Towards Preheating after Inflation :
Inflaton Fragmentation, Oscillon Formation & Decay
Numerical Simulations of Early Universe :
Sources of GWs @ NORDITA 2025

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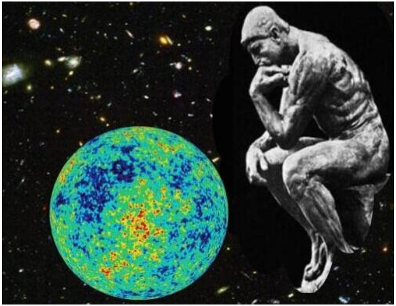
(Quantum) Initial conditions for structure

Initial $\zeta(\vec{x})$ $\longrightarrow$ CMB $\longrightarrow$ LSS (gravitational instability)

Properties of (initial) primordial fluctuations:



1. **Adiabatic & Super-Hubble $\zeta(\vec{x})$**
2. Almost **scale-invariant**

$$\boxed{\mathcal{P}_\zeta = A_S \left(\frac{k}{k_*}\right)^{n_S - 1}} \quad k_* = 0.05\,\mathrm{Mpc}^{-1}$$

$$A_S \simeq 2\times 10^{-9}\,,\ n_s - 1 \simeq -0.035$$

3. Nearly **Gaussian** (Variance: $\sigma \simeq 10^{-4}$)

$$\boxed{\mathbf{P}[\zeta] = \mathcal{B}\exp\left[\frac{-\zeta^2}{2\sigma^2}\left(1 + f_{\rm NL}\,\zeta + \ldots\right)\right]} \quad (f_{\rm NL} \to 0)$$

→ **LSS, CMB ⇒ Large-scale primordial fluctuations**

→ **Origin ⇒ Quantum fluctuations during Inflation**

Single-field Slow-roll Inflation (Successful Framework!)

$$\textbf{System} = \textbf{Gravity}\ (g_{\mu\nu}) + \textbf{Scalar Field}\ (\phi)$$

$$S[g_{\mu\nu},\phi] = \int \mathrm{d}^4x\,\sqrt{-g}\,\left(\frac{m_p^2}{2}R - \frac{1}{2}\partial_\mu\phi\,\partial_\nu\phi\,g^{\mu\nu} - V(\phi) + \ldots\right)$$

$$\mathrm{d}s^2 = -\beta^2(t)\,\mathrm{d}t^2 + a^2(t)\left[\left(e^{-2\Psi(t,\vec{x})}\,\delta_{ij} + 2\,h_{ij}(t,\vec{x})\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$

Two types of of inflationary fluctuations ($m \ll H$) –

1) **Comoving Curvature Perturbations:** $-\zeta(t,\vec{x}) = \Psi + \left(\frac{H}{\dot{\phi}}\right)\delta\varphi$

2) **Transverse & traceless Tensor Perturbations:** $h_{ij}(t,\vec{x})$

Slow-roll regime: (**Slow terminal motion of ϕ**) $\epsilon_1,\ |\epsilon_2| \ll 1$

$$\epsilon_1 = -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{3m_p^2H^2}\,;\quad \epsilon_2 = \frac{\mathrm{d}\ln\epsilon_1}{\mathrm{d}N}$$

Power-spectra: Linear Perturbation Theory

Slow-roll Primordial power-spectrum on **large scales** –

(CMB pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$)

Scalar power spectrum

$$\mathcal{P}_\zeta(k) = \frac{1}{8\pi^2}\left(\frac{H}{m_p}\right)^2 \frac{1}{\epsilon_1} = A_S\left(\frac{k}{k_*}\right)^{n_S - 1}$$

Scalar spectral index

$$\boxed{n_s - 1 = -2\,\epsilon_1 - \epsilon_2} \ll 1$$

Tensor power spectrum

$$\mathcal{P}_\mathcal{T}(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2 = A_\mathcal{T}\left(\frac{k}{k_*}\right)^{n_\mathcal{T}}$$

Tensor spectral index

$$\boxed{n_\mathcal{T} = -2\,\epsilon_1} \ll 1$$

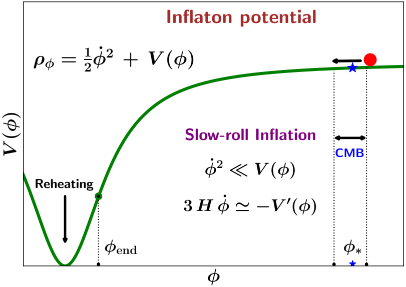
Tensor-to-scalar ratio: $\boxed{r = \frac{A_\mathcal{T}}{A_S} = 16\,\epsilon_1} \ll 1$

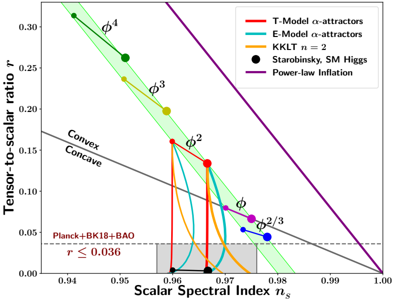
$\Rightarrow$ **Tiny fluctuations** that are **nearly scale-invariant**

Observational Constraints (Planck Legacy+BICEP/Keck)

$$A_s = 2.1 \times 10^{-9}\ ; \quad n_s - 1 \in [-0.043, -0.024]\ ;$$

$$\boxed{r \leq 0.036 \quad \Rightarrow \quad H^{\rm inf} \leq 10^{13}\ {\rm GeV} \quad \Rightarrow \quad E^{\rm inf} \leq 10^{16}\ {\rm GeV}}$$





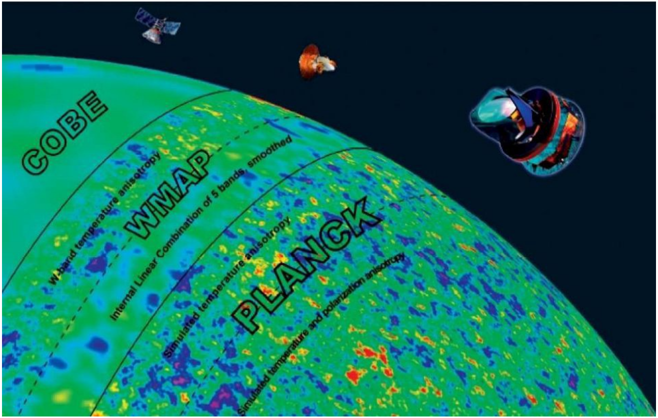
Latest CMB Data [BICEP/Keck + Planck] $\Rightarrow$ On CMB scales:

Single-field slow-roll paradigm of Inflation &

Concave, *e.g.* asymptotically-flat potentials!

**Planck(2018); **BICEP/Keck(2021); **SSM & Sahni(2022), **Bhatt, SSM *et.al.*(2022)

Graceful-Exit from Inflation → hot Big Bang



→ **Origin of the fluctuations in the plasma** (✓)

Inflationary quantum fluctuations

→ **Origin of the constituents of the plasma (hot Big Bang) ?**

Inflaton Decay & Reheating

What happened to other fields during inflation?

- Observations favour '**single-field slow-roll**' inflation.
- '**Cold inflationary paradigm**:' $\Rightarrow \; \rho_\chi \,, \; \rho_\psi \ll \rho_\varphi$

  and Negligible coupling to external fields $\boldsymbol{g^2, h \ll 1}$

$$\boldsymbol{S}[\boldsymbol{\varphi}, \boldsymbol{\chi}, \boldsymbol{\psi}] = -\int \mathrm{d}^4x\sqrt{-g}\Big[\; \frac{1}{2}\,\partial_\mu\varphi\,\partial^\mu\varphi + V(\varphi) + \cancelto{0}{\frac{1}{2}\partial_\mu\chi\partial^\mu\chi} + \cancelto{0}{\frac{1}{2}m_{0\chi}^2\chi^2}$$

$$+ \quad \cancelto{0}{\bar{\psi}\left(i\gamma^\mu\partial_\mu + m_{0\psi}\right)\psi}$$

$$+ \quad \cancelto{0}{\frac{1}{2}\boldsymbol{g^2}\,\boldsymbol{\varphi^2}\,\boldsymbol{\chi^2}} + \cancelto{0}{\boldsymbol{h}\,\boldsymbol{\psi}\bar{\boldsymbol{\psi}}\boldsymbol{\varphi}} + \ldots\Big]$$

$\Rightarrow$ **particle production during inflation can be neglected**.

- **Effects of the small coupling?**
  1. **Primordial Non-Gaussianity**: inflaton interactions.
  2. Decay of the inflaton field: **Reheating the universe**.

General Reheating Dynamics

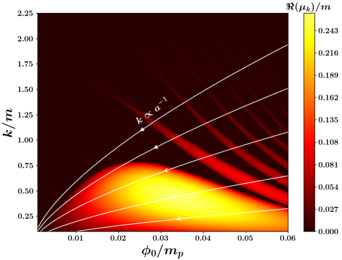
1) **Preheating — Non-perturbative inflaton decay:** (Early stages)

$$\mathcal{L}_{\rm int} \supset -\frac{1}{2} g^2 \varphi^2 \chi^2 \text{ for } g^2 \geq 4\left(\frac{m}{\phi_0}\right)^2$$

⇒ **Broad-band resonance** for

⇒ **Explosive growth** $\boxed{n_\chi \propto e^{\mu_k t}}$

$m \simeq 10^{-5}\, m_p\,,\ \phi_0 \simeq 0.2\, m_p \Rightarrow \boxed{g^2 \geq 10^{-8}}$



2) **Perturbative inflaton decay:** (certainly at Late times)

$$\varphi\varphi \longrightarrow \chi\chi \ \text{ for } \ g^2 < 10^{-8}\,; \ \ \varphi \longrightarrow \bar{\psi}\psi \ \text{ for } \ h \lesssim 10^{-2}$$

3) **Coherent Oscillations:**

For $h,\, g \sim 0$, the **inflaton condensate oscillates for a long time**

$$\boxed{\phi(t) = \phi_0(t)\, \cos(mt)\,; \quad \langle w_\phi \rangle \simeq 0 \ \Rightarrow \ \rho_\phi \propto a^{-3}}$$

⇒ universe remains **condensate-dominated** × (**Not Correct!**)

We have ignored **two important effects**:

→ **Gravitational clustering: (Very late times)**

Metric fluctuations are important on time scales

$$t \sim \mathcal{O}(H^{-1}) \gg m^{-1}$$

$$\boxed{\mathrm{d}s^2 = -e^{-2\Psi(t,\vec{x})}\mathrm{d}t^2 + a^2(t)\left[\left(e^{2\Psi(t,\vec{x})}\,\delta_{ij} + h_{ij}(t,\vec{x})\right)\mathrm{d}x^i\mathrm{d}x^j\right]}$$

→ **Asymptotically flat potentials:**

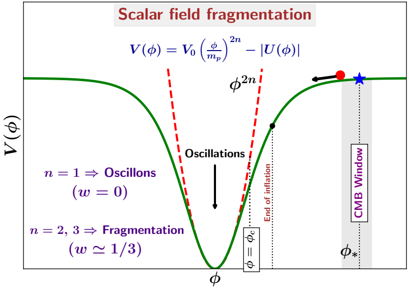
(CMB observations)

$$V(\phi) = V_0\left(\frac{\phi}{m_p}\right)^{2n} - |U(\phi)|$$

have '**attractive self-interaction**'

⇒ **Scalar field fragmentation**

⇒ **Cosmological Solitons : Oscillons**



**Amin *et. al* & Lozanov *et. al* (2010-2020)

Existence of quasi-Solitons: Oscillons

→ **Self-supported, localised, long-lived non-linear 'solitary' configurations**.

→ **Solitons are ubiquitous in nature**!

(**1834 J. S. Russell: solitary wave in a canal in Edinburgh!**)

(Appearing in fluids, smoke rings, condensed matter physics, optics, HEP, topological defects and Cosmology.)

→ **Oscillons are oscillating non-topological pseudo-solitons**!

→ **Analytical results** based on **small-amplitude oscillations**

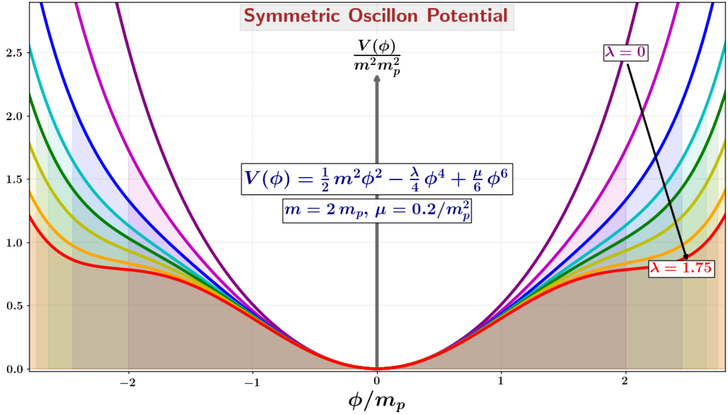
$$\boldsymbol{V}(\boldsymbol{\varphi}) \approx \frac{1}{2}\, m^2 \boldsymbol{\varphi}^{\mathbf{2}} - \frac{\lambda}{4}\, \boldsymbol{\varphi}^{\mathbf{4}} + \frac{\mu}{6}\, \boldsymbol{\varphi}^{\mathbf{6}}$$

→ Supports **Oscillon-like solution** of the form

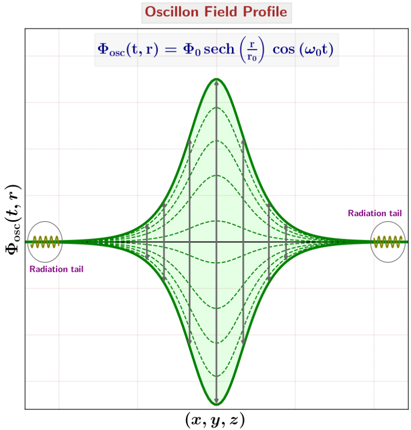
$$\boldsymbol{\varphi}_{\mathbf{osc}}(\boldsymbol{t}, \boldsymbol{r}) \approx \boldsymbol{\Phi}(\boldsymbol{r}) \cos(\boldsymbol{\omega}_{\mathbf{0}}\, \boldsymbol{t}) + \ldots\,; \qquad \boxed{\boldsymbol{\Phi}(\boldsymbol{r}) \approx \boldsymbol{\Phi}_{\mathbf{0}}\, \mathrm{sech}\left(\frac{\boldsymbol{r}}{\boldsymbol{r}_{\mathbf{0}}}\right)}$$

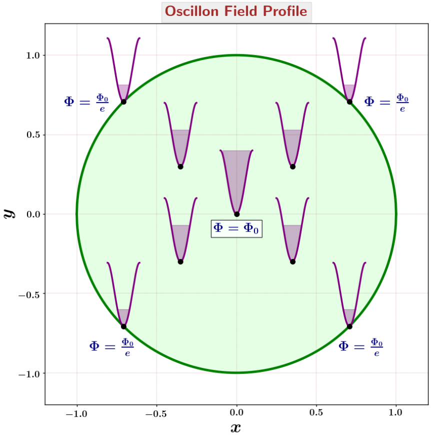
****Rajaraman(1987), **Gleiser** ***et. al*****; **Amin** ***et. al*** **; **Mahbub, SSM (2023)**

Potential $$\boxed{V(\Phi) = \frac{1}{2}\, m^2 \Phi^2 - \frac{\lambda}{4}\, \Phi^4 + \frac{\mu}{6}\, \Phi^6} \;;\quad \mu\, m^2 > \lambda^2$$



Oscillon field $$\Phi(t,r) \approx \phi_0 \,\mathrm{sech}\left(\frac{r}{r_0}\right) \cos(\omega_0 t)$$





Existence of Oscillons ?

1. **Oscillons exit as analytic (stationary) solutions**

(of post-inflationary oscillations around asymptotically flat potentials)

(a) For symmetric plateau potentials:

→ **small-amplitude oscillations**

$$V(\varphi) \approx \frac{1}{2}\, m^2 \boldsymbol{\varphi}^2 - \frac{\lambda}{4}\, \mu\, \boldsymbol{\varphi}^4 + \frac{g}{6}\, \lambda\, \boldsymbol{\varphi}^6$$

→ Supports **Oscillon-like solution** of the form

$$\Phi(t, r) = \phi_0\ \mathbf{sech}\left(\frac{r}{r_0}\right)\ \mathbf{cos}\,(\omega_0 t)$$

(b) For asymmetric plateau potentials?

$$V(\varphi) \approx \frac{1}{2}\, m^2 \boldsymbol{\varphi}^2 - \frac{1}{3}\, \mu\, \boldsymbol{\varphi}^3 + \frac{1}{4}\, \lambda\, \boldsymbol{\varphi}^4$$

2. **Can they form dynamically ?**

(starting from natural conditions at the end of inflation)

*Copeland *et. al*(1995); *Amin *et. al*(2011); *Mahbub, SSM(2023); *Kim, McDonald

Self-resonance and inflaton fragmentation

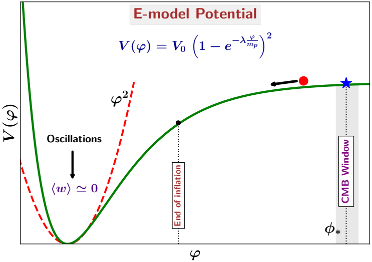
In the linear regime, Fourier mode functions satisfy

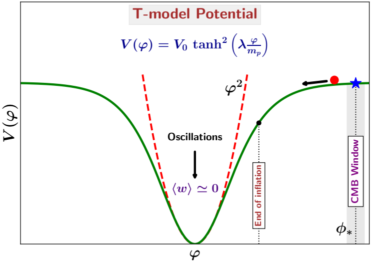
$$\delta\ddot{\varphi}_k + 3H\delta\dot{\varphi}_k + \left[\frac{k^2}{a^2} + V_{,\phi\phi}(\phi)\right]\delta\varphi_k = 0$$

$\Rightarrow$ Exponential growth of inflaton fluctuations $\delta\varphi_k(t) \propto e^{\mu_k m t}$

Band structure **similar to Mathieu resonance** for quadratic case

$$V(\phi) = \frac{1}{2}m^2\phi^2 - |U(\phi)| \quad (\textbf{E-Model} \;\&\; \textbf{T-Model})$$





**Lozanov, Amin(2017); **Shafi, Copeland, Mahbub, SSM, Basak (2024)

Non-linear dynamics: $\mathcal{C}$osmo$\mathcal{L}$attice

Fully non-linear field equations

$$\ddot{\tilde{\varphi}} + 3\tilde{H}\tilde{\varphi} - \frac{\tilde{\nabla}^2}{a^2}\,\tilde{\varphi} + \tilde{V}_{,\tilde{\varphi}} = 0$$

$$\tilde{H} \equiv \frac{\dot{a}}{a} = \frac{1}{3m_p^2}\left\langle \widetilde{K}_{\tilde{\varphi}} + \widetilde{G}_{\tilde{\varphi}} + \tilde{V}(\tilde{\varphi}) \right\rangle$$

Where $\tilde{t} = m\,t$; $\tilde{x} = m\,x$; $\tilde{\varphi}, \tilde{\chi} = \frac{1}{\beta}\,\frac{\varphi,\chi}{m_p}$; $\tilde{F} = \frac{F}{\beta^2 m^2 m_p^2}$

$$\widetilde{K}_{\tilde{\varphi}} = \frac{1}{2}\left(\frac{\partial\tilde{\varphi}}{\partial\tilde{t}}\right)^2 \; ; \quad \widetilde{G}_{\tilde{\varphi}} = \frac{1}{2a^2(\tilde{t})}\left[\left(\frac{\partial\tilde{\varphi}}{\partial\tilde{x}}\right)^2 + \left(\frac{\partial\tilde{\varphi}}{\partial\tilde{y}}\right)^2 + \left(\frac{\partial\tilde{\varphi}}{\partial\tilde{z}}\right)^2\right]$$

$$\tilde{\rho}_{\tilde{\varphi}} = \widetilde{K}_{\tilde{\varphi}} + \widetilde{G}_{\tilde{\varphi}} + \tilde{V}(\tilde{\varphi})\,; \quad \tilde{p}_{\tilde{\varphi}} = \widetilde{K}_{\tilde{\varphi}} - \frac{1}{3}\widetilde{G}_{\tilde{\varphi}} - \tilde{V}(\tilde{\varphi})$$

Lattice specifications:

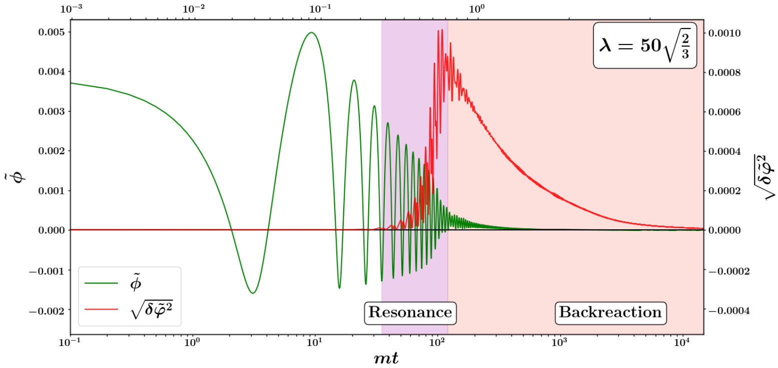
$$N = 128^3\,; \quad 0.05\,m^{-1} \le k \le 5\,m^{-1}$$

**Figueroa *et. al* (2020, 2021); **Mahbub, SSM (2023)

Self-resonance and Inflaton Fragmentation

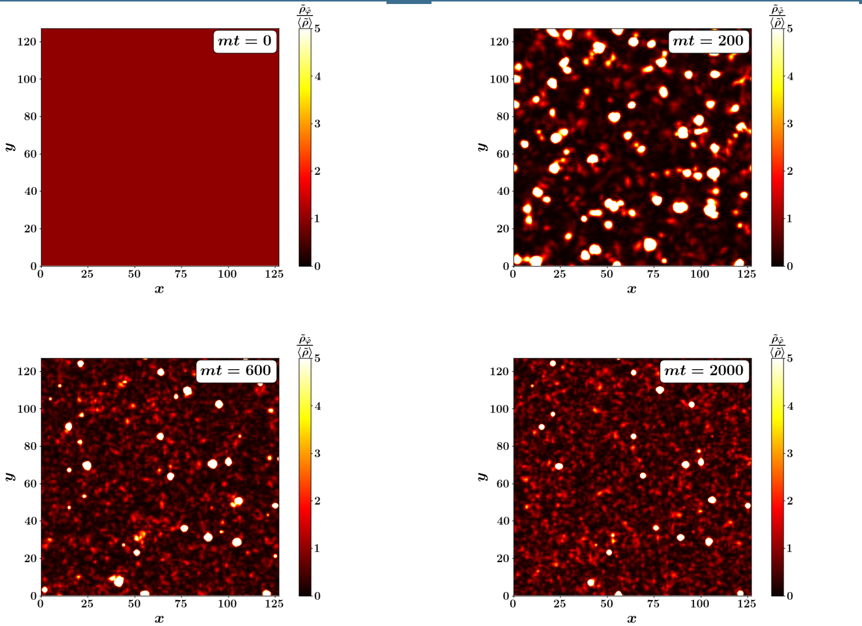
Strong self-resonance ⇒ Inflaton fragmentation

(Asymmetric E-Model potential)

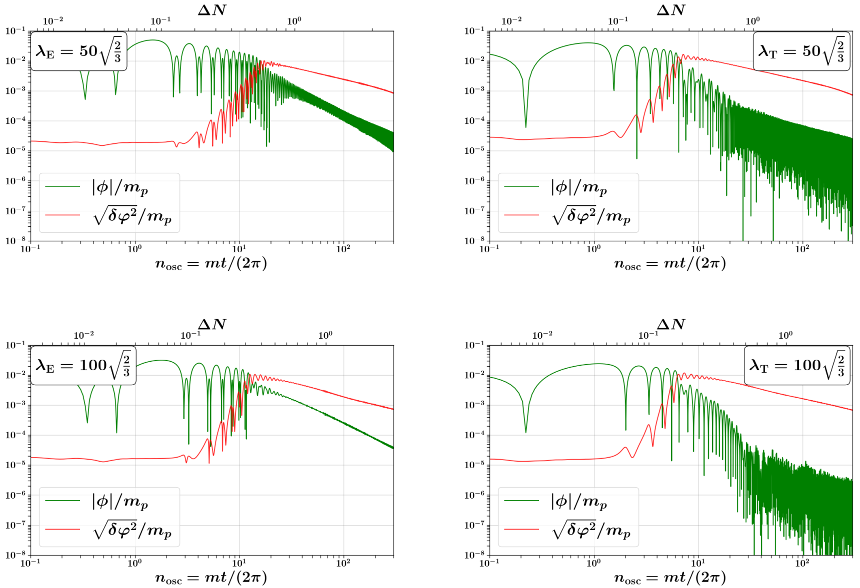


**Mahbub, SSM (2023); **Shafi, Copeland, Mahbub, SSM, Basak (2024)

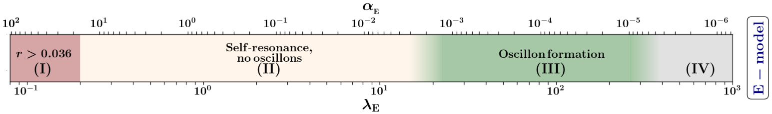
Oscillon formation in real time (Asymmetric)

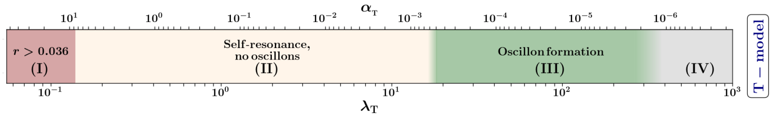


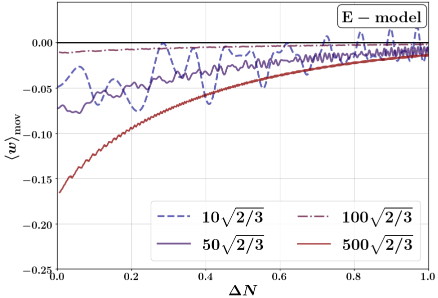
Inflaton Fragmentation within a Single e-fold

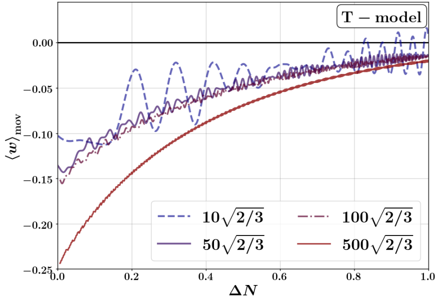


Islands of Parameter Space in λ





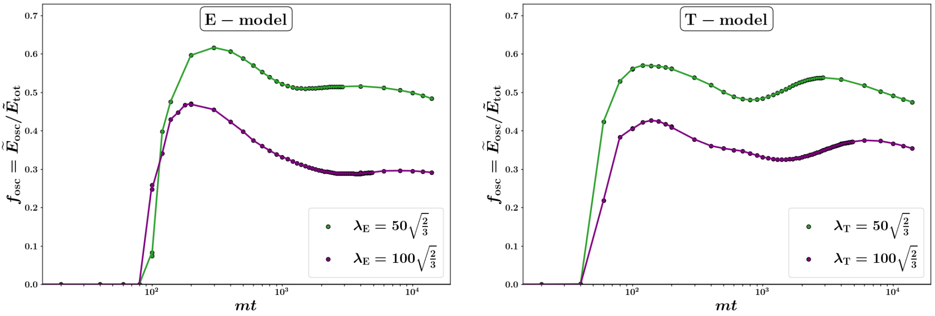




Fractional Energy Density of Oscillons

Energy/Mass fraction

$$f_{\rm osc} \equiv \frac{E_{\rm osc}}{E_{\rm tot}} = \frac{\int_{\delta\rho_\varphi \gtrsim 4\bar{\rho}_\varphi} \mathrm{d}^3\boldsymbol{x}\,\rho_\varphi(\boldsymbol{x},t)}{\int \mathrm{d}^3\boldsymbol{x}\,\rho_\varphi(\boldsymbol{x},t)}$$



$\gtrsim$ 40% **of the total density** $\Rightarrow$ **Significant!**

**Mahbub, SSM (2023)

Conclusions (so far)

1. **Oscillons do form** after inflation in absence of external coupling starting from **generic initial conditions**.
2. Oscillons form **for both Symmetric and Asymmetric plateau potentials**.

Important Questions

1. What is the **lifetime of oscillons**? How do they **decay?**
2. We have **ignored external coupling;** $g \to 0$

   What happens if $g \neq 0$ **? Do oscillons form?**

Our latest work!

**Hertzberg (2010); **Zhang, Amin, Copeland, Saffin, Lozanov (2020)

(P)reheating via Oscillon decay

Formation and decay of oscillons after inflation in the presence of an external coupling, Part-I: Lattice simulations

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JCAP 10 (2024) 082 [e-print arXiv: 2406.00108]

Preheating via Self & External Resonance

→ For inflaton decay, $|\phi(t)| \gg \delta\varphi(t,\vec{x}),\ \chi(t,\vec{x})$

$$\begin{aligned}
\varphi(t,\vec{x}) &= \boldsymbol{\phi}(t) + \boldsymbol{\delta}\varphi(t,\vec{x}) \\
\chi(t,\vec{x}) &= \bar{\chi}(t) \rightarrow 0 + \boldsymbol{\delta\chi}(t,\vec{x}) \quad (\boldsymbol{\chi} \ \ \textbf{field is in vacuum state})
\end{aligned}$$

→ At the end of inflation, $\rho_\phi \gg \rho_\chi,\ \rho_{\delta\varphi}$ (**Condensate dominated**)

→ Resulting **equations of dynamics in the linear regime**

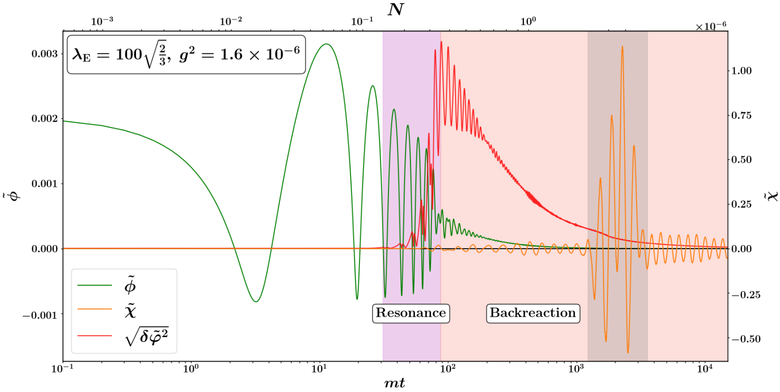
$$\begin{aligned}
\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) &= 0 \\
\boldsymbol{\delta\ddot{\varphi}_k + 3H\delta\dot{\varphi}_k + \left[\frac{k^2}{a^2} + V_{,\phi\phi}(\phi)\right]\delta\varphi_k} &= 0 \quad \textbf{Self} - \textbf{resonance} \\
\boldsymbol{\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\frac{k^2}{a^2} + g^2\,\phi^2\right]\chi_k} &= 0 \quad \textbf{External} - \textbf{resonance}
\end{aligned}$$

and the **Hubble parameter** $H^2 \simeq \frac{1}{3\,m_p^2}\left[\frac{1}{2}\,\dot{\phi}^2 + V(\phi)\right]$

****SSM Lecture Notes (2024)**

Self-resonance and Oscillon decay for $g \neq 0$

Self-resonance → Oscillons → χ production



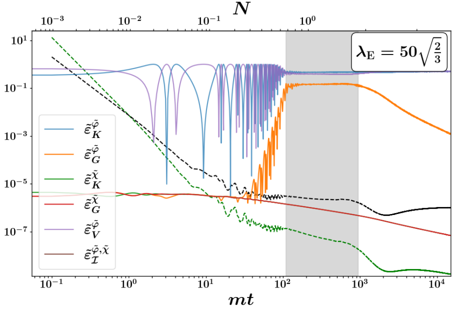
Asymmetric E-model potential

**Mahbub, SSM (2023); **Shafi, Copeland, Mahbub, SSM, Basak (2024)

Evolution of energy density components

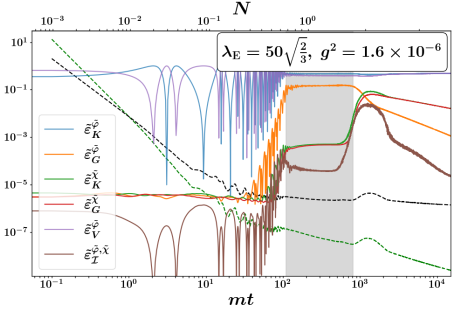
Absence of external interaction

Long-lived Oscillons



Presence of external interaction

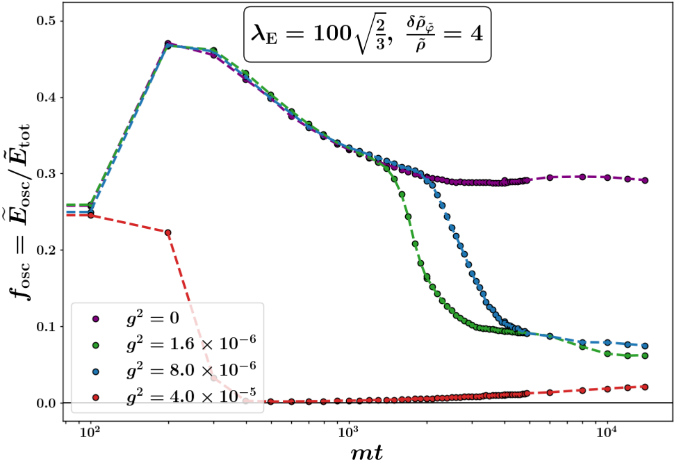
oscillon decay into χ



(**Production of χ-particles due to oscillon decay**!)

**Shafi, Copeland, Mahbub, SSM, Basak (2024)

Energy(Mass) Fraction of Oscillons



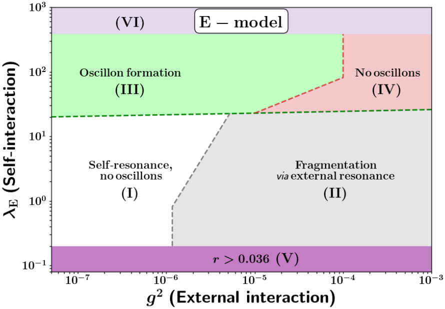
Reduction in f_{osc} due to χ-production

**Shafi, Copeland, Mahbub, SSM, Basak (2024)

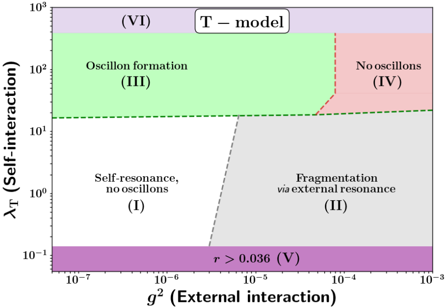
Islands of the Extended Parameter Space $\{\lambda,\, g^2\}$

External Coupling $\boxed{\mathcal{I}(\varphi, \chi) = \frac{1}{2} g^2 \varphi^2 \chi^2}$

$$V_{\rm E}(\varphi) = V_{0\rm E}\, \left(1 - e^{-\boldsymbol{\lambda}_{\rm E} \frac{\varphi}{m_p}}\right)^2 \qquad V_{\rm T}(\varphi) = V_{0\rm T}\, \tanh^2\left(\boldsymbol{\lambda}_{\rm T} \frac{\varphi}{m_p}\right)$$



(**E-Model** $\boldsymbol{\lambda}_{\rm E}$ *vs* $\boldsymbol{g^2}$)



(**T-Model** $\boldsymbol{\lambda}_{\rm T}$ *vs* $\boldsymbol{g^2}$)

Summary

1. **What happens if $g \neq 0$? Do oscillons form?**

   YES! (Preheating via Oscillon decay into χ)

2. **Lifetime of oscillons? How does an oscillon decay?**

   Ongoing work (analytical)!

3. **More realistic interactions?**

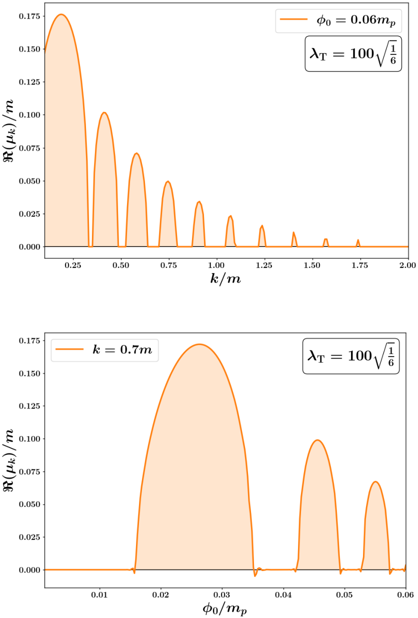
   Ongoing work (Lattice)!

Phenomenological implications of oscillons?

- High Frequency GWs from Preheating
- Late-time Gravitational clustering and GWs
- Primordial Black Holes
- Oscillons post-reheating
- Oscillons in scalar field (fuzzy) dark matter

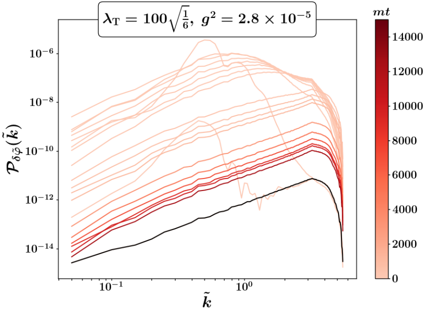
Extra Slides

Structure of the resonance with $\{\lambda, g^2\}$

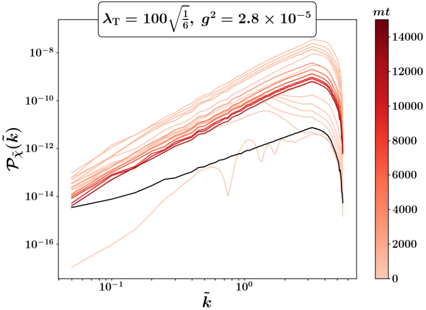


Power spectra of fluctuations: Resonance

Inflaton $\delta\varphi$-fluctuations



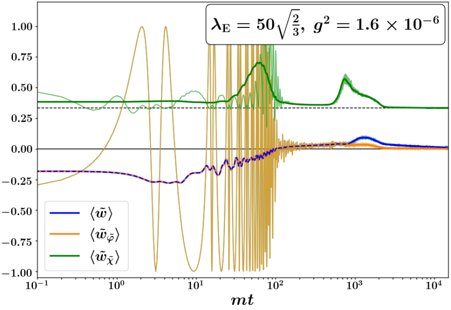
Offspring χ-fluctuations



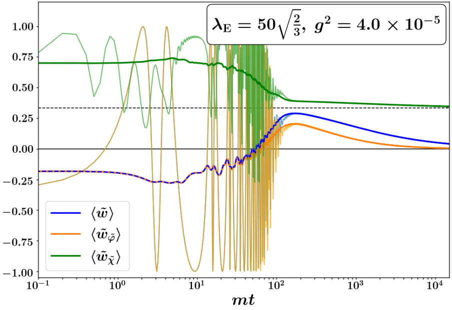
**Shafi, Copeland, Mahbub, SSM, Basak (2024)

Evolution of Equation of State

Low external interaction



Large external interaction



$\langle w_\varphi \rangle \to 0$ asymptotically

**Shafi, Copeland, Mahbub, SSM, Basak (2024)