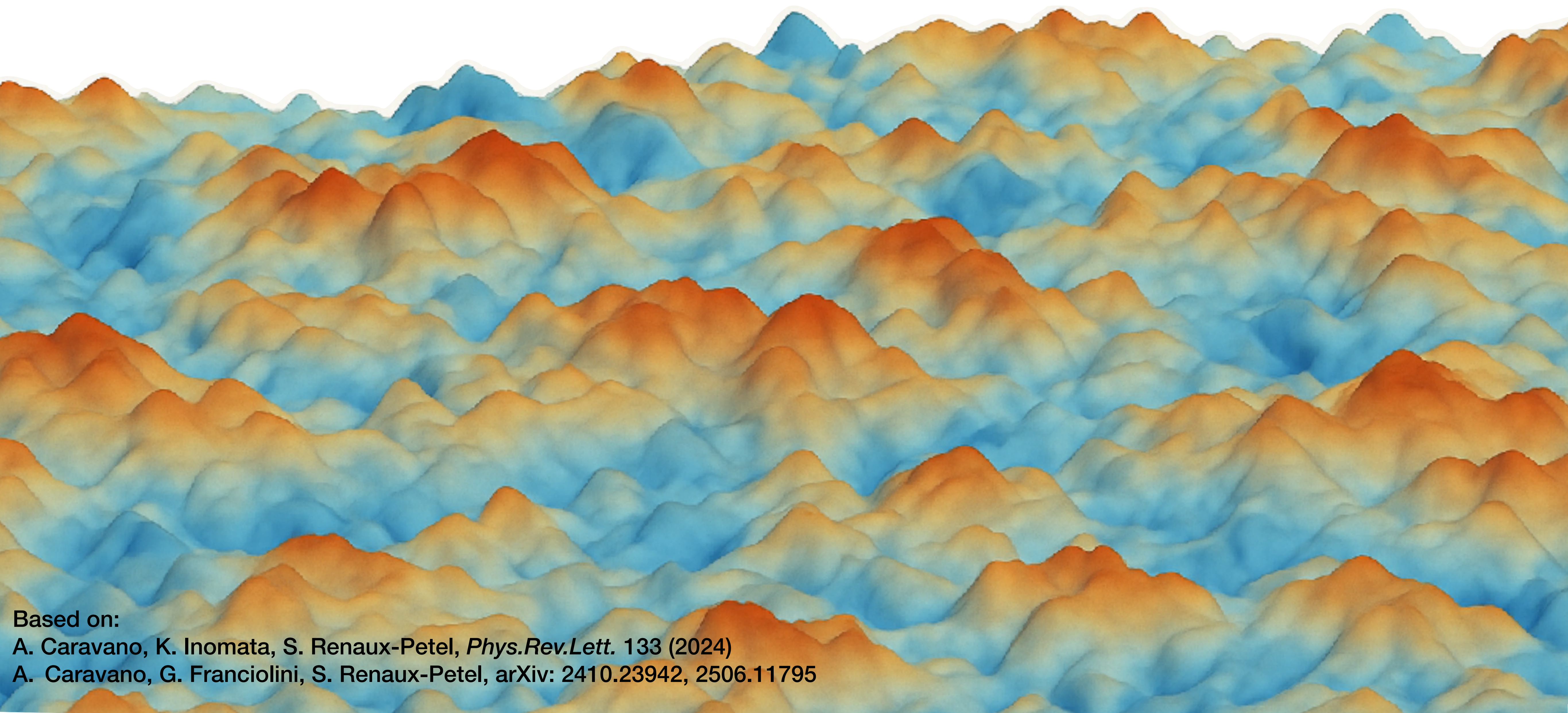
Backreaction and cosmic butterflies

Non-Perturbative Insights into the Small-Scale Physics of Inflation

Angelo Caravano (He/Him) - Institut d'Astrophysique de Paris



Based on:

A. Caravano, K. Inomata, S. Renaux-Petel, *Phys.Rev.Lett.* 133 (2024)

A. Caravano, G. Franciolini, S. Renaux-Petel, arXiv: 2410.23942, 2506.11795

Roadmap

0) Introduction and motivation:

1) Lattice simulations of inflation

AC, Komatsu, Lozanov, Weller — 2102.06378, 2110.10695, 2204.12874

AC — 2209.13616

2506.11797

2) Small-scale physics: Inflationary butterfly effect

2.1) Oscillatory potential

2.2) Ultra-slow-roll inflation

AC, K. Inomata, S. Renaux-Petel — 2403.12811

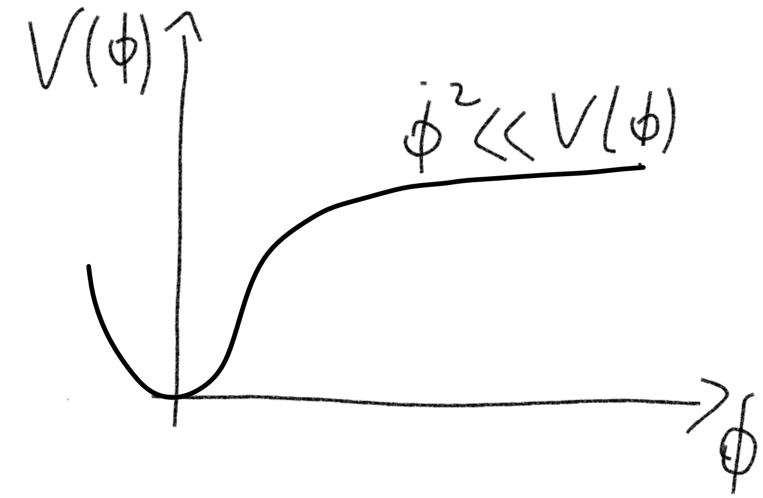
AC, G. Franciolini, S. Renaux-Petel — 2410.23942

2506.11795

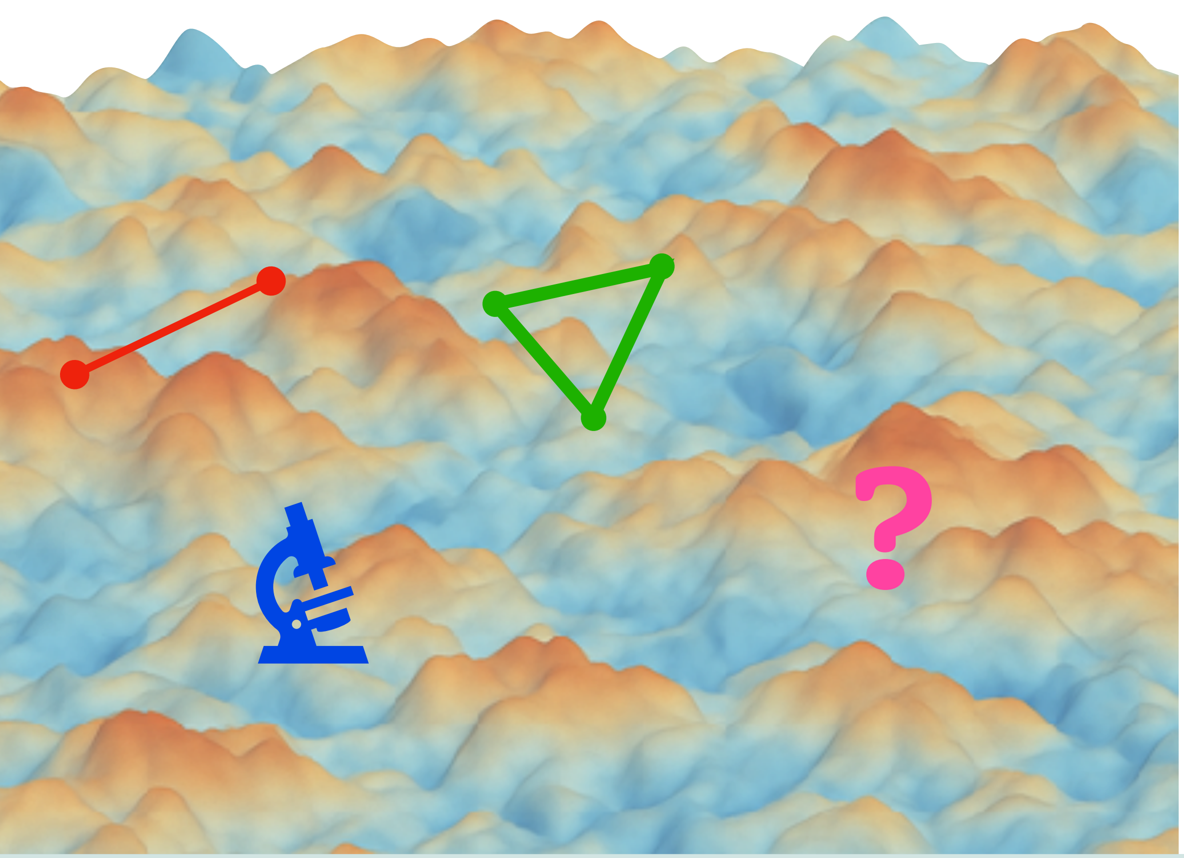
Inflation

[**A. Guth**, Phys. Rev. D 23 (1981) 347.]
[**K. Sato**, Mon. Not. Roy. Astron. Soc. 195 (1981) 467.]
[**A.D. Linde**, Adv. Ser. Astrophys. Cosmol. 3 (1987) 149.]
…

Single-field slow-roll inflation is compatible with all current observations



$\zeta =$ super-horizon curvature perturbation



- ✔ Nearly scale invariant $\mathscr{P}_\zeta(k) = \mathscr{P}_\zeta(k_0)\left(\frac{k}{k_0}\right)^{n_s-1}$, $n_s \simeq 0.97$
- ✔ Gaussian $\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \simeq 0$
- ✔ Very small $\zeta \simeq \sim 10^{-5}$
- ✔ (Slow-roll) suppressed tensor-to-scalar ratio $r = 16\epsilon \simeq 8M_{\rm Pl}\left(\frac{V'}{V}\right)^2$

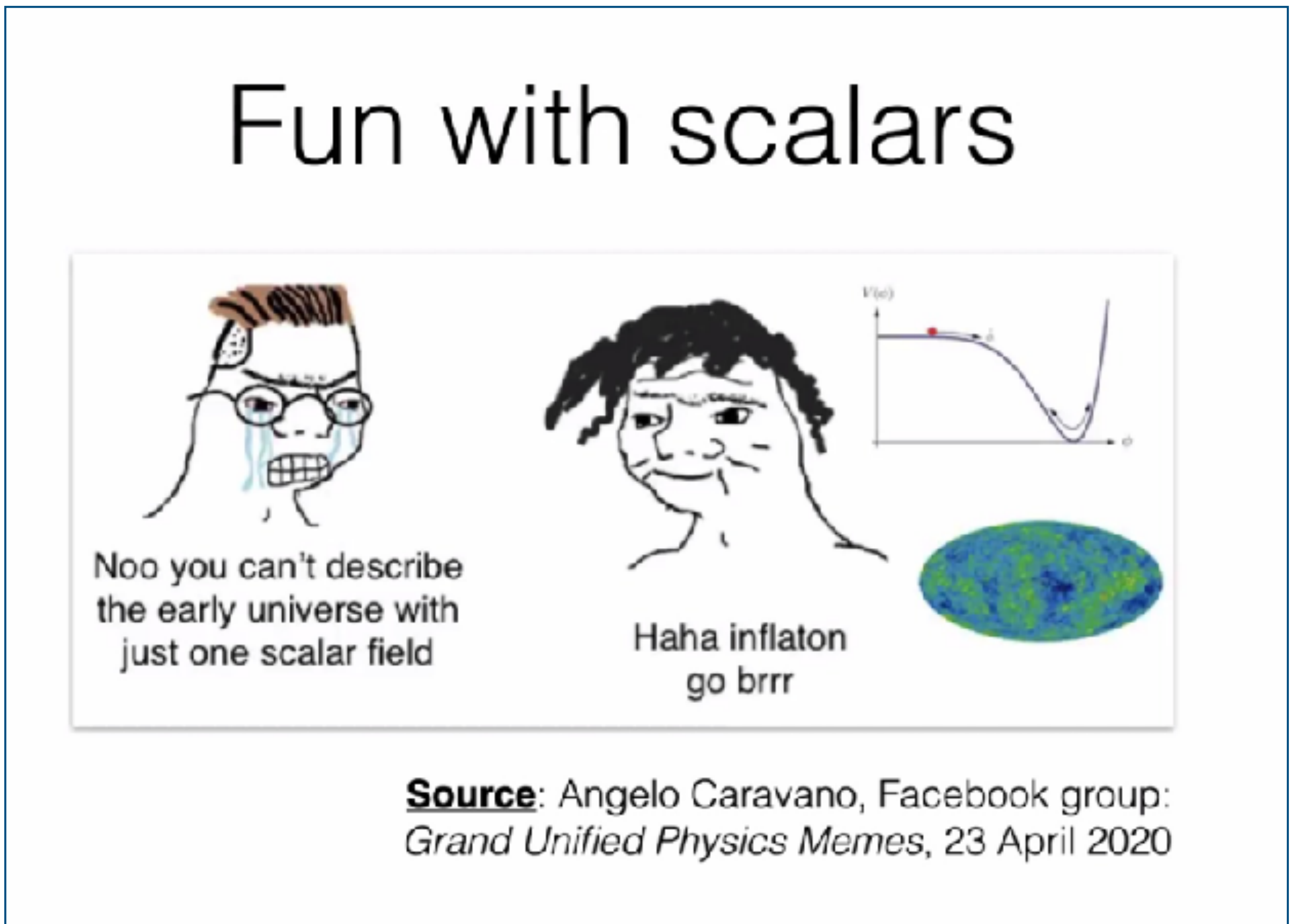
Beyond single-field slow-roll inflation

Why going beyond the “simple” single-field model?

Beyond single-field slow-roll inflation

Why going beyond the “simple” single-field model?

Slide from Alejandro Jenkins’s presentation:



Beyond single-field slow-roll inflation

1. Fundamental motivation (Up → Bottom):

UV sensitivity: flatness of slow-roll potential is hard to control.

cutoff $\Lambda \to \tilde{\Lambda}$ $\Rightarrow$ $M_{Pl}^2 \frac{\Delta V''}{V} \sim \frac{M_{Pl}^2}{\tilde{\Lambda}^2} \gg 1$

Two avenues:

1.1) Hints for some extra symmetry → new physics

1.2) Deviations from slow-roll

Beyond single-field slow-roll inflation

2. Phenomenological motivation (Bottom → Up):

Can we learn more about inflation from the data?

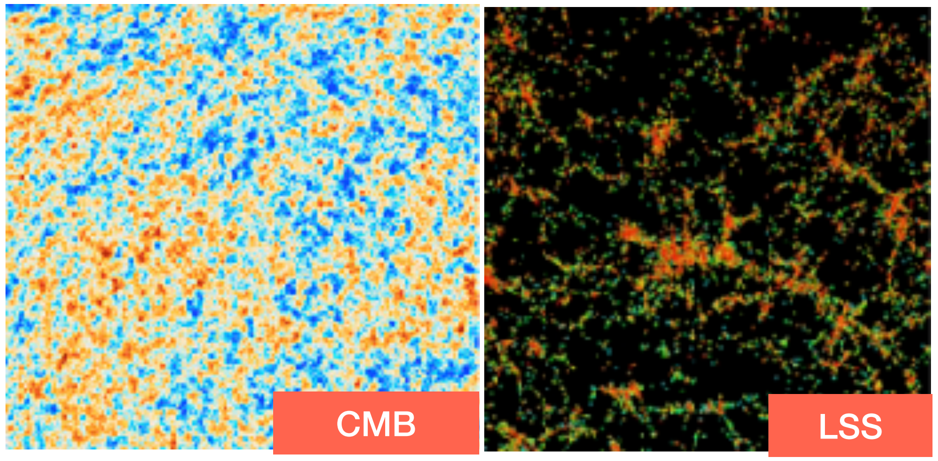
Example: non-Gaussianity

Interacting theory ϕ, ψ, A_μ

↓

non-Gaussianity

$$\frac{\delta\rho}{\rho}$$

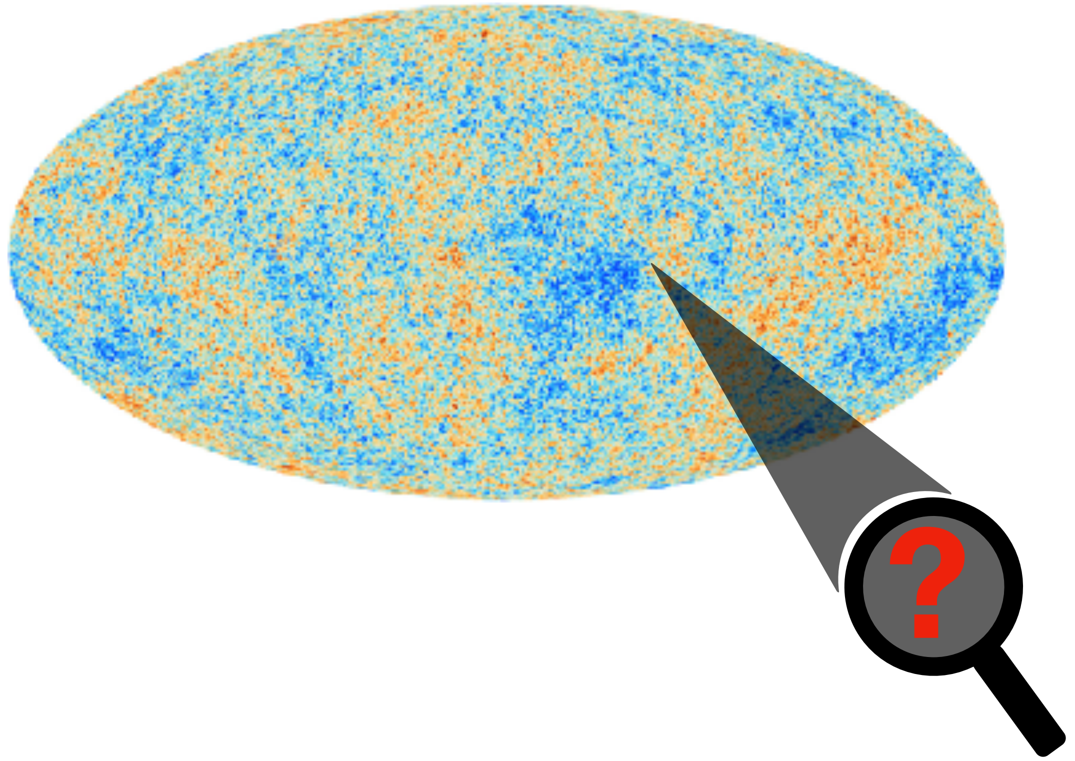


Beyond single-field slow-roll inflation

2. Phenomenological motivation (Bottom → Up):

Inflation generates fluctuations at scales $\sim e^{40}$ smaller than CMB scales

What is the physics of inflation at scales $\lambda \ll \lambda_{CMB}$?

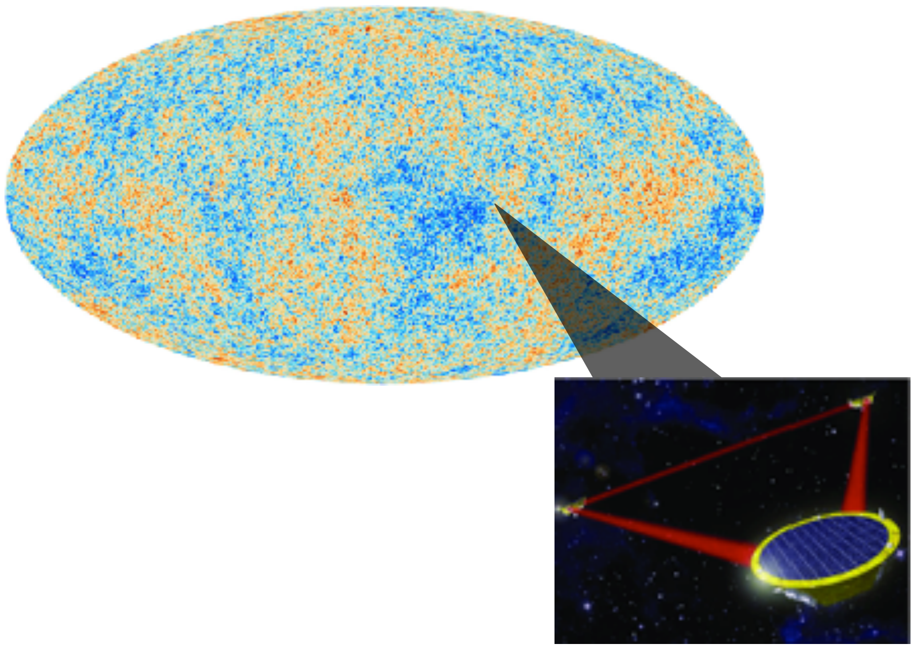


Inflation at small scales

Inflation generates fluctuations at scales $\sim e^{40}$ smaller than CMB scales

What is the physics of inflation at scales $\lambda \ll \lambda_{CMB}$?

Thanks to **gravitational waves interferometers**, we now have an observational windows on these scales



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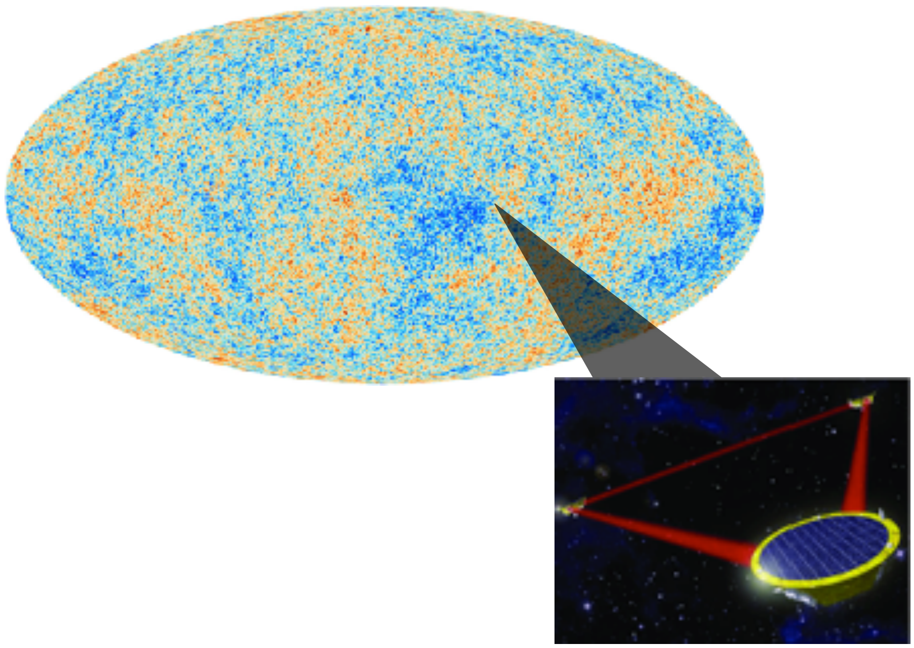


Thanks to **gravitational waves interferometers**, we now have an observational windows on these scales

For sizeable effect, however:

$$\mathscr{P}_\zeta \sim 10^{-2} - 10^{-4} \quad \longrightarrow \quad \boxed{\zeta \sim 10^{-1} - 10^{-2}}$$

$$\gg \mathscr{P}_{\zeta,\mathrm{CMB}} \sim 10^{-9}$$

nonlinear/non-perturbative physics?

(See ongoing debate on loops)

Roadmap

1) Lattice simulations of inflation

AC, Komatsu, Lozanov, Weller

2102.06378
2110.10695
2204.12874

AC

2209.13616
2506.11797

Lattice simulations of inflation

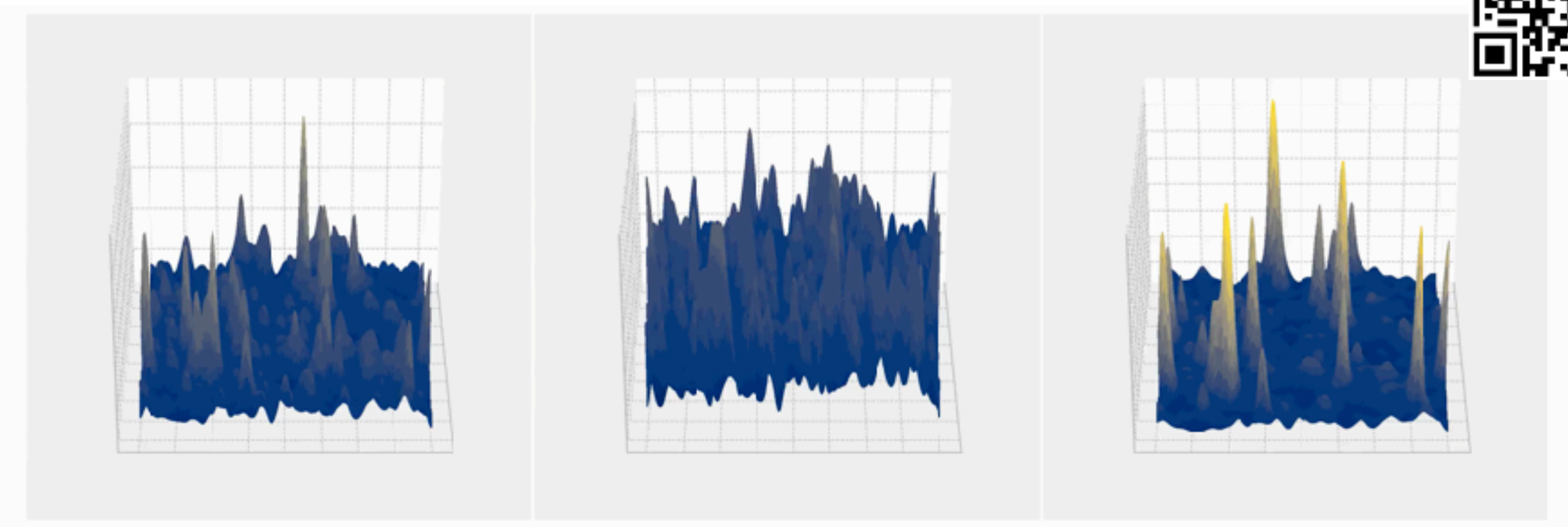
[AC 2506.11797]

- Lattice simulations: known tool to study non-perturbative cosmological phenomena.

  Examples: reheating, cosmological phase transitions

My goal: Develop lattice techniques for inflation

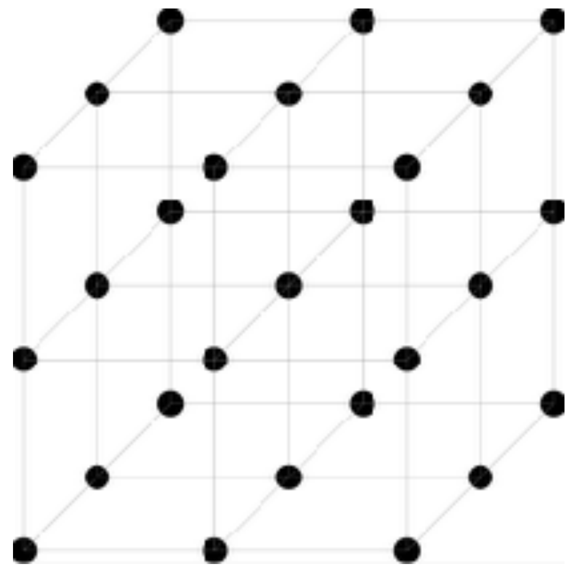
Public code: *InflationEasy:* A C++ Lattice Code for Inflation



Lattice simulations

Put the continuous inflationary universe on a discrete cubic lattice:

$\phi(\vec{x}, t)$ →



$\phi_i(t)$

$$\phi(\vec{x}, t) = \bar{\phi}(t) + \delta\phi(\vec{x}, t)$$

& perturbation theory on $\delta\phi$

→

Nonlinear evolution of ϕ_i

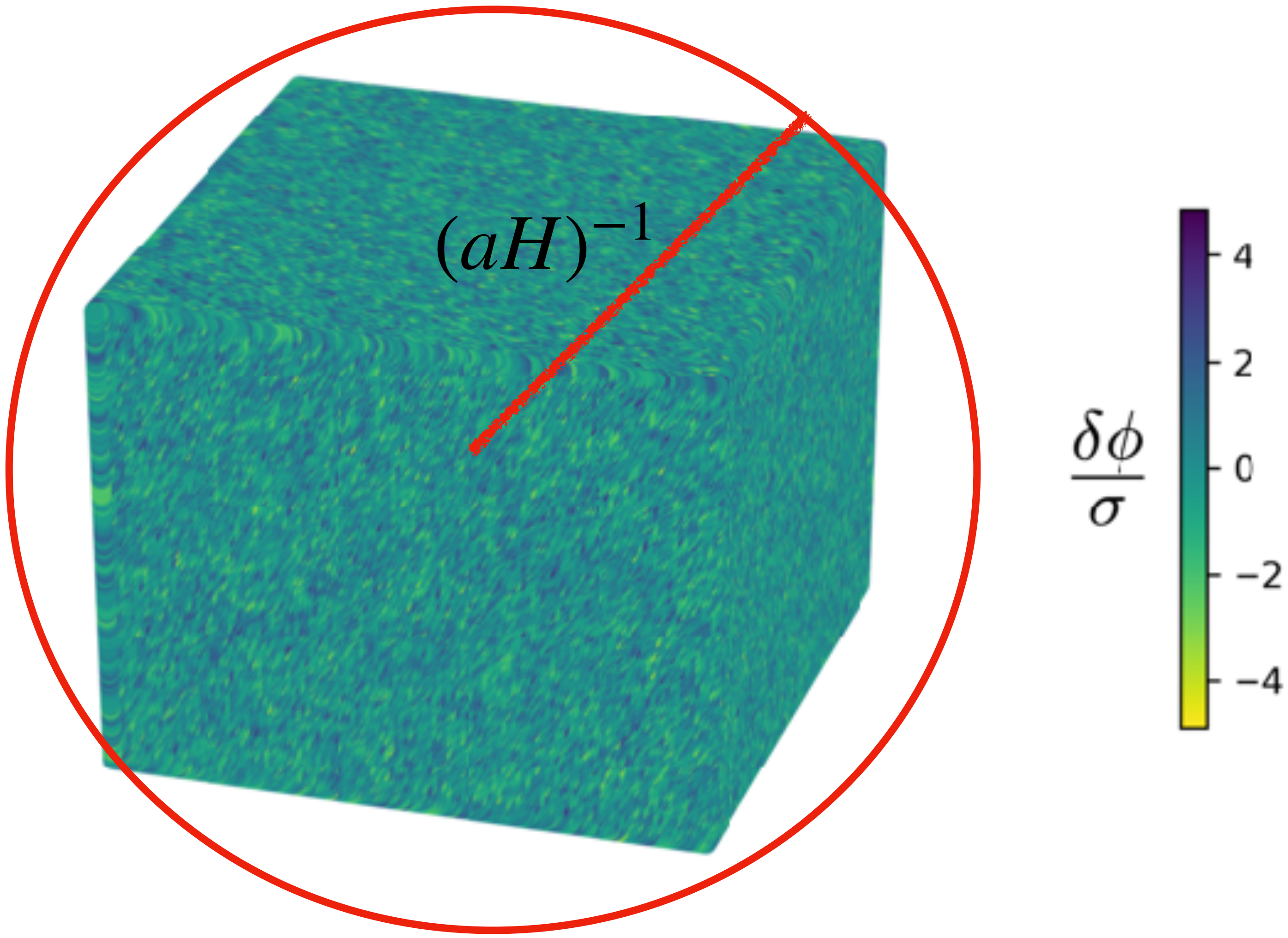
Numerically solve the classical eqs:

$$\frac{\partial \mathscr{L}}{\partial \phi_i} = \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{\phi}_i}\right)$$

Lattice simulations of inflation

[AC 2506.11797]

Start with quantum fluctuations on sub-horizon box:



Lattice simulations of inflation

[**AC** 2506.11797]

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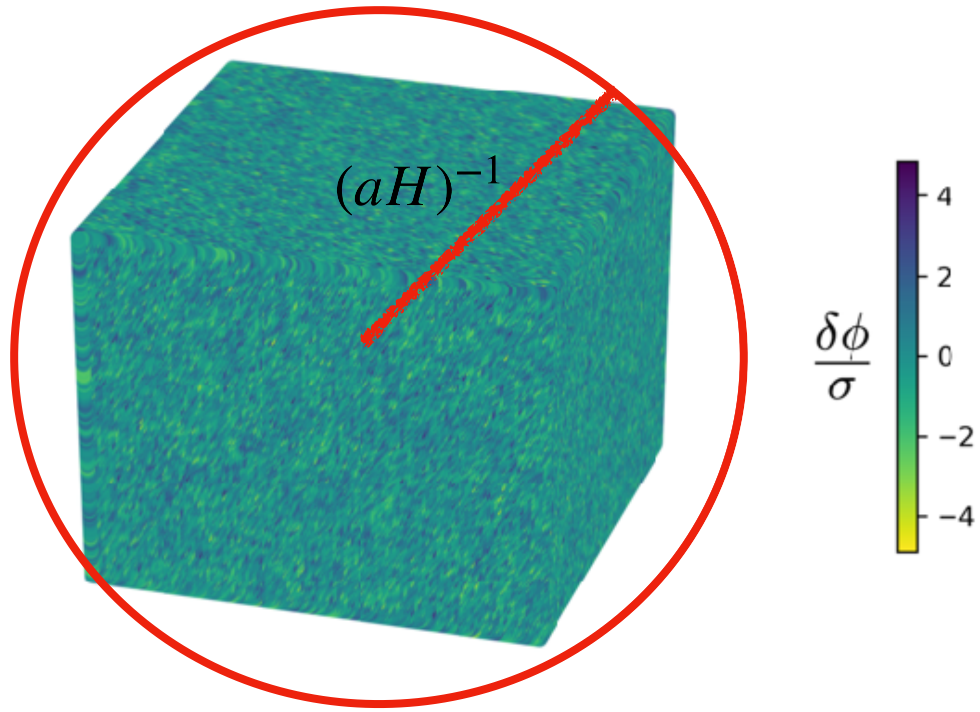


$$\hat{\phi}(\vec{n}) = \sum_{\vec{m}} \left[\hat{a}_{\vec{m}}\, u(\vec{\kappa}_{\vec{m}})\, e^{i\frac{2\pi}{N}\vec{n}\cdot\vec{m}} + \hat{a}^{\dagger}_{\vec{m}}\, u^{\dagger}(\vec{\kappa}_{\vec{m}})\, e^{-i\frac{2\pi}{N}\vec{n}\cdot\vec{m}} \right]$$

$$u(\vec{\kappa}) = \frac{L^{3/2}}{a\sqrt{2\omega_{\vec{\kappa}}}} e^{-i\omega_{\vec{\kappa}}\tau}$$

"Discrete Bunch Davies" [**AC+** 2102.06378]

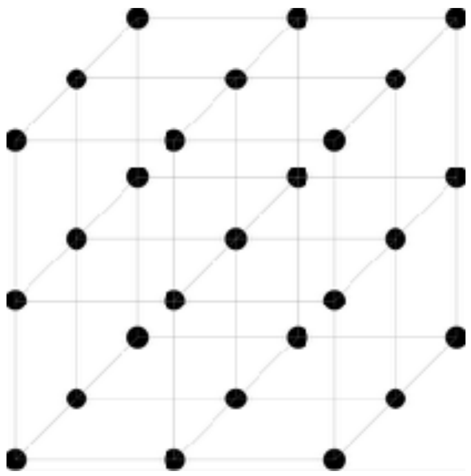
$$\hat{a}_{\vec{m}} = e^{i2\pi\hat{Y}_{\vec{m}}} \sqrt{-\ln(\hat{X}_{\vec{m}})/2},$$

$\hat{X}_{\vec{m}}, \hat{Y}_{\vec{m}}$ uniform randoms between 0 and 1: **"stochastic" approximation of quantum noise**

Lattice approach: evolution

[AC 2506.11797]

Solve numerically for all lattice points:

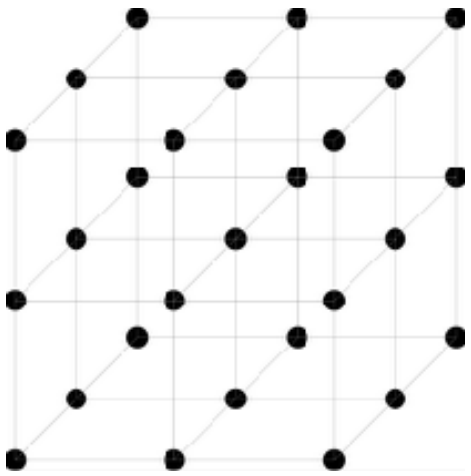


$$\phi''(\vec{n}) + 2H\phi'(\vec{n}) - \nabla^2\phi(\vec{n}) + a^2\frac{\partial V}{\partial \phi}(\vec{n}) = 0$$

+ Friedmann equation for scale factor $\frac{d^2a}{d\tau^2} = \frac{1}{6}\left(\langle\rho\rangle - 3\langle p\rangle\right) a^3$

Lattice approach: evolution

Solve numerically for all lattice points:



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+ Friedmann equation for scale factor $\frac{d^2a}{d\tau^2} = \frac{1}{6}\left(\langle\rho\rangle - 3\langle p\rangle\right)a^3$

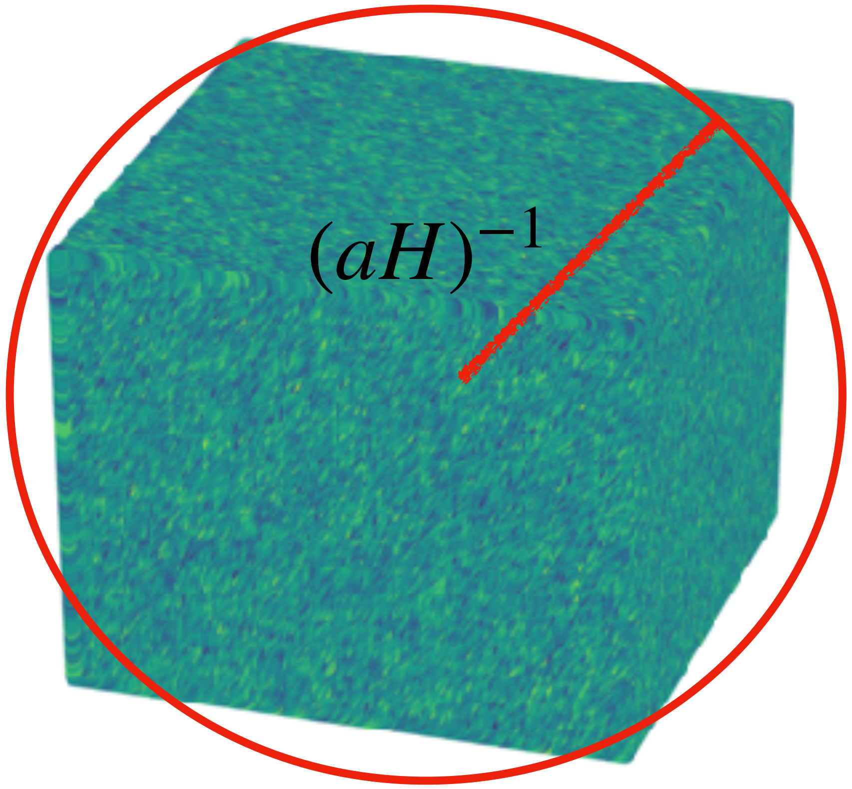
Assuming **unperturbed metric** $ds^2 = a^2(-d\tau^2 + d\vec{x}^2)$ because:

- $\delta g_{ij} \equiv 0$ (spatially flat gauge)
- $\delta g_{0\mu} \propto \epsilon = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\dot{\phi}^2}{M_{\rm Pl}^2 H^2} \to 0$, known as "decoupling limit" of gravity $M_{\rm Pl} \to \infty$

C. Cheung et al. [0709.0293]
S. R. Behbahani et al. [1111.3373]
P. Creminelli et al. [2401.10212]
…

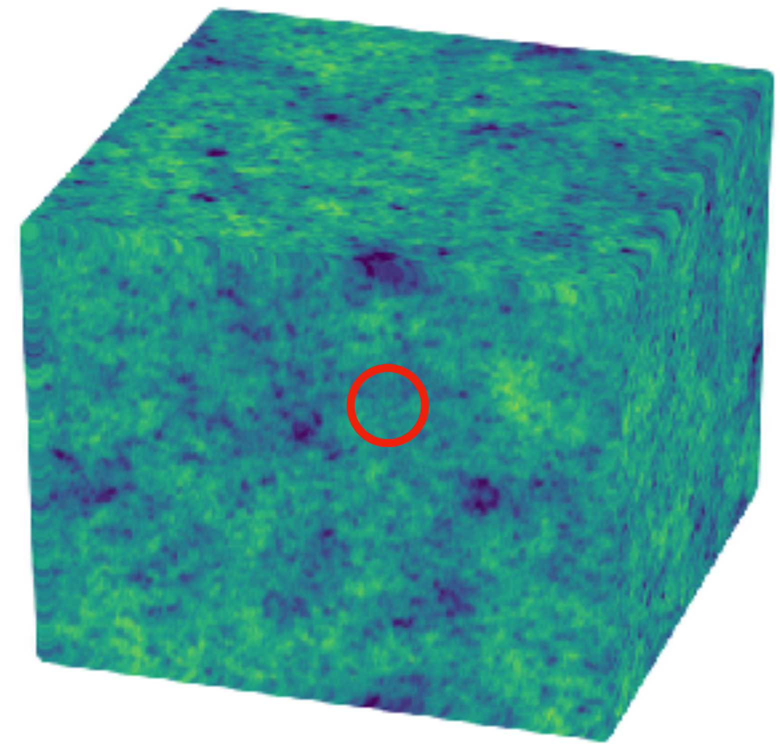
Lattice simulations of Inflation

[AC 2506.11797]



"sub-horizon" box

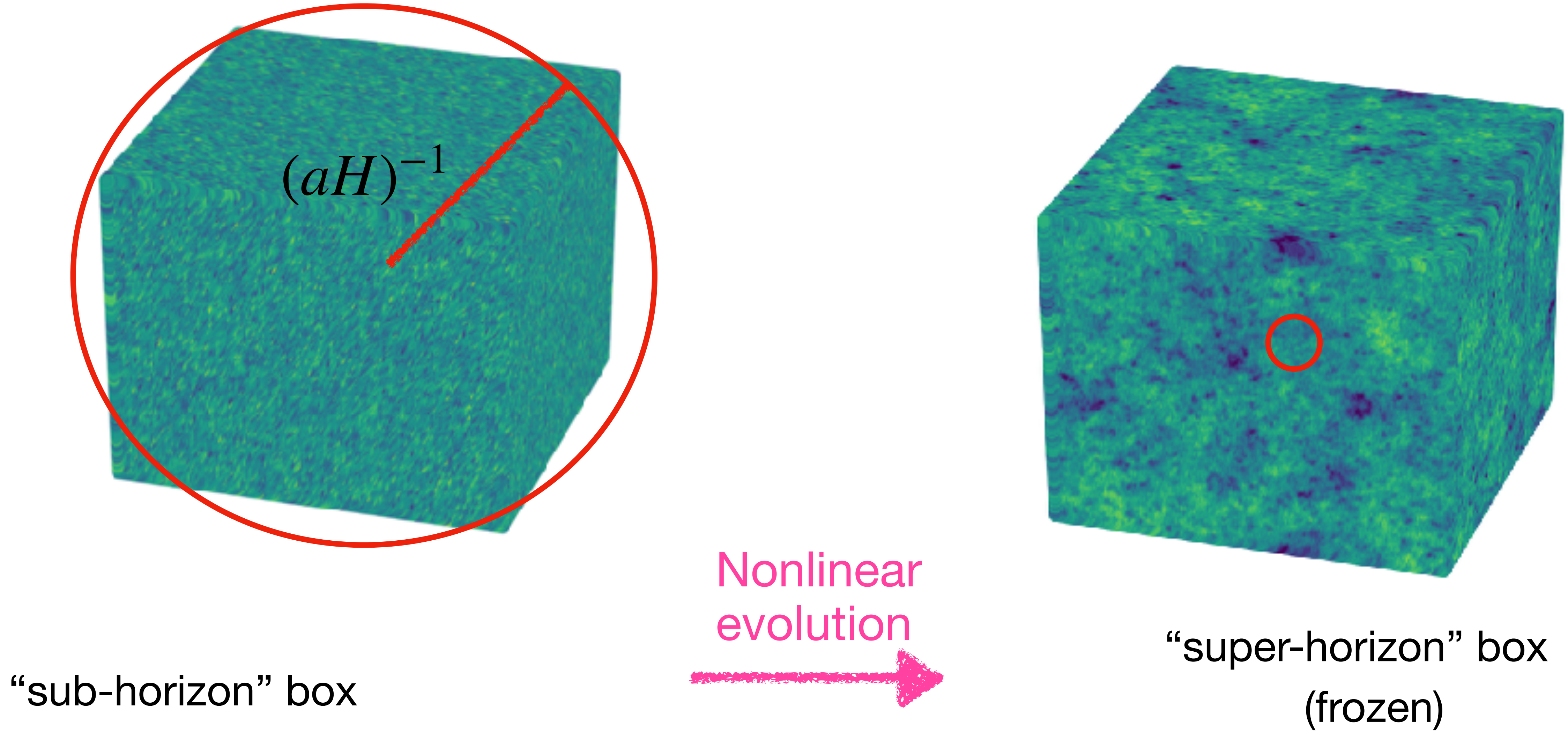
Nonlinear
evolution



"super-horizon" box
(frozen)

Lattice simulations of Inflation

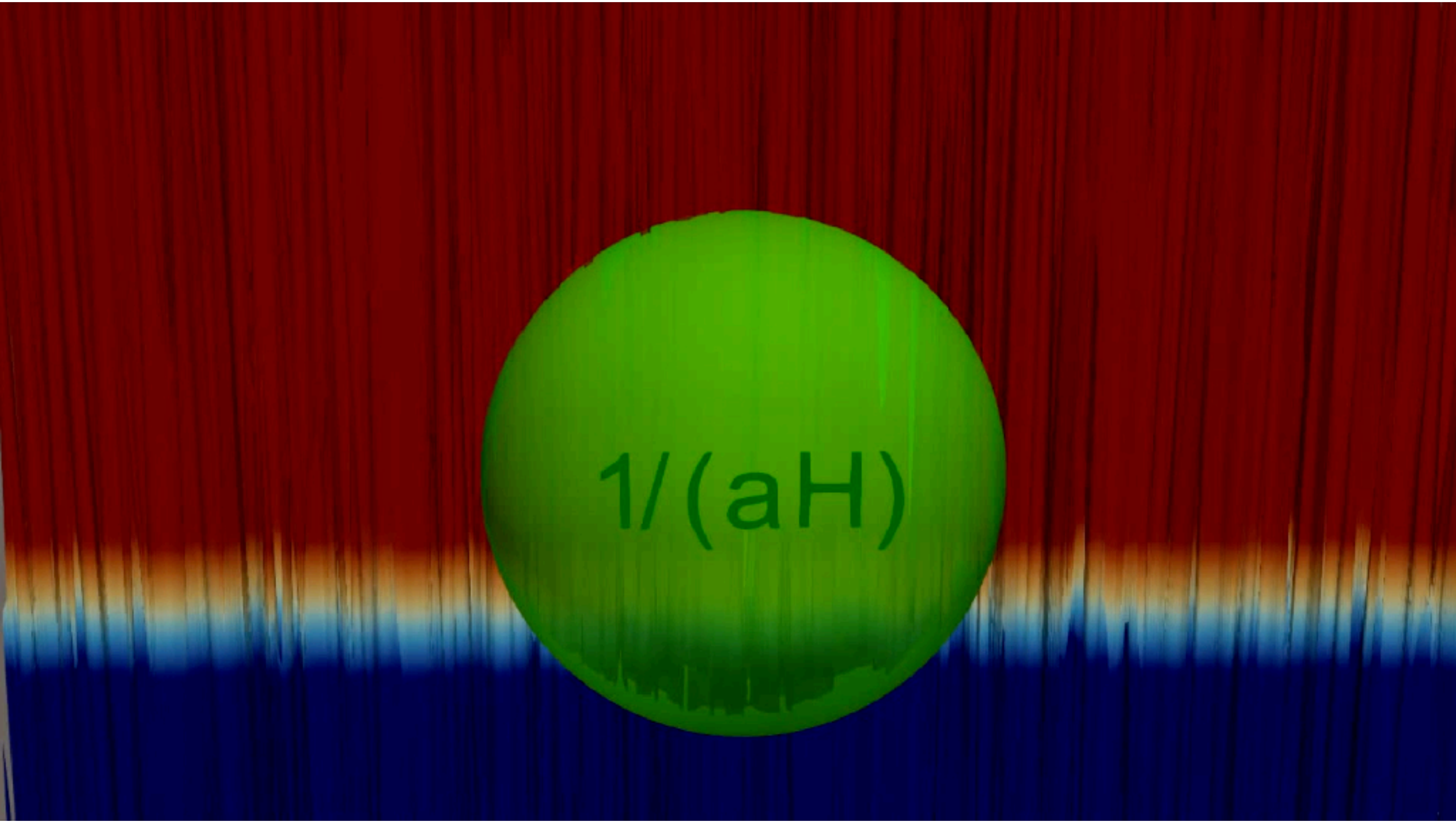
[AC 2506.11797]



- Key point: non-perturbative $\phi(\vec{x}, t) \neq \bar{\phi}(t) + \delta\phi(\vec{x}, t)$

- Assumptions: 1) Neglect gravitational interaction fixed metric $ds^2 = a(\tau)\left(-d\tau^2 + d\vec{x}^2\right)$

  2) Semi-classical approach (neglect quantum tunneling, interference, etc…)


1/(aH)

Roadmap

2) Small-scale physics: Inflationary butterfly effect

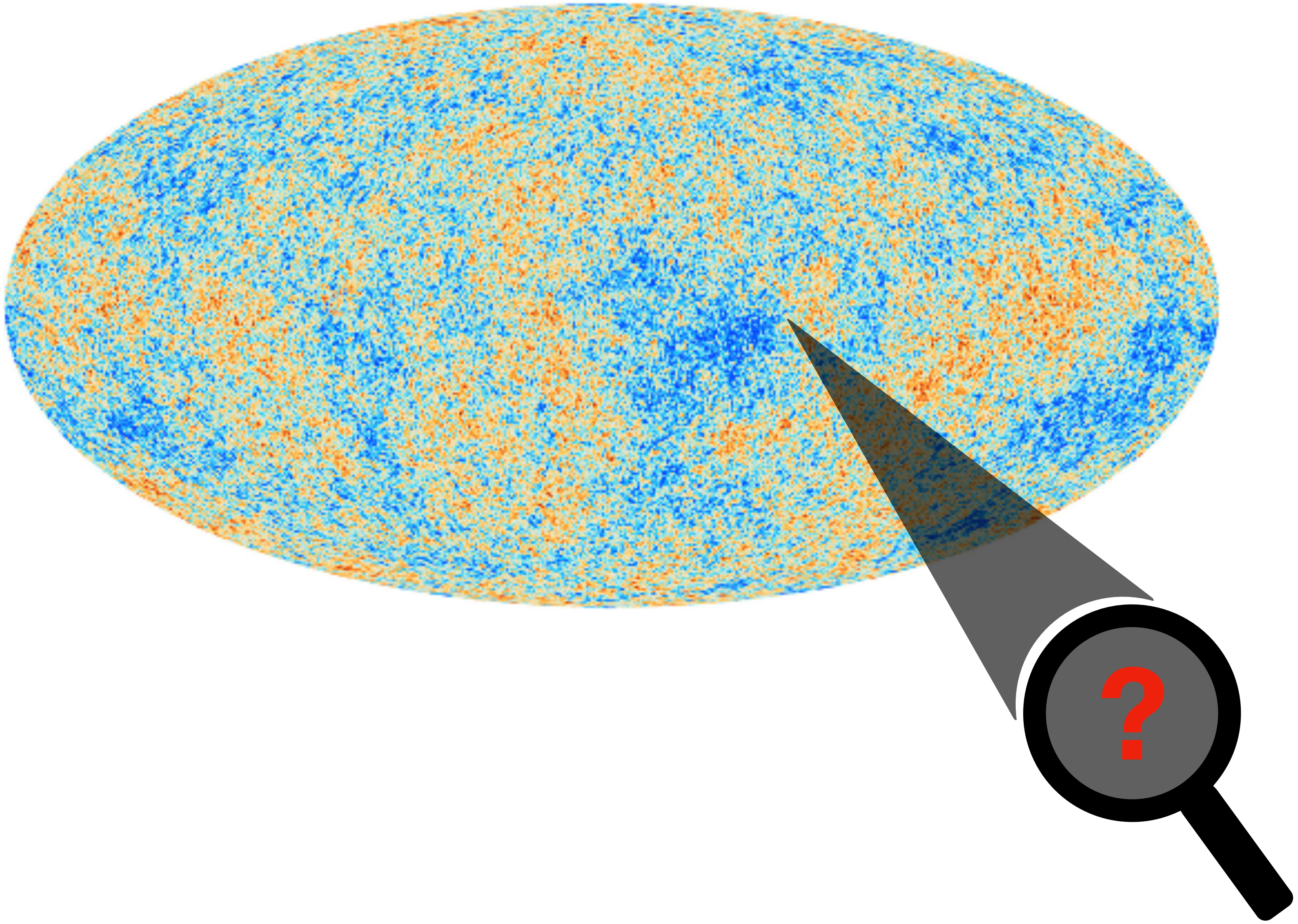
2.1) Oscillatory potential

AC, K. Inomata, S. Renaux-Petel

2403.12811

The early Universe at small scales

What is the physics of inflation at scales $\lambda \ll \lambda_{CMB}$?



Inflation generates fluctuations at scales $\sim e^{40}$ smaller than CMB scales

Inflation on small scales

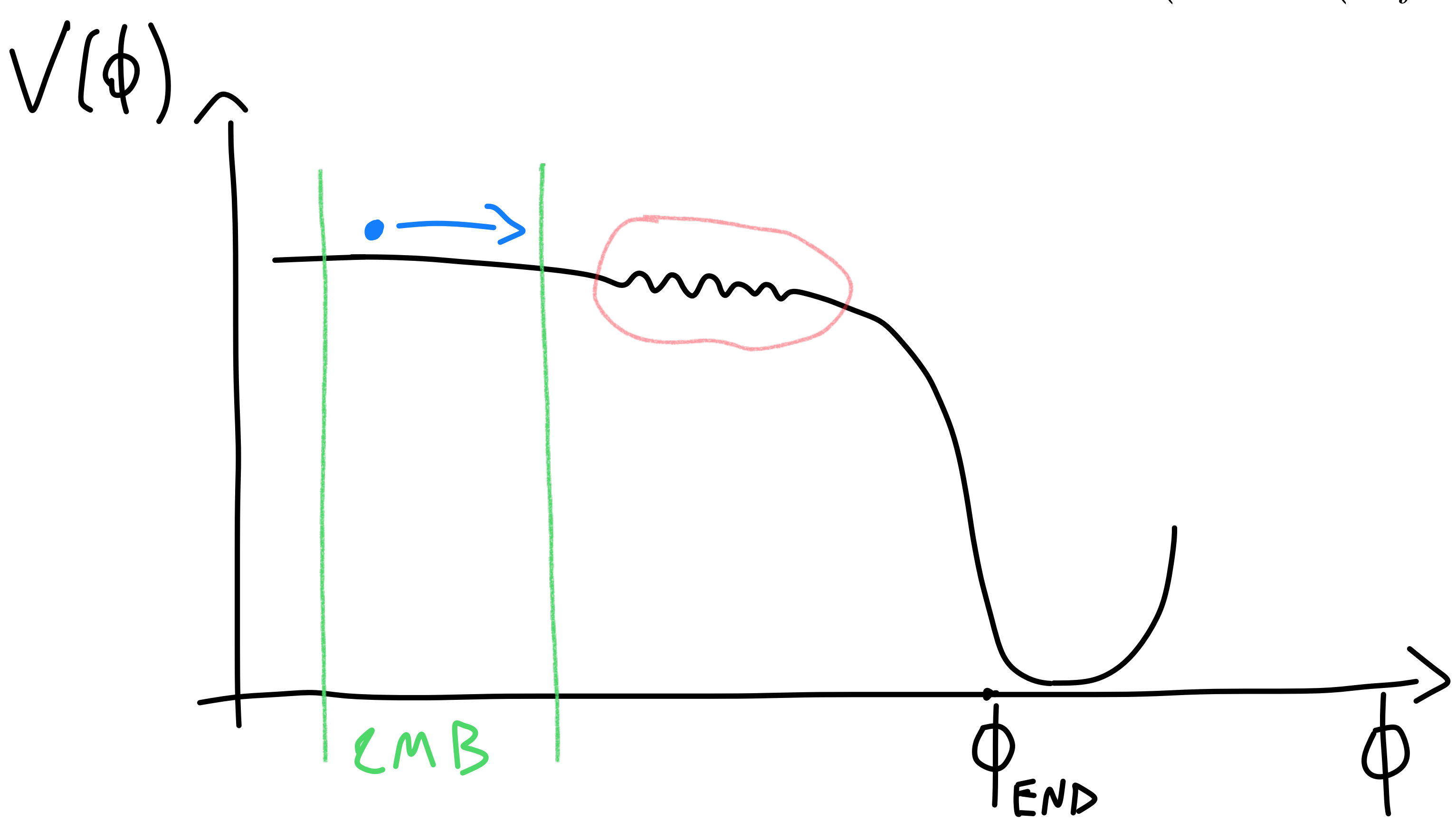
Toy model: a **small-scale modification** of the inflaton potential

$$V(\phi) = V_{sr}(\phi) + \Lambda^4 W(\phi)\left[\cos\left(\frac{\phi-\phi_0}{f}\right) - 1\right]$$

Slow-roll potential

Localised oscillation

$$W(\phi) = \frac{1}{4}\left(1+\tanh\left(\frac{\phi-\phi_0}{f}\right)\right)\left(1+\tanh\left(\frac{\phi_0-\phi+\Delta\phi}{f}\right)\right)$$



Oscillatory potential

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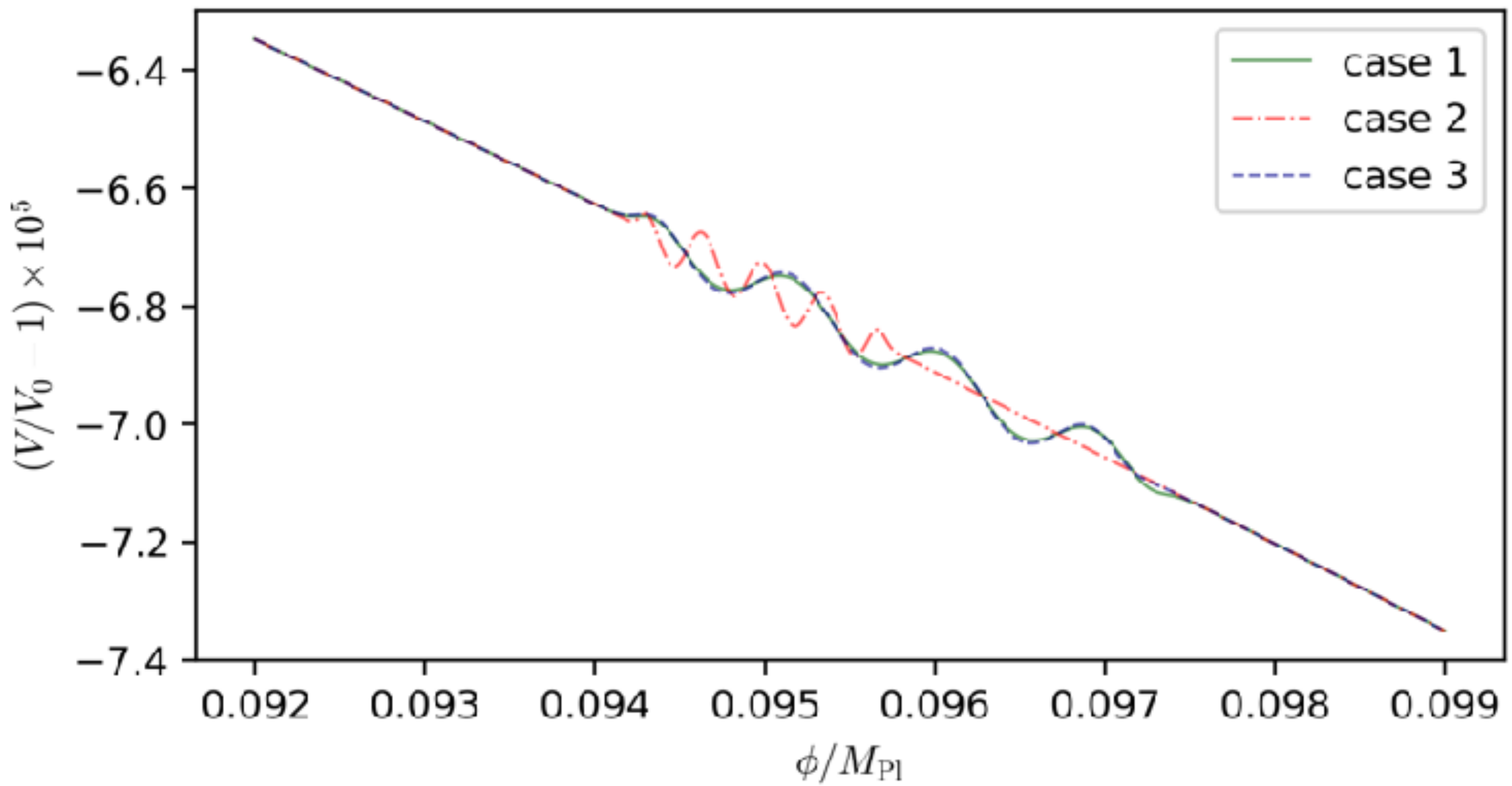
oscillation → parametric resonance → exponential growth of perturbations → observable!

Oscillatory potential

$$V(\phi) = V_{sr}(\phi) + \Lambda^4 W(\phi)\left[\cos\left(\frac{\phi - \phi_0}{f}\right) - 1\right]$$

Let's consider the following three cases:

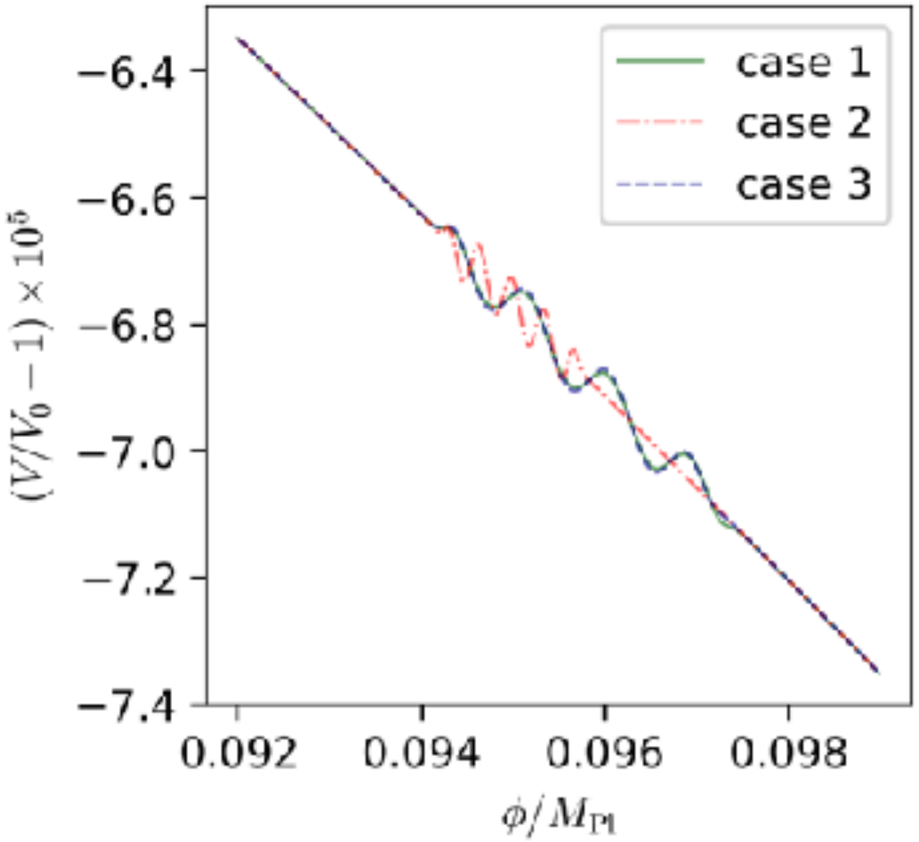


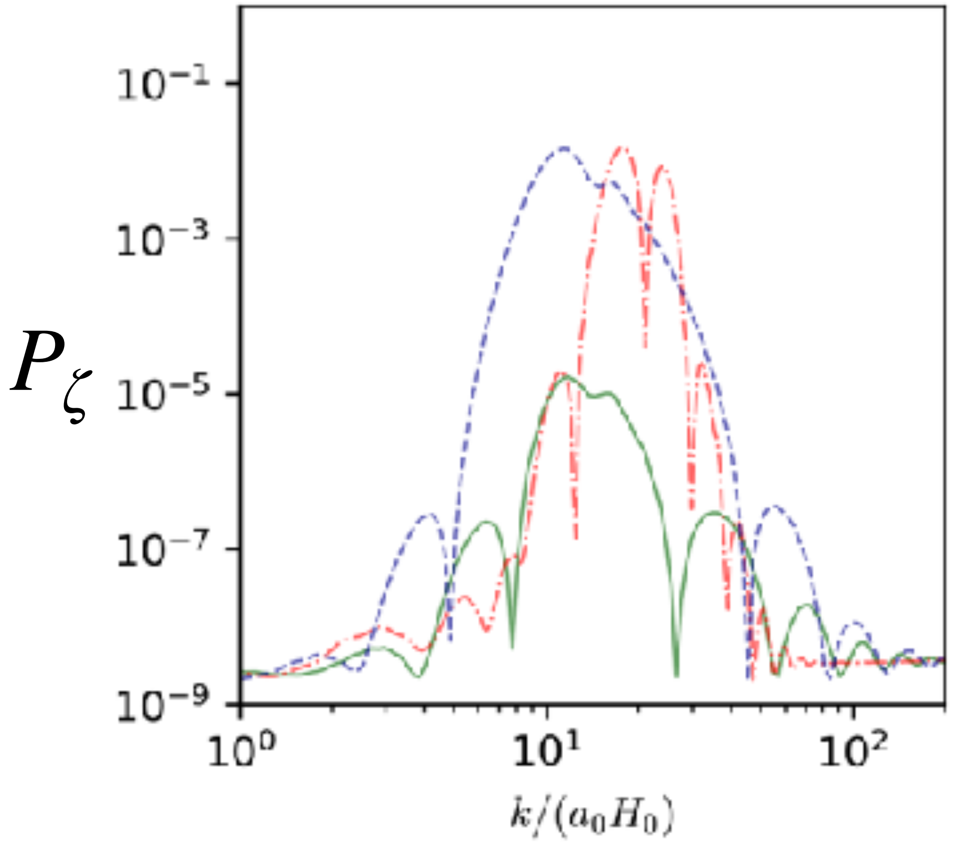
Oscillatory potential

$$V(\phi) = V_{sr}(\phi) + \Lambda^4 W(\phi)\left[\cos\left(\frac{\phi-\phi_0}{f}\right) - 1\right]$$

The feature induces a growth of the power spectrum:

Linear prediction:





Case 1: $P_\zeta \simeq 10^{-5}$

Case 2: $P_\zeta \simeq 10^{-2}$

Case 3: $P_\zeta \simeq 10^{-2}$

Oscillatory potential

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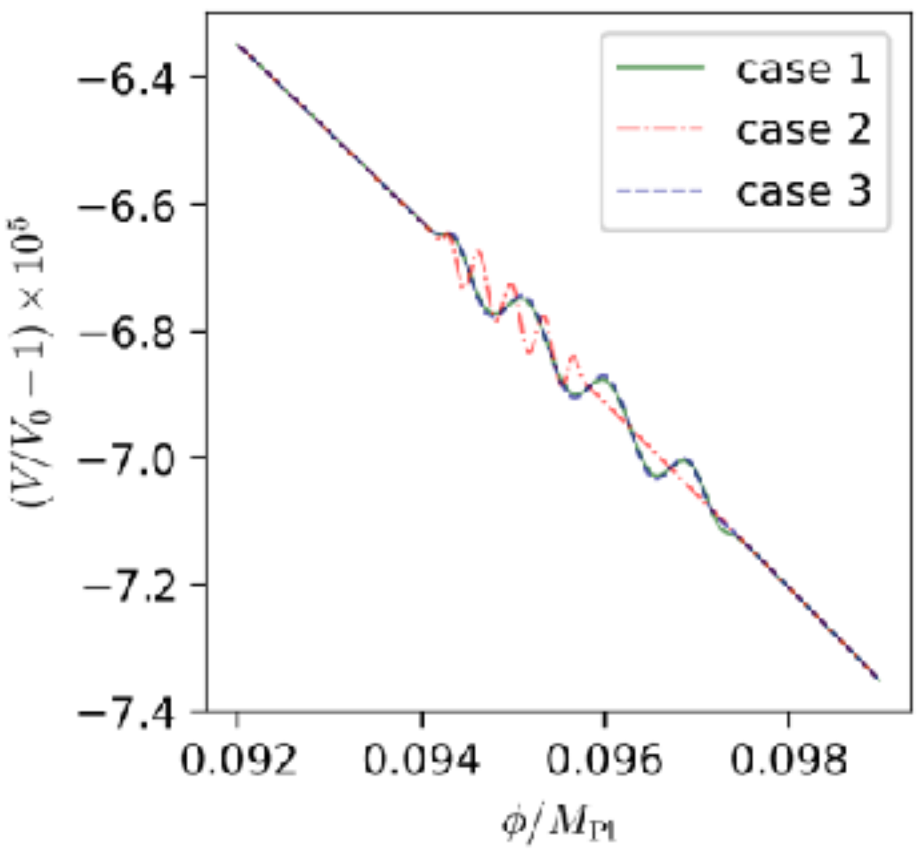
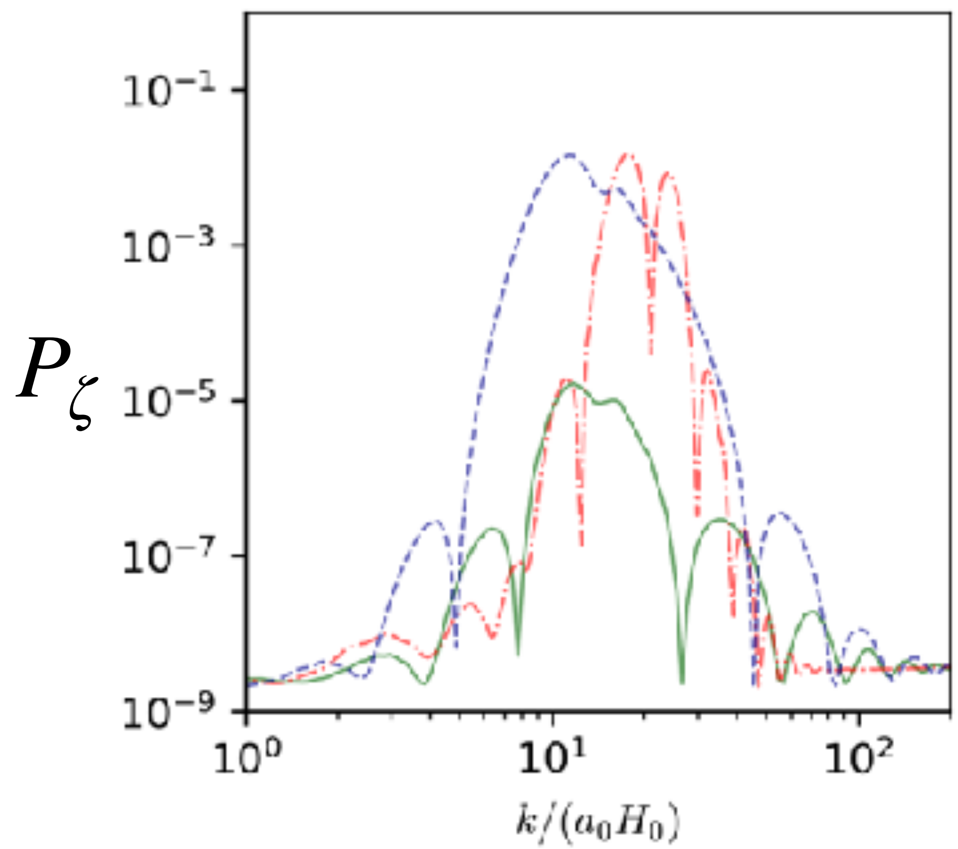




Case 1: $P_\zeta \simeq 10^{-5}$

Case 2: $P_\zeta \simeq 10^{-2}$

Case 3: $P_\zeta \simeq 10^{-2}$

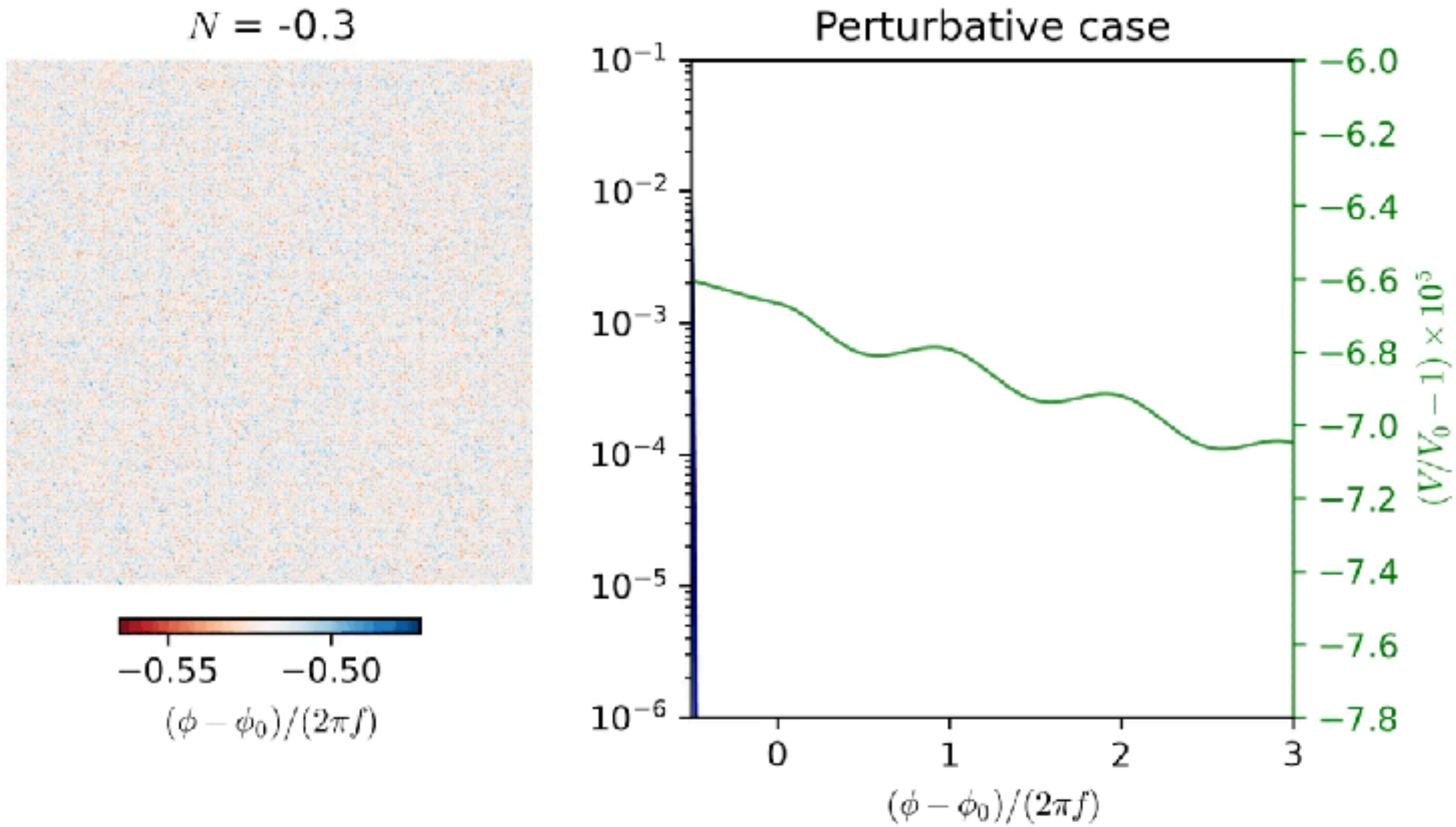
[K. Inomata, M. Braglia, X. Chen, S. Renaux-Petel 2211.02586]

$$P_{\zeta,1-\rm loop} \gtrsim P_{\zeta,\rm tree}$$

In case 3 and 2, but not 1

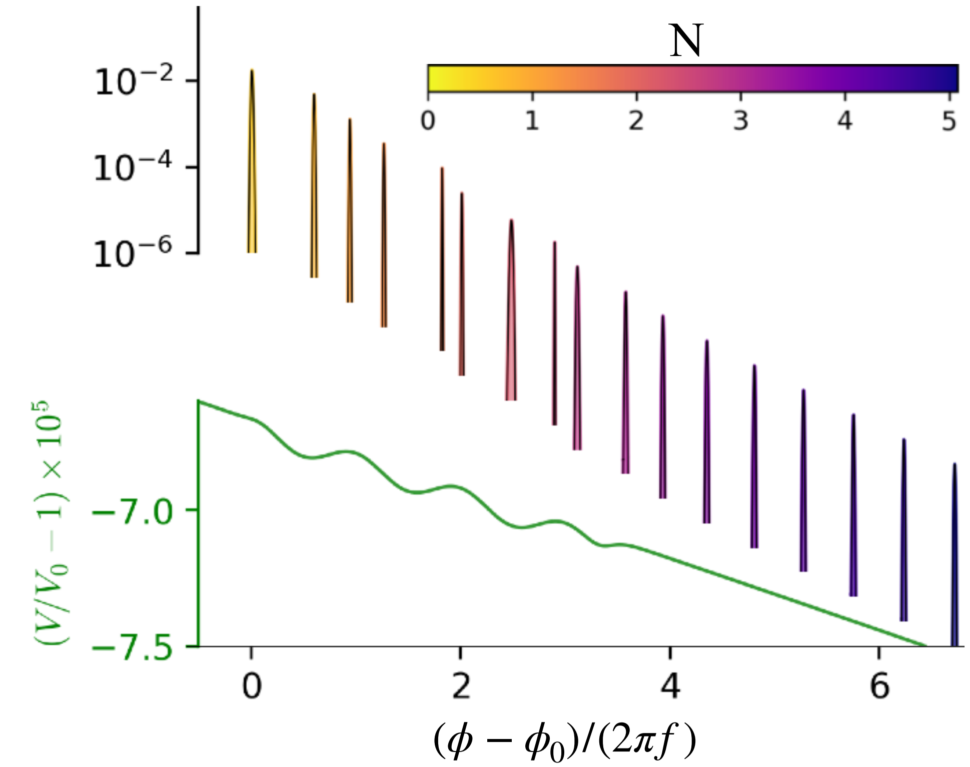
Case 1. $(P_\zeta \sim 10^{-5})$

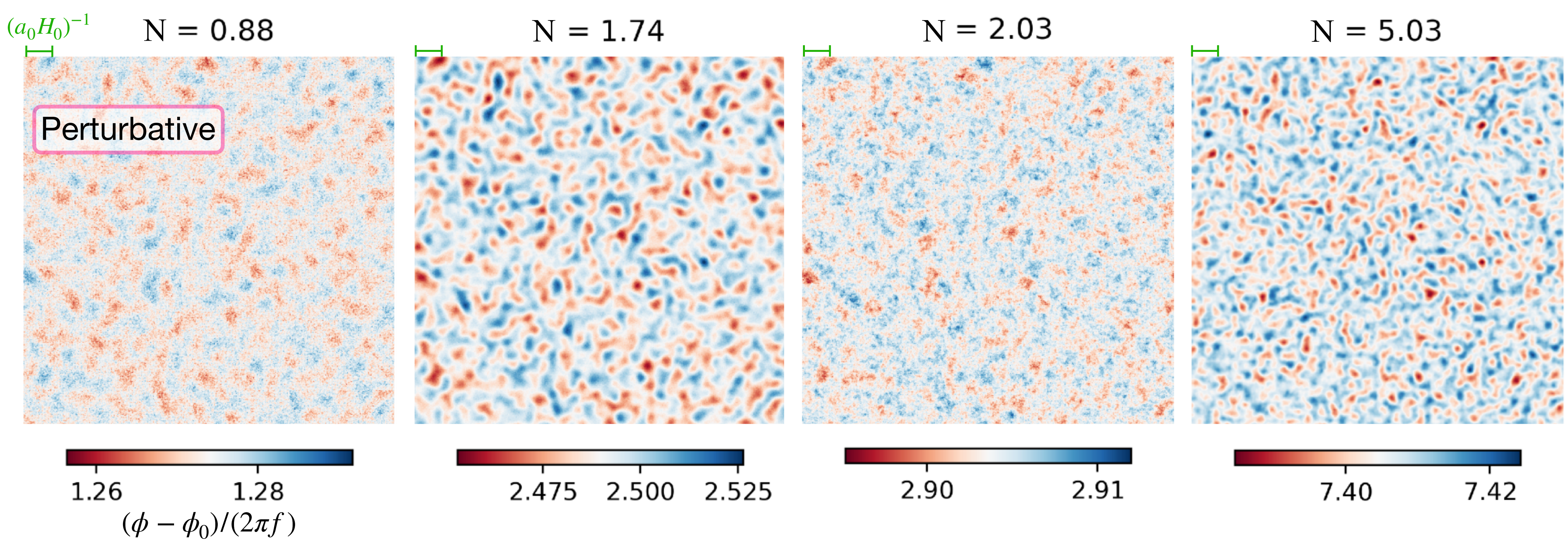
Case 1 is perturbative



Case 1. ($P_\zeta \sim 10^{-5}$)

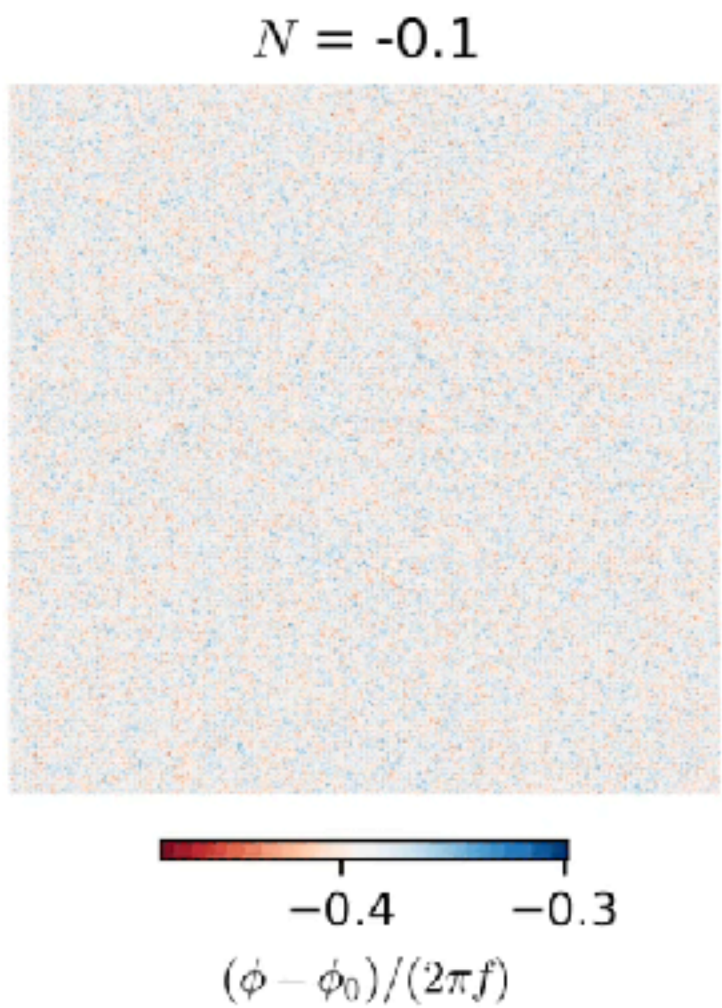
Case 1 is perturbative

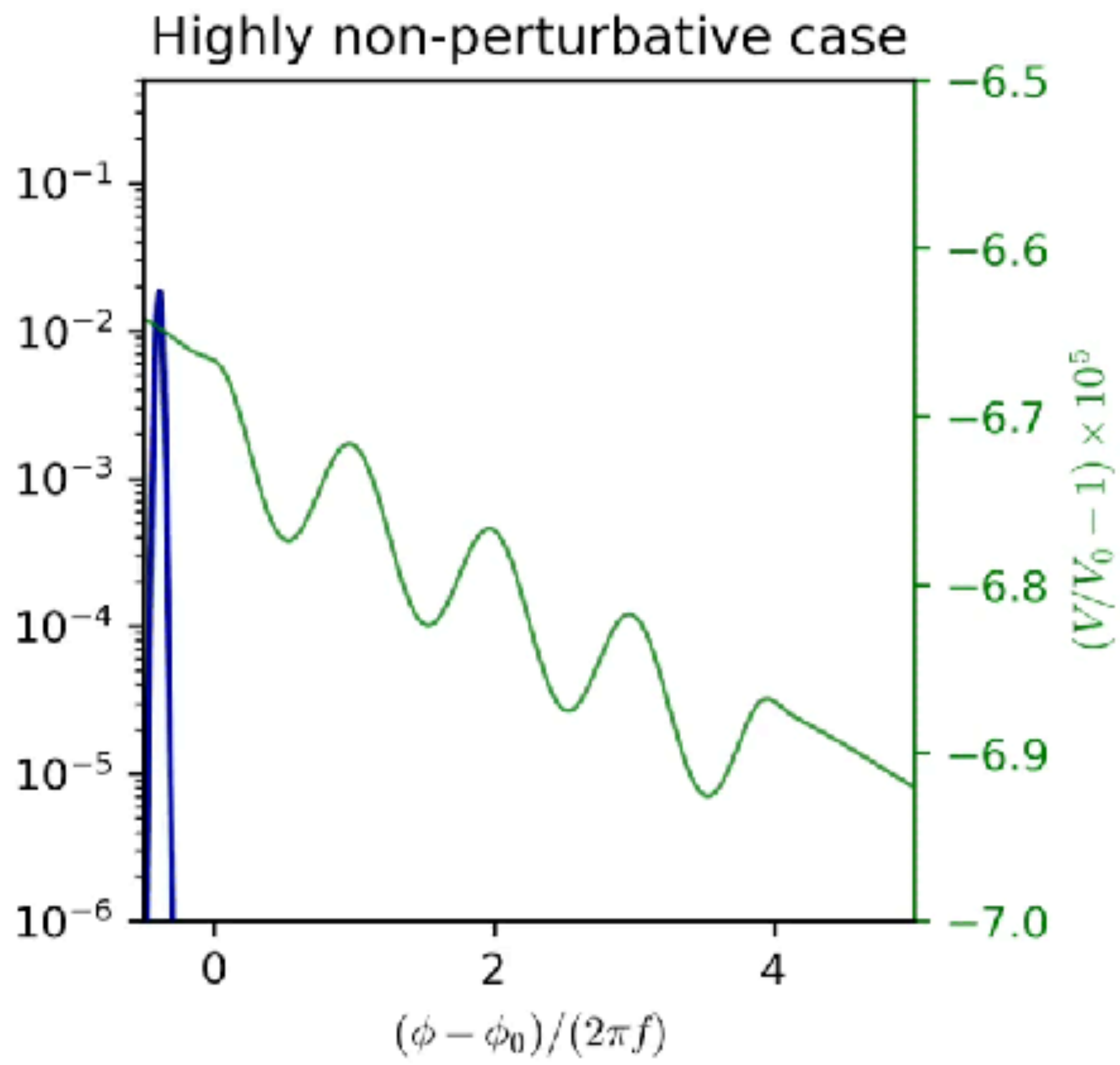




Case 2. $(P_\zeta \sim 10^{-2})$

Case 2 is highly non-perturbative:
Inflaton is stuck inside the oscillatory potential





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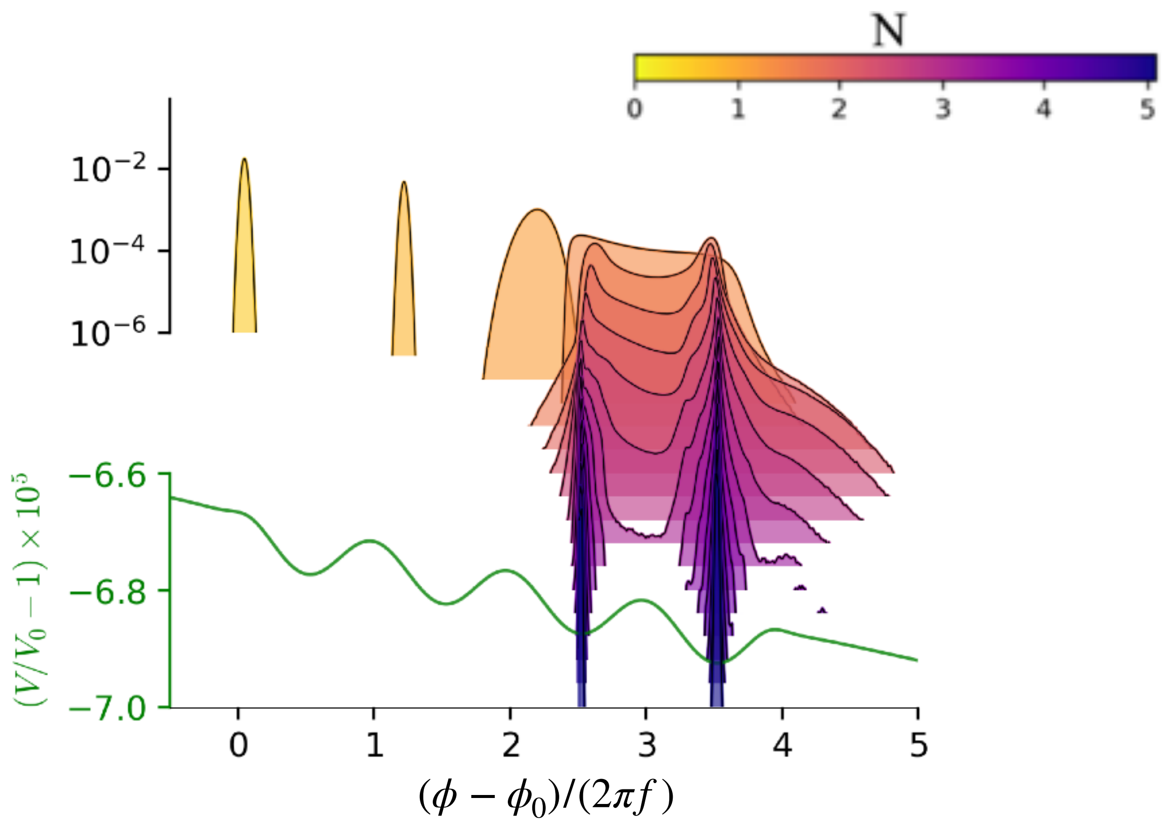
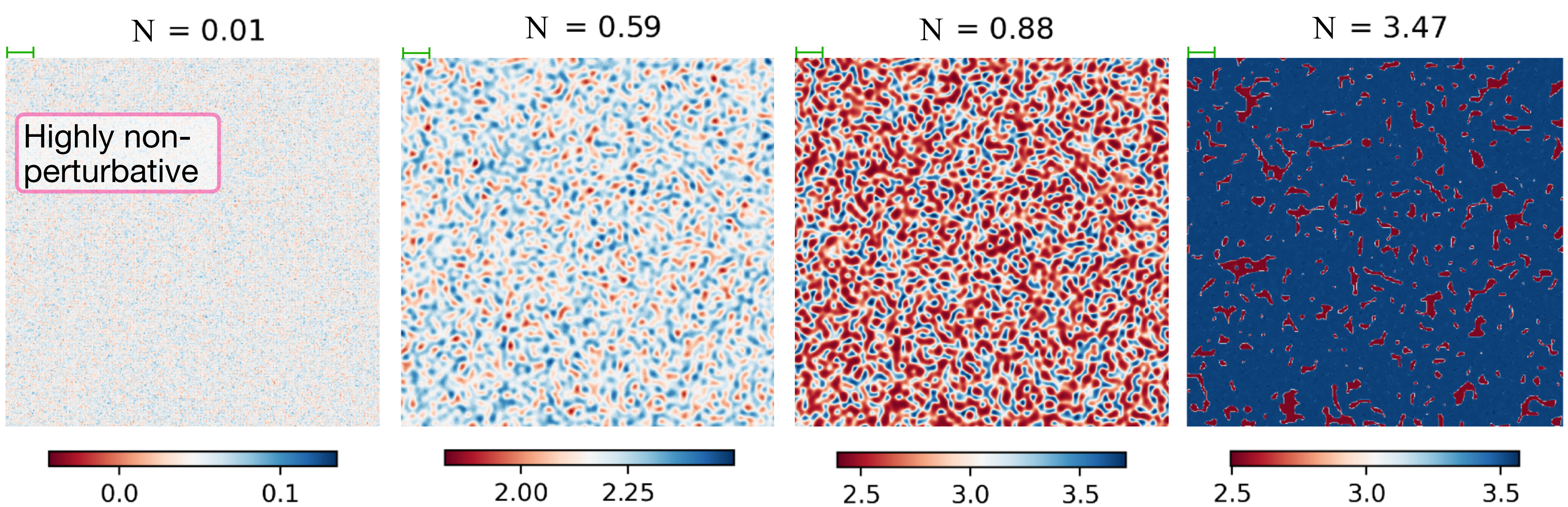
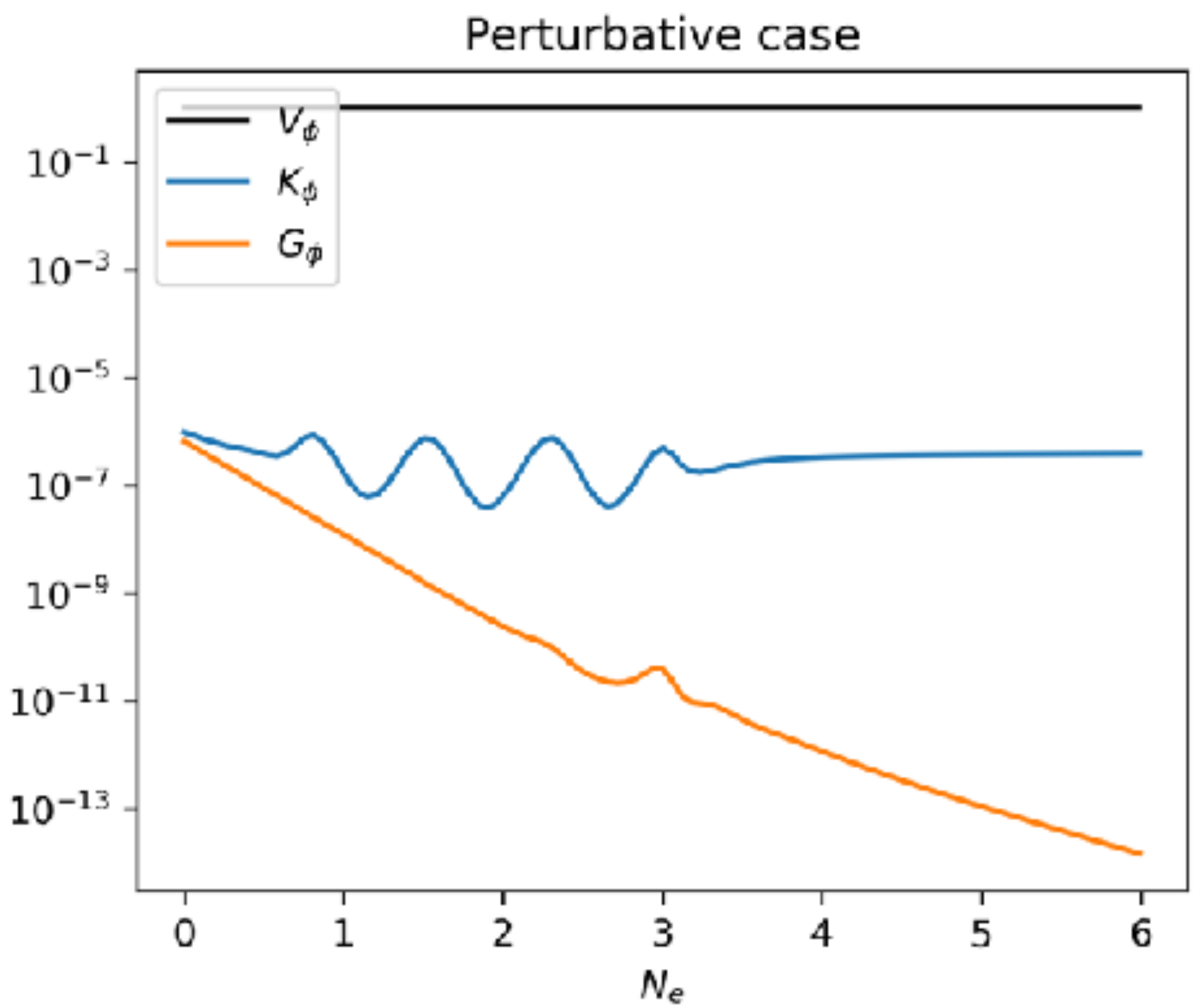


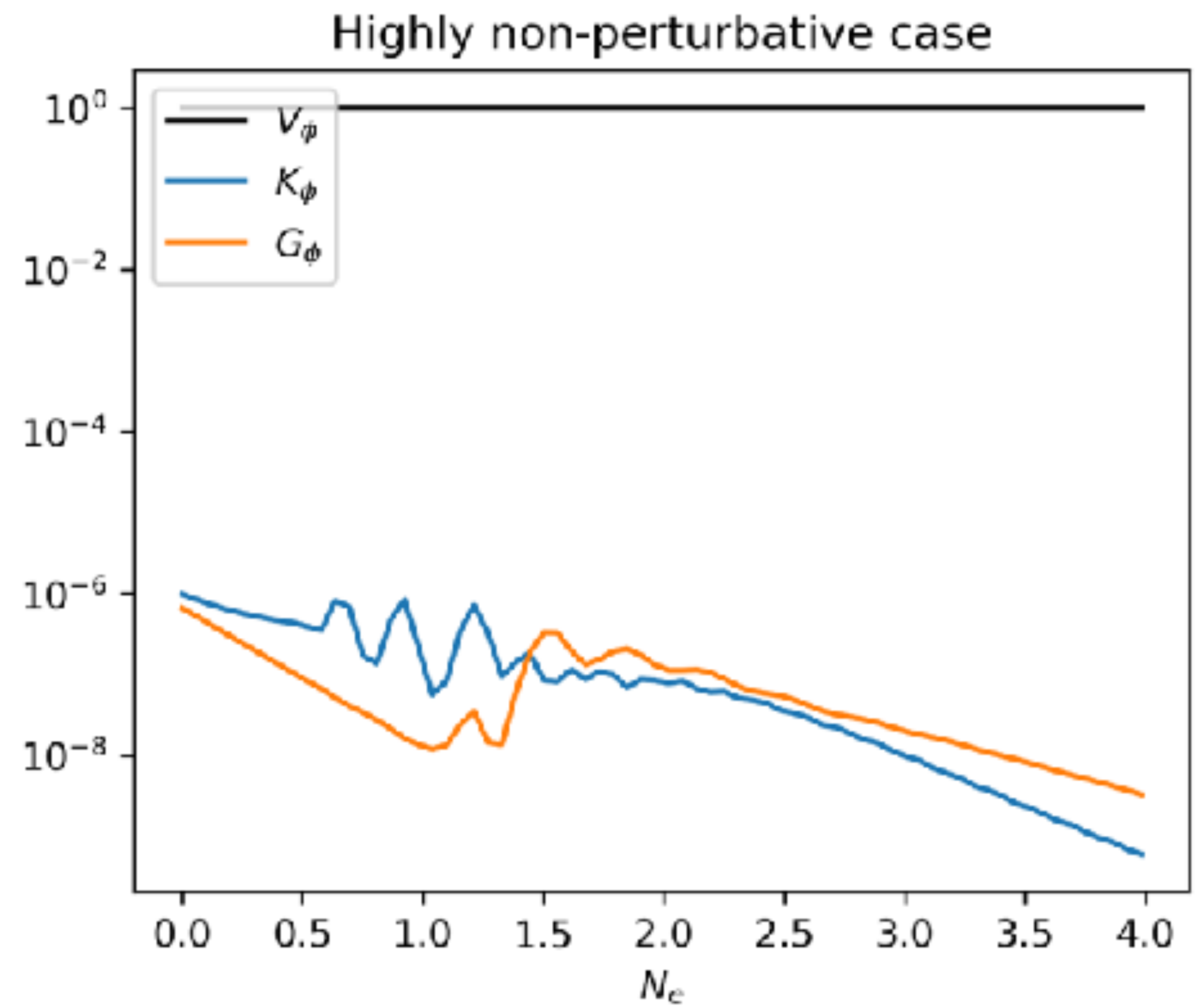


AC, S. Renaux-Petel, K. Inomata [2403.12811]

Case 2. $(P_\zeta \sim 10^{-2})$

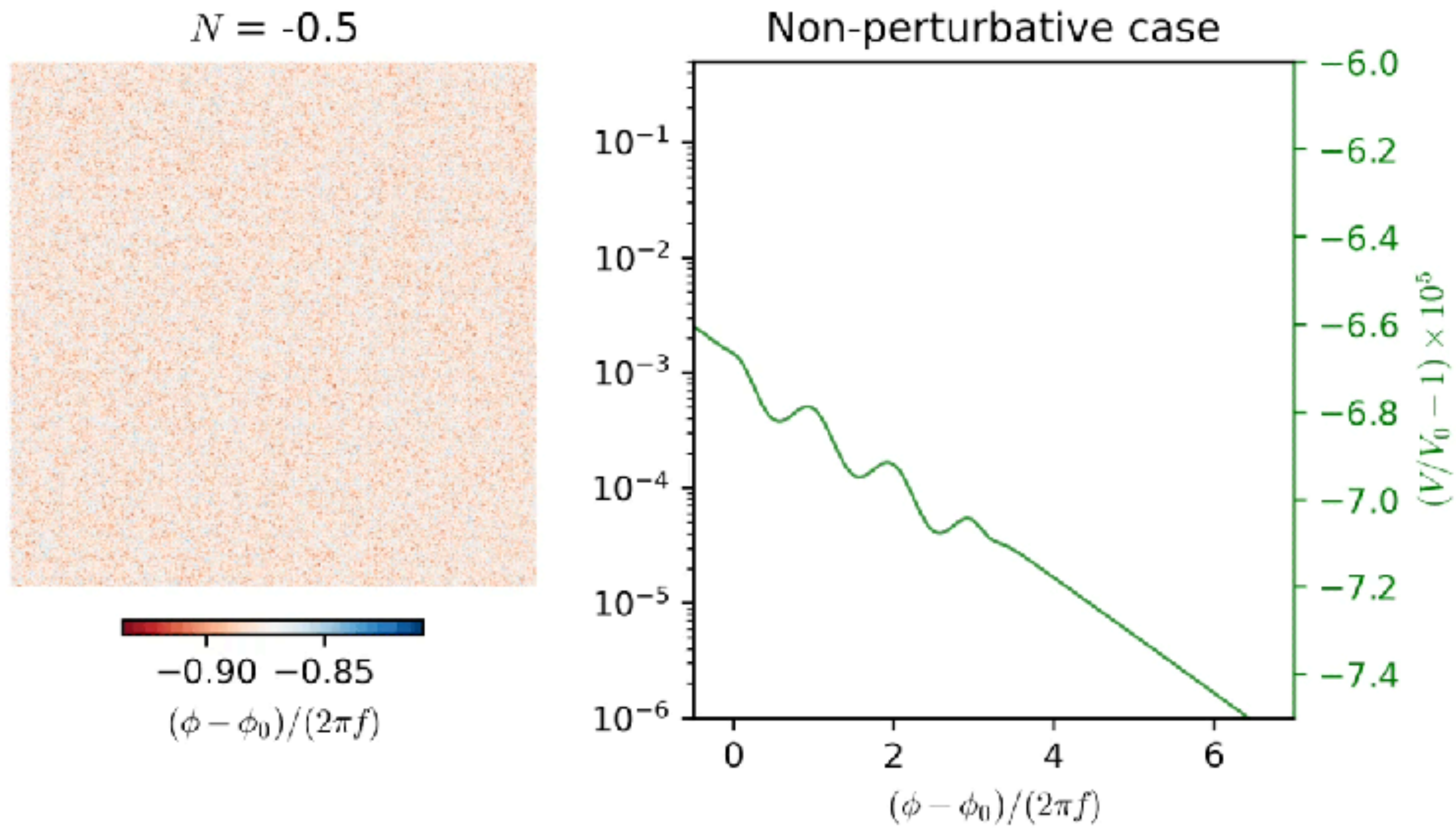
Let's look at the energy:





Case 3. $(P_\zeta \sim 10^{-2})$

Case 3: Only some patches are stuck in the resonant potential!
The rest continues slow-rolling



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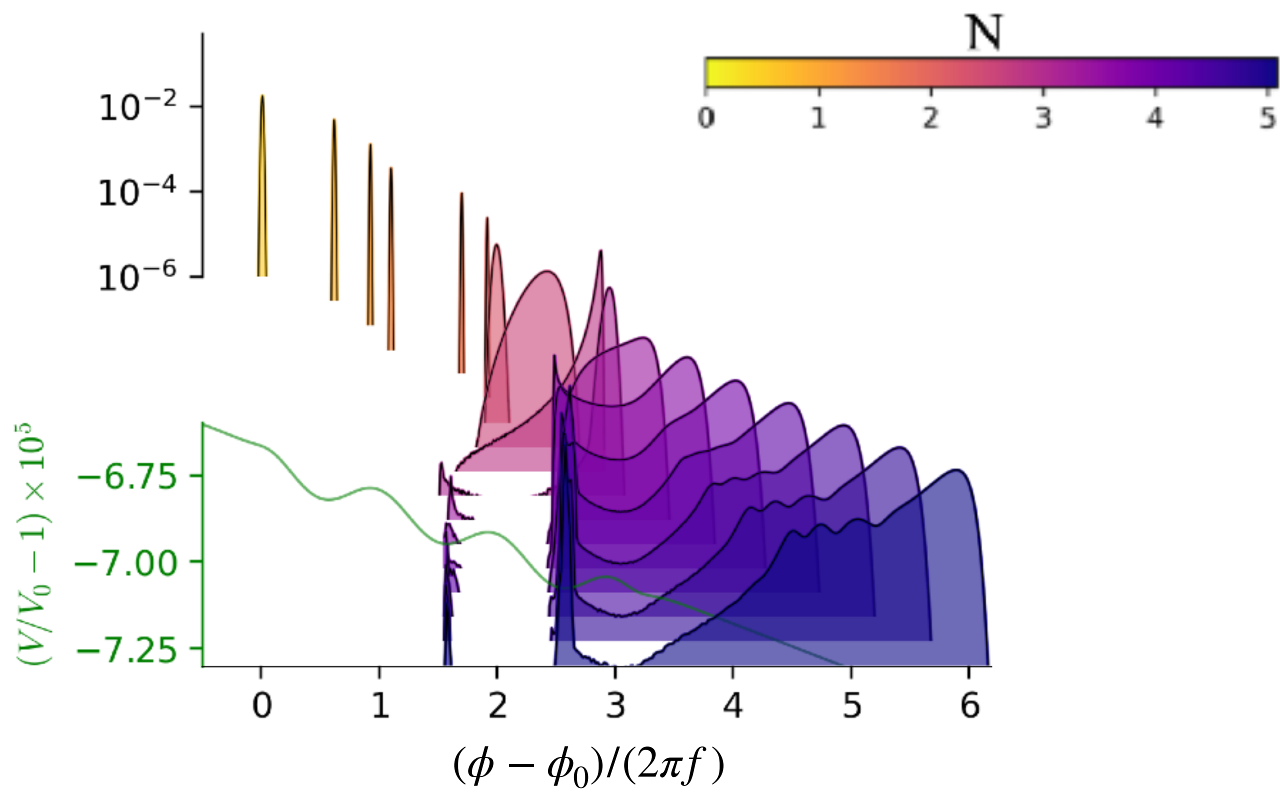
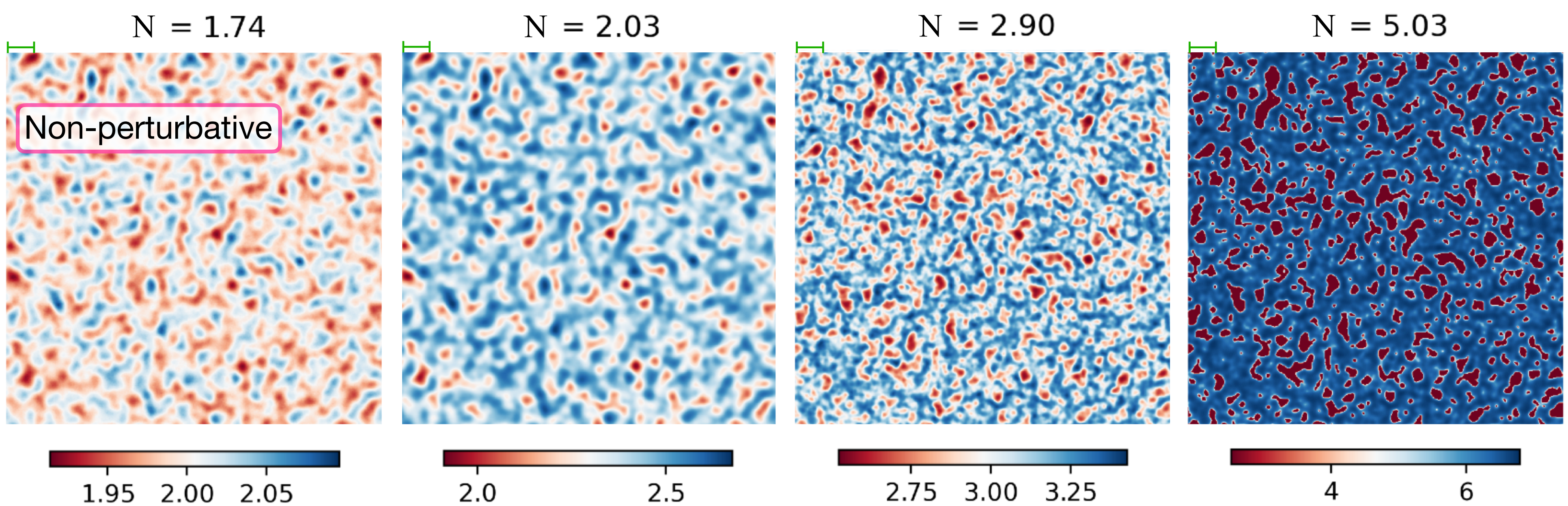


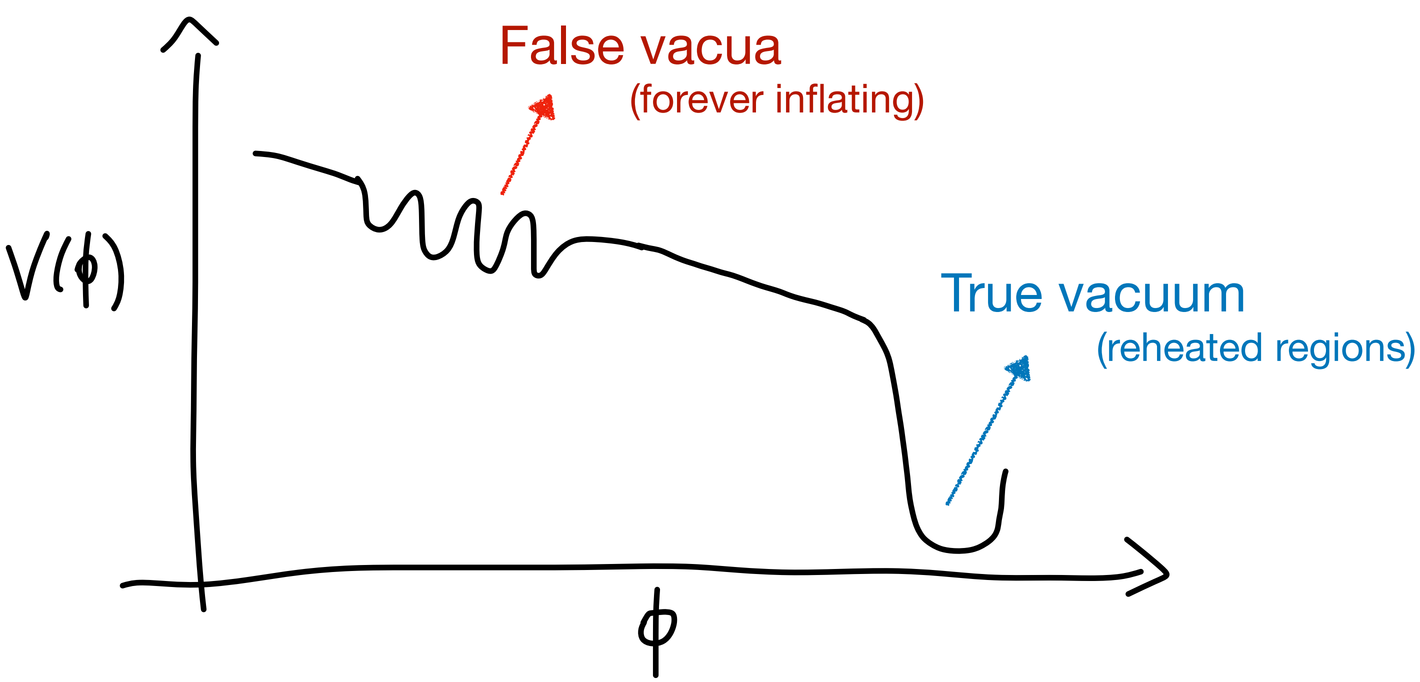


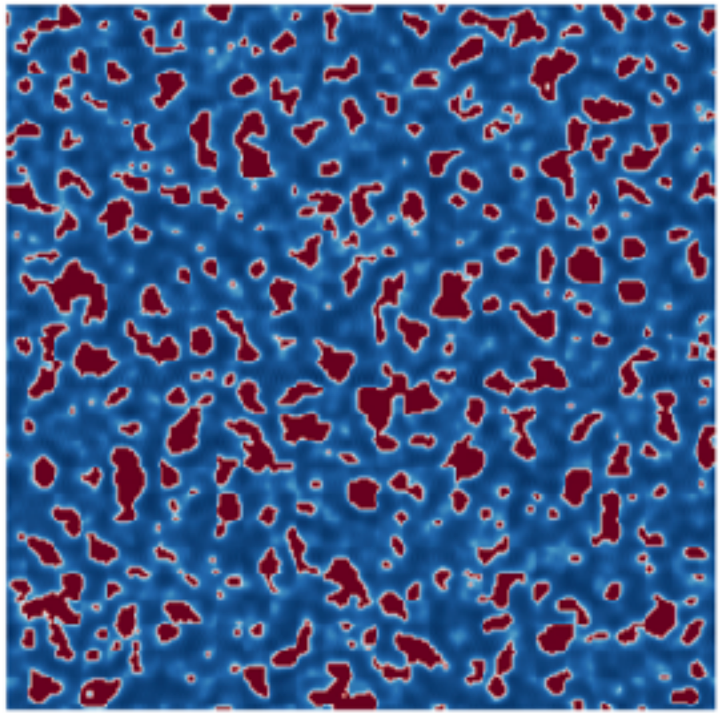
Case 3: inflaton trapping

Case 3:

What happens to the trapped regions at the end of inflation?

Their fate is <u>analogous to false vacuum trapping</u>.





Inflaton trapping and PBHs

Case 3:

What happens to the trapped regions at the end of inflation?

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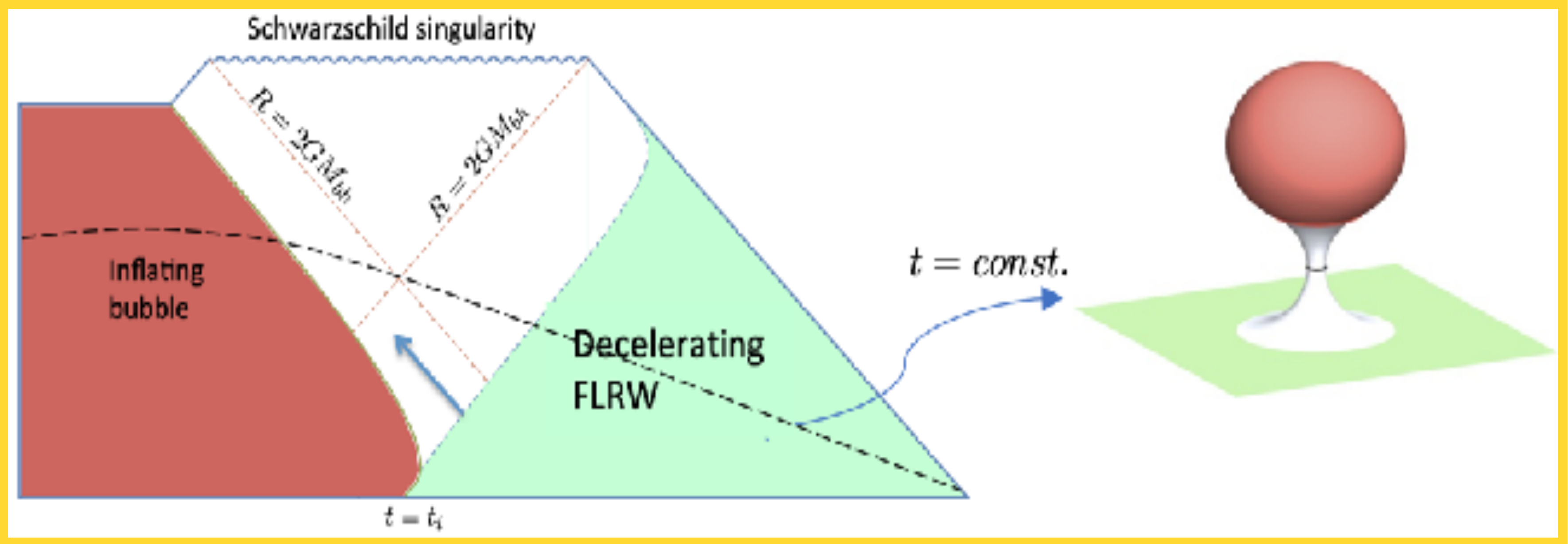


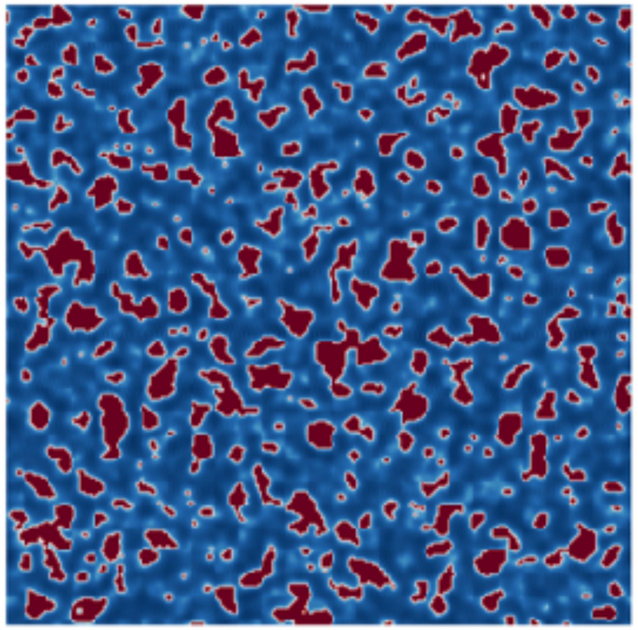


Figure credit:
[J. Garriga, A. Vilenkin, J. Zhang arXiv:1512.01819]

The trapped regions become PBHs at the end of inflation! (in the form of baby universes)

PBH abundance

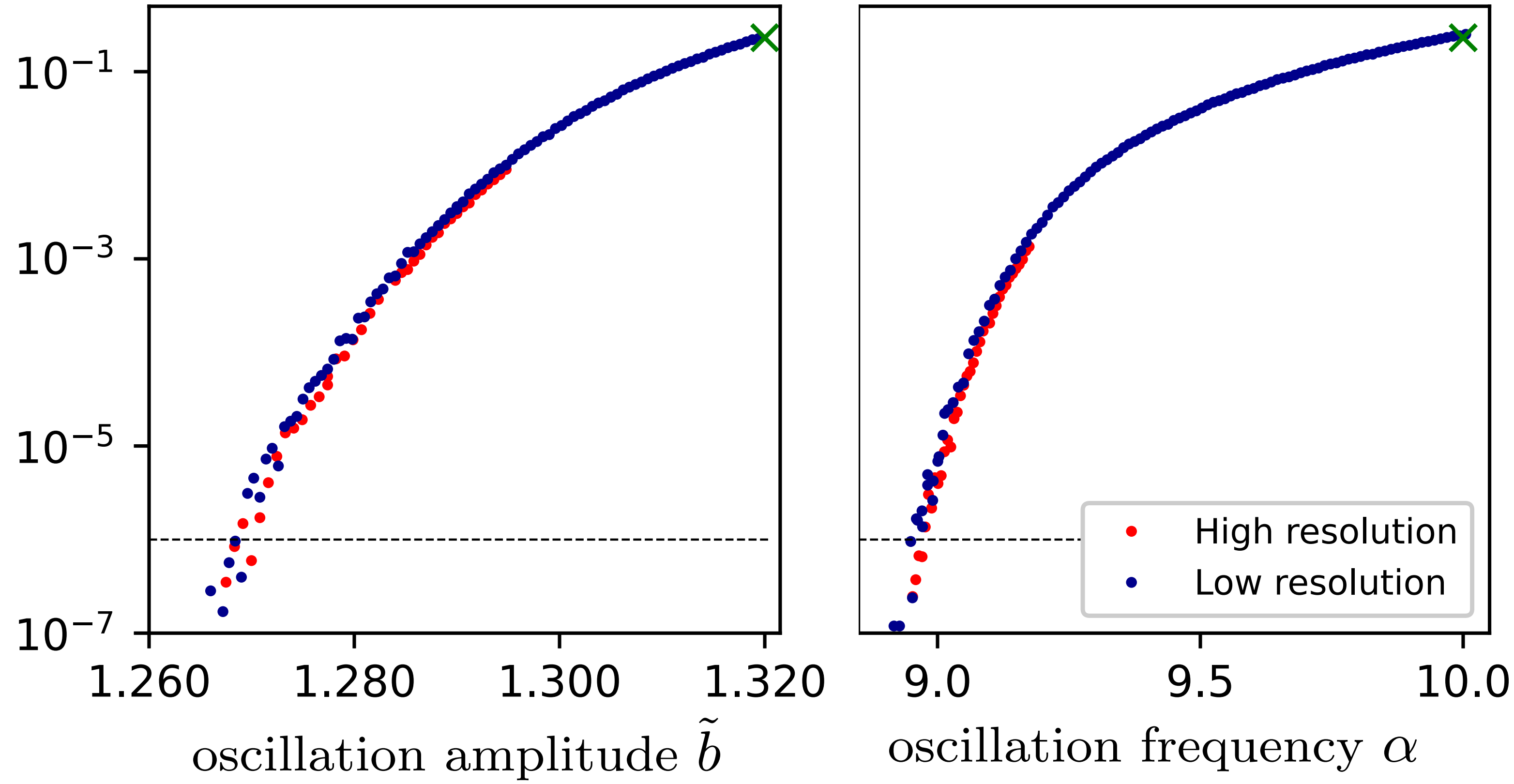
Case 3:

The trapped regions become PBHs at the end of inflation!

How many PBHs?

Mass fraction in PBHs at the time of formation

Trapped volume fraction $\mathcal{F}_t = V_{\rm trapped}/V_{\rm total} \simeq \beta$



500 lattice simulations in this plot

Inflationary Butterfly Effect

Lorenz (1972):

“Can the Flap of a Butterfly’s Wings in Brazil Set Off a Tornado in Texas?” [1]

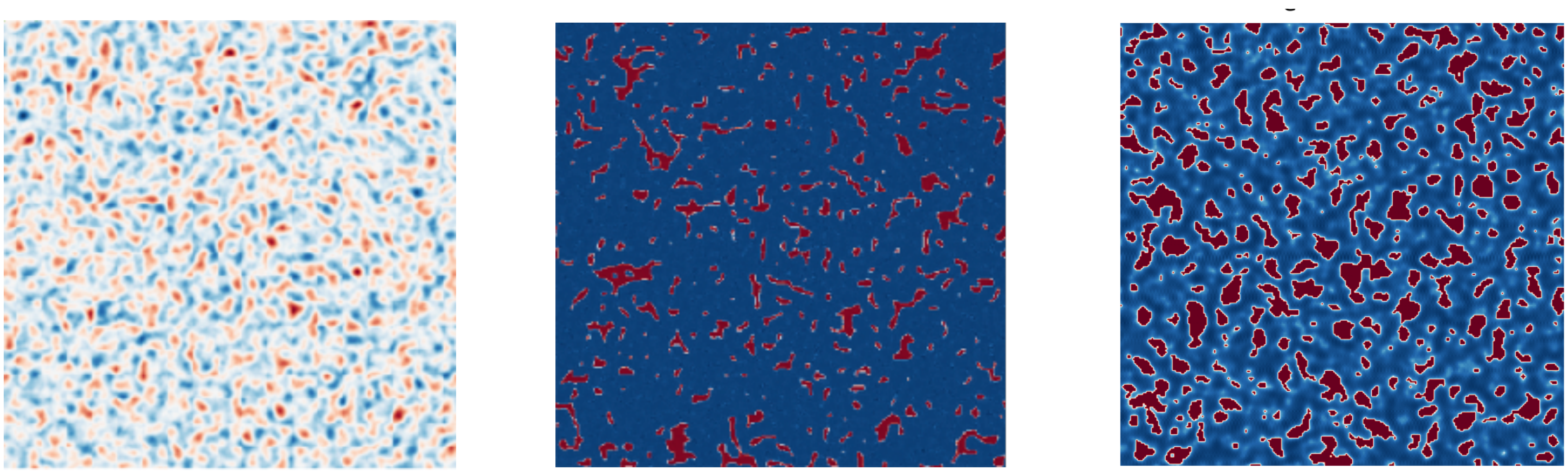
Can tiny, small-scale quantum fluctuations affect the dynamics of the entire Universe?

[1]: E. N. Lorenz, American Association for the Advancement of Science (1972).

Inflationary Butterfly Effect

Main lesson:

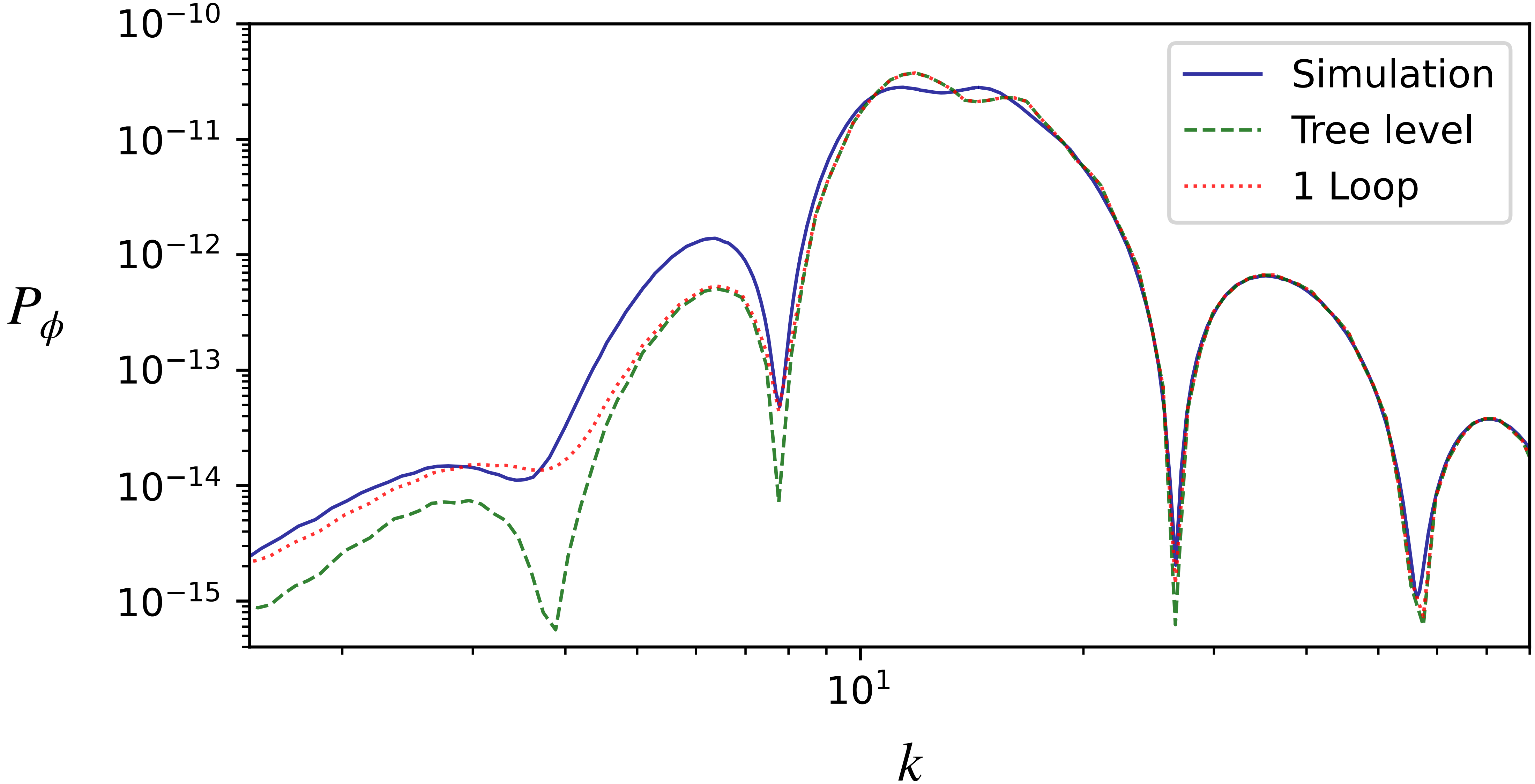
<u>Non-perturbative physics</u> at small scales can have drastic effects on the inflationary dynamics when $\mathscr{P}_\zeta \sim 10^{-2}$



Loop effects

In the perturbative setup (case 1),

first **quantitative comparison** between full nonlinear, tree-level and 1-loop



AC, S. Renaux-Petel, K. Inomata [2403.12811]

Loop effects

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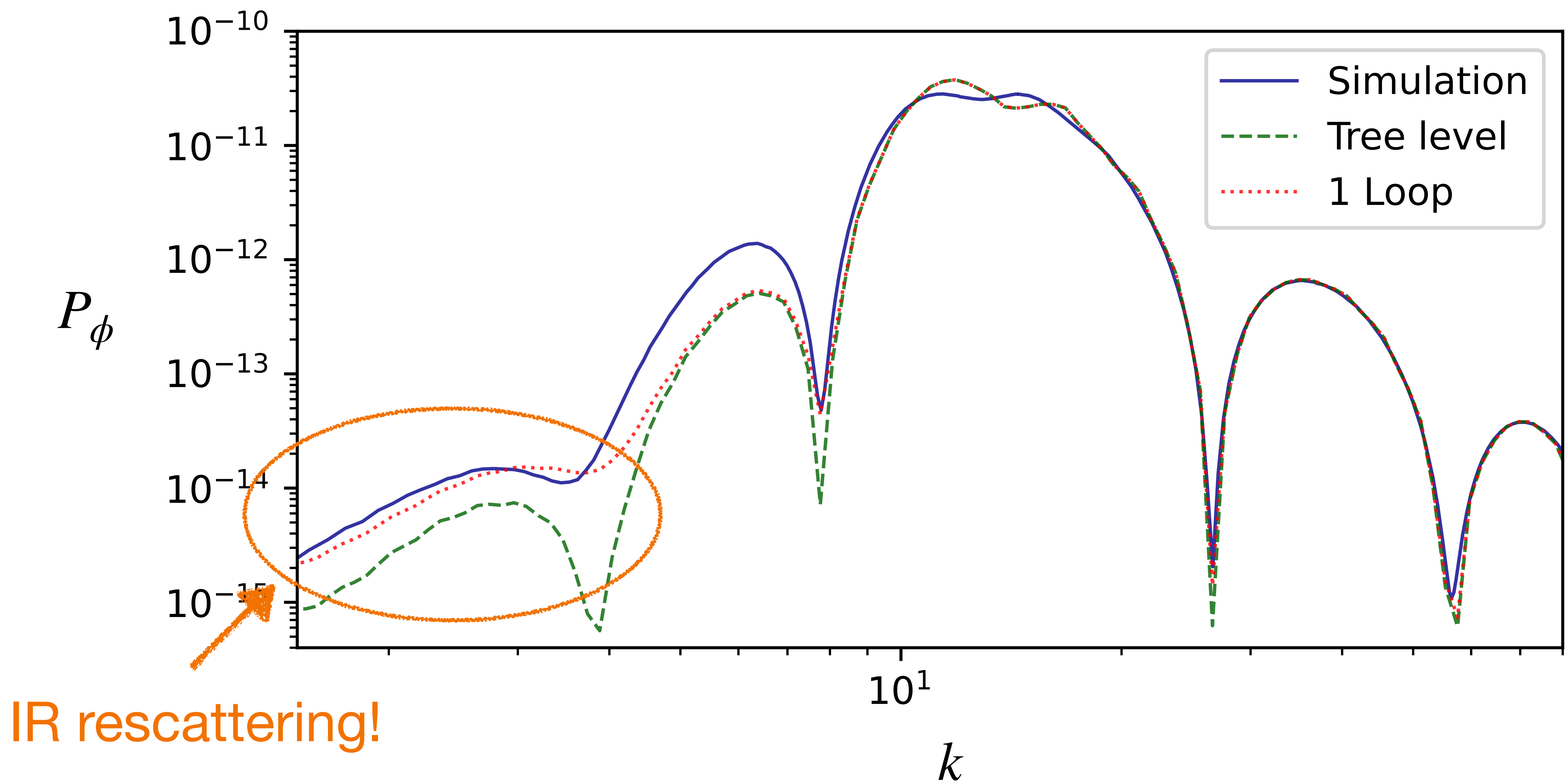

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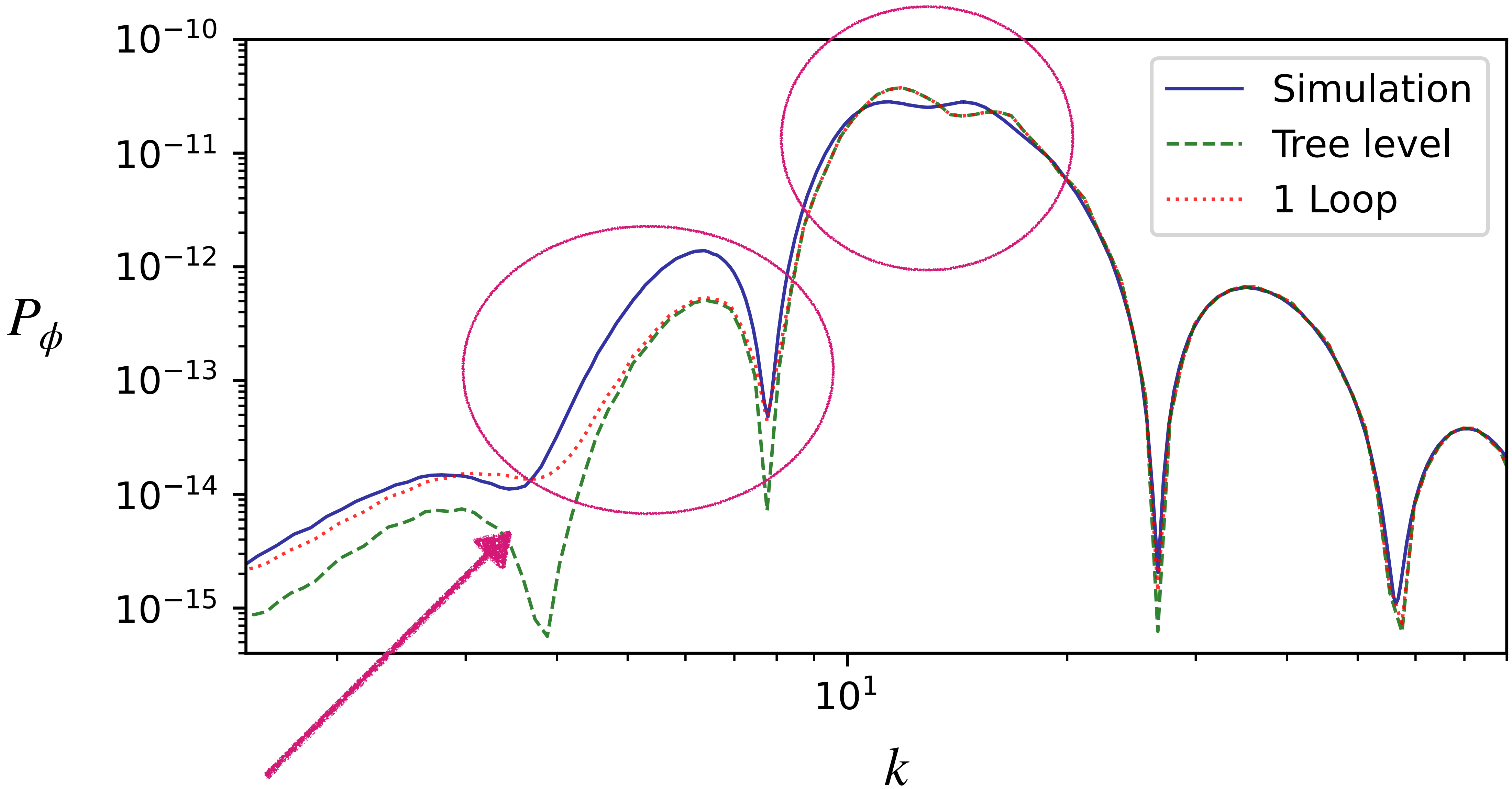


Beyond 1-loop?? Other corrections?

Roadmap

2) Small-scale physics: Inflationary butterfly effect

2.1) Oscillatory potential

2.2) Ultra-slow-roll inflation

AC, K. Inomata, S. Renaux-Petel 2403.12811

AC, G. Franciolini, S. Renaux-Petel 2410.23942 2506.11795

AC, G. Franciolini, S. Renaux-Petel, [2410.23942]
[2506.11795]

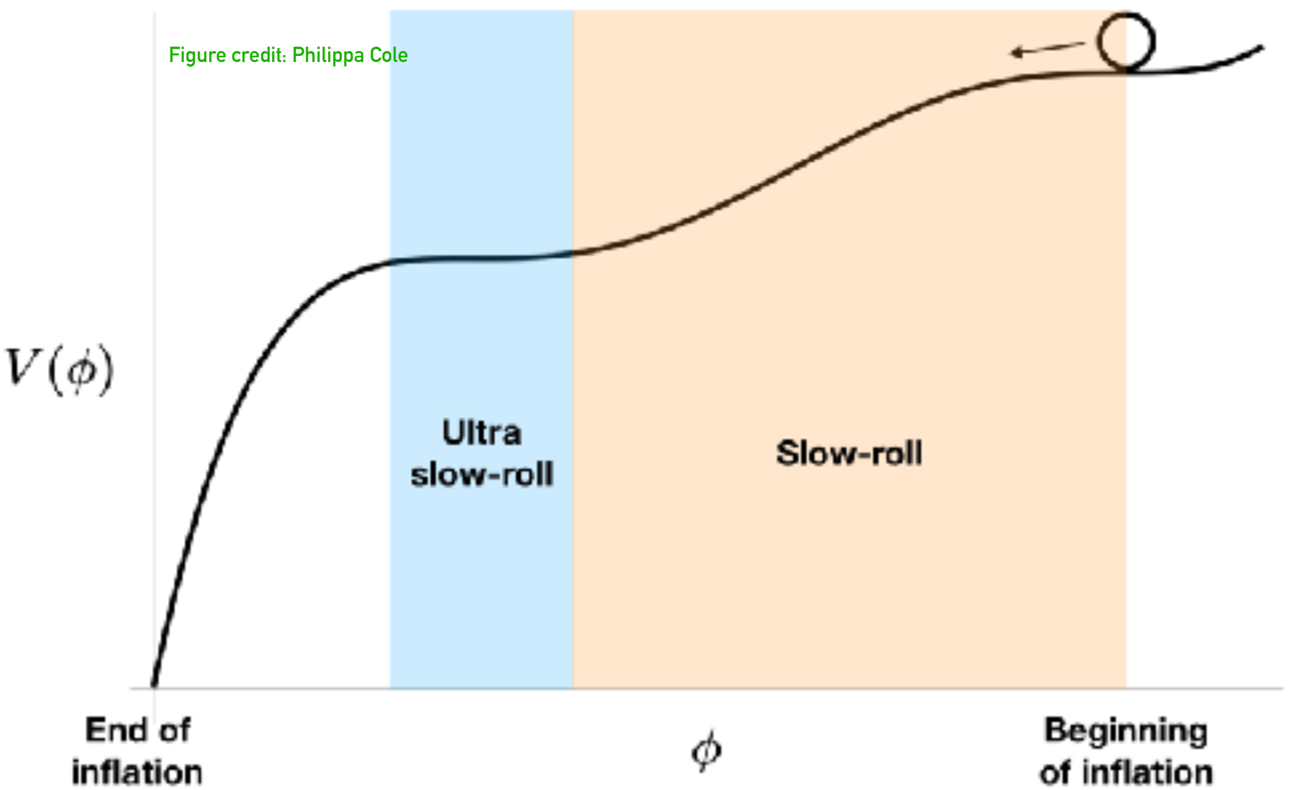
Ultra-Slow-Roll inflation

A well-known mechanism to enhance density fluctuations is an inflection point

Fluctuations amplified via a
deceleration of the inflaton

$$\epsilon_H = -\frac{\dot{H}}{H^2} \ll 1$$

$$|\eta_H| = \frac{\dot{\epsilon}_H}{H\epsilon_H} \sim 1$$



So-called "ultra slow-roll" phase

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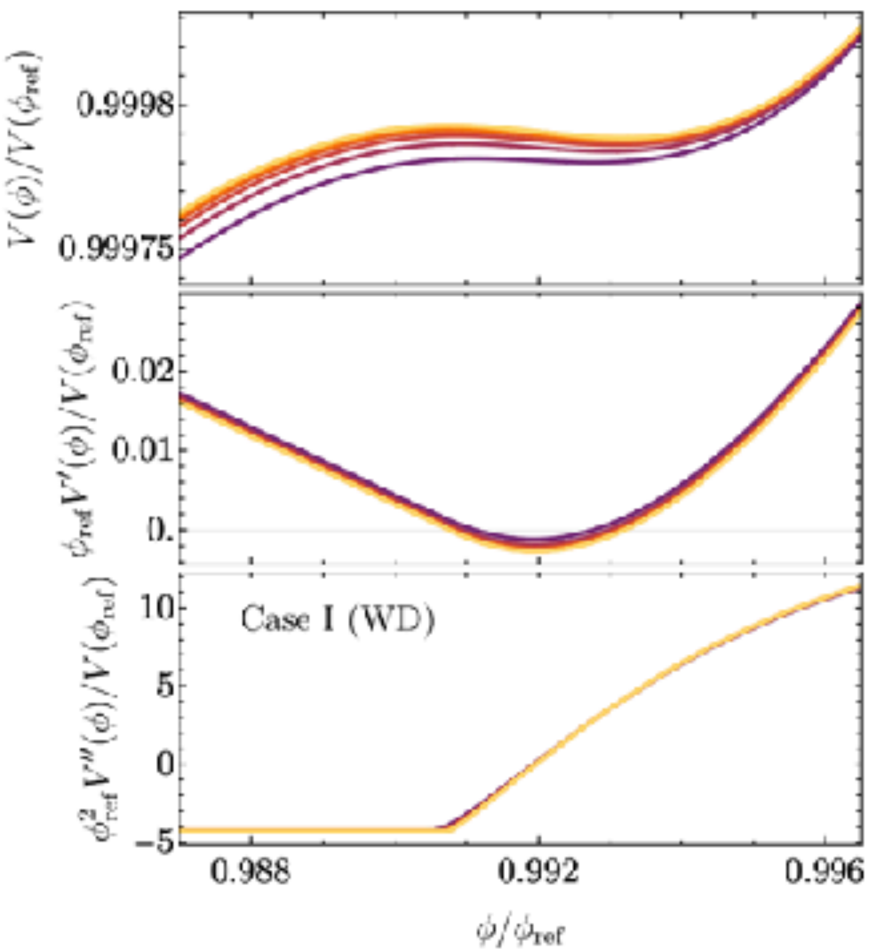
Ultra-Slow-Roll inflation

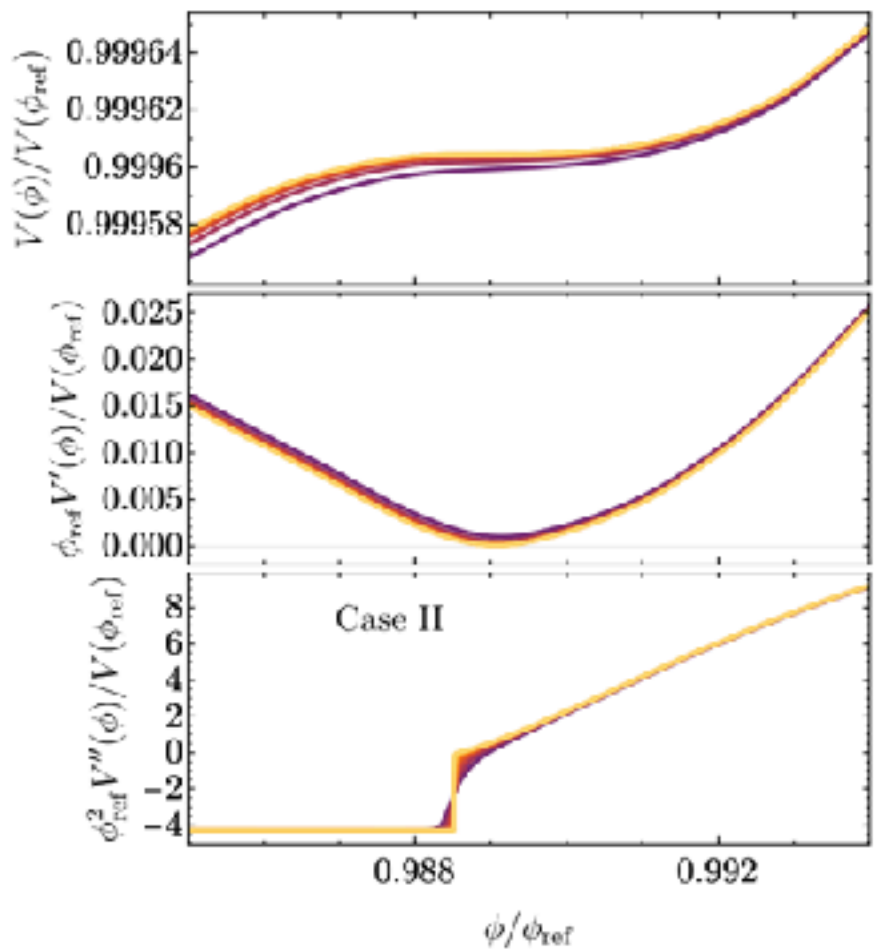
A systematic study of USR potentials:

Case 1



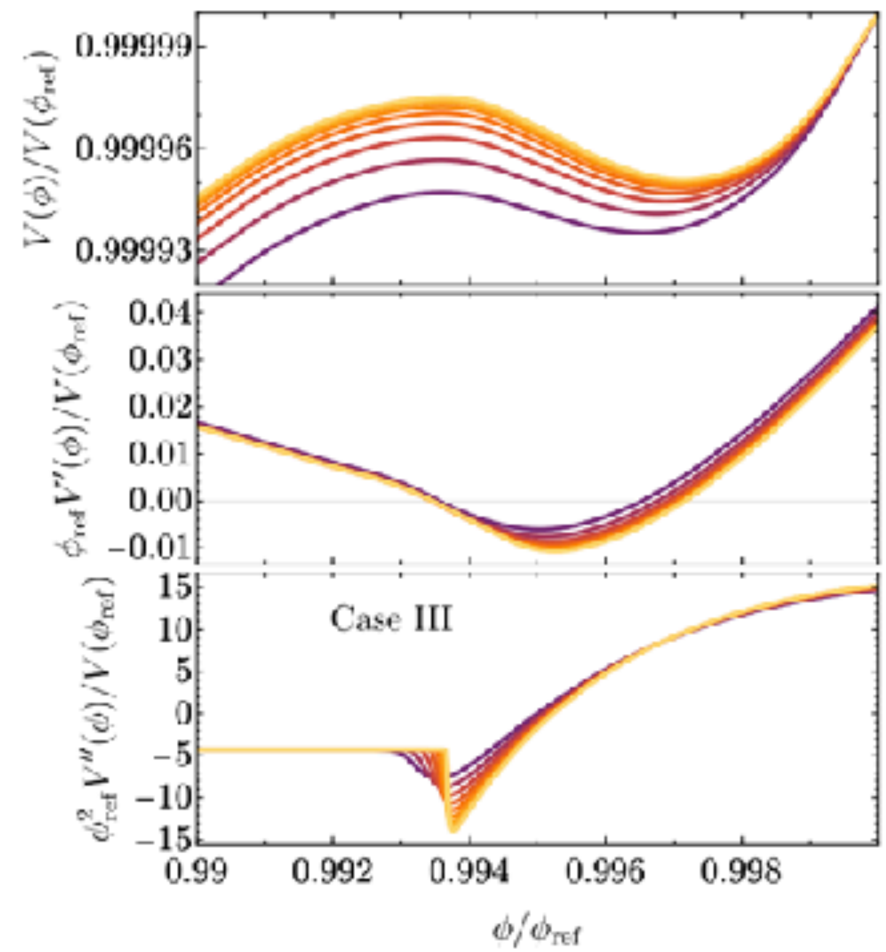
$$\frac{\partial^3 V(\phi)}{\partial \phi^3} \sim 0$$

Case 2



$$\frac{\partial^3 V(\phi)}{\partial \phi^3} > 0$$

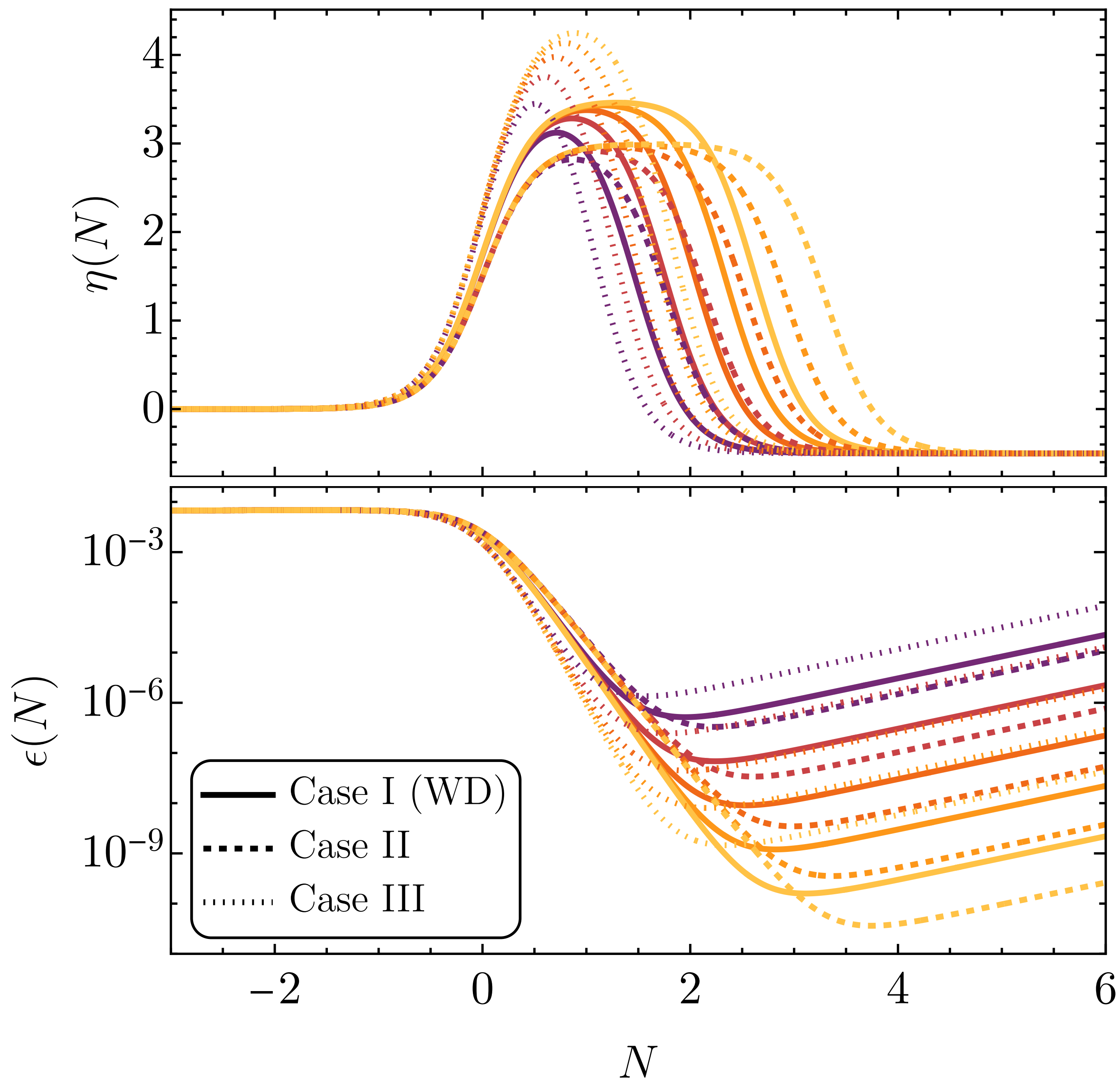
Case 3



$$\frac{\partial^3 V(\phi)}{\partial \phi^3} < 0$$

Ultra-Slow-Roll inflation

A systematic study of USR potentials:



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Ultra-Slow-Roll inflation

$\frac{\partial^3 V(\phi)}{\partial \phi^3}$ is the leading self-interaction of the inflaton:

$$V(\bar{\phi} + \delta\phi) = \sum_n \frac{\delta\phi^n}{n!} \frac{\partial^n V(\phi)}{\partial \phi^n}\Big|_{\bar{\phi}}$$

Case 1

$$\frac{\partial^3 V(\phi)}{\partial \phi^3} \sim 0$$

↓

Free theory
Aka "Wands duality"

Case 2

$$\frac{\partial^3 V(\phi)}{\partial \phi^3} > 0$$

↓

Repulsive
self-interaction

Case 3

$$\frac{\partial^3 V(\phi)}{\partial \phi^3} < 0$$

↓

Attractive
self-interaction

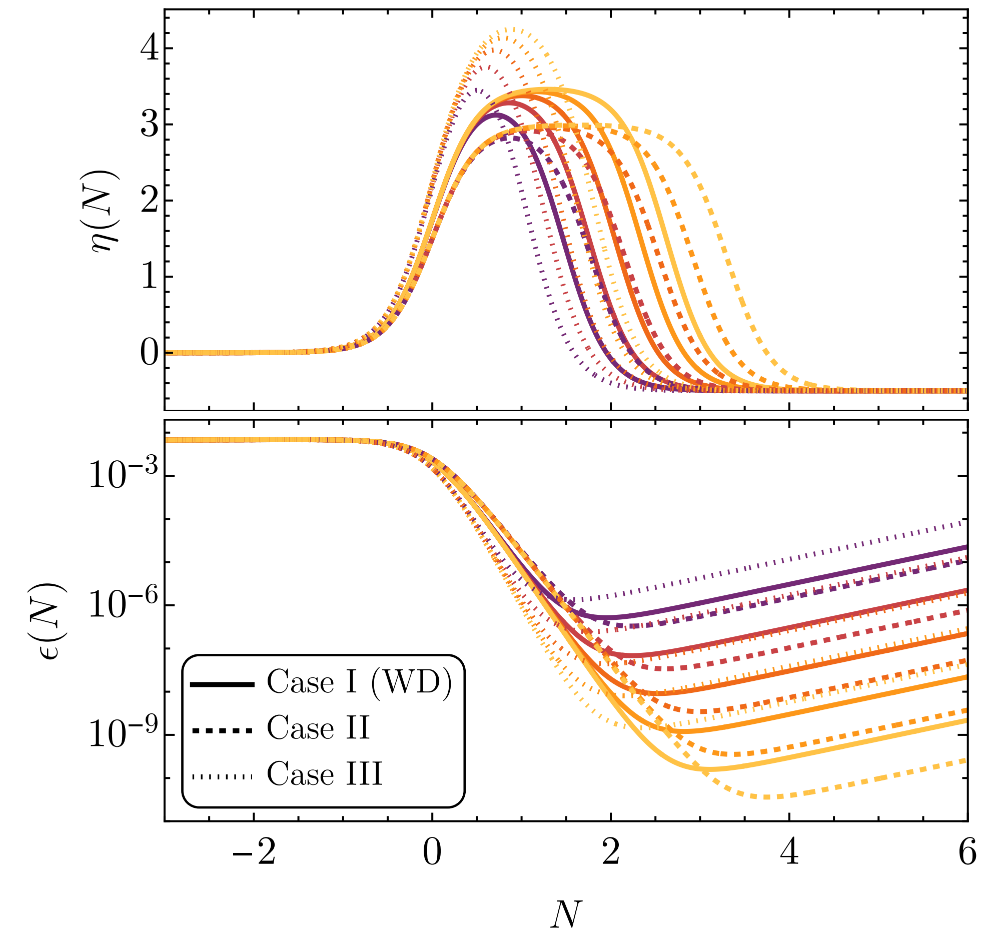
Ultra-Slow-Roll inflation

Wands duality: [D. Wands (1998)]

Evolution of scalar field perturbation is invariant (dual) under the transformation of the background:

$$\eta \to 3 - \eta$$

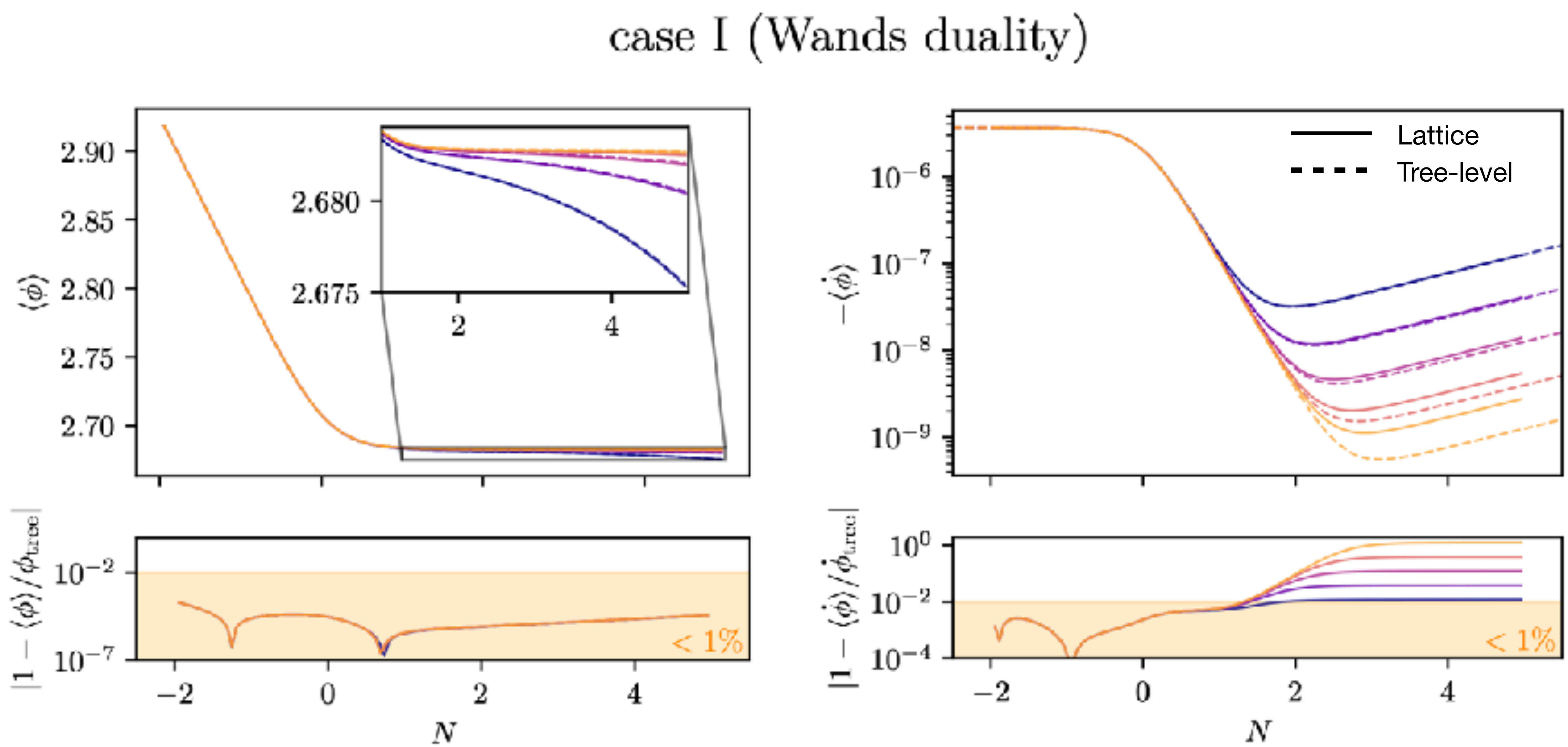
Our potential in case 1 is constructed so that $\eta_{USR} = 3 - \eta_{SR,2}$, so the theory is approximately free



Ultra-Slow-Roll inflation

Result #1:

We find **backreaction**, i.e. an effect of fluctuations on the background evolution



This is a new effect!

Ultra-Slow-Roll inflation

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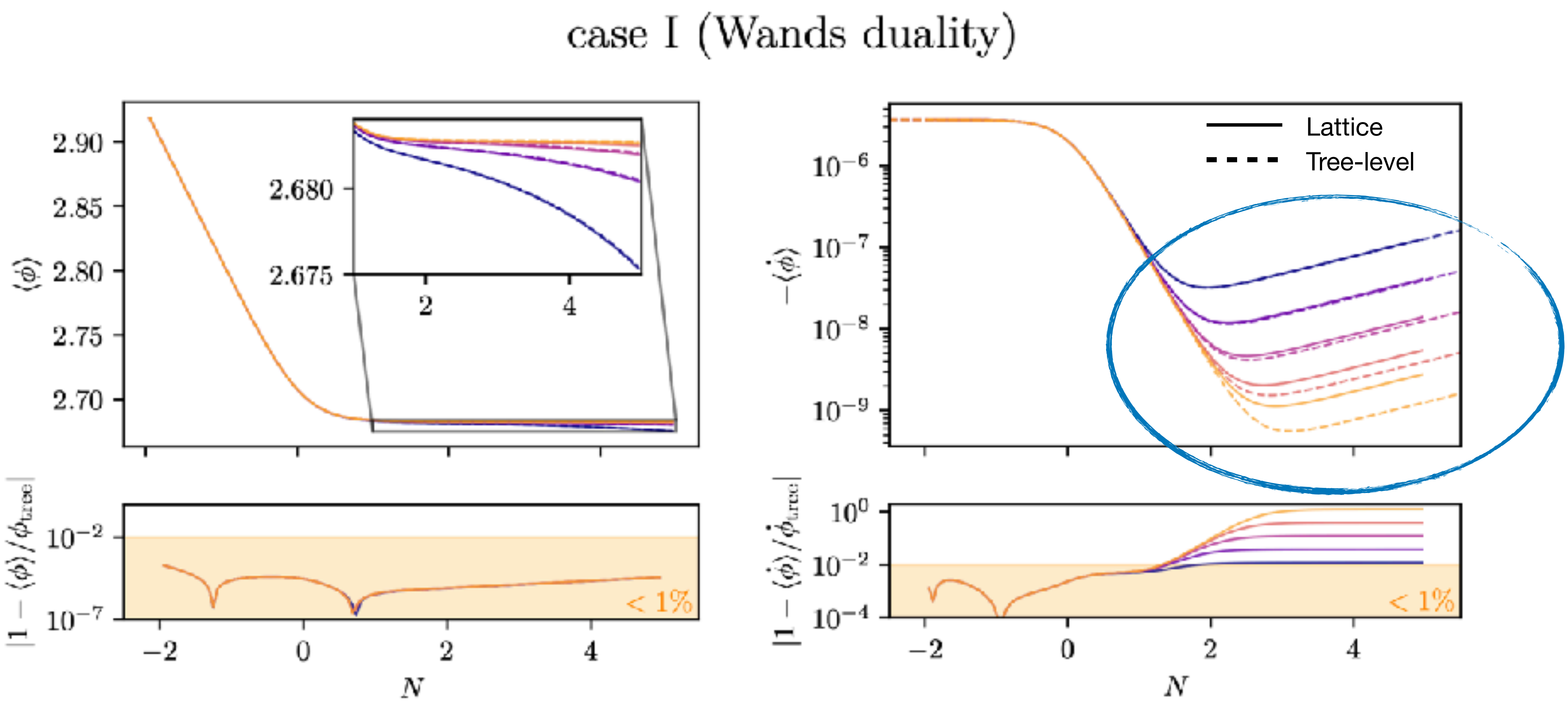

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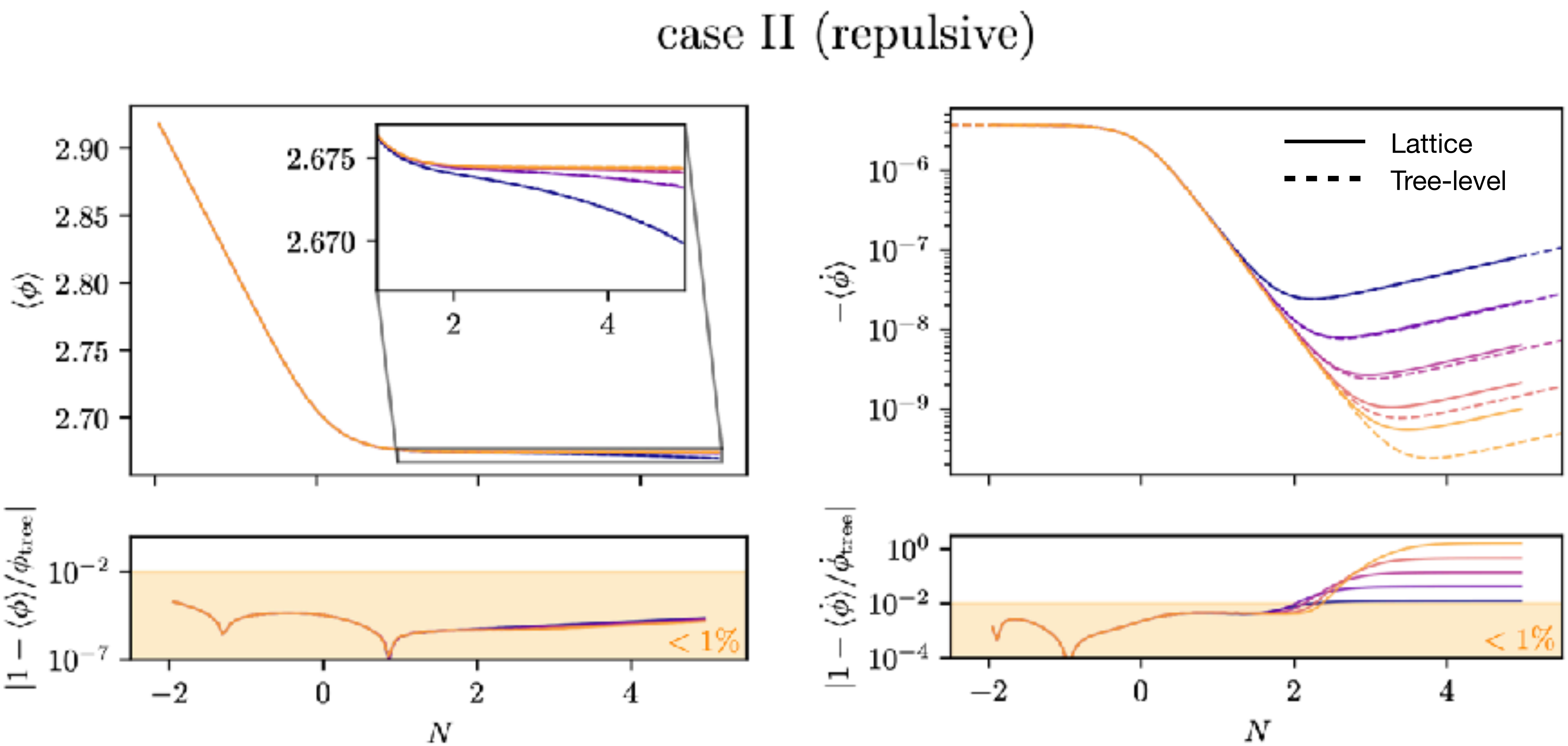


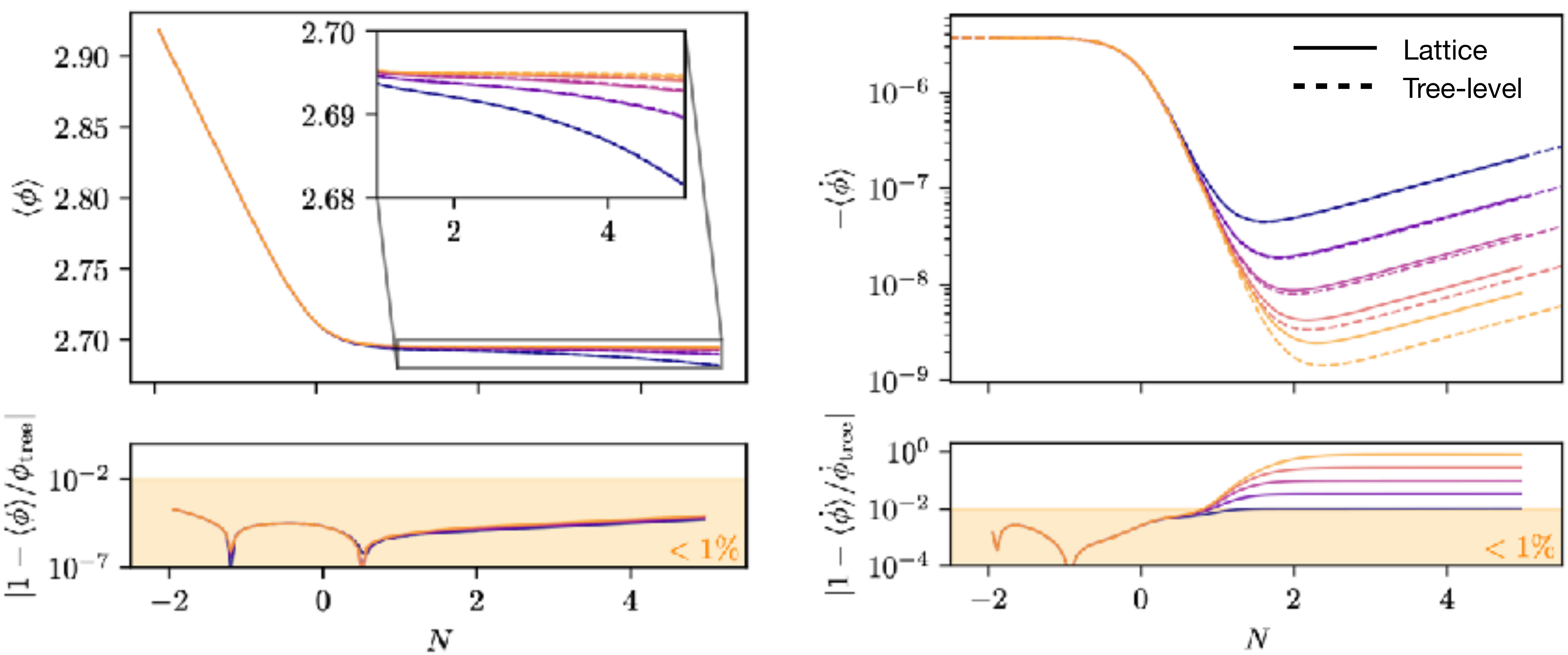
This is a new effect!

Ultra-Slow-Roll inflation

Result #1:

We find **backreaction**, i.e. an effect of fluctuations on the background evolution

case III (attractive)



This is a new effect!

Ultra-Slow-Roll inflation

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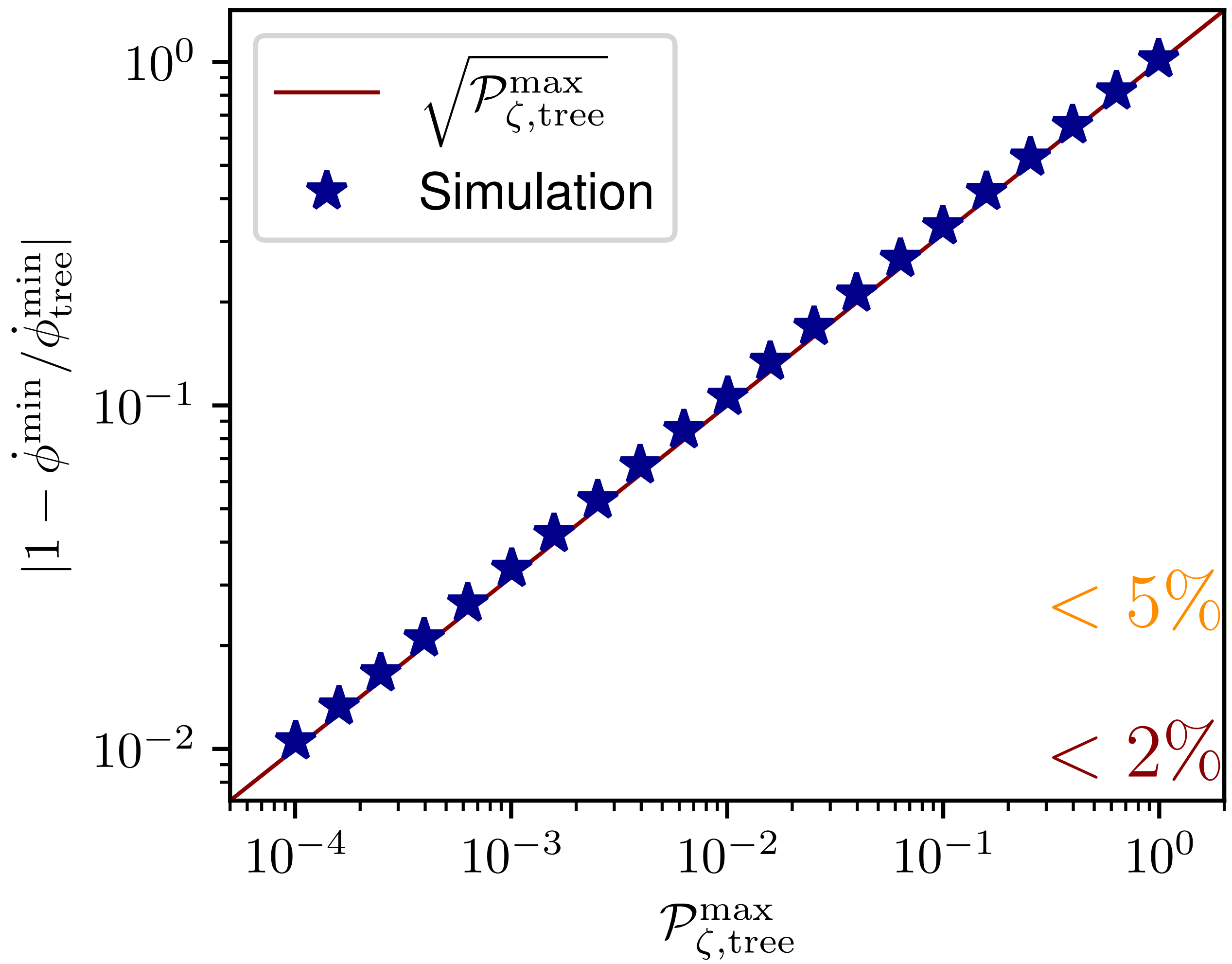
Result #1:

We find **backreaction**, i.e. an effect of fluctuations on the background evolution

Backreaction follows a simple fitting formula: $\dot{\phi} = \dot{\phi}_{\rm tree}\left(1 + \sqrt{\mathcal{P}^{\rm max}_{\zeta,\rm tree}}\right)$



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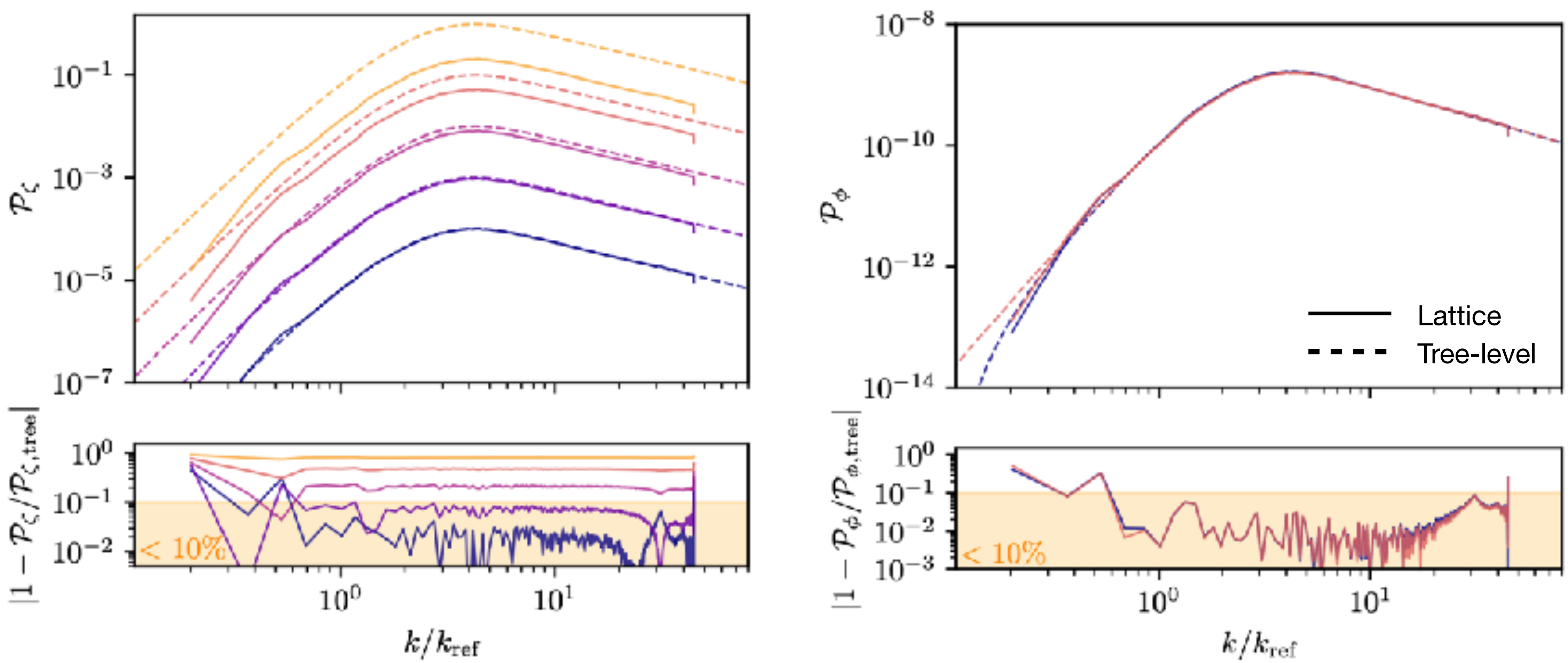
Ultra-Slow-Roll inflation

Result #2:

How **nonlinearity** affects inflaton fluctuations

Attention: $\zeta \equiv -H\frac{\delta\phi}{\dot{\phi}} = \zeta_{\text{lin}}$

case I (Wands duality)



Ultra-Slow-Roll inflation

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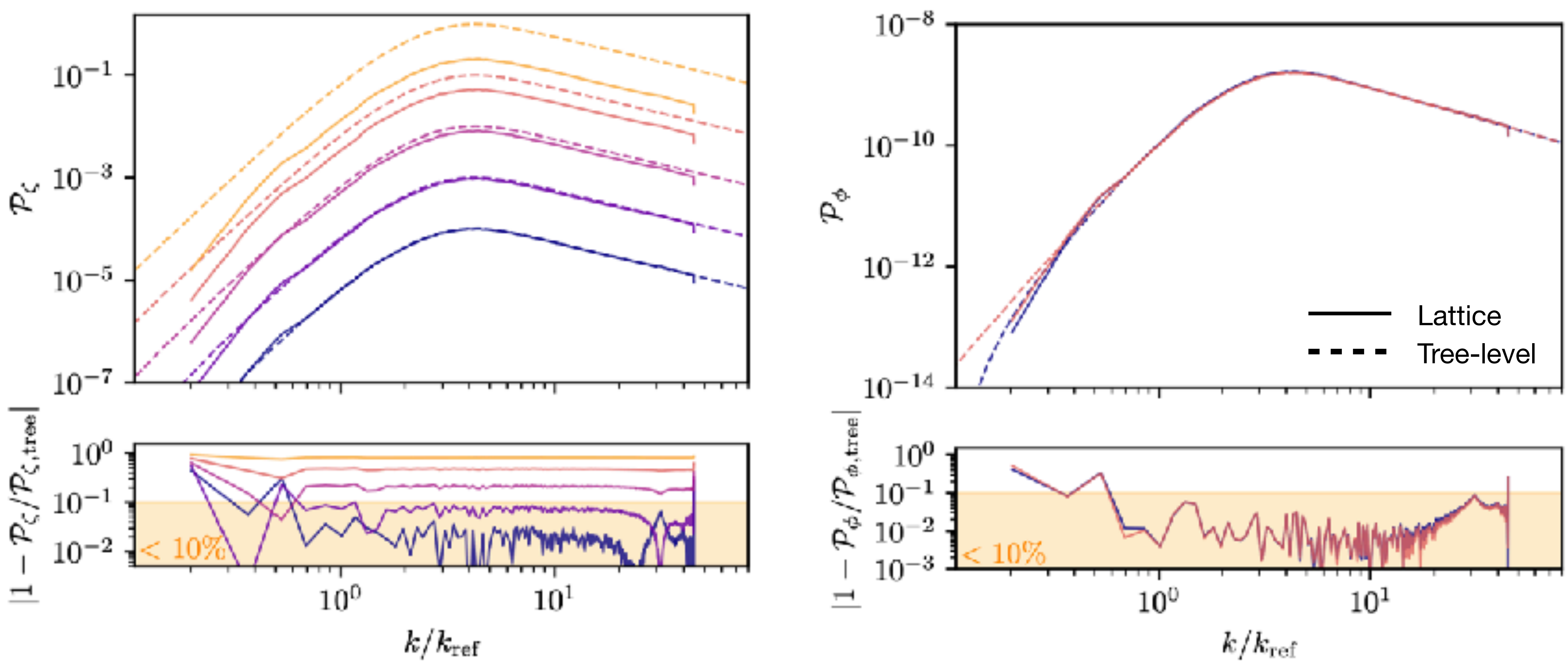

Ultra-Slow-Roll inflation

Result #2:

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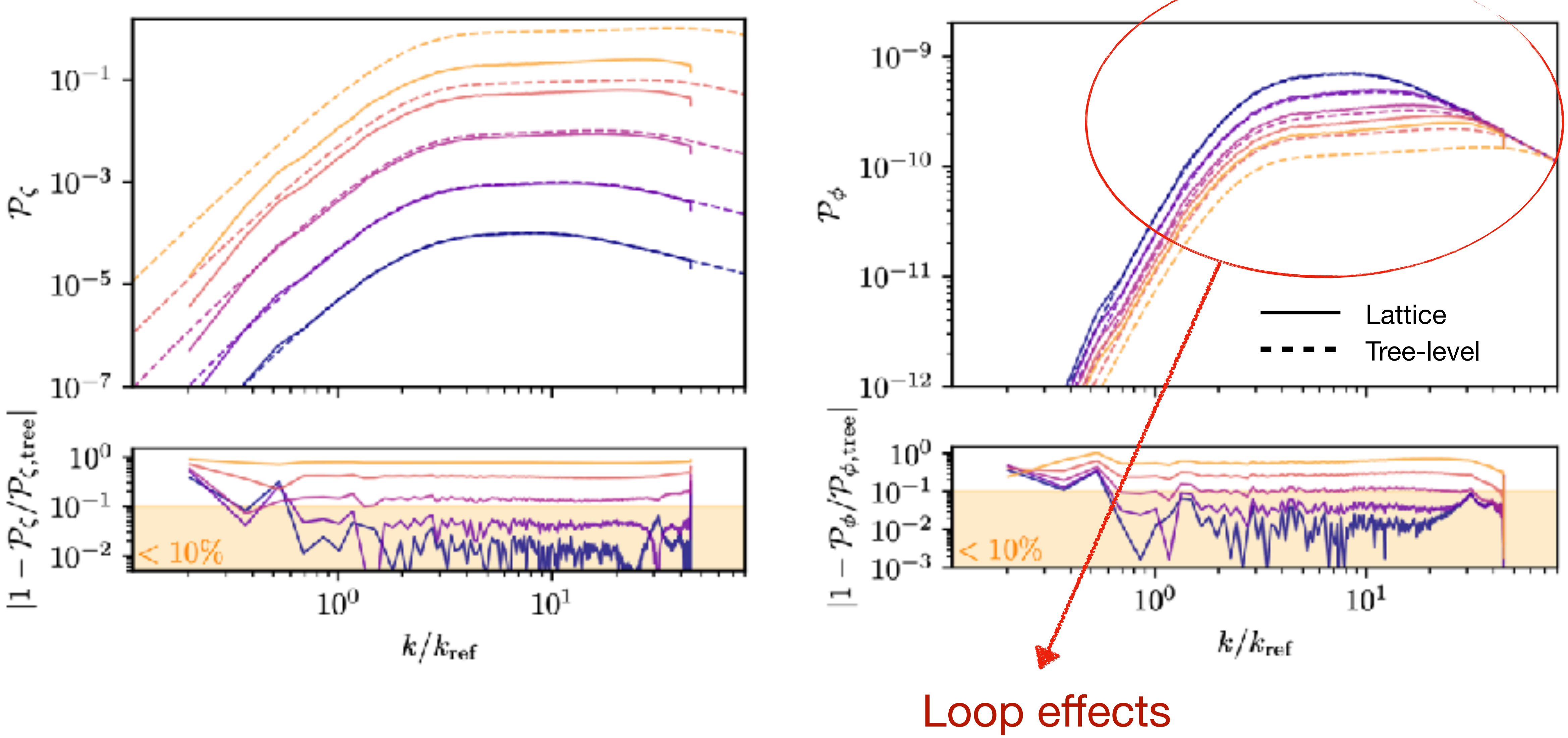
case II (repulsive)



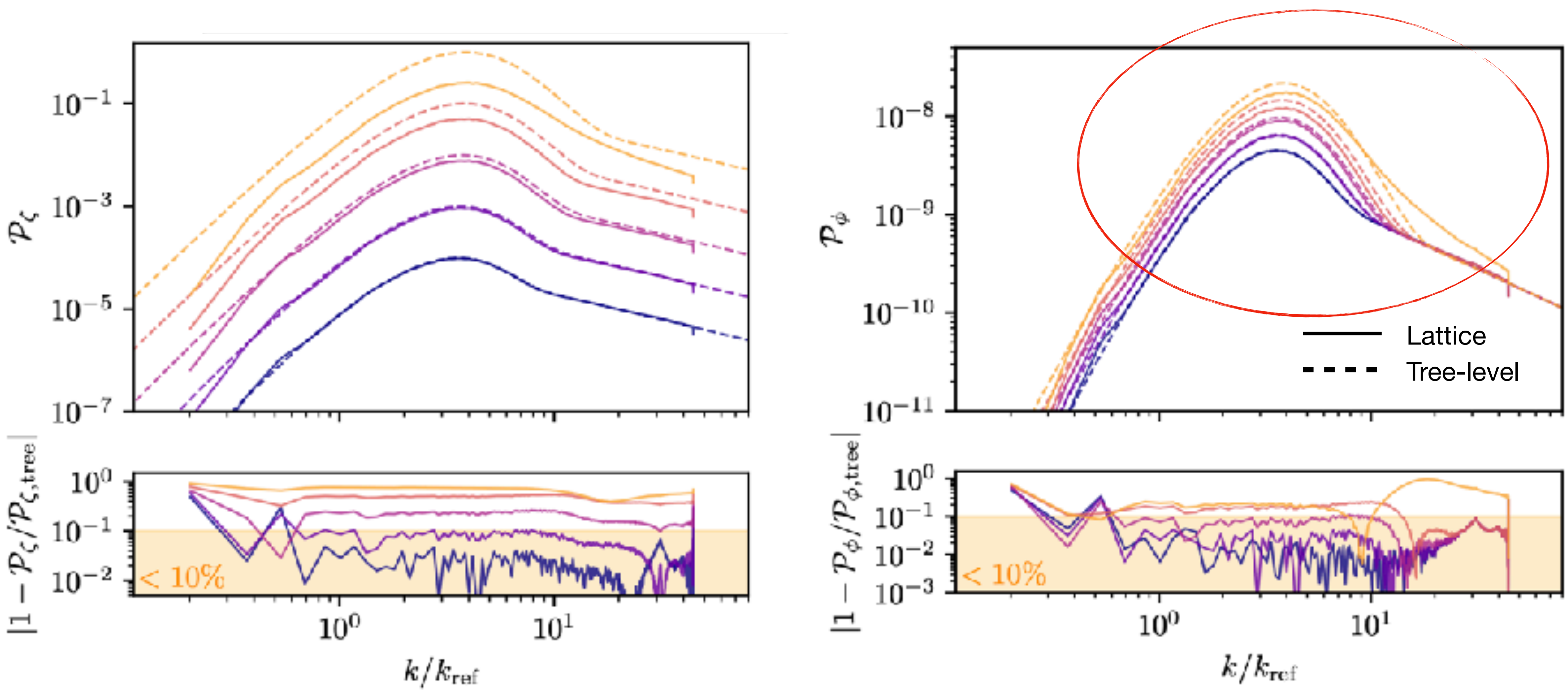
Ultra-Slow-Roll inflation

Result #2:

Attention: $\zeta \equiv -H\frac{\delta\phi}{\dot{\phi}} = \zeta_{\text{lin}}$

How **nonlinearity** affects inflaton fluctuations

case III (attractive)



Ultra-Slow-Roll inflation

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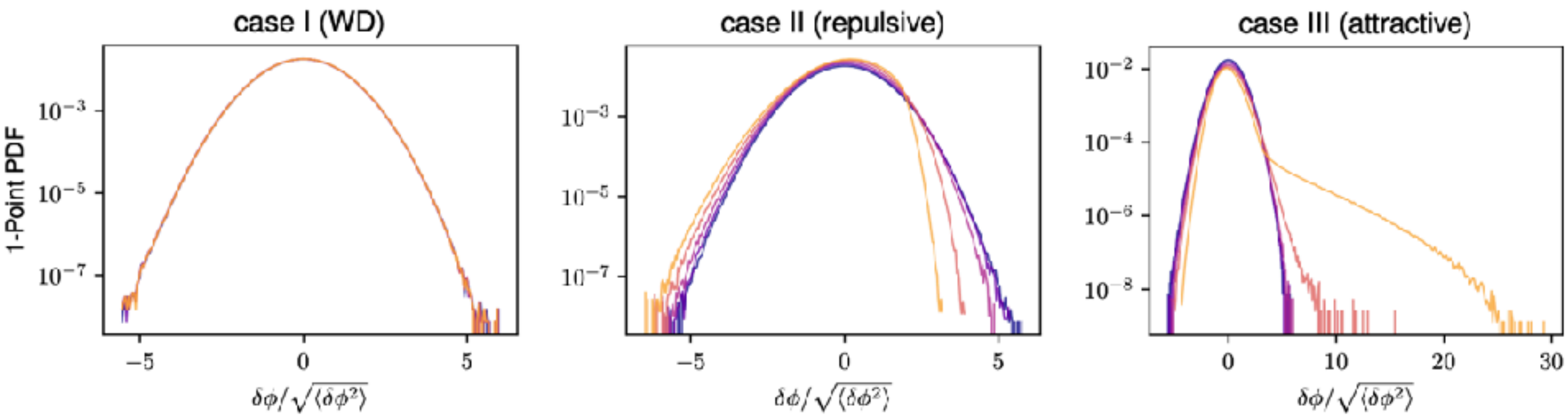
Result #2:

How **nonlinearity** affects inflaton fluctuations



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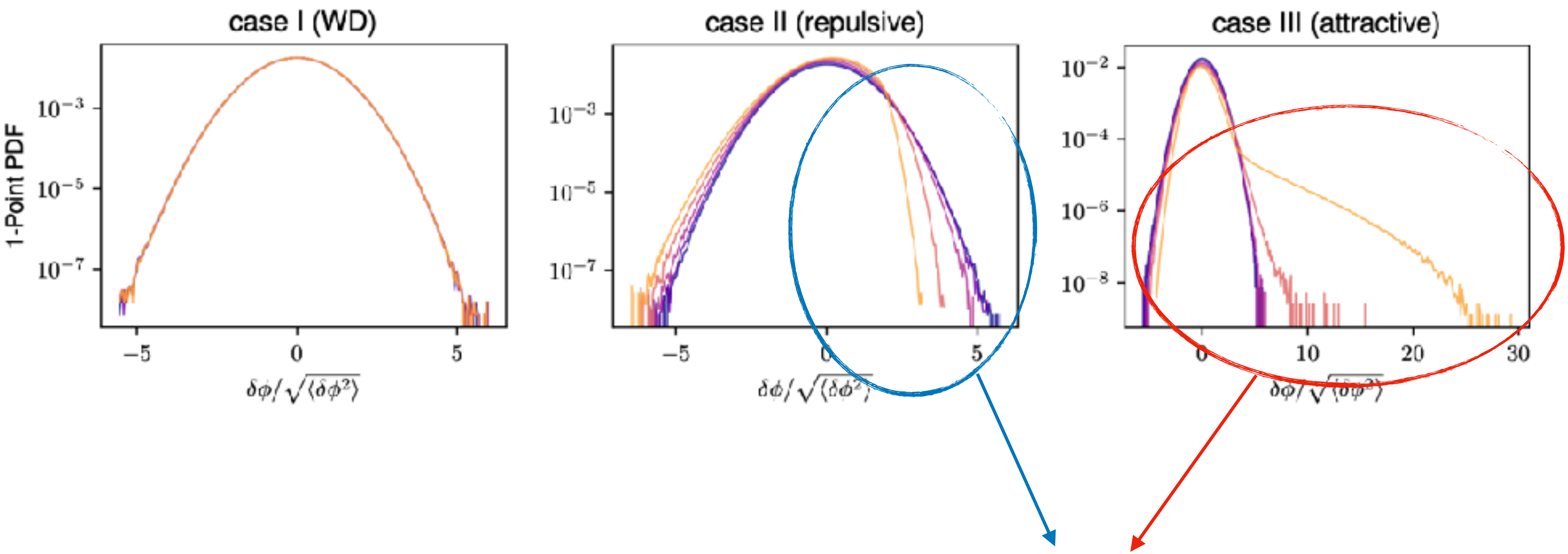

repulsive/**attractive** nature

Ultra-Slow-Roll inflation

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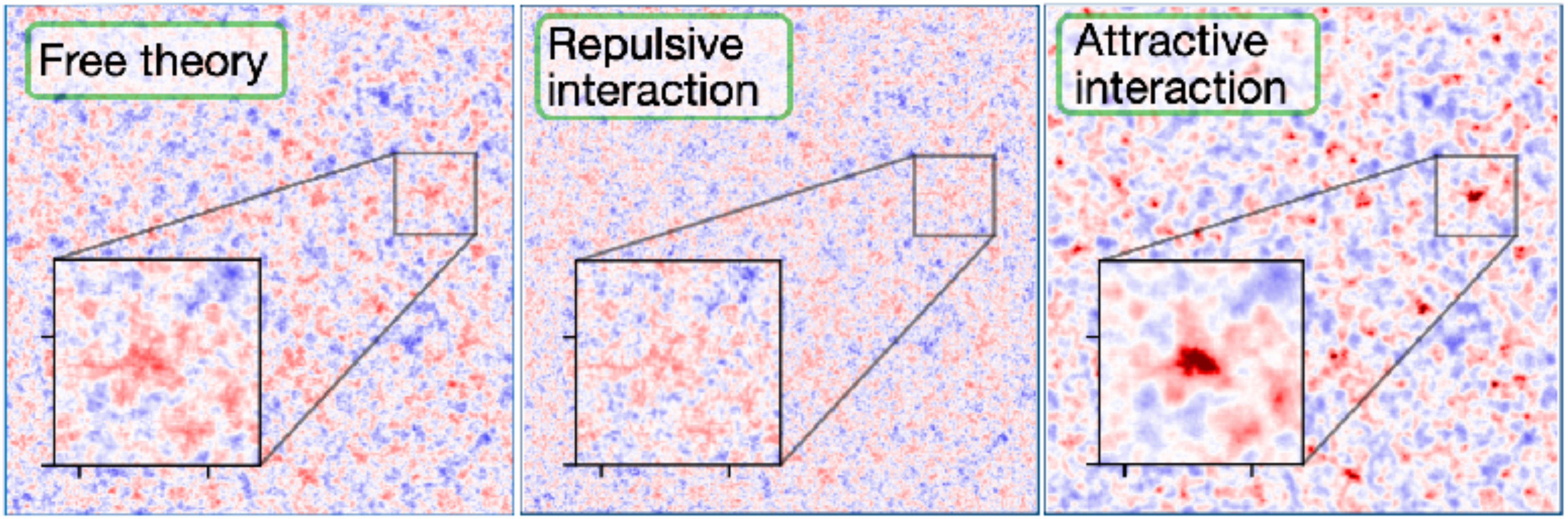
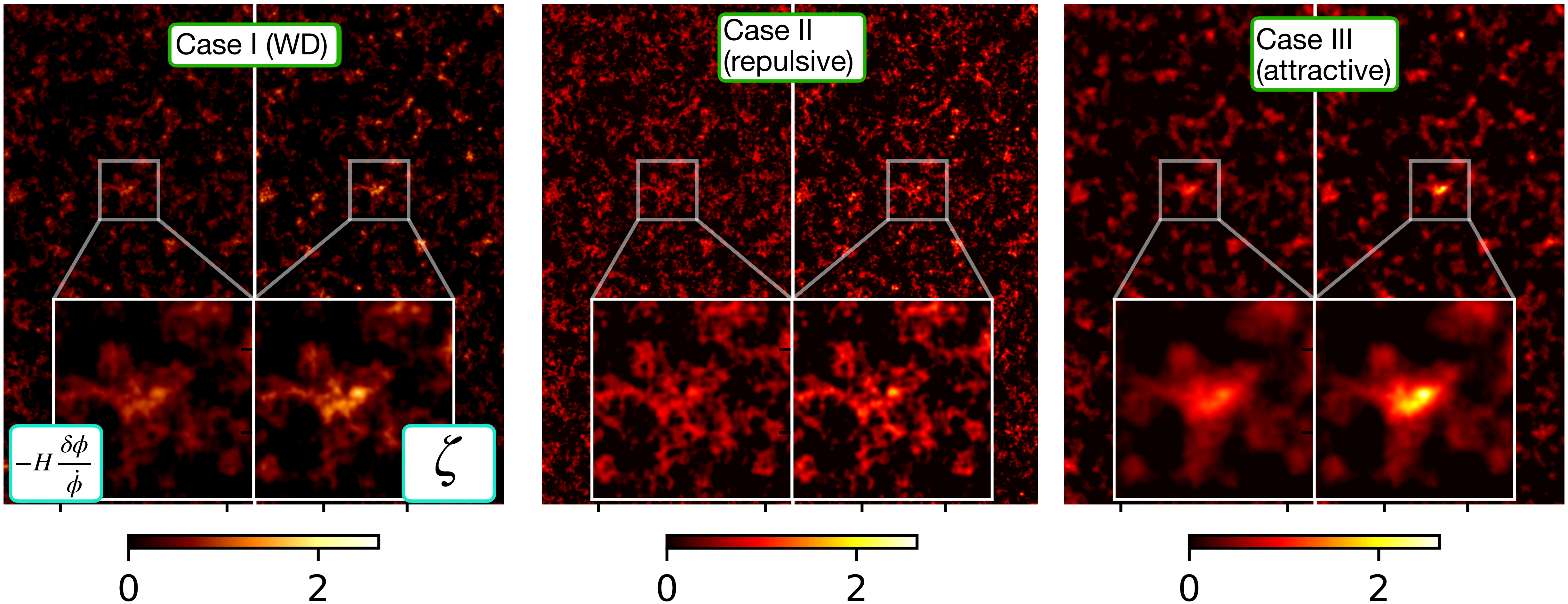
Self-interactions matter:



Ultra-Slow-Roll inflation

So far, we only looked at $\zeta_{\rm lin} = -H\dfrac{\delta\phi}{\dot{\phi}}$

We calculate ζ in a fully nonlinear way using a δN technique applied to simulation data



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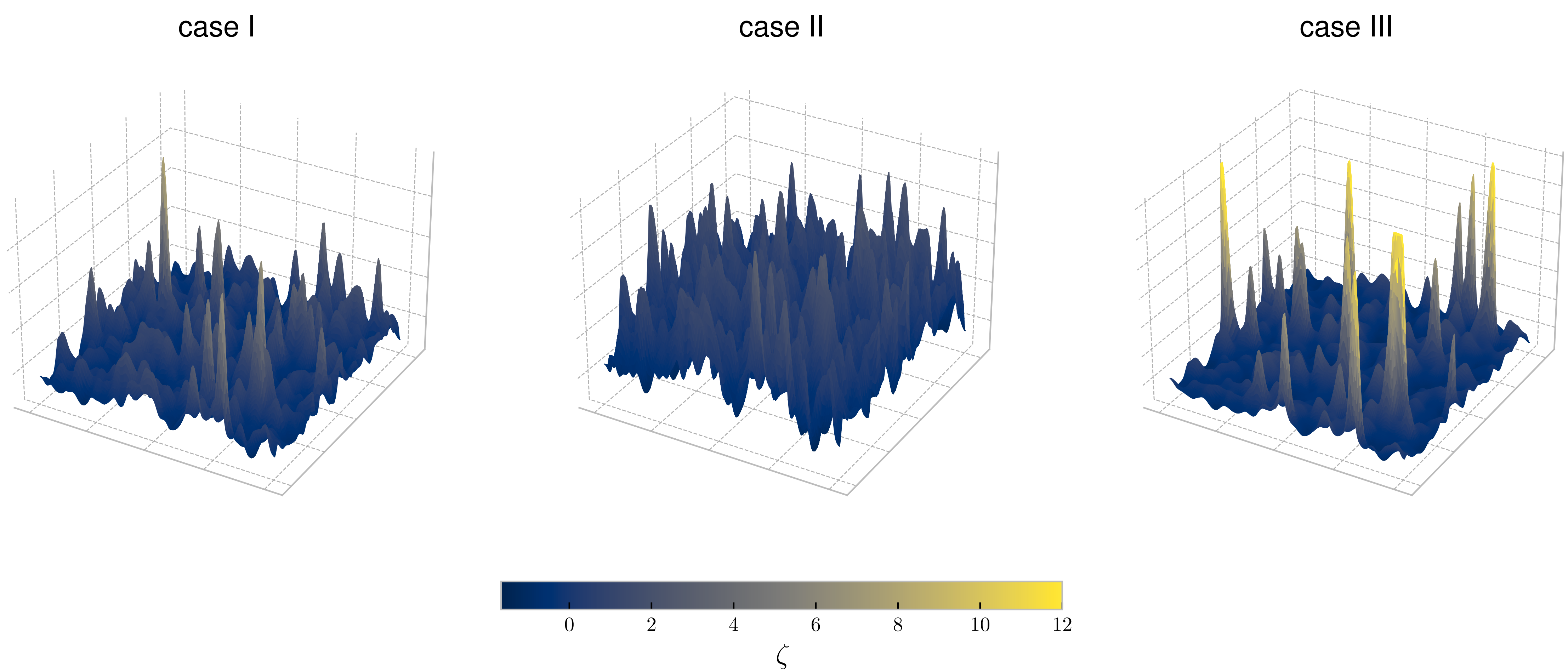

Ultra-Slow-Roll inflation

So far, we only looked at $\zeta_{\rm lin} = -H\dfrac{\delta\phi}{\dot\phi}$

We calculate ζ in a fully nonlinear way using a δN technique applied to simulation data

In all our models,
η_{III} = constant.

→

$$\zeta(\vec{x}) = \frac{1}{\eta}\log(1 + \eta\,\zeta_{\rm lin}(\vec{x}))$$

Ultra-Slow-Roll inflation

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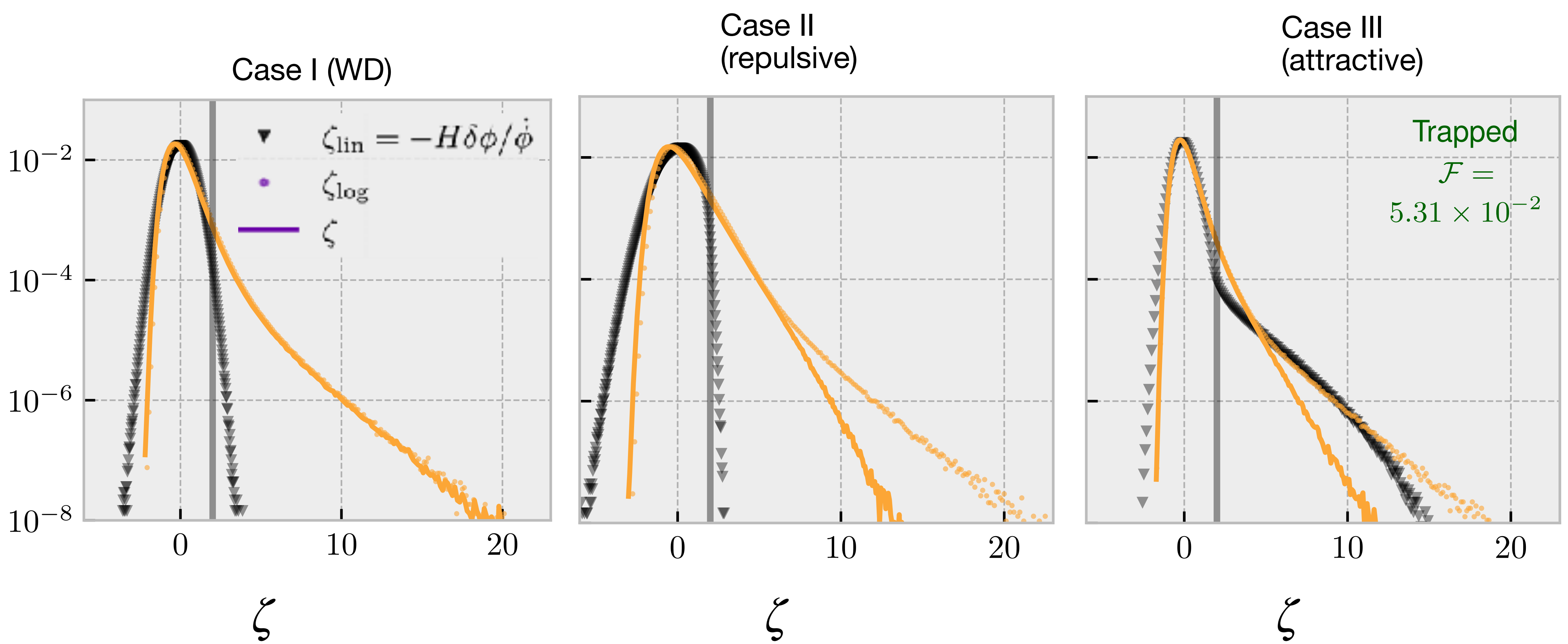
So far, we only looked at $\zeta_{\rm lin} = -H\dfrac{\delta\phi}{\dot{\phi}}$

It is interesting to see how the logarithmic relation breaks for very large fluctuations:



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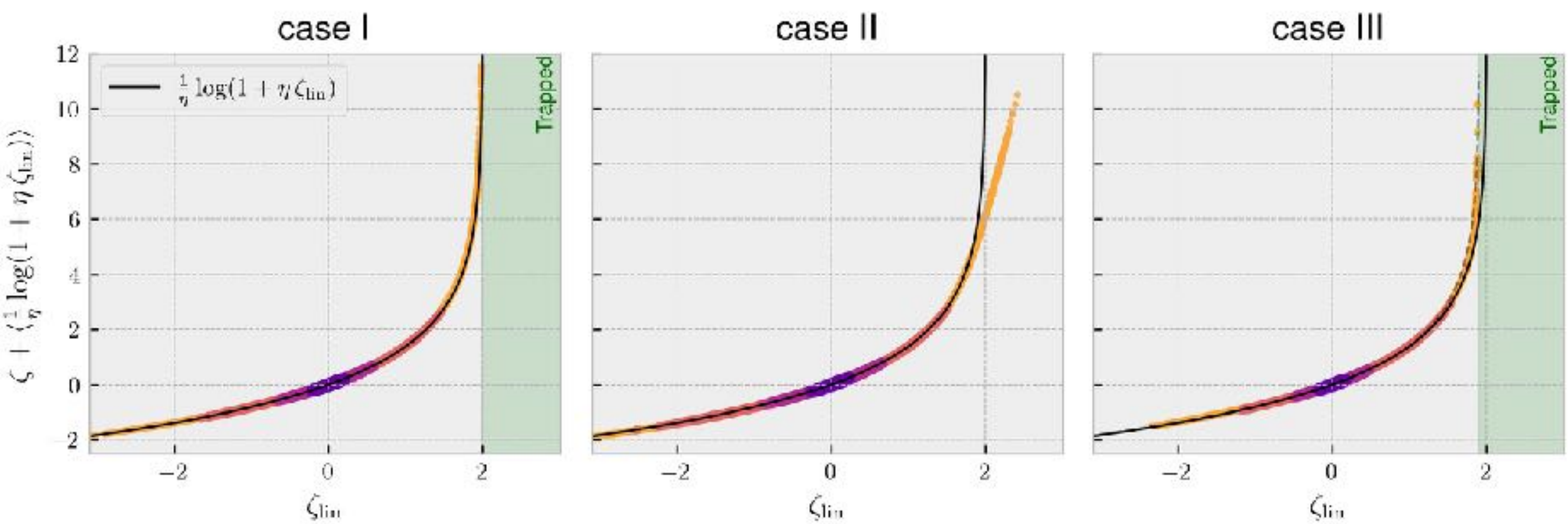

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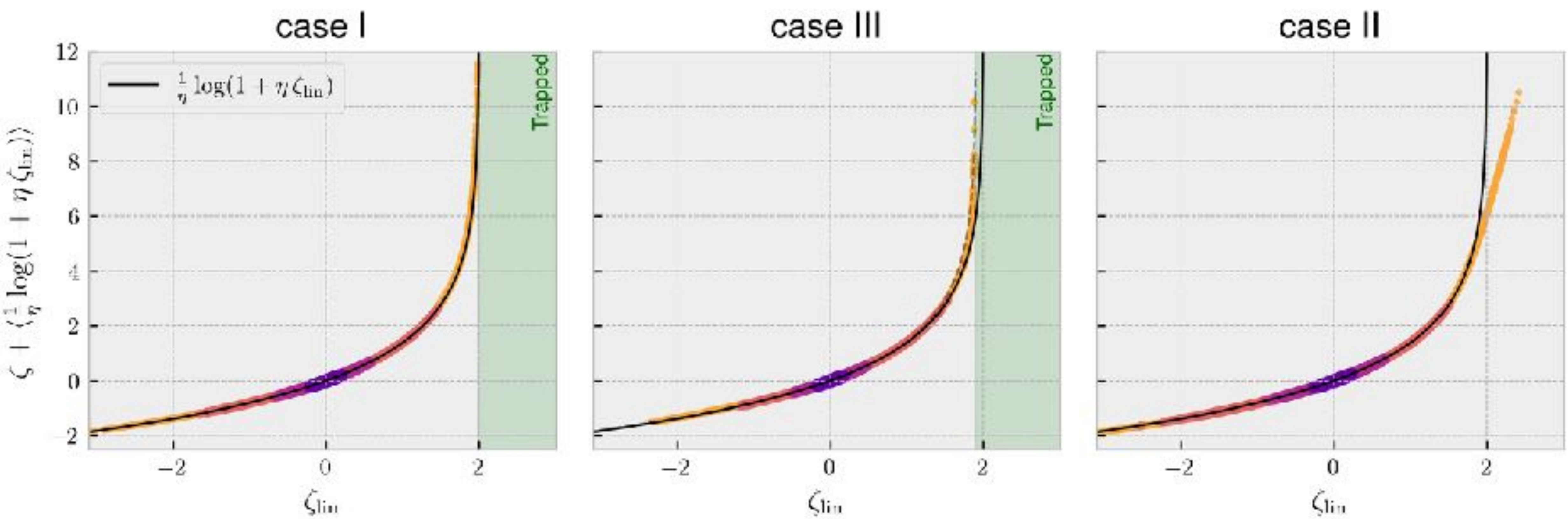


What goes wrong with the log relation: **nonlinear $\neq$ nonperturbative**

The notion of a unique background is lost

Summary

- Lattice simulations of inflation are a **new technique**, made publicly available

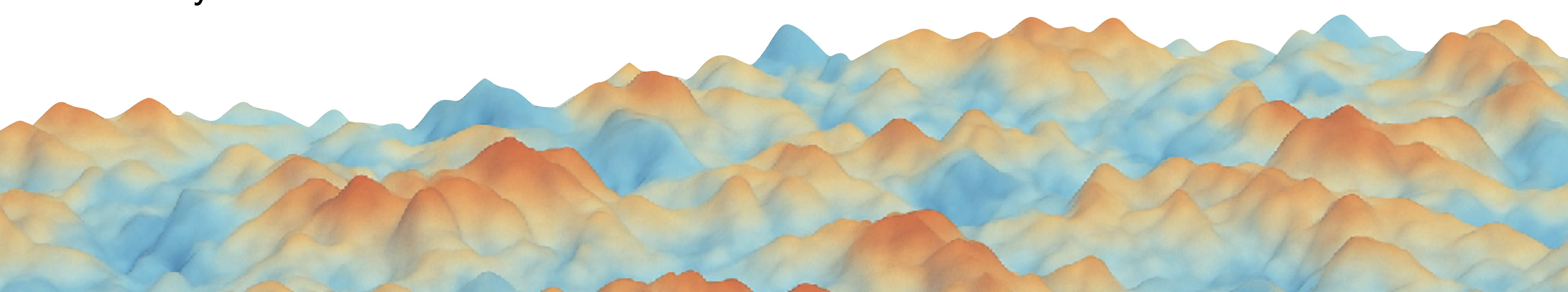
- We can finally know what happens **when perturbation theory breaks** down during inflation.

  Extremely relevant for probing the small scale physics of inflation, but there are a lot of other applications! (See axion inflation)

- What's next?

  Develop techniques to calculate **measurable quantities** directly from the simulation (e.g GW spectrum).

  Stay tuned for more!

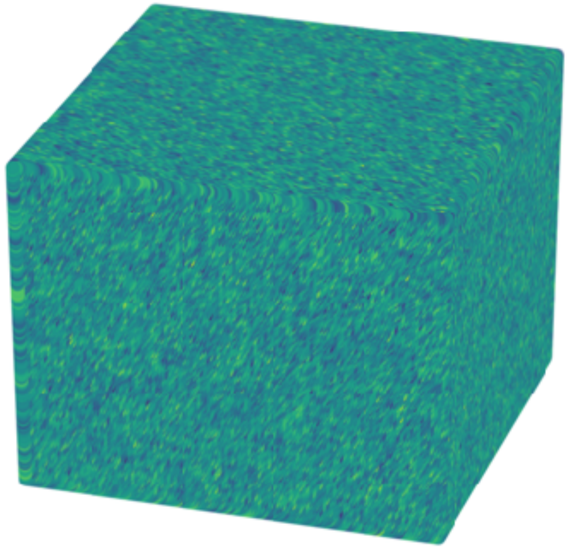



Thank you for the attention!

Backup slides

Lattice simulation: initial conditions

[AC 2209.13616]



- $$\hat{\phi}(\vec{n}) = \sum_{\vec{m}} \left[\hat{a}_{\vec{m}}\, u(\vec{\kappa}_{\vec{m}})\, e^{i\frac{2\pi}{N}\vec{n}\cdot\vec{m}} + \hat{a}^{\dagger}_{\vec{m}}\, u^{\dagger}(\vec{\kappa}_{\vec{m}})\, e^{-i\frac{2\pi}{N}\vec{n}\cdot\vec{m}} \right]$$

  $\vec{n}$ = lattice site, $\quad n_i, m_i \in 1,\ldots,N\,. \quad \vec{\kappa}_{\vec{m}} = \frac{2\pi}{L}\vec{m}$

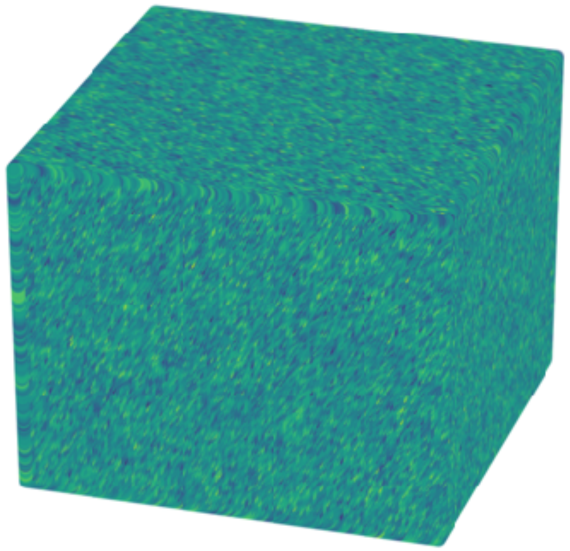
-

-

$$\hat{a}_{\vec{m}} = e^{i2\pi \hat{Y}_{\vec{m}}} \sqrt{-\ln(\hat{X}_{\vec{m}})/2},$$

Lattice approach: initial conditions

[AC 2209.13616]



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  $\vec{n}$ = lattice site, $n_i, m_i \in 1,\ldots,N$. $\vec{\kappa}_{\vec{m}} = \frac{2\pi}{L}\vec{m}$

- Discrete Bunch-Davies spectrum: [AC+ 2102.06378]

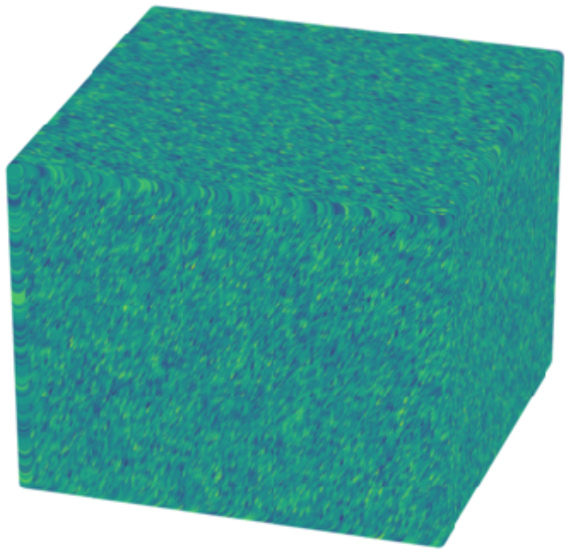
  $$u(\vec{\kappa}) = \frac{L^{3/2}}{a\sqrt{2\omega_{\vec{\kappa}}}} e^{-i\omega_{\vec{\kappa}}\tau}, \qquad \omega_{\vec{\kappa}}^2 = k_{\rm eff}^2(\vec{\kappa}) + m^2 \neq \kappa^2 + m^2$$

  (discrete dispersion relation)

- $$\hat{a}_{\vec{m}} = e^{i2\pi \hat{Y}_{\vec{m}}} \sqrt{-\ln(\hat{X}_{\vec{m}})/2},$$

Lattice approach: initial conditions

[AC 2209.13616]



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  $\vec{n}$ = lattice site, $\quad n_i, m_i \in 1,\ldots,N\,. \quad \vec{\kappa}_{\vec{m}} = \frac{2\pi}{L}\vec{m}$

- Discrete Bunch-Davies spectrum: [AC+ 2102.06378]

$$u(\vec{\kappa}) = \frac{L^{3/2}}{a\sqrt{2\omega_{\vec{\kappa}}}} e^{-i\omega_{\vec{\kappa}}\tau},$$

$$\omega^2_{\vec{\kappa}} = k^2_{\rm eff}(\vec{\kappa}) + m^2 \neq \kappa^2 + m^2$$ (discrete dispersion relation)

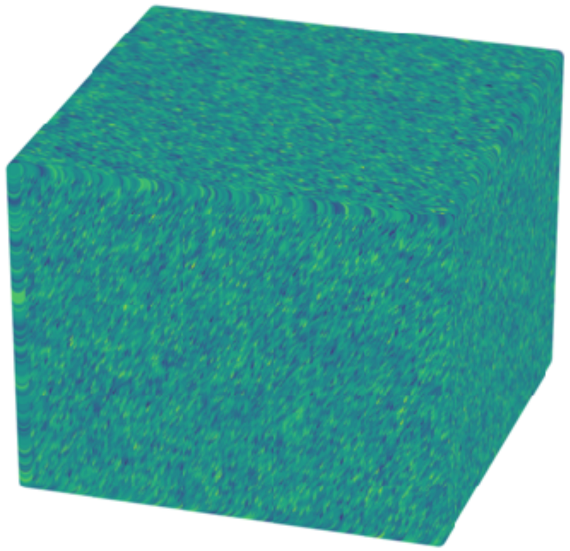
$$k^2_{\rm eff}(\vec{\kappa}_{\vec{m}}) = \frac{4}{(dx)^2}\left[\sin^2\left(\frac{\pi m_1}{N}\right) + \sin^2\left(\frac{\pi m_2}{N}\right) + \sin^2\left(\frac{\pi m_3}{N}\right)\right].$$

-

$$\hat{a}_{\vec{m}} = e^{i2\pi\hat{Y}_{\vec{m}}}\sqrt{-\ln(\hat{X}_{\vec{m}})/2},$$

Lattice approach: initial conditions

[AC 2209.13616]



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  $\vec{n}$ = lattice site, $\quad n_i, m_i \in 1,\ldots,N\,. \quad \vec{\kappa}_{\vec{m}} = \frac{2\pi}{L}\vec{m}$

-

- Stochastic approximation:

$$\hat{a}_{\vec{m}} = e^{i2\pi\hat{Y}_{\vec{m}}}\sqrt{-\ln(\hat{X}_{\vec{m}})/2},$$

$\hat{X}_{\vec{m}}, \hat{Y}_{\vec{m}}$ uniform randoms between 0 and 1

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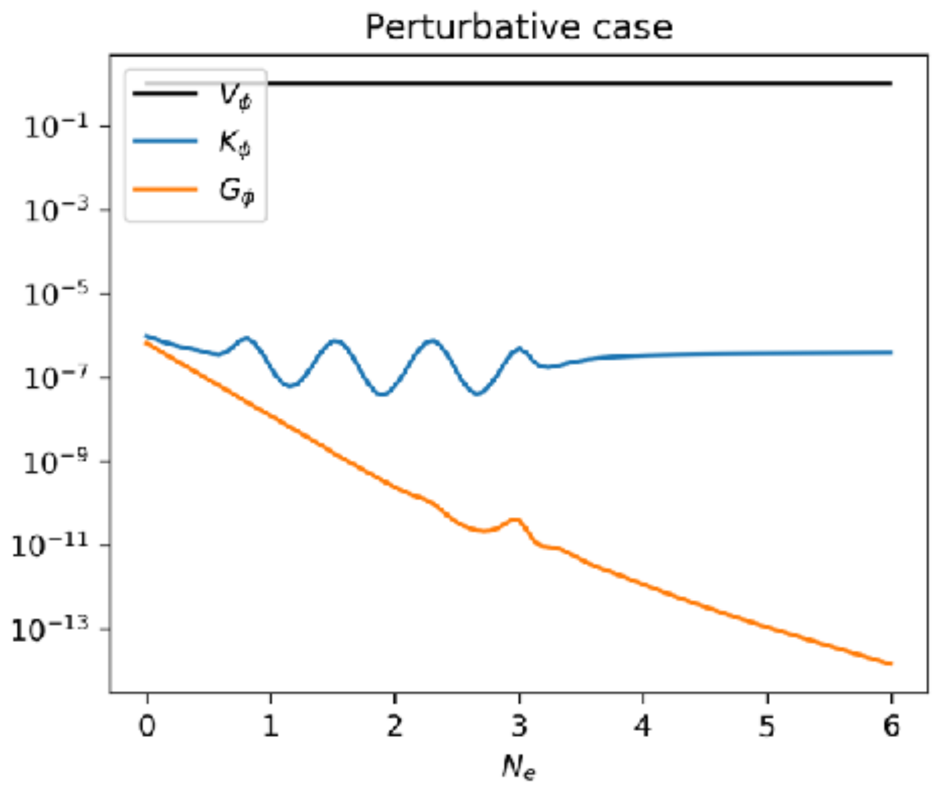
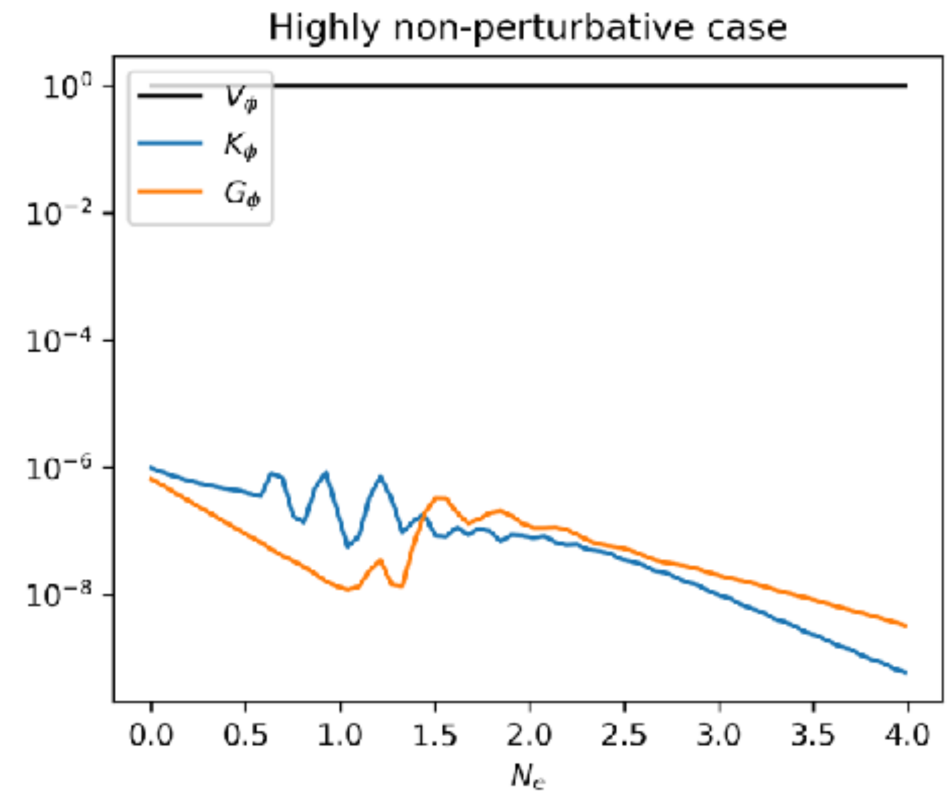
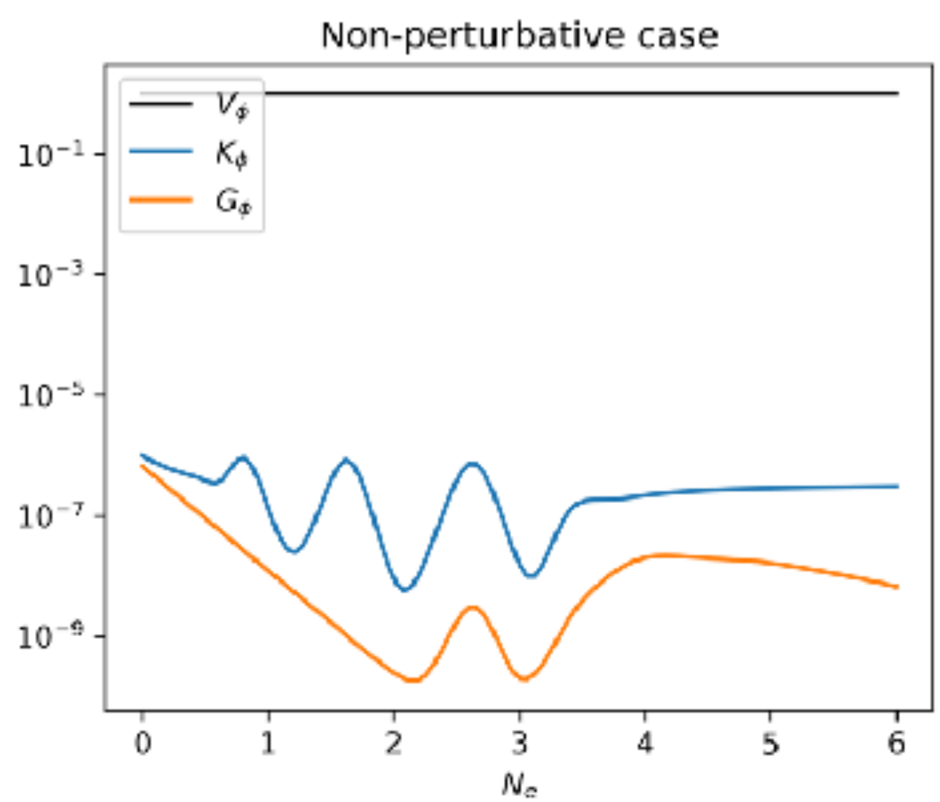
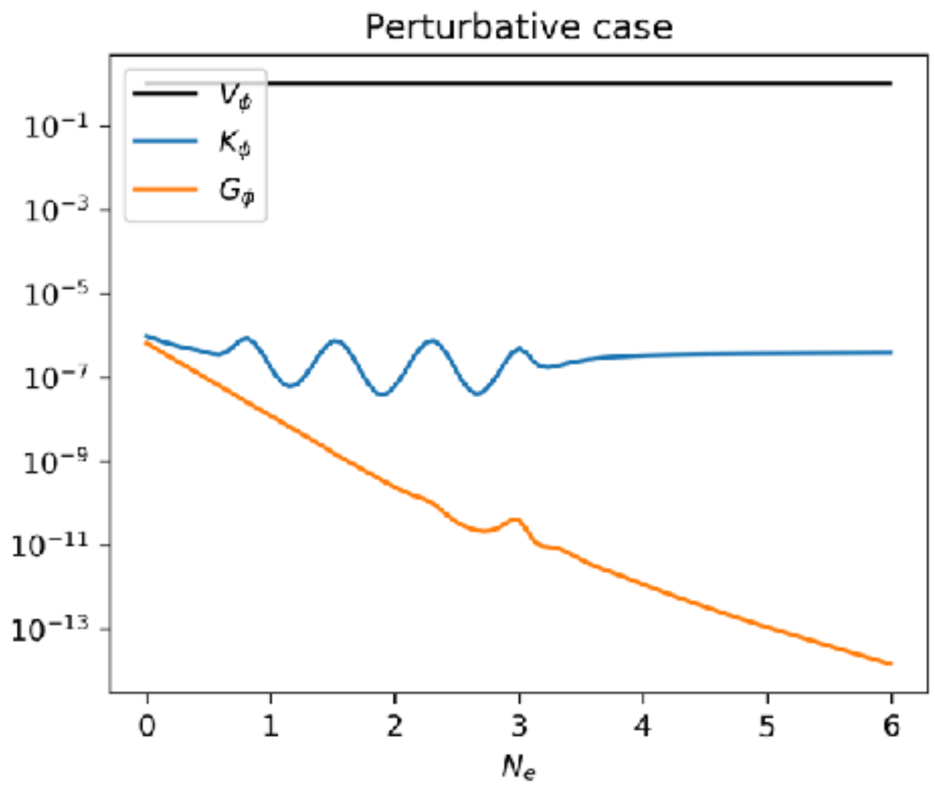


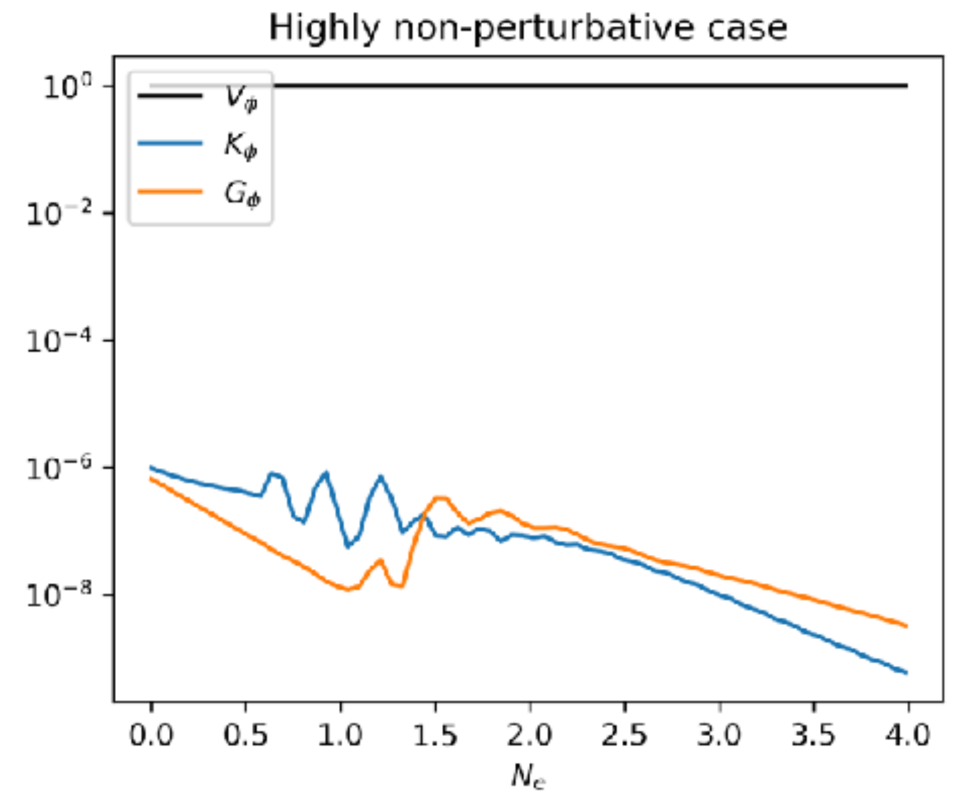


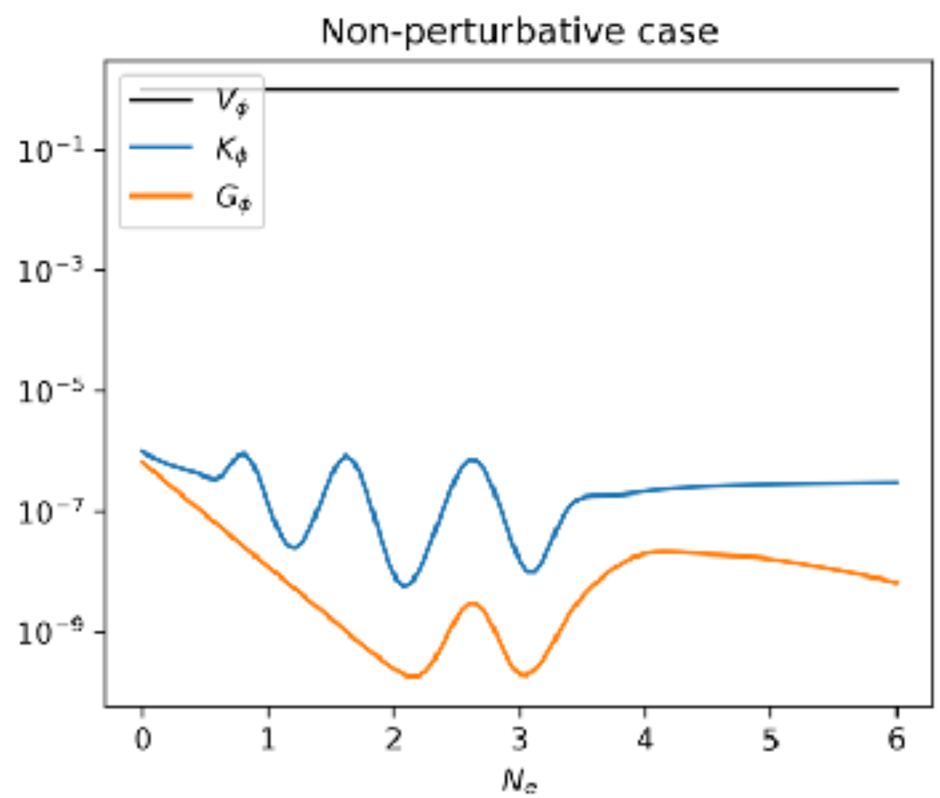


AC, S. Renaux-Petel, K. Inomata [2403.12811]

Energy contributions in oscillatory potentials







Energy contributions in USR

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