Equation of state during (p)reheating and its observational implications

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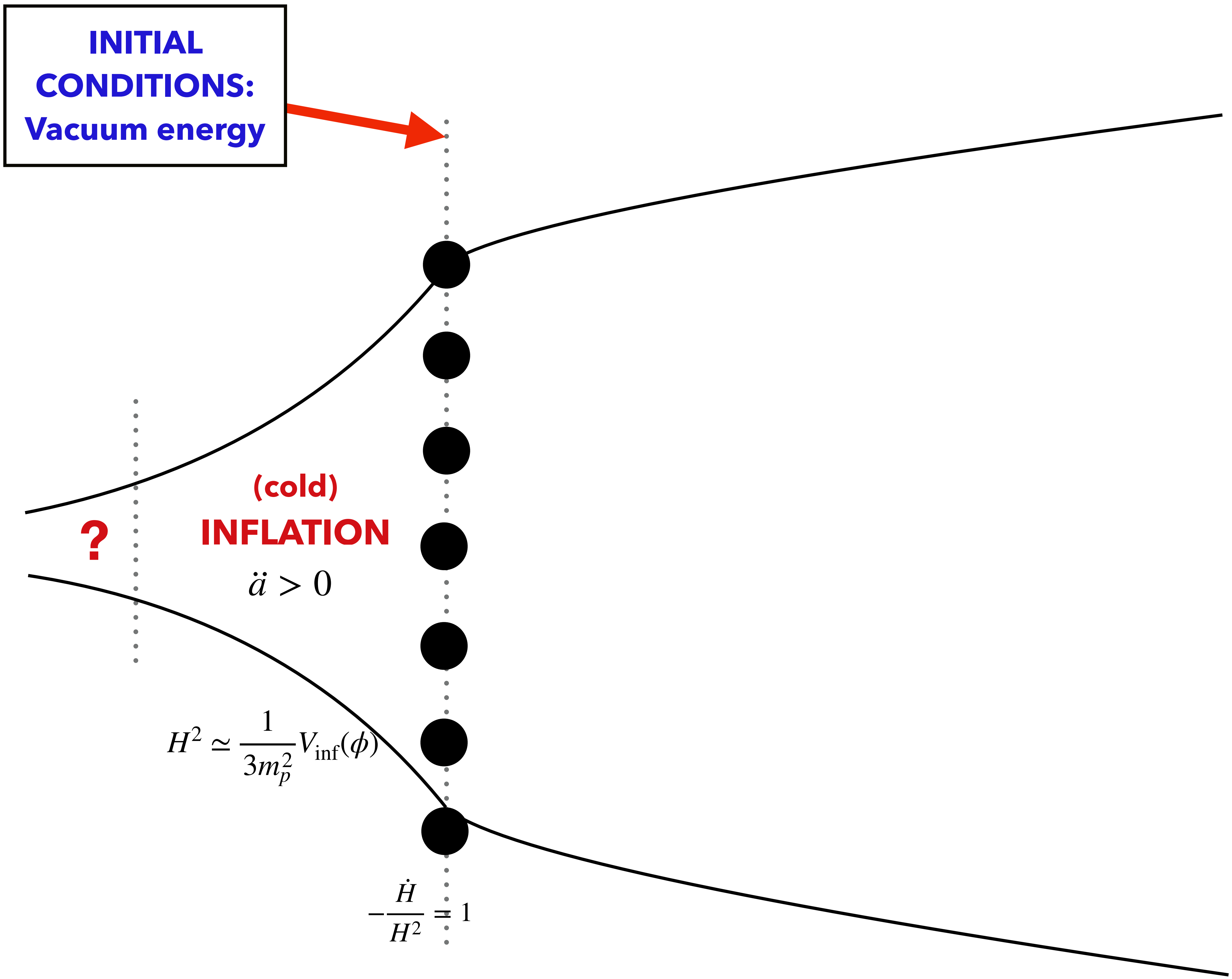
Numerical simulations of Early Universe sources of GWs - NORDITA - 1st August 2025

History of the early universe



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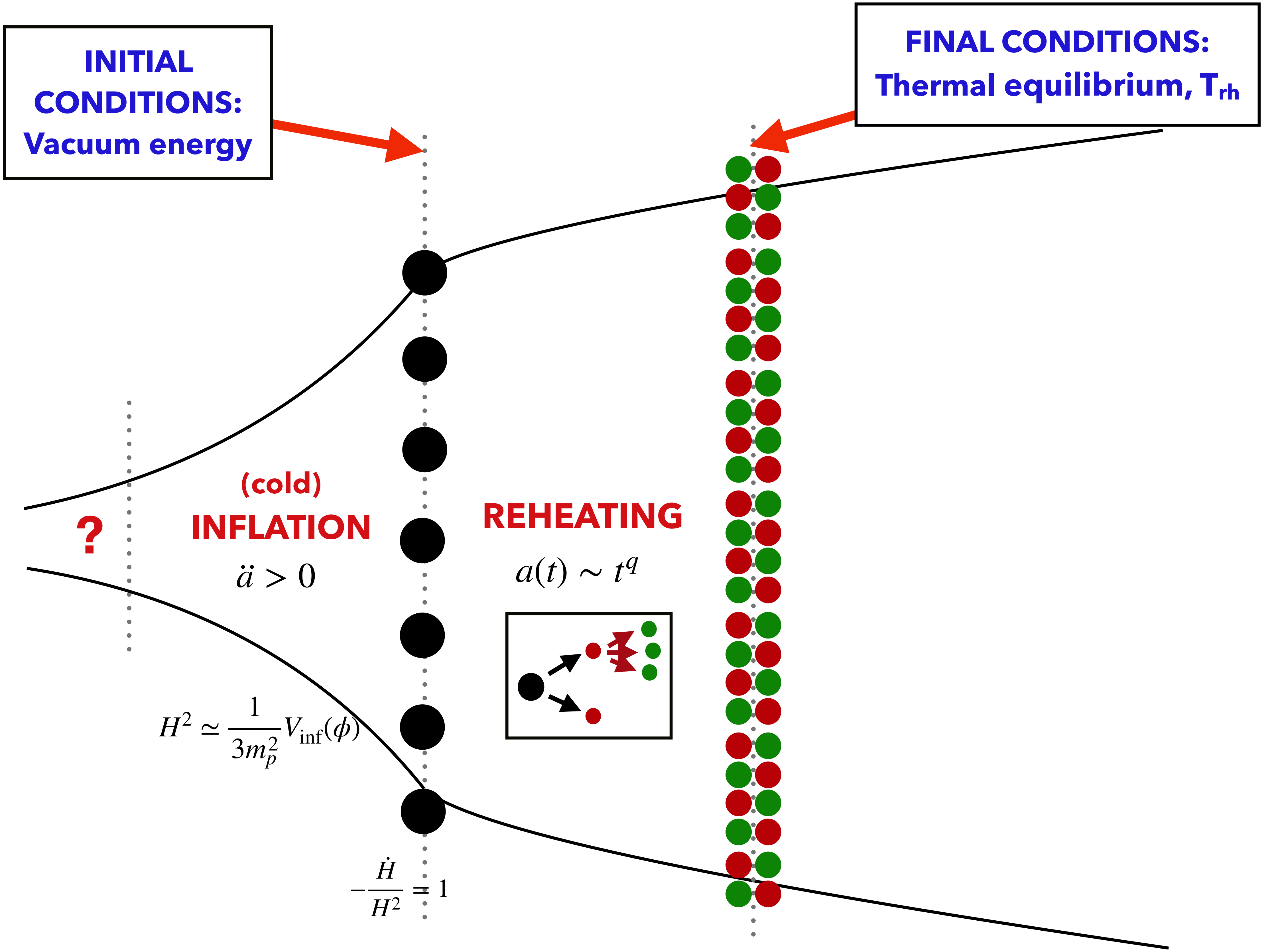
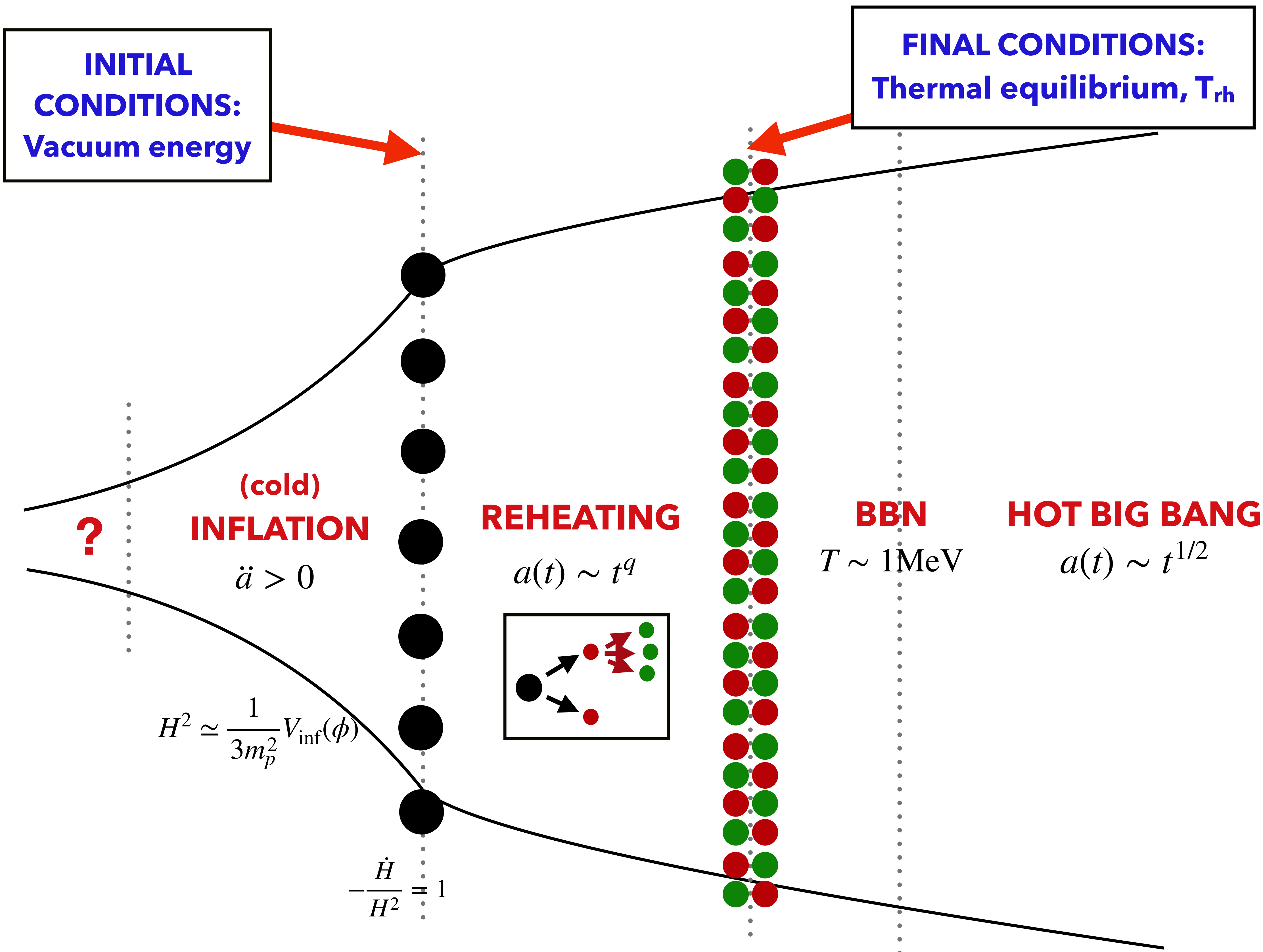
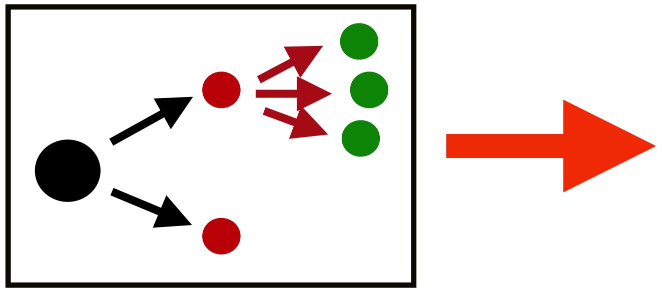


History of the early universe



Reheating



$$\mathcal{L} = \mathcal{L}(\phi, \varphi_i, \psi_j, A_\mu, h_{\mu\nu}, \ldots)$$ **??**

- **REHEATING** must guarantee a complete energy transfer from the inflaton to a thermal distribution of SM Particles before BBN.

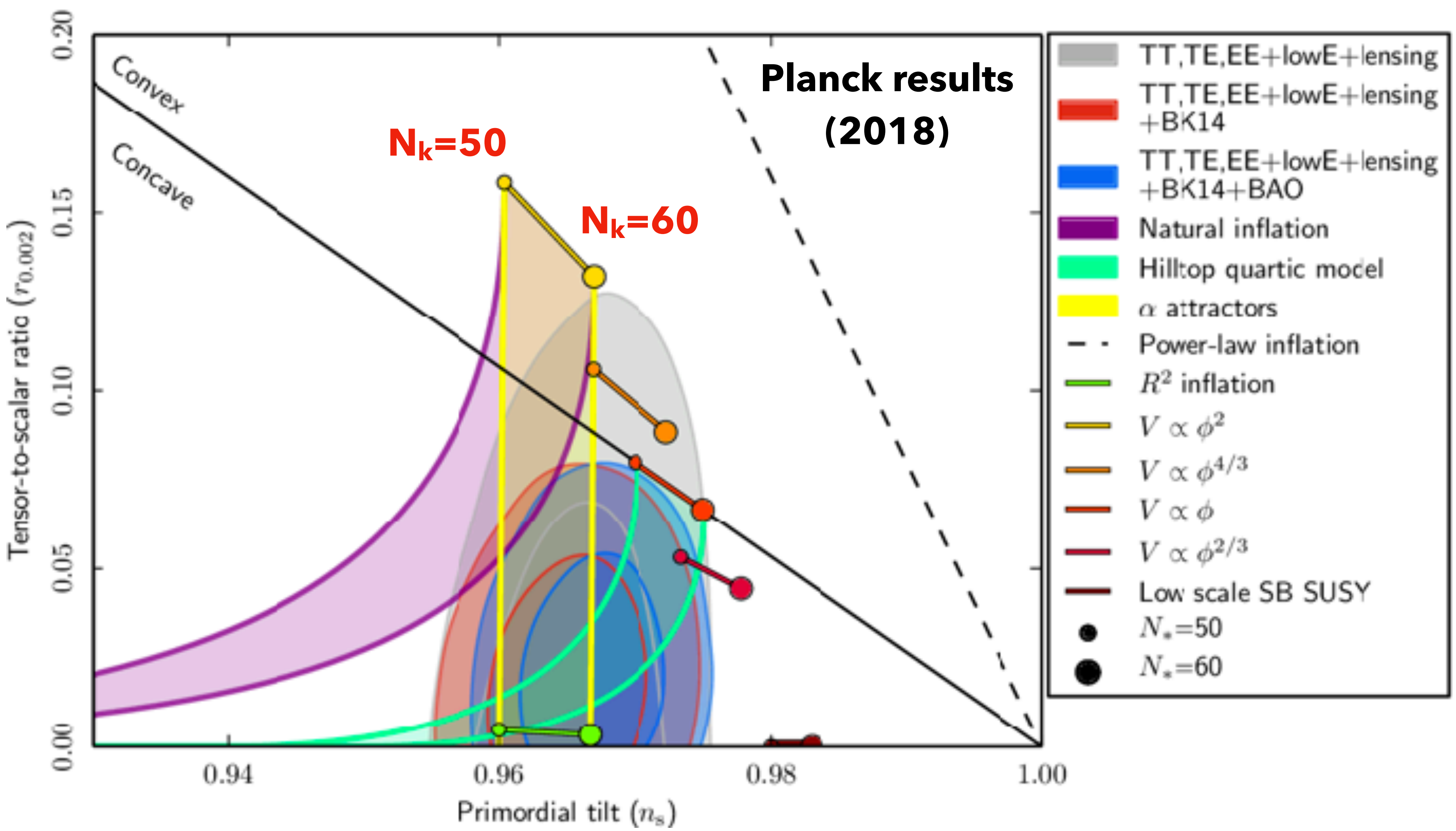
- **PREHEATING** (field instabilities due to non-perturbative effects) may dominate the initial post-inflationary dynamics and lead to inflaton fragmentation.

WHEN DOES THE UNIVERSE BECOME RADIATION DOMINATED?

- Characterizing the equation of state during (p)reheating is crucial to determine inflationary CMB observables and the redshift of SGWBs from the early universe.

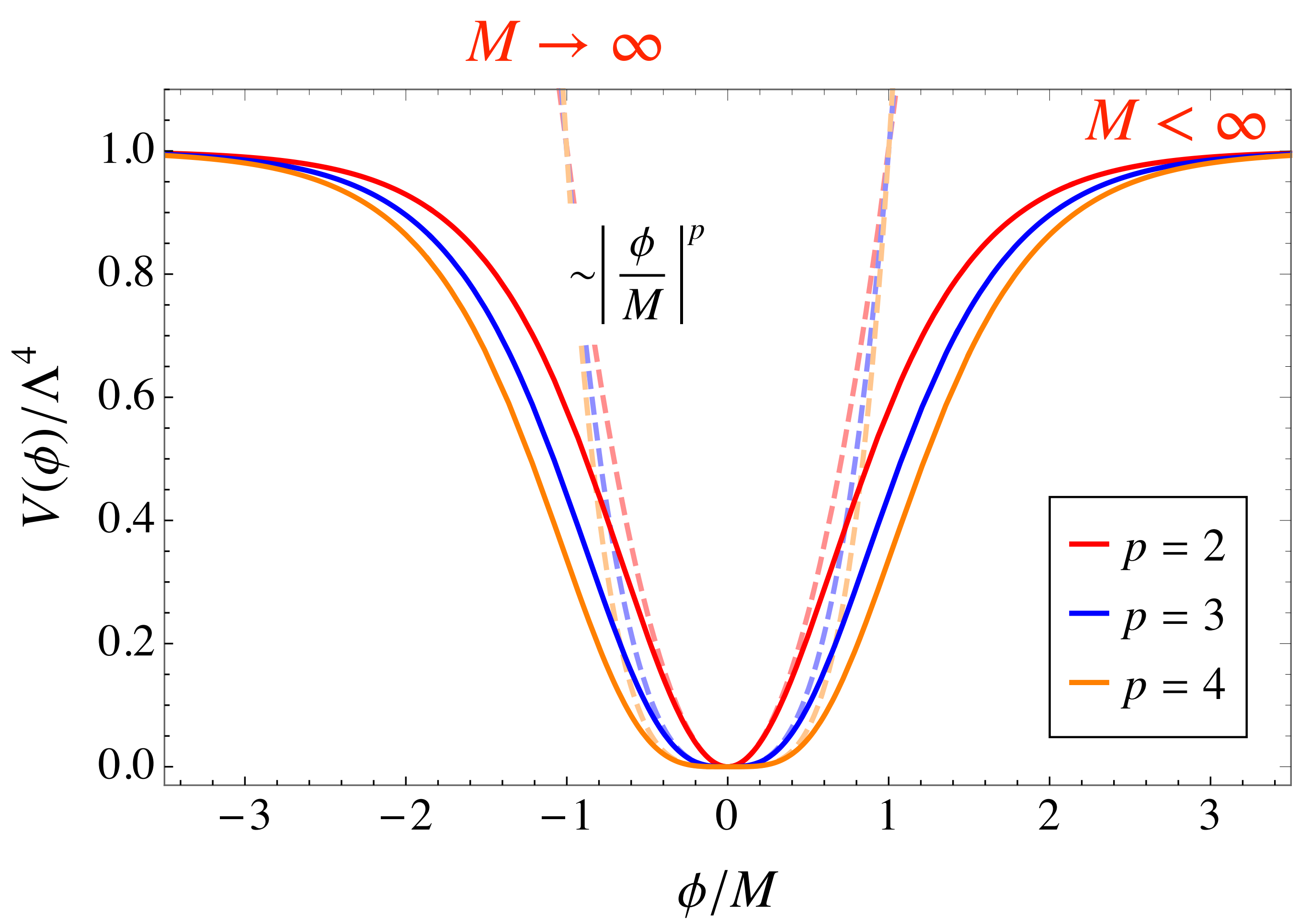
Inflationary potentials

N_k: number e-folds from the horizon crossing of the pivot scale (k_*=0.05 Mpc^{-1}) till the end of inflation



α-attractor T-model

$$V(\phi) = \frac{\Lambda^4}{2}\tanh^2\left(\frac{|\phi|}{M}\right)$$



- **CMB constraints:** $r_{0.05} < 0.036$ → $M \lesssim 10 m_p$

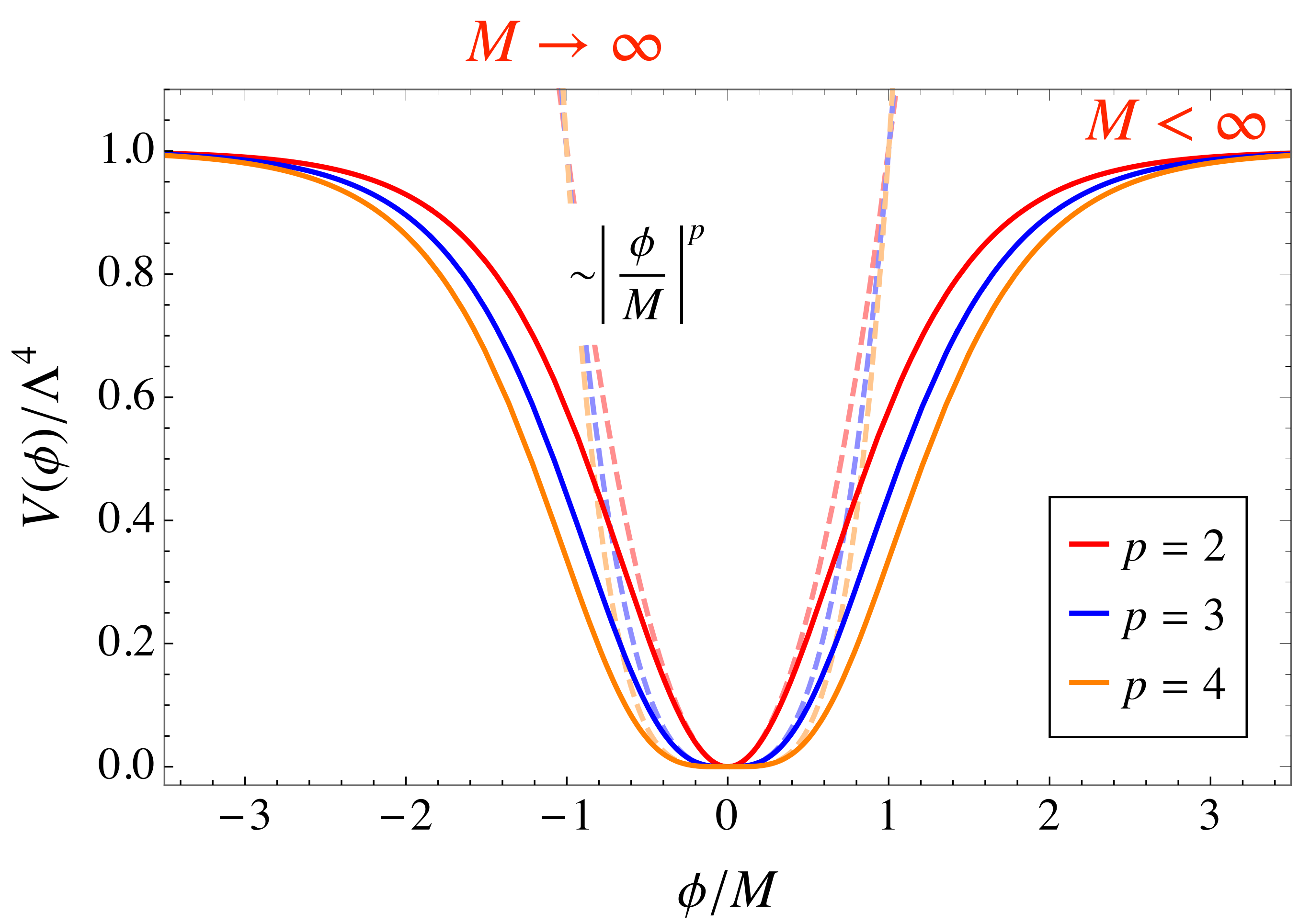
Planck (2018)

α-attractor T-model

$$V(\phi) = \frac{\Lambda^4}{2}\tanh^2\left(\frac{|\phi|}{M}\right)$$

$$V(\phi) = \frac{1}{2}\frac{\Lambda^4}{M^2}|\phi|^2 + \mathcal{O}(|\phi|^4)\ldots$$

quadratic approximation



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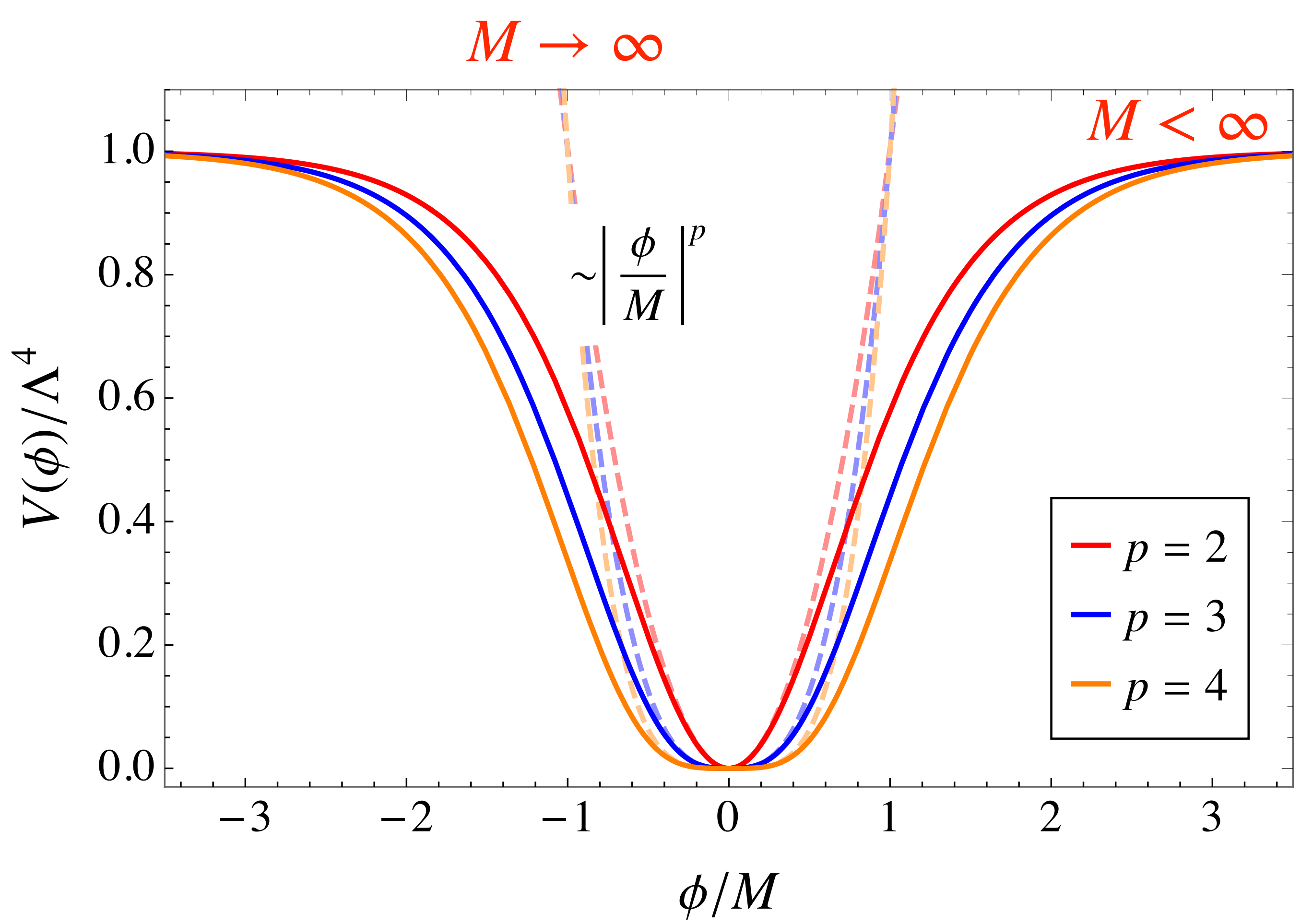
quadratic approximation



- **End of inflation:** $\epsilon_V(\phi_*) \equiv 1 \;\rightarrow\; \phi_* = \frac{1}{2}M \operatorname{arcsinh}\left(\sqrt{8}m_{\rm pl}/M\right)$
- **Inflection point:** $V_{,\phi\phi}(\phi_i) \equiv 0 \;\rightarrow\; \phi_{\rm i} = M \operatorname{arcsinh}\left(\sqrt{1/2}\right)$

Monomial approximation valid if: $\phi_{\rm i} > \phi_* \;\rightarrow\; M \gtrsim 1.6 m_{\rm pl}$

Potential

The need to reheat the universe naturally implies the necessity of interactions between the inflaton and other fields:

$$V(\phi, X) = \frac{\Lambda^4}{2}\tanh^2\left(\frac{\phi}{M}\right) + \frac{1}{2}h\phi X^2 + \frac{1}{2}g^2\phi^2 X^2 + \frac{1}{4}\lambda X^4$$

ϕ inflaton

X daughter field

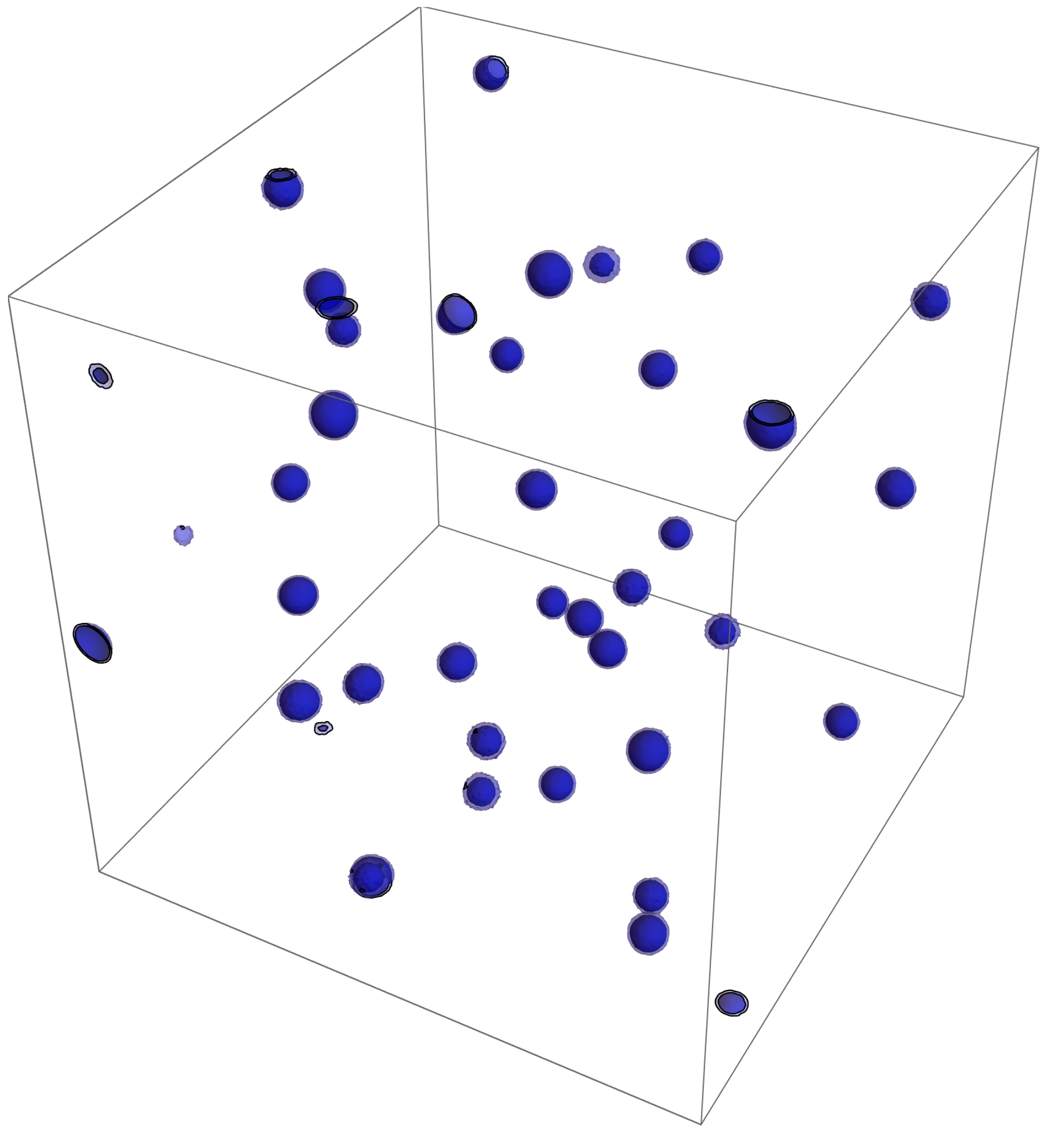
- $\frac{1}{2}h\phi X^2$: perturbative decay + preheating effects
- $\frac{1}{2}g^2\phi^2 X^2$: only preheating effects
- $\frac{1}{4}\lambda X^4$: necessary to stabilize the potential for h>0

Inflaton fragmentation

$$V(\phi, X) = \frac{\Lambda^4}{2}\tanh^2\left(\frac{\phi}{M}\right) + \frac{1}{2}h\phi X^2 + \frac{1}{2}g^2\phi^2X^2 + \frac{1}{4}\lambda X^4$$

$$M \lesssim 0.1 m_{\rm p}$$

Inflaton oscillates over the plateau region of the potential: inflaton fragments into **oscillons**



Amin et al (2011),
Zhou et al (2013),
Antusch et al (2017),
Lozanov & Amin (2019)
…

Inflaton fragmentation

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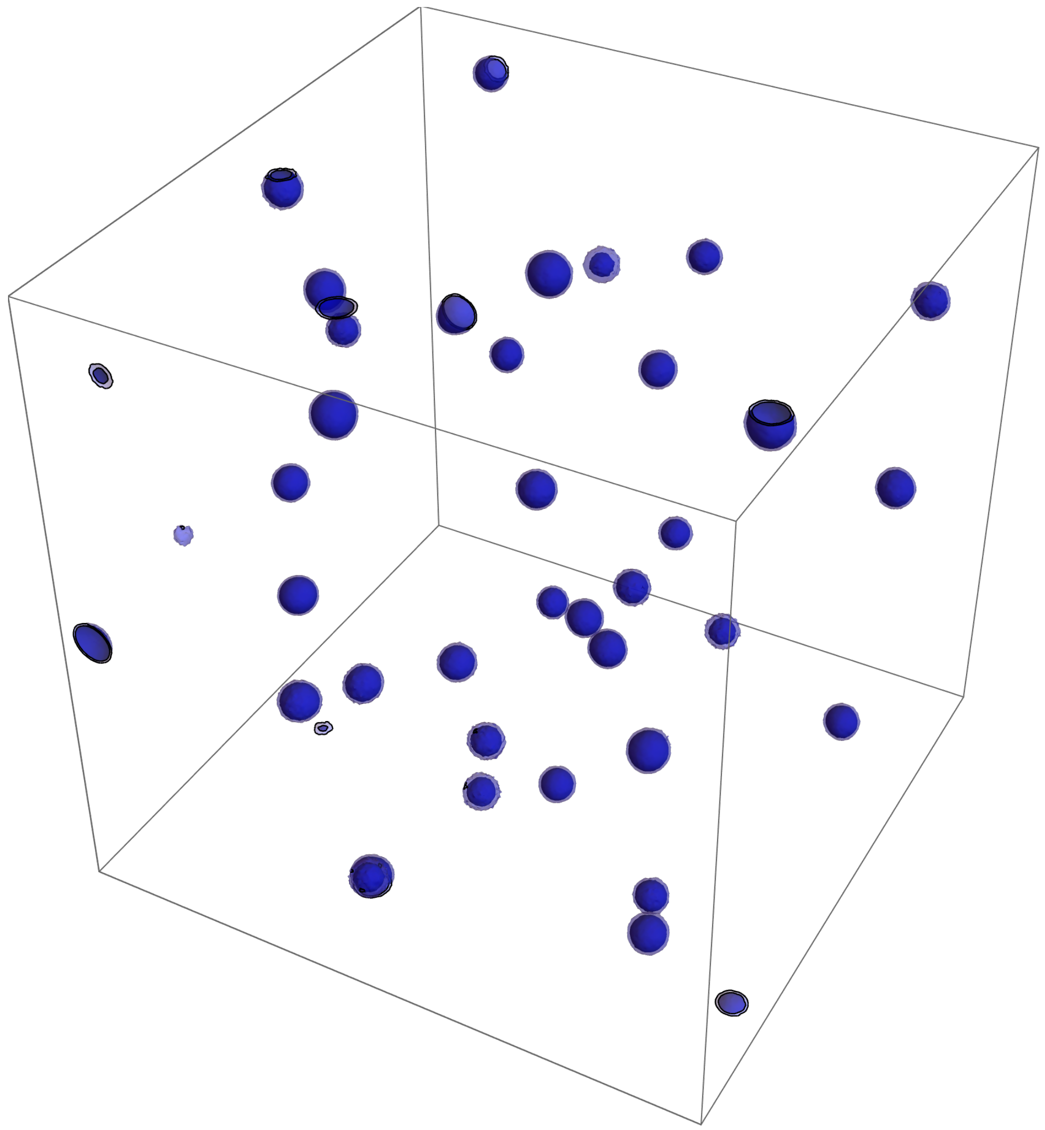
$$M \lesssim 0.1 m_{\rm p}$$

In the presence of inflaton interactions to other fields, **oscillons may form and survive** (or not) depending on the coupling strength

see S.S. Mishra talk!



Antusch & Orani (2015)
Shafi, Copeland, Mahbub, Mishra, Basak (2024)
Li, Yamaguchi, Zhang (2025)

Inflaton fragmentation

$$V(\phi, X) = \frac{\Lambda^4}{2}\tanh^2\left(\frac{\phi}{M}\right) + \frac{1}{2}h\phi X^2 + \frac{1}{2}g^2\phi^2 X^2 + \frac{1}{4}\lambda X^4$$

$$M \gtrsim m_{\rm p}$$

inflaton oscillations take place in the quadratic part of the potential, and **oscillons do not form**

we take the monomial approximation from now on

Inflaton fragmentation

$$V(\phi, X) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}h\phi X^2 + \frac{1}{2}g^2\phi^2X^2 + \frac{1}{4}\lambda X^4$$

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↓

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Inflaton fragmentation

$$V(\phi, X) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}h\phi X^2 + \frac{1}{2}g^2\phi^2X^2 + \frac{1}{4}\lambda X^4$$

In the absence of interactions, the **inflaton survives as an oscillating homogeneous condensate** with a decaying amplitude.

$$\phi(t) \sim \frac{\phi_*}{a^{3/2}}\cos(m_\phi t)$$

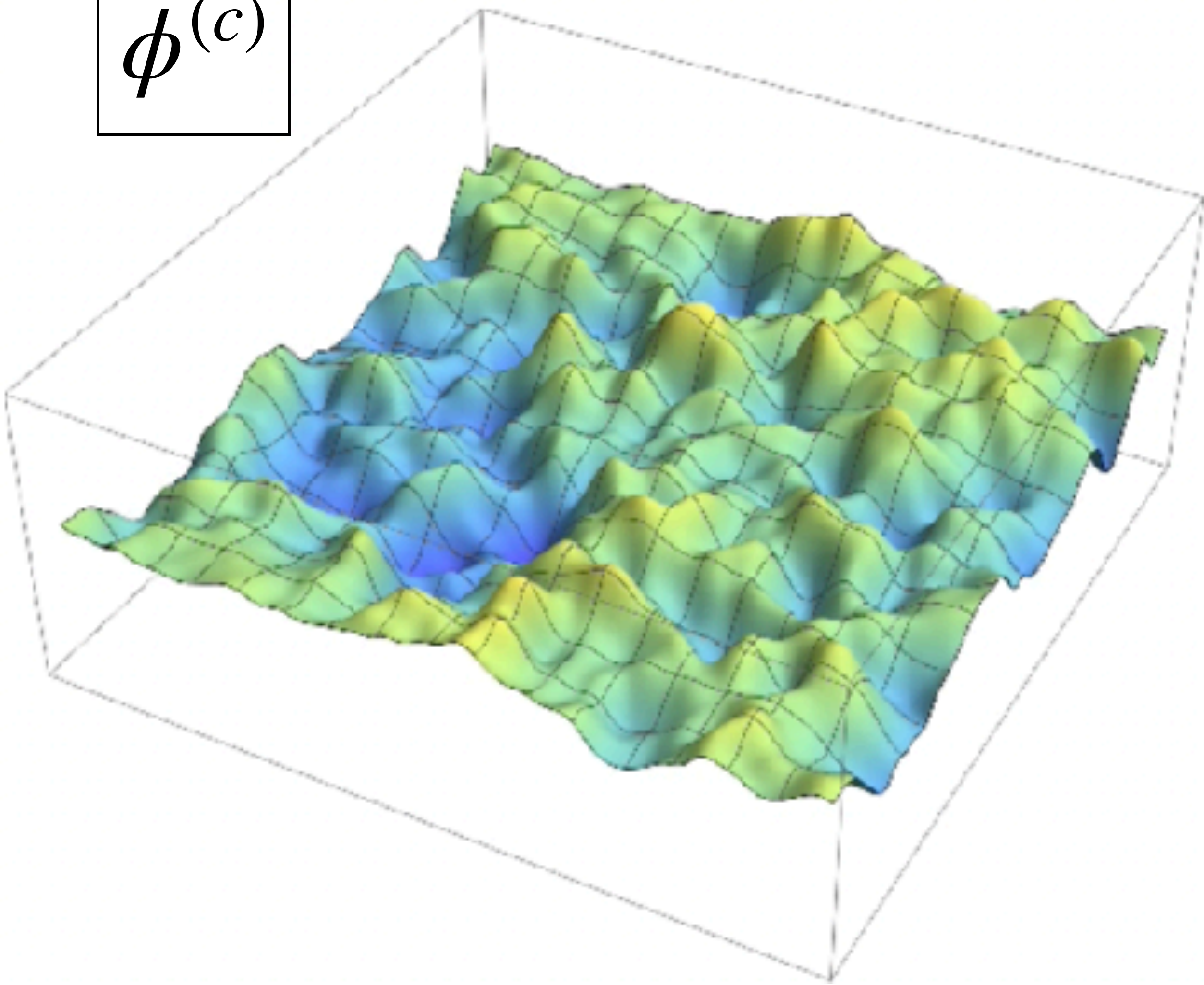
The universe behaves as **matter-dominated.**

$$\bar{w}_{\text{hom}} \equiv \frac{\langle p_\phi \rangle_{\text{osc}}}{\langle \rho_\phi \rangle_{\text{osc}}} = 0$$

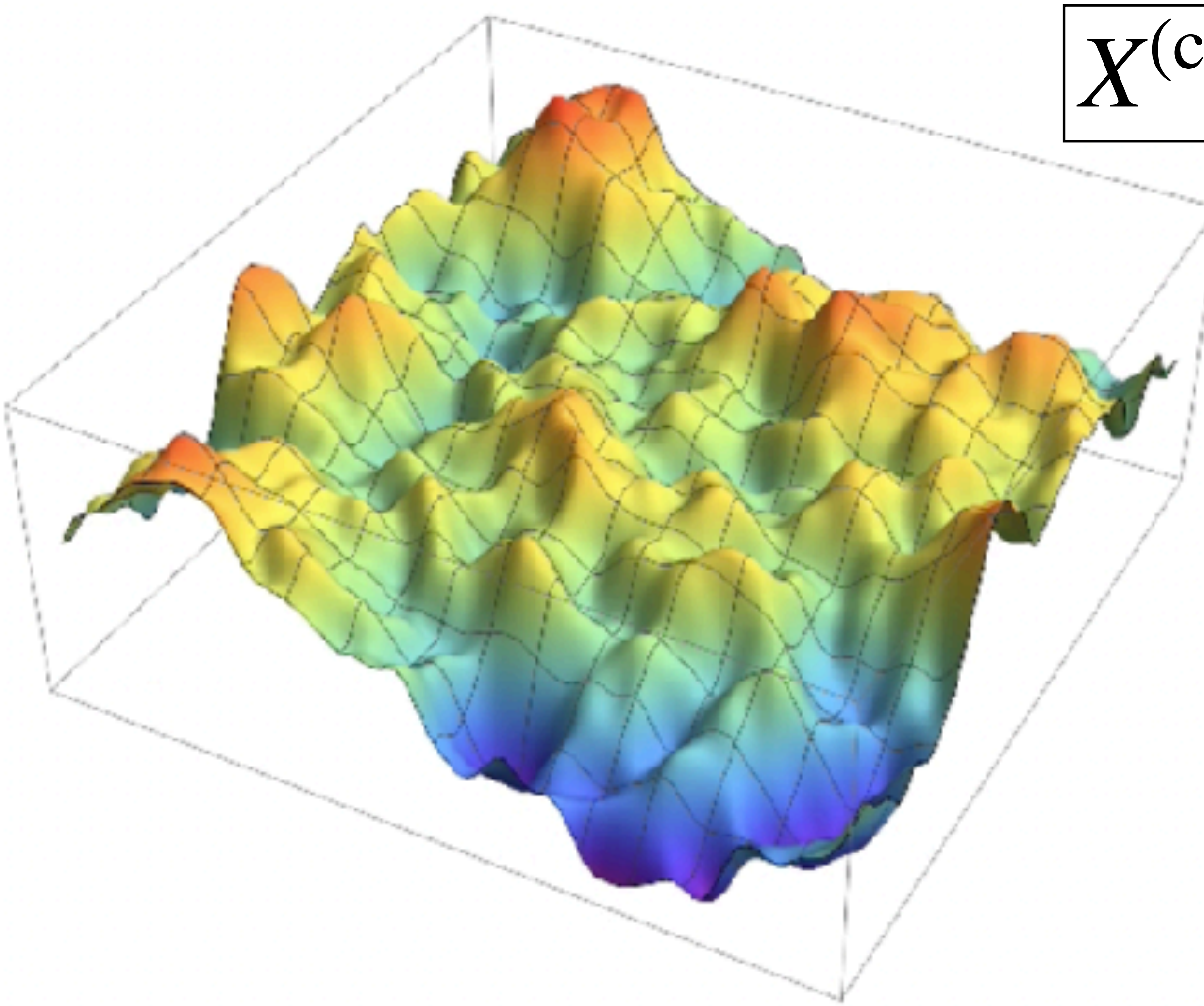
Inflaton fragmentation

$$V(\phi, X) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}h\phi X^2 + \frac{1}{2}g^2\phi^2X^2 + \frac{1}{4}\lambda X^4$$

$\phi^{(c)}$



$X^{(c)}$



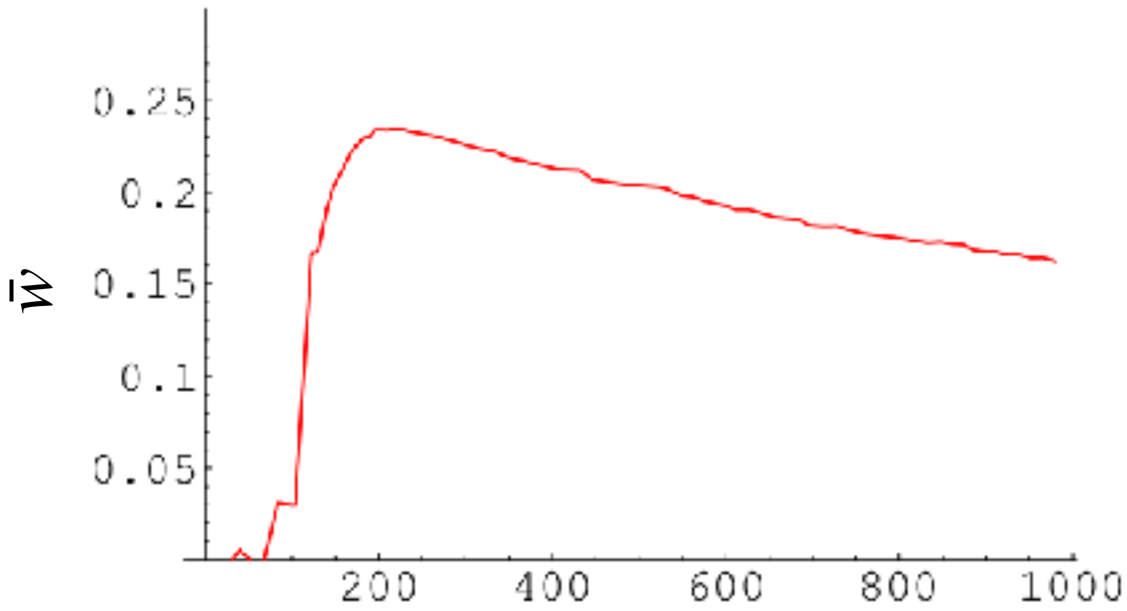
The inflaton fragments by resonant effects
for sufficiently strong interactions

Inflaton fragmentation

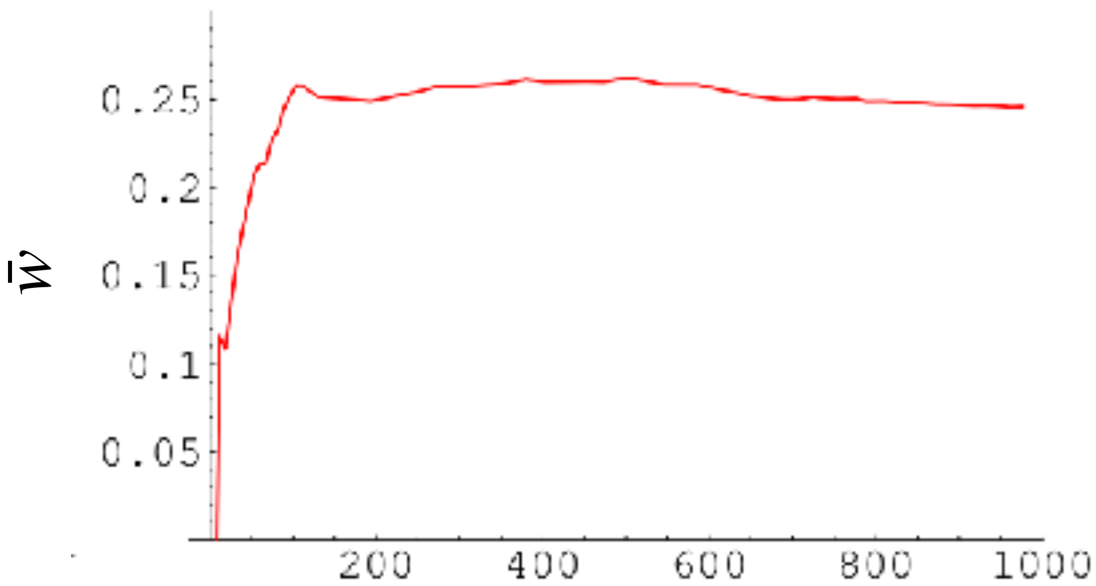
How does the equation of state evolve after inflaton fragmentation?

Dufaux, Felder, Kofman, Peloso, Podolsky (2006):

$$\frac{1}{2}g^2\phi^2 X^2 + \frac{1}{4}\lambda X^4$$



$$\frac{1}{2}h\phi X^2 + \frac{1}{4}\lambda X^4$$



$\bar{w} \to 0$ at late times

$\bar{w}$ at late times?

Antusch, Figueroa, Marschall, F. T. (2020, 2021)
Antusch, Marschall, F. T. (2022)

EoS during preheating with trilinear interactions

$$V(\phi, X) \simeq \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}h\phi X^2 + \frac{1}{4}\lambda X^4$$

Antusch, Marschall & F. T.
(2507.13465)

1. Equation of state characterization

a) **Linearized analysis** → Initial (linear) preheating

b) **Lattice simulations** → Later (non-linear) evolution

c) **Boltzmann equations** → Final perturbative decay

2. Observational implications

a) **Gravitational waves from preheating**

b) **Inflationary CMB observables**

EoS during preheating with trilinear interactions

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Antusch, Marschall & F. T.
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1. Equation of state characterization

a) **Linearized analysis** → Initial (linear) preheating

Preheating: natural variables

➤ Let us define (dimensionless) **natural variables**:

- **Fields:** $\varphi \equiv \frac{1}{\phi_*} a^{3/2} \phi\,; \qquad \chi \equiv \frac{1}{\phi_*} a^{3/2} X$

  ϕ_* : Inflaton amplitude at the end of inflation

- **Time and space:** $u \equiv m_\phi \int_{t_*}^{t} dt'; \qquad \vec{y} \equiv m_\phi \vec{x}$

  m_ϕ : Oscillation frequency at the end of inflation

➤ **EQUATIONS OF MOTION:**

Inflaton: $$\varphi'' - a^{-2} \nabla^2_{\vec{y}} \varphi + \left(1 + \cancel{F(u)}\right) \varphi + \frac{1}{2} \tilde{q}^{(h)} \chi^2 = 0$$

Daughter field: $$\chi'' - a^{-2} \nabla^2_{\vec{y}} \chi + \left(\tilde{q}^{(h)} \varphi + \tilde{q}^{(\lambda)} \chi^2 + \cancel{F(u)}\right) \chi = 0$$

$$F(u) \equiv -\frac{3}{4}\left(\frac{a'}{a}\right)^2 - \frac{3}{2}\frac{a''}{a} \sim u^{-2}$$

COUPLING PARAMETERS:

$$\tilde{q}^{(h)}(a) \equiv q_*^{(h)} a^{-3/2}\,; \qquad q_*^{(h)} \equiv \frac{h\phi_*}{m_\phi^2}$$

$$\tilde{q}^{(\lambda)}(a) \equiv q_*^{(\lambda)} a^{-3}\,; \qquad q_*^{(\lambda)} \equiv \frac{\lambda\phi_*^2}{m_\phi^2}$$

Preheating: natural variables

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➤ **EQUATIONS OF MOTION:**

Inflaton: $$\varphi'' - a^{-2} \nabla^2_{\vec{y}} \varphi + \varphi + \frac{1}{2} \tilde{q}^{(h)} \chi^2 = 0$$

Daughter field: $$\chi'' - a^{-2} \nabla^2_{\vec{y}} \chi + \left(\tilde{q}^{(h)} \varphi + \tilde{q}^{(\lambda)} \chi^2 \right) \chi = 0$$

COUPLING PARAMETERS:

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$$F(u) \equiv -\frac{3}{4} \left(\frac{a'}{a} \right)^2 - \frac{3}{2} \frac{a''}{a} \sim u^{-2}$$

Preheating: linearized analysis

➤ **Linear decomposition**:
$$\begin{cases} \varphi(\vec{y}, u) \equiv \bar{\varphi}(u) + \delta\varphi(\vec{y}, u) \\ \chi(\vec{y}, u) \equiv \delta\chi(\vec{y}, u) \end{cases}$$

➤ **Equations of motion** (ignoring quartic self-interaction)**:**

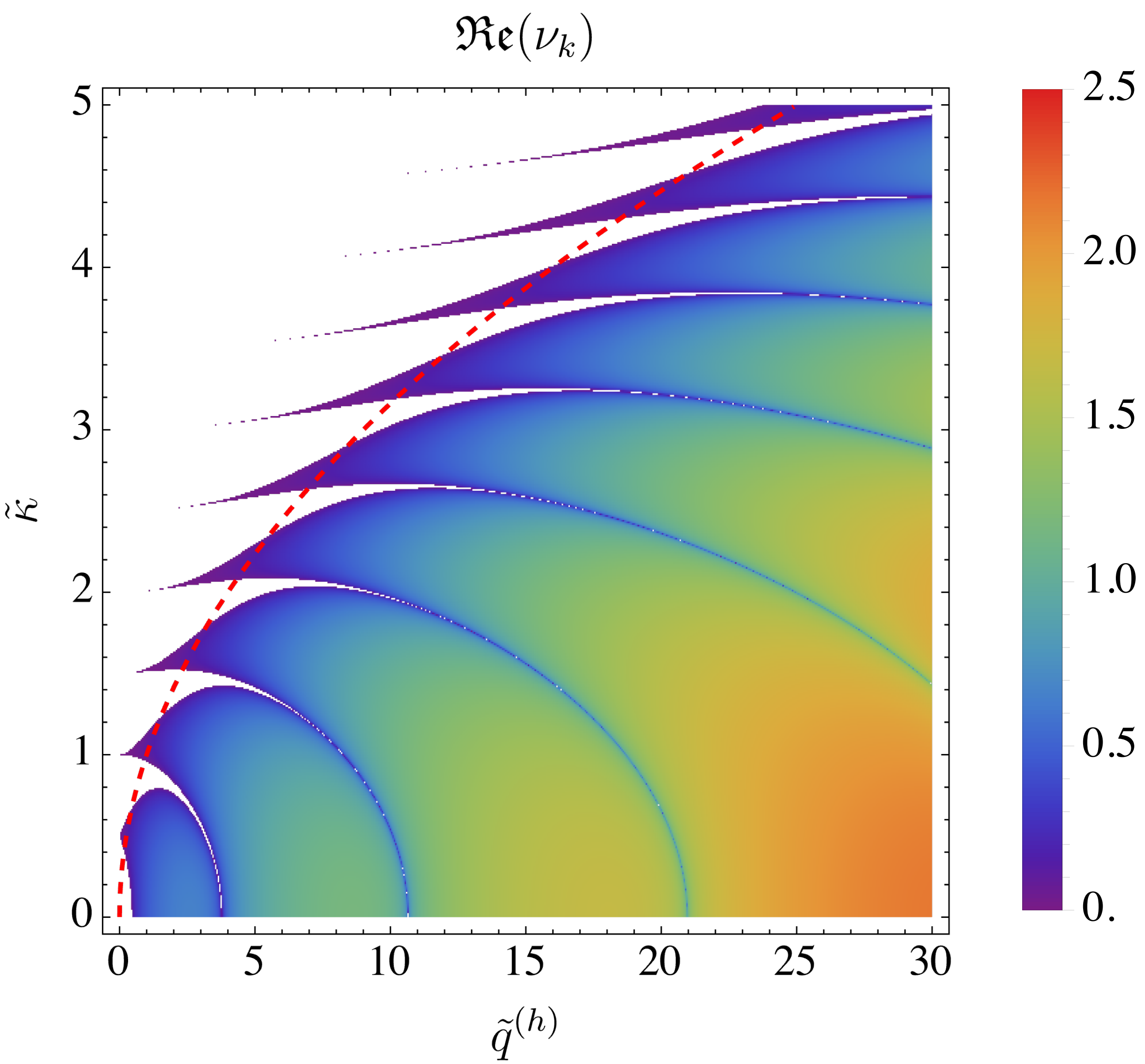
- Order 0: $\bar{\varphi}'' + \bar{\varphi} = 0 \longrightarrow \boxed{\bar{\varphi} \simeq \cos(u)}$

- Order 1: $\boxed{\delta\chi_k'' + \tilde{\omega}_{k,\chi}^2 \delta\chi_k \simeq 0; \quad \tilde{\omega}_{k,\chi} \equiv \sqrt{\tilde{\kappa}^2(a) + \tilde{q}^{(h)}(a)\bar{\varphi}}}$ $\qquad \tilde{\kappa}(a) \equiv \dfrac{k}{a m_\phi}$

During half of each oscillation, $\bar{\varphi} < 0$, and the fluctuations get excited through **tachyonic resonance.**

Preheating: linearized analysis

$$\delta\chi_k'' + \tilde{\omega}_{k,\chi}^2 \delta\chi_k \simeq 0; \qquad \tilde{\omega}_{k,\chi} \equiv \sqrt{\tilde{\kappa}^2(a) + \tilde{q}^{(h)}(a)\bar{\varphi}} \quad \longrightarrow \quad |\chi_{\mathbf{k}}|^2 \sim e^{2\nu_\kappa(\tilde{q}^{(h)})u}$$



- Modes $\tilde{\kappa} < \sqrt{\tilde{q}^{(h)}}$ line mostly excited by **tachyonic resonance.**

Tachyonic resonance ends when:

$$\tilde{q}^{(h)}(a) \equiv q_*^{(h)} a^{-3/2} < \frac{1}{6}$$

- Few modes $\tilde{\kappa} > \sqrt{\tilde{q}^{(h)}}$ excited due to **parametric resonance** $[\tilde{\omega}'_{k,\chi} \gg \tilde{\omega}_{k,\chi}{}^2]$ (much weaker!)

Preheating: linearized analysis

- Tachyonic resonance leads to an **amplification of field fluctuations.**

  If tachyonic resonance survives long enough, the inflaton **fragments, and the EoS deviates from w=0.**

- We can incorporate the **quartic self-interaction** by solving the field mode equations in the Hartree-Fock approximation:

$$
\begin{cases}
\bar{\varphi}'' + \bar{\varphi} + \frac{1}{2}\tilde{q}^{(h)}\langle\chi^2\rangle = 0 \\
\delta\chi_k'' + (\tilde{\kappa}^2 + \tilde{q}^{(h)}\bar{\varphi} + 3\tilde{q}^{(\lambda)}\langle\chi^2\rangle)\delta\chi_k = 0 \\
+ \text{ Friedmann eqs.}
\end{cases}
\quad \rightarrow
$$

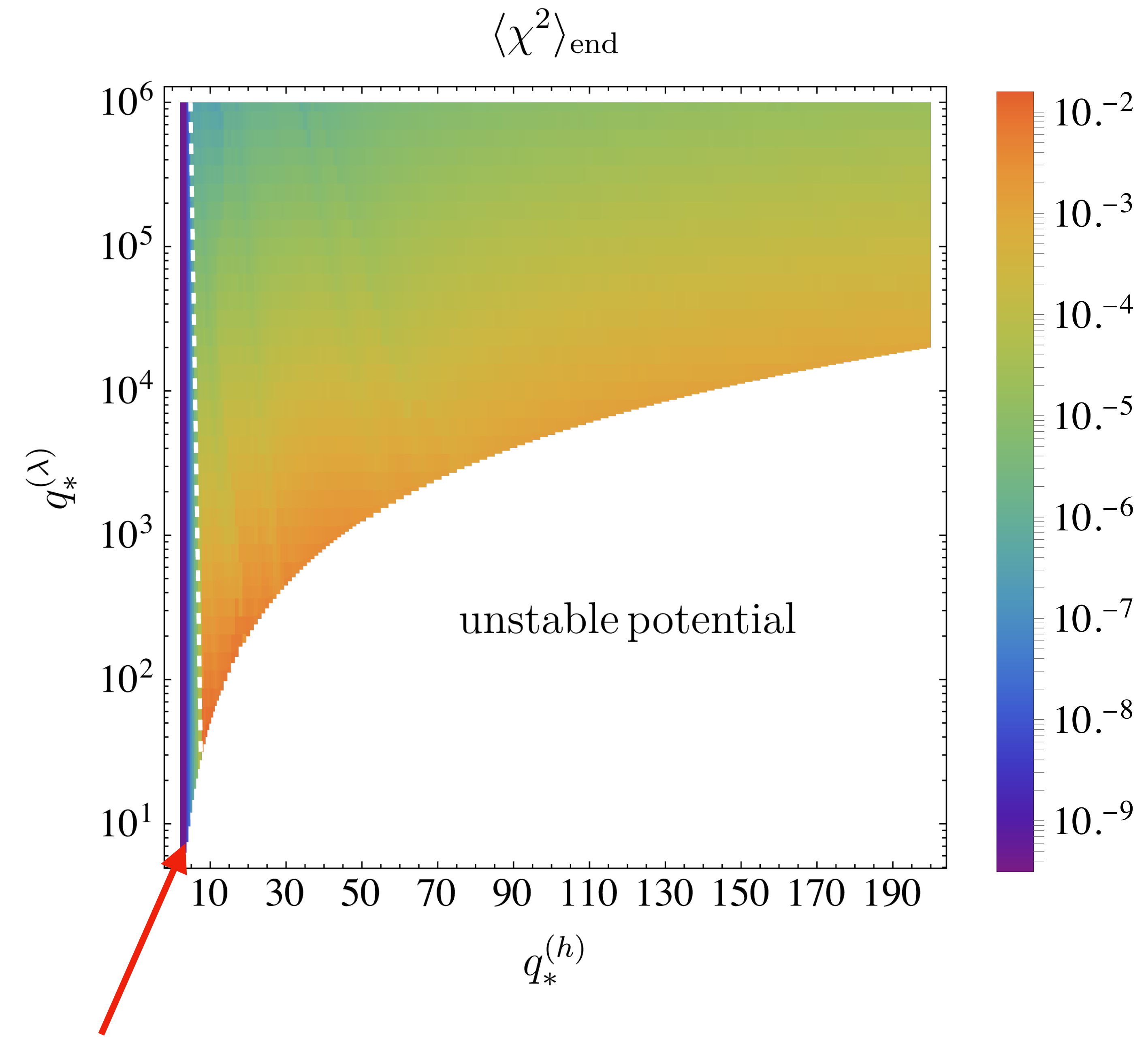
For which values of

$$\{q_*^{(h)}, q_*^{(\lambda)}\}$$

does the inflaton fragment?

Preheating: linearized analysis

Daughter field variance when tachyonic resonance ends



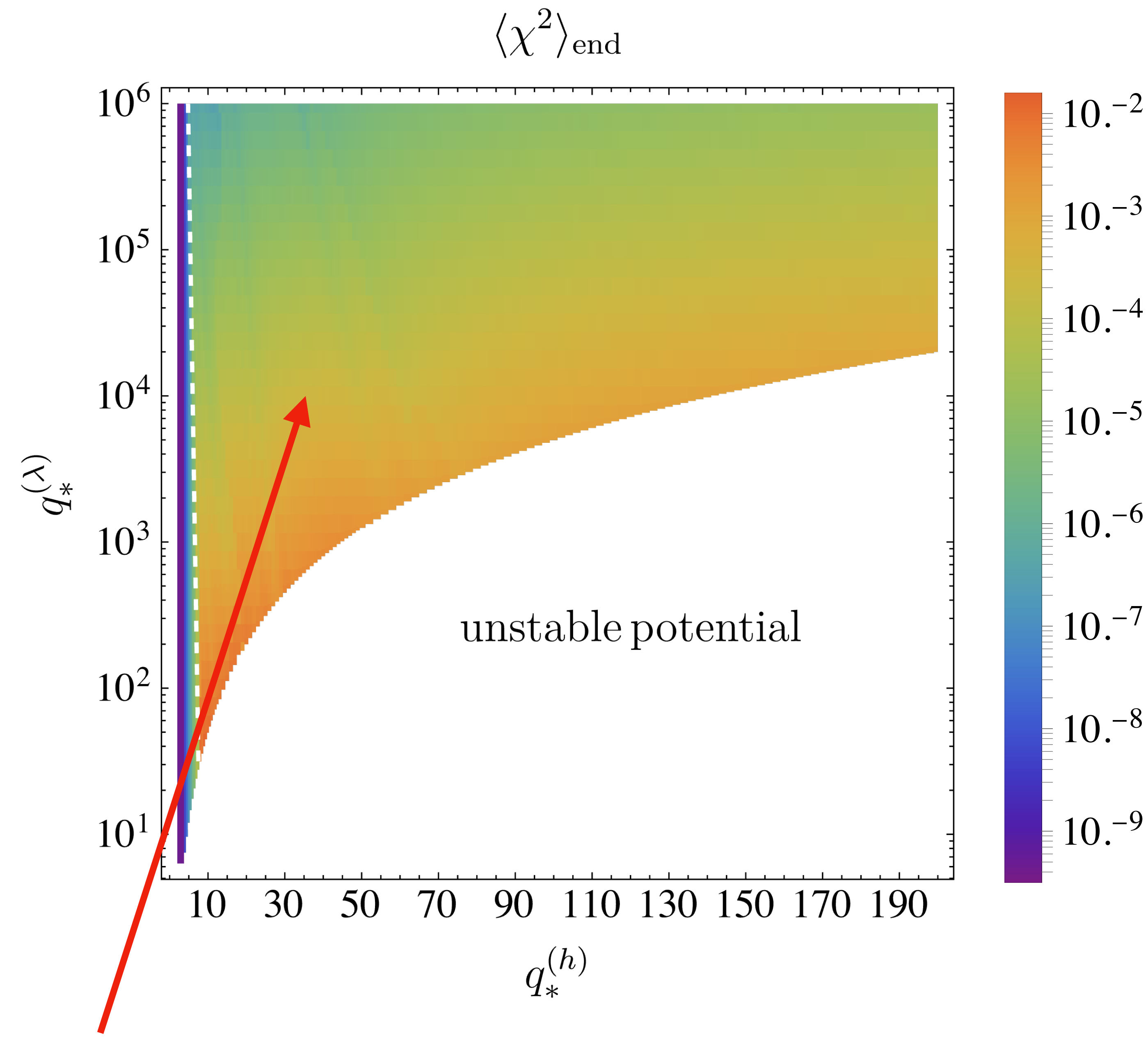
$q_*^{(h)} \lesssim 10$: Tachyonic resonance ends before the inflaton fragments: $\langle\chi^2\rangle_{\rm end} \ll 1$

Fluctuations remain subdominant.

EoS remains $\bar{w} = 0$ until perturbative reheating.

Preheating: linearized analysis

Daughter field variance when tachyonic resonance ends

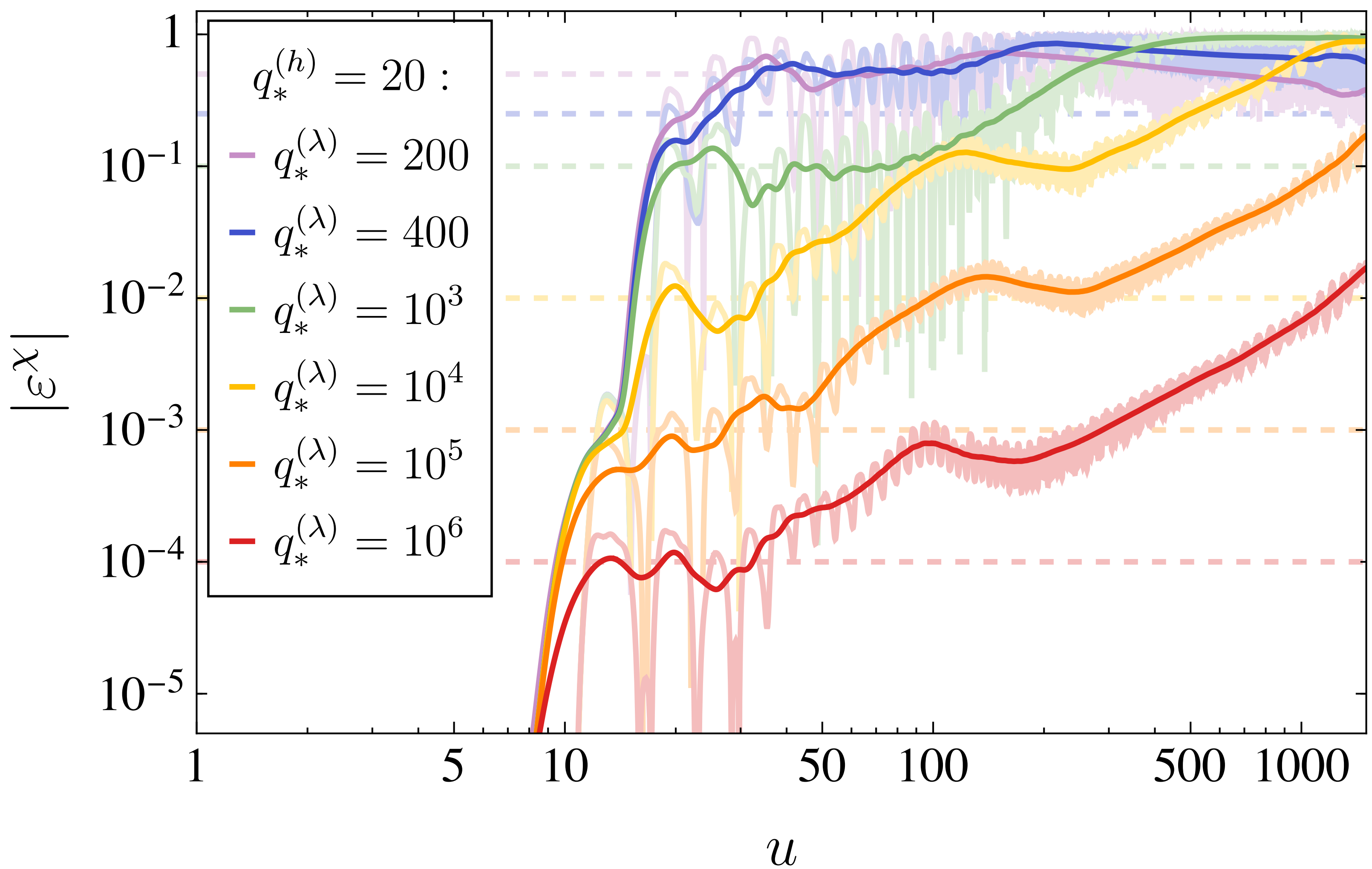


$q_*^{(h)} \gtrsim 10$: Daughter field variance grows tachyonically until:

$$\langle\chi^2\rangle_{\rm end} \approx \frac{\tilde{q}^{(h)}}{3\tilde{q}^{(\lambda)}} \sim \frac{q_*^{(h)}}{3q_*^{(\lambda)}} a_{\rm end}^{3/2} \ll 1$$

Preheating: linearized analysis

Fraction of energy stored in the daughter field



Further (slower) growth at later times for all self-interaction strengths

$$\langle \chi^2 \rangle (u \to \infty) > \langle \chi^2 \rangle_{\text{end}}$$

The inflaton **fragments for all self-interaction strenghts**, as long as $q_*^{(h)} \gtrsim 10.$

- - - - - - - : $\langle \chi^2 \rangle_{\text{end}} \approx \dfrac{q_*^{(h)}}{3 q_*^{(\lambda)}} a_{\text{end}}^{3/2}$

EoS during preheating with trilinear interactions

$$V(\phi, X) \simeq \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}h\phi X^2 + \frac{1}{4}\lambda X^4$$

Antusch, Marschall & F. T.
(2507.13465)

1. Equation of state characterization

a) **Linearized analysis** → Initial (linear) preheating

b) **Lattice simulations** → Later (non-linear) evolution

Our set-up

- In order to fully capture the non-linearities of the system, we simulate the following potential with **CosmoLattice** in 2+1 dimensions:

$$V(\phi, X) \simeq \frac{\Lambda^4}{2}\tanh^2\left(\frac{\phi}{M}\right) + \frac{1}{2}h\phi X^2 + \frac{1}{4}\lambda X^4$$

we set $M = 5m_p$.

CosmoLattice
Daniel G. Figueroa
Adrien Florio
Francisco Torrentí
Wessel Valkenburg

- We characterize the **equation of state** evolution for ~**10 e-folds**:

$$w \equiv \frac{p}{\rho} = \varepsilon_{\rm k} - \frac{1}{3}\varepsilon_{\rm g} - \varepsilon_{\rm p}$$

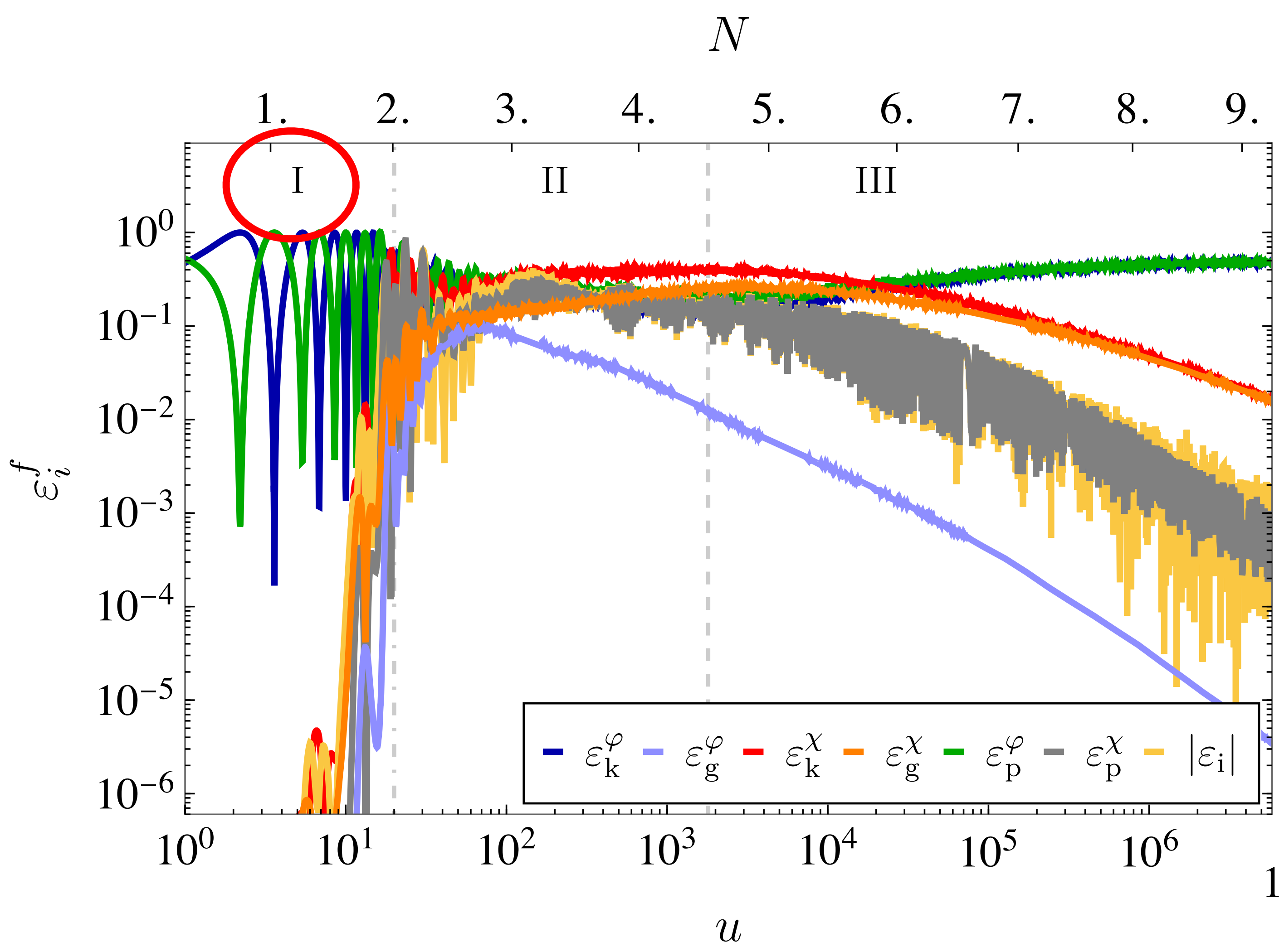
Kinetic: $\varepsilon_{\rm k} \equiv \varepsilon_{\rm k}^{\varphi} + \varepsilon_{\rm k}^{\chi}$

Gradient: $\varepsilon_{\rm g} \equiv \varepsilon_{\rm g}^{\varphi} + \varepsilon_{\rm g}^{\chi}$

Potential: $\varepsilon_{\rm p} \equiv \varepsilon_{\rm p}^{\varphi} + \varepsilon_{\rm p}^{\chi} + \varepsilon_{\rm i}$

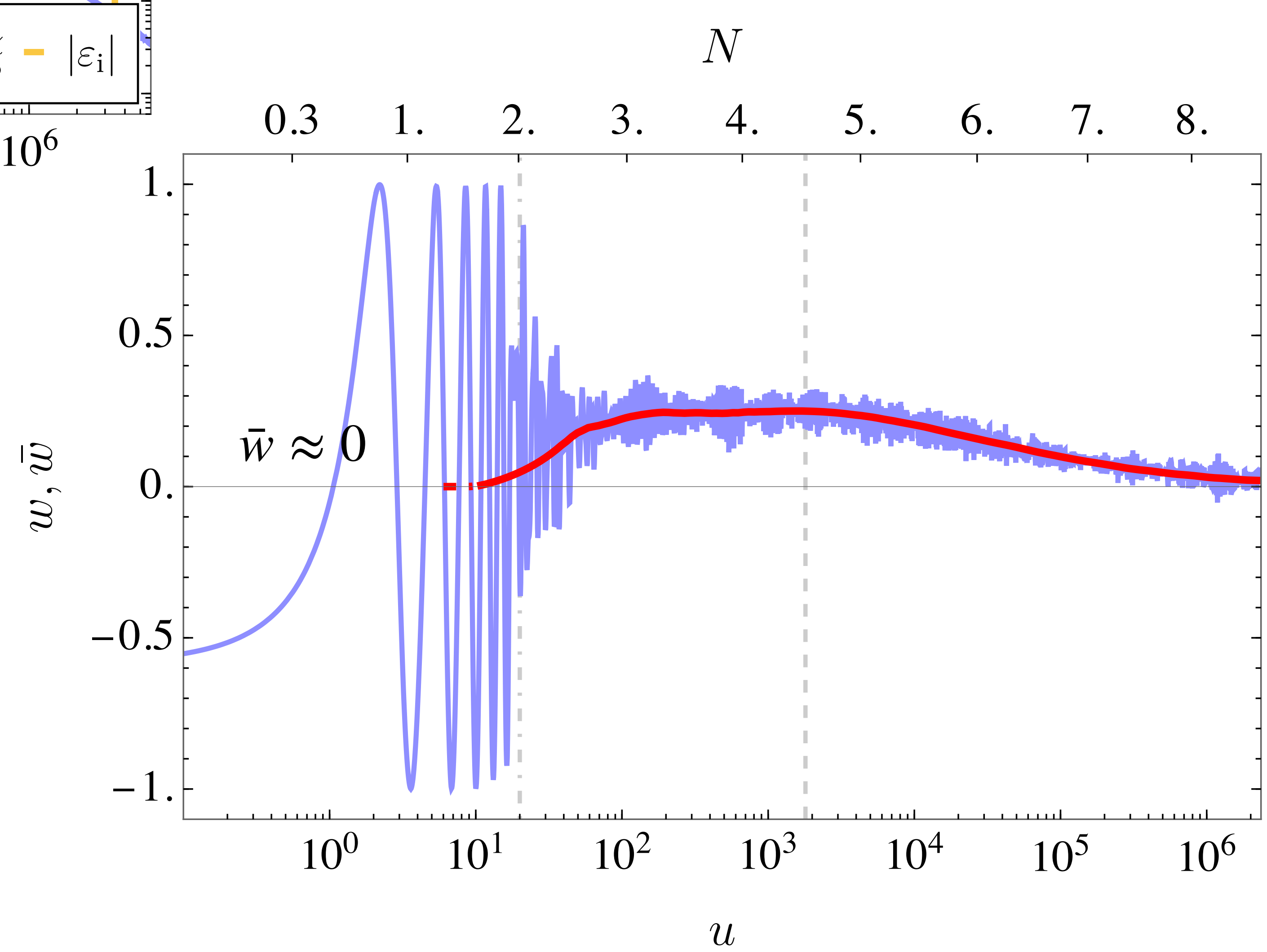
ε_α : Fraction of energy density in each component

Lattice simulations of preheating



$q_*^{(h)} = 50$

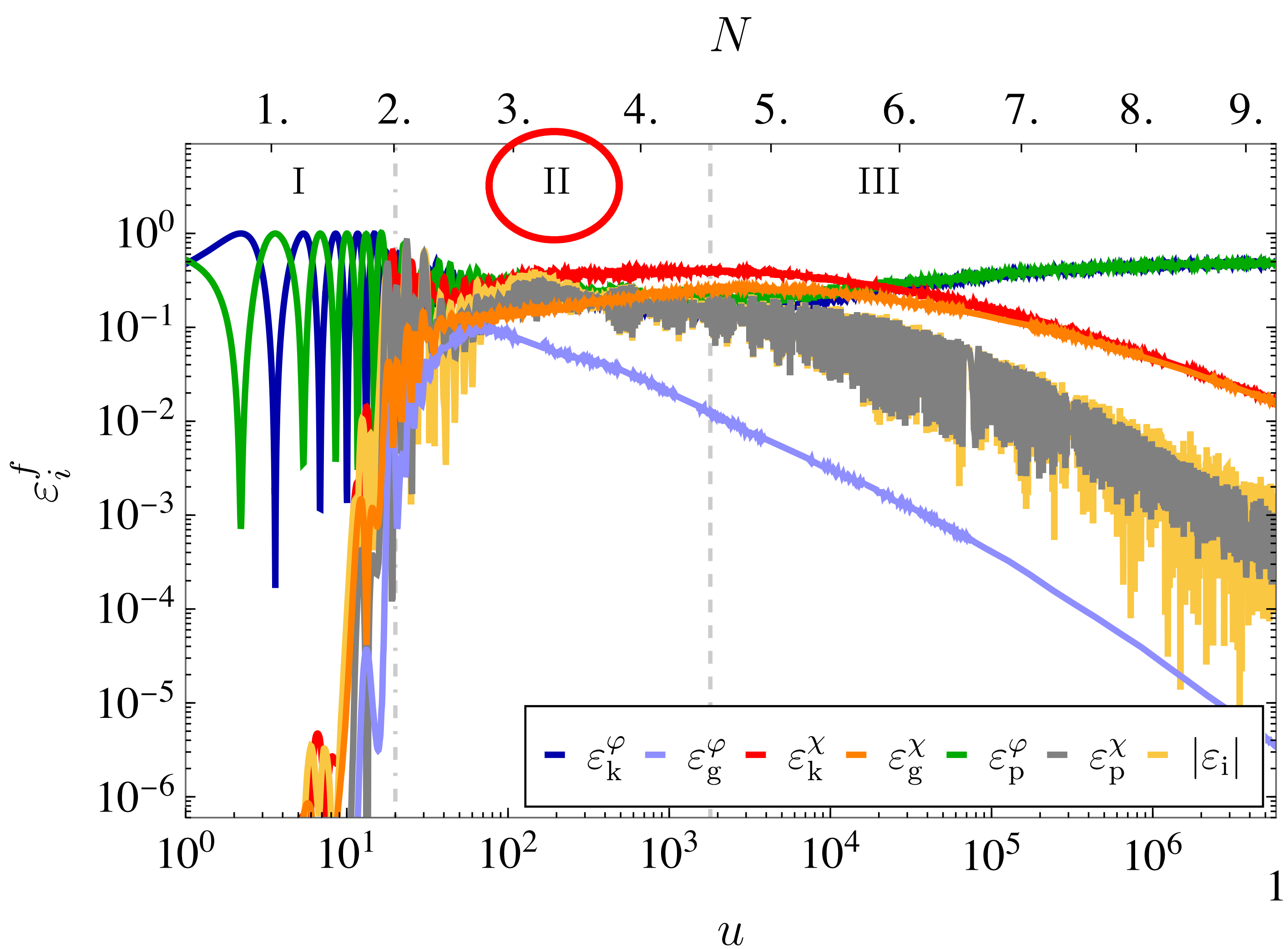
$q_*^{(\lambda)} = 2500$



Daughter field fluctuations amplified resonantly.

Inflaton fluctuations amplified through backreaction effects.

Lattice simulations of preheating



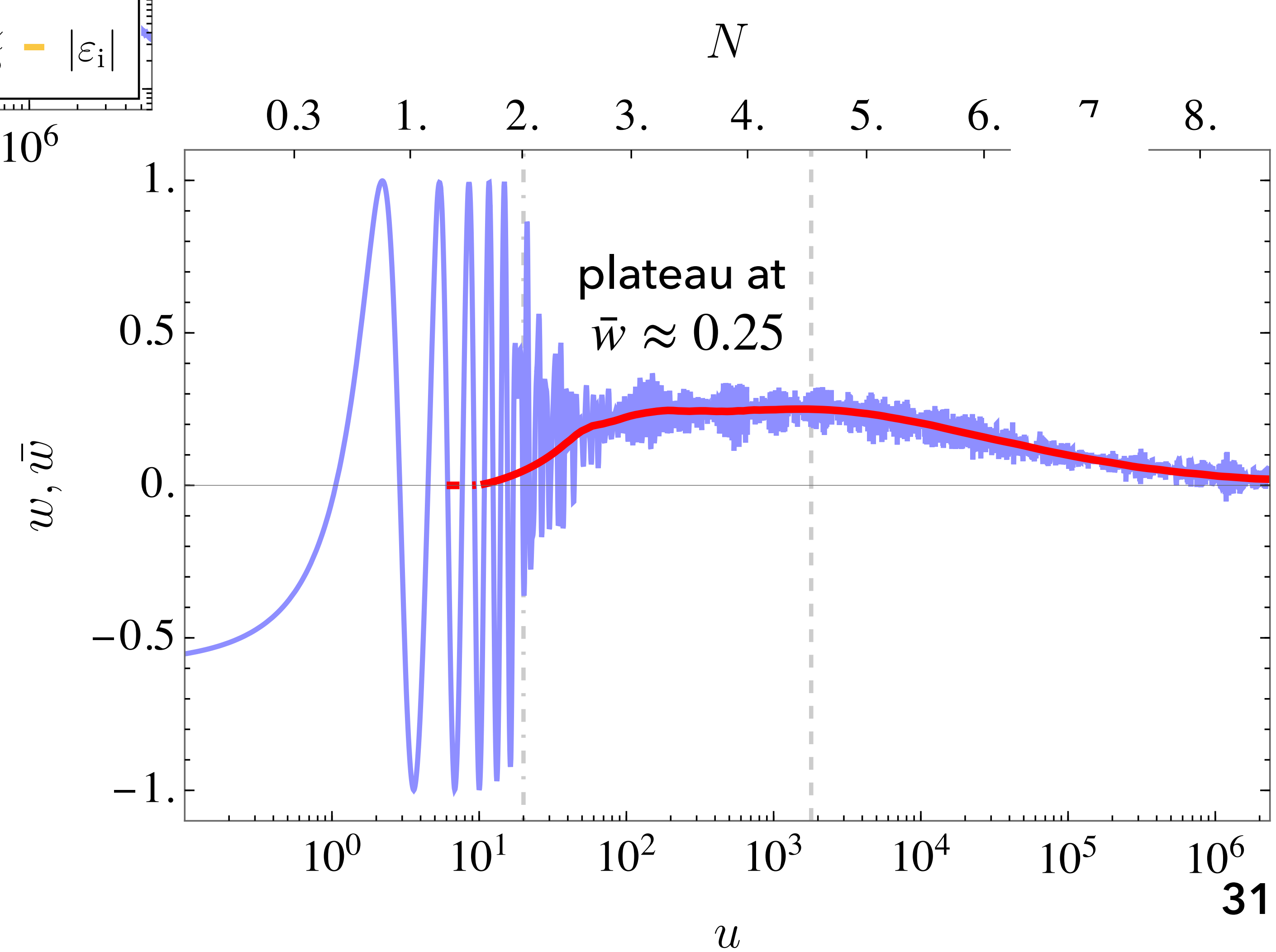
$q_*^{(h)} = 50$

$q_*^{(\lambda)} = 2500$

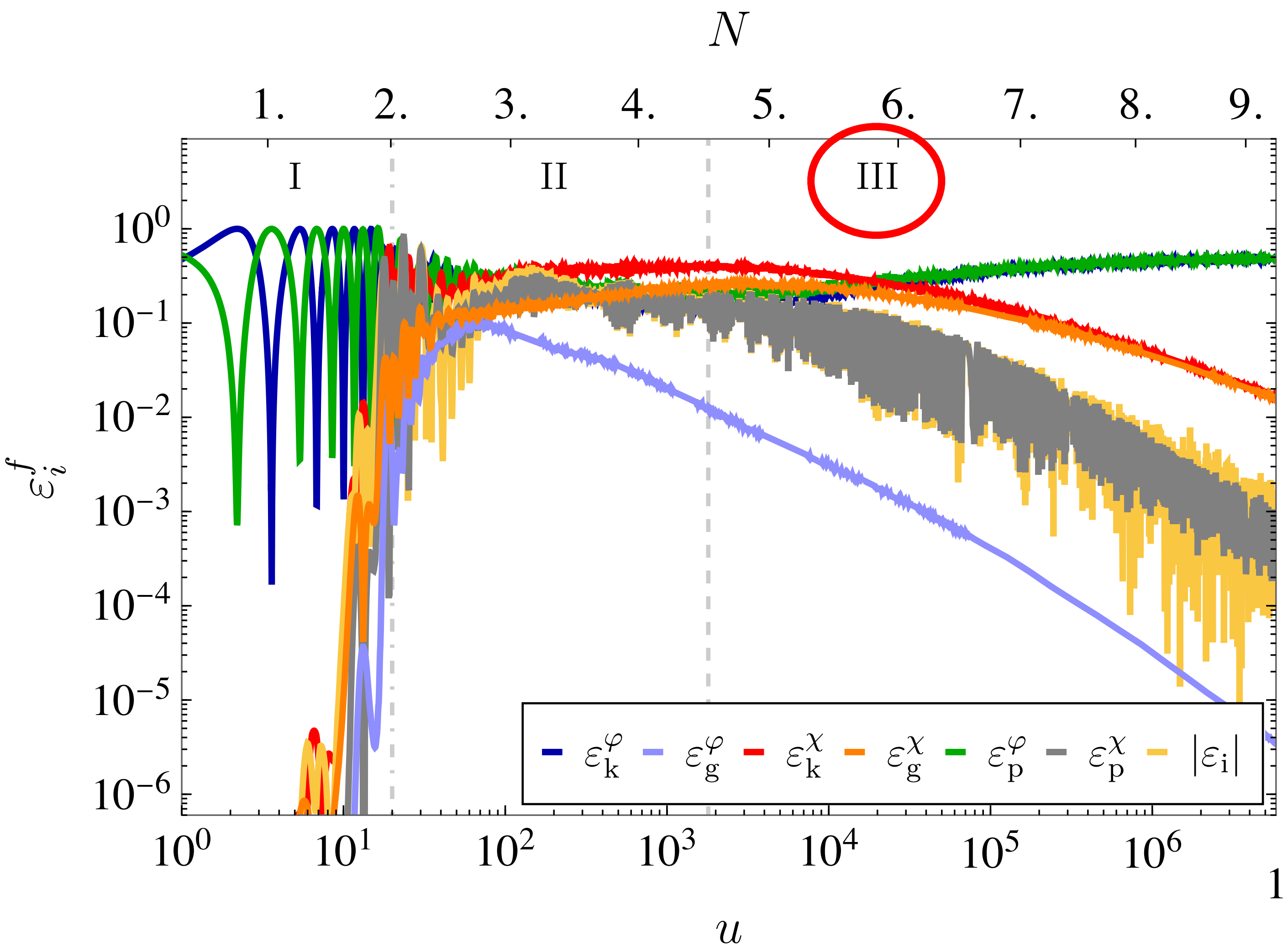
$\varepsilon_g^\varphi \downarrow$: tachyonic resonance has ended

$\varepsilon_g^\chi \uparrow$: self-resonance of daughter field due to $\propto X^4$ term

$$\varepsilon_g \equiv \varepsilon_g^\varphi + \varepsilon_g^\chi \approx \text{const}$$



Lattice simulations of preheating



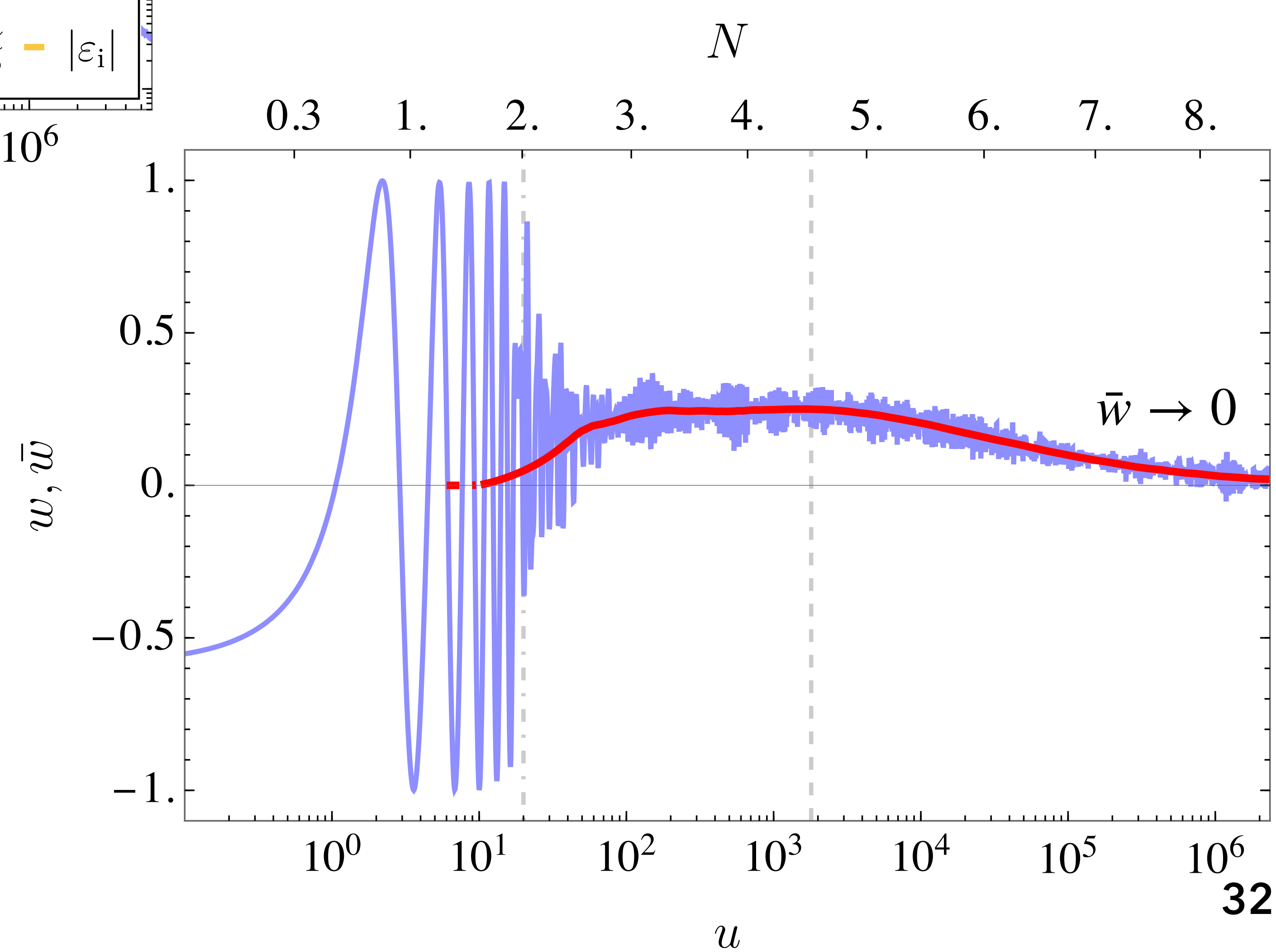
$$q_*^{(h)} = 50$$
$$q_*^{(\lambda)} = 2500$$

$$\tilde{q}^{(h)}, \tilde{q}^{(\lambda)} \ll 1$$

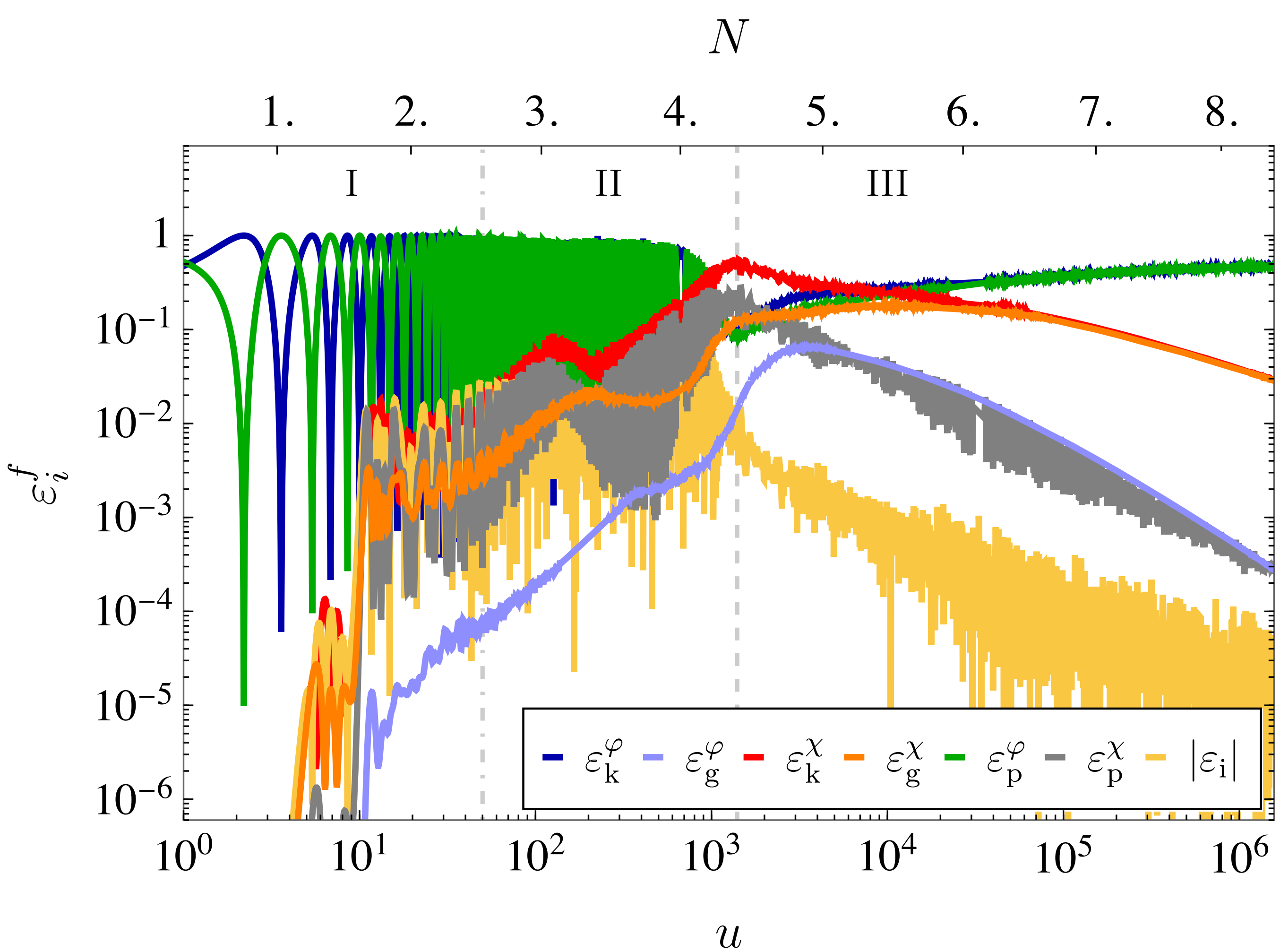
Resonant effects finished.

Fluctuations dilute as radiation.

Inflaton homogeneous mode dilutes as matter.

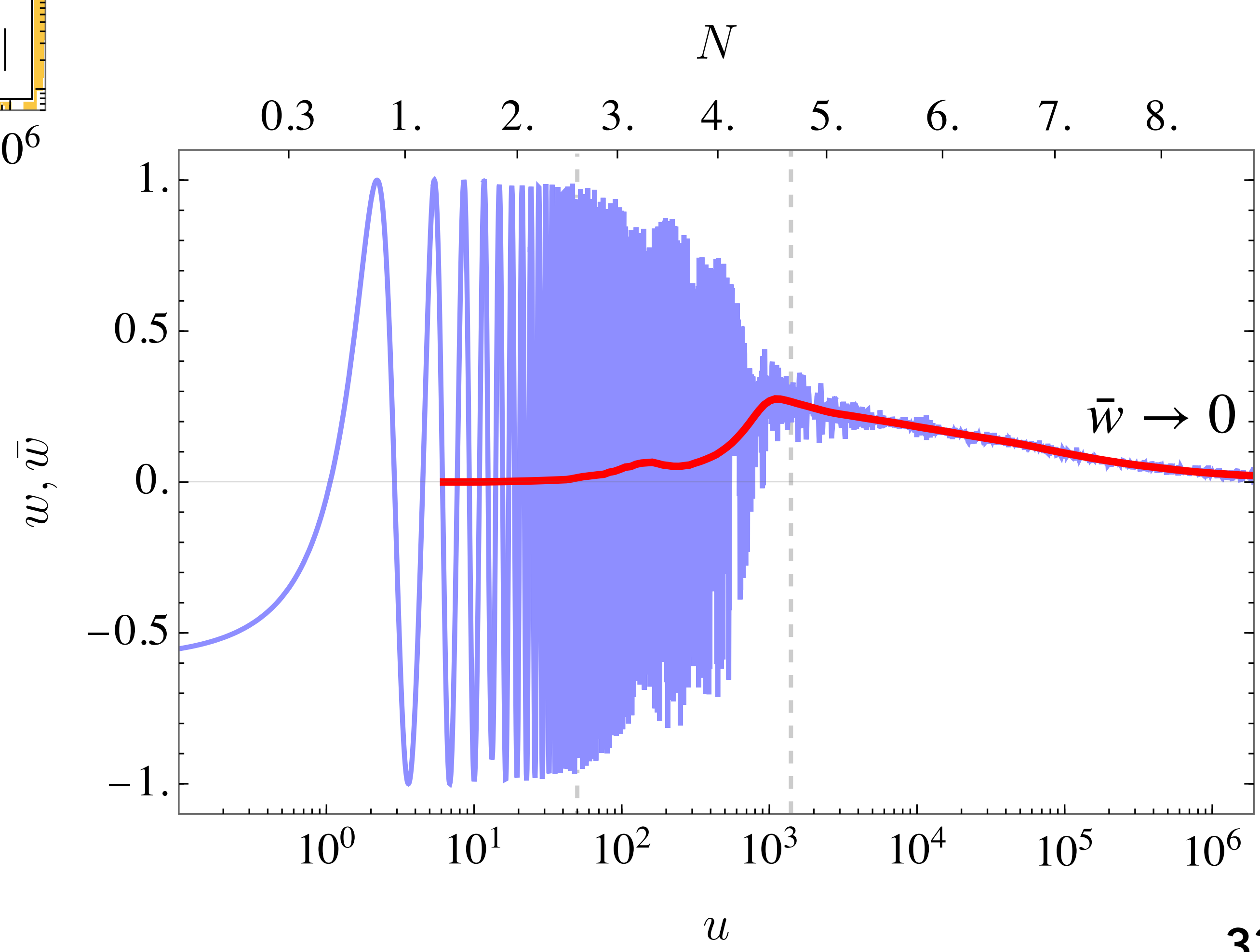


Lattice simulations of preheating

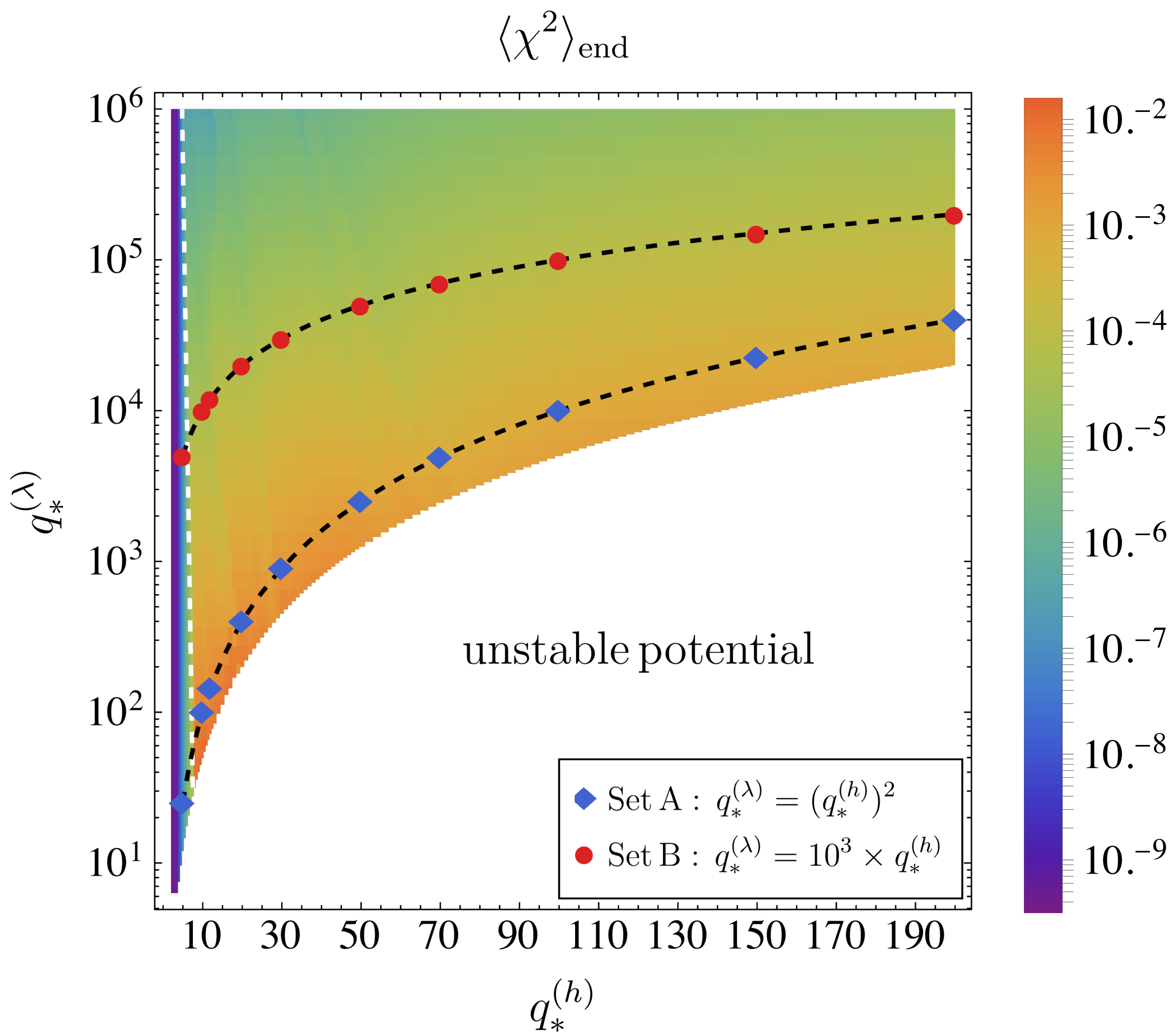


$$q_*^{(h)} = 50$$
$$q_*^{(\lambda)} = 50000$$

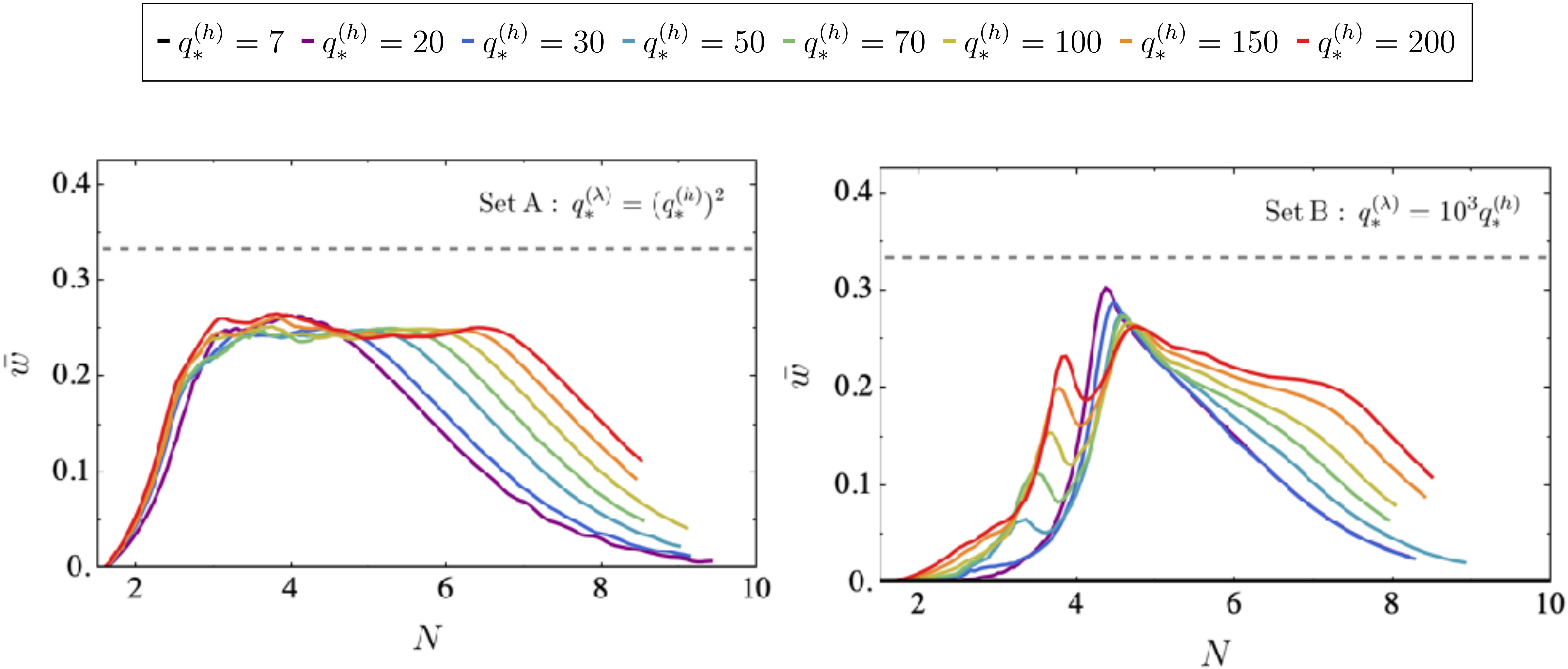
For **stronger self-interactions**, there is no plateau in the EoS evolution.



Lattice simulations of preheating



Lattice simulations of preheating



- ➤ Preheating **never achieves a radiation-dominated universe.**
- ➤ After preheating ends, we recover a **matter-dominated universe** dominated by the **homogeneous inflaton.**

EoS during preheating with trilinear interactions

$$V(\phi, X) \simeq \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}h\phi X^2 + \frac{1}{4}\lambda X^4$$

Antusch, Marschall & F. T. (2507.13465)

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c) **Boltzmann equations** → Final perturbative decay

Perturbative reheating

- The trilinear introduction introduces a **perturbative decay channel** $\phi \to XX$ with the following decay rate:

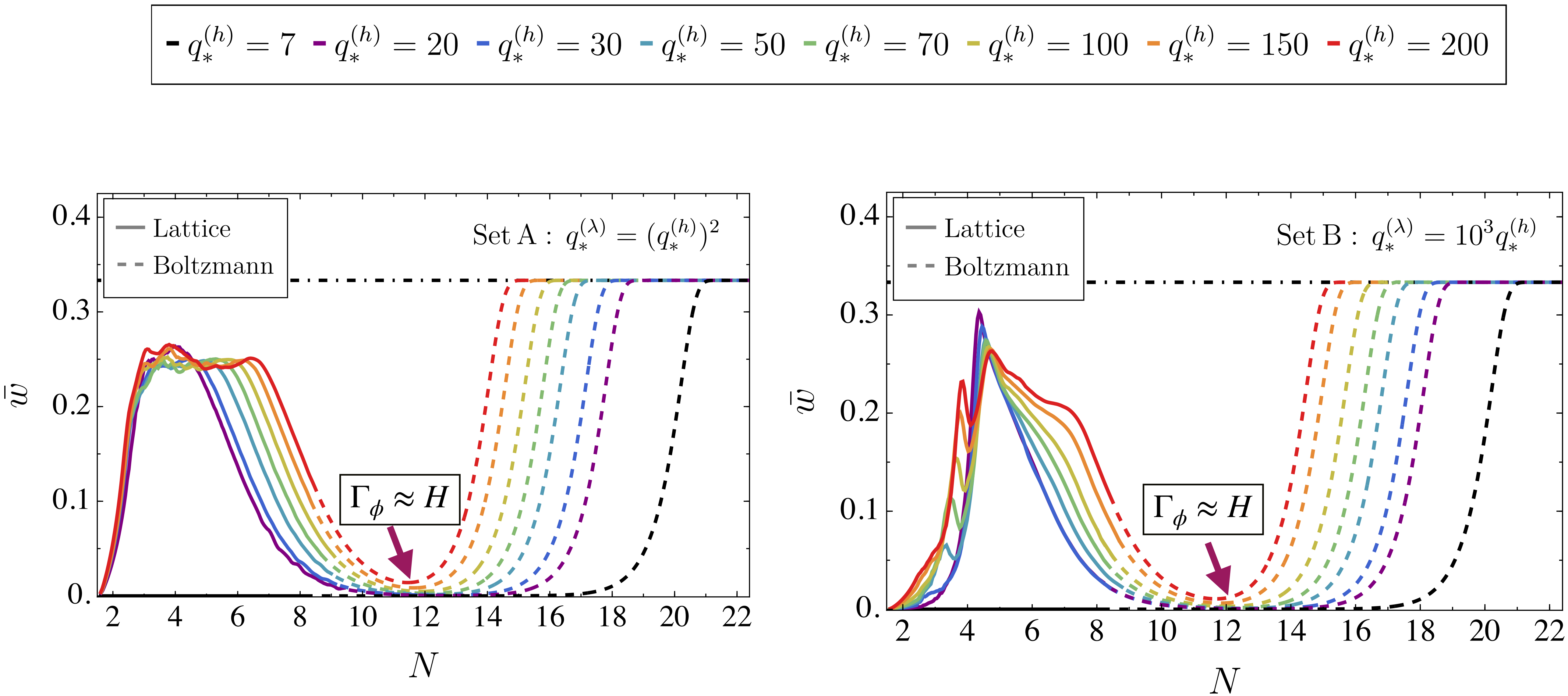
$$\Gamma_\phi = \frac{h^2}{32\pi m_\phi}\,; \qquad \text{which becomes effective when} \quad H \approx \Gamma_\phi$$

- We solve the **Boltzmann equations** from the end of the lattice simulation until the completion of perturbative reheating:

$$\begin{cases} \dot\rho_\varphi + 3H\rho_\varphi + \Gamma_\phi \rho_\varphi = 0 \\ \dot\rho_\chi + 4H\rho_\chi - \Gamma_\phi \rho_\varphi = 0 \\ H^2 = \dfrac{1}{3m_{\rm pl}^2}(\rho_\varphi + \rho_\chi) \end{cases}$$

with Mathematica!!

Equation of state (lattice + boltzmann)



Radiation-domination is achieved 14 - 21 e-folds **after the end of inflation**

(for the considered coupling strenghts)

EoS during preheating with trilinear interactions

$$V(\phi, X) \simeq \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}h\phi X^2 + \frac{1}{4}\lambda X^4$$

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(2507.13465)

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2. Observational implications

a) Gravitational waves from preheating

Gravitational waves from preheating

- We can compute the GW spectrum at the time $u_{\rm f}$ **when GW production ends** (with e.g. 3+1 lattice simulations):

$$\Omega^{(\rm f)}_{\rm gw}(k, u_{\rm f}) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm gw}}{d\log k}(k, u_{\rm f})$$

- In order to compute the **frequencies and amplitudes of the GW spectrum measured today**, we need to redshift it:

$$f_{\rm gw} \simeq 4 \cdot 10^{10} \epsilon_{\rm f}^{1/4} \frac{k}{a_{\rm f} H_{\rm f}} \left(\frac{H_{\rm f}}{m_p}\right)^{1/2} {\rm Hz} \ ; \qquad h_0^2 \Omega_{\rm gw} \simeq 1.6 \cdot 10^{-5} \, \epsilon_{\rm f} \, \Omega^{(\rm f)}_{\rm gw}$$

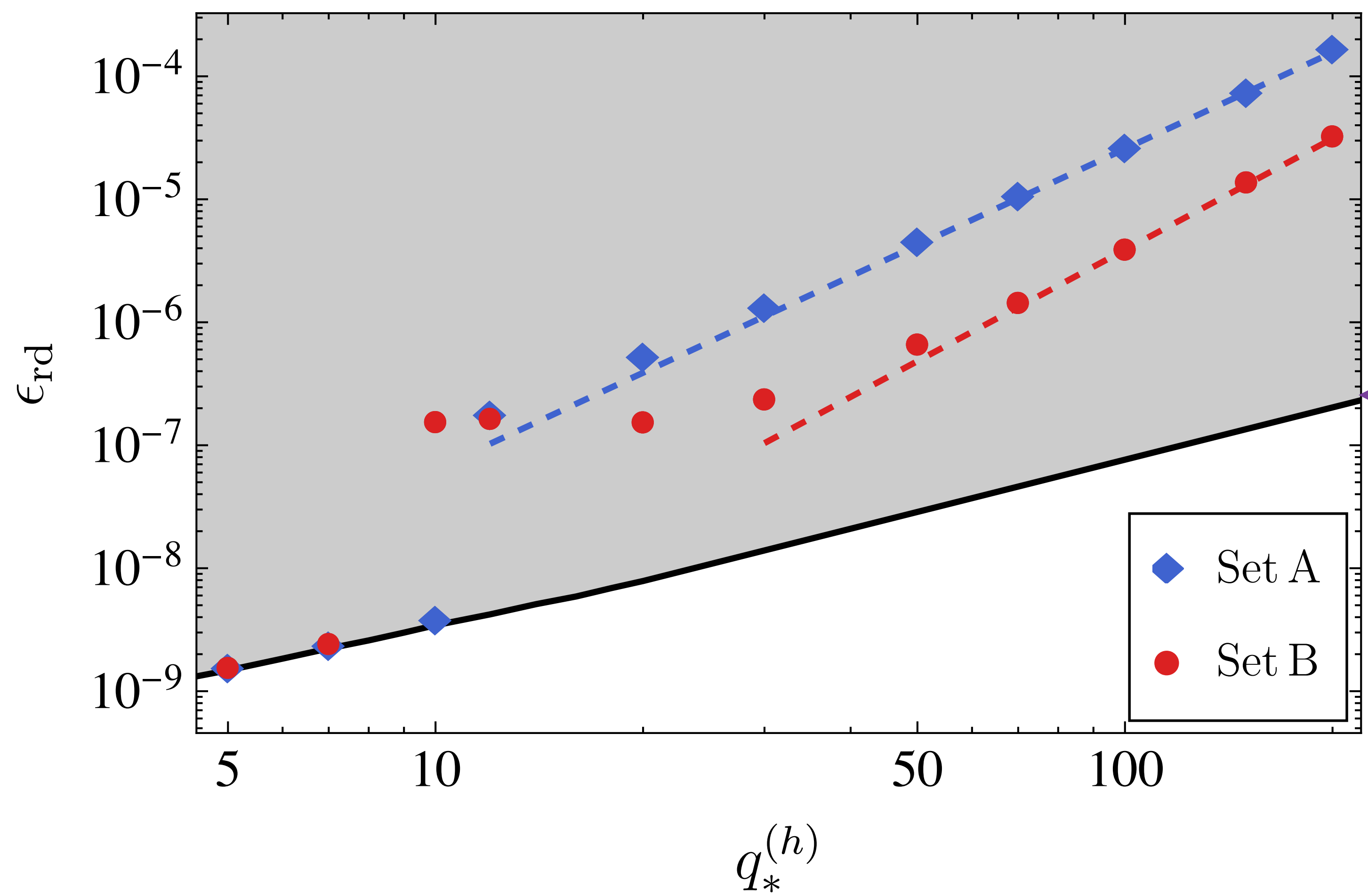
where $\epsilon_{\rm f}$ is the "**suppression factor**" from time $u_{\rm f}$ until radiation-domination:

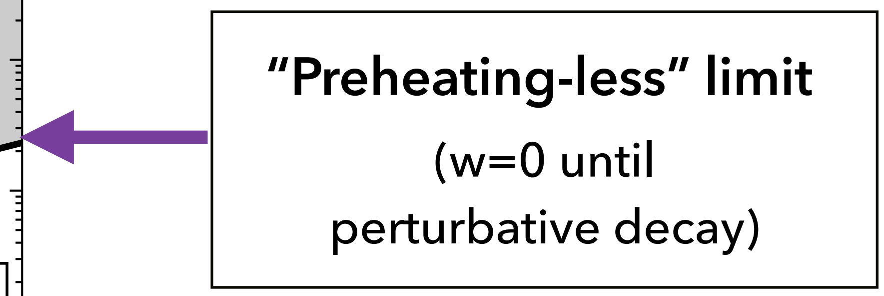
$$\epsilon_{\rm f} \equiv \left(\frac{a_{\rm f}}{a_{\rm rd}}\right)^{1-3\bar{w}_{\rm f}} \begin{cases} < 1 & \text{if MD} \\ = 1 & \text{if RD} \\ > 1 & \text{if KD} \end{cases}$$

[$a_{\rm f} = a(u_{\rm f})$
$a_{\rm rd} = a(u_{\rm rd})$
$\bar{w}_{\rm f}$: average e.o.s]

GW suppression factor

- **GW suppression factors** as a function of the coupling strength, from the end of inflation until radiation domination:

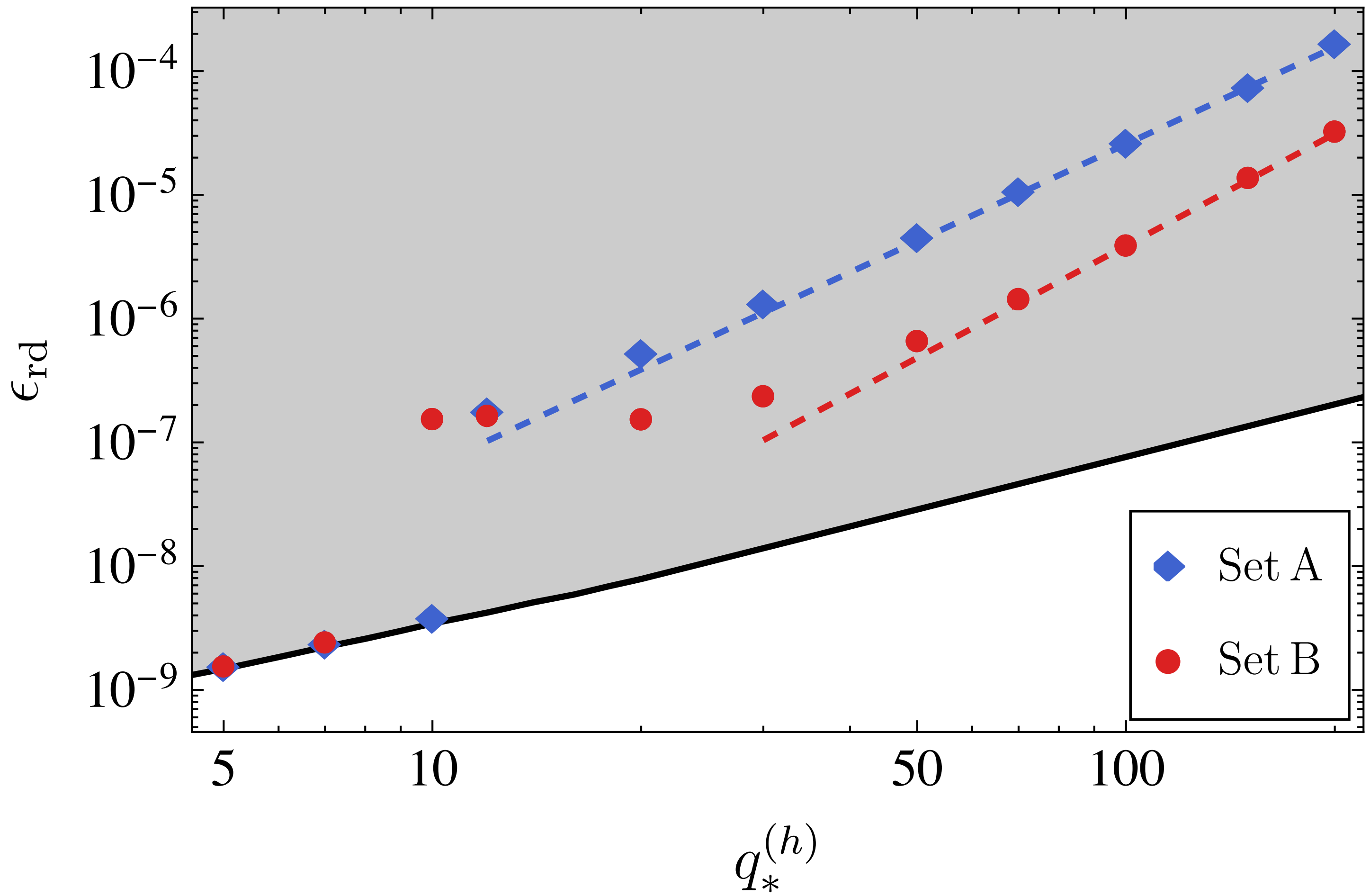




[**NOTE:** $\epsilon_{\rm rd} \equiv \left(\frac{a_{\rm end}}{a_{\rm rd}}\right)^{1-3\bar{w}_{\rm rd}} = \epsilon_{\rm i} \cdot \epsilon_{\rm f}$; with $\epsilon_{\rm i} \equiv \left(\frac{a_{\rm end}}{a_{\rm f}}\right)^{1-3\bar{w}_{\rm i}} \sim 0.1$]

GW suppression factor

➤ **GW suppression factors** as a function of the coupling strength, from the end of inflation until radiation domination:



Set A:

$$\epsilon_{\rm rd} \simeq (8.4 \pm 0.1) \cdot 10^{-8} \left(\frac{q_*^{(h)}}{10}\right)^{2.60\pm0.01}$$

Set B:

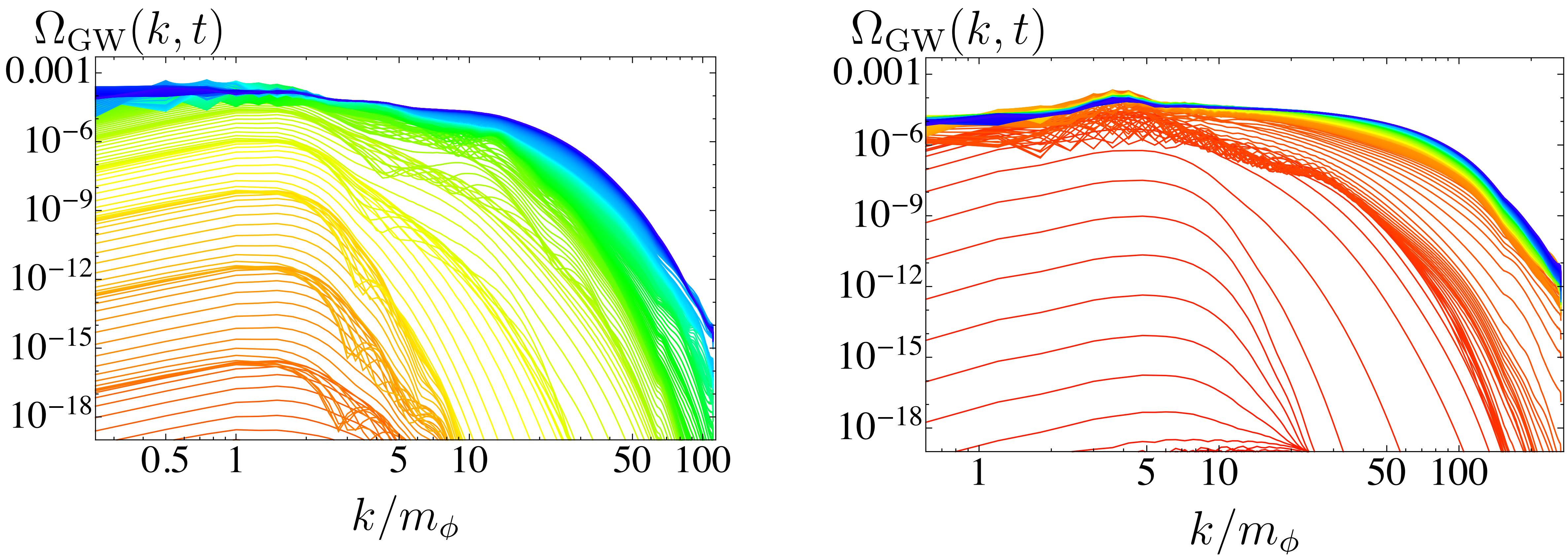
$$\epsilon_{\rm rd} \simeq (3.8 \pm 0.3) \cdot 10^{-9} \left(\frac{q_*^{(h)}}{10}\right)^{3.01\pm0.03}$$

[**NOTE:** $\epsilon_{\rm rd} \equiv \left(\frac{a_{\rm end}}{a_{\rm rd}}\right)^{1-3\bar{w}_{\rm rd}} = \epsilon_{\rm i} \cdot \epsilon_{\rm f}$; with $\epsilon_{\rm i} \equiv \left(\frac{a_{\rm end}}{a_{\rm f}}\right)^{1-3\bar{w}_{\rm i}} \sim 0.1$]

Gravitational waves from preheating

- **3+1-dimensional lattice simulations** have been used to parametrize the main peak of the GW spectrum **(Cosme, Figueroa & Loayza, JCAP 05 (2023) 023)**:

$$f_{\rm gw} \simeq (1.0 \pm 0.1) \cdot 10^8 \, \epsilon_{\rm f}^{1/4} \left(\frac{q_*^{(h)}}{10} \right)^{0.52\pm0.04} \text{Hz} \,; \qquad h_0^2 \Omega_{\rm gw}^{(0)} \simeq (2.67 \pm 0.5) \cdot 10^{-9} \, \epsilon_{\rm f} \left(\frac{q_*^{(h)}}{10} \right)^{-0.43\pm0.07}$$



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- Combining their results with our parametrization for $\epsilon_{\rm f}$, we get:

$$f_{\rm gw} \simeq (3.2 \pm 0.4) \cdot 10^6 \left(\frac{q_*^{(h)}}{10} \right)^{1.14\pm0.04} \text{Hz}$$

$$h_0^2 \Omega_{\rm gw}^{(0)} \simeq (2.8 \pm 0.9) \cdot 10^{-15} \left(\frac{q_*^{(h)}}{10} \right)^{2.05\pm0.09}$$

The signal gets suppressed up to six orders of magnitude
and **shifted to the IR by one-two orders of magnitude.**

EoS during preheating with trilinear interactions

$$V(\phi, X) \simeq \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}h\phi X^2 + \frac{1}{4}\lambda X^4$$

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b) Inflationary CMB observables

Inflationary CMB observables

- Inflationary predictions depend on **N**$_k$: number e-folds from the horizon crossing of the pivot scale (k_{CMB}=0.05 Mpc^{-1}) till the end of inflation

$$N_k \equiv \ln\frac{a_{\rm end}}{a_k} \simeq \frac{1}{m_{\rm pl}^2}\int_{\phi_k}^{\phi_{\rm end}} \frac{V}{V_{,\phi}}|d\phi|$$

- Its value can be determined with knowledge of the post-inflationary EoS:

$$\frac{k_{\rm CMB}}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0} = e^{-N_k}\frac{a_{\rm end}}{a_{\rm rd}}\frac{a_{\rm rd}}{a_{\rm eq}}\frac{a_{\rm eq}}{a_0}\frac{H_k}{H_0}$$

$$\left[\frac{a_0}{a_{\rm eq}} = (1+z_{\rm eq}) \simeq 3387 \qquad \frac{a_{\rm rd}}{a_{\rm eq}} = \left(\frac{\rho_{\rm rd}}{\rho_{\rm eq}}\right)^{-1/4} \qquad \rho_{\rm eq} = 6\Omega_{m,0} m_{\rm pl}^2 H_0^2 (1+z_{\rm eq})^3\right]$$

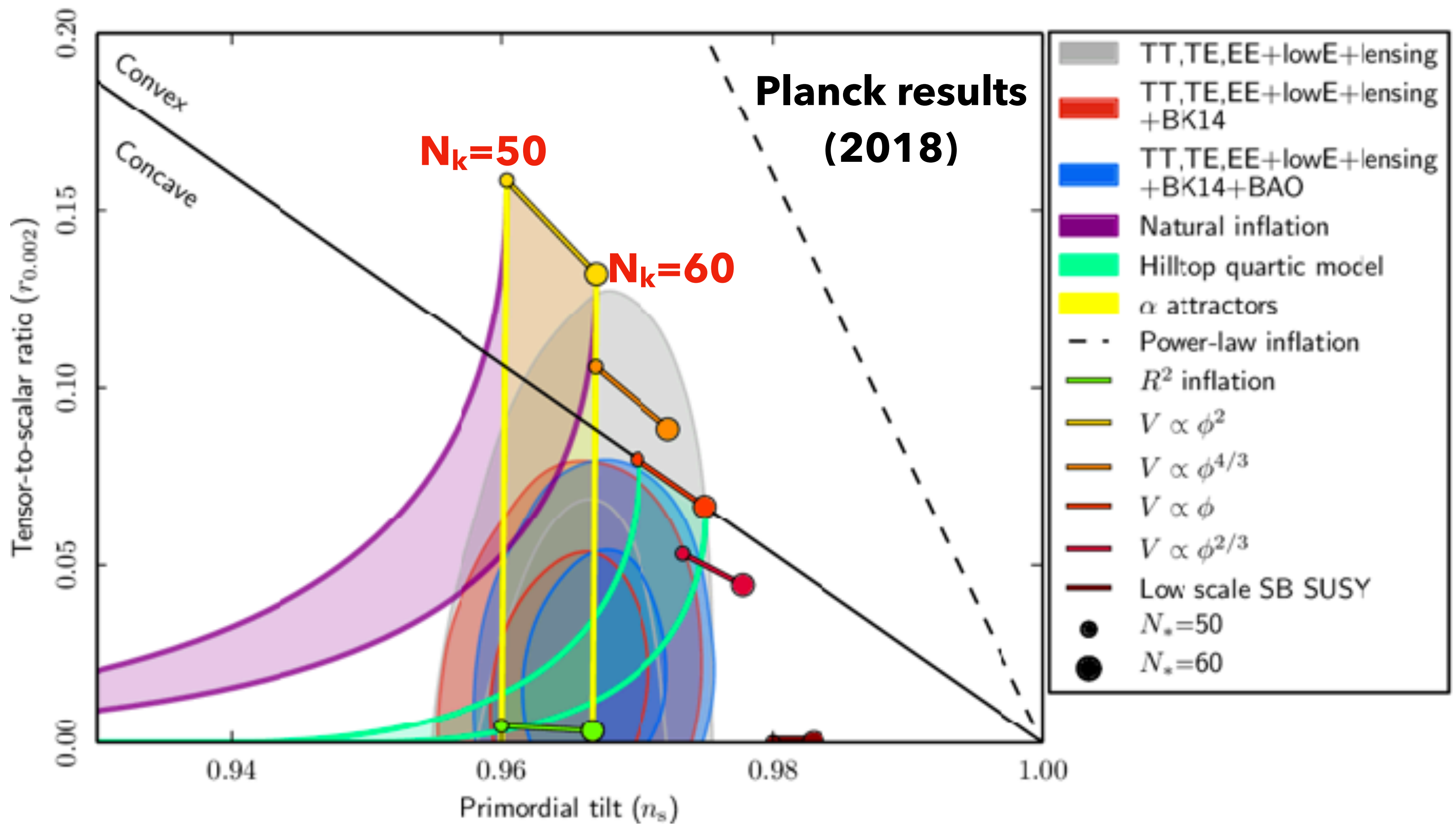
$$\boxed{N_k \simeq 61.5 + \frac{1}{4}\ln\frac{V^2(\phi_k)}{m_{\rm pl}^4\rho_{\rm end}} - \frac{1-3\bar{w}_{\rm rd}}{4}N_{\rm rd} - \frac{1}{12}\ln g_{\rm th}}$$

$$\left[\bar{w}_{\rm rd} = \frac{1}{N_{\rm rd}}\int_0^{N_{\rm rd}} dN' w(N')\right]$$

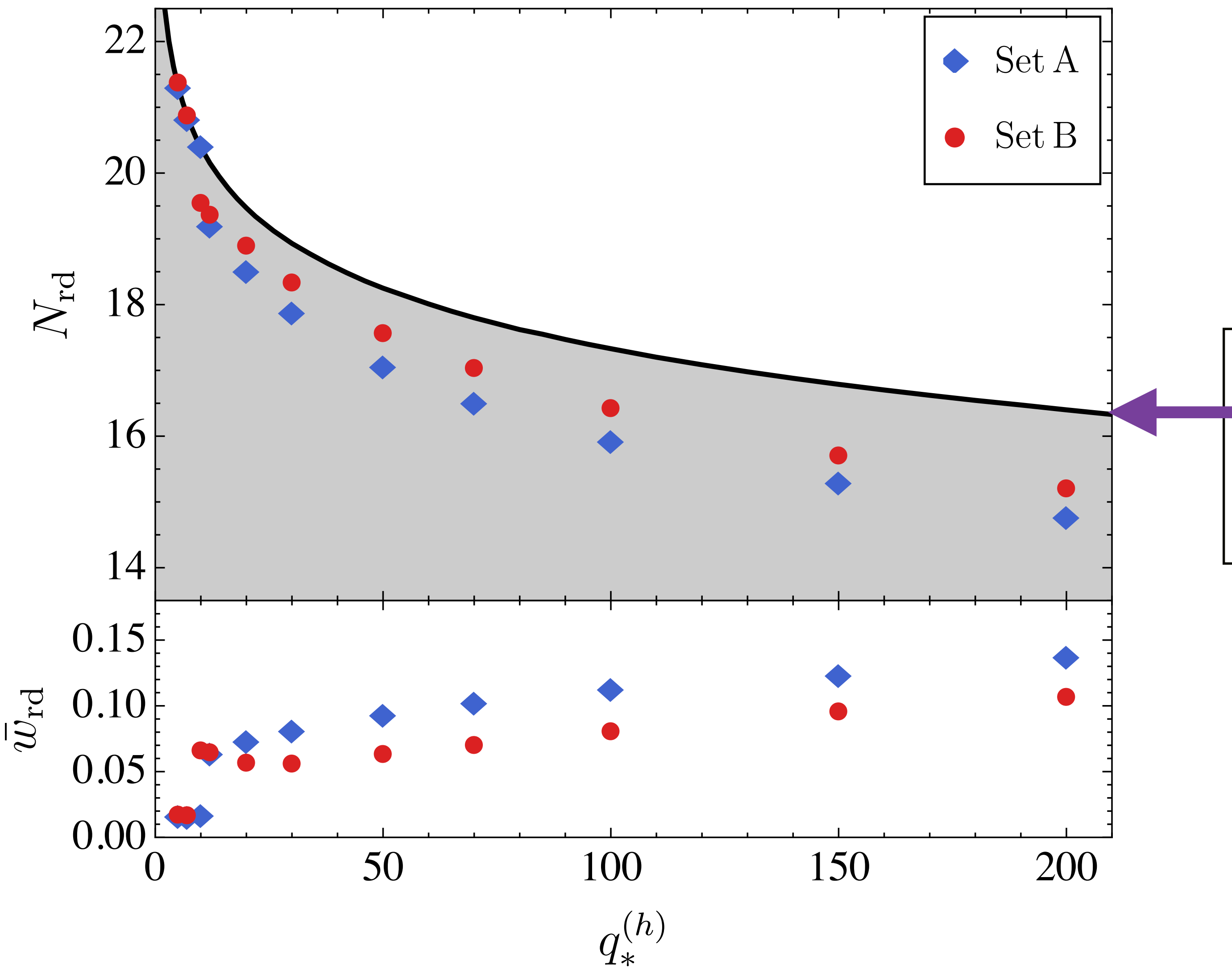
Inflationary CMB observables

Theoretical predictions for inflationary observables (n_s and r) **depend on the post-inflationary evolution of the equation of state**

$$N_k \simeq 61.5 + \frac{1}{4}\ln\frac{V^2(\phi_k)}{m_{\rm pl}^4\rho_{\rm end}} - \frac{1-3\bar{w}_{\rm rd}}{4}N_{\rm rd} - \frac{1}{12}\ln g_{\rm th}$$



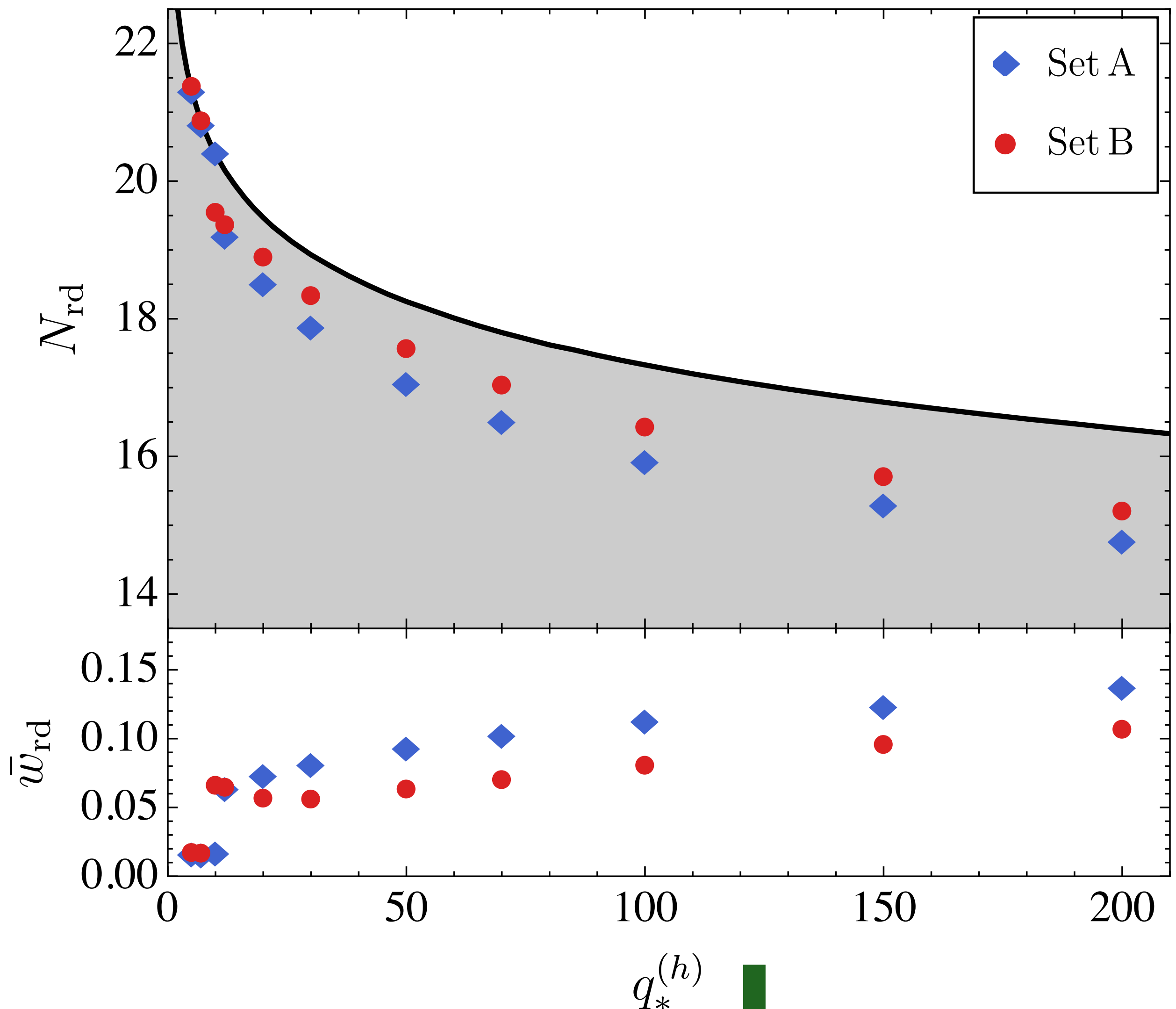
Inflationary CMB observables



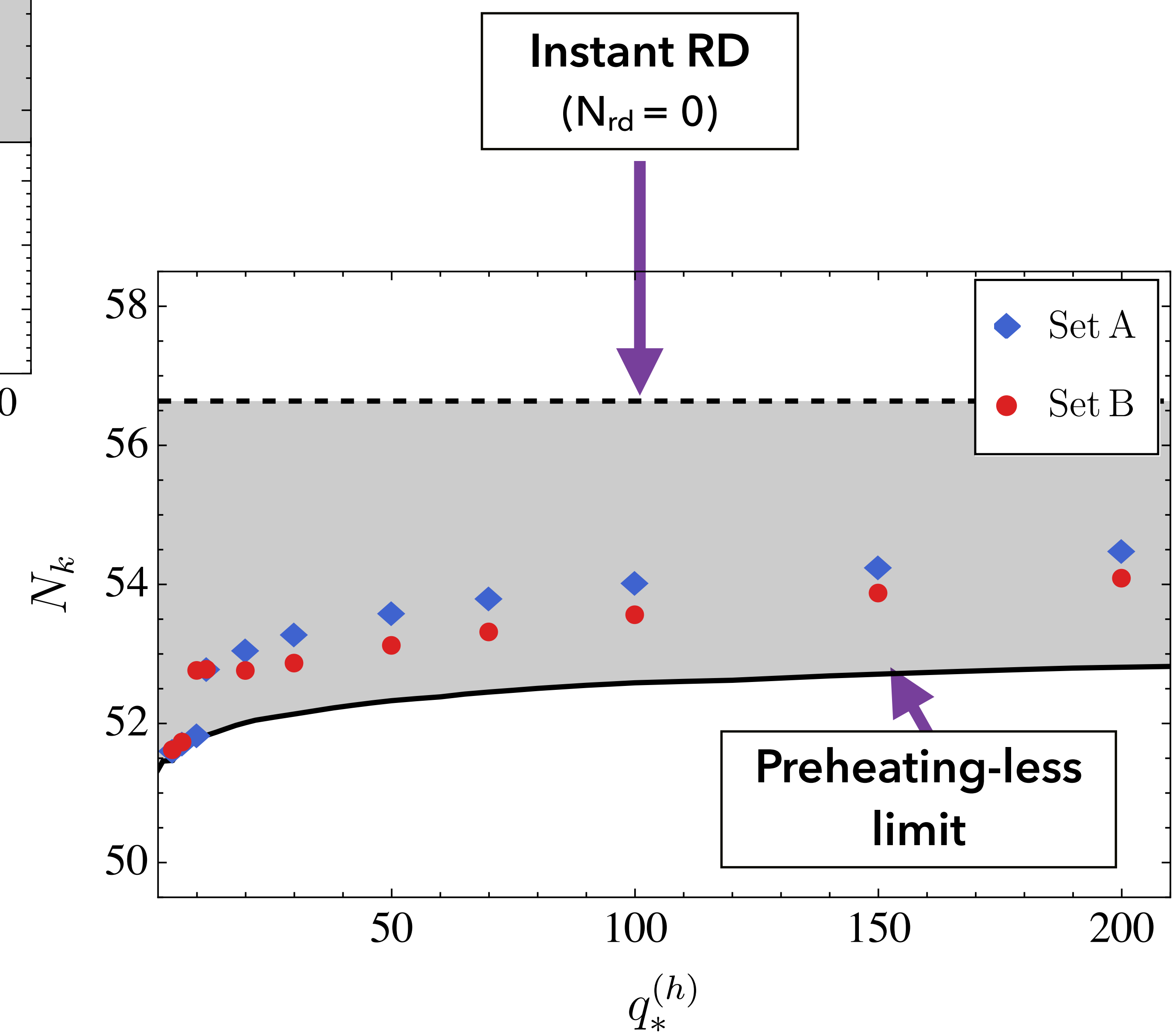
Preheating-less limit

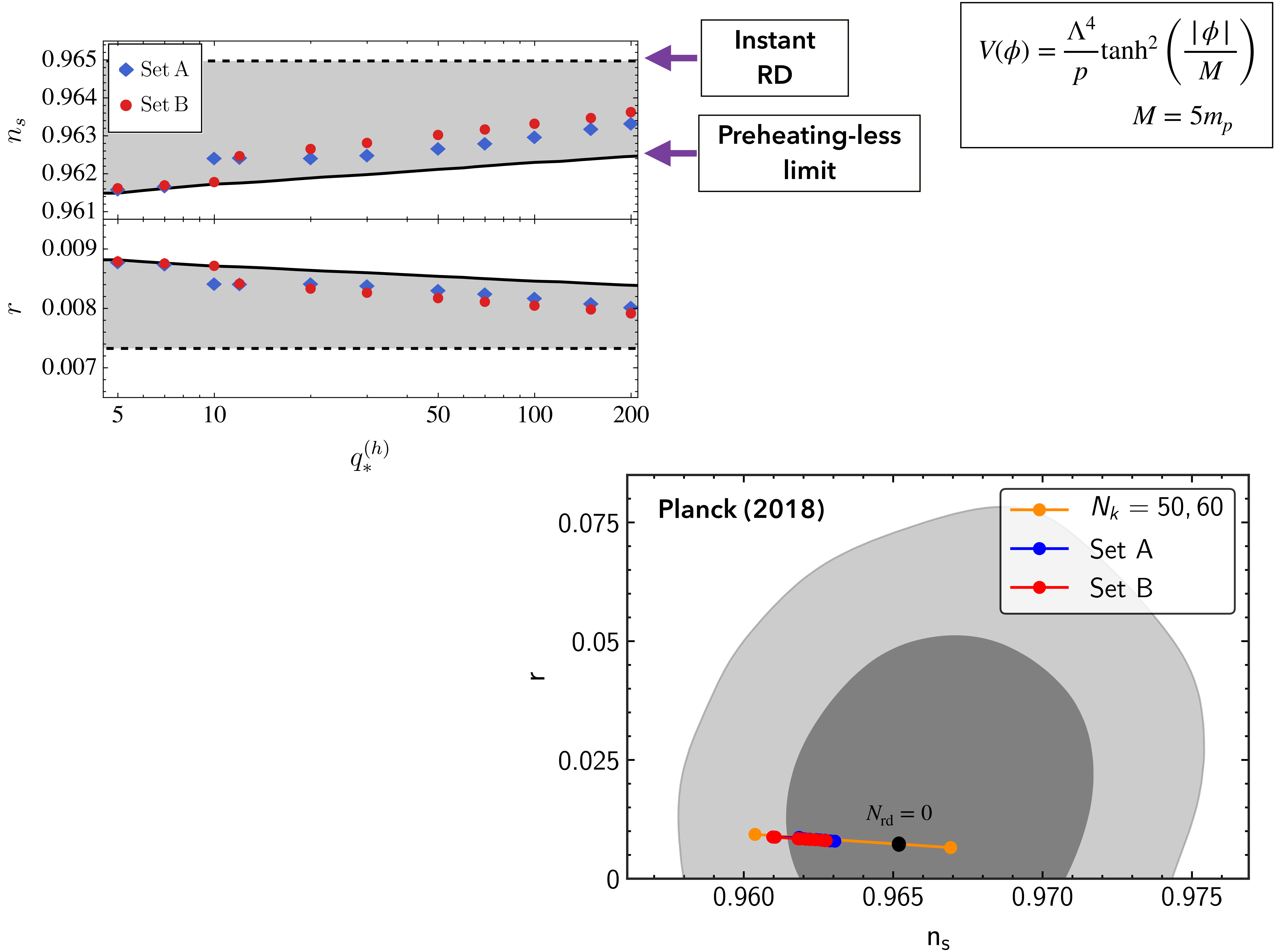
(w=0 until perturbative decay)

Inflationary CMB observables



$$N_k \simeq 61.5 + \frac{1}{4}\ln\frac{V^2(\phi_k)}{m_{\rm pl}^4\rho_{\rm end}} - \frac{1-3\bar{w}_{\rm rd}}{4}N_{\rm rd} - \frac{1}{12}\ln g_{\rm th}$$





Instant RD
Preheating-less limit
$V(\phi) = \frac{\Lambda^4}{p}\tanh^2\left(\frac{|\phi|}{M}\right)$
$M = 5m_p$
n_s
r
Set A
Set B
0.965
0.964
0.963
0.962
0.961
0.009
0.008
0.007
5
10
50
100
200
$q_*^{(h)}$
Planck (2018)
$N_k = 50, 60$
Set A
Set B
$N_{\rm rd} = 0$
0.075
0.05
0.025
0
0.960
0.965
0.970
0.975
n_s
r

Conclusions

- If your inflationary potential has a quadratic minimum , the equation of state is expected to return to $\bar{w} \to 0$ after preheating ends.

- This may potentially led to a severe suppression of any SGWB produced during inflation or (p)reheating.

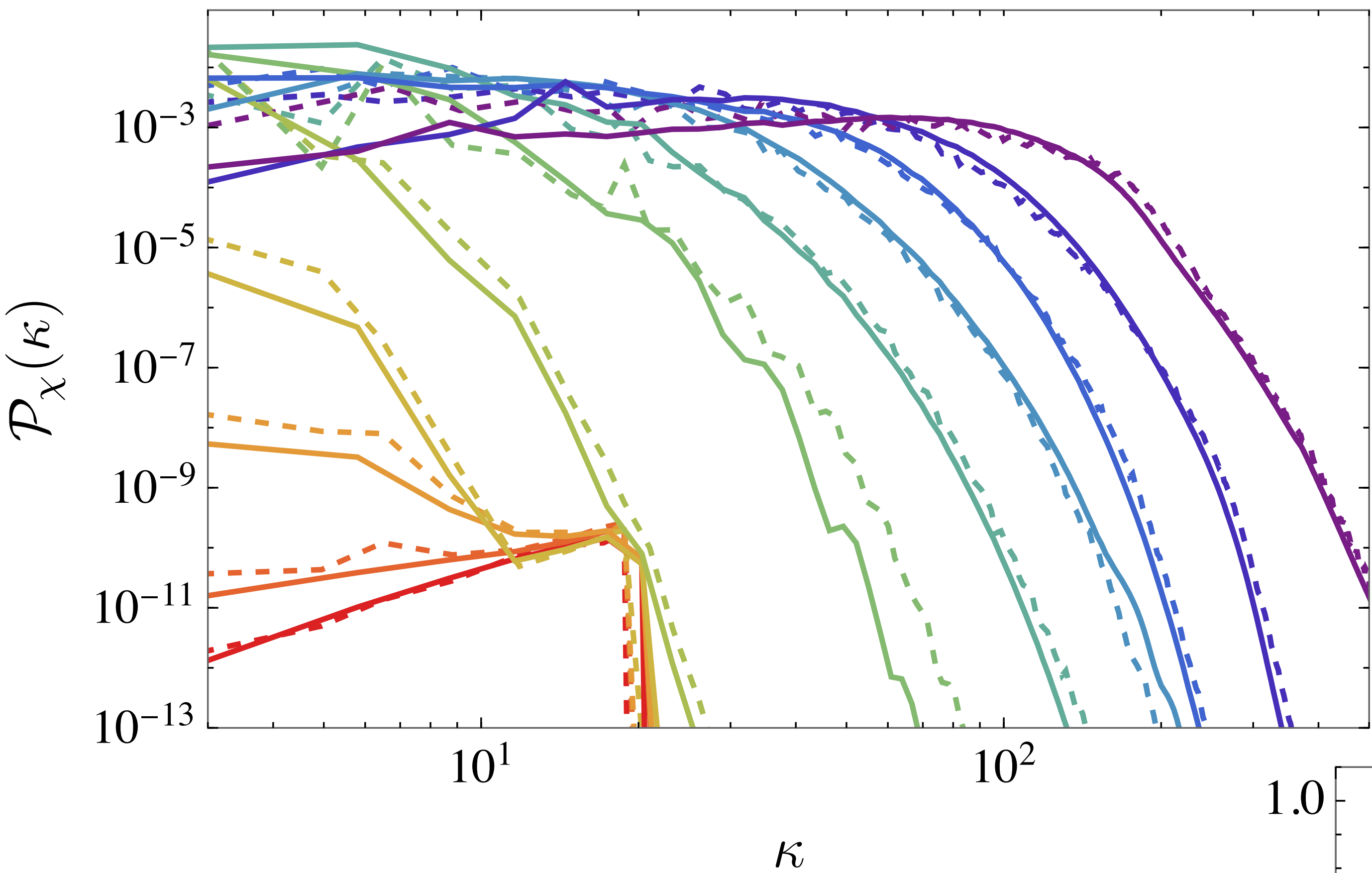
- The characterization of the equation of state until radiation domination also allows to accurately compute inflationary CMB observables.

CHECK YOUR EQUATION OF STATE!

Thank you!

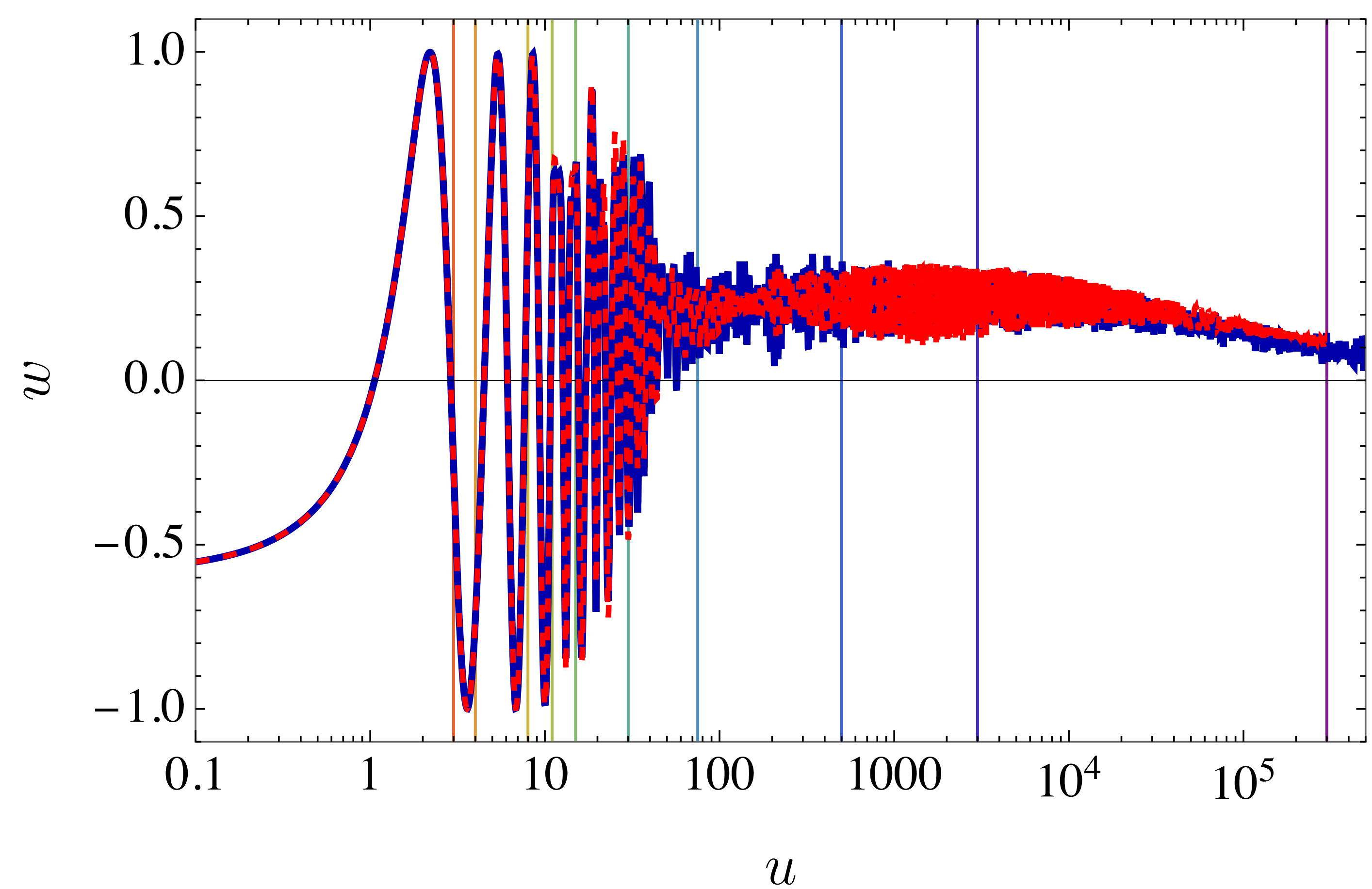
Backup slides

Comparison of 2+1 and 3+1 simulations

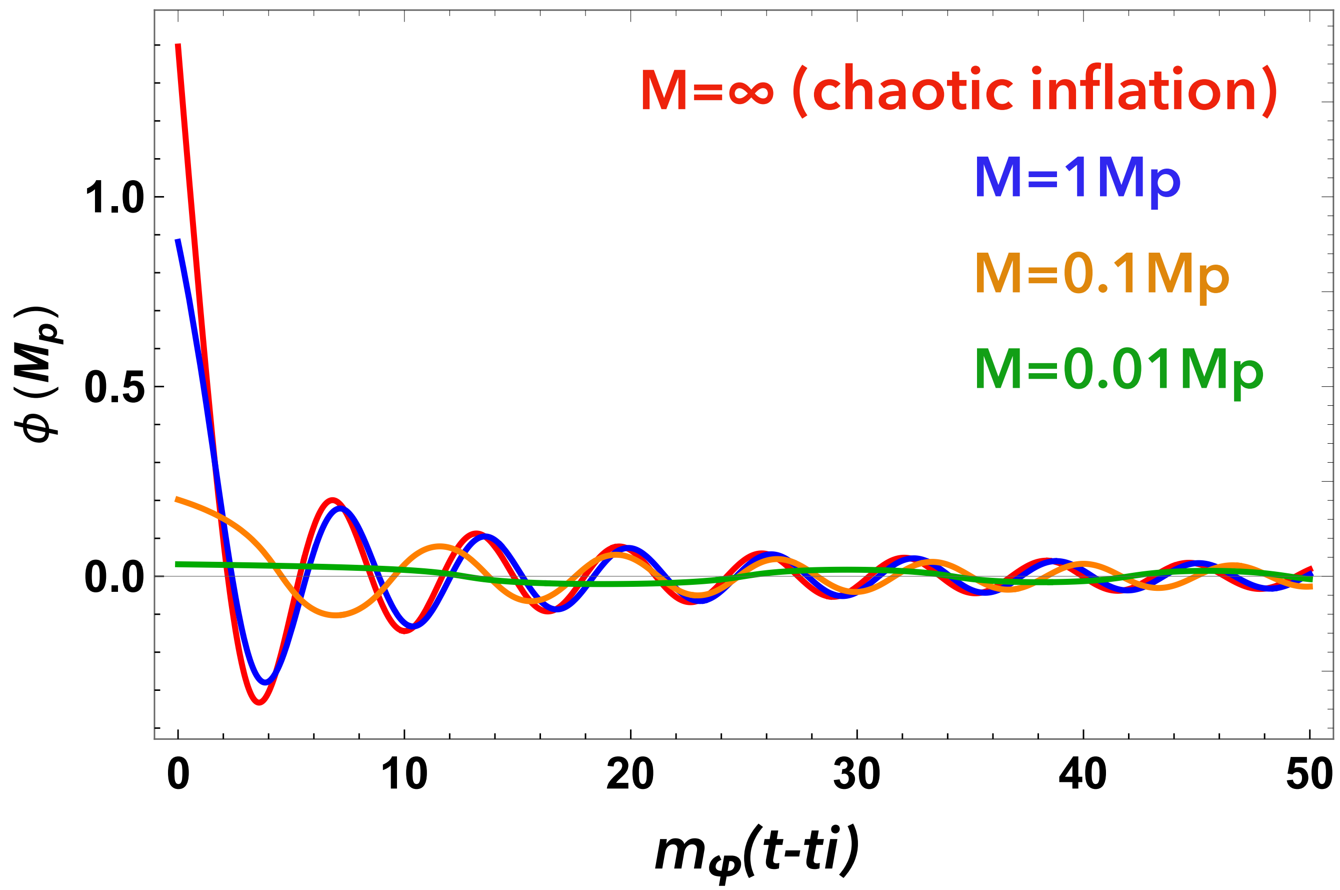


— 2+1 dimensions

- - - - 3+1 dimensions



Post-inflationary oscillations



TWO REGIMES:

$$M \gtrsim M_p \rightarrow V''(|\phi_i|) > 0$$

$$M \lesssim M_p \rightarrow V''(|\phi_i|) < 0$$

for M $\gtrsim M_p$, we can approximate the potential with a power-law form

$\phi_i \equiv \phi(t_i)$ inflaton amplitude at the end of inflation

Frequency of oscillations: (M $\gtrsim M_p$)

$$\Omega^2_{\text{osc}} \approx \omega_*^2 \left(t/t_i\right)^{2-4/p} \quad , \quad \omega_*^2 \equiv \frac{p}{M^p}\Lambda^4\phi_i^{p-2}$$

Inflationary predictions for alpha-att. T-model