Numerical simulations of early universe sources of Gravitational Waves

2025/07/30 - Nordita, Stockholm

GW background signals from Axion Inflation: lattice calculation

2303.17436 (*PRL 131 (2023) 15, 151003*)
2411.16368 *(PRD 111 (2025) 6, 063545)*
2505.19950
And ongoing work

Joanes Lizarraga (UPV/EHU)
in collaboration with Daniel G. Figueroa, Nicolás Loayza, Carmelo López-Mediavilla, Ander Urio and Jon Urrestilla

ıtroduction

The model: **Axion-inflation** $+\ \dfrac{\phi}{\Lambda}F\tilde{F}$

Shift symmetry $\phi \to \phi + c$

[K. Freese, J. A. Frieman, A. V. Olinto (PRL 65,3233 1990)]
...

Why so interesting?

- Very efficient energy transport Axion -> Gauge fields
- Chiral instability $|A^{+}| \gg |A^{-}|$

Anber, L. Sorbo (0908.4089)]
k, L. Sorbo (1109.0022)]
naby, E. Pajer, M. Peloso (1110.3327)]

Very rich phenomenology:

- PBH
- GWs
- Non Gaussianities
- ...

[N. Barnaby, M. Peloso (1011.1500)]
[A. Linde, S. Mooji, E. Pajer (1212.1693)]
[E. Bugaev, P. Klimai (1312.7435)]
[N. Bartolo et al (LISA) (1610.06481)]
[J. G. Bellido, M. Peloso, C. Unal (1610.037
.....

he model

ynamical equations: @ FLRW

$$\ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\vec{E}\cdot\vec{B}$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{\alpha_\Lambda}{a m_p}\left(\dot{\phi}\vec{B} - \vec{\nabla}\phi\times\vec{E}\right)$$

$$\ddot{a} = -\frac{a}{3m_p^2}\left(2\rho_{\rm K} - \rho_{\rm V} + \rho_{\rm EM}\right)$$

onstraint equations:

$$\vec{\nabla}\cdot\vec{E} = -\frac{\alpha_\Lambda}{a m_p}\vec{\nabla}\phi\cdot\vec{B}$$

$$H^2 = \frac{1}{3m_p^2}\left(\rho_{\rm K} + \rho_{\rm G} + \rho_{\rm V} + \rho_{\rm EM}\right)$$

$$V = \frac{1}{2}m^2\phi^2$$

$$m/m_{\rm p} = 6.16\cdot 10^{-}$$

$$\alpha_\Lambda/m_p = 1/\Lambda$$

$\phi \to$ axion-like scalar field

$A_\mu \to$ dark photon

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

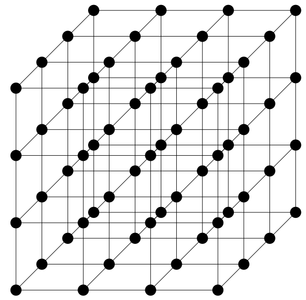
$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$

ur lattice study

[D.G. Figueroa, JLi, A. Urio, J. Urrestilla (2303.17436)]
[D.G. Figueroa, JLi, N. Loayza, A. Urio, J. Urrestilla (2411.16368)]

1- Full equations of motion: inhomogeneities allowed!

$$\vec{E}\cdot\vec{B} \rightarrow \vec{\nabla}^2\phi$$
$$\vec{E}\cdot\vec{B} \rightarrow \vec{\nabla}\phi\times\vec{E}$$



2- Lattice gauge-invariant and shift symmetric.

[D. G. Figueroa, M. Shaposhnikov (1705.09629)]
[J. R. C. Cuissa , D. G. Figueroa (1812.03132)]

CosmoLattice

Alternative proposals:
aravano, E. Komatsu, K. D. Lozanov, J. Weller (2204.12874)]
arma, A. Brandenburg, K. Subramanian, A. Vikman(2411.04854)]

[D. G. Figueroa, A. Florio, F. Torrenti & W. Valkenburg (2006.1512
[D. G. Figueroa, A. Florio, F. Torrenti & W. Valkenburg (2102.010

3- Strong backreaction during and until the end of inflation

From the linear regime → $\epsilon_H = 1$ →

iscretisation procedure

[D. G. Figueroa, M. Shaposhnikov (1705.096

Continuum

$$\frac{\phi}{\Lambda}\vec{E}\cdot\vec{B}$$

The only one that satisfies all conditions

Lattice

$$\sum_i \frac{\phi}{\Lambda} E_i^{(2)} B_i^{(4)}$$

Want to preserve:

1- Gauge transformations

$$A_\mu \to A_\mu + \partial_\mu \alpha(x)$$

2- Bianchi identities

$\vec{\nabla}\times\vec{E} = \dot{\vec{B}}$ $\quad \vec{\nabla}\cdot$ … 0

3- Topological nature … coupling

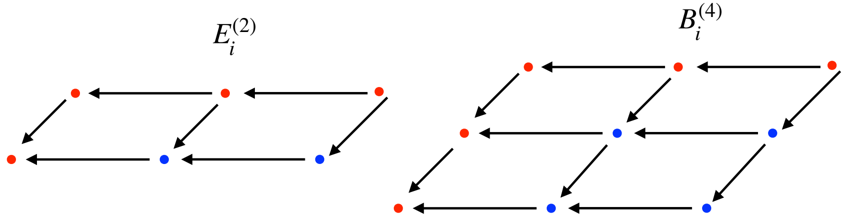
$F_{\mu\nu}\tilde{F}^{\mu\nu} =$ …

Exact lattice shift symmetry

4- Continuum limit to $\mathcal{O}(dx^2)$

$$E_i^{(2)} \equiv \frac{1}{2}(E_i + E_{i,-i}) \quad @ \quad n$$

$$B_i^{(4)} \equiv \frac{1}{4}(B_i + B_{i,-j} + B_{i,-k} + B_{i,-j-k}) \quad @ \quad n$$



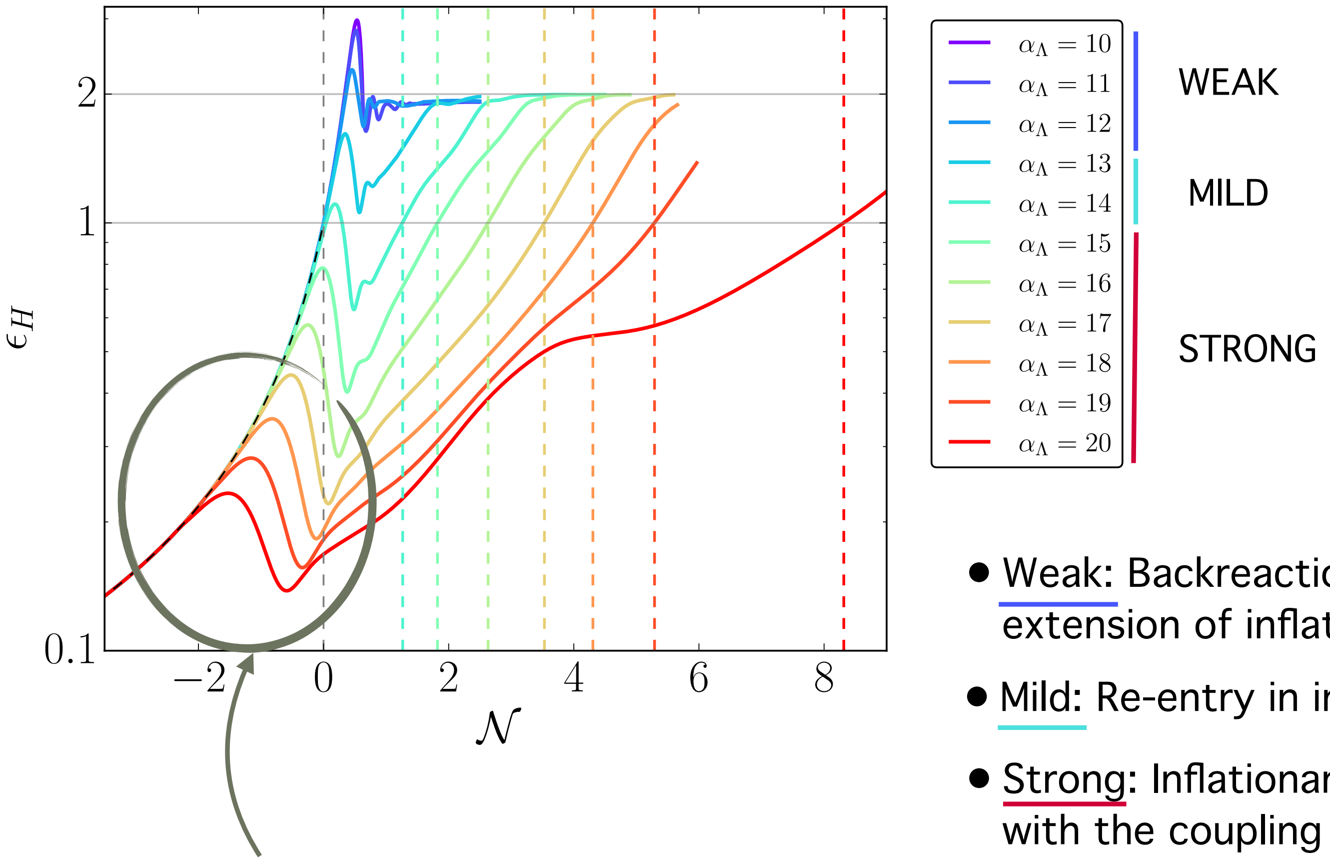
)utline

1. Uncovering the strong backreaction regime
    1. General features
    2. (Electro)magnetic slow-roll
    3. UV sensitivity

2. The effect of the potential

3. GW generation and detectability prospects

. Uncovering the strong backreaction regime

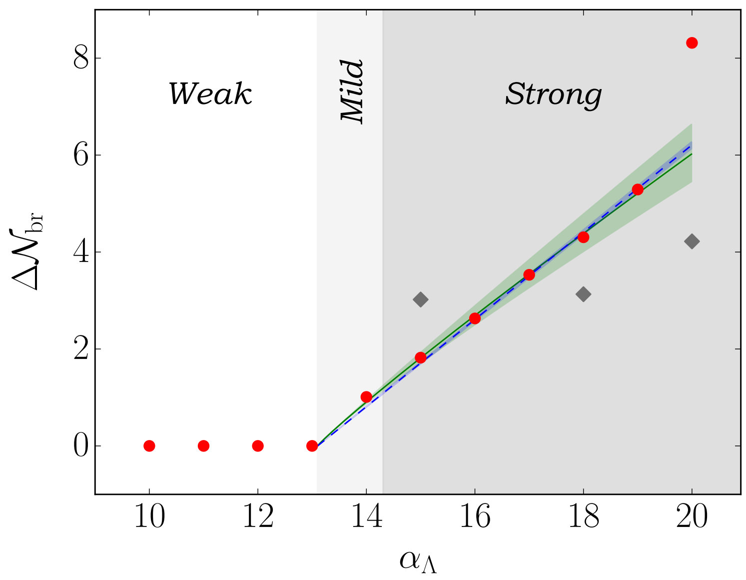


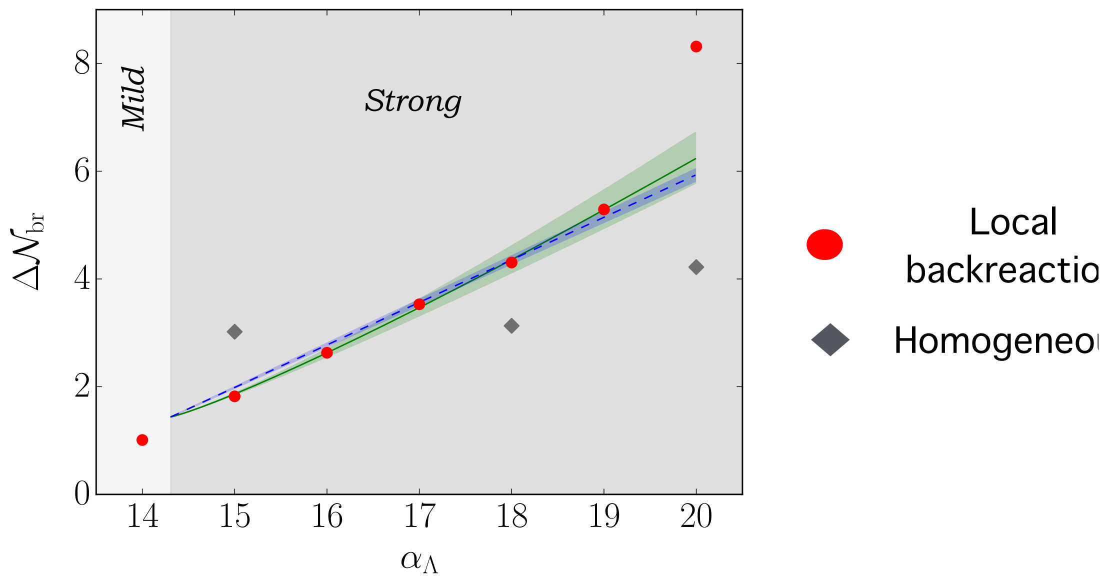
Bump before standard slow-roll end, earlier for higher couplings

Preheating:
[Adshead et al (1909.12842), (1909.12843)...]
[J. R. C. Cuissa and D. G. Figueroa (1812.03132)]

- Weak: Backreaction only after end inflation, no extension of inflation.
- Mild: Re-entry in inflation.
- Strong: Inflationary period exponentially grows with the coupling value!

.1 General features



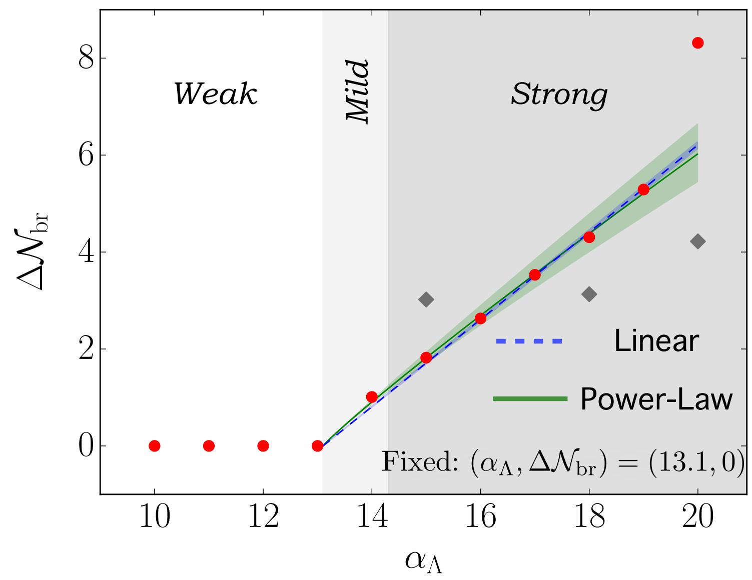


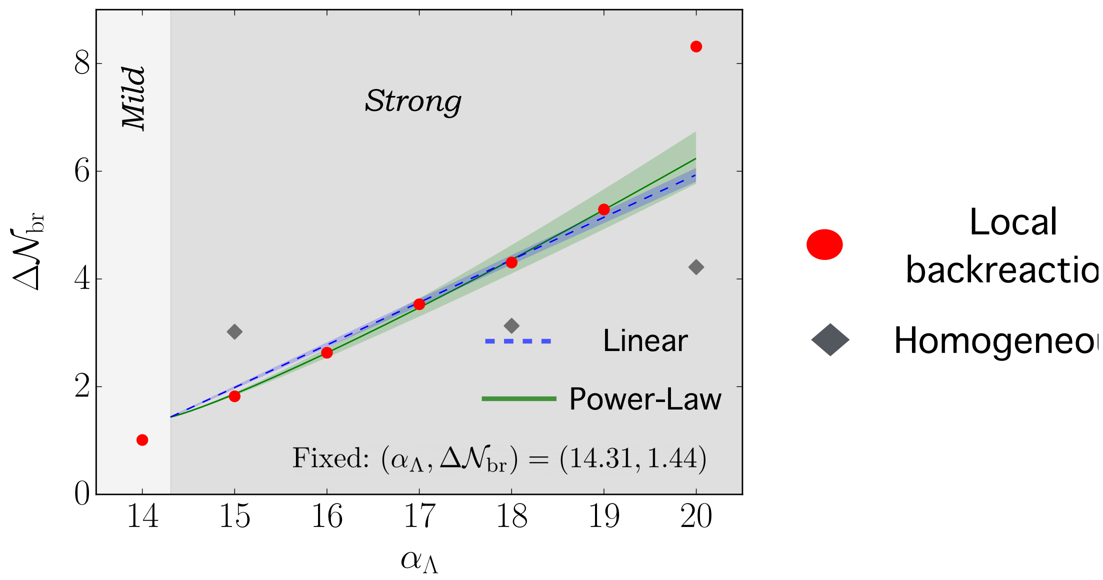
Local backreaction:
Inflationary period extends
~ linearly respect to the
coupling

VS

Homogeneous backreaction:
Inflationary period is clustered
~ 3-4 e-folds

.1 General features





Linear fits approximately supported, but preferred mild curvature

Extrapolations

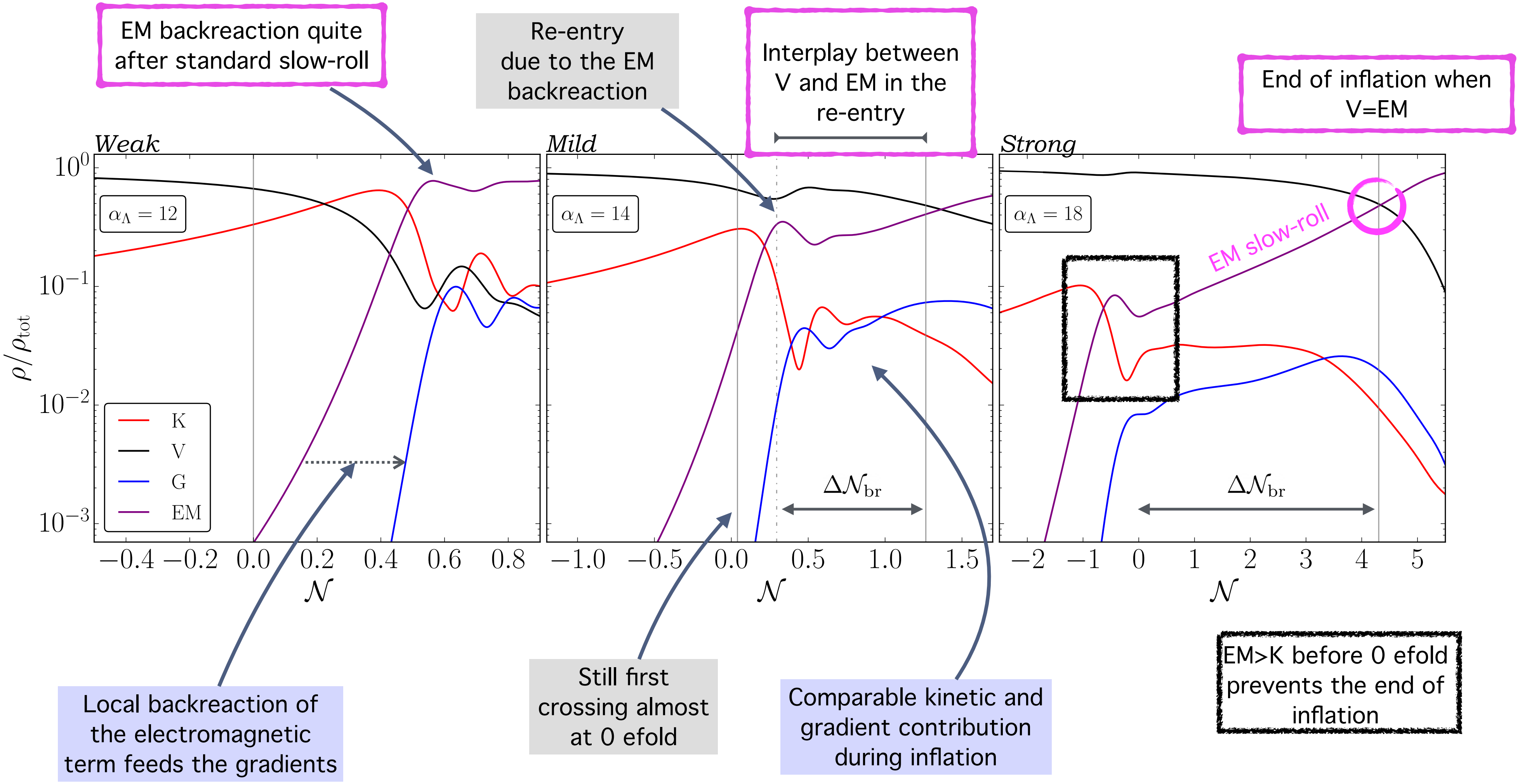
$\alpha_\Lambda = 25 \rightarrow \Delta\mathcal{N}_{br} \sim 10 \; - \; 12$

$\alpha_\Lambda = 30 \rightarrow \Delta\mathcal{N}_{br} \sim 15 \; - \; 18$

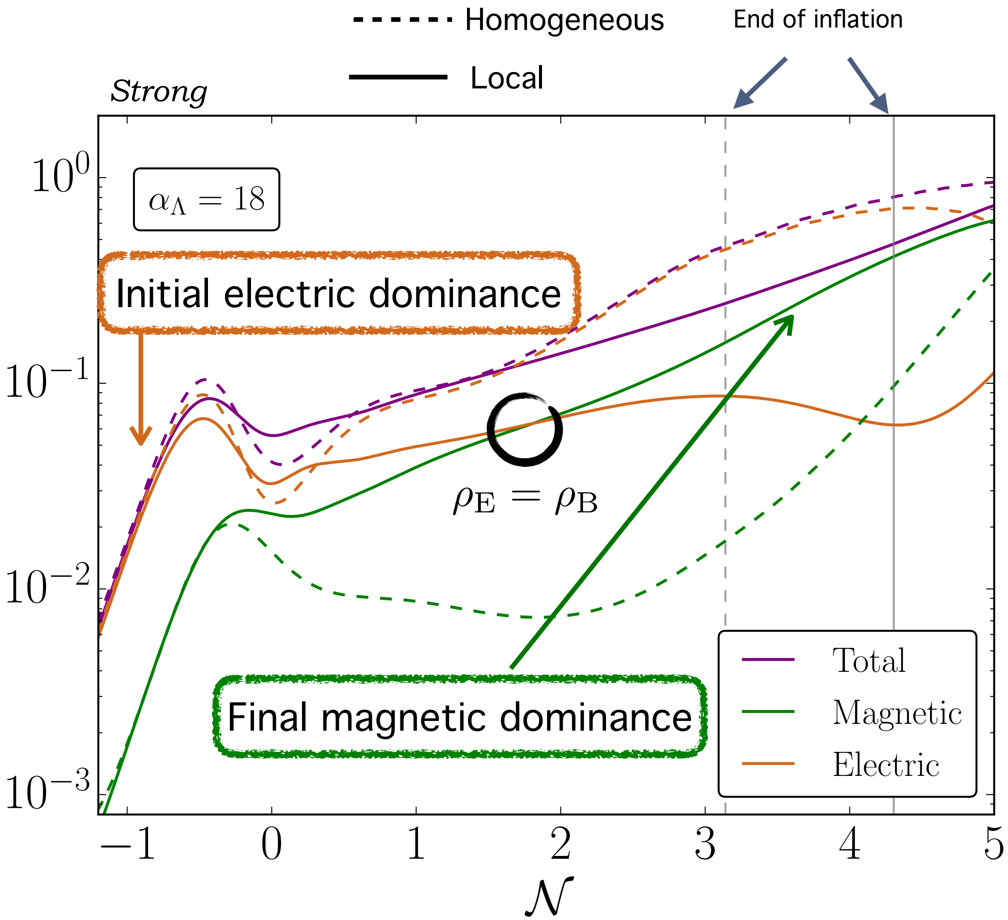
$\alpha_\Lambda = 35 \rightarrow \Delta\mathcal{N}_{br} \sim 18 \; - \; 25$

.1 General features

$\epsilon_H = 1 + (2\rho_{\rm K} - \rho_{\rm V} + \rho_{\rm EM})/\rho_{\rm t\epsilon}$

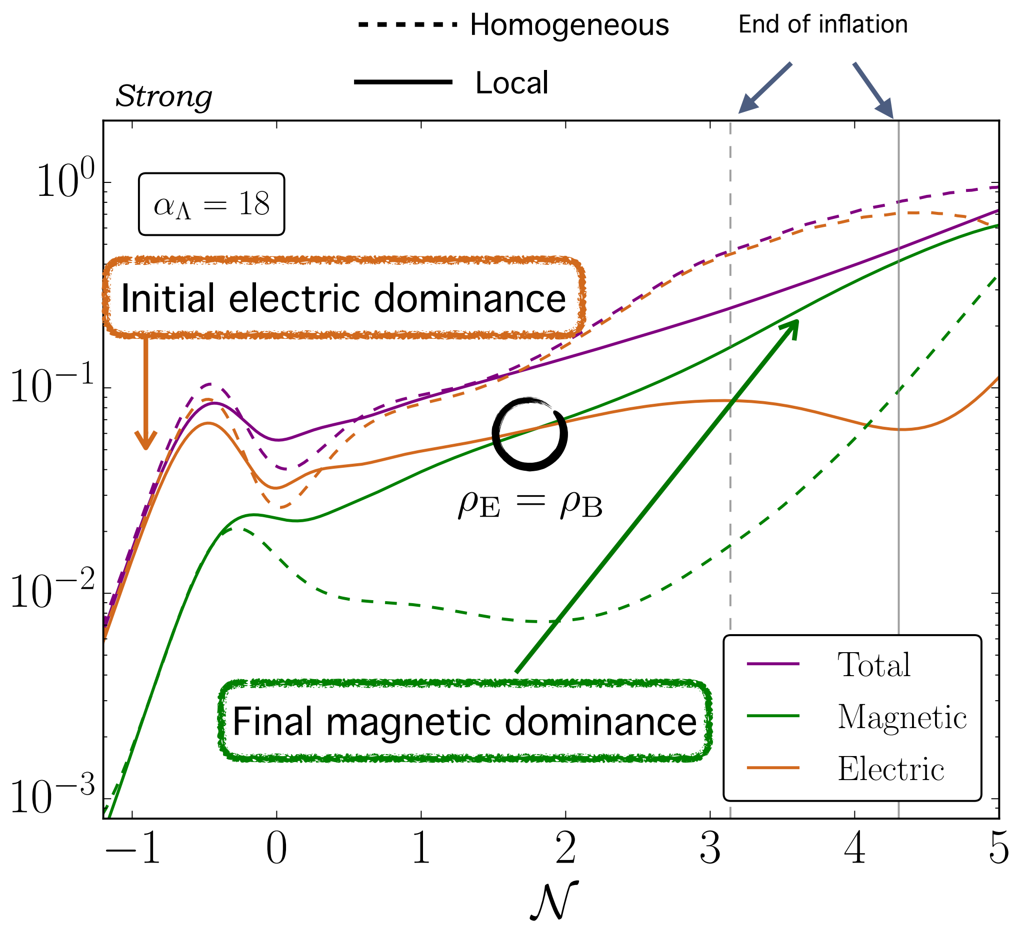


.2 (Electro)magnetic slow roll

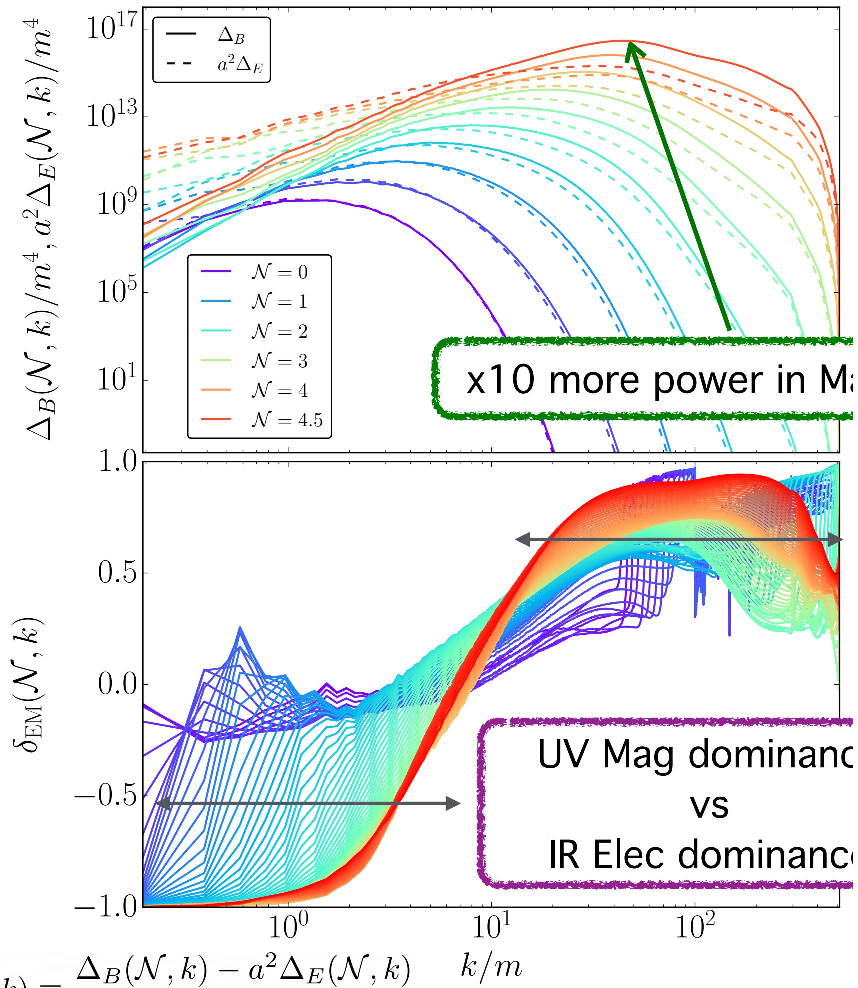


Homogeneous: complete electric dominance

.2 (Electro)magnetic slow roll



Homogeneous: complete electric dominance

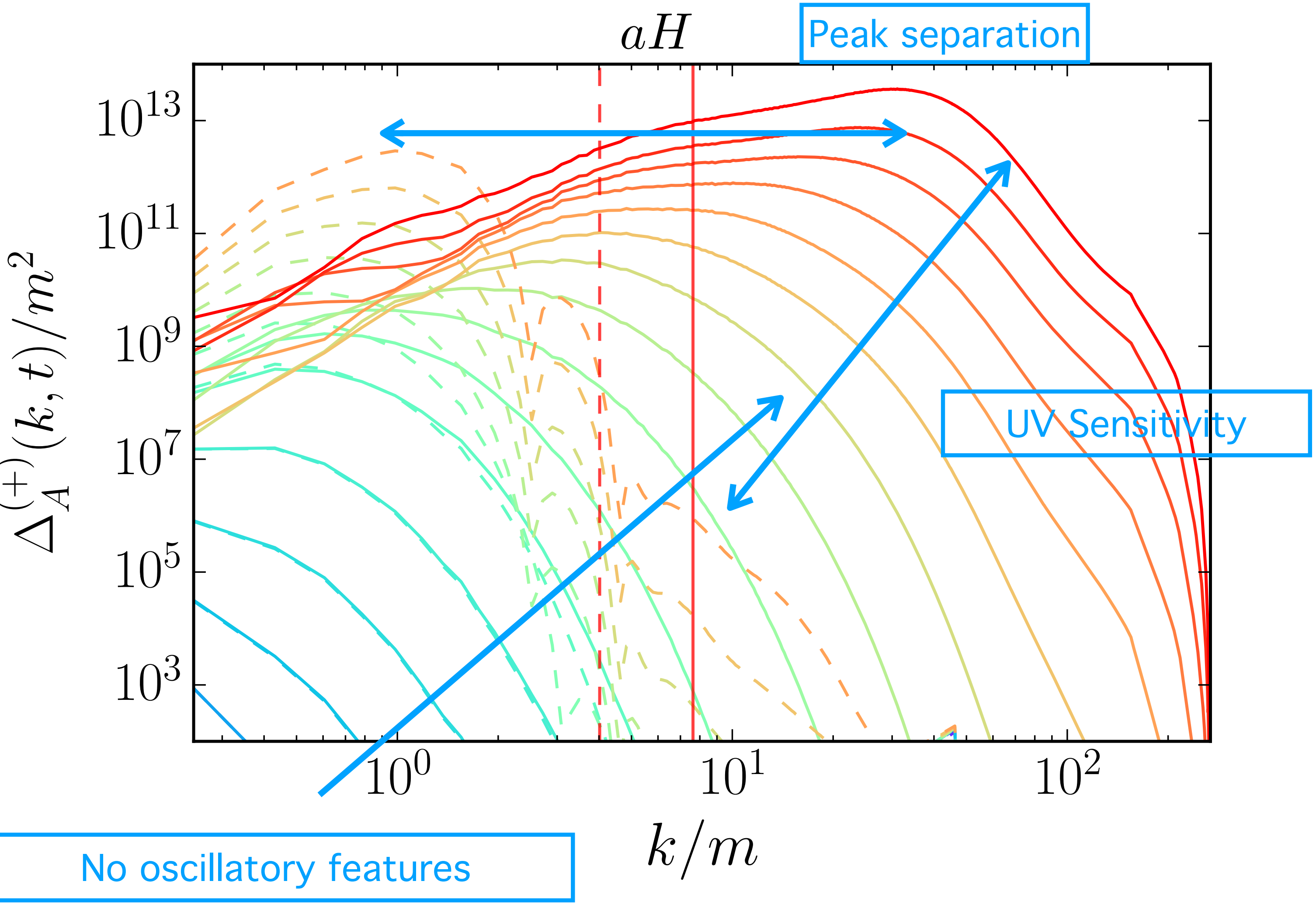


$$\delta_{EM}(\mathcal{N},k) \equiv \frac{\Delta_B(\mathcal{N},k) - a^2\Delta_E(\mathcal{N},k)}{\Delta_B(\mathcal{N},k) + a^2\Delta_E(\mathcal{N},k)}$$

.3 Full vs Homogeneneous: PS

$_\Lambda = 18$

$^{(+)}_{A} = \frac{k^3}{2\pi^2}|A^{(+)}|^2$



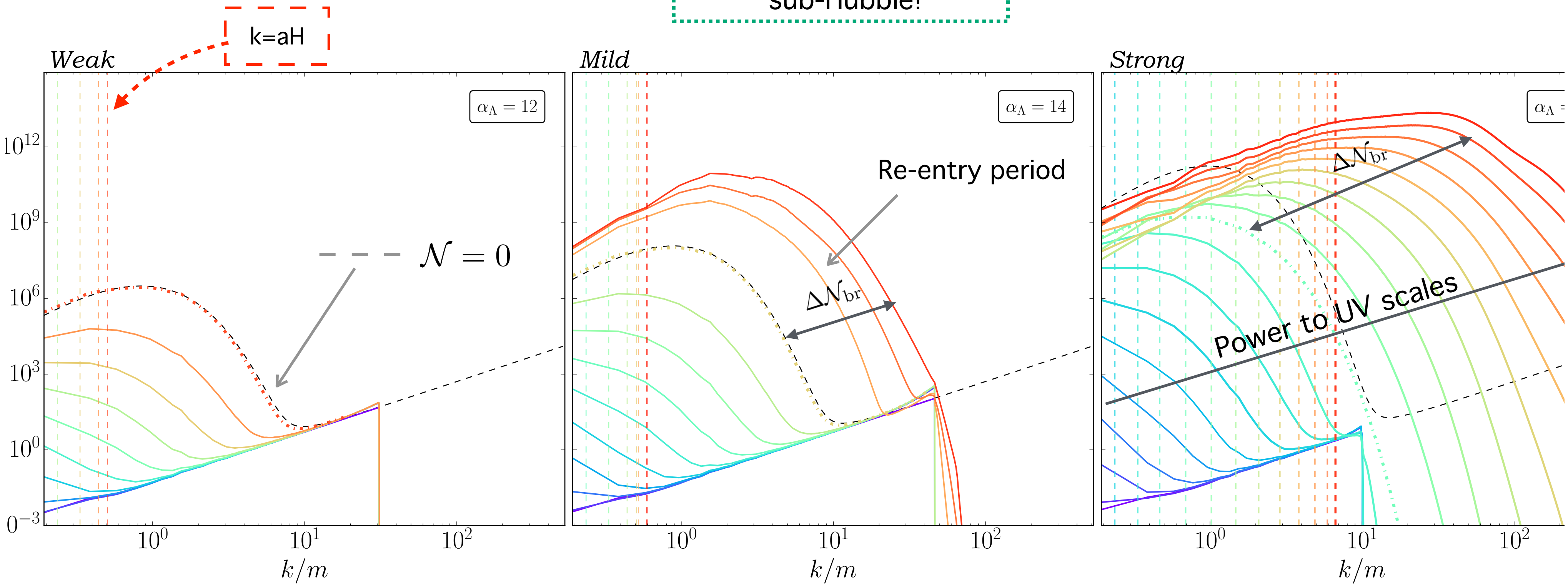
Full

Homogeneous

$\langle \vec{E} \cdot \vec{B} \rangle_{\rm L}$

.4 UV sensitivity

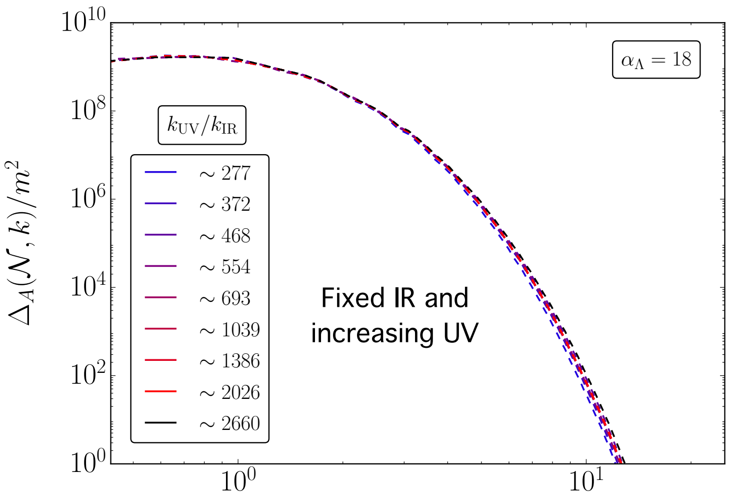
Peak always remains sub-Hubble!



No noticeable changes during inflation	During the re-entry period, excitation moves towards UV scales	Considerable amount of extra e-folds: extreme UV sensitivity
$N = 320, \quad k_{\rm UV}/k_{\rm IR} \sim 280$	$N = 480, \quad k_{\rm UV}/k_{\rm IR} \sim 415$	$N = 3072, \quad k_{\rm UV}/k_{\rm IR} \sim 2660$

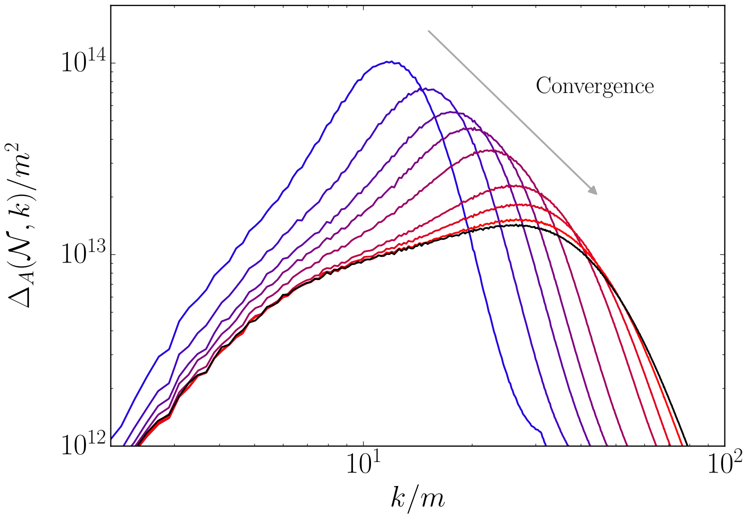
.4 UV sensitivity: convergence?

Lack of UV resolution distorts the power spectrum, and the self-consistent background evolution



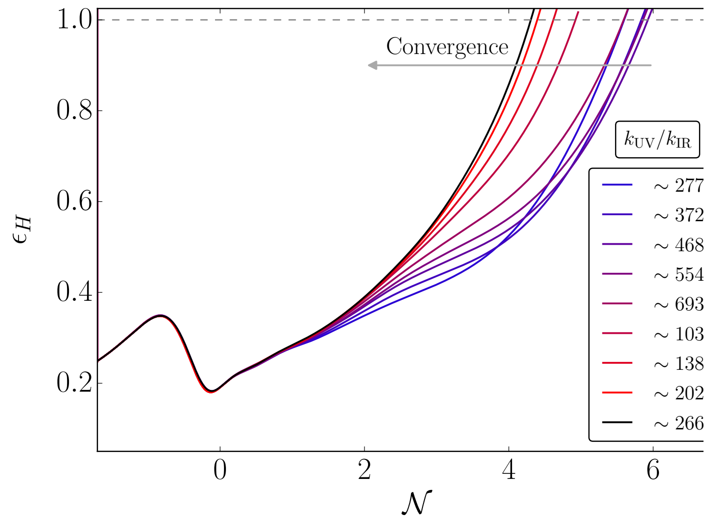
$\mathcal{N} = 0$

Common evolution

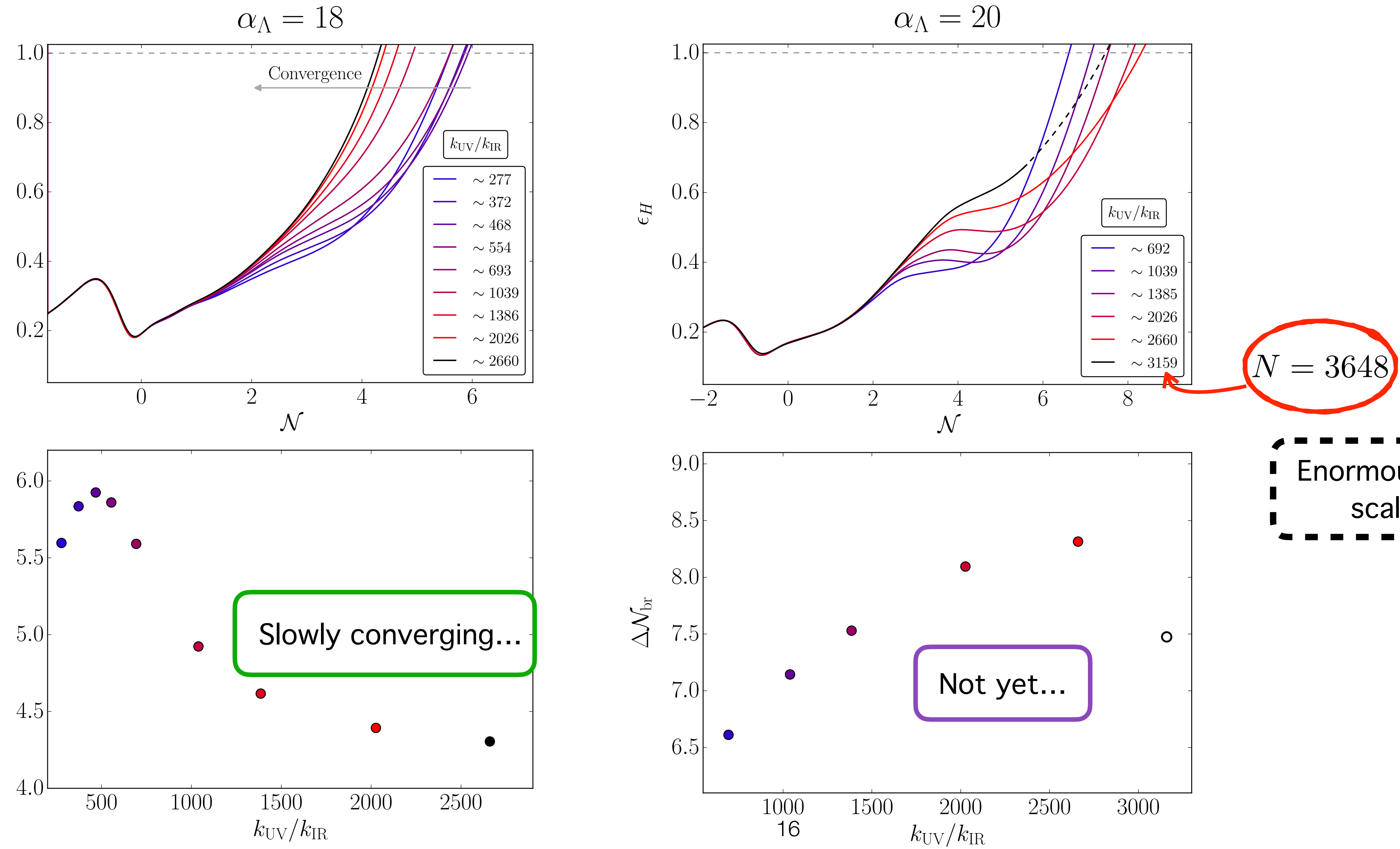


$\mathcal{N} = \mathcal{N}_{\rm end}$

During backreaction, different evolution depending on UV coverage



.4 UV sensitivity: convergence?



Enormous separation scales required!

ummary & Conclusions 1

- Local physics and gradients play a crucial role in U(1)-axion inflation
- Strong backreaction leads to notable inflation extension
- This phase is related to "(electro)magnetic slow-roll"
- The dependance of the extension with the coupling is ~ linear
- UV scales more relevant for strong backreaction:
  - Bad separation of scales $\longrightarrow$ fake physics of high couplings

What if we go beyond $V(\phi) = \frac{1}{2}m^2\phi^2$?

. The effect of the potential

Many single-field inflationary models

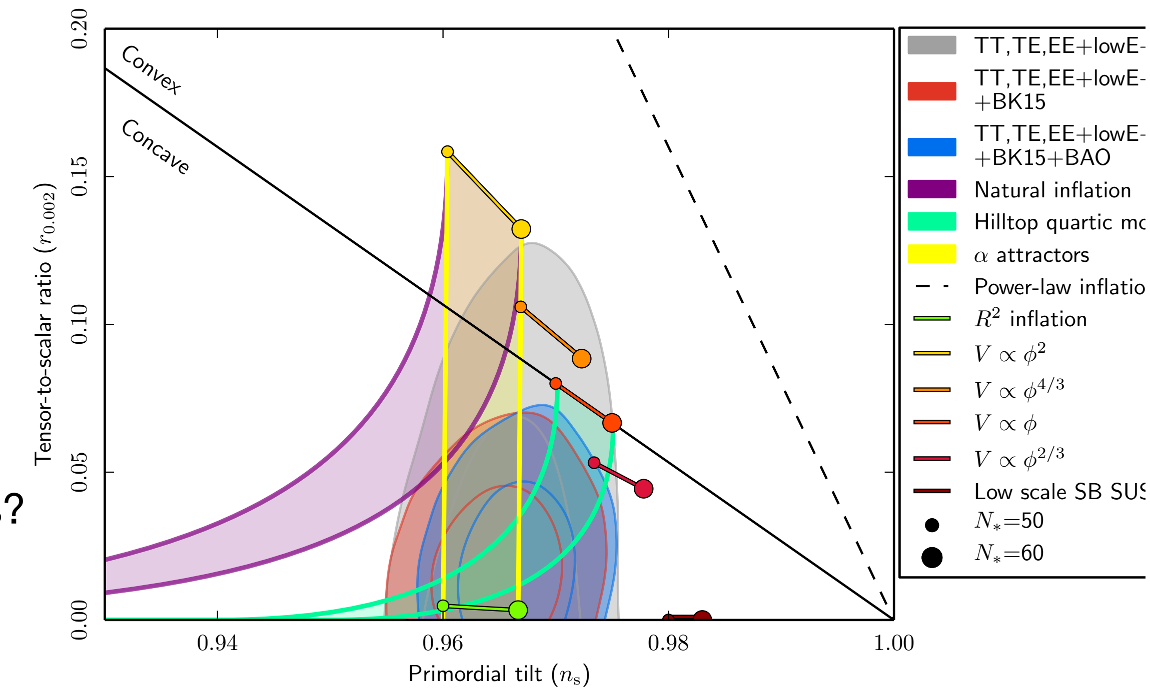
Low tensor-to-scalar ratio favoured

U(1) gauge-axion inflation models commonly:

- Quadratic potentials
- Natural Inflation potential

Why go beyond?

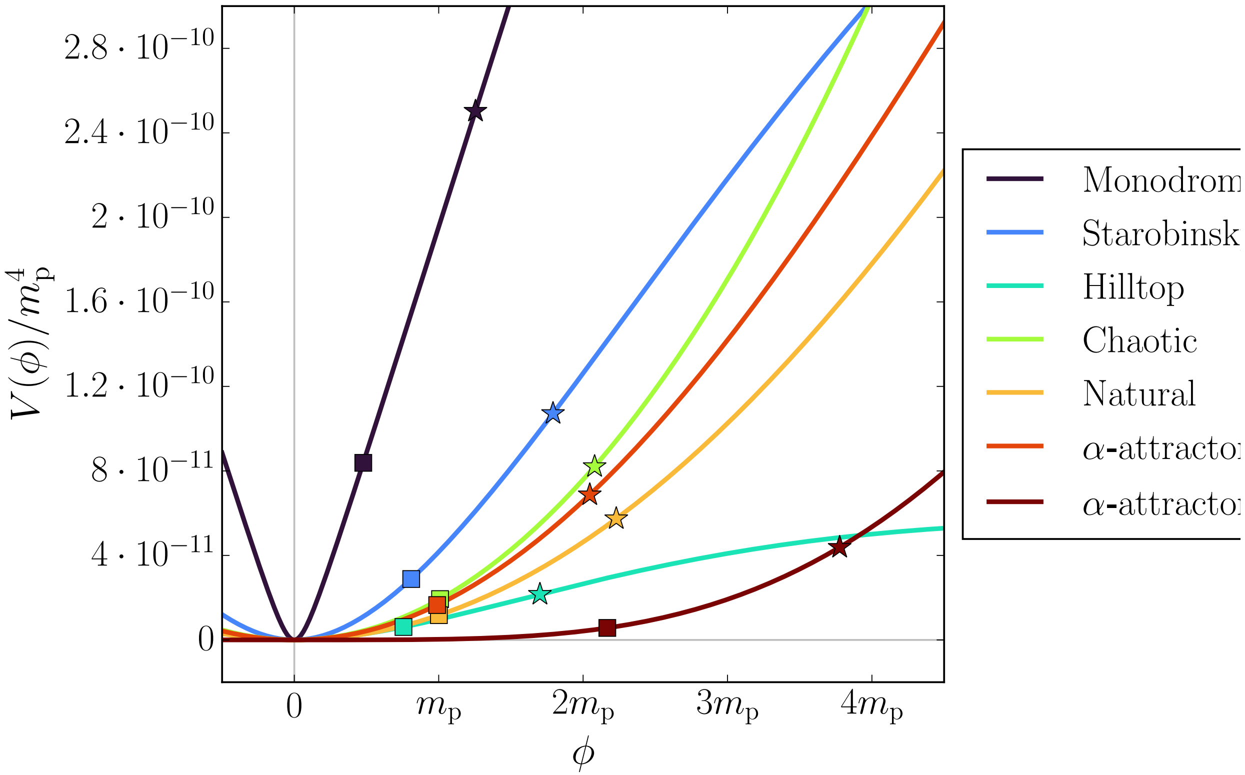
- Dependance on the potential of new results?
- Range of validity of homogenous backreaction approach?

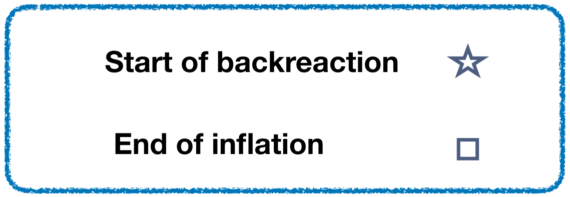


)ur set

We consider a diverse set of potentials:

- Traditional: Natural, Chaotic
- Favoured: α-attractors, Starobinsky
- UV motivated: Monodromy
- Others: Hilltop





trong Bacreation Limit (SBR)

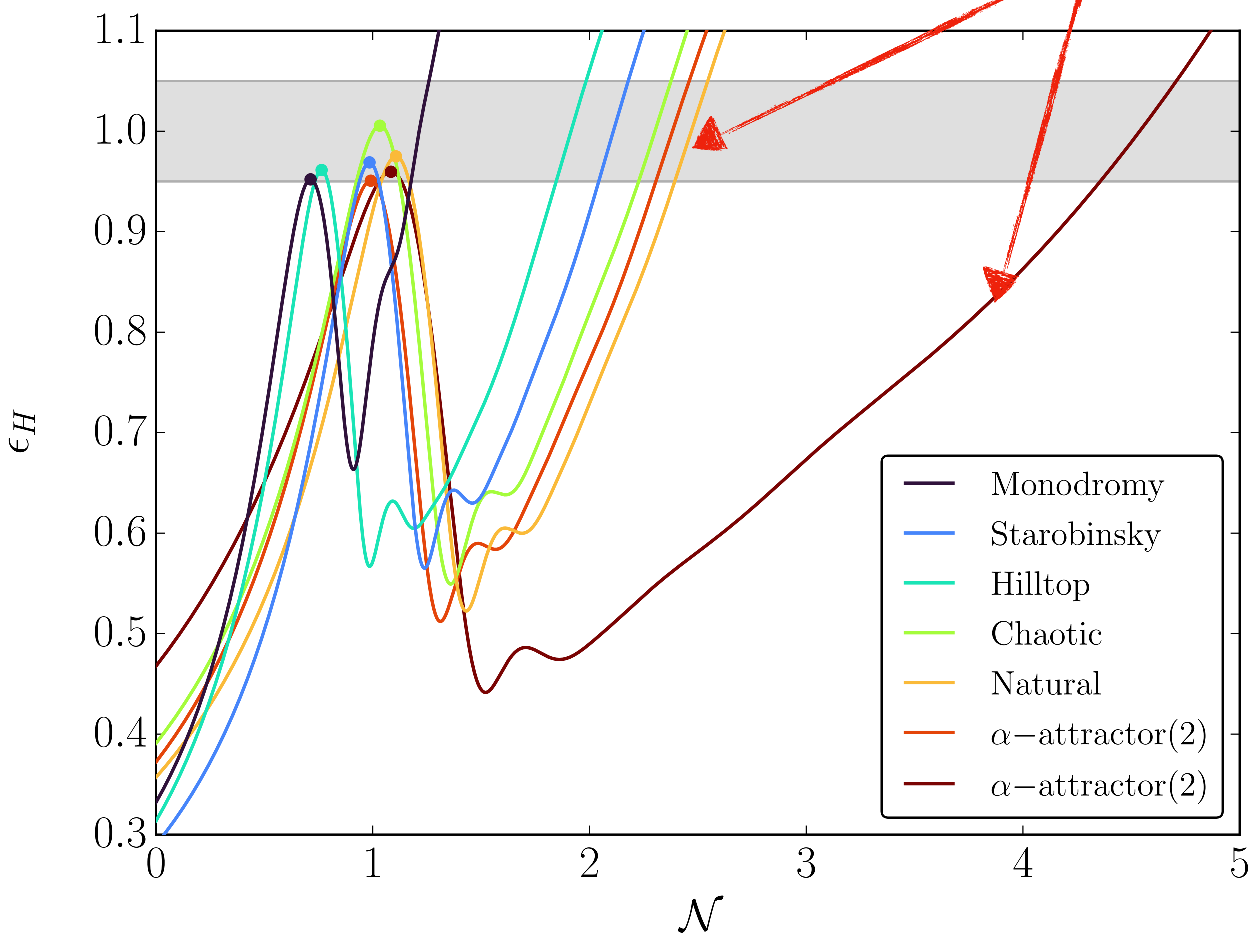
Quantitative differences already present

Nhat coupling value?

- SBR limit
  Between mild and strong

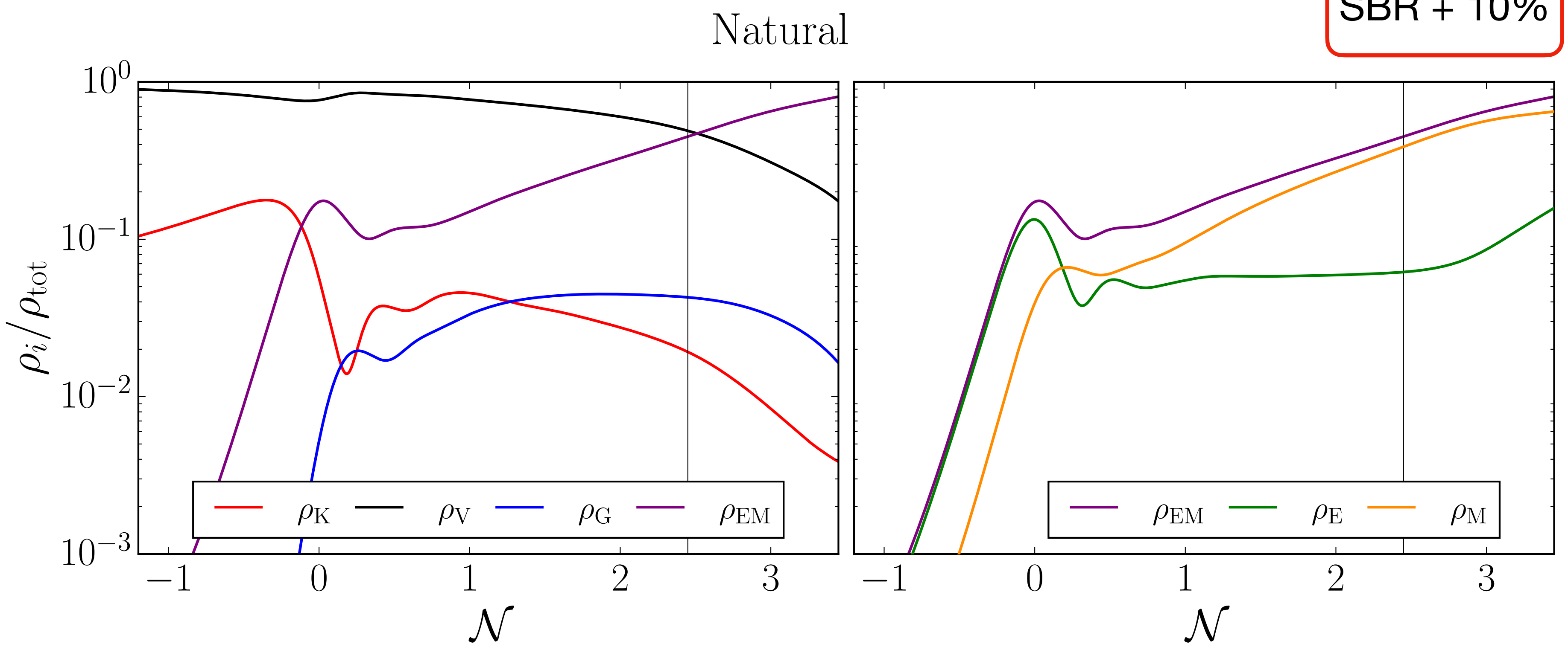
  and +5%, +10%, +15%
  and +20%

3etter comparison with
'elative values



.1 Dynamics

SBR + 10%



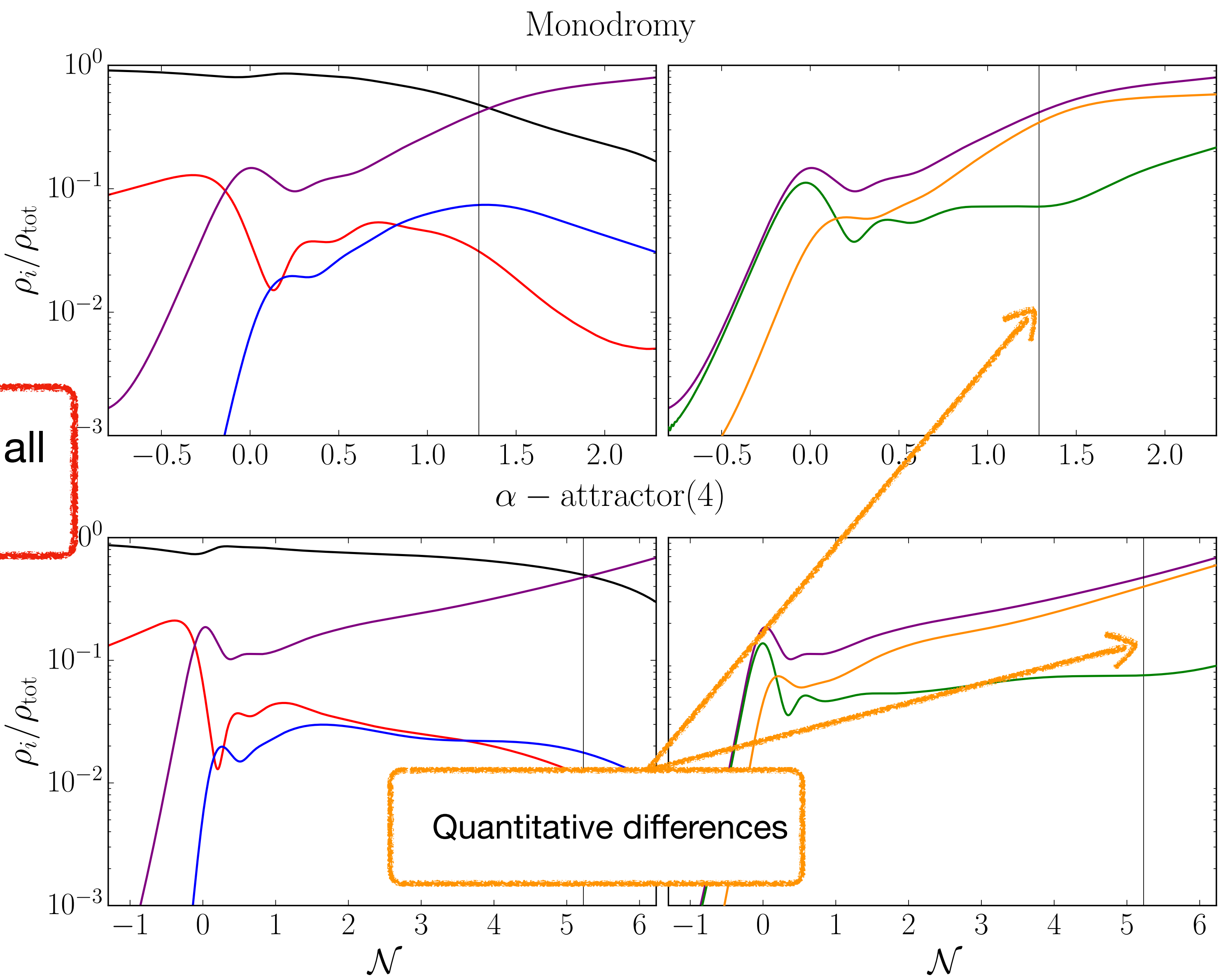
EM Slow-roll
Gradients dominate over Kinetic
Extra inflation ✔

Magnetic dominance in extra eFolds ✔

.1 Dynamics

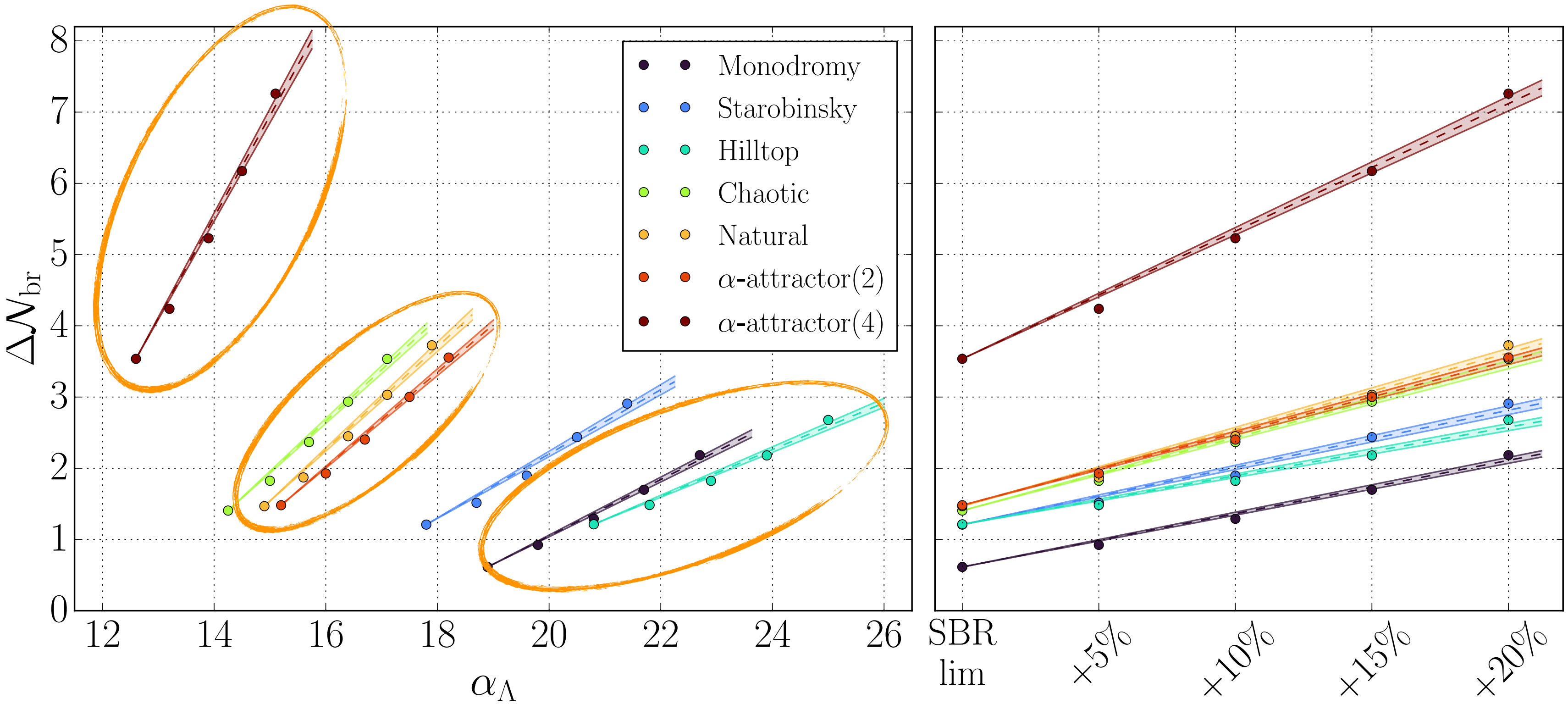
SBR + 10%

ualitatively same for all potential



Quantitative differences

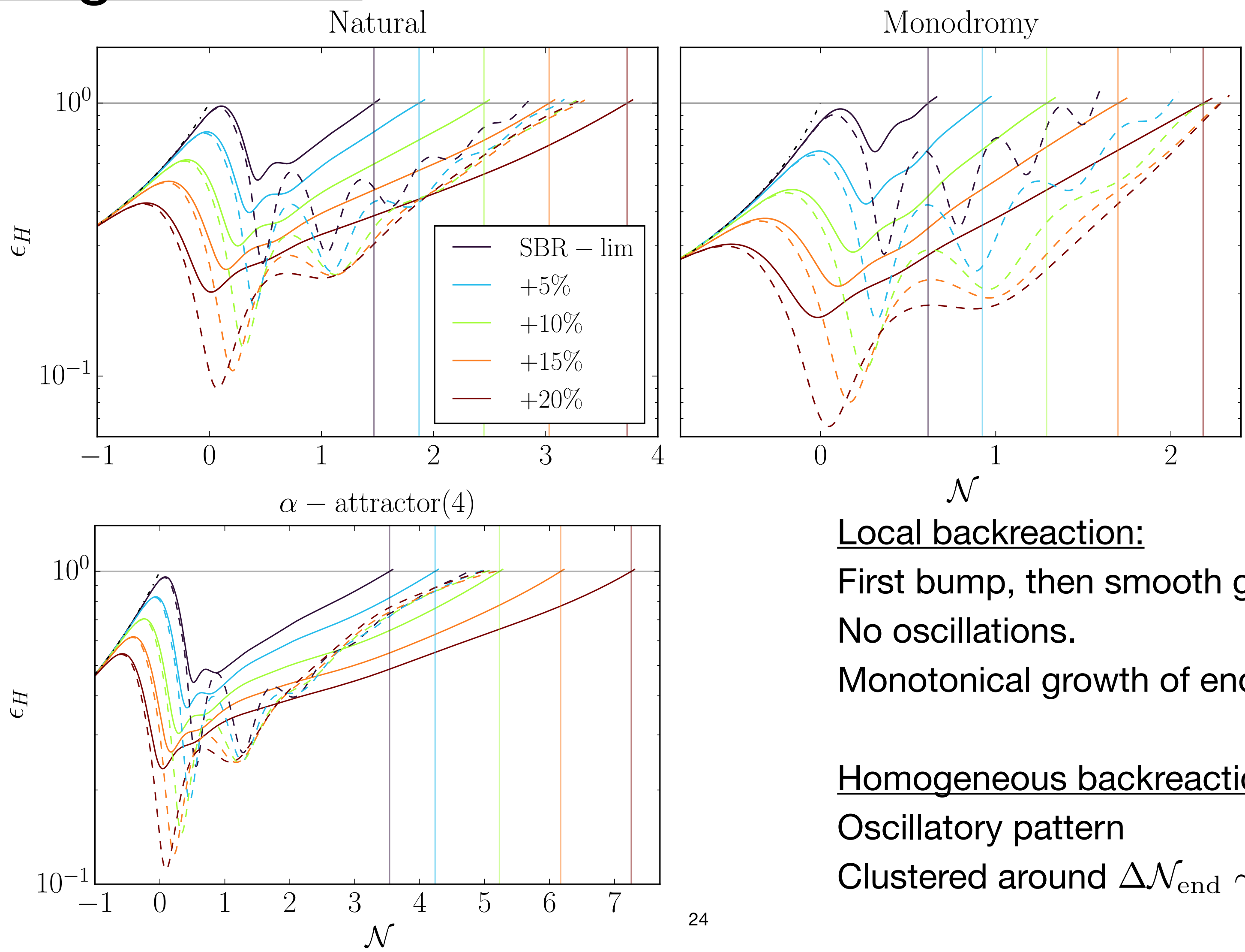
.2 Parametrisation



~ Linear growth universal

Slope depends on the slope… of the potential

.2 Homogeneous?



Local backreaction:

First bump, then smooth growth for all cases

No oscillations.

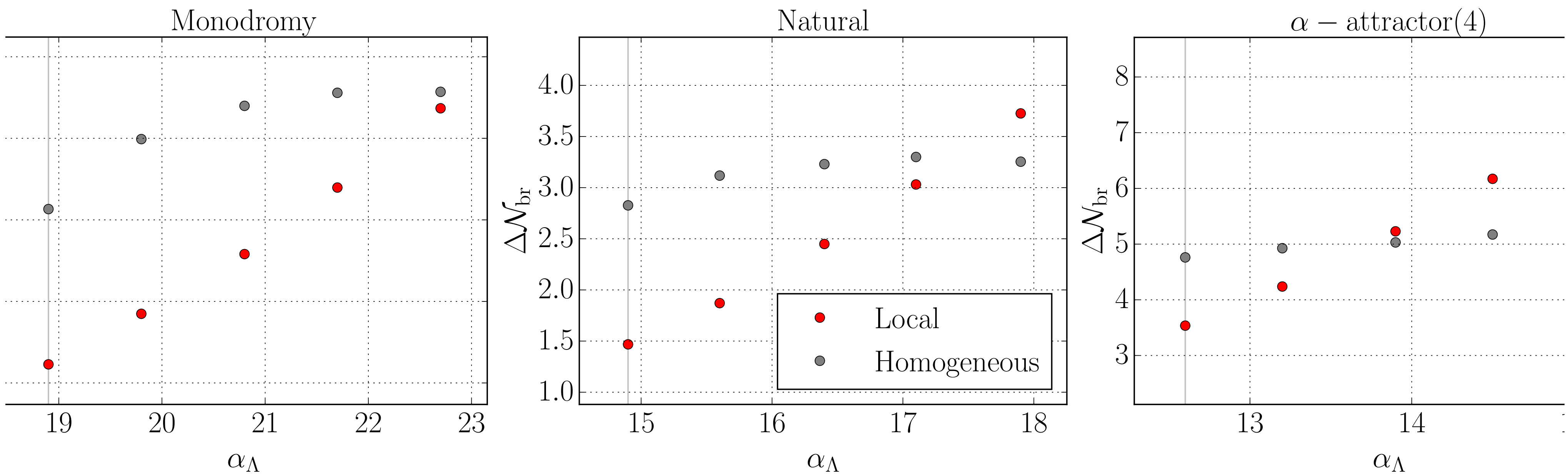
Monotonical growth of end of inflation $\mathcal{N}_{\rm end}$

Homogeneous backreaction:

Oscillatory pattern

Clustered around $\Delta\mathcal{N}_{\rm end} \sim 1{\rm eF}$

.2 Homogeneous?

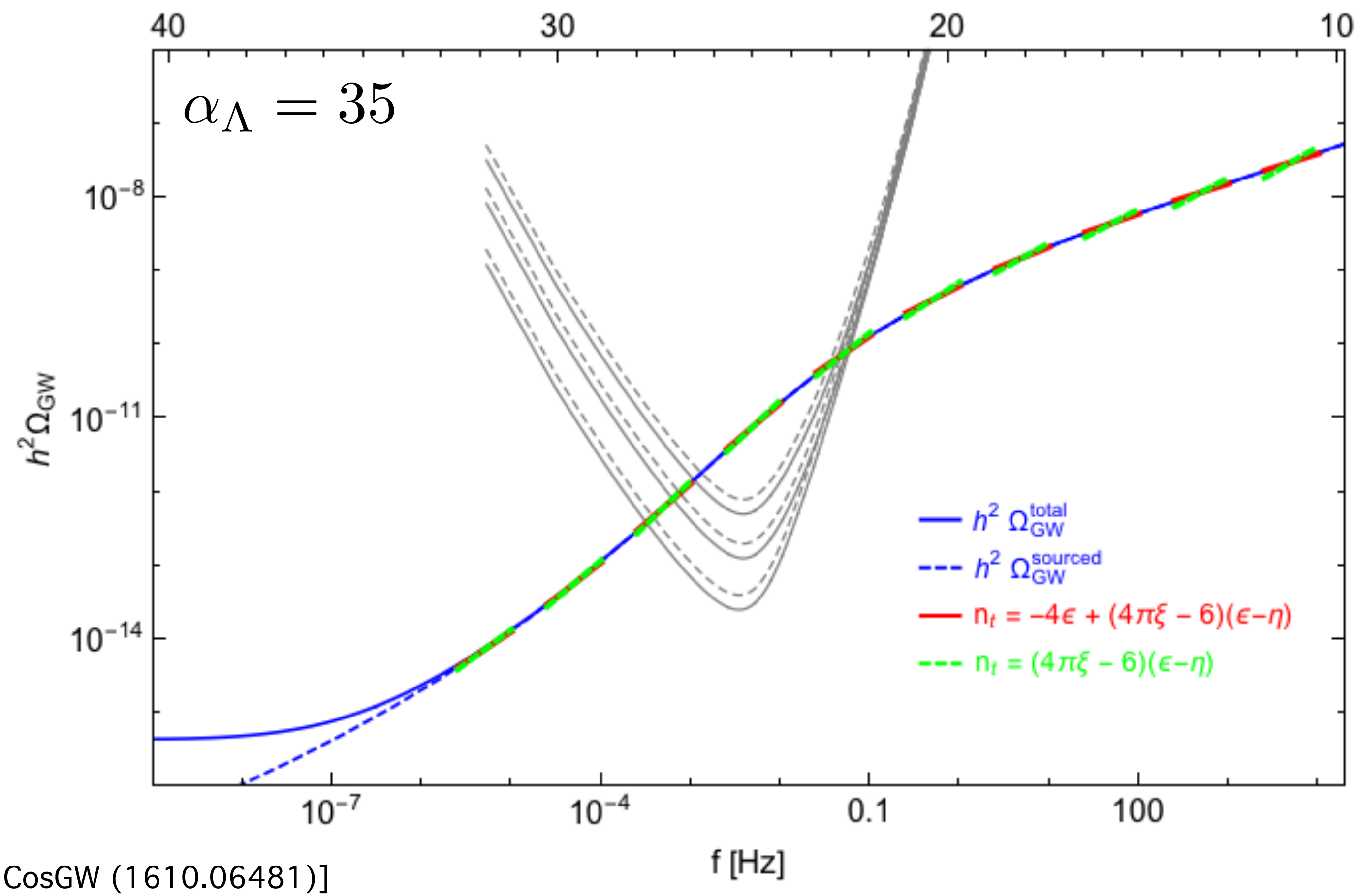


No linear growth in homogeneous
Clustering ~1eF

ummary & Conclusions 2

- Universal behaviour
  - Extension of the inflationary phase
  - Magnetic slow-roll
  - Scale dependence and UV sensitivity (not shown)
- Homogeneous does not give a good description in any case
- Case by case studies required, relevant for phenomenology: GWs, CMB scales (?)

. GW generation @ inflation: Asses possible observability



. CosGW (1610.06481)]

GW signal from particle production during axion inflation

- Relevant parameters?
- Any insight from local backreaction procedure? (Lattice)
- Would the signal be detectable?

. GW generation

Continuum

$$_j(\mathbf{x},t) + 3H\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = \frac{2}{m_p^2}\Pi_{ij}^{\rm TT}(\mathbf{x},t)$$

$$^{\rm TT}_{ij} = (\partial_i\phi\partial_j\phi - E_iE_j - \frac{1}{a^2}B_iB_j)^{\rm TT}$$

Lattice

$$\ddot{u}_{ij}(\mathbf{x},t) + 3H\dot{u}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}u_{ij}(\mathbf{x},t) = \frac{2}{m_p^2}\Pi_{ij}^{\rm eff}(\mathbf{x},$$

[J. Garcia-Bellido, D.G. Figueroa, A. Sastre (0707.08

$$h_{ij}(\mathbf{k},t) = \Lambda^{\rm L}_{ij,kl}(\hat{\mathbf{k}})u_{kl}(\mathbf{k},t)$$

In Fourier
Recover physical dc

$$\Lambda^{\rm L}_{ij,kl}(\hat{\mathbf{k}}) = P^{\rm L}_{ik}(\hat{\mathbf{k}})P^{\rm L}_{jl}(\hat{\mathbf{k}}) - \frac{1}{2}P^{\rm L}_{ij}(\hat{\mathbf{k}})P^{\rm L}_{kl}(\hat{\mathbf{k}})$$

TT pro

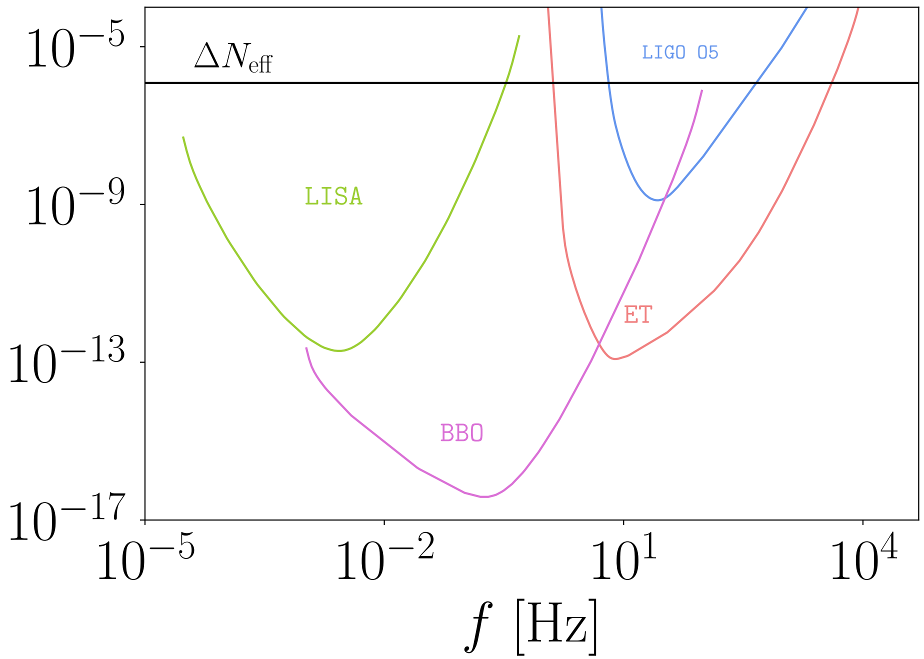
CosmoLattice
A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

–Technical Note II: Gravitational Waves–
Written on May 6, 2022
(Corrected on June 20, 2023)

CosmoLattice
A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

– Technical Note III –
Gravitational Waves from U(1) Gauge Theories
Written on June 16, 2023

. GW generation @ inflation: procedure



Possible observability at low frecuencies:
deep inside inflation

Strategy:

1- Consider a set of strong couplings

$$\alpha_\Lambda = 15,\ 20,\ 25,\ 30,\ 35$$

2- Divide spectral range into ... lly superimpos pat...

3- Every patch ... ub-Hubble to very Hubble

4- Extract GW power spectrum

5- Concatenate all patches: whole cosmic evolution

Work in progress

1st step

. GW generation @ inflation: procedure

ome notes on the procedure:

1- Preheating bounds $\alpha_\Lambda \lesssim 14$

[Adshead et al (1909.12842) & (1909.12843)]

Might be affected by different phenomena at the end on inflation (PBH dominance…)
We keep open the parameter window

2- Well inside inflation, negligible backreation.
Backreaction less solution valid until onset of non-linearities

3- GW generated only through the tachyonic excitation of the gauge modes

Back to

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

$$\ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\vec{E}\cdot$$

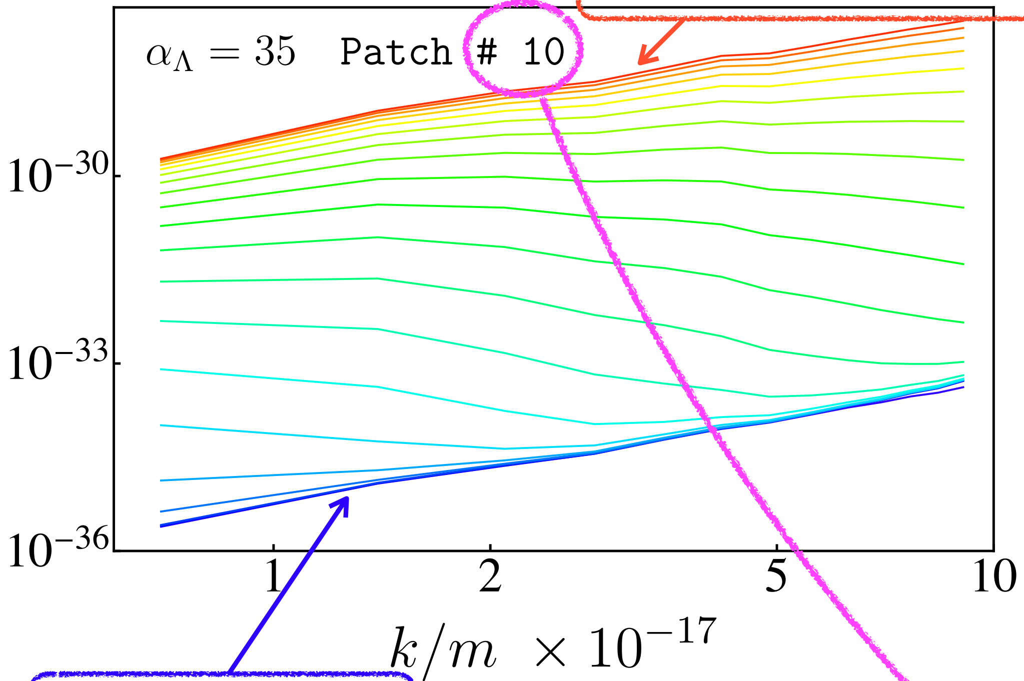
$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{\alpha_\Lambda}{a m_p}\left(\dot{\phi}\vec{B} - \vec{\nabla}\phi \times\right.$$

$$\ddot{a} = -\frac{a}{3m_p^2}\left(2\rho_{\rm K} - \rho_{\rm V} + \rho_{\rm EM}\right)$$

$$\Pi_{ij}^{\rm eff} \approx E_i E_j - \frac{1}{a^2} B_i B_j$$

. GW generation @ inflation: procedure

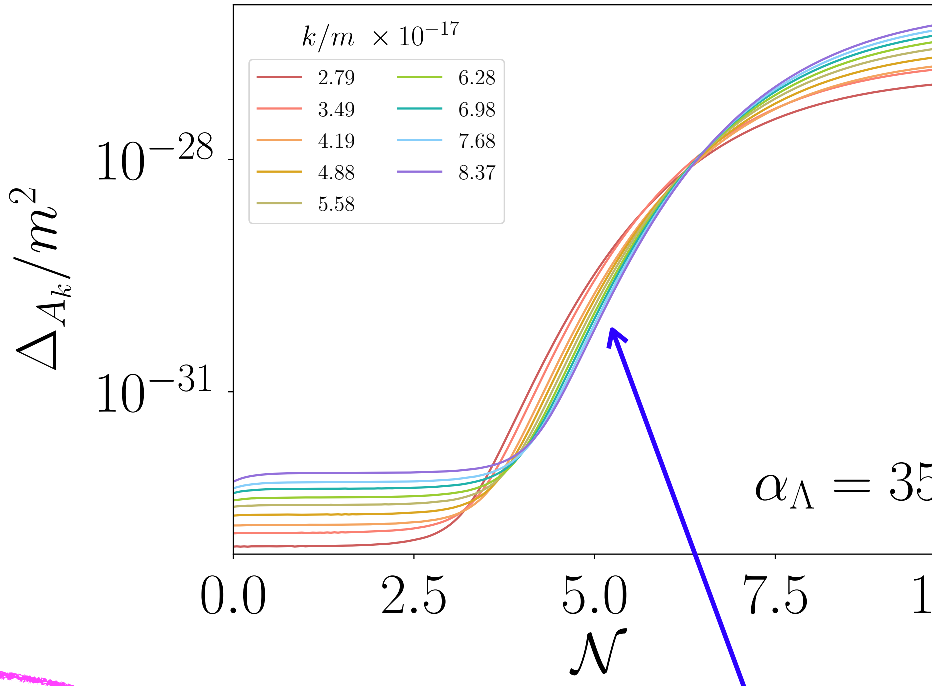
super-Hubble excitation
$\Delta\mathcal{N} = 10$



IC: vacuum solution
$\propto k^2$

~30-40 patches of
N~200 in total until end of
slow roll inflation

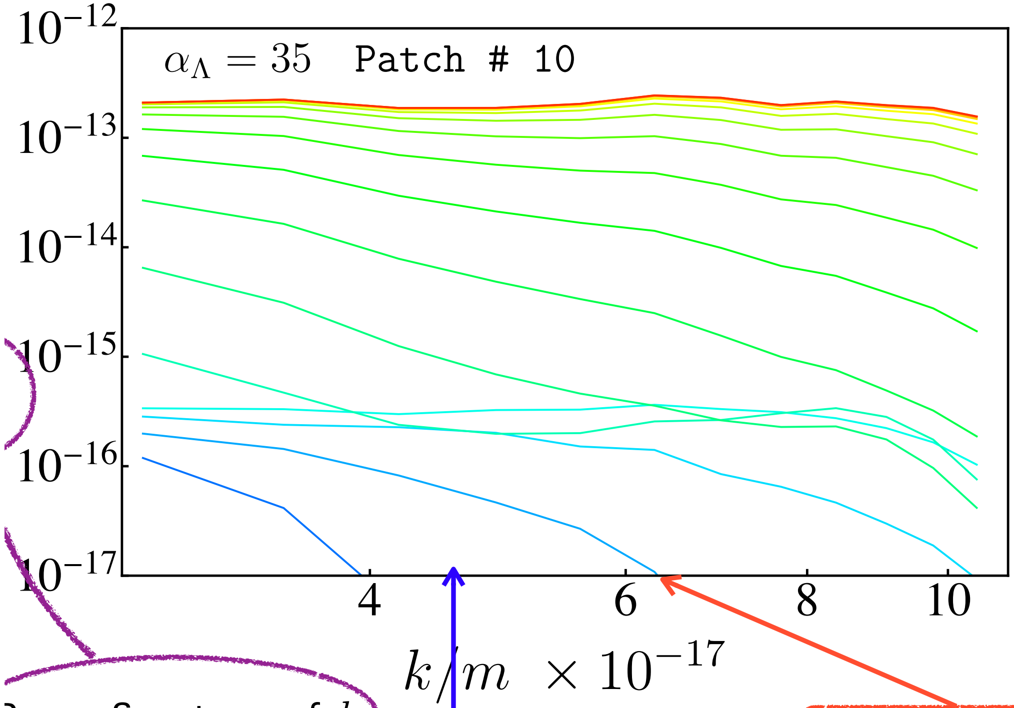
Mode by mode evolution

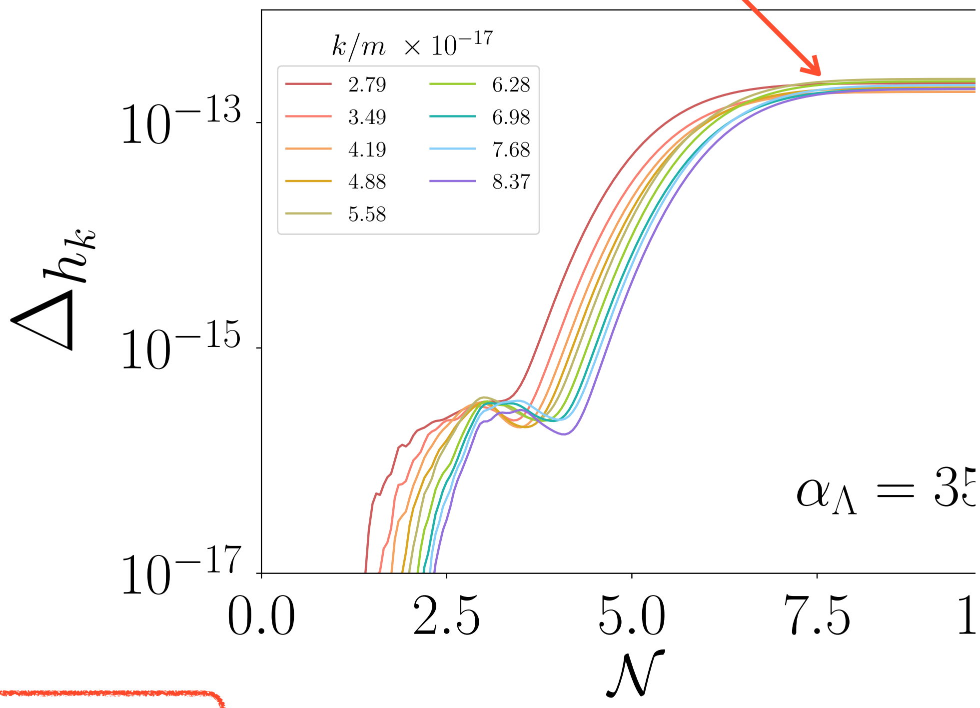


Tachyonic growth

. GW generation @ inflation: procedure

Saturation at super-Hubble scales



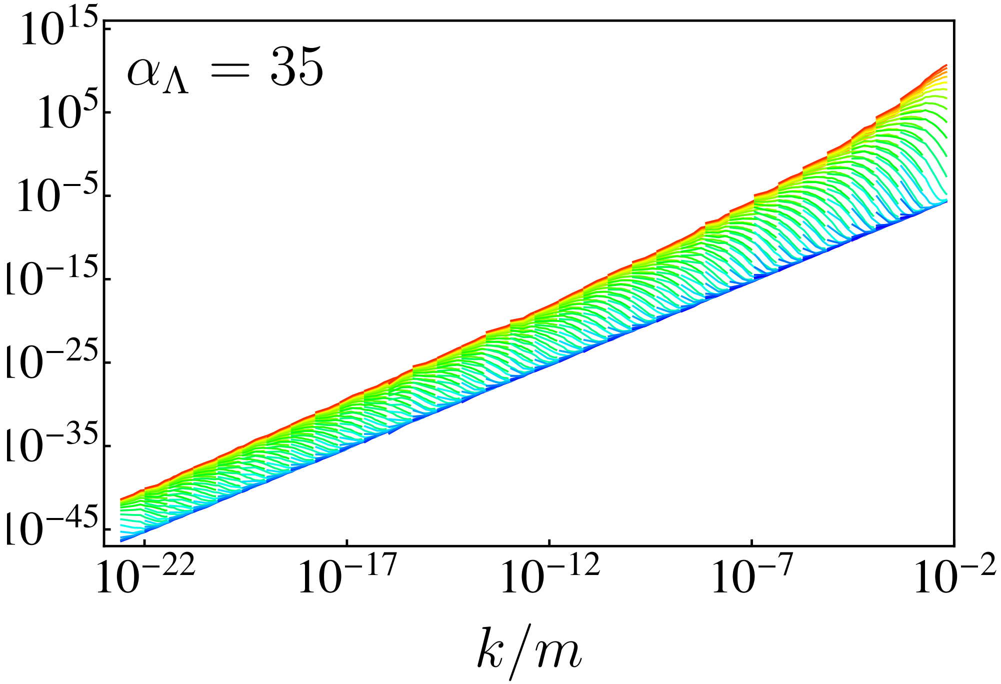


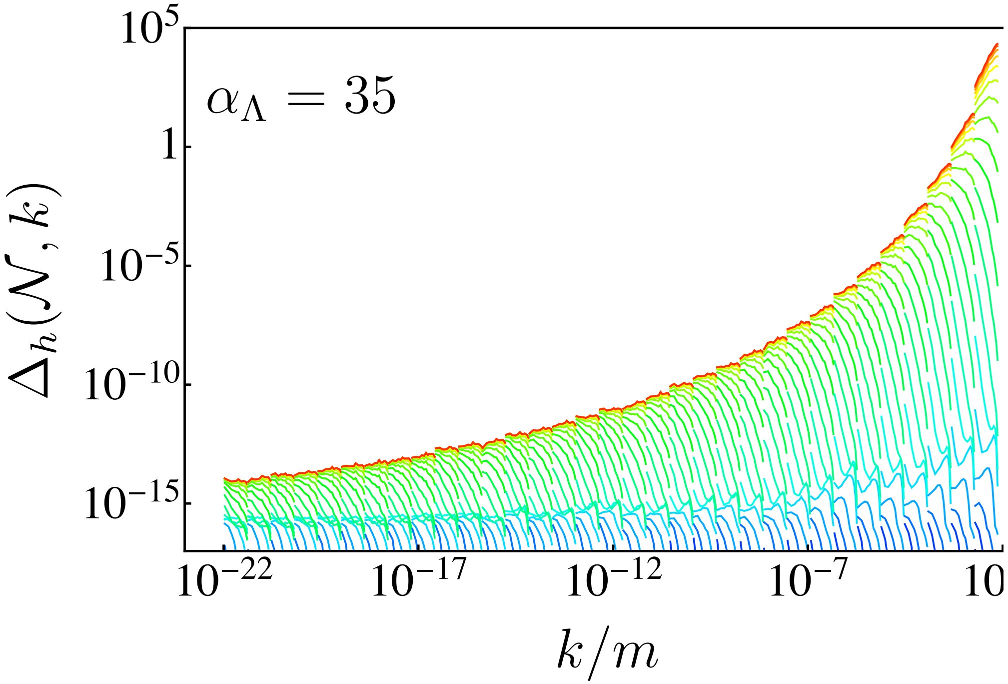
Power Spectrum of h

ICs starting at 0

Sequential cut-off in the source:
remove non-classical contribution

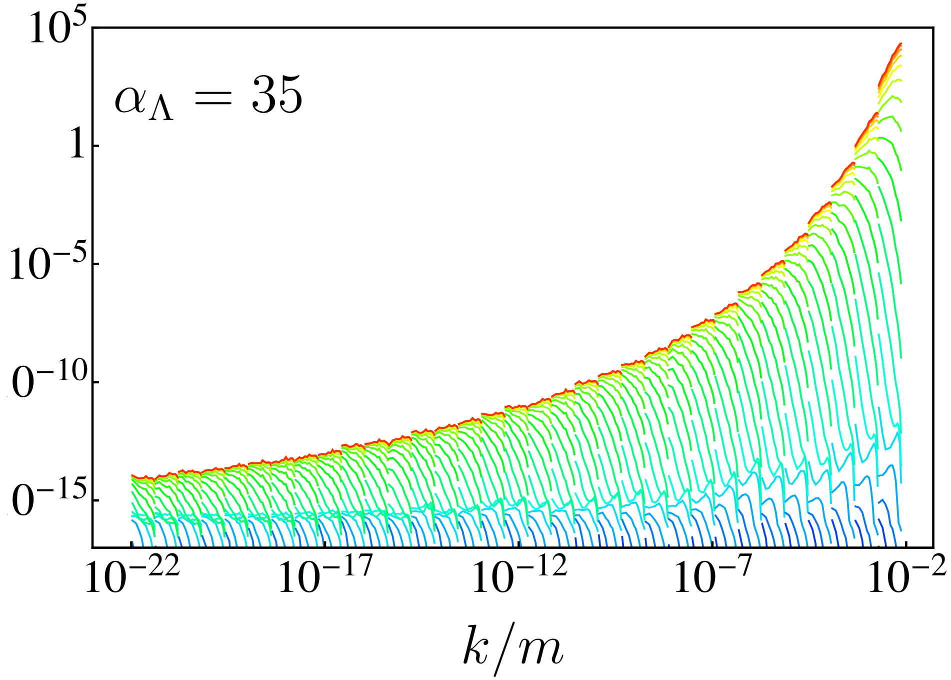
. GW generation @ inflation: procedure

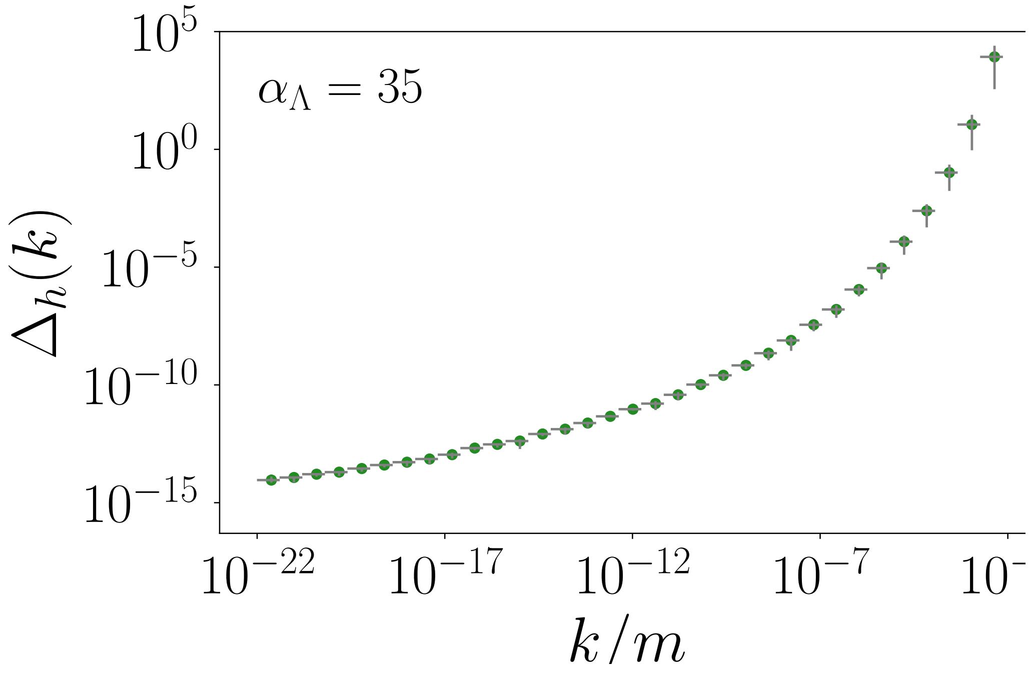




. GW generation @ inflation: procedure

Re-cast information:
saturated super-Hubble value
with “error” bands for every patch

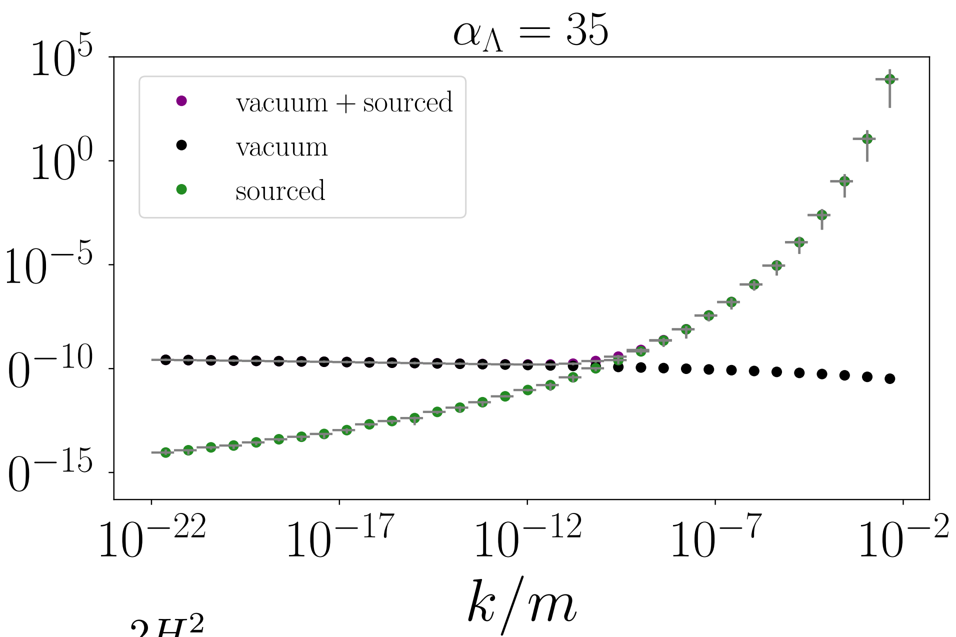


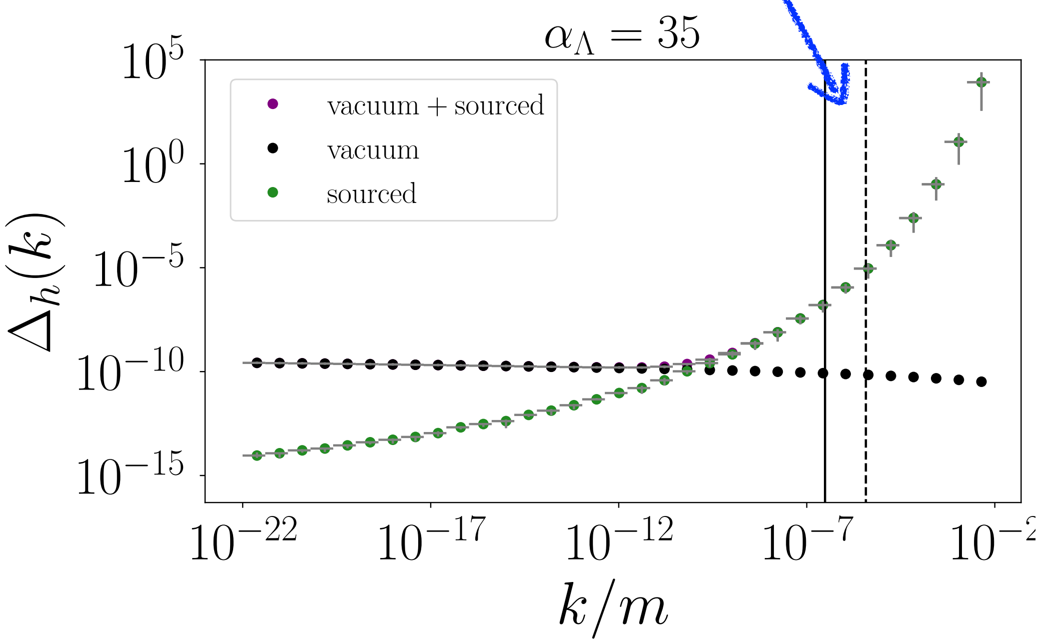


. GW generation @ inflation: procedure

How does this compare with the vacuum prediction?

Onset of non-linearities
EM ~1%





$$ {}^{\rm vac}_{h} = \frac{2H^2}{\pi^2 m_{\rm pl}^2} $$

All the way up to end of slow roll inflation

. GW generation @ inflation: today

rom comoving wavenumber to frequency domain

edshift affected by:

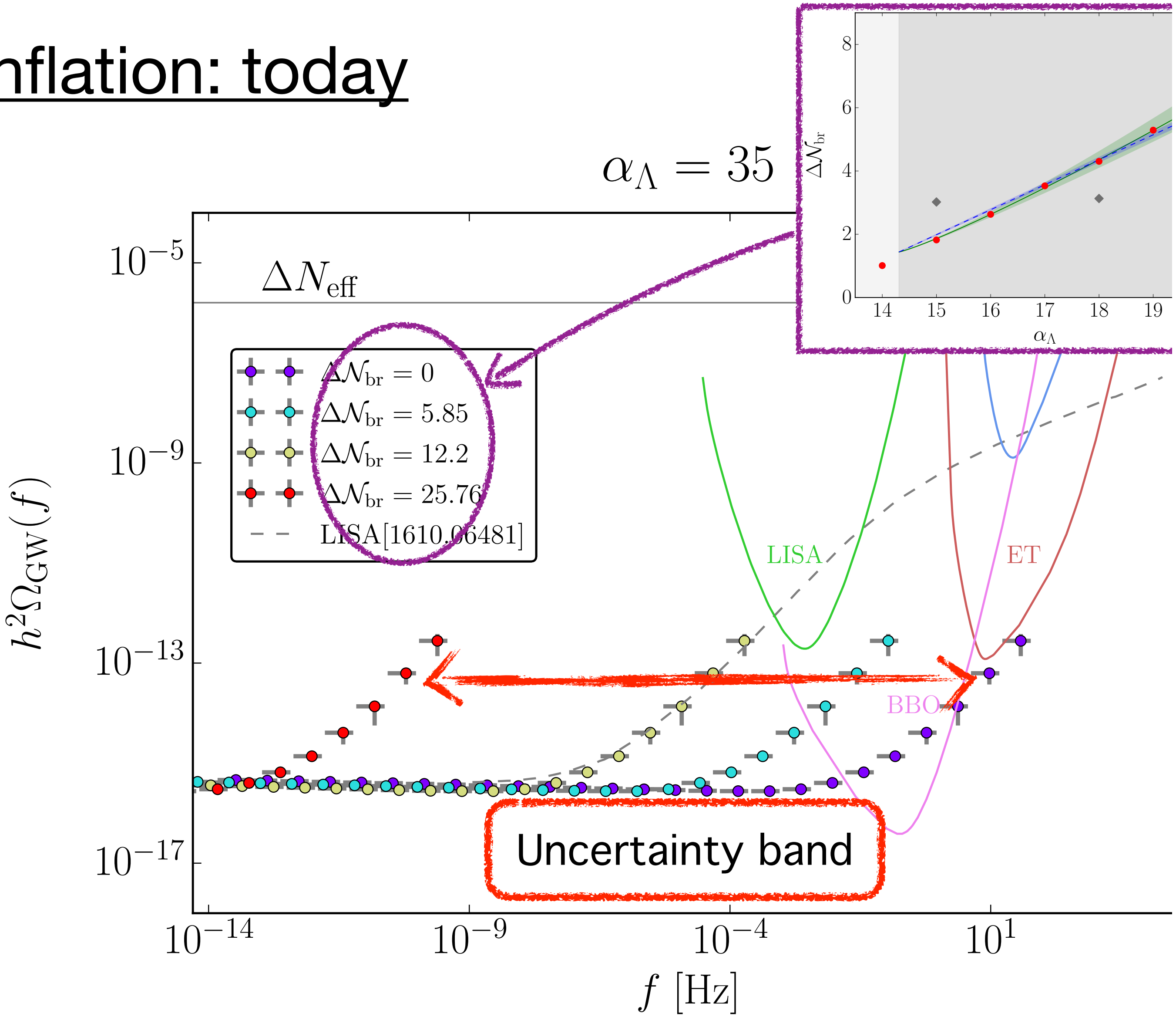
1- Number of extra e-folds in inflation

$$\Delta\mathcal{N}_{\rm br}$$

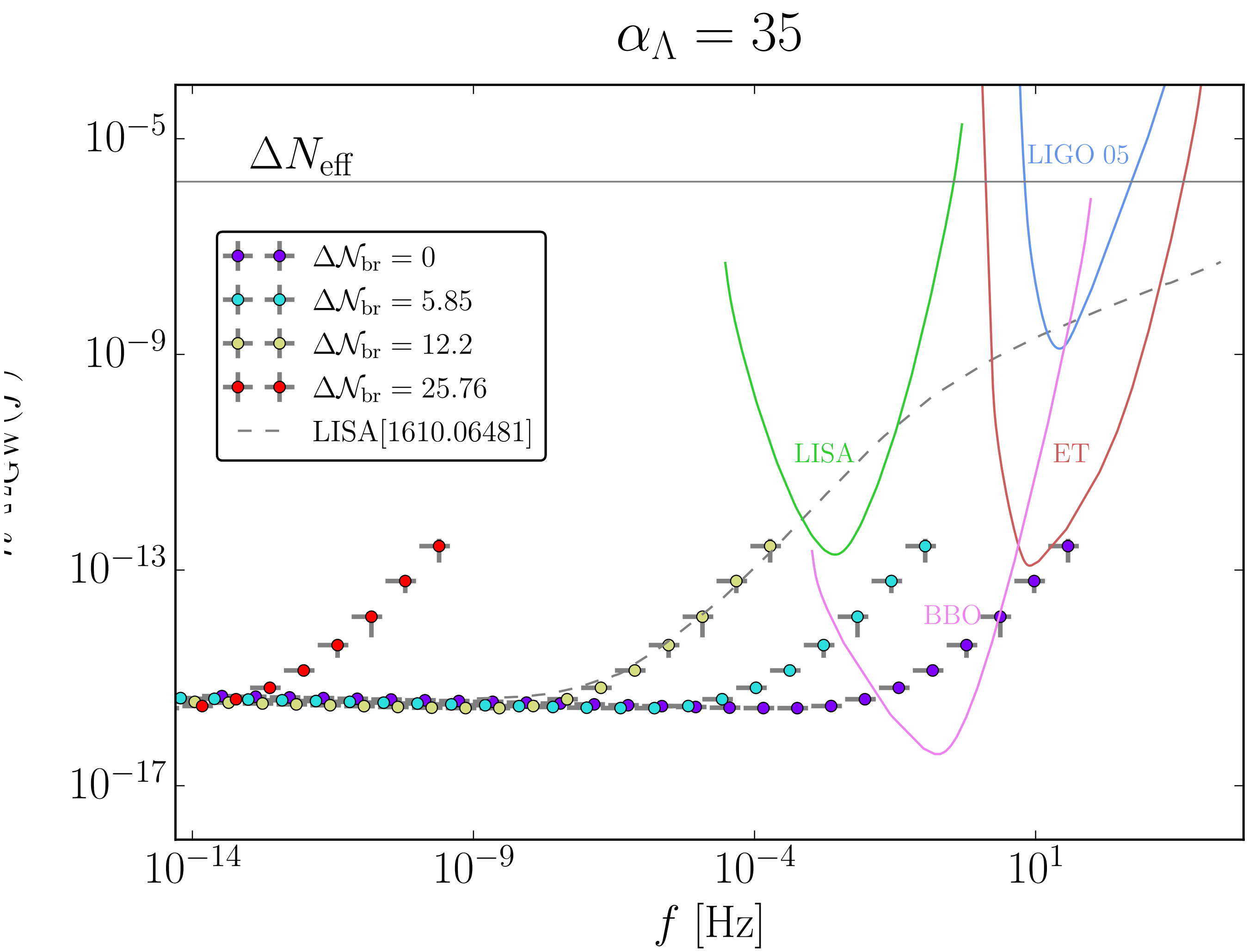
2- Details of evolution from the end of inflation to RD

$$\Delta\mathcal{N}_{\rm RD} \sim 2$$

Exact numbers only from lattice simulations



. GW generation @ inflation: today



Huge uncertainty in the prediction

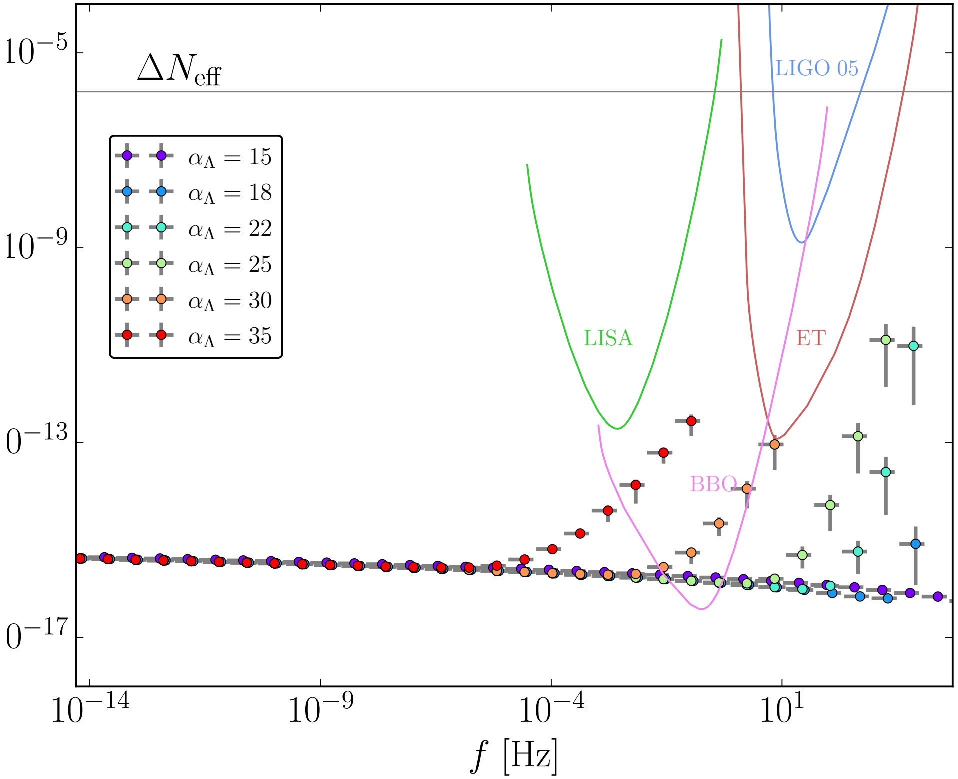
Detectability subject to strong backreaction deta until the end of inflation

Dedicated lattice studies needed

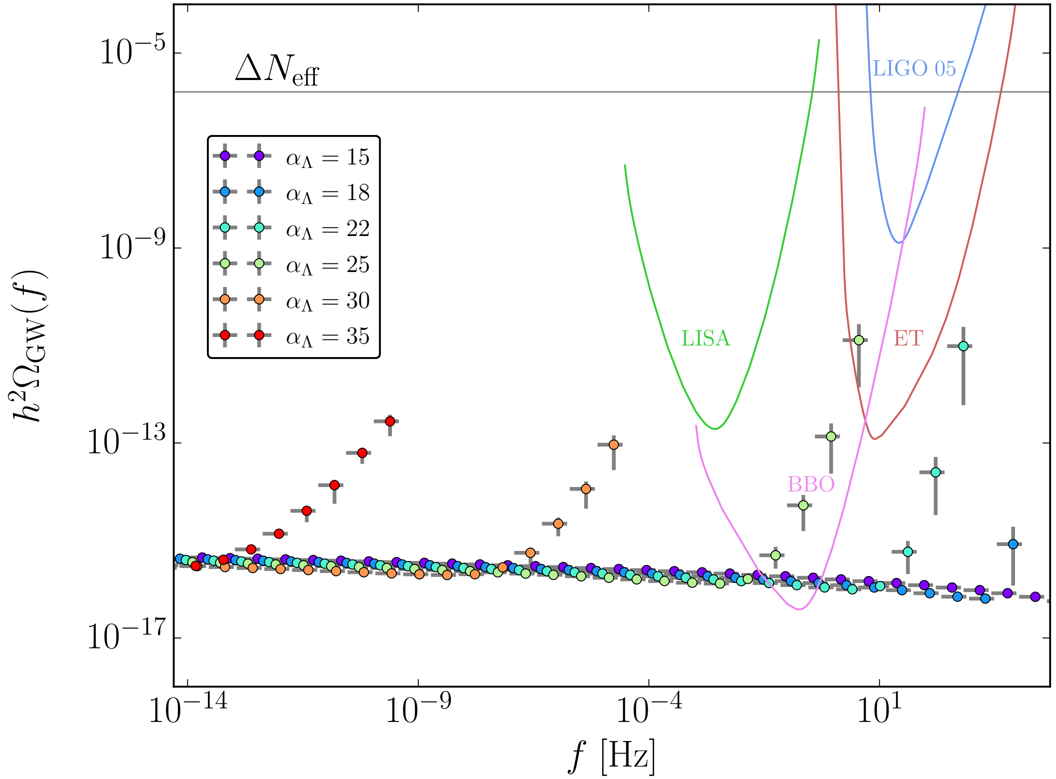
. GW generation @ inflation: today

Subject t
non-linearit

Lower limit



Upper limit



ummary & Conclusions 3: GWs in axion inflation

ealistic detectability prospects are subject to:

Accurate gravitational wave (GW) production modelling.

Accurate understanding of cosmic history:

- Non-trivial effects during strong backreaction not only affect high frequencies, but the whole signal.
- Dedicated full lattice simulation for strong couplings are required.
- We intend to apply the same patching procedure (technically difficult)

Chirality?

Preheating bounds?

inal remarks and questions

- Electromagnetic slow roll as a featuring characteristic of SBR
- General and independent of the potential
  - Some potentials might be used to test the range of validity of such regime
  - What's beyond EM SR for even larger couplings?
- GW detection prospects depend strongly on the full local backreaction dynamics
  - Indirectly: extension of inflation
  - Directly: full spectrum shape
- Many new questions (less certainty), not the end of the story
  - Case by case study required: potentials? Couplings?
  - Exact predictions only from lattice simulations.
    - Also in other models: SU(2)...