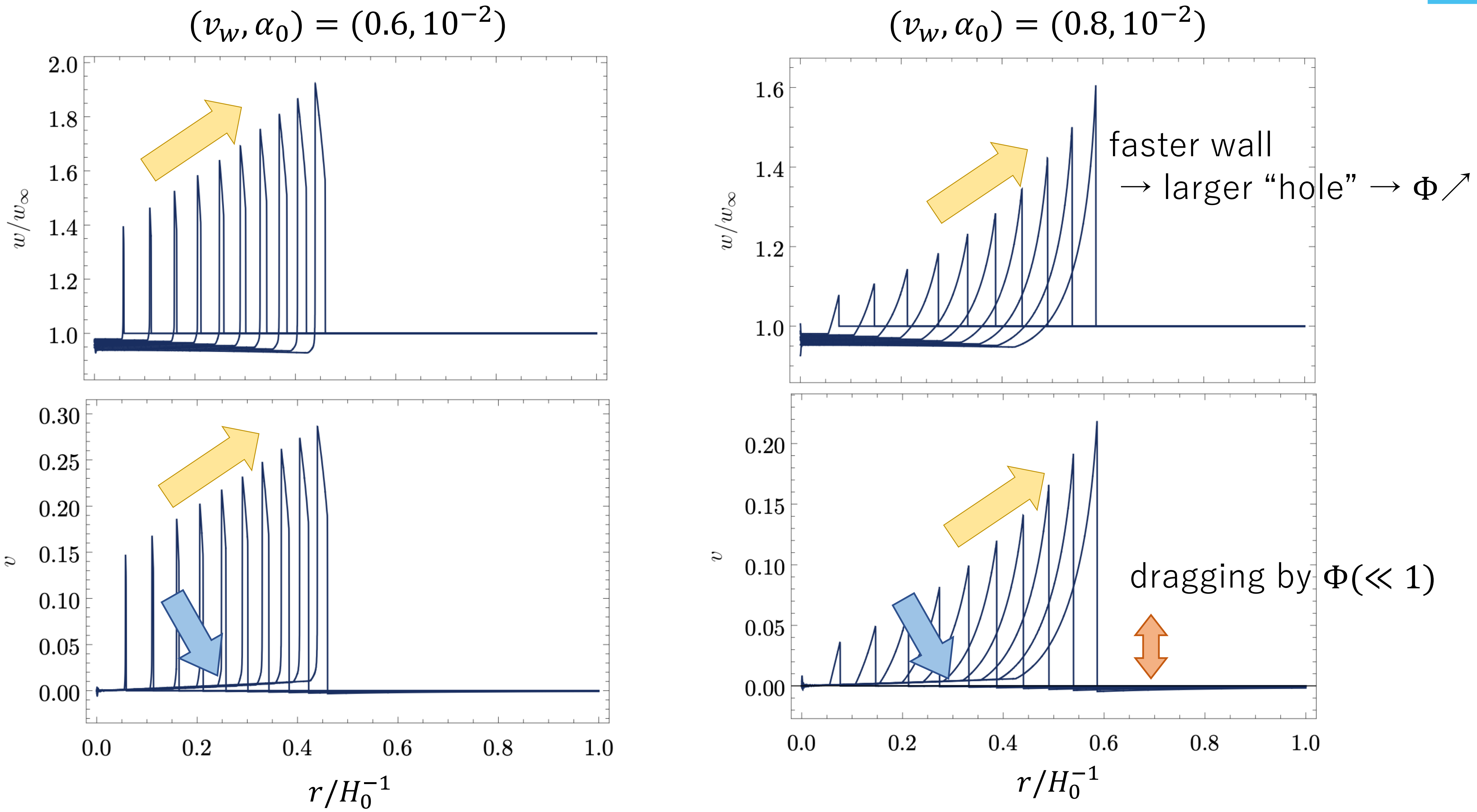
$a(t = H_0^{-1}) \sim 1.8 \quad \rightarrow \quad \alpha(t = H_0^{-1}) \sim 0.1$

Effect of evolving $\alpha(t)$:

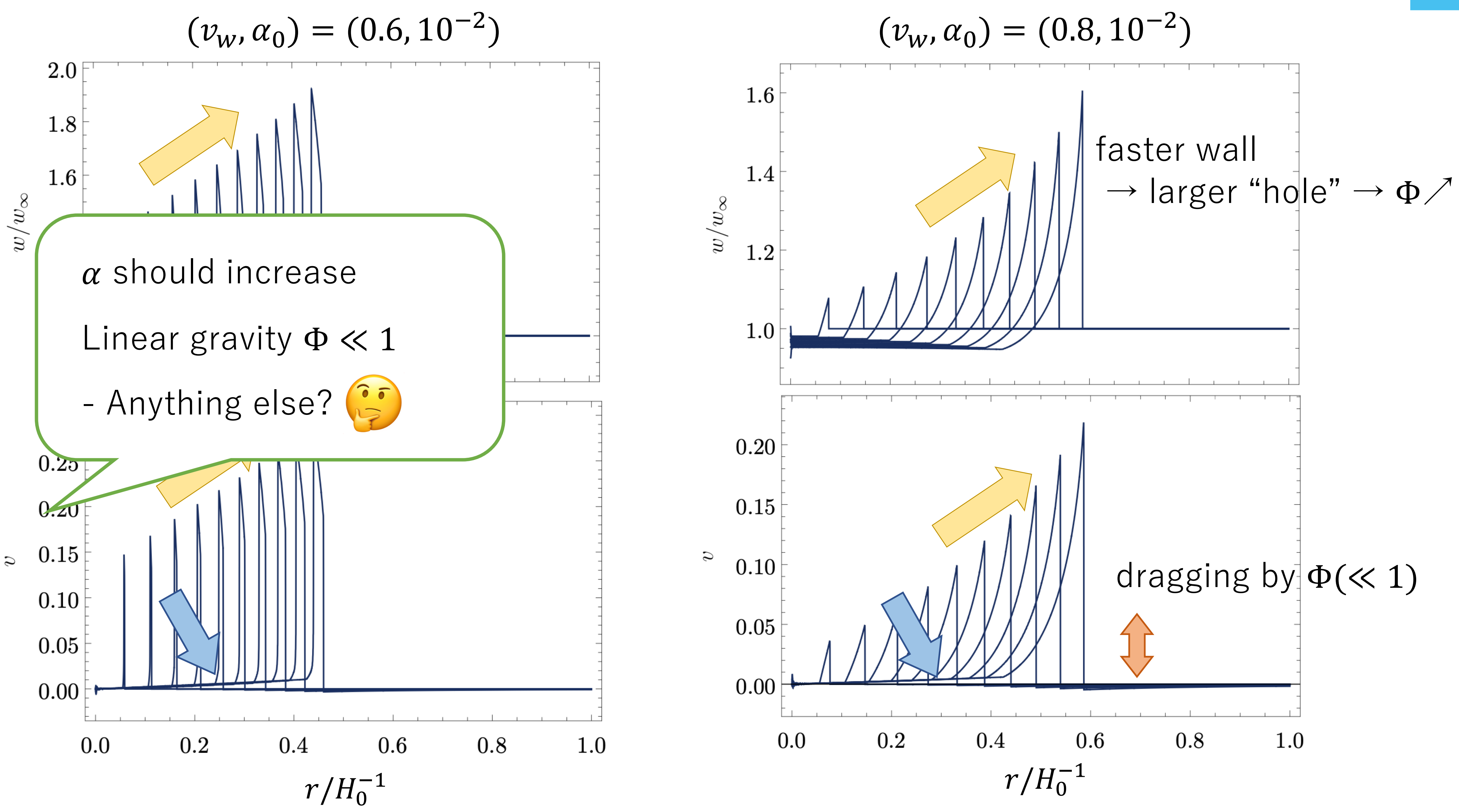
$\rightarrow$ **velocity tail** inside the bubble

(flow into emptier region)





$(v_w, \alpha_0) = (0.6, 10^{-2})$
$(v_w, \alpha_0) = (0.8, 10^{-2})$
w/w_∞
v
r/H_0^{-1}
faster wall
→ larger "hole" → Φ↗
dragging by Φ(≪ 1)



$(v_w, \alpha_0) = (0.6, 10^{-2})$
$(v_w, \alpha_0) = (0.8, 10^{-2})$
w/w_∞
v
r/H_0^{-1}
faster wall
→ larger "hole" → Φ↗
dragging by $\Phi(\ll 1)$
α should increase
Linear gravity $\Phi \ll 1$
- Anything else?

- **Deviation from Minkowski b.g. solutions..!!**

flat: self-similarity in r/t $\rightarrow$ r/η for cosmo?

- compare with the flat case at the same α

Thinning of fluid shell (both v & w)

※ analytic study in Cai & Wang 2022

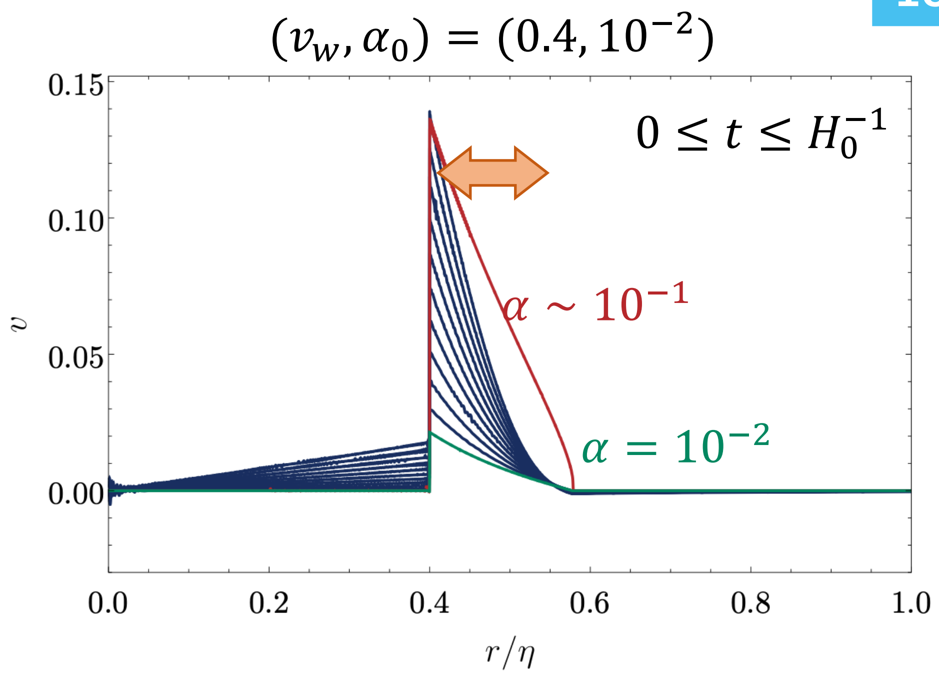
→ similar behavior late time limit of fluid profile

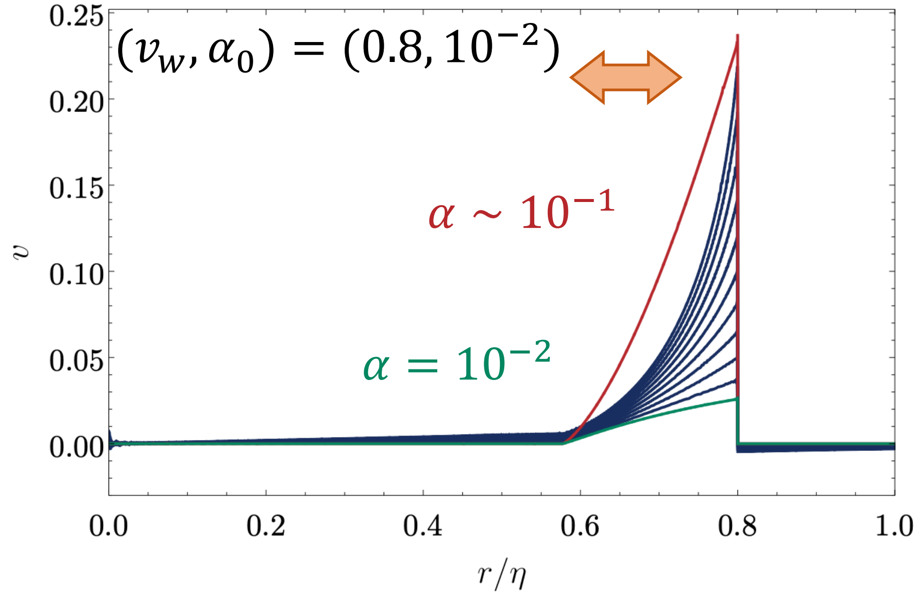
- Implication to SGWB spectrum?

・smaller ΔR_* $\rightarrow$ shift characteristic frequency

・decrease in kinetic energy fraction

$$\kappa(t) \equiv \frac{3}{\epsilon\, r_w^3(t)} \int dr\, r^2 w v^2 \gamma^2$$ → reduces amplitude





• Deviation from Minkowski b.g. solutions..!!

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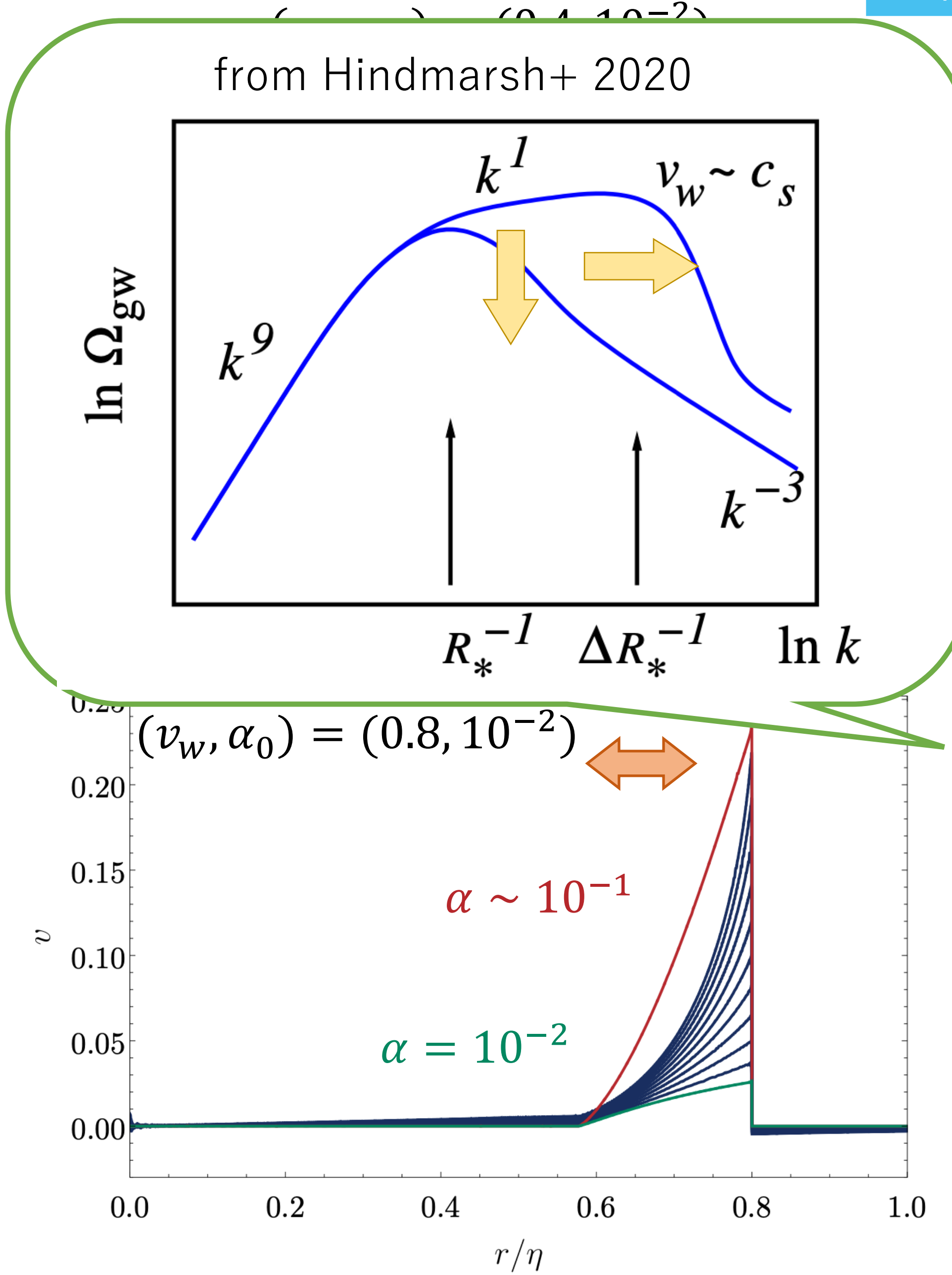
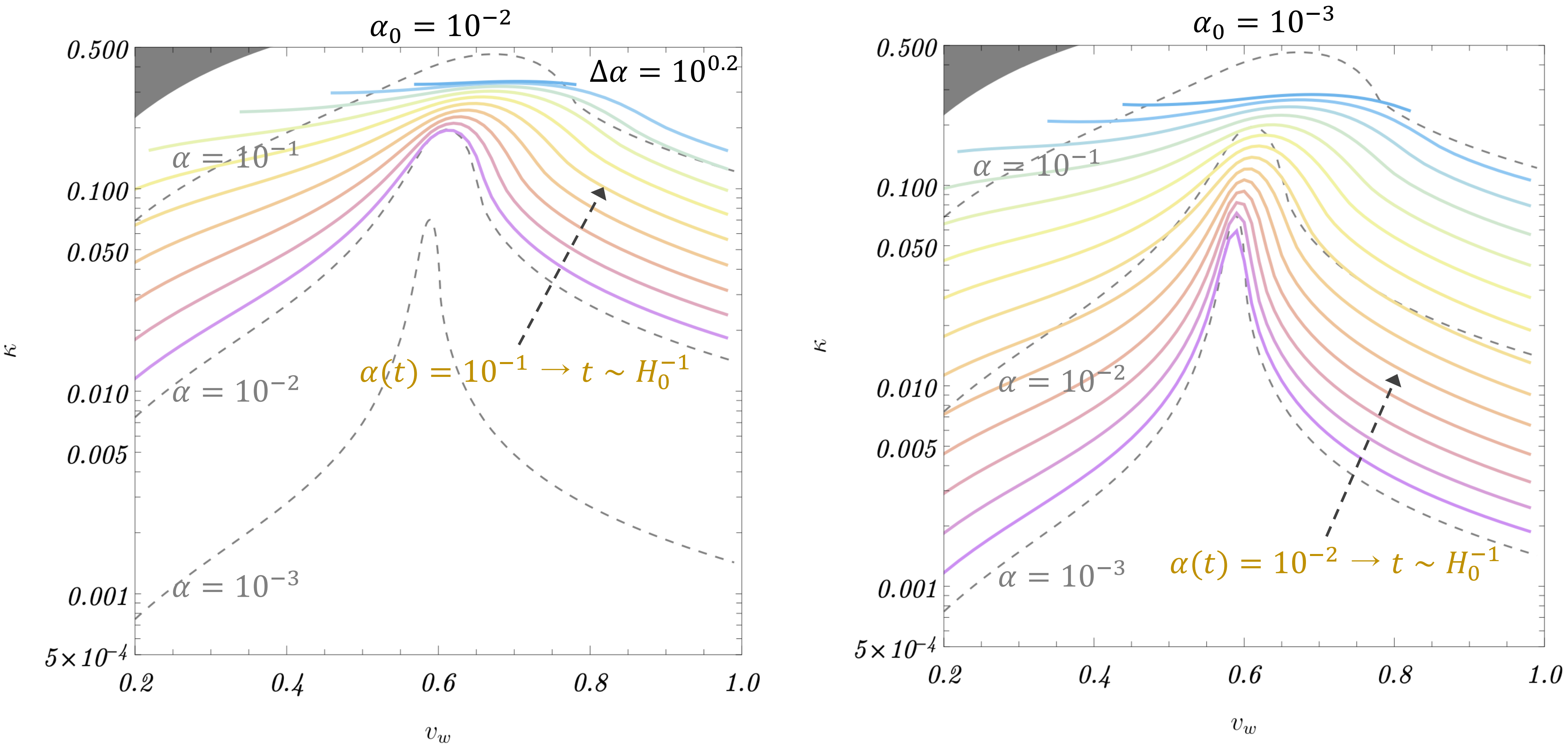


- Kinetic energy fraction (※ reduction of the same order for $\alpha_0 = 10^{-1}$)



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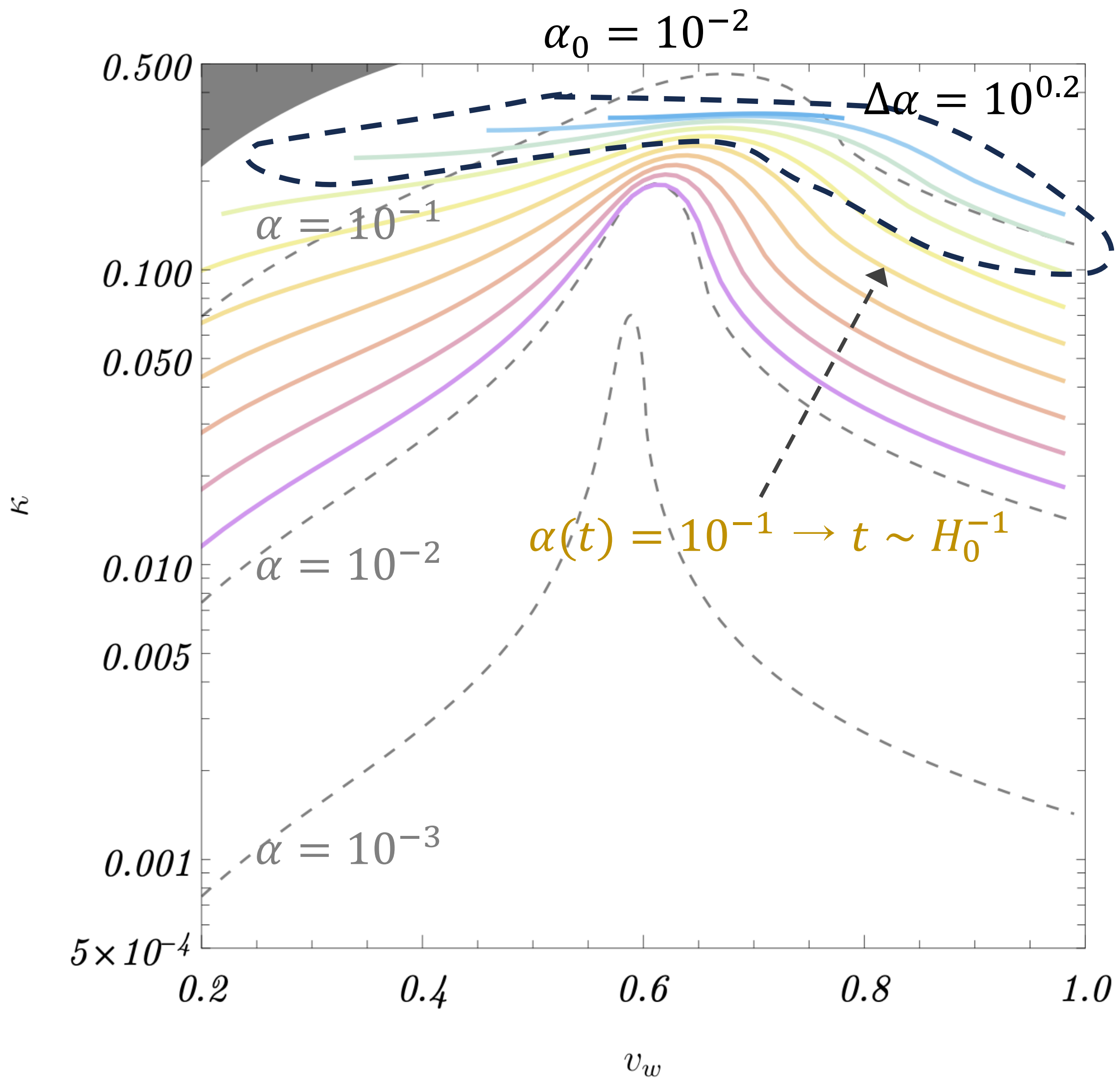
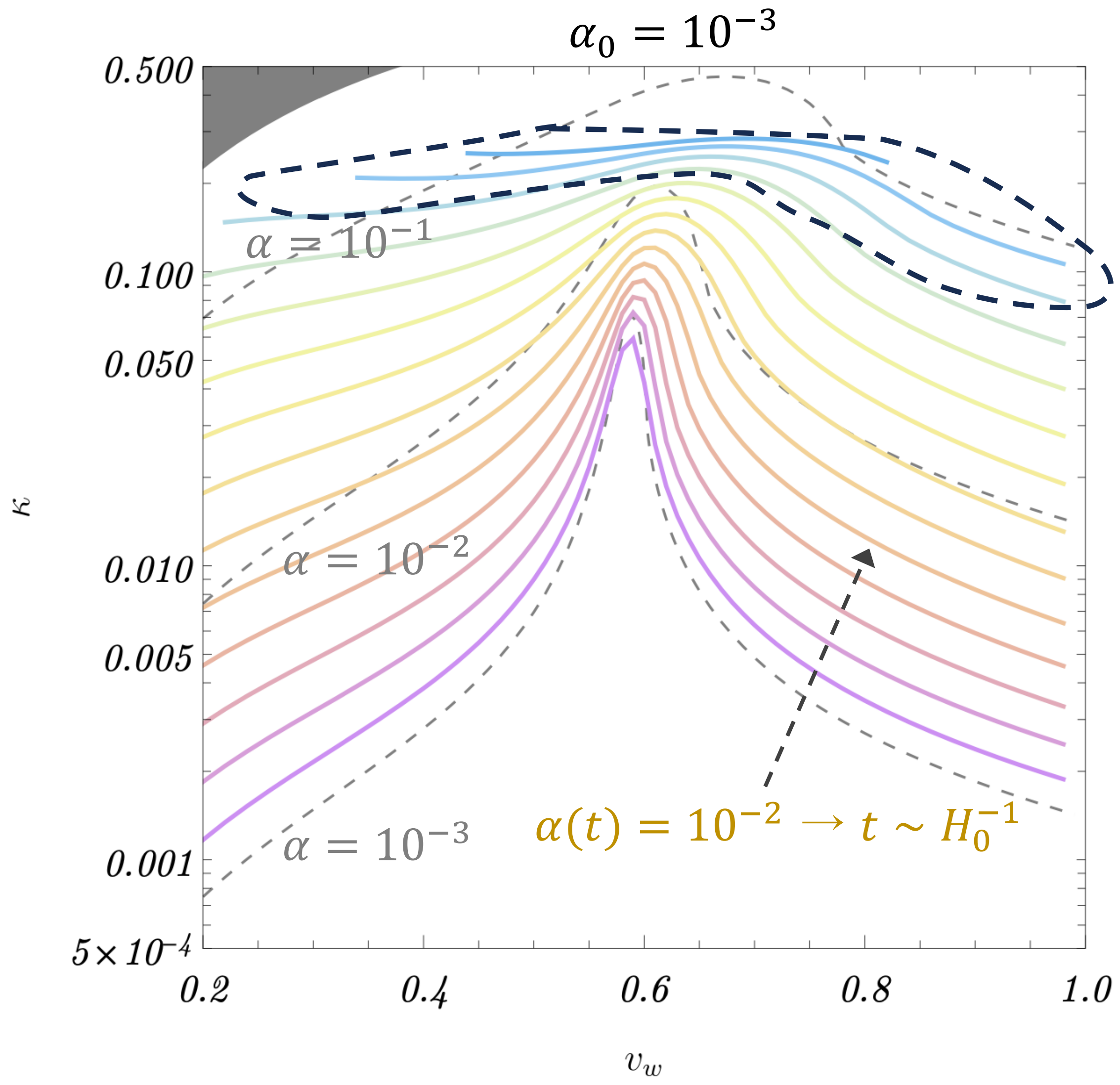




- Edge of validity…?

~~$\Phi \ll 1$~~ large $\alpha(t)$ and $r_w(t)$ at later time

Larger v_w & α_0

→ earlier break down

~~$\gamma^2 l_c^2 H^2 \ll 1$~~

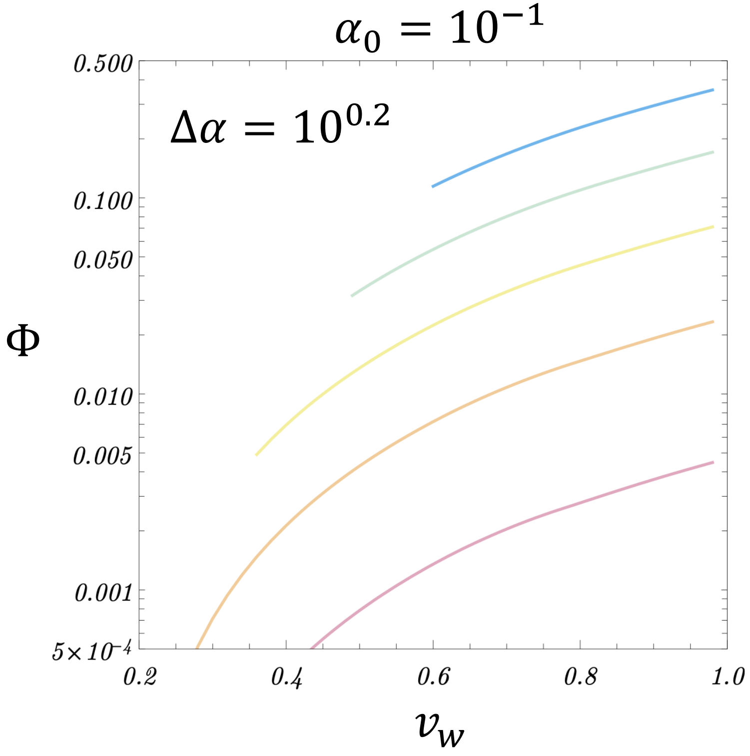
distortion by Φ

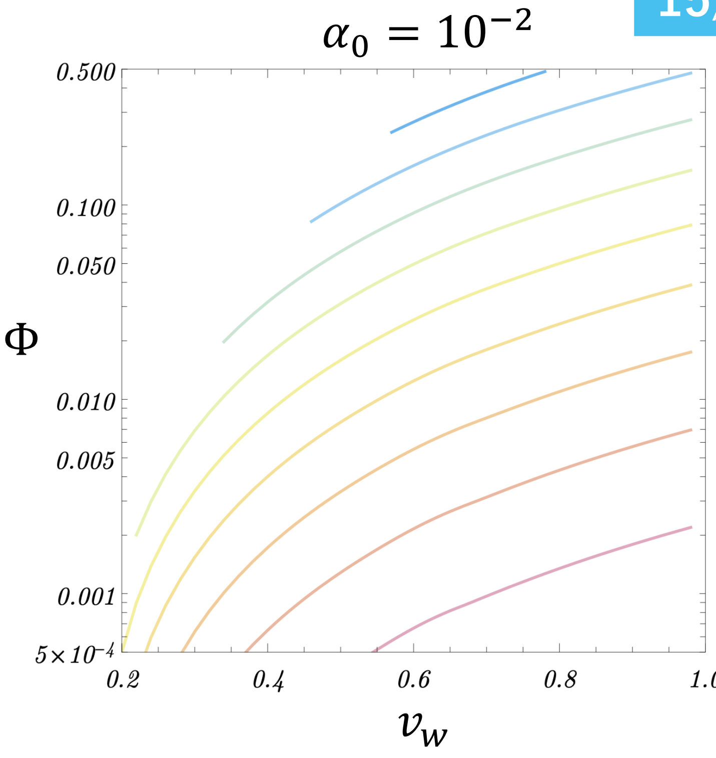
over horizon scale

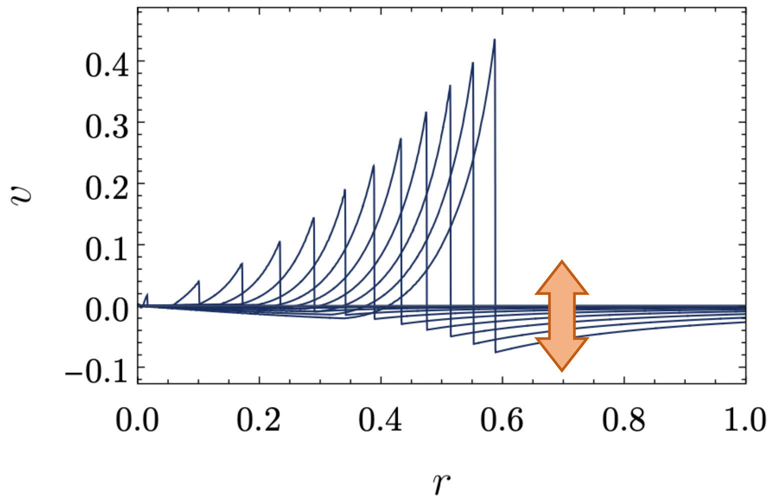
→ need for full GR/NR

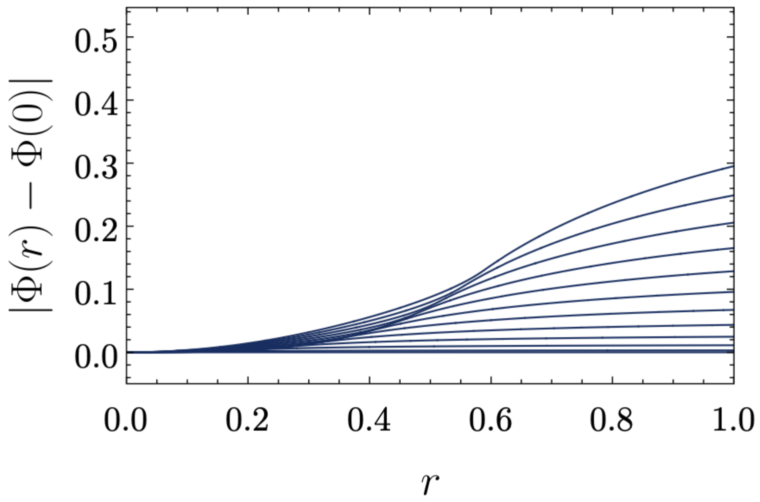
Johnson+ 2011

Giombi & Hindmarsh 2023

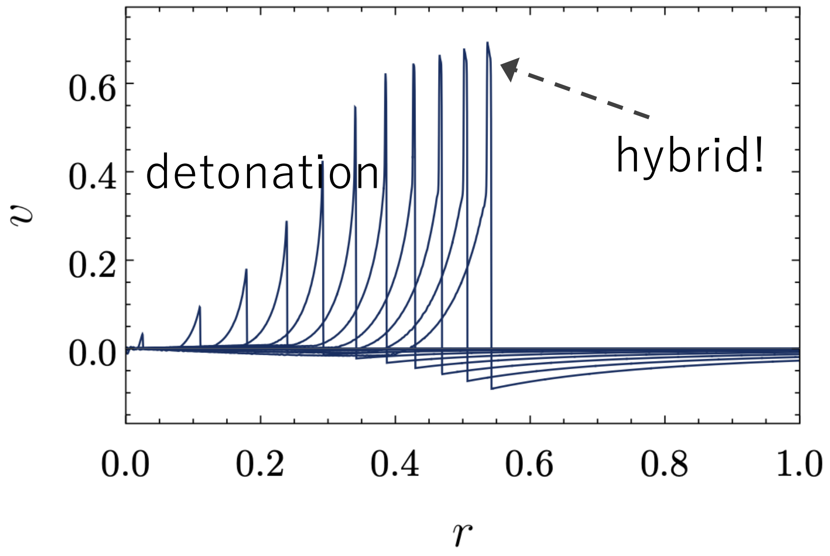


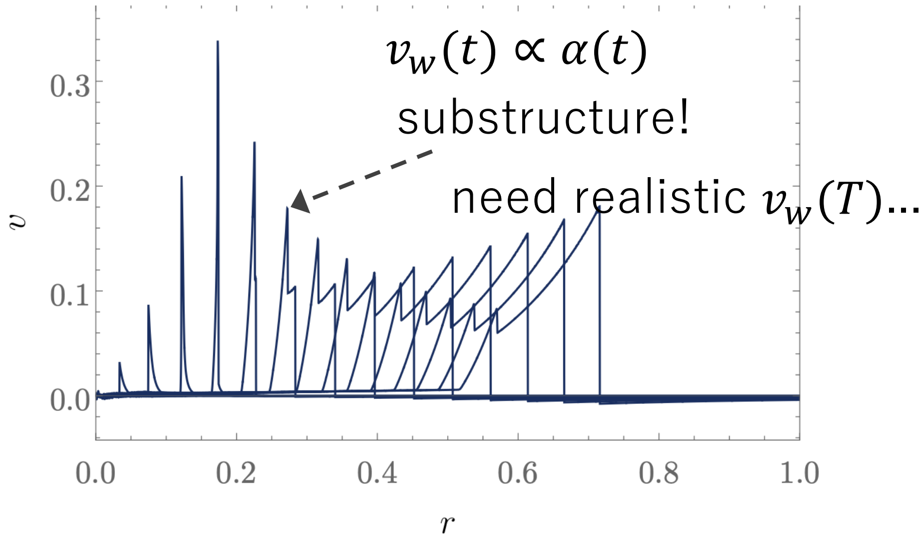




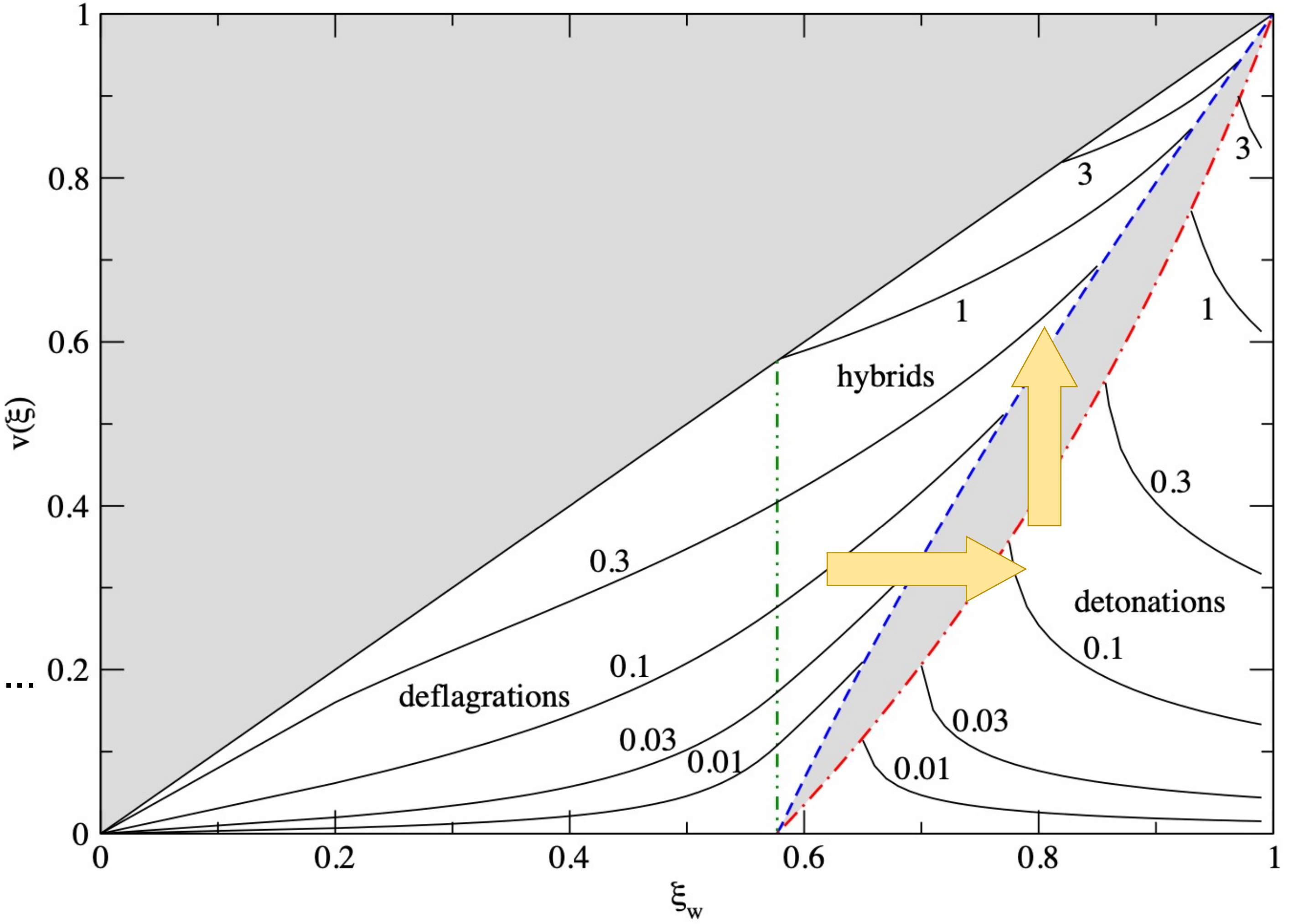


- (Potentially interesting) dynamical behavior





Phase diagram from Espinosa+ 2010



Outline

- ➢Need for large bubble SGWB templates!
- ➢Higgsless scheme for Minkowski background
- ➢Accommodating gravity: “cosmological hydrodynamics”
- ➢Summary & Outlook

Summary and Outlook

- SGWB templates for large bubbles – GR effects?
  → **hydrodynamics + weak gravity in expanding spacetime**.

- Fluid profile around a bubble modified by $\alpha(t) \propto a^4(t)$
  → thinning of fluid shell would affect SGWB amplitude/spectrum

- Improvements we need!
  → full GR implementation for larger v_w & α_0?
  → modeling $v_w(T)$?
  → colliding bubbles, 3D results...?

August 8th, 2025

Gravitational effects on sound waves:

-a perturbative approach for large bubbles in cosmological FOPTs-

Numerical Simulations of Early Universe sources of GWs @ NORDITA

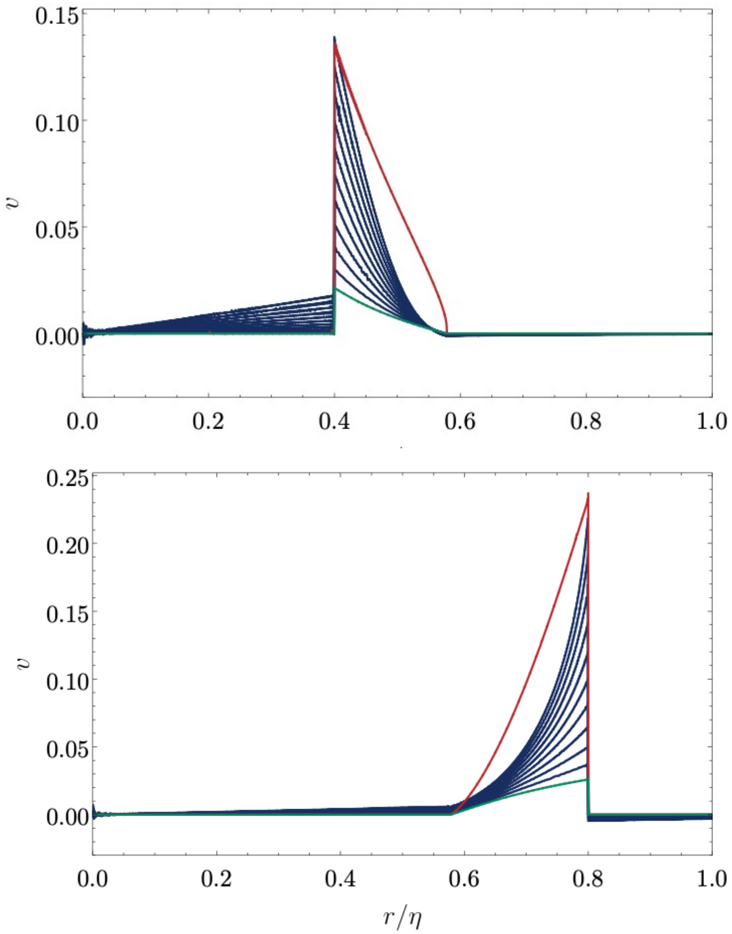
Jun'ya Kume (University of Padova)

Based on JCAP 02 (2025) 057 (arXiv:2408.10770)

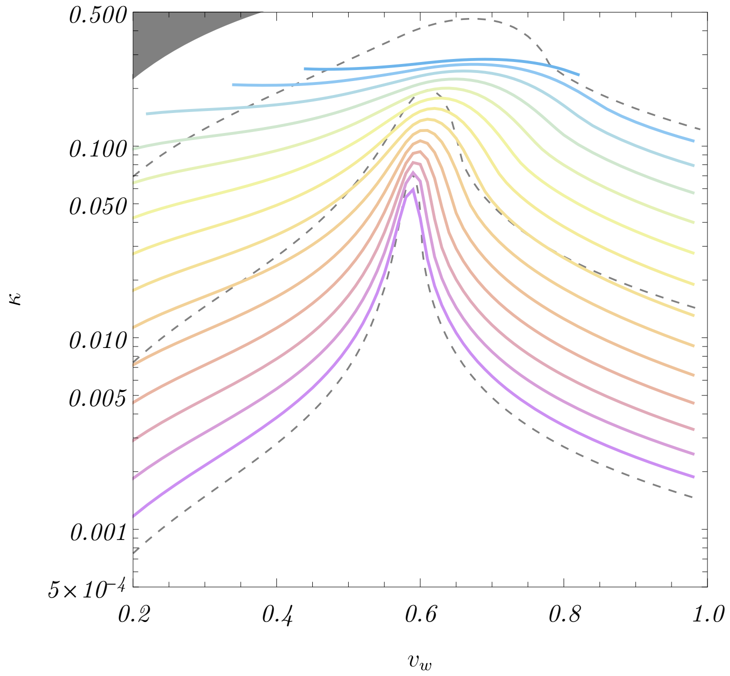
with Ryusuke Jinno (U. Kobe)

Fluid profile around large bubbles

see R. Jinno & J. Kume arXiv:2408.10770 for more details.

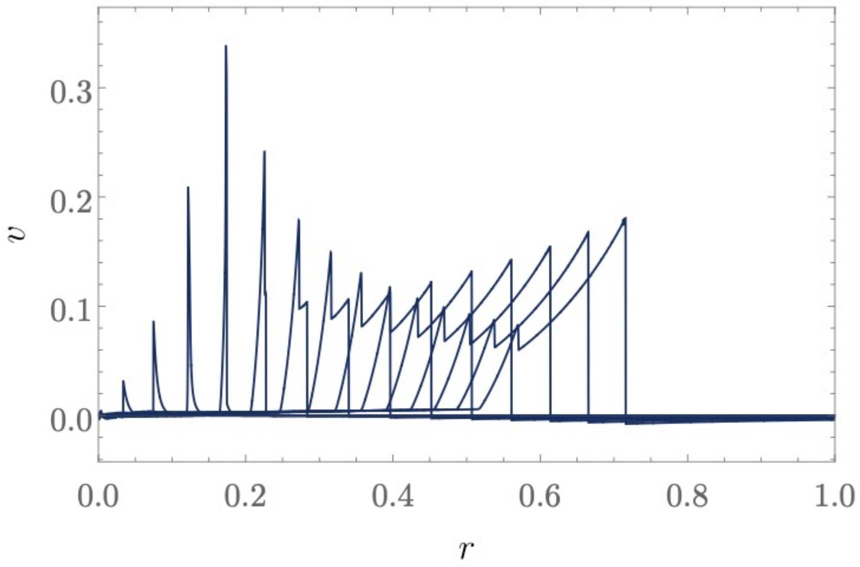


Thinning of fluid shell
→ reduction in kinetic energy fraction



lowering temperature
→ acceleration of the wall

transition in the fluid profile



Difference between gauge inv. quantities

$$\Phi_{\rm ZSG} = \Phi + \dot{\chi}, \qquad \Psi_{\rm ZSG} = \Psi + H\chi, \qquad \delta w_{\rm ZSG} = \delta w - 4aHw_b\left(v_0 + \frac{\chi}{a}\right)$$

$v_0(t,r)$ (satisfying $v(t,r) = \partial_r v_0(t,r)$) → identical!

Suppression we expect

$$\dot{\chi} \sim H\chi \ll H l_c v, \qquad aHw_b\left(v_0 + \frac{\chi}{a}\right) \sim aHw_b\left(\frac{l_c v}{a} + \frac{\chi}{a}\right) \sim H l_c w_b v,$$

→ $l_c \sim$ (shell thickness) $\ll H^{-1}$

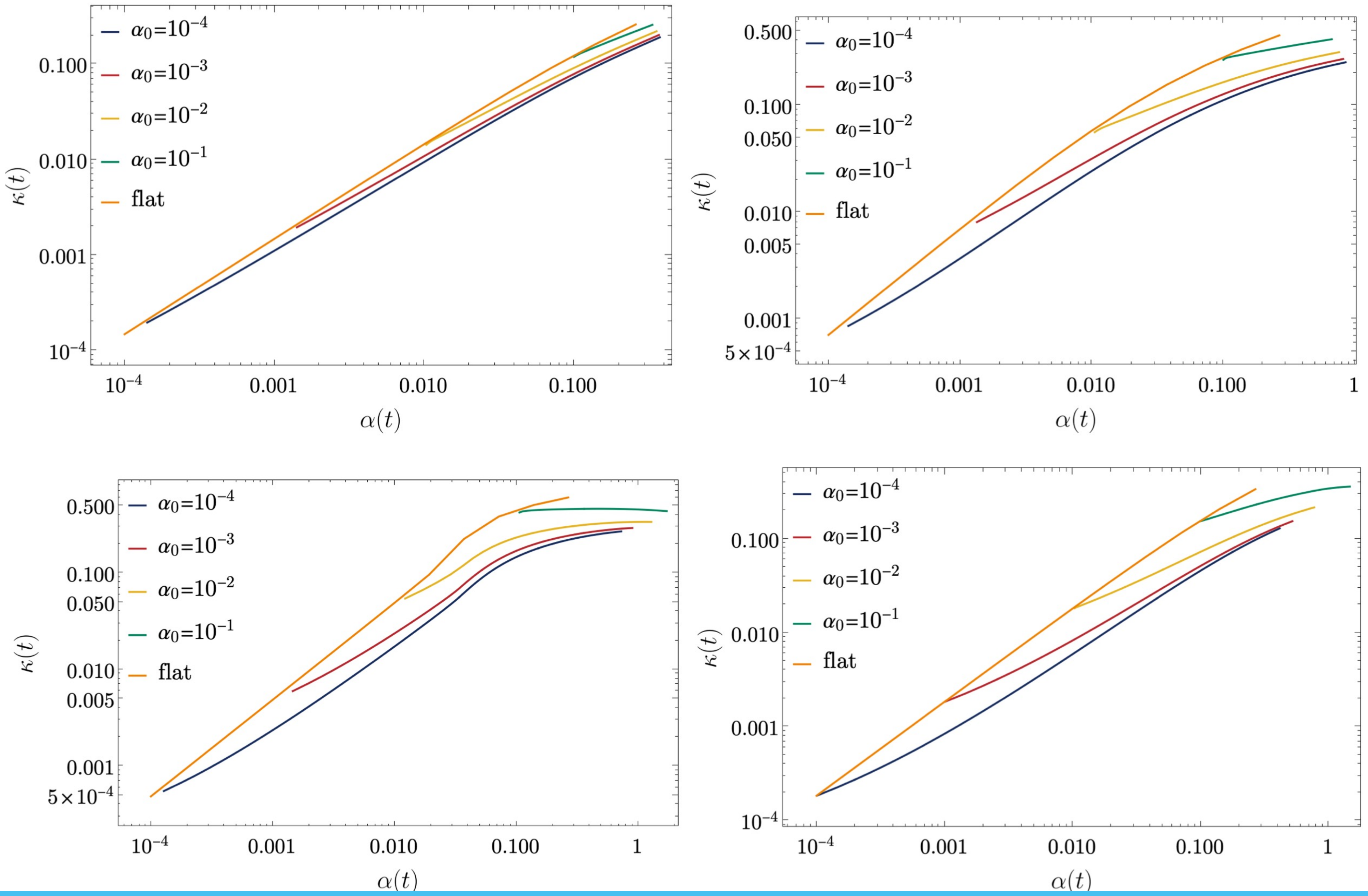
Full equations

$$\left[\frac{\partial}{\partial t}+\frac{1}{a(1+2\varphi)}\left(\mathcal{N}v^k+\frac{c}{a}\chi^k\right)\nabla_k\right]\varrho+\left(\varrho+\frac{p}{c^2}\right)\left\{\mathcal{N}\left(3\frac{\dot{a}}{a}-\kappa\right)\right.$$

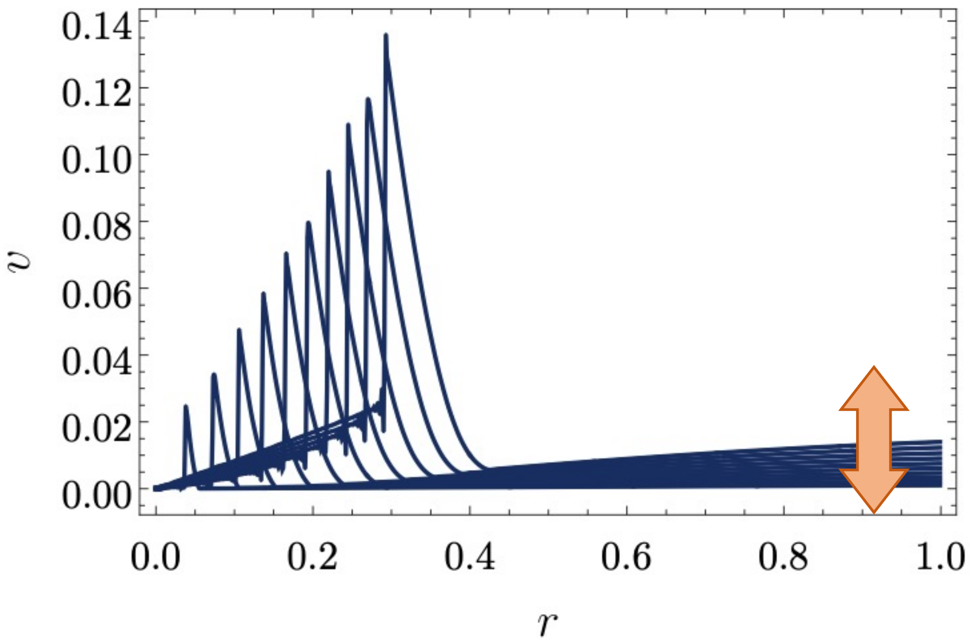
$$\left.+\frac{(\mathcal{N}v^k)_{,k}}{a(1+2\varphi)}+\frac{\mathcal{N}v^k\varphi_{,k}}{a(1+2\varphi)^2}+\frac{1}{\gamma}\left[\frac{\partial}{\partial t}+\frac{1}{a(1+2\varphi)}\left(\mathcal{N}v^k+\frac{c}{a}\chi^k\right)\nabla_k\right]\gamma\right\}=0.$$

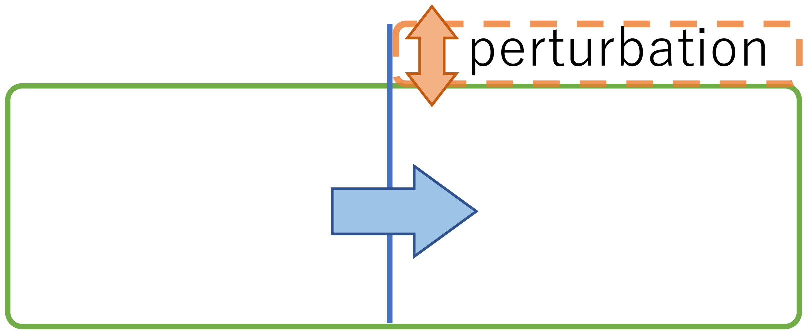
$$\frac{1}{a\gamma}\left[\frac{\partial}{\partial t}+\frac{1}{a(1+2\varphi)}\left(\mathcal{N}v^k+\frac{c}{a}\chi^k\right)\nabla_k\right](a\gamma v_i)+v^k\nabla_i\left(\frac{c\chi_k}{a^2(1+2\varphi)}\right)$$

$$+\frac{c^2}{a}\mathcal{N}_{,i}-\left(1-\frac{1}{\gamma^2}\right)\frac{c^2\mathcal{N}\varphi_{,i}}{a(1+2\varphi)}+\frac{1}{\varrho+\frac{p}{c^2}}\left\{\frac{\mathcal{N}}{a\gamma^2}p_{,i}+\frac{v_i}{c^2}\left[\frac{\partial}{\partial t}+\frac{1}{a(1+2\varphi)}\left(\mathcal{N}v^k+\frac{c}{a}\chi^k\right)\nabla_k\right]p\right\}=0.$$


α_0=10^{-4}
α_0=10^{-3}
α_0=10^{-2}
α_0=10^{-1}
flat
$\kappa(t)$
$\alpha(t)$
0.100
0.010
0.001
10^{-4}
10^{-4}
0.001
0.010
0.100
α_0=10^{-4}
α_0=10^{-3}
α_0=10^{-2}
α_0=10^{-1}
flat
$\kappa(t)$
$\alpha(t)$
0.500
0.100
0.050
0.010
0.005
0.001
5×10^{-4}
10^{-4}
0.001
0.010
0.100
1
α_0=10^{-4}
α_0=10^{-3}
α_0=10^{-2}
α_0=10^{-1}
flat
$\kappa(t)$
$\alpha(t)$
0.500
0.100
0.050
0.010
0.005
0.001
5×10^{-4}
10^{-4}
0.001
0.010
0.100
1
α_0=10^{-4}
α_0=10^{-3}
α_0=10^{-2}
α_0=10^{-1}
flat
$\kappa(t)$
$\alpha(t)$
0.100
0.010
0.001
10^{-4}
10^{-4}
0.001
0.010
0.100
1

Comparison between different background choices:





$$a(t)\frac{dr_w}{dt} = v_w$$