Perturbative cosmological phase transitions in a broad temperature range[*]

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Numerical simulations of early universe sources of GWs
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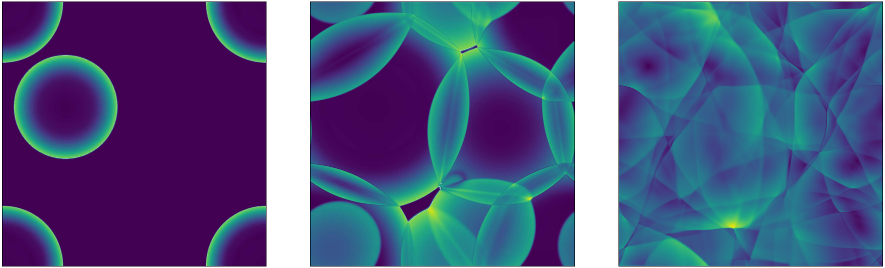
[*] Based on collaboration of F. Bernardo, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite-temperature affect gravitational-wave predictions*, [2503.18904], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory*, Phys. Rev. D **110** (2024) 096006 [2405.18349]. Supported by the SNSF under grant PZ00P2-215997.

The thermal history of electroweak symmetry breaking

If strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

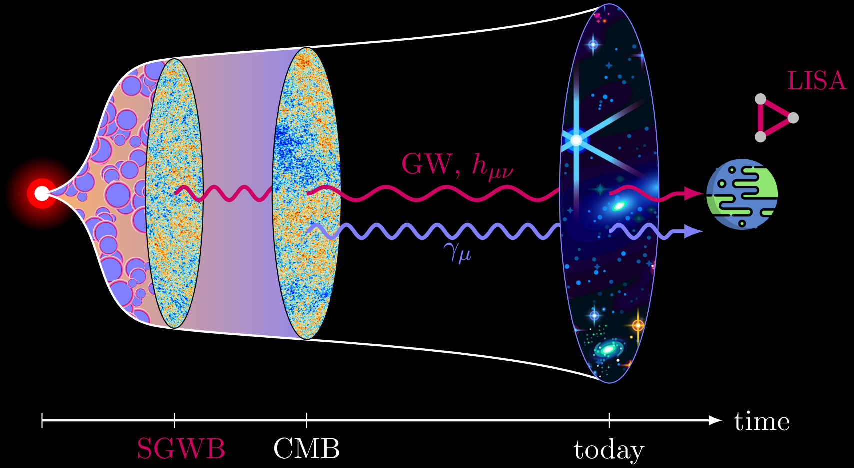
- ▷ Nucleation, growth, and collision of bubbles
- ▷ Generation of Baryon asymmetry of the universe



figures by D. Cutting, M. Hindmarsh, and D. J. Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions*, Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

Gravitational Waves (GWs) can be

sourced by early-universe phase transitions caused by new physics.



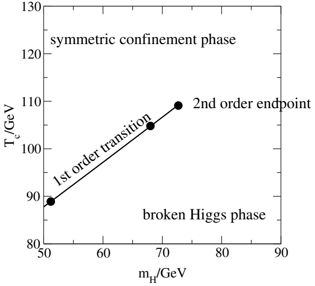
Cosmic Microwave Background (CMB)
Stochastic GW Background (SGWB)
Laser Interferometer Space Antenna (LISA)

Extended thermal history of EW symmetry breaking

In Standard Model, EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions.

Study **BSM physics** near EW scale in context of phase transitions:

- ▷ Light fields strongly coupled to Higgs
- ▷ Collider targets. BSM testing pipeline: Collider phenomenology



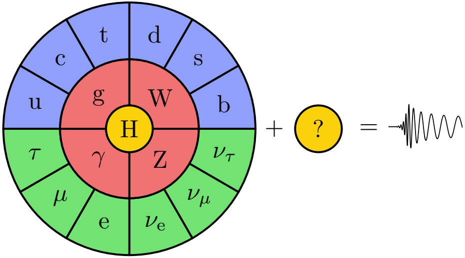
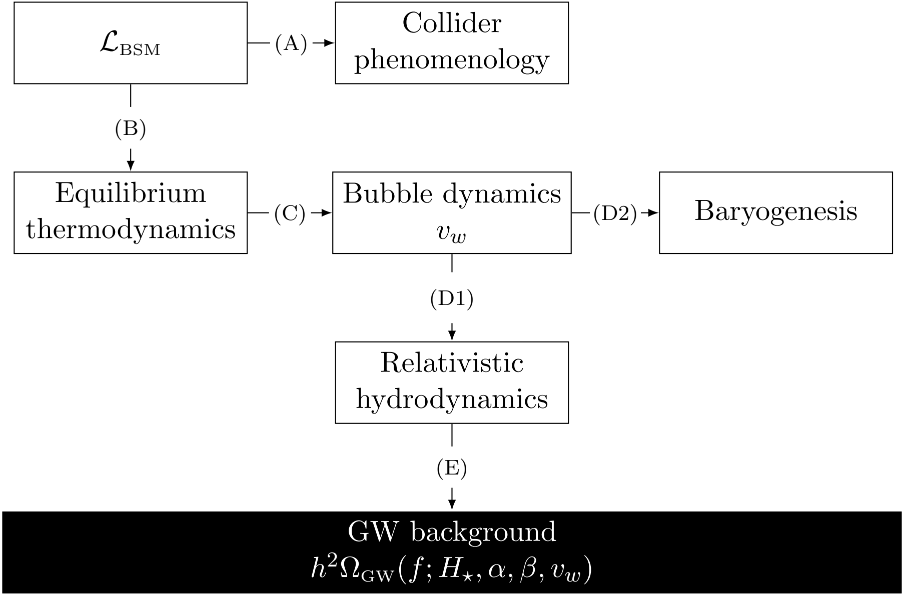


figure by M. Laine, *Electroweak phase transition beyond the standard model*, in *4th International Conference on Strong and Electroweak Matter*, pp. 58–69, 6, 2000 [hep-ph/0010275]

Uncertainties of the gravitational wave pipeline



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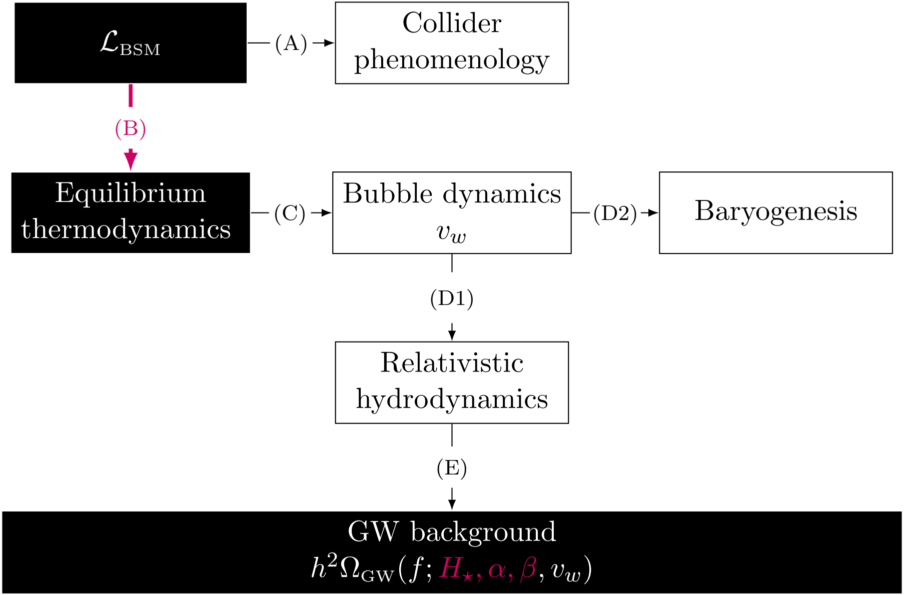

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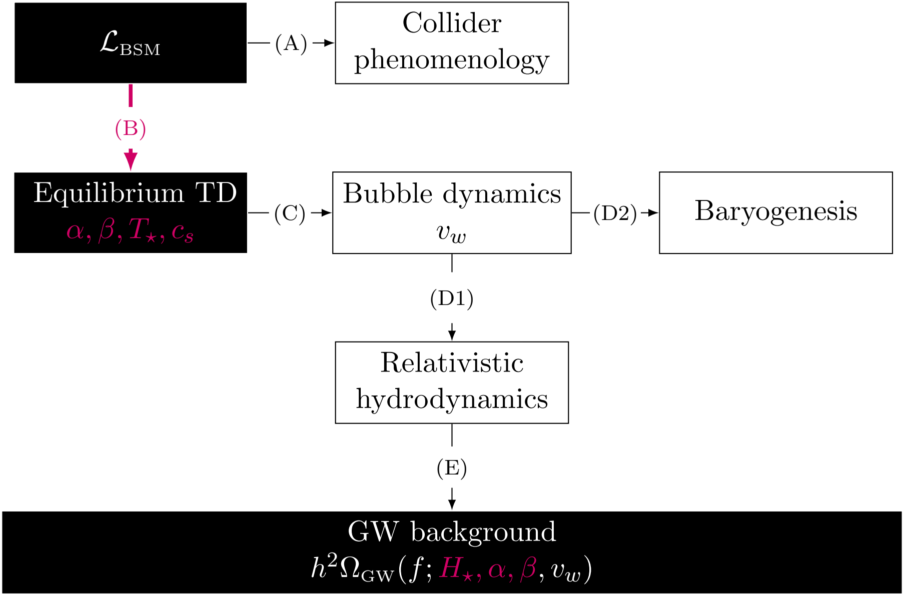
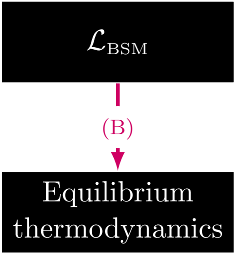
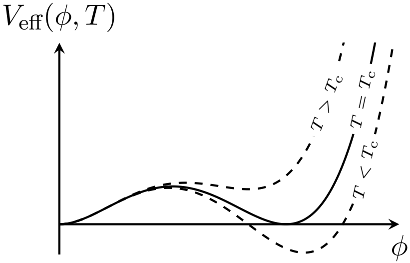


(B): The effective potential V_{eff} in perturbation theory[1]

encodes equilibrium thermodynamics as function of BSM parameters.
Origin of uncertainty.





[1] R. Jackiw, *Functional evaluation of the effective potential*, Phys. Rev. D **9** (1974) 1686

(B): The effective potential V_{eff} in perturbation theory

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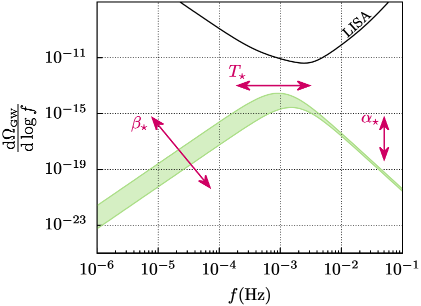
Origin of uncertainty.

$T_\star$ reference temperature of the transition ($T_\star = T_{\mathrm{n}}, T_{\mathrm{p}}$),

α phase transition strength,

β/H inverse duration of the transition,



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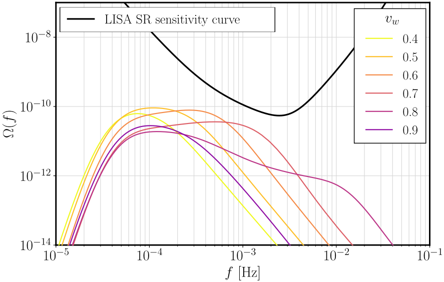
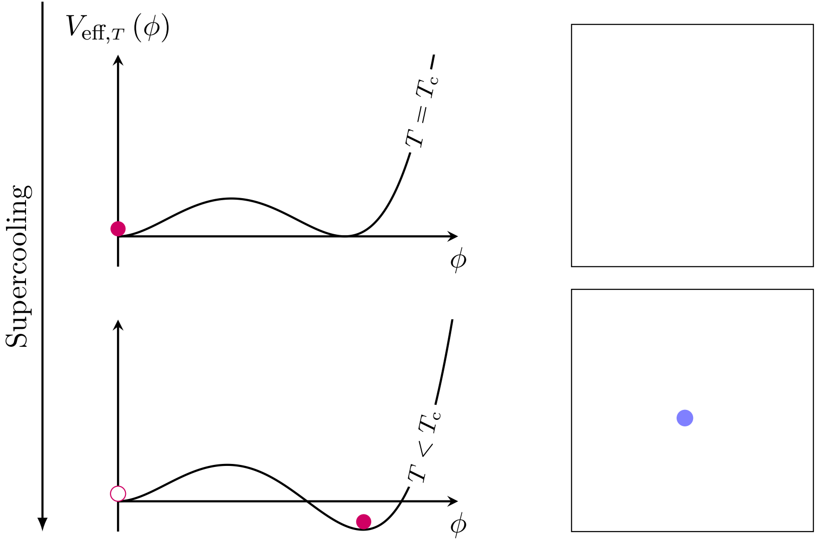
Origin of uncertainty.

- $T_\star$ reference temperature of the transition ($T_\star = T_{\rm n}, T_{\rm p}$),
- α phase transition strength,
- β/H inverse duration of the transition,
- v_w Iteratively solve coupled fluid, scalar field, Boltzmann equations[2]



[2] Cf. talk by **Jorinde v.d. Vis** on **Tue 9:30**, A. Ekstedt, O. Gould, J. Hirvonen, *et al.*, *How fast does the WallGo? A package for computing wall velocities in first-order phase transitions*, JHEP **04** (2025) 101 [2411.04970], C. Gowling and M. Hindmarsh, *Observational prospects for phase transitions at LISA: Fisher matrix analysis*, JCAP **10** (2021) 039 [2106.05984]

Nucleation rate and transition reference scale



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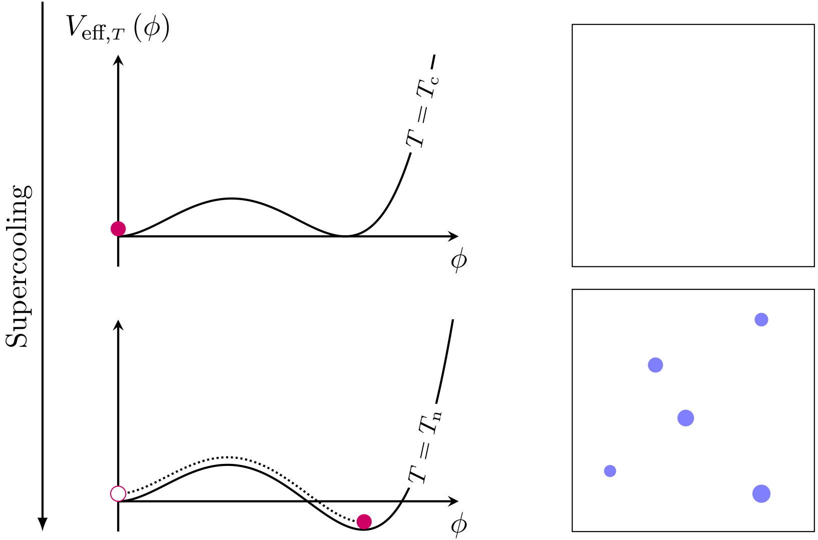

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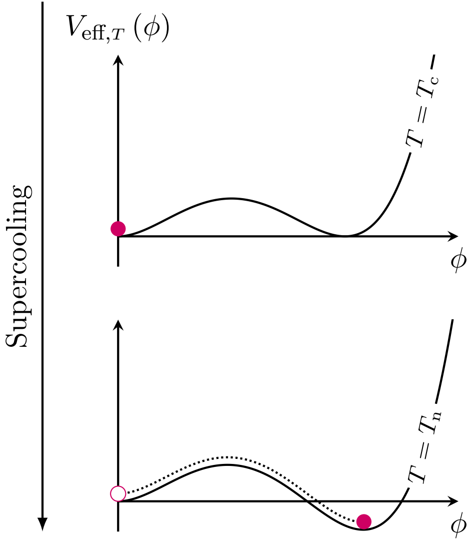

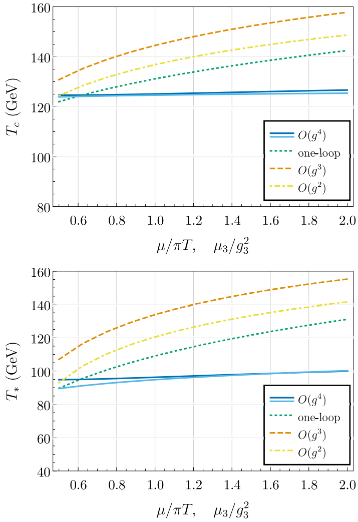

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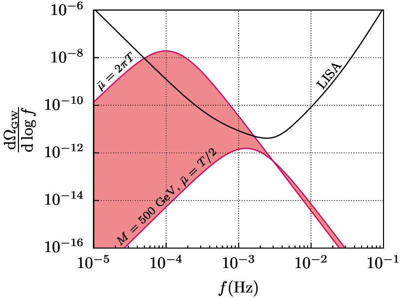
Theoretical predictions are not robust

$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes[3] as $\Omega_{\rm GW}$ depends strongly on the transition temperature, $T_\star$, in simulation fits:

$$\Omega_{\rm GW} \propto \frac{(\Delta V_\star)^2}{T_\star^8}\,.$$

Vary RG scale $\bar{\mu}$ in SM extensions:

- ▷ SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$



[3] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

[4] S. Biondini, P. Schicho, and T. V. I. Tenkanen, *Strong electroweak phase transition in t-channel simplified dark matter models*, JCAP **10** (2022) 044 [2207.12207]

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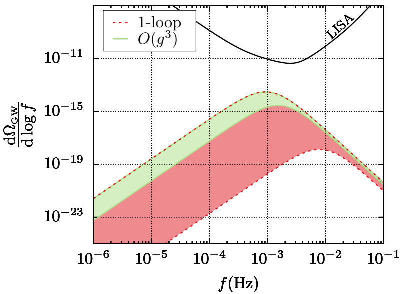
Vary RG scale $\bar{\mu}$ in SM extensions:

- SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$
- xSM: SM + singlet
- DM models[4]



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The effective potential in a broad temperature range

Inspect limiting cases from total potential[5]

$$V_{\rm eff}^{\rm res} = V_{\rm eff}^{\rm res} - V_{\rm eff}^{\rm res,soft} + V_{\rm eff}^{\rm res,soft} = \underbrace{\left(V_{\rm eff}^{\rm naive} - V_{\rm eff}^{\rm naive,soft}\right)}_{\text{IR safe}} + \underbrace{V_{\rm eff}^{\rm res,soft}}_{\text{IR safe}} .$$

At low-T, approach vacuum Coleman-Weinberg (CW) limit

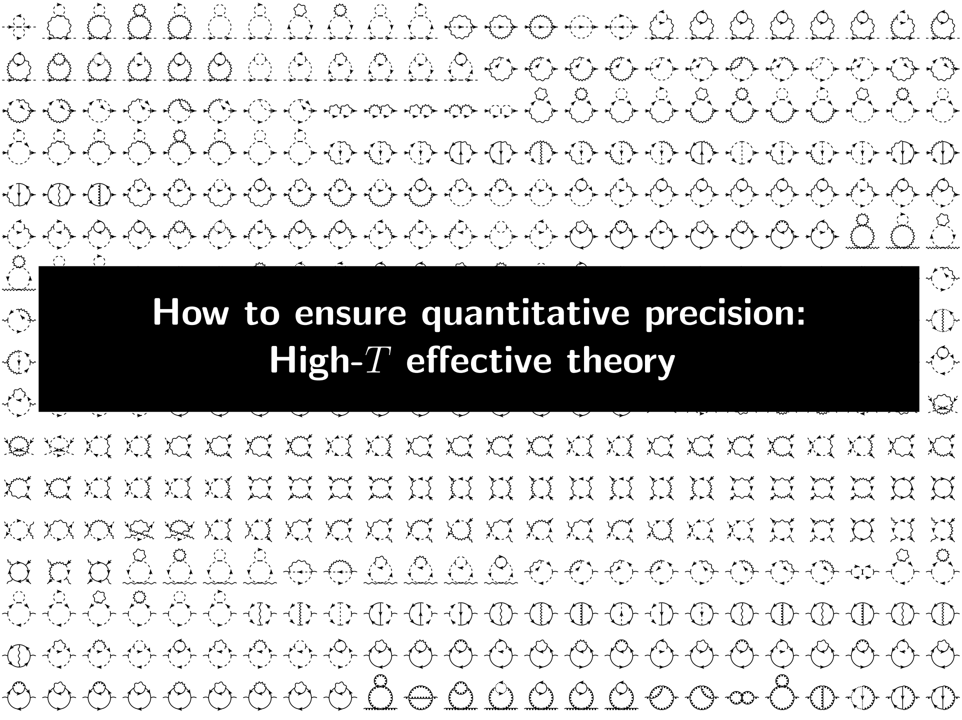
$$V_{\rm eff}^{\rm res} = V_{\rm eff}^{\rm naive}\Big|_{T\to 0} = V_{\rm eff}^{\rm CW} .$$

At high-T, can relate potential to EFT for zero-mode

$$V_{\rm eff}^{\rm res}\Big|_{M/T\ll 1} = V_{\rm eff}^{\text{high-}T\text{ EFT}} .$$

This talk: construct, test limits, extend validity of high-T EFT.

[5] P. Navarrete, R. Paatelainen, K. Seppänen, and T. V. I. Tenkanen, *Cosmological phase transitions without high-temperature expansions,* [2507.07014]



How to ensure quantitative precision: High-T effective theory

Perturbative phase transitions need scale hierarchies

for quantum effects ΔV_{fluct} to influence the tree-level potential

$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}}\ .$$

Assume particle χ couples to the SM via $g^2 \Phi^\dagger \Phi \chi^\dagger \chi$. If $M_\chi \gg m_\Phi$, integrating out χ introduces Higgs-mass corrections of the form:

$$(\Delta m_\Phi^2)\Phi^\dagger\Phi =$$

$$\sim g^2 M_\chi^2 \Phi^\dagger \Phi\,, \qquad \frac{(\Delta m_\Phi^2)}{m_\Phi^2} = g^2 \left[\frac{M_\chi}{m_\Phi}\right]^2 .$$

Relevant operators ($\sigma > 0$) in the IR get large UV contributions and

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 \left[\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}}\right]^\sigma \overset{!}{\gtrsim} 1 \quad \Rightarrow \begin{cases} \text{strong coupling} & g^2 \gtrsim 1 \\ \text{scale hierarchy} & \frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \sim \left[\frac{1}{g^2}\right]^{\frac{1}{\sigma}} \gg 1 \end{cases}$$

Multi-scale hierarchy in hot classicalizing gauge theories

Evaluated **Matsubara sums** yield Bose(Fermi) distribution. Asymptotically high T and weak $g \ll 1$: effective expansion parameter

$$\epsilon_{\rm B} = g^2 n_{\rm B}(E) = \frac{g^2}{e^{E/T}-1} \approx \frac{g^2 T}{E} \,.$$

Differs from weak coupling g^2. Fermions are IR-safe $g^2 n_{\rm F}(E) \sim g^2/2$.

$$E \sim \begin{cases} \pi T & \textit{hard}\ \text{scale} \\ gT & \textit{soft}\ \text{scale} \\ g^{3/2}T & \textit{supersoft}\ \text{scale} \\ g^2 T/\pi & \textit{ultrasoft}\ \text{scale} \end{cases}$$

quantum theory (hard, soft scales)

symmetry breaking (supersoft, ultrasoft scales)

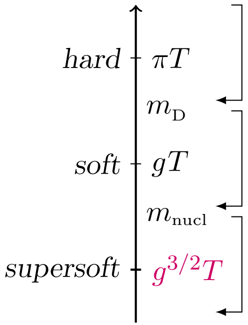
Limit: Confinement-like behavior in ultrasoft sector $g^2 n_{\rm B}(g^2 T) \sim \mathcal{O}(1)$. Ultrasoft bosons are non-perturbative at finite T: Linde IR problem.[6]

[6] A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas*, Phys. Lett. B **96** (1980) 289, O. Gould and T. V. I. Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions*, JHEP **01** (2024) 048 [2309.01672]

Effective Field Theory (EFT): Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.

Precision thermodynamics of non-Abelian gauge theories as QCD and (EW) phase transition[7] using e.g. `DRalgo`.[8] Two `step` procedure:



$\mathcal{L}_{\text{Full}}$, $(d+1)$-dim

`step1`: Dimensional Reduction

$\mathcal{L}_d$, d-dim

`step2`: Integrate out $m_{B_0} \sim m_{\text{D}}$ and $m_{B_i} \sim g\phi$

$\overline{\mathcal{L}}_d$, d-dim, HEFT-like

[7] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

[8] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, `DRalgo`*: A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

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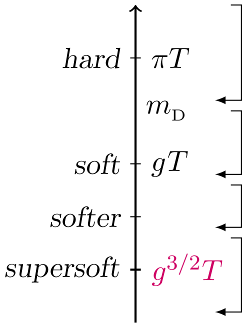

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`step2.1`: Integrate out $m_{B_0} \sim m_{\mathrm{D}}$

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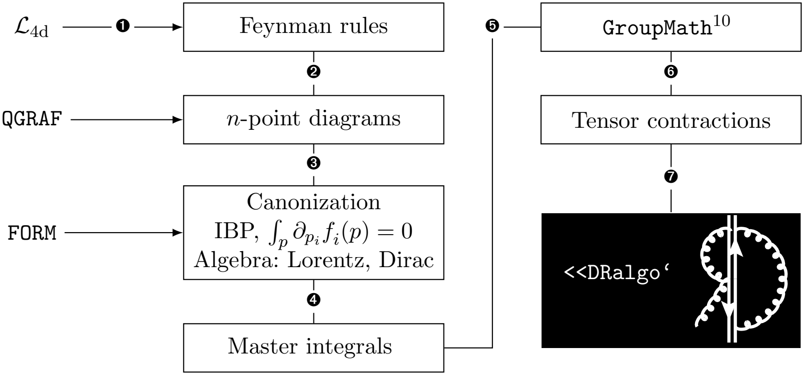
State-of-the-art dim-4 operator Mathematica package DRalgo.[9]



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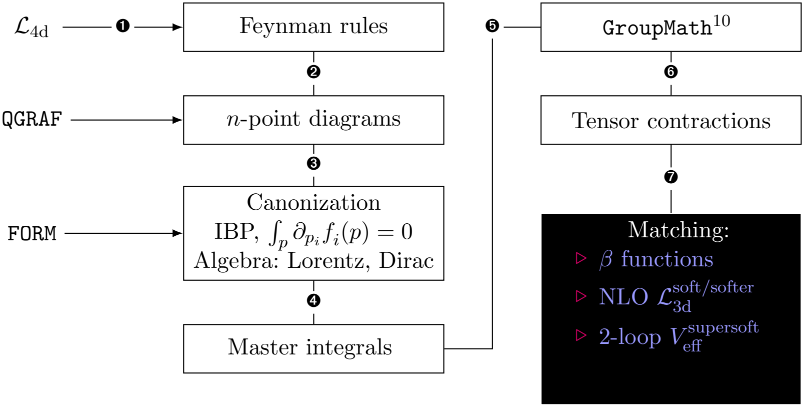

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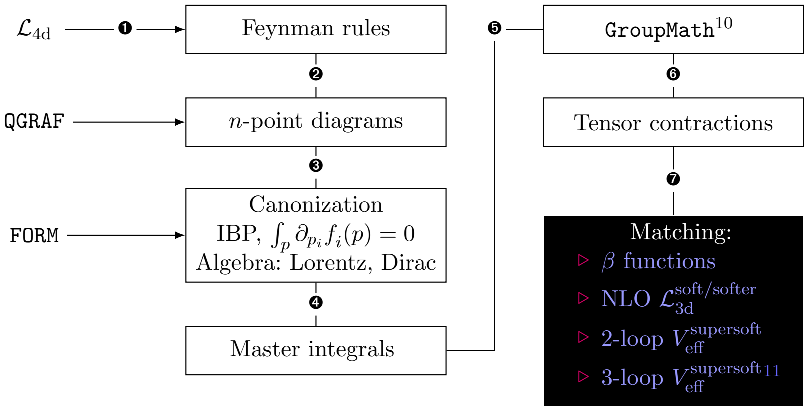

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Completing the perturbative program for phase transitions

❶ $\mathcal{L}^{3d}$ (hard-to-soft matching)

❷ $V_{\rm eff}^{3d}$ (soft-to-supersoft matching)

❸ Thermodynamics

Constructing the supersoft EFT

Two approaches to a *final* (nucleation) EFT

$$\mathcal{L}_{4\mathrm{d}} \to \mathcal{L}_{3\mathrm{d}}^{\mathrm{soft}} \to \mathcal{L}_{3\mathrm{d}}^{\mathrm{softer}} \to V_{\mathrm{eff}}^{\mathrm{supersoft}} \; ,$$

In the broken phase, utilize different hierarchies among Lorentz scalars, temporal scalars, and vectors:

$$m^2_{A_0} \big[\sim (gT)^2\big] \overset{\texttt{step2.1}}{\gg} m^2_{A_i} \big[\sim (g_3\phi)^2\big] \overset{\texttt{step2.2}}{\gg} m^2_3 \big[\sim \lambda_3 \phi^2\big] \; ,$$

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In the symmetric phase, the hierarchy is flipped

$$m_{A_0}^2\big[\sim (gT)^2\big] \gg m_3^2\big[\sim (g^{3/2}T)^2\big] \overset{\texttt{step2}}{\gg} m_{A_i}^2\big[\sim (g^2T)^2\big] \,.$$

Limits of GW predictions form cosmological phase transitions

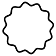
The SM-like $3d$ EFT $(\mathrm{SU}(2)$+Higgs)

describes the thermodynamics[12] of several parent $4d$ theories:

$$\mathcal{L}_{\text{3d}}^{\text{soft}} = \frac{1}{4}F_{ij}^{a}F_{ij}^{a} + (D_i\Phi)^{\dagger}(D_i\Phi) + (\partial_i A_0^a)^2 + V(\Phi) \ ,$$

$$V(\Phi) = m_3^2\Phi^{\dagger}\Phi + m_{\text{D}}^2 A_0^a A_0^a + \lambda_3(\Phi^{\dagger}\Phi)^2 + h_3(\Phi^{\dagger}\Phi)A_0^a A_0^a + \dots \ .$$

If $m_{A_i} \sim g_3\phi \gg m_3$, integrating out vector boson introduces LO barrier

$V_{\text{LO}}(\Phi) =$ • $+$  .

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[12] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

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If $m_{A_i} \sim g_3\phi \gg m_3$, integrating out vector boson introduces LO barrier

$$V_{\rm LO}(\Phi) = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 - \frac{g_3^3}{2\pi}\Big(\frac{\Phi^\dagger\Phi}{2}\Big)^{3/2}\ .$$

[12] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

[13] O. Gould and T. V. I. Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions*, JHEP **01** (2024) 048 [2309.01672],

The SM-like $3d$ EFT ($\mathrm{SU}(2)$+Higgs)

describes the thermodynamics[12] of several parent $4d$ theories:

$$\mathcal{L}_{\rm 3d}^{\rm softer} = \frac{1}{4}F_{ij}^aF_{ij}^a + (D_i\Phi)^\dagger(D_i\Phi) + V(\Phi)\ ,$$
$$V(\Phi) = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2\ .$$

If $m_{A_i} \sim g_3\phi \gg m_3$, integrating out vector boson introduces LO barrier

$$V_{\rm LO}(\Phi) \to y\Phi^\dagger\Phi + x(\Phi^\dagger\Phi)^2 - \frac{1}{2\pi}\Big(\frac{\Phi^\dagger\Phi}{2}\Big)^{3/2}\ .$$

Since $x \sim \frac{m_3^2}{m_{A_i}^2} \ll 1$, and at the phase transition $y \sim 1/x$, we strictly[13] x-pand the perturbative series using 3d EFT dimensionless couplings

$$x \equiv \frac{\lambda_3}{g_3^2}\ , \qquad\qquad y \equiv \frac{m_3^2}{g_3^4}\ .$$

[12] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

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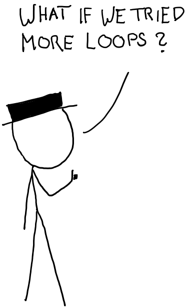
❷ $V_{\text{eff}}^{3\text{d}}$ (soft-to-supersoft matching)

Integrating out vector bosons in two steps up to 2-loops with `DRalgo`

$$m_{A_0}^2\left[\sim (gT)^2\right] \overset{\texttt{step2.1}}{\gg} m_{A_i}^2\left[\sim (g_3\phi)^2\right] \overset{\texttt{step2.2}}{\gg} m_3^2\left[\sim \lambda_3\phi^2\right] \ .$$

Focus on `step2.2` and add last perturbative orders $\mathrm{N}^3\mathrm{LO}$ and $\mathrm{N}^4\mathrm{LO}$.[14]



[14] A. Ekstedt, O. Gould, and J. Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order,* Phys. Rev. D **106** (2022) 036012 [2205.07241], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory,* Phys. Rev. D **110** (2024) 096006 [2405.18349]

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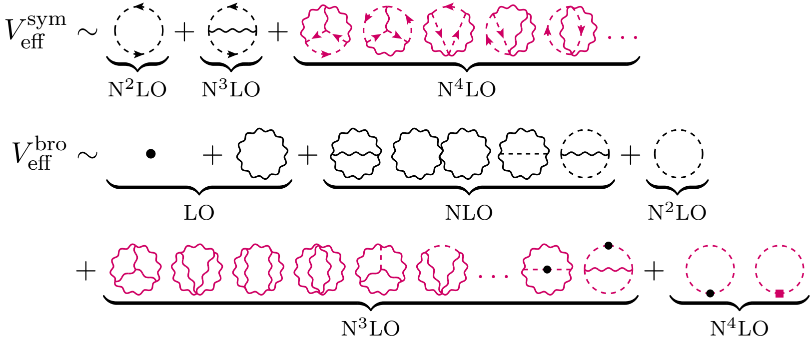

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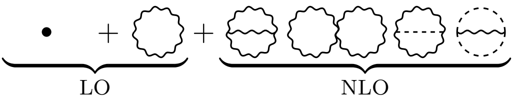
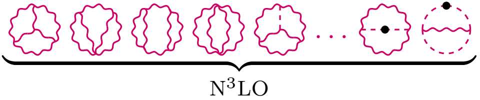
Focus on `step2.2` and add last perturbative orders N^{3}LO and N^{4}LO.[14]
In strict EFT expansion this organization can also be understood as:

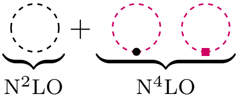
$$S^{\text{tree}}_{\text{supersoft}} = \int_{\mathbf{x}} \frac{1}{2}(\partial_i s)^2 Z_s + $$



$$+$$



$$S^{\text{1-loop}}_{\text{supersoft}} = $$



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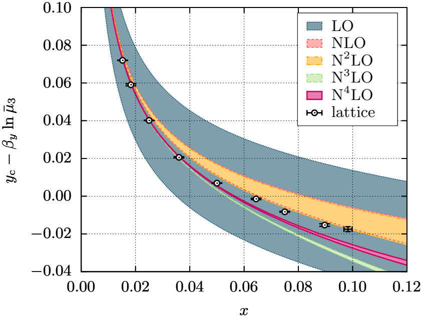
❸ Thermodynamics (for SU(2) + Higgs)

By using $F \sim V_{\text{eff}}(\phi_{\text{min}})$, determine the critical mass y_{c} (or T_{c})

$$\Delta F(y_{\text{c}}(x), x) = [F_{\text{bro}} - F_{\text{sym}}](y_{\text{c}}(x), x) = 0\,,$$

and the scalar *condensates*[15]

$$\Delta\langle\Phi^\dagger\Phi\rangle \equiv \frac{\partial}{\partial y}\Delta F\,, \qquad \Delta\langle(\Phi^\dagger\Phi)^2\rangle \equiv \frac{\partial}{\partial x}\Delta F\,.$$



[15] Lattice data: K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], O. Gould, S. Güyer, and K. Rummukainen, *First-order electroweak phase transitions: a nonperturbative update,* [2205.07238]

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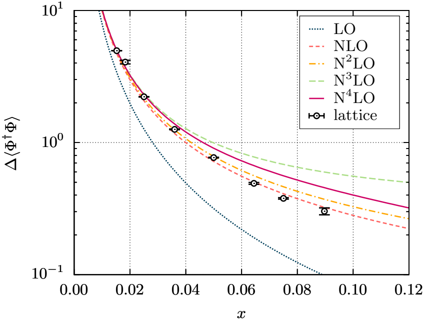

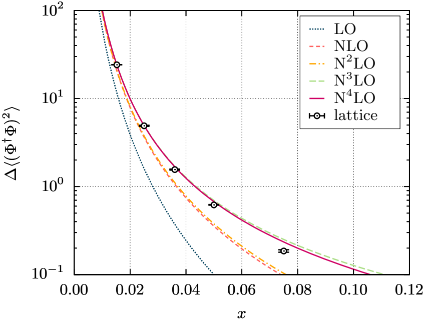

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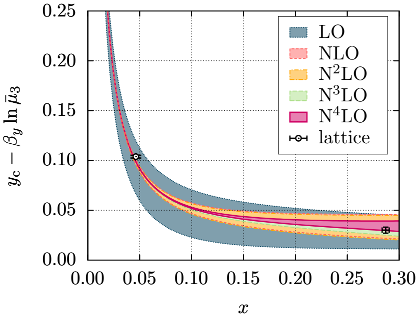


[15] Lattice data: K. Kajantie, M. Karjalainen, M. Laine, and J. Peisa, *Three-dimensional U(1) gauge + Higgs theory as an effective theory for finite temperature phase transitions,* Nucl. Phys. B **520** (1998) 345 [hep-lat/9711048], S. Mo, J. Hove, and A. Sudbo, *The Order of the metal to superconductor transition,* Phys. Rev. B **65** (2002) 104501 [cond-mat/0109260]

Predicting gravitational waves

Thermodynamics enters the GW spectrum through the strength and inverse duration of the transition, $h^2\Omega_{\rm GW}(f; H_\star, \alpha, \beta, v_w)$:

$$\alpha \sim \frac{\mathrm{d}\,\Delta F(y_{\rm c}, x)}{\mathrm{d}\ln T} = \left(\frac{\mathrm{d}\,y_{\rm c}}{\mathrm{d}\ln T}\right)\Delta\langle\Phi^\dagger\Phi\rangle + \left(\frac{\mathrm{d}\,x}{\mathrm{d}\ln T}\right)\Delta\langle(\Phi^\dagger\Phi)^2\rangle\,,$$

$$\frac{\beta}{H} = -\frac{\mathrm{d}\ln\Gamma}{\mathrm{d}\ln T}\,.$$

$\implies$ [UV] $\times$ [IR] factorization.[16]

Completed perturbative predictions for:	α at N^{4}LO,
	β/H at N^{2}LO.

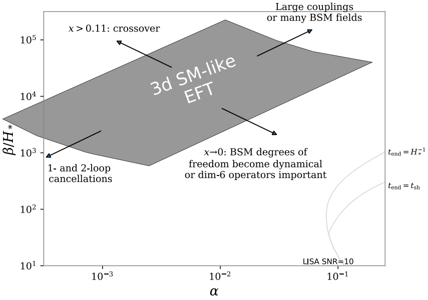
Limit: last perturbative order is N^{4}LO.

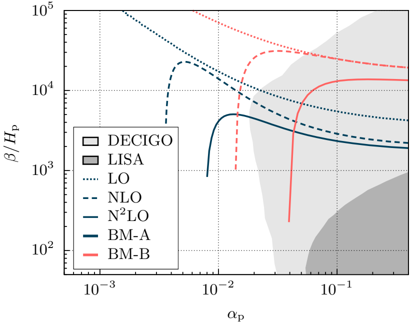
Todo: final perturbative correction for thermal bubble nucleation rate.

[16] O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition,* Phys. Rev. D **100** (2019) 115024 [1903.11604]

$\alpha/(\beta/H)$ rhombus of SM-like EFT with no prospect for large SNR.[17]
Better access interesting LISA SNR by increasing loop order:

BM-A xSM with weakly portal-coupled singlet (decoupled)
BM-B xSM with strongly portal-coupled singlet





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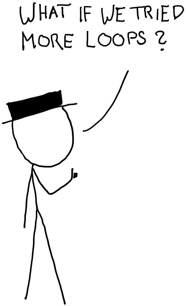
Limitations of GW predictions form cosmological phase transitions

Dimension-six operators in U(1) + Higgs



So far truncated operators at high T at dimension 4:

$$S^{\rm 3d}_{\rm soft} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}^{\rm 3d}_{\rm soft} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\},$$

$$S^{\rm 3d}_{\rm softer} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}^{\rm 3d}_{\rm softer} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(m_{\rm D})^n} \right\}.$$

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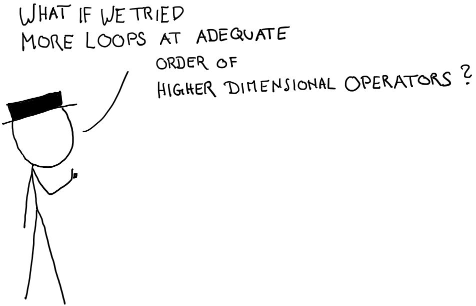

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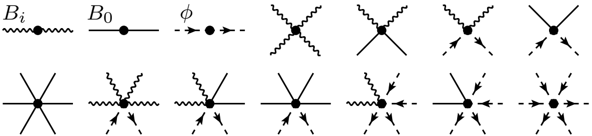
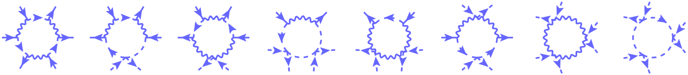
Vertex structures

$\mathcal{L}_{\text{soft}}$ is non-super-renormalizable.



Determine Wilson coefficients $\alpha_i(d)$ in d-dimensions:

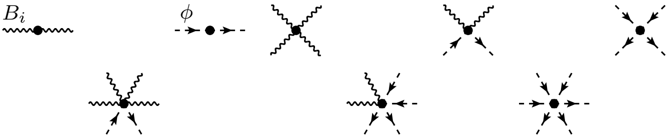
- Evaluate (2–6)-point vertices at one-loop order



- Field redefinitions

Wilson coefficients are gauge-parameter (ξ) independent order-by-order.
Now focus on $c_6(\phi^\dagger\phi)^3$ effect.

$\mathcal{L}_{\text{softer}}$ is non-super-renormalizable.



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Marginal operators at dimension six

The soft-scale marginal operator is suppressed at $\mathcal{O}(g^6)$

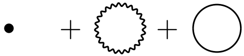
$$c_6 = \frac{\zeta_3}{32\pi^4}\Big(g^6 - \frac{31}{30}g^4\lambda + 5\,g^2\lambda^2 + \frac{20}{3}\lambda^3\Big) + \mathcal{O}(g^8)\,.$$

The softer-scale marginal operator is enhanced at $\mathcal{O}(g^3)$

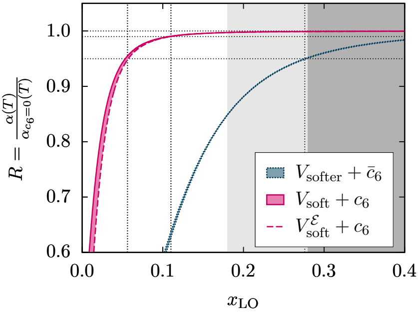
$$\bar{c}_6 = c_6 + \frac{\sqrt{3}\,g^3}{8\pi}\Big(\frac{m_{\rm D}^{\rm LO}}{m_{\rm D}}\Big)^3\big(1 - x_{\rm LO}\big) + \mathcal{O}(g^4)\,.$$

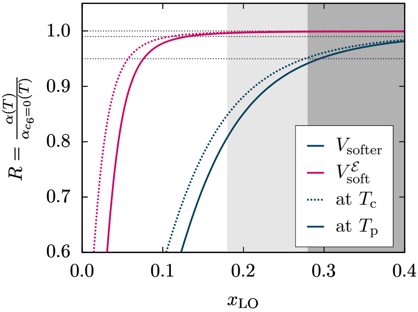
Validity of EFT

Leading-order effective potential given by

$$V_{\rm soft}^{\rm LO}(\Phi) =$$  .

Effect of c_6 shows broad window of high-T validity for soft EFT:[18]





[18] F. Bernardo, P. Klose, P. Schicho, and T. V. I. Tenkanen, *Higher-dimensional operators at finite-temperature affect gravitational-wave predictions,* [2503.18904]

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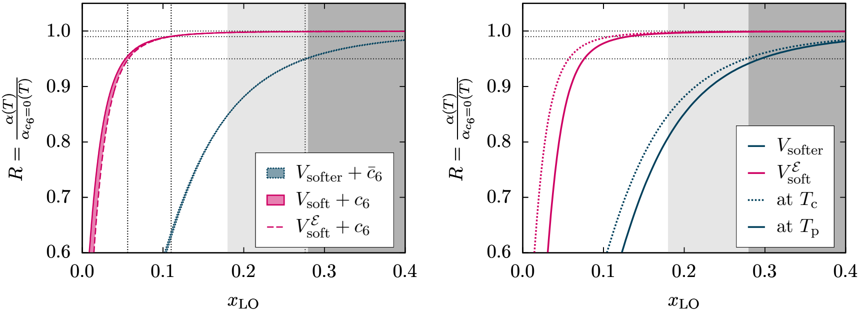

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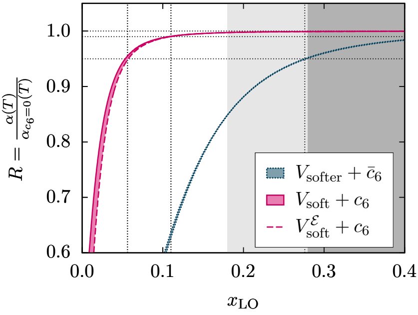

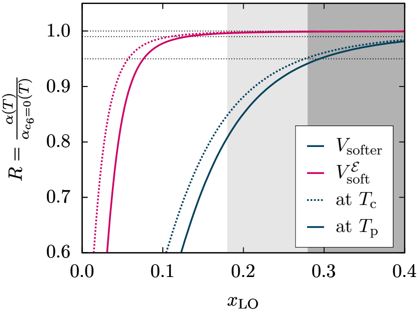

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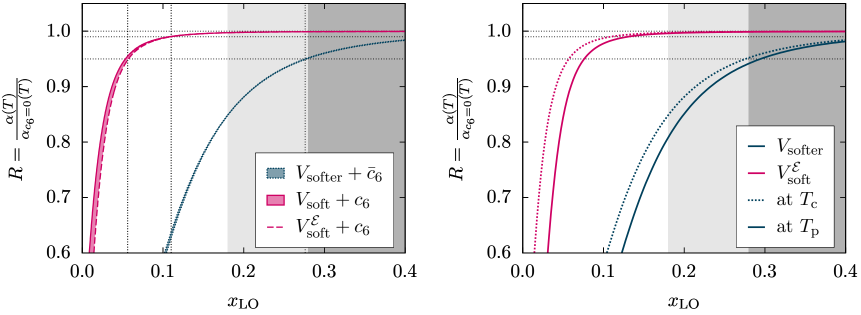

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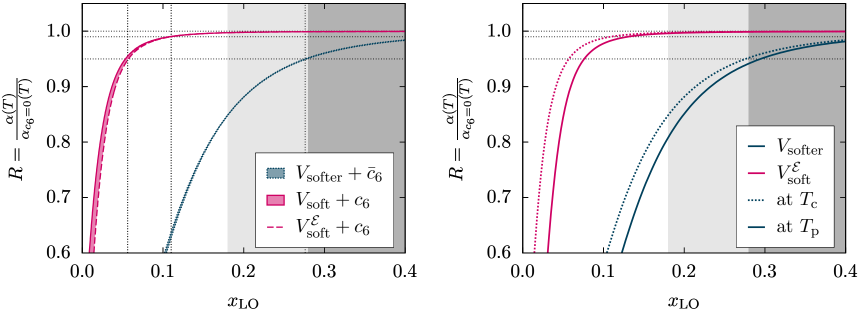

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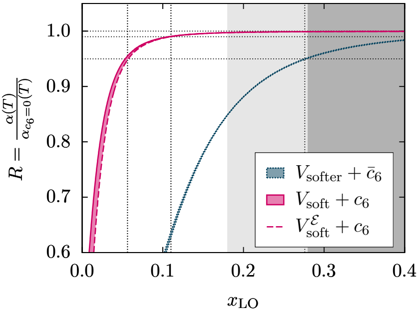

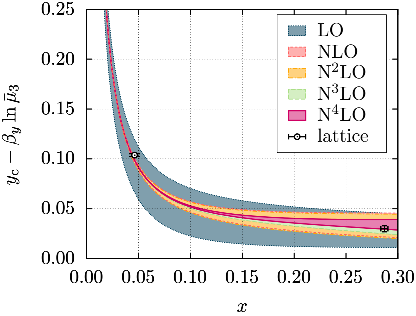

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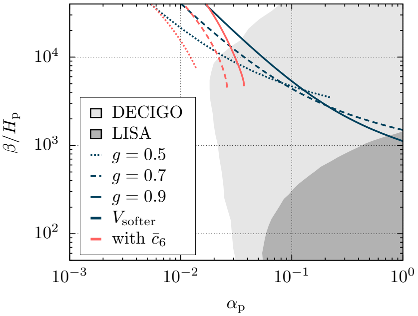
Gravitational wave prospects

Soft scale enhances phase-transition strength, $\alpha(T)$.
High-T expansion is compromised for regime relevant for LISA:



Limitation: conventional lattice results of (dim-4) super-renormalizable 3d EFT do not describe hard/soft-scale driven transitions.[19]

Need 4d full theory lattice simulations for a reliable description?

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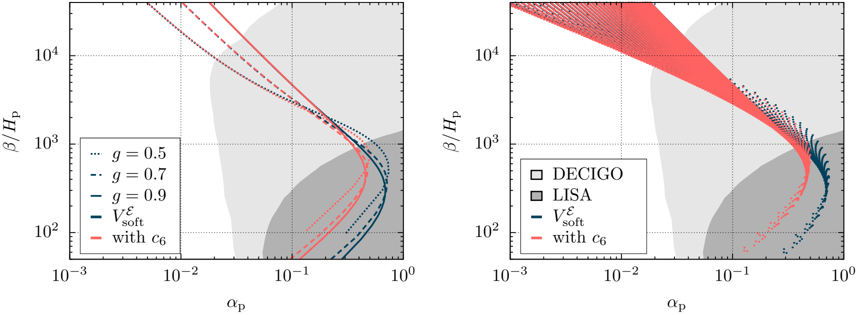

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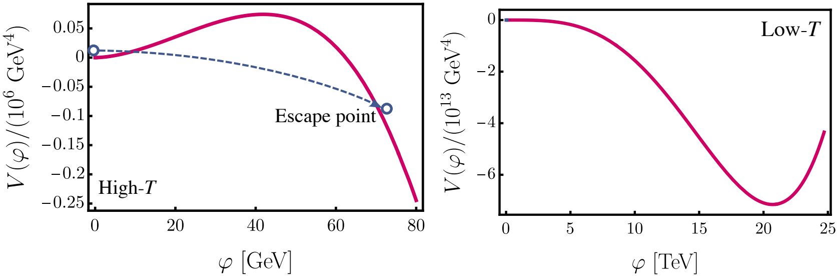
exhibit strong supercooling and phase transition. Barrier until low-T

$$m_\varphi^2(T) = \left[\mu_0^2\right] + m_T^2 \,.$$

Trapped field φ in false vacuum φ_{F} until $T_{\mathrm{p}} \ll T_{\mathrm{c}}$. Split computation:[20]

High-T: Small field regime $M(\varphi) < T$ use 3D EFT
Low-T: Large field regime $M(\varphi) > T$ use vacuum potential



[20] M. Kierkla, P. Schicho, B. Swiezewska, T. V. I. Tenkanen, and J. van de Vis, *Finite-temperature bubble nucleation with shifting scale hierarchies*, JHEP **07** (2025) 153 [2503.13597], M. Kierkla, B. Swiezewska, T. V. I. Tenkanen, and J. van de Vis, *Gravitational waves from supercooled phase transitions: dimensional transmutation meets dimensional reduction*, JHEP **02** (2024) 234 [2312.12413], cf. talk by **Marek Lewicki** on **Mon 9:30**

Phase transitions beyond high-T

The formalism ($\texttt{3DEFT}^{+}$) to extend V_{eff}

requires the full effective potential[21]

$$V_{\text{eff}}^{\text{res}} = V_{\text{eff}}^{\text{res}} - V_{\text{eff}}^{\text{res,soft}} + V_{\text{eff}}^{\text{res,soft}} = \left(V_{\text{eff}}^{\text{naive}} - V_{\text{eff}}^{\text{naive,soft}}\right) + V_{\text{eff}}^{\text{res,soft}} \,.$$

Use (hot) Loop-Tree Duality (`hotLTD`)[22] to evaluate IR safe

$$\Delta V_{\text{eff}}^{\text{hard}} = V_{\text{eff}}^{\text{naive}} - V_{\text{eff}}^{\text{naive,soft}} \,.$$

[21] P. Navarrete, R. Paatelainen, K. Seppänen, and T. V. I. Tenkanen, *Cosmological phase transitions without high-temperature expansions*, [2507.07014]

[22] (i) Subtract UV divergences, (ii) Matsubara sums → 3D integrand, (iii) Monte Carlo spatial integration.

The formalism ($\mathtt{3DEFT}^+$) to extend V_{eff}

requires the full effective potential[21]

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Use (hot) Loop-Tree Duality (`hotLTD`)[22] to evaluate IR safe

$$\Delta V_{\text{eff}}^{\text{hard}} = \left(\frac{1}{2}\sumint_P \ln\left(P^2 + M_\phi^2\right) - V_{\text{eff}}^{\text{naive,soft}}\right) .$$

[21] P. Navarrete, R. Paatelainen, K. Seppänen, and T. V. I. Tenkanen, *Cosmological phase transitions without high-temperature expansions*, [2507.07014]

[22] (i) Subtract UV divergences, (ii) Matsubara sums $\rightarrow$ 3D integrand, (iii) Monte Carlo spatial integration.

The formalism ($\texttt{3DEFT}^+$) to extend V_{eff}

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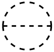
Use (hot) Loop-Tree Duality ($\texttt{hotLTD}$)[22] to evaluate IR safe

$$\Delta V_{\text{eff}}^{\text{hard}} = \bigg(\frac{1}{2}\sum\!\!\!\!\!\!\int_P \ln\big(P^2 + M_\phi^2\big) - V_{\text{eff}}^{\text{naive,soft}}\bigg) \,.$$

Evaluate soft part $V_{\text{eff}}^{\text{naive,soft}}$ in 3D EFT with coupling and IR expansion of masses, using unresummed propagators with $X_3 = X + \delta X_3$:

$V_{\text{eff}}^{(1)\text{naive,soft}} \supset$  $= \dfrac{T}{2} \displaystyle\int_{\mathbf{p}} \ln\left(p^2 + M_\phi^2\right) \,,$

$V_{\text{eff}}^{(2)\text{naive,soft}} \supset$  $= \dfrac{g^2}{12} S_{3d}(M_\phi) + \delta g_3^2[\dots] + \delta M_3^2[\dots] + \dots \,.$

[21] P. Navarrete, R. Paatelainen, K. Seppänen, and T. V. I. Tenkanen, *Cosmological phase transitions without high-temperature expansions*, [2507.07014]

[22] (i) Subtract UV divergences, (ii) Matsubara sums → 3D integrand, (iii) Monte Carlo spatial integration.

The formalism ($\texttt{3DEFT}^+$) to extend V_{eff}

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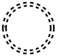
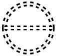
Use (hot) Loop-Tree Duality ($\texttt{hotLTD}$)[22] to evaluate IR safe

$$\Delta V_{\text{eff}}^{\text{hard}} = \left(\frac{1}{2} \sum\!\!\!\!\!\!\!\int_P \ln\big(P^2 + M_\phi^2\big) - V_{\text{eff}}^{\text{naive,soft}} \right) .$$

Evaluate soft part $V_{\text{eff}}^{\text{res,soft}}$ in 3D EFT without coupling nor IR expansion of masses, using resummed propagators with $X_3 = X + \delta X_3$:

$$V_{\text{eff}}^{(1)\text{res,soft}} \supset$$  $$= \frac{T}{2} \int_{\mathbf{p}} \ln\left(p^2 + M_3^2\right) \,,$$

$$V_{\text{eff}}^{(2)\text{res,soft}} \supset$$  $$= \frac{g_3^2}{12T} S_{3d}(M_3) \,.$$

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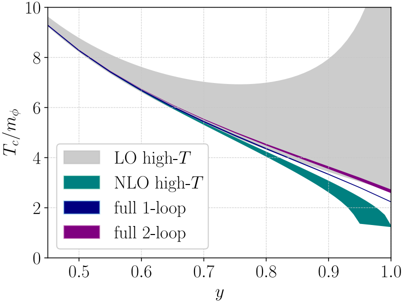
[21] P. Navarrete, R. Paatelainen, K. Seppänen, and T. V. I. Tenkanen, *Cosmological phase transitions without high-temperature expansions*, [2507.07014]

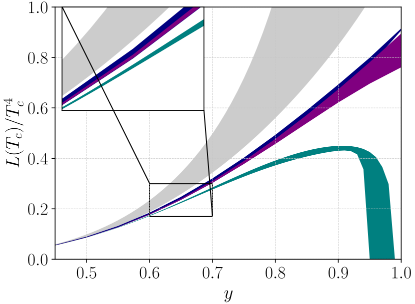
[22] (i) Subtract UV divergences, (ii) Matsubara sums → 3D integrand, (iii) Monte Carlo spatial integration.

Proof of principle example: Higgs-Yukawa model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \sigma\phi + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{3!}g\phi^3 + \frac{1}{4!}\lambda\phi^4 + \bar{\psi}(\not{\partial} + m_\psi)\psi + y\phi\bar{\psi}\psi\,.$$

For small T/m_ϕ high-T breaks down but the full result remains robust.





Alternatives: full thermal integrals,[23] tadpole resummation,[24] ...

[23] M. Laine, M. Meyer, and G. Nardini, *Thermal phase transition with full 2-loop effective potential*, Nucl. Phys. B **920** (2017) 565 [1702.07479]

[24] D. Curtin, J. Roy, and G. White, *Gravitational waves and tadpole resummation: Efficient and easy convergence of finite temperature QFT*, Phys. Rev. D **109** (2024) 116001 [2211.08218]

Conclusions

Precision thermodynamics of BSM theories:

- reliably describe cosmological FOPT and GW production,
- practical approach: **Effective Theories** + **universality**.

Reaching perturbative limits and overcoming limitations:

- higher dimensional operators at $\mathcal{O}(g^6)$ in the $3d$ EFT,
- purely perturbative $\mathcal{O}(g^6)$ contributions from hard scale πT,
- final perturbative corrections for bubble nucleation rate,

Going beyond high-T:

- 3DEFT$^+$ New frameworks with large range of validity,
- 4d full theory lattice simulations to test reliable description.