Origin of Chiral Magnetic Effect, Production of Turbulence and Generation of Large-Scale Magnetic Fields

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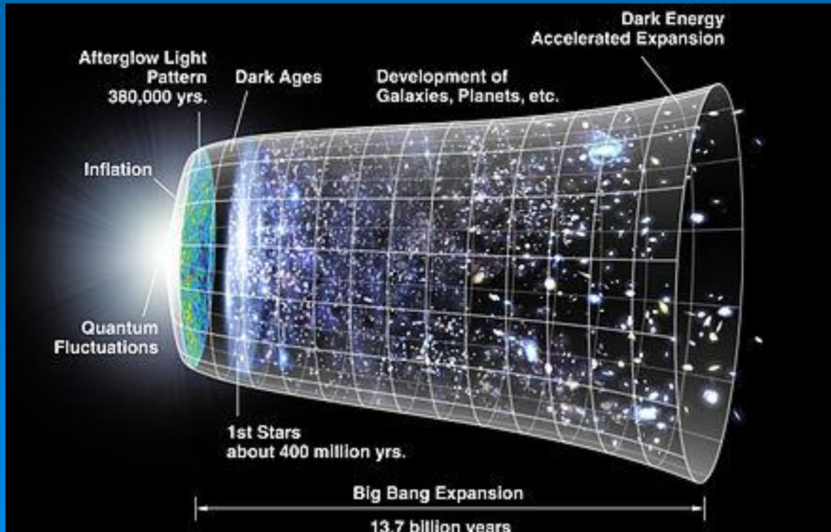
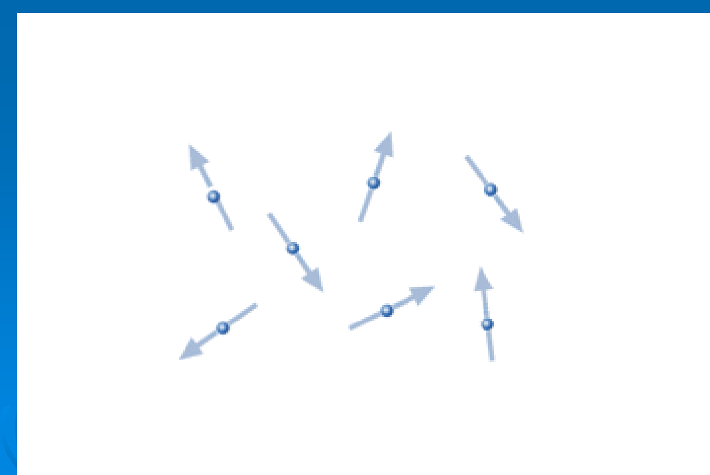
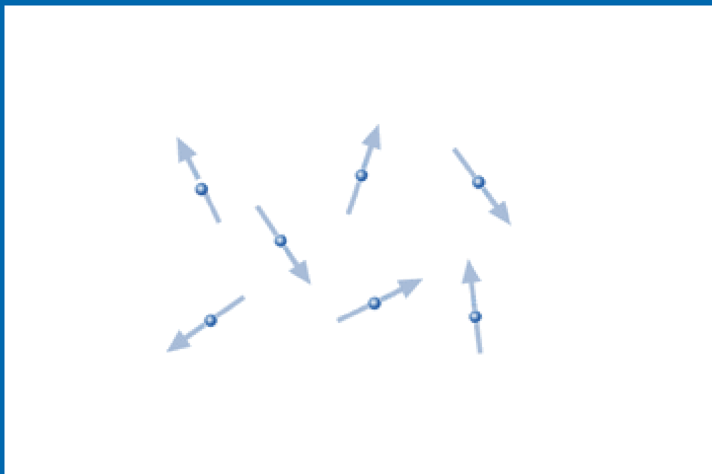




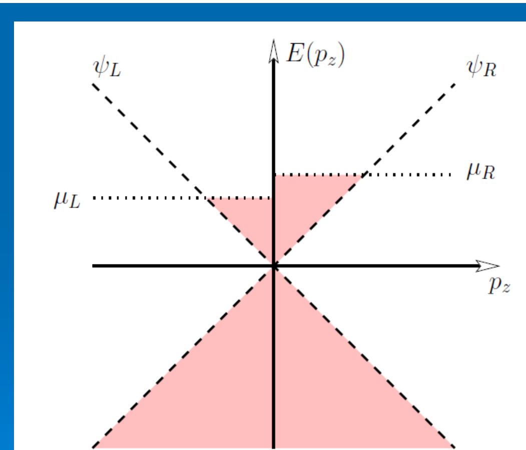
Chiral Magnetic Effect

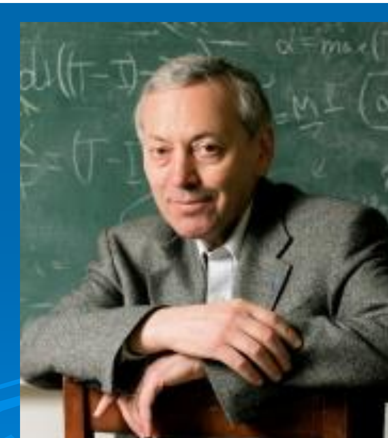
$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{U} \times \boldsymbol{B} + \eta\, \mu_5 \boldsymbol{B} - \eta\, \boldsymbol{\nabla} \times \boldsymbol{B}]\,,$$

1. We consider fermions in magnetic field.
2. The particles with the momentum ***p*** along magnetic field have positive projection of spin onto momentum (the right-chiral particles).
3. The particles with the momentum ***p*** opposite to magnetic field have negative projection of the spin onto momentum (the left-chiral particles).



Vilenkin, A. 1980, Phys. Rev., D22, 3080





$$Q_5 = \text{Number of left particles} - \text{Number of right particles}$$

$$\frac{dQ_5}{dt} = \frac{2\alpha_{\rm em}}{\pi\hbar}\int d^3x\, \boldsymbol{E}\cdot\boldsymbol{B} = -\frac{\alpha_{\rm em}}{\pi\,\hbar c}\frac{d}{dt}\int d^3x\, \boldsymbol{A}\cdot\boldsymbol{B},$$

$$f_L(p) = \frac{1}{\exp\left(\frac{c|p_z|-\mu_L}{k_B T}\right)+1}, \quad p_z < 0\,,$$

$$f_R(p) = \frac{1}{\exp\left(\frac{c|p_z|-\mu_R}{k_B T}\right)+1}, \quad p_z > 0\,,$$

$$\alpha_{\rm em} \equiv \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

$$J_\mu = \frac{eB}{(2\pi\hbar)^2}\Big[\int_0^\infty \frac{dp_z}{2\pi\hbar}\,\psi^\dagger_{p_z}\gamma_z\psi_{p_z} f_R(p_z) + \int_{-\infty}^0 \frac{dp_z}{2\pi\hbar}\,\psi^\dagger_{p_z}\gamma_z\psi_{p_z} f_L(p_z)\Big]\,,$$

$$\mu_5^{\rm phys} \equiv \mu_{\rm L}^{\rm phys} - \mu_{\rm R}^{\rm phys}\,,$$

$$J_{\rm CME} = \frac{\alpha_{\rm em}}{\pi\hbar}\mu_5^{\rm phys}\boldsymbol{B},$$

Chiral Magnetic Effect

Vilenkin, A. 1980, Phys. Rev., D22, 3080

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Applications:

1. Astrophysics:
   - the early Universe,
   - proto-neutron stars;
2. Heavy ion collisions;
3. *Condensed Matter Physics*
   - new materials (Weyl semimetals);

Equations in Chiral MHD: Small Magnetic Diffusion

Rogachevskii, I., Ruchayskiy, O., Boyarsky, A., FrÖhlich, J., Kleeorin, N., Brandenburg, A., & Schober, J., 2017, Astrophys. J. 846, 153

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c} \boldsymbol{J}_{\rm tot} + \frac{1}{c} \frac{\partial \boldsymbol{E}}{\partial t},$$

$$\nabla \cdot \boldsymbol{E} = 4\pi \varrho_{\rm tot},$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c} \frac{\partial \boldsymbol{B}}{\partial t},$$

$$\nabla \cdot \boldsymbol{B} = 0,$$

$$\boldsymbol{J}_{\rm tot} = \boldsymbol{J}_{\rm Ohm} + \boldsymbol{J}_{\Theta}$$

$$\boldsymbol{J}_{\rm Ohm} = \sigma \left(\boldsymbol{E} + \frac{1}{c} \boldsymbol{U} \times \boldsymbol{B} \right)$$

Fröhlich & Pedrini (2000, 2002).

$$\varrho_{\rm tot} = \varrho_{\rm el} - \boldsymbol{B} \cdot \boldsymbol{\nabla} \Theta.$$

$$\boldsymbol{J}_{\rm CME} \to \boldsymbol{J}_{\Theta} = \boldsymbol{B} \frac{\partial \Theta}{\partial t} + c\, \boldsymbol{\nabla} \Theta \times \boldsymbol{E}.$$

Quasi-neutrality:

$$\varrho_{\rm tot} = 0.$$

Electric and Magnetic Fields for Small Magnetic Diffusion in Chiral MHD

$$\boldsymbol{E} = -\frac{1}{c} \left[\boldsymbol{U} \times \boldsymbol{B} + \eta \left(\mu_5 \boldsymbol{B} - \boldsymbol{\nabla} \times \boldsymbol{B} \right) \right] + {\rm O}(\eta^2),$$

$$\eta \equiv \frac{c^2}{4\pi\sigma}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \left[\boldsymbol{U} \times \boldsymbol{B} + \eta\, \mu_5 \boldsymbol{B} - \eta\, \boldsymbol{\nabla} \times \boldsymbol{B} \right],$$

Equations in Chiral MHD

Chiral Magnetic Effect: $J_{\rm CME} = \frac{\alpha_{\rm em}}{\pi\hbar}\mu_5^{\rm phys} B,$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times[\boldsymbol{U}\times\boldsymbol{B}+\eta(\mu_5\boldsymbol{B}-\boldsymbol{\nabla}\times\boldsymbol{B})],$$

$$\rho\frac{D\boldsymbol{U}}{Dt} = (\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}-\boldsymbol{\nabla}p+\boldsymbol{\nabla}\cdot\boldsymbol{\tau},$$

$$\frac{D\rho}{Dt} = -\rho\boldsymbol{\nabla}\cdot\boldsymbol{U},$$

together with the evolution equation of μ_5:

$$\frac{D\mu_5}{Dt} = \mathcal{D}_5(\mu_5)+\lambda\eta[\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})-\mu_5\boldsymbol{B}^2]-\mu_5\boldsymbol{\nabla}\cdot\boldsymbol{U}.$$

$$D/Dt \equiv \partial/\partial t + \boldsymbol{U}\cdot\boldsymbol{\nabla},$$

Applications:

1. Astrophysics:
   the early Universe,
   proto-neutron stars;
2. Heavy ion collisions;
3. *Condensed Matter Physics*
   new materials
   (Weyl semimetals);

$$\mu_5 \equiv (4\alpha_{\rm em}/\hbar c)\mu_5^{\rm phys}$$

$$\mu_5^{\rm phys} \equiv \mu_{\rm L}^{\rm phys} - \mu_{\rm R}^{\rm phys},$$

$$\lambda = 3\hbar c\left(\frac{8\alpha_{\rm em}}{k_B T}\right)^2$$

Conservation Law: Magnetic Helicity + Chiral Chemical Potential

$$\frac{\partial}{\partial t}\left(\frac{\lambda}{2}\boldsymbol{A}\cdot\boldsymbol{B}+\mu_5\right)+\boldsymbol{\nabla}\cdot\left[\frac{\lambda}{2}(\boldsymbol{E}\times\boldsymbol{A}-\boldsymbol{B}\,\Phi)-D_5\boldsymbol{\nabla}\mu_5\right]=0,$$

Conservation Law:

Magnetic Helicity + Chiral Chemical Potential

$$\frac{\partial}{\partial t}\left(\frac{\lambda}{2}\boldsymbol{A}\cdot\boldsymbol{B}+\mu_5\right)+\boldsymbol{\nabla}\cdot\left[\frac{\lambda}{2}(\boldsymbol{E}\times\boldsymbol{A}-\boldsymbol{B}\,\Phi)-D_5\boldsymbol{\nabla}\mu_5 \qquad \iota\right]=0,$$

Magnetic Helicity Density:

$$\frac{\partial \boldsymbol{A}\cdot\boldsymbol{B}}{\partial t}+\boldsymbol{\nabla}\cdot(\boldsymbol{E}\times\boldsymbol{A}+\boldsymbol{B}\,\Phi)=-2\boldsymbol{E}\cdot\boldsymbol{B}$$

$$\boldsymbol{E}\cdot\boldsymbol{B}\propto\eta$$

$$\eta\to 0$$

Chiral Chemical Potential:

$$\frac{\partial(2\mu_5/\lambda)}{\partial t}+\boldsymbol{\nabla}\cdot[-(2D_5/\lambda)\,\boldsymbol{\nabla}\mu_5]=2\boldsymbol{E}\cdot\boldsymbol{B},$$

$$\lambda=3\hbar c\left(\frac{8\alpha_{\rm em}}{k_B T}\right)^2$$

$$\boldsymbol{E}=-\boldsymbol{U}\times\boldsymbol{B}+\eta\boldsymbol{\nabla}\times\boldsymbol{B}-\eta\,\mu_5\,\boldsymbol{B}+\mathrm{O}(\eta^2)$$

Chiral Dynamo Instability

Joyce and Shaposhnikov, PRL 79, 1193 (1997)

$$\mu_{\rm eq} \equiv \mu_0 = {\rm const\ and\ } \boldsymbol{U}_{\rm eq} = 0.$$

$$\boldsymbol{B}(t,x,z) = B_y(t,x,z)\boldsymbol{e}_y + \boldsymbol{\nabla} \times [A(t,x,z)\boldsymbol{e}_y]$$

$$\gamma = |v_\mu\, k| - \eta k^2,$$

Chiral velocity

$$v_\mu \equiv \eta\mu_{5,0}$$

$$\gamma^{\rm max} = v_\mu^2/4\eta,$$

$$k^{\rm max} = \frac{1}{2}|\mu_0|.$$

$$\frac{\partial A(t,x,z)}{\partial t} = v_\mu\, B_y + \eta\Delta A,$$

$$\frac{\partial B_y(t,x,z)}{\partial t} = -v_\mu\, \Delta A + \eta\Delta B_y,$$

Nonlinear dynamo

Schober, Rogachevskii, Brandenburg, et al., ApJ **858**, 124 (2018)

Nonlinear dynamos Equations

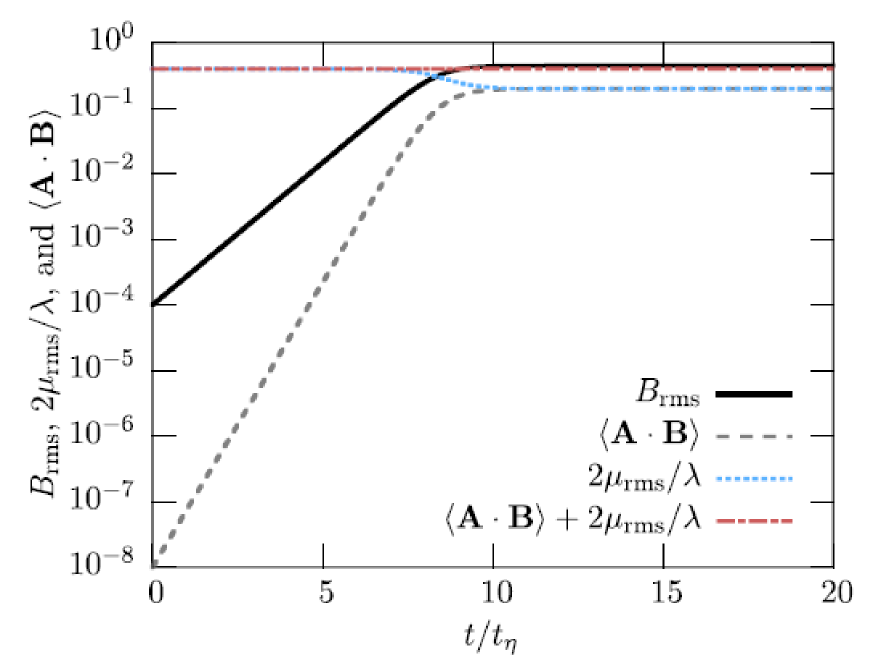
$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{U} \times \boldsymbol{B} + \eta(\mu_5 \boldsymbol{B} - \boldsymbol{\nabla} \times \boldsymbol{B})],$$

$$\rho \frac{D\boldsymbol{U}}{Dt} = (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{\nabla} p + \boldsymbol{\nabla}\cdot\boldsymbol{\tau},$$

$$\frac{D\rho}{Dt} = -\rho \boldsymbol{\nabla} \cdot \boldsymbol{U},$$

together with the evolution equation of μ_5:

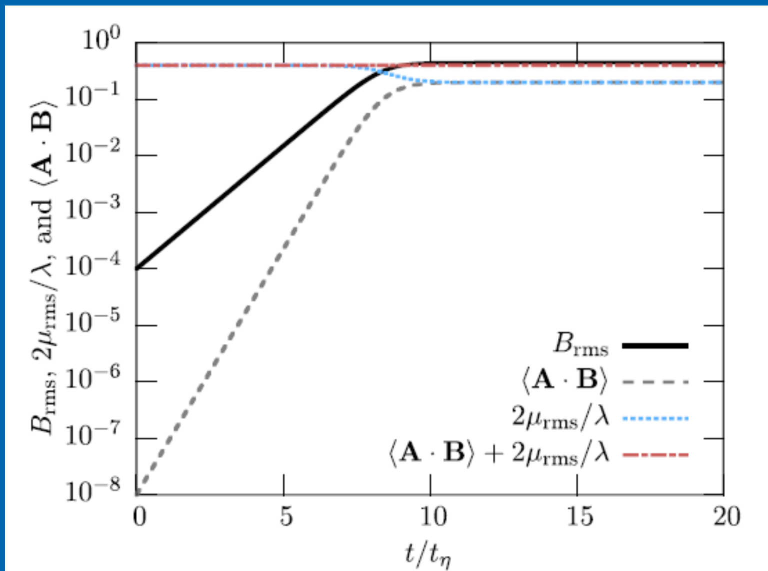
$$\frac{D\mu_5}{Dt} = \mathcal{D}_5(\mu_5) + \lambda\eta[\boldsymbol{B}\cdot(\boldsymbol{\nabla} \times \boldsymbol{B}) - \mu_5 \boldsymbol{B}^2] - \mu_5 \boldsymbol{\nabla} \cdot \boldsymbol{U}.$$



$$D/Dt \equiv \partial/\partial t + \boldsymbol{U}\cdot\boldsymbol{\nabla},$$

Chiral-Magnetically Produced Turbulence and Inverse Energy Cascade

Schober, Rogachevskii, Brandenburg, et al. ApJ. 858, 124 (2018)



$$\gamma = |v_\mu\, k| - \eta k^2,$$

$$v_\mu \equiv \eta\mu_{5,0}$$

$$\gamma^{\max} = v_\mu^2/4\eta,$$

$$k^{\max} = \frac{1}{2}|\mu_0|.$$

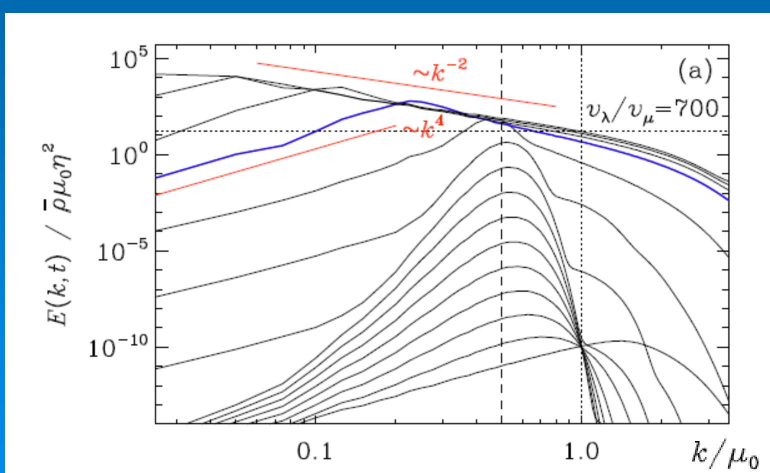
$$\langle \boldsymbol{B}^2\rangle\, \xi_{\rm M} \lesssim \mu_0/\lambda,$$

$$½\lambda\, \langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle + \langle \mu_5\rangle = \text{const} \equiv \mu_0$$

Brandenburg et al., ApJL 845, L21 (2017)

$$v_\lambda = \mu_5/(\overline{\rho}\lambda)^{1/2},$$

$$E_{\rm M}(k,t) = C_\mu\, \overline{\rho}\, \mu_5^3 \eta^2 k^{-2}$$



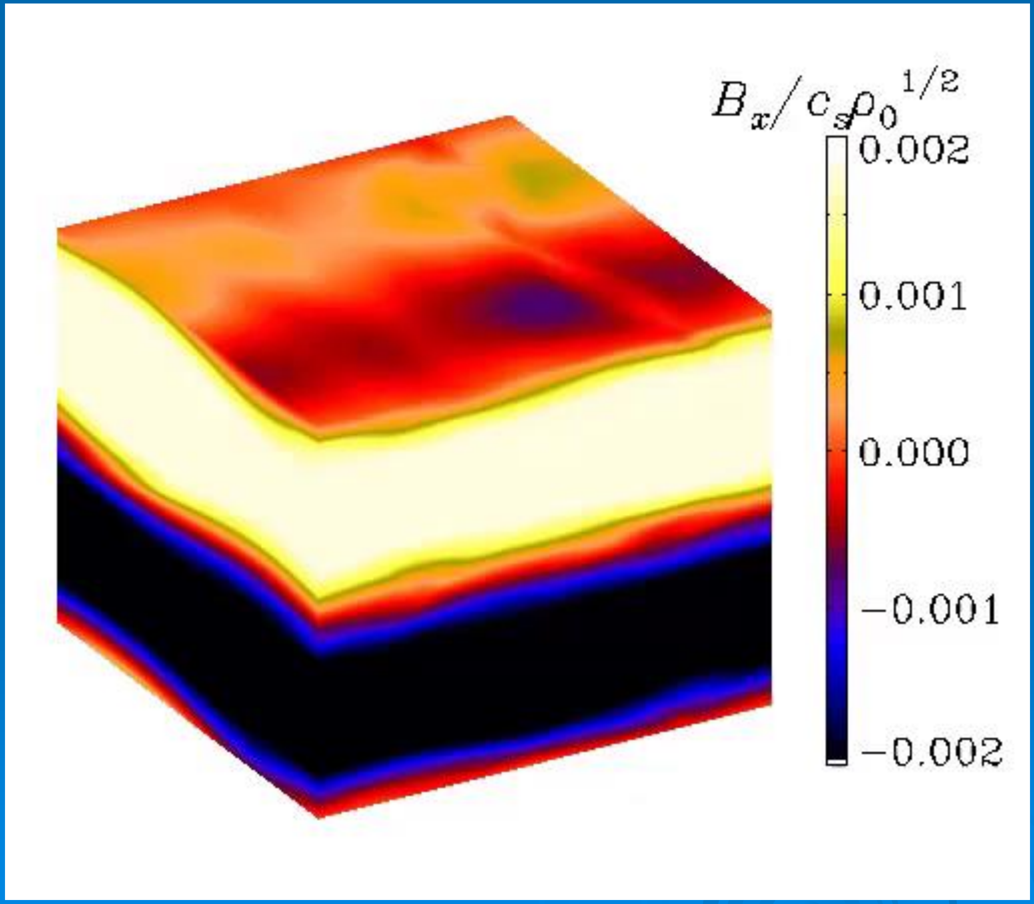
Inverse Energy cascade has been predicted by Boyarsky, Fröhlich, Ruchayskiy, PRL (2012)

$$\frac{\partial}{\partial t}\left(\frac{\lambda}{2}\boldsymbol{A}\cdot\boldsymbol{B} + \mu_5\right) + \boldsymbol{\nabla}\cdot\left[\frac{\lambda}{2}(\boldsymbol{E}\times\boldsymbol{A} - \boldsymbol{B}\,\Phi) - D_5\boldsymbol{\nabla}\mu_5\right] = 0,$$

Generation of Large-Scale Magnetic Field: Mean-Field Theory

Magnetically Produced Turbulence and Generation of Large-Scale Magnetic Field

Rogachevskii et al., ApJ **846**, 153 (2017)
Brandenburg et al., ApJL **845**, L21 (2017)
Schober et al., ApJ **858**, 124 (2018); GAFD 113, 107-130 (2019); GAFD 114, 106-129 (2020).



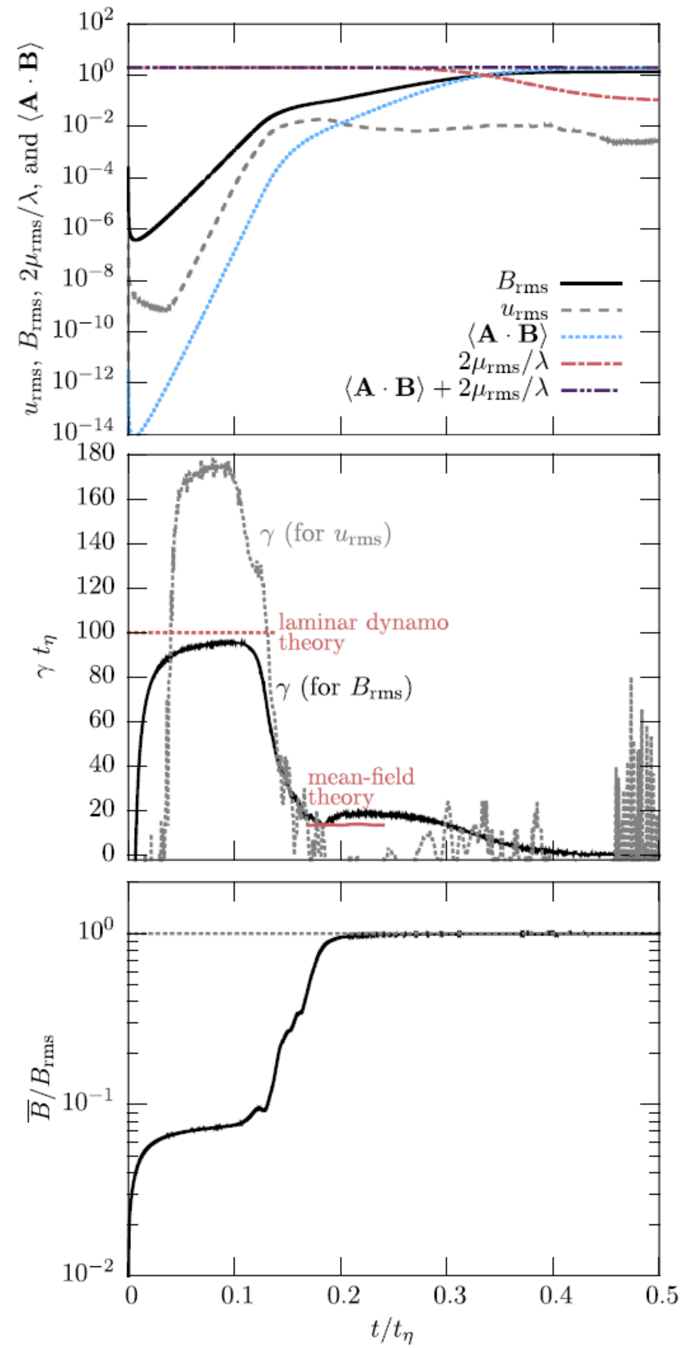


Figure 9. Chiral magnetically driven turbulence. Time evolution for different quantities.

Mean-Field Theory in MHD

Mean-Field Approach: $B = \overline{B} + b, \quad U = \overline{U} + u,$

- **Induction equation for mean magnetic field:**

$$\frac{\partial \overline{B}}{\partial t} = \nabla \times \left(\overline{U} \times \overline{B} + \langle u \times b \rangle \right) + \eta \Delta \overline{B},$$

$$\overline{B} = \langle B \rangle$$

$$\overline{U} = \langle U \rangle$$

- **Turbulent electromotive force:** $\mathcal{E} = \langle u \times b \rangle,$

$$\mathcal{E} = \alpha \overline{B} + V^{\mathrm{eff}} \times \overline{B} - \eta_T (\nabla \times \overline{B}),$$

$$\alpha = -\frac{\tau_0}{3} \langle u \cdot (\nabla \times u) \rangle .$$

$$V^{\mathrm{eff}} = -\frac{1}{2} \nabla \eta_T .$$

$$\eta_T = \frac{\tau_0}{3} \langle u^2 \rangle .$$

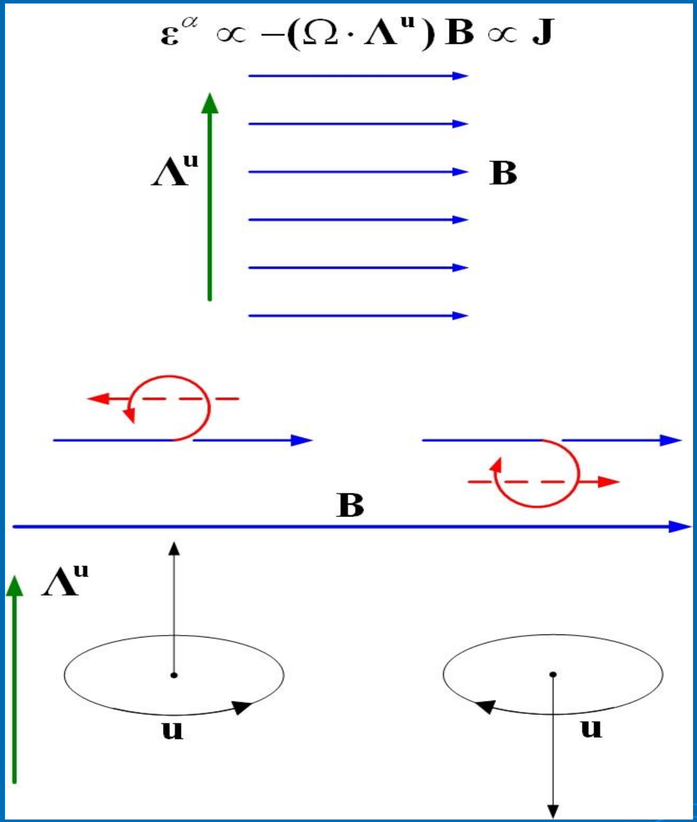
Steenbeck, Krause, Rädler (1966)

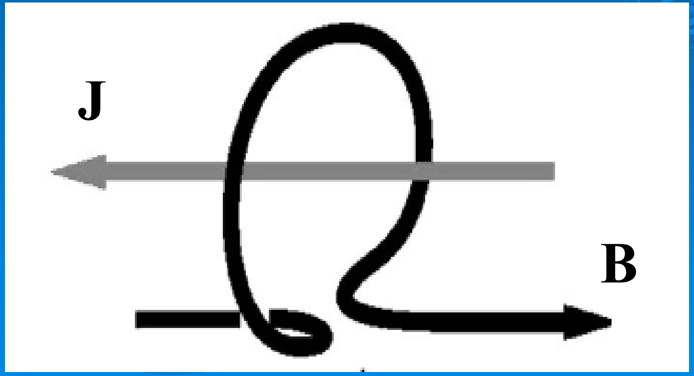
Physics of the Kinetic Alpha-Effect

Parker (1955); Steenbeck, Krause, Rädler (1966)

$$\alpha = -\frac{\tau_0}{3} \langle u \cdot (\nabla \times u) \rangle .$$

- The α - effect is related to the kinetic helicity in a density stratified or inhomogeneous rotating turbulence.
- The deformations of the magnetic field lines are caused by upward and downward rotating turbulent eddies.
- The stratification of turbulence breaks a symmetry between the upward and downward eddies.
- Therefore, the total effect of the upward and downward eddies on the mean magnetic field does not vanish and it creates the mean electric current parallel to the original mean magnetic field.





Algebraic Nonlinearities in Mean-Field Dynamo

- **Induction equation for mean magnetic field:**

$$\frac{\partial \overline{B}}{\partial t} = \nabla\times\left(\overline{U}\times\overline{B} + \langle u\times b\rangle\right) + \eta\,\Delta\overline{B},$$

- **Nonlinear turbulent electromotive force (algebraic nonlinearity):**

$$\mathcal{E}\left(\overline{B}\right) = \alpha\left(\overline{B}\right)\overline{B} + V^{\rm eff}\left(\overline{B}\right)\times\overline{B} - \eta_{T}\left(\overline{B}\right)\left(\nabla\times\overline{B}\right),$$

Iroshnikov (1970); Rüdiger (1974)

$$\alpha(\overline{B}) = \frac{\alpha_{\rm K}}{1+\overline{B}^2/\overline{B}_{\rm eq}^2},$$

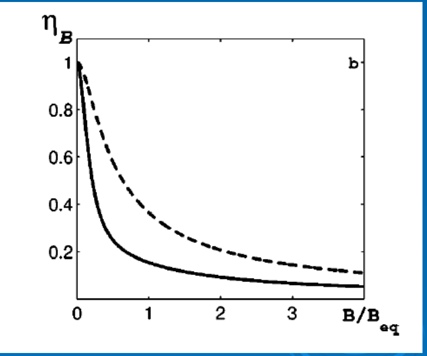
$$\overline{B}_{\rm eq}^2 = \mu_0\,\overline{\rho}\langle u^2\rangle,$$

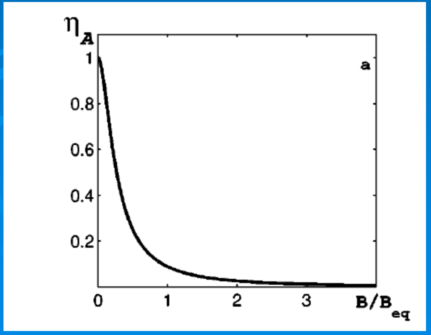
Nonlinear turbulent electromotive force:

Rüdiger and Kichatinov (1993; 1994) $\mathrm{Rm} \ll 1$

Field, Blackman, Chou (1999)
Rogachevskii, Kleeorin (2000; 2001; 2004; 2006) $\mathrm{Rm} \gg 1$

Turbulent diffusion





Methods for Derivations

- **Quasi-Linear Approach** or Second-Order Correlation Approximation (SOCA) or First-Order Smoothing Approximation (FOSA)

  Rm << 1 , Re << 1

  Steenbeck, Krause, Rädler (1966); Roberts, Soward (1975); Moffatt (1978)

- **Path-Integral Approach** (delta-correlated in time random velocity field or short yet finite correlation time)

  Zeldovich, Molchanov, Ruzmaikin, Sokoloff (1988)
  Rogachevskii, Kleeorin (1997)

  $$\mathrm{St} = \frac{\tau}{\ell/u} \ll 1$$

- **Tau-approaches** (spectral tau-approximation, minimal tau-approximation) – third-order or high-order closure

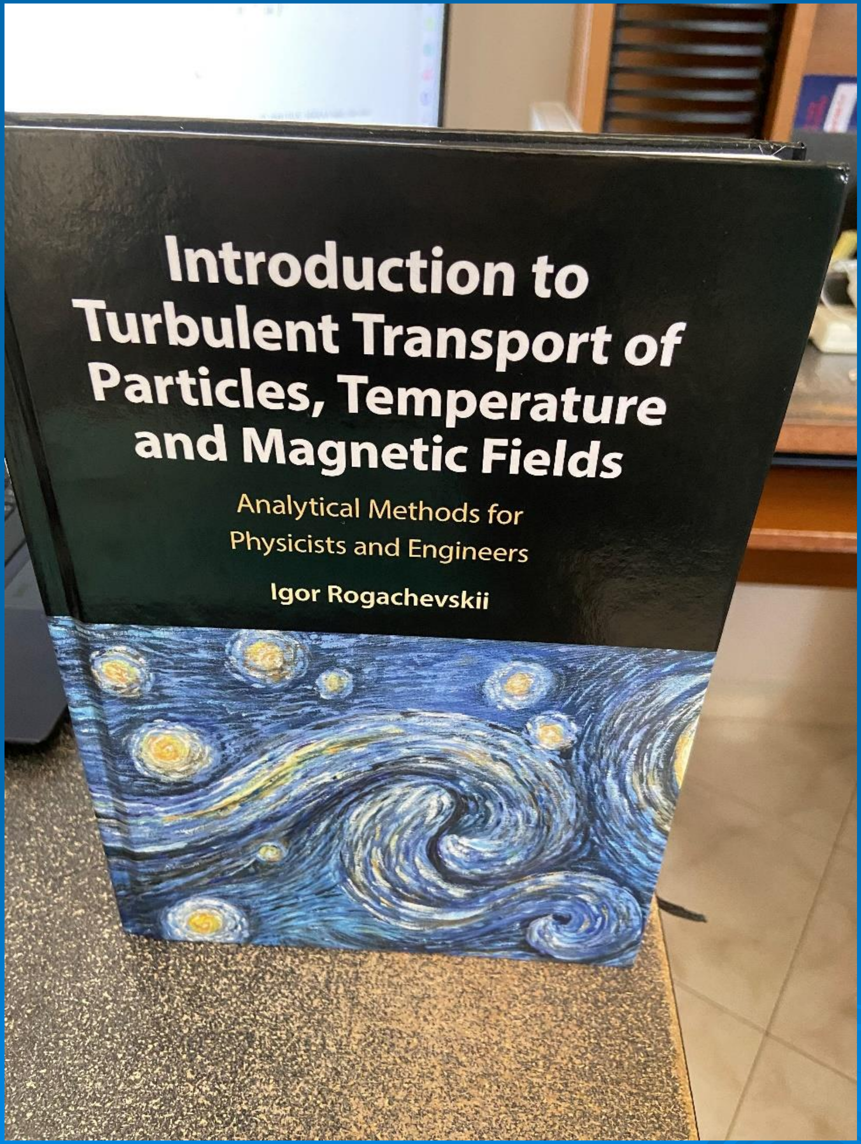
  Re >> 1 and Rm >> 1

  Pouquet, Frisch, Leorat (1976); Kleeorin, Rogachevskii, Ruzmaikin (1990)
  Rogachevskii, Kleeorin (2000; 2001; 2003); Blackman, Field (2002);
  Rädler, Kleeorin, Rogachevskii (2003)

- **Renormalization Procedure** (renormalization of viscosity, diffusion, electromotive force and other turbulent transport coefficients) - there is no separation of scales

  Moffatt (1981; 1983); Kleeorin, Rogachevskii (1994)

I. Rogachevskii, "Introduction to Turbulent Transport of Particles, Temperature and Magnetic Fields" (Cambridge University Press, Cambridge, 2021).



- **Various analytical methods are applied in this book:**
- **Mean-field approach;**
- **Multi-scale approach;**
- **Dimensional analysis;**
- **Quasi-linear approach;**
- **Tau approach;**
- **Path-integral approach;**
- **Analyses based on the budget equations.**
- **One-way and two-way couplings between turbulence and particles, or temperature, or magnetic fields are described.**

•Table of Contents:

Dynamic Nonlinearity in Mean-Field Dynamo

- **Induction equation for mean magnetic field:**

$$\frac{\partial \overline{B}}{\partial t} = \nabla\times\left(\overline{U}\times\overline{B} + \langle u\times b\rangle\right) + \eta\,\Delta\overline{B},$$

$$\mathcal{E}\left(\overline{B}\right) = \alpha\left(\overline{B}\right)\overline{B} + V^{\mathrm{eff}}\left(\overline{B}\right)\times\overline{B} - \eta_{T}\left(\overline{B}\right)\left(\nabla\times\overline{B}\right),$$

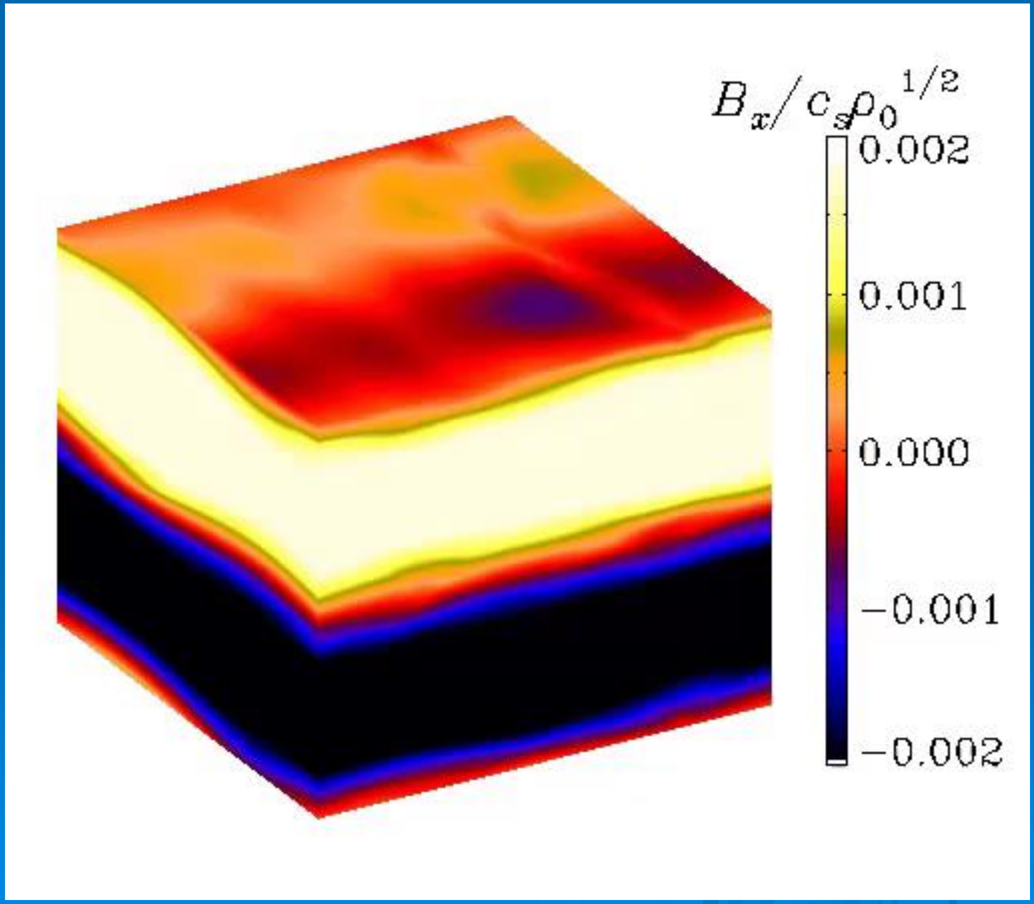
- **Total (kinetic + magnetic) nonlinear alpha effects (dynamic nonlinearity):**

$$\alpha\left(\overline{B}\right) = -\frac{\tau_{\mathrm{c}}}{3}\langle u\cdot(\nabla\times u)\rangle + \frac{\tau_{\mathrm{c}}}{3\mu_0\,\overline{\rho}}\langle b\cdot(\nabla\times b)\rangle,$$

A. Pouquet, U. Frisch, and J. Leorat, J. Fluid Mech. 77, 321 (1976)

Magnetically Produced Turbulence and Generation of Large-Scale Magnetic Field

Rogachevskii et al., ApJ **846**, 153 (2017)
Brandenburg et al., ApJL **845**, L21 (2017)
Schober et al., ApJ **858**, 124 (2018); GAFD 113, 107-130 (2019); GAFD 114, 106-129 (2020).



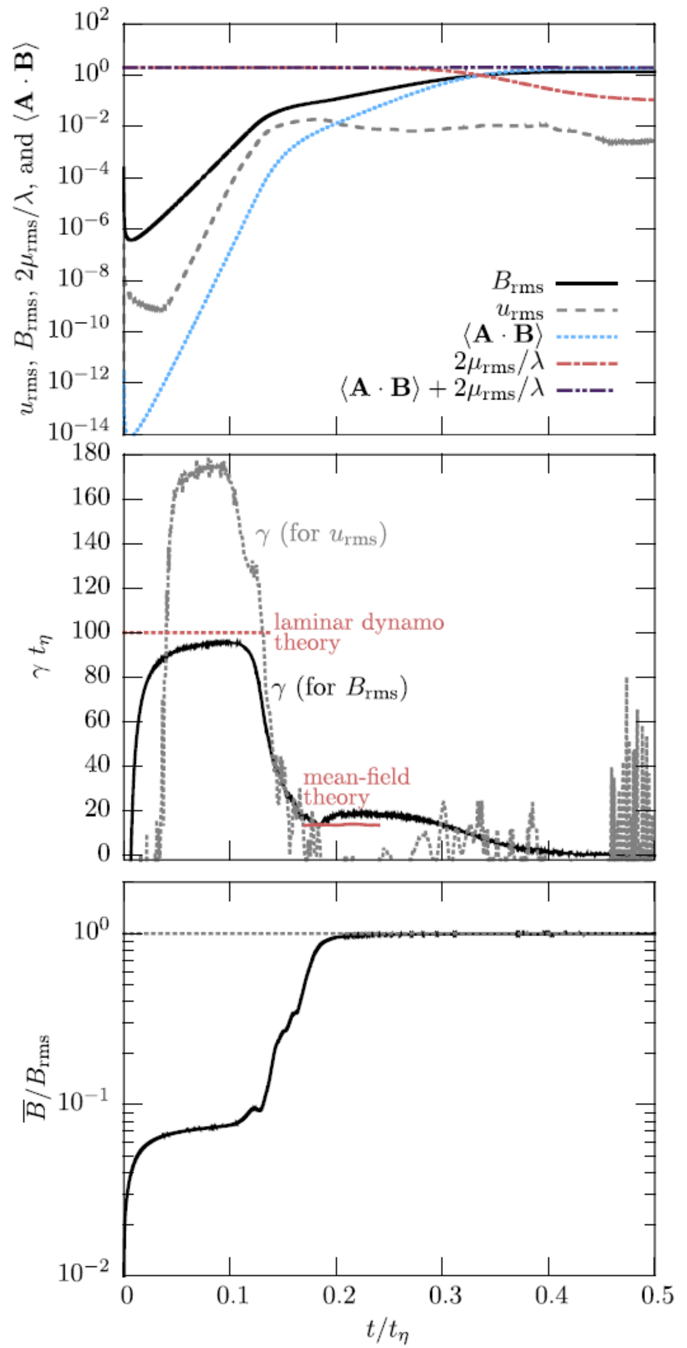


Figure 9. Chiral magnetically driven turbulence. Time evolution for different quantities.

New Kind of Chiral Alpha Effect (Dimensional Analysis)

Rogachevskii, I., Ruchayskiy, O., Boyarsky, A., Fröhlich, J., Kleeorin, N., Brandenburg, A., & Schober, J., 2017, Astrophys. J. 846, 153

- **Magnetic fluctuations:**

$$\frac{\partial b}{\partial t} = (\overline{B} \cdot \nabla) u + \overline{v}_\mu \nabla \times b + \ldots,$$

$$b_{\rm tang} = \tau \, (\overline{B} \cdot \nabla) u,$$

- **Chiral magnetic fluctuations:**

$$\overline{v}_\mu = \overline{\rho} \, \overline{\mu}_5$$

$$b_\mu = \tau \, \overline{v}_\mu \nabla \times b_{\rm tang} = \tau^2 \, \overline{v}_\mu \nabla \times \, [(\overline{B} \cdot \nabla) u],$$

- **Chiral electromotive force** $\overline{\mathcal{E}}^\mu \equiv \overline{u \times b_\mu}$:

$$\overline{\mathcal{E}}_i^\mu = \left(\tau^2 \, \overline{v}_\mu \, \overline{u_n \nabla_i \nabla_j u_n}\right) \overline{B}_j \equiv \alpha_{ij}^\mu \, \overline{B}_j .$$

- **Chiral alpha effect:**

$$\alpha_{ij}^\mu = \overline{v}_\mu \, \tau^2 \, \overline{u_n \nabla_i \nabla_j u_n} = -\overline{v}_\mu \int \tau^2(k) \, k_i k_j \, \langle u^2 \rangle_{\boldsymbol{k}} \, dk,$$

$$\alpha_\mu = -\frac{2}{3} \overline{v}_\mu \ln {\rm Re}_M .$$

New Kind of Chiral Alpha Effect (quasi-linear approach and tau approach)

- **Magnetic fluctuations:**

$$\frac{\partial b}{\partial t} = (\overline{B} \cdot \nabla) u + \overline{v}_\mu \nabla \times b + ...,$$

$$\overline{v}_\mu = \overline{\rho}\,\overline{\mu}_5$$

- **Chiral magnetic fluctuations:**

$$b_\mu = \tau\, \overline{v}_\mu \nabla \times b_{\rm tang} = \tau^2\, \overline{v}_\mu \nabla \times \left[(\overline{B} \cdot \nabla) u\right],$$

- **Chiral electromotive force** $\overline{\mathcal{E}}^\mu \equiv \overline{u \times b_\mu}$ **:**

$$\overline{\mathcal{E}}_i^\mu = \left(\tau^2\, \overline{v}_\mu\, \overline{u_n \nabla_i \nabla_j u_n}\right)\, \overline{B}_j \equiv \alpha_{ij}^\mu\, \overline{B}_j.$$

- **Chiral alpha effect:**

$$\alpha_{ij}^\mu = \overline{v}_\mu\, \tau^2\, \overline{u_n \nabla_i \nabla_j u_n} = -\overline{v}_\mu \int \tau^2(k)\, k_i k_j\, \langle u^2 \rangle_k\, dk,$$

Quasi-linear approach: Rm<<1

$$\alpha_\mu = -\frac{(q-1)}{3(q+1)}\, {\rm Re}_{_M}^2\, \overline{v}_\mu.$$

Tau-approach: Rm>>1

$$\alpha_\mu = -\frac{2}{3} \overline{v}_\mu \ln {\rm Re}_{_M}.$$

How is it possible to produce chiral asymmetry?

$$\langle \mu_5 \rangle (t_0) = 0.$$

Schober J., Rogachevskii I., Brandenburg A., Phys. Rev. Lett. 128, 065002 (2022); Phys. Rev. D 105, 043507 (2022).

$$\mu_5 \equiv (4\alpha_{\rm em}/\hbar c)\mu_5^{\rm phys}$$

$$\mu_5^{\rm phys} \equiv \mu_{\rm L}^{\rm phys} - \mu_{\rm R}^{\rm phys},$$

Equations in Chiral MHD

Chiral Magnetic Effect: $J_{\rm CME} = \frac{\alpha_{\rm em}}{\pi\hbar} \mu_5^{\rm phys} B,$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{U} \times \boldsymbol{B} + \eta(\mu_5 \boldsymbol{B} - \boldsymbol{\nabla} \times \boldsymbol{B})],$$

$$\rho \frac{D\boldsymbol{U}}{Dt} = (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{\nabla} p + \boldsymbol{\nabla}\cdot\boldsymbol{\tau},$$

$$\frac{D\rho}{Dt} = -\rho \boldsymbol{\nabla} \cdot \boldsymbol{U},$$

together with the evolution equation of μ_5:

$$\frac{D\mu_5}{Dt} = \mathcal{D}_5(\mu_5) + \lambda\eta[\boldsymbol{B}\cdot(\boldsymbol{\nabla} \times \boldsymbol{B}) - \mu_5 \boldsymbol{B}^2] - \mu_5 \boldsymbol{\nabla} \cdot \boldsymbol{U}.$$

$$D/Dt \equiv \partial/\partial t + \boldsymbol{U}\cdot\boldsymbol{\nabla},$$

Applications:

1. Astrophysics:
   the early Universe,
   proto-neutron stars;
2. Heavy ion collisions;
3. *Condensed Matter Physics*
   new materials
   (Weyl semimetals);

$$\mu_5 \equiv (4\alpha_{\rm em}/\hbar c)\mu_5^{\rm phys}$$

$$\mu_5^{\rm phys} \equiv \mu_{\rm L}^{\rm phys} - \mu_{\rm R}^{\rm phys},$$

$$\lambda = 3\hbar c \left(\frac{8\alpha_{\rm em}}{k_B T}\right)^2$$

Conservation Law: Magnetic Helicity + Chiral Chemical Potential

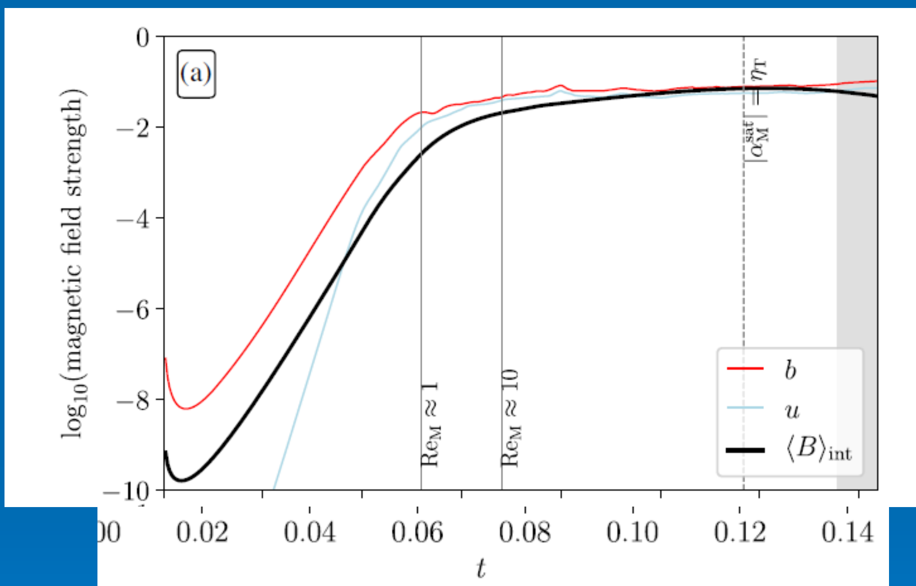
$$\frac{\partial}{\partial t}\left(\frac{\lambda}{2}\boldsymbol{A}\cdot\boldsymbol{B} + \mu_5\right) + \boldsymbol{\nabla}\cdot\left[\frac{\lambda}{2}(\boldsymbol{E} \times \boldsymbol{A} - \boldsymbol{B}\,\Phi) - D_5 \boldsymbol{\nabla}\mu_5 \quad \right] = 0,$$

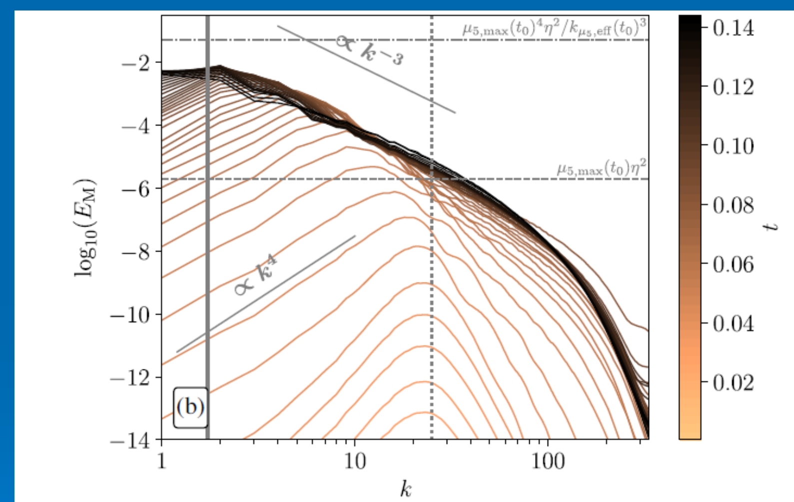
Evolution of Magnetic Field and Magnetic Energy Spectrum

Schober J., Rogachevskii I., Brandenburg A., Phys. Rev. Lett. 128, 065002 (2022); Phys. Rev. D 105, 043507 (2022).

Initial random distributions of μ_5 at different wave numbers with a power-law spectrum and zero mean value: $\langle \mu_5 \rangle (t_0) = 0.$

$$\gamma_5 = \frac{\eta \mu_{5,\mathrm{max}}^2}{4},$$





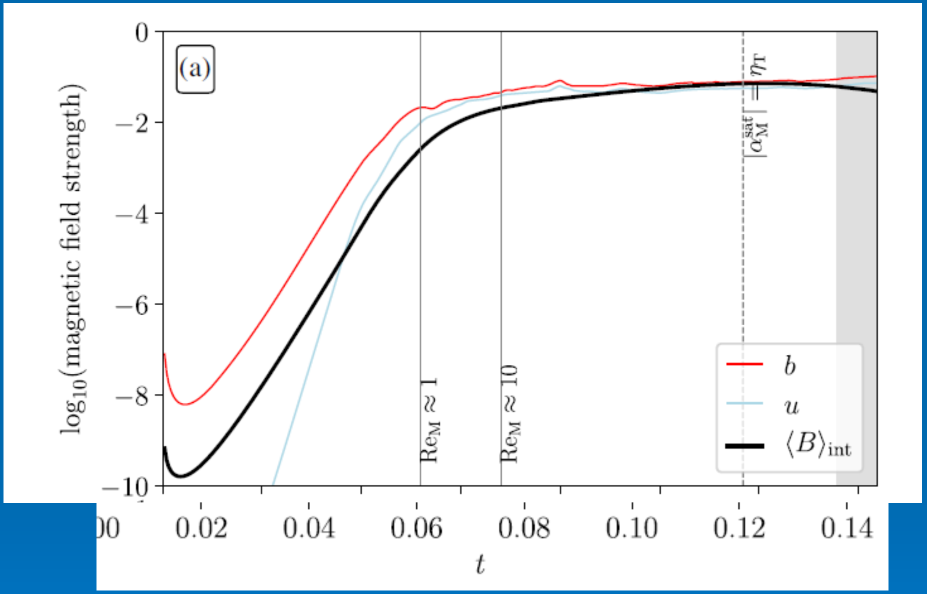
Magnetic field:

B_{rms}	$(2 \int E_{\mathrm{M}}(k) \mathrm{d}k)^{1/2}$	Rms magnetic field strength
b	$(2 \int_{k_5}^{k_{\max}} E_{\mathrm{M}}(k) \mathrm{d}k)^{1/2}$	Field strength of small-scale magnetic fluctuations
$\langle B \rangle_{\mathrm{int}}$	$\left(\frac{\int E_{\mathrm{M}}(k)^2 \mathrm{d}k}{\int E_{\mathrm{M}}(k) \mathrm{d}k} \right)^{1/2}$	Magnetic field strength on the integral scale of turbulence

Small-Scale Chiral Dynamo Instability

Schober J., Rogachevskii I., Brandenburg A., Phys. Rev. Lett. 128, 065002 (2022); Phys. Rev. D 105, 043507 (2022).

Initial random distributions of μ_5 at different wave numbers with a power-law spectrum and zero mean value: $\langle \mu_5 \rangle (t_0) = 0.$



$$\gamma_5 = \frac{\eta \mu_{5,\max}^2}{4},$$

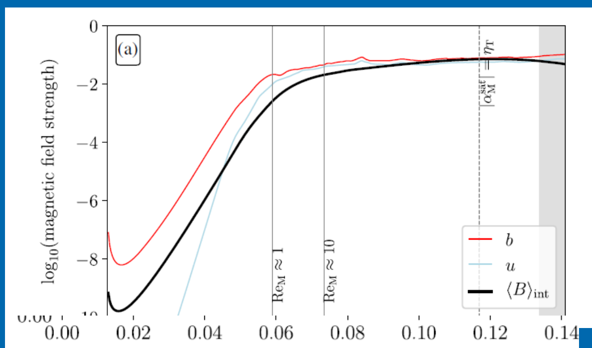
$$k_5 / k_{\mu_5,\mathrm{eff}} \gtrsim 10$$

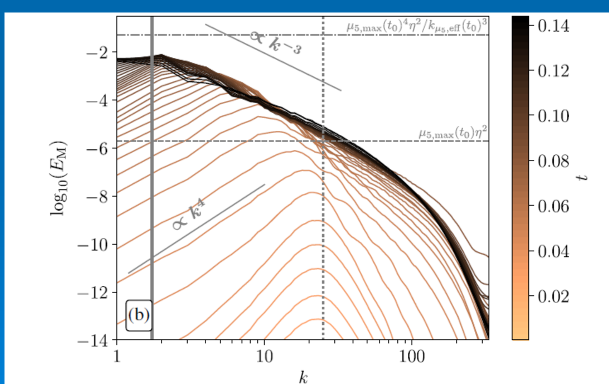
$$k_5(t) = \mu_{5,\max}(t)/2$$

$$k_{\mu_5,\mathrm{eff}}^{-1}(t) = \langle \mu_5^2 \rangle^{-1} \int k^{-1} E_5(k) \mathrm{d}k.$$

Chiral Dynamo, Production of Turbulence and Mean-Field Dynamo

Schober J., Rogachevskii I., Brandenburg A., Phys. Rev. Lett. 128, 065002 (2022); Phys. Rev. D 105, 043507 (2022).



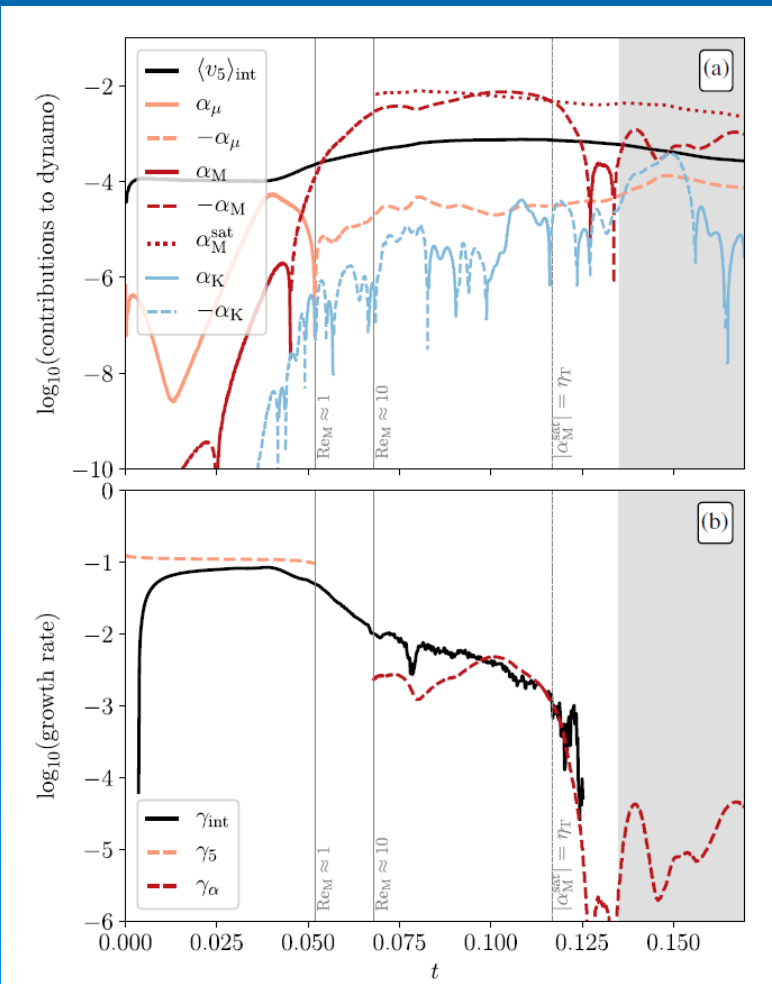


- The initial fluctuations of the chiral chemical potential with a wide range of scales produce magnetic fluctuations by the chiral dynamo.
- Fluctuations of the chiral chemical potential on larger scales serve as a mean field for fluctuations on smaller scales, so that the chiral dynamo instability excites magnetic fluctuations and produces small-scale magnetic helicity.
- Simultaneously, the chiral dynamo drives turbulence by the Lorentz force and enhances the fluid and magnetic Reynolds numbers.
- When Reynolds numbers are large enough, the mean-field dynamo instability is excited and amplifies a large-scale magnetic field

Dominant Magnetic Alpha Effect in Chiral Mean-Field Dynamo

Schober J., Rogachevskii I., Brandenburg A., Phys. Rev. Lett. 128, 065002 (2022); Phys. Rev. D 105, 043507 (2022).

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{U} \times \boldsymbol{B} + \eta\, \mu_5 \boldsymbol{B} - \eta\, \boldsymbol{\nabla} \times \boldsymbol{B}] \,,$$



Turbulent Electromotive Force

$$\overline{\boldsymbol{\mathcal{E}}} \equiv \overline{\boldsymbol{u} \times \boldsymbol{b}} = \alpha_M \overline{\boldsymbol{B}} - \eta_T (\boldsymbol{\nabla} \times \overline{\boldsymbol{B}})$$

(i) Magnetic Alpha Effect

$$\alpha_M = 2(q-1)/(q+1)\tau_c \chi_c$$

Current Helicity

$$\chi_c = \langle \boldsymbol{b} \cdot (\boldsymbol{\nabla} \times \boldsymbol{b}) \rangle_{\rm int}$$

(ii) Kinetic Alpha Effect

$$\alpha_K = -(1/3)\tau_c \chi_K$$

Kinetic Helicity

$$\chi_K = \langle \boldsymbol{u} \cdot \boldsymbol{\omega} \rangle_{\rm int}$$

(iii) Mean $\overline{v}_5$ Effect

(iv) α_μ - Effect

$$\alpha_\mu = -\frac{2}{3} \overline{v}_5 \ln(\mathrm{Re_M})$$

$$\overline{v}_5 = \overline{\rho}\, \overline{\mu}_5$$

Budget Equation for Magnetic Helicity:

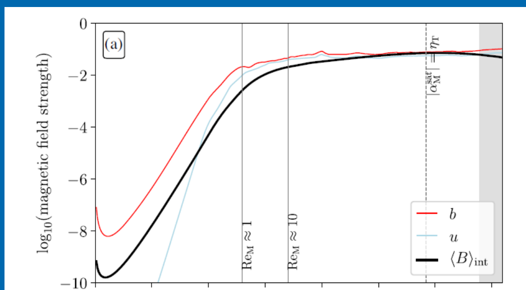
$$\frac{\partial}{\partial t} \overline{\boldsymbol{a} \cdot \boldsymbol{b}} + \mathrm{div} \boldsymbol{F} = 2 \overline{v}_5 \overline{\boldsymbol{b}^2} - 2 \overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} - 2\eta \overline{\boldsymbol{b}(\boldsymbol{\nabla} \times \boldsymbol{b})},$$

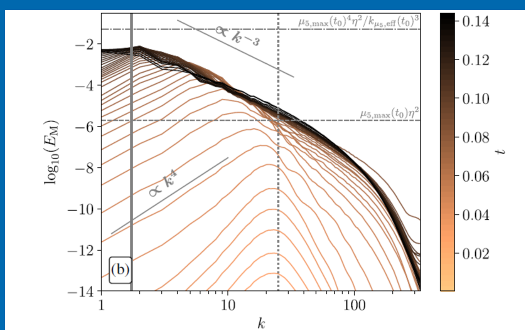
$$\alpha_{\rm M}^{\rm sat} = \eta \bar{\mu}_5 \frac{\overline{\boldsymbol{b}^2}}{\overline{\boldsymbol{B}}^2}$$

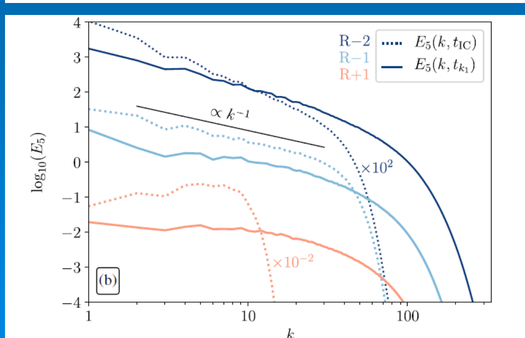
$$2 \overline{v}_5 \overline{\boldsymbol{b}^2} - 2 \alpha_{\rm M} \overline{\boldsymbol{B}}^2,$$

Chiral Dynamo, Production of Turbulence and Mean-Field Dynamo

Schober J., Rogachevskii I., Brandenburg A., Phys. Rev. Lett. 128, 065002 (2022);
Phys. Rev. D 105, 043507 (2022).







- The initial fluctuations of the chiral chemical potential with a wide range of scales produce magnetic fluctuations by the chiral dynamo. Indeed, fluctuations of the chiral chemical potential on larger scales serve as a mean field for fluctuations on smaller scales, so that the chiral dynamo instability excites magnetic fluctuations and produces small-scale magnetic helicity.

- Simultaneously, the chiral dynamo drives turbulence magnetically by the Lorentz force and enhances the fluid and magnetic Reynolds numbers.

- When Reynolds numbers are large enough, the mean-field dynamo instability is excited and amplifies a large-scale magnetic field that is well described by the magnetic alpha effect caused by current helicity.

- During the mean-field dynamo phase, the power spectra develop a universal shape for magnetic energy $E_M \propto k^{-3}$ and the chiral chemical potential $E_5 \propto k^{-1}$.

How is it possible to produce chiral asymmetry?

$$\mu_5 = 0$$

Schober J., Rogachevskii I., Brandenburg A.,
Phys. Rev. Lett. **132**, 065101 (2024);
Phys. Rev. D, **110**, 043515 (2024)

Chiral Magnetic Wave

$$\frac{\partial B}{\partial t} = (B_0 \cdot \nabla) U - B_0 (\nabla \cdot U) + \eta \Delta B$$
$$+ v_\mu (\nabla \times B - \mu_0 \Delta U) ,$$
$$\rho_0 \frac{\partial U}{\partial t} = (B_0 \cdot \nabla) B - \nabla (p + B_0 \cdot B) ,$$
$$\frac{\partial \rho}{\partial t} = -\rho_0 (\nabla \cdot U) ,$$
$$\frac{\partial \tilde{\mu}_5}{\partial t} = \lambda \left[\eta B_0 \cdot (\nabla \times B) - 2 v_\mu B_0 \cdot B - \eta \tilde{\mu}_5 B_0^2 \right]$$
$$- C_5 (B_0 \cdot \nabla) \mu ,$$
$$\frac{\partial \mu}{\partial t} = - C_\mu (B_0 \cdot \nabla) \tilde{\mu}_5 .$$

Alfven wave

$$\omega_A = \boldsymbol{k} \cdot \boldsymbol{v}_A$$

$$v_A = B_0 / \sqrt{\rho}$$

Chiral magnetic wave:

$$\Omega_{\mathrm{CMW}} = \sqrt{C_5 \, C_\mu} \, |\boldsymbol{k} \cdot B_0| ,$$

Kharzeev and Yee (2011)

The dispersion relation for the MHD waves is not modified by the chiral magnetic wave, and vise versa.

However, the MHD waves can play a source for the chiral magnetic wave.

Chiral Asymmetry and Dynamo Produced by Inhomogeneous Fluctuations of Chemical Potential

$$k_5/k_{\mu_5,\mathrm{eff}} \gtrsim 10,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{U} \times \boldsymbol{B} + \eta\,(\mu_5 \boldsymbol{B} - \boldsymbol{\nabla} \times \boldsymbol{B})],$$

$$\rho \frac{D\boldsymbol{U}}{Dt} = (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{\nabla} p + \boldsymbol{\nabla}\cdot(2\nu\rho \mathbf{S}),$$

$$\frac{D\rho}{Dt} = -\rho\,\boldsymbol{\nabla}\cdot\boldsymbol{U},$$

$$\frac{D\mu}{Dt} = -\mu\,\boldsymbol{\nabla}\cdot\boldsymbol{U} - C_\mu(\boldsymbol{B}\cdot\boldsymbol{\nabla})\mu_5 - \mathcal{D}_\mu\,\nabla^4\mu,$$

$$\frac{D\mu_5}{Dt} = -\mu_5\,\boldsymbol{\nabla}\cdot\boldsymbol{U} - C_5(\boldsymbol{B}\cdot\boldsymbol{\nabla})\mu - \mathcal{D}_5\,\nabla^4\mu_5 + \lambda\eta\left[\boldsymbol{B}\cdot(\boldsymbol{\nabla} \times \boldsymbol{B}) - \mu_5 \boldsymbol{B}^2\right],$$

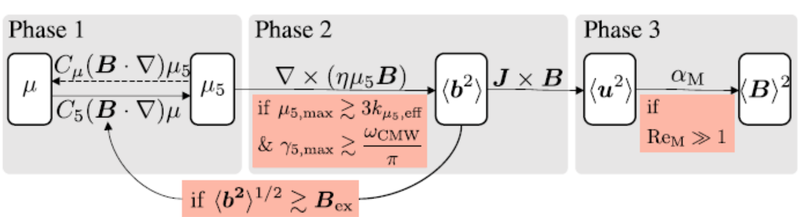
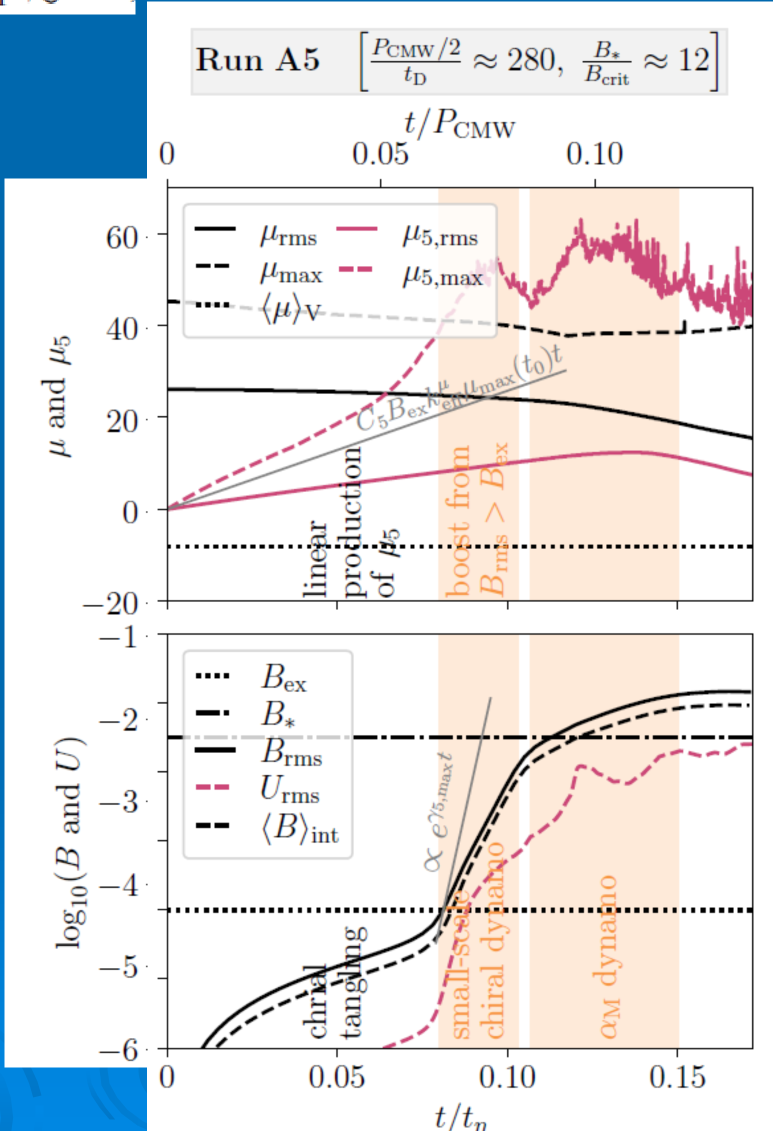


FIG. 1. Illustration of the dynamics in a plasma with autonomous production of chiral asymmetry (Phase 1), accompanied by a generation of fluctuations in the magnetic field $\langle b^2\rangle^{1/2}$ (Phase 2) and the velocity field $\langle u^2\rangle^{1/2}$. If the magnetic Reynolds number becomes larger than unity, the magnetic α effect produces a large-scale magnetic field $\langle \boldsymbol{B}\rangle$ (Phase 3).



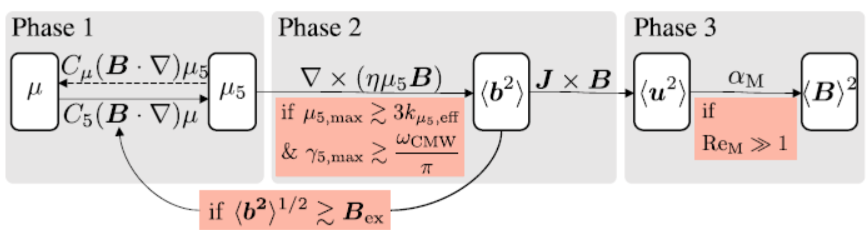
Small-Scale Chiral Dynamo Instability

$$\gamma_5 = \frac{\eta \mu_{5,\max}^2}{4},$$

$$k_5 / k_{\mu_5,\mathrm{eff}} \gtrsim 10$$

$$k_{\mu_5,\mathrm{eff}}^{-1}(t) = \langle \mu_5^2 \rangle^{-1} \int k^{-1} E_5(k) \mathrm{d}k.$$

$$k_5(t) = \mu_{5,\max}(t)/2$$



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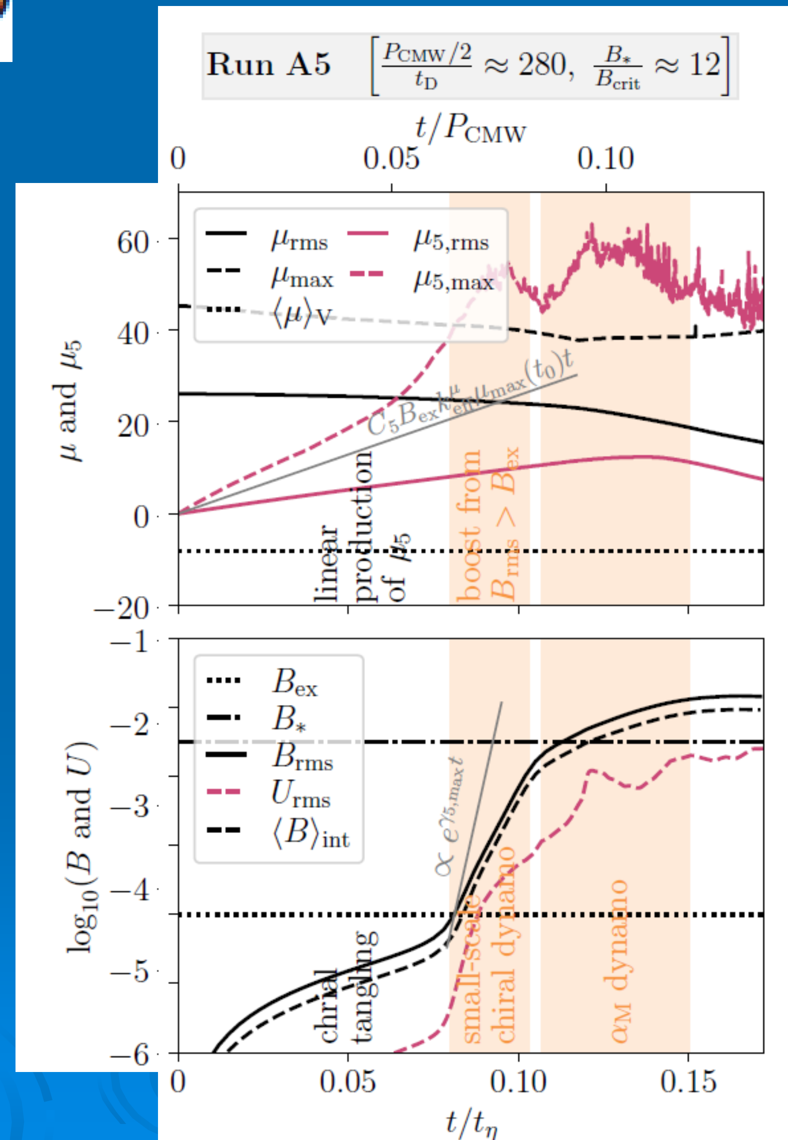


Mean-Field Dynamo Instability

$$\gamma_\alpha^{\max} = \frac{(\eta\langle\mu_5\rangle + \alpha_{\rm M})^2}{4(\eta+\eta_{\rm T})}.$$

$$\alpha_{\rm M} = \tau_{\rm c}\,\langle \boldsymbol{b}\cdot(\boldsymbol{\nabla}\times\boldsymbol{b})\rangle$$

$$\chi_c = \langle \boldsymbol{b}\cdot(\boldsymbol{\nabla}\times\boldsymbol{b})\rangle_{\rm int} \approx \langle \boldsymbol{a}\cdot\boldsymbol{b}\rangle_{\rm int} k_{\rm int}^2$$

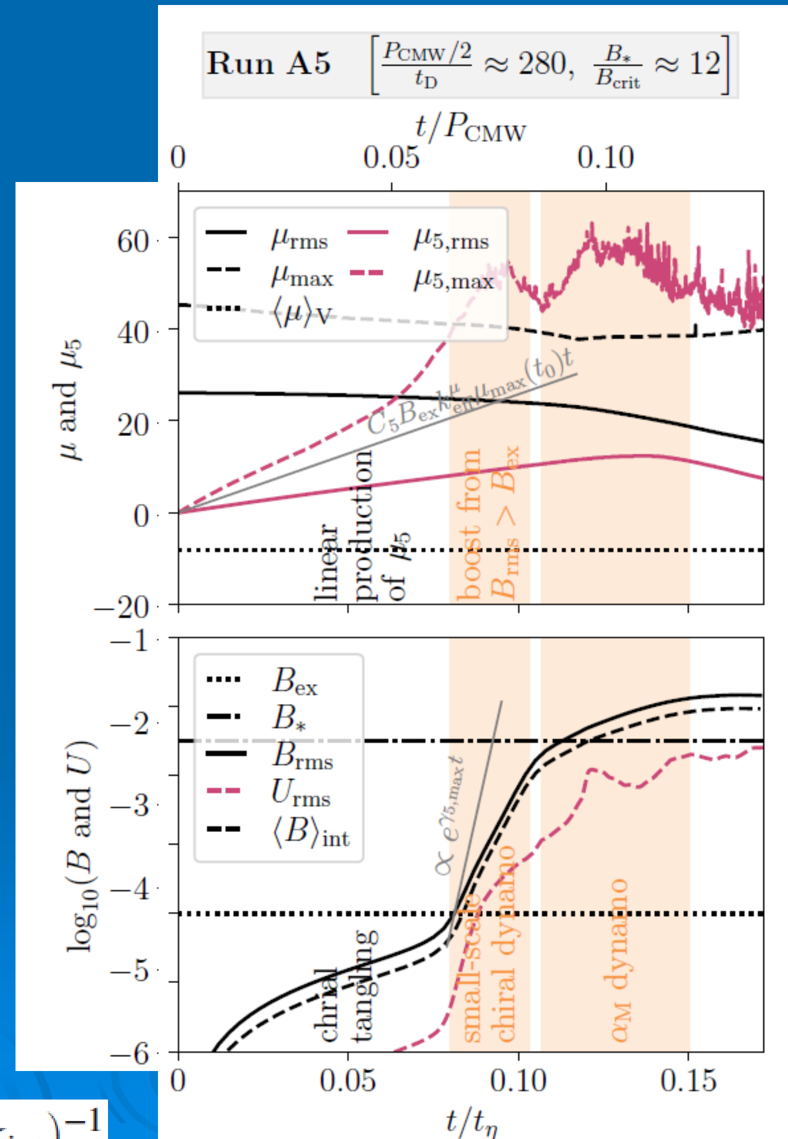
$$\gamma_\alpha = (\eta\langle\mu_5\rangle + \alpha_{\rm M})k - (\eta+\eta_{\rm T})k^2.$$

Budget Equation for Magnetic Helicity:

$$\frac{\partial}{\partial t}\langle \boldsymbol{a}\cdot\boldsymbol{b}\rangle + \nabla\cdot\boldsymbol{F} = 2\eta\langle\mu_5\rangle\langle\boldsymbol{b}^2\rangle - 2\langle\boldsymbol{\mathcal{E}}\rangle\cdot\langle\boldsymbol{B}\rangle - 2\eta\langle\boldsymbol{b}(\boldsymbol{\nabla}\times\boldsymbol{b})\rangle,$$

$$2\eta\langle\mu_5\rangle\langle\boldsymbol{b}^2\rangle - 2\alpha_{\rm M}\langle\boldsymbol{B}\rangle^2$$

$$\alpha_{\rm M}^{\rm sat} = \eta\langle\mu_5\rangle\langle\boldsymbol{b}^2\rangle/\langle\boldsymbol{B}\rangle^2$$



$$\tau_{\rm c} \approx (U_{\rm A} k_{\rm int})^{-1}$$

Chiral Asymmetry and Dynamo Produced by Inhomogeneous Fluctuations of Chemical Potential

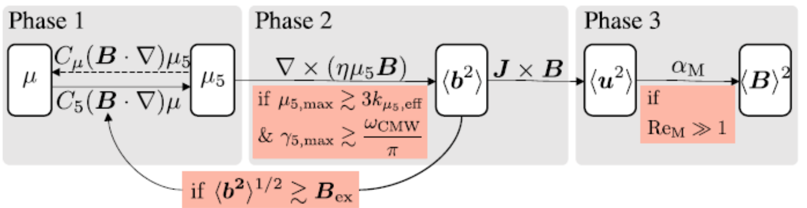
$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{U} \times \boldsymbol{B} + \eta\,(\mu_5 \boldsymbol{B} - \boldsymbol{\nabla} \times \boldsymbol{B})],$$

$$\rho \frac{D\boldsymbol{U}}{Dt} = (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{\nabla} p + \boldsymbol{\nabla}\cdot(2\nu\rho\mathbf{S}),$$

$$\frac{D\rho}{Dt} = -\rho\,\boldsymbol{\nabla}\cdot\boldsymbol{U},$$

$$\frac{D\mu}{Dt} = -\mu\,\boldsymbol{\nabla}\cdot\boldsymbol{U} - C_\mu(\boldsymbol{B}\cdot\boldsymbol{\nabla})\mu_5 - \mathcal{D}_\mu\,\nabla^4\mu,$$

$$\frac{D\mu_5}{Dt} = -\mu_5\,\boldsymbol{\nabla}\cdot\boldsymbol{U} - C_5(\boldsymbol{B}\cdot\boldsymbol{\nabla})\mu - \mathcal{D}_5\,\nabla^4\mu_5 + \lambda\eta\left[\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B}) - \mu_5 \boldsymbol{B}^2\right],$$



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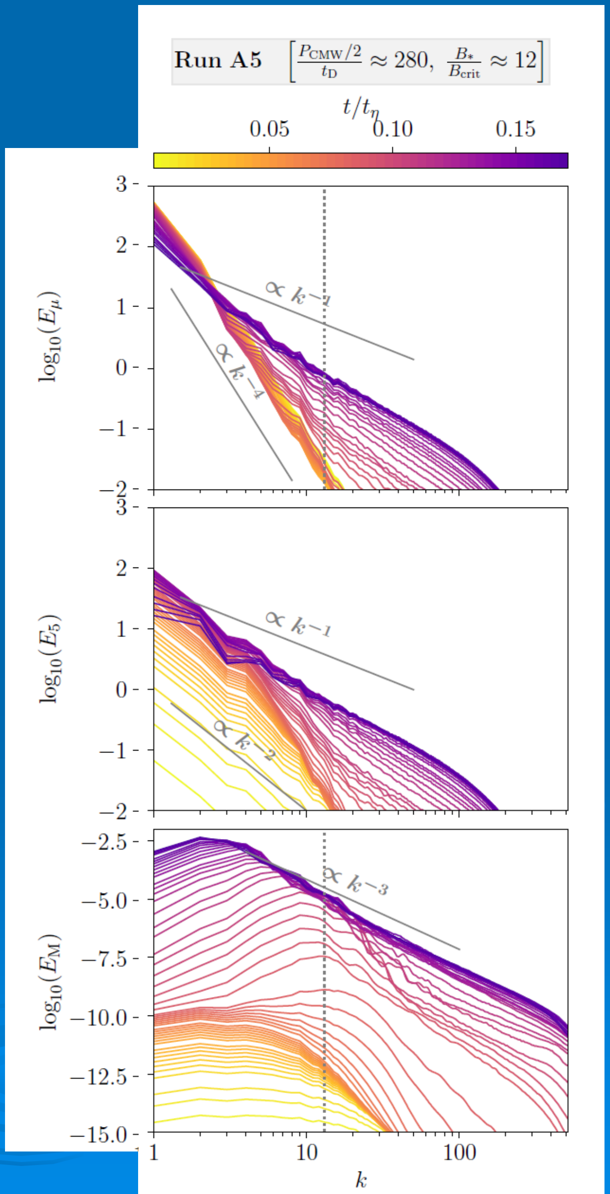


Gravitational Waves caused by Chiral Magnetic Effect

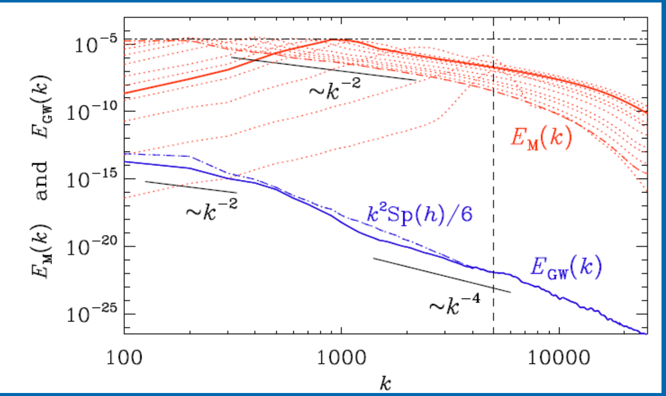
A. Brandenburg, Y. He, T. Kahniashvili, M. Rheinhardt, J. Schober, 2021,
ApJ 911, 110.

$$\left(\partial_t^2 - c^2\nabla^2\right) h_{ij} = \frac{16\pi G}{ac^2} T_{ij}^{\mathrm{TT}}$$

- From fluid motions
  $T_{ij} = \left(p/c^2 + \rho\right) \gamma^2 u_i u_j + p\delta_{ij}$
  Relativistic equation of state:
  $p = \rho c^2/3$
- From magnetic fields:
  $T_{ij} = -B_i B_j + \delta_{ij} B^2/2$

- h_{ij} are rescaled $h_{ij} = a h_{ij}^{\mathrm{phys}}$
- Comoving spatial coordinates ∇ and conformal time t
- Comoving stress-energy tensor components T_{ij}
- Radiation-dominated epoch such that $a'' = 0$

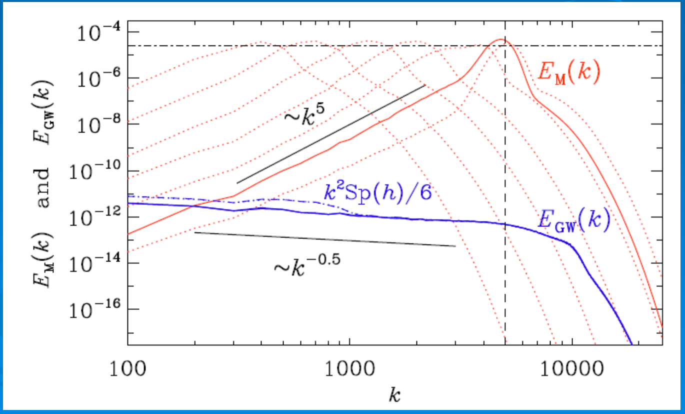
$\eta k_1 < v_\mu < v_\lambda$ (regime I).



$v_\lambda = \mu_{50}/\lambda^{1/2}, \qquad v_\mu = \mu_{50}\eta.$

$E_{\mathrm{GW}} \propto E_{\mathrm{M}}/k^2$

$\eta k_1 < v_\lambda < v_\mu$ (regime II).



Conclusions

- Chiral asymmetry (non-zero chiral chemical potential) can arise from an initially fluctuating inhomogeneous chiral chemical potential with zero mean, or from an initially fluctuating inhomogeneous chemical potential.
- The chiral dynamo: (a) generates magnetic fluctuations and small-scale magnetic helicity and (b) drives turbulence via the Lorentz force.
- Sufficiently strong turbulence is produced to activate a mean-field dynamo that is described by the magnetic alpha effect caused by current helicity. The growing large-scale magnetic field is saturated by the turbulent magnetic diffusion
- During the mean-field dynamo phase, the power spectra develop a universal shape for magnetic energy $E_M \propto k^{-3}$ and the chiral chemical potential $E_5 \propto k^{-1}$.

References

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DNS + energetic analysis of chiral turbulence

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DNS + chiral magnetic waves+chiral turbulence

- Schober, J., Rogachevskii, I., Brandenburg, A., 2022, PRL 128, 065002 (2022); PRD 105, 043507 (2022); $\langle\mu_5\rangle(t_0) = 0.$
- Schober, J., Rogachevskii, I., Brandenburg, A., 2024, PRL 132, 065101 (2024); PRD 110, 043515 (2024); $\mu_5 = 0$

THE END