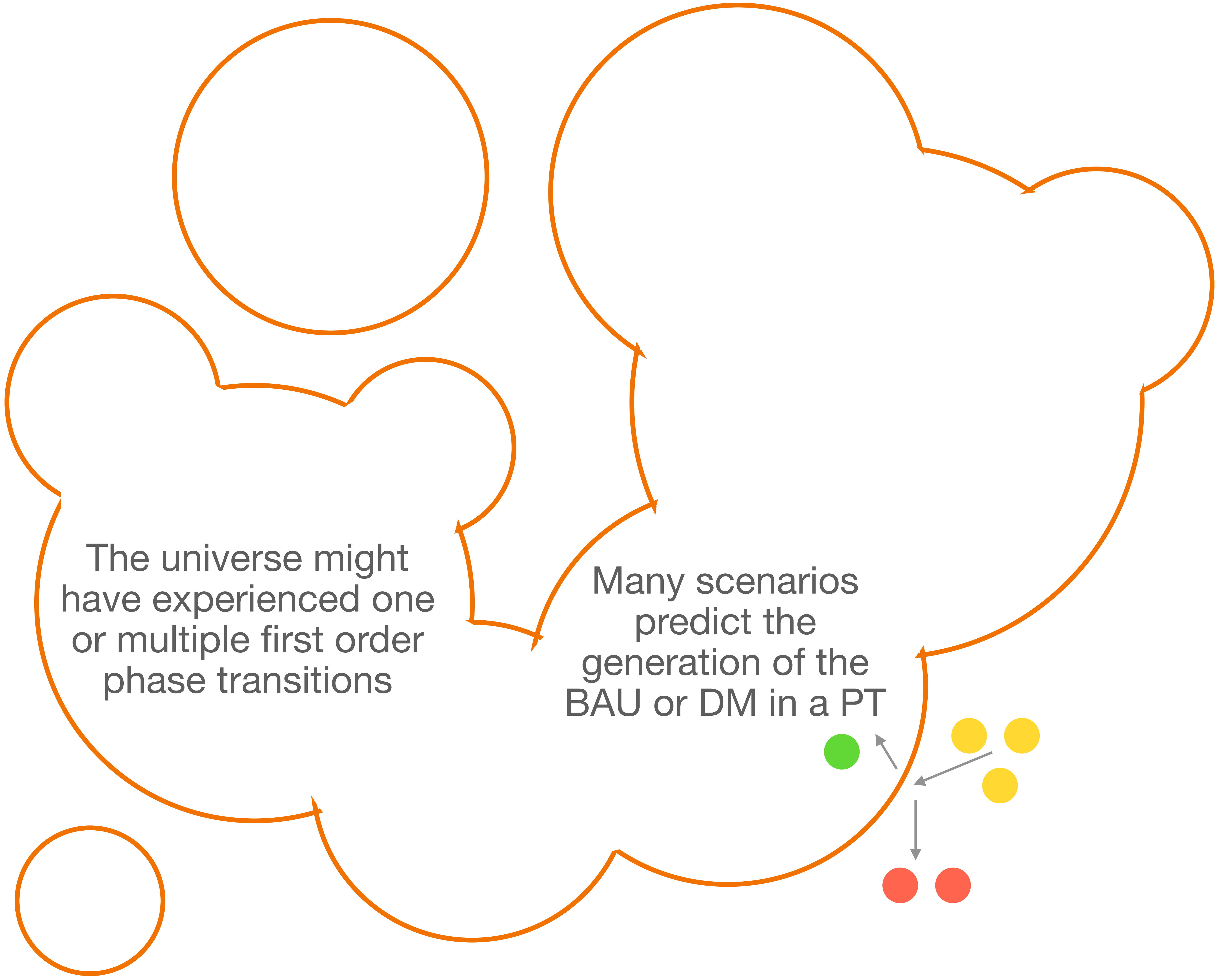
The bubble wall velocity in cosmological phase transitions

Numerical Simulations of Early Universe Sources of Gravitational Waves

Based on
Ekstedt, Gould, Hirvonen, Laurent, Niemi, Schicho, JvdV https://arxiv.org/abs/2411.04970
Ekstedt, Gould, Hirvonen, Laurent, Niemi, Schicho, JvdV In progress

Jorinde van de Vis, August 5, 2025


The universe might have experienced one or multiple first order phase transitions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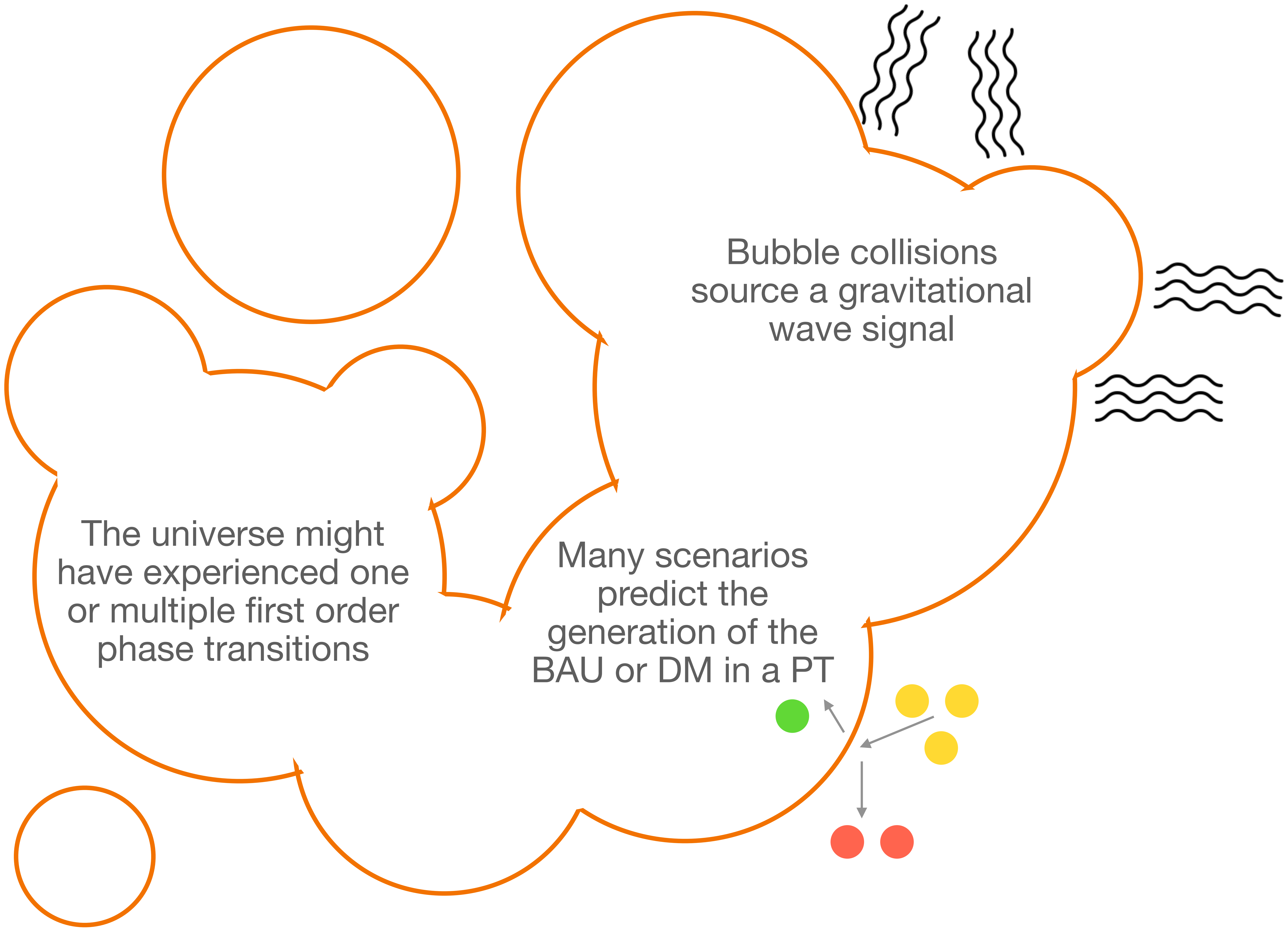
Many scenarios predict the generation of the BAU or DM in a PT


Bubble collisions source a gravitational wave signal
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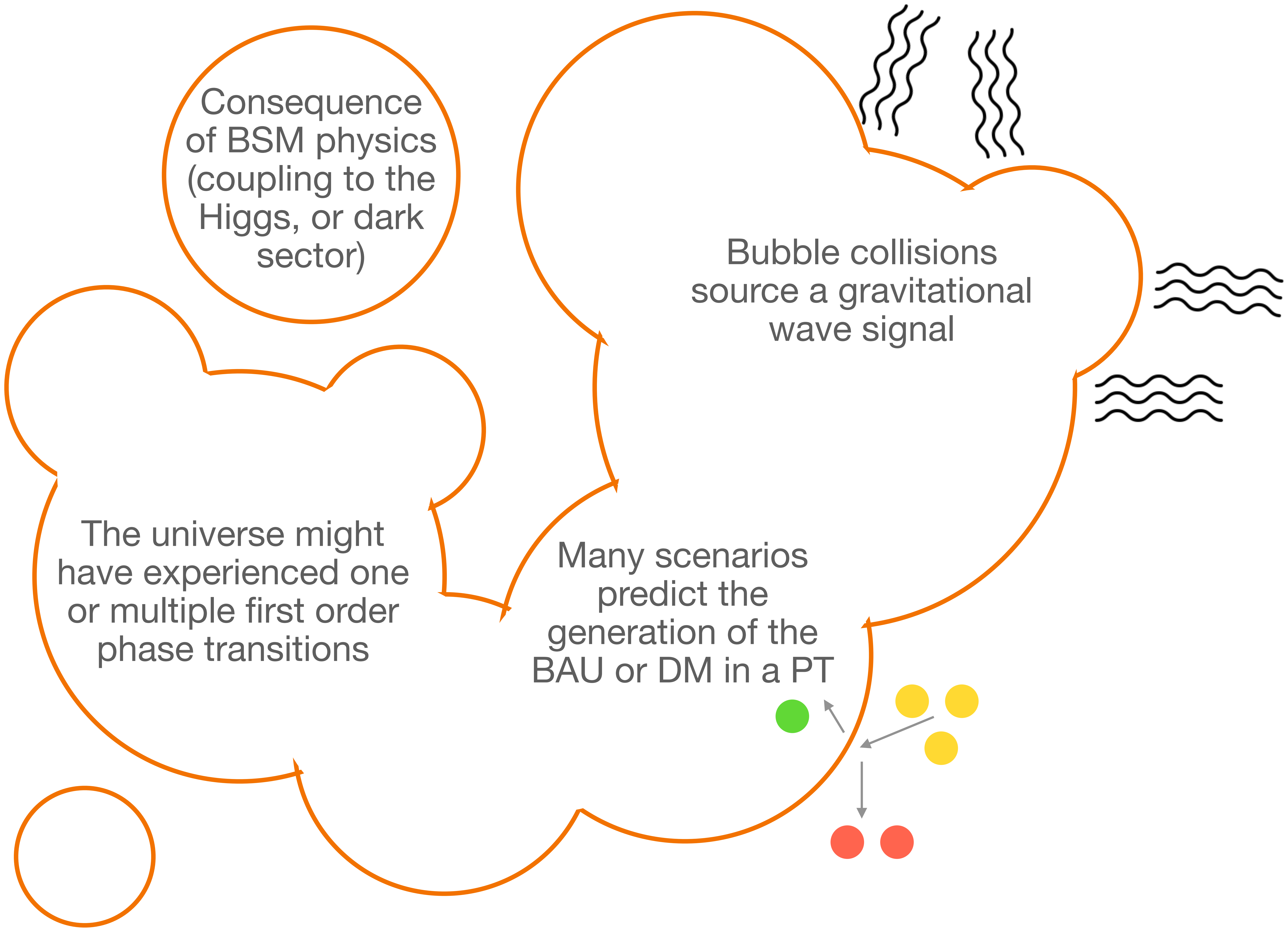
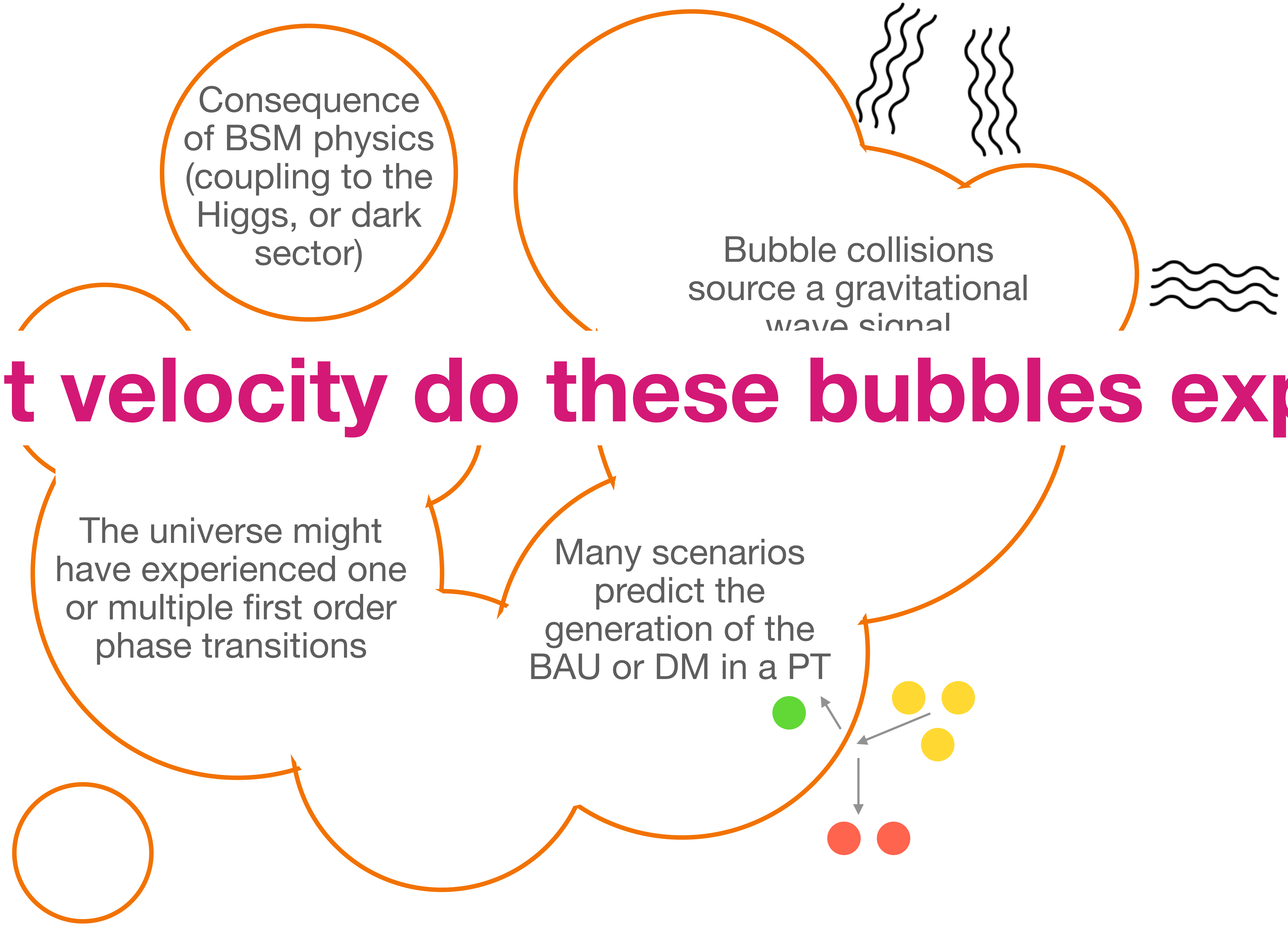
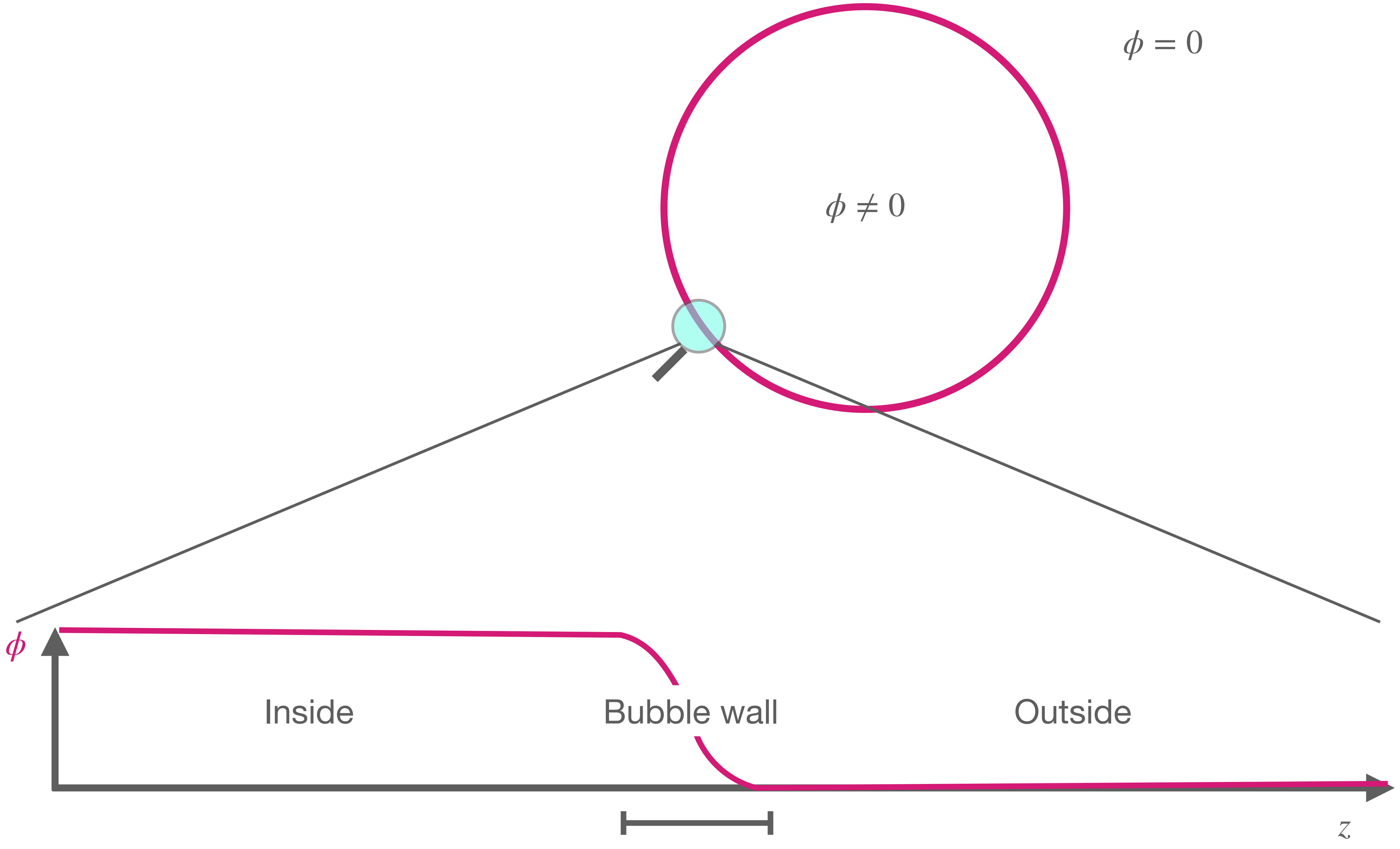

Consequence of BSM physics (coupling to the Higgs, or dark sector)
The universe might have experienced one or multiple first order phase transitions
Bubble collisions source a gravitational wave signal
Many scenarios predict the generation of the BAU or DM in a PT

With what velocity do these bubbles expand?



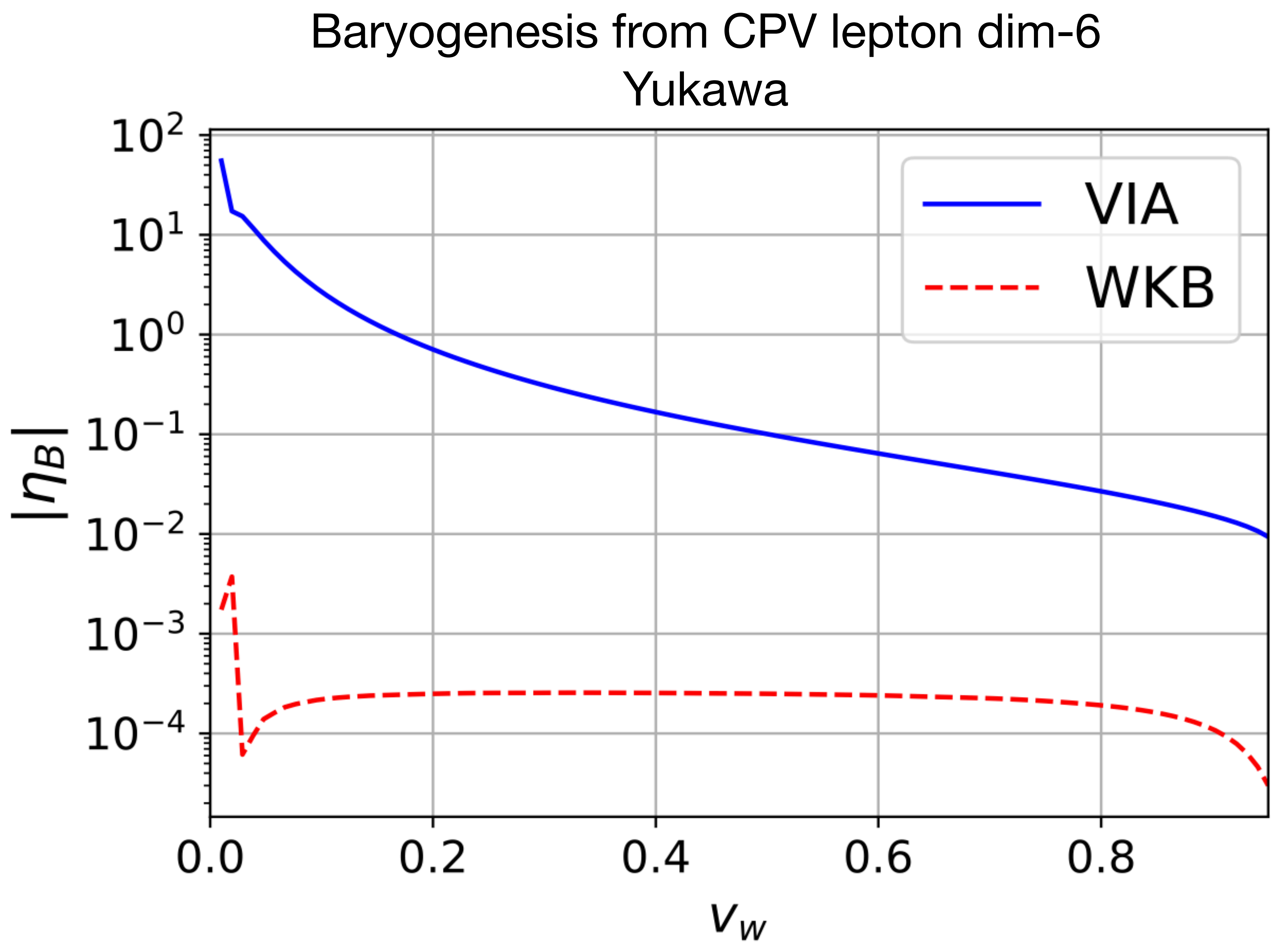
Bubble wall velocity v_w

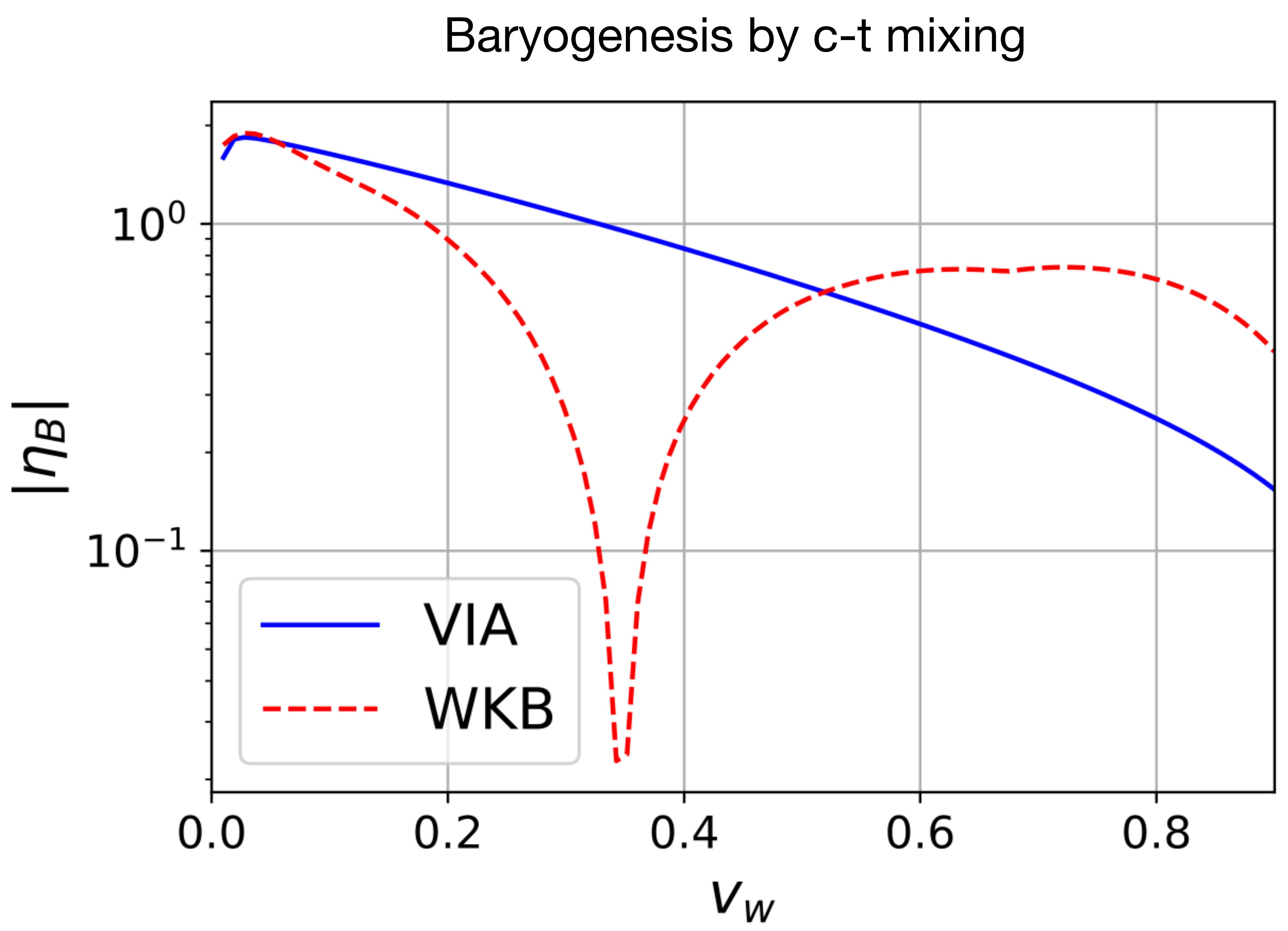
- The *bubble wall* is the region where the field(s) interpolate between the high- and low-temperature vacuum
- Vacuum energy release provides outward force; plasma causes friction
- When the forces balance, bubbles reach a terminal expansion velocity: v_w
- … or they keep accelerating until they collide (but not in this talk)



FOPT phenomenology depends on v_w

Baryon asymmetry

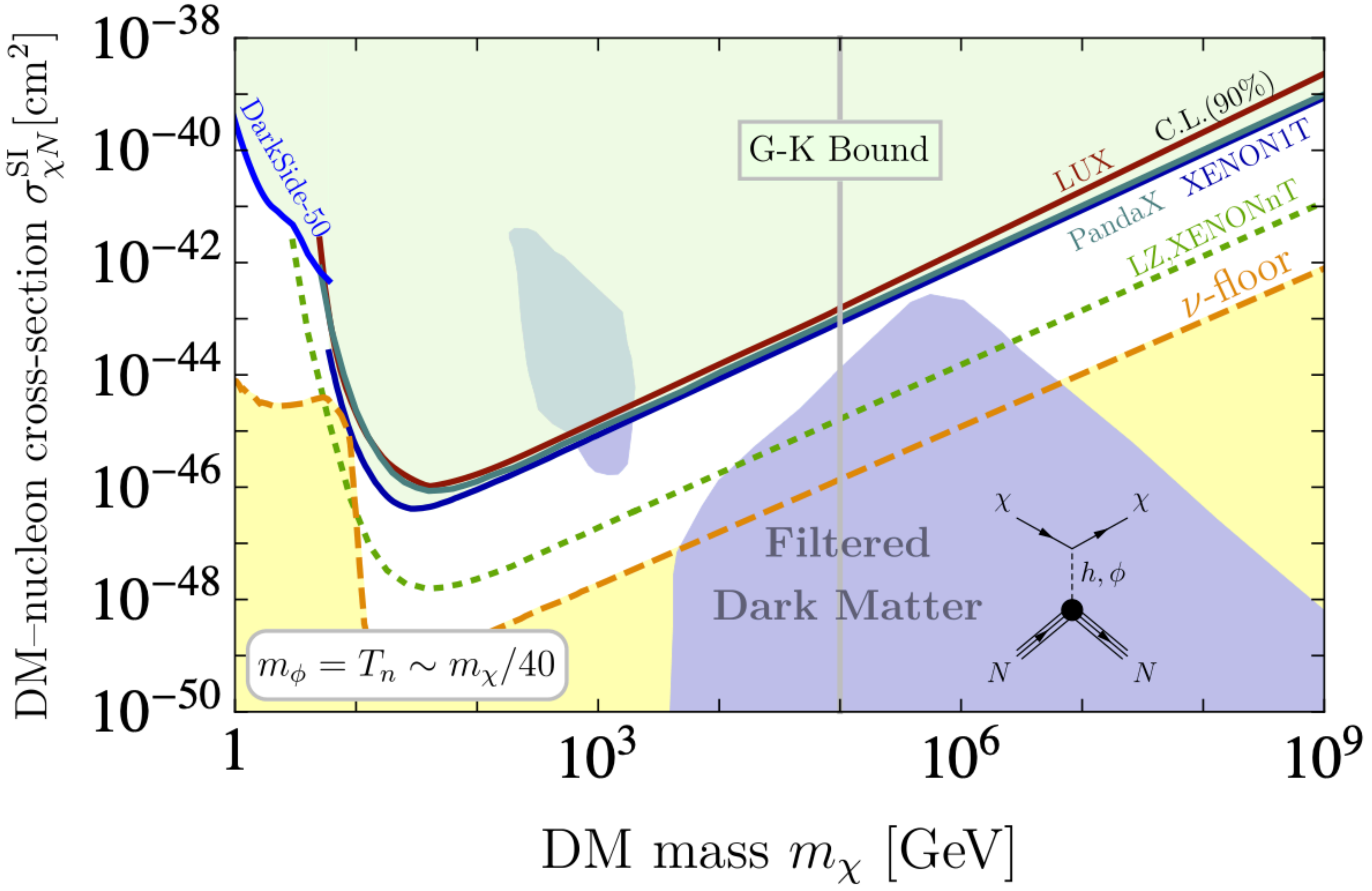


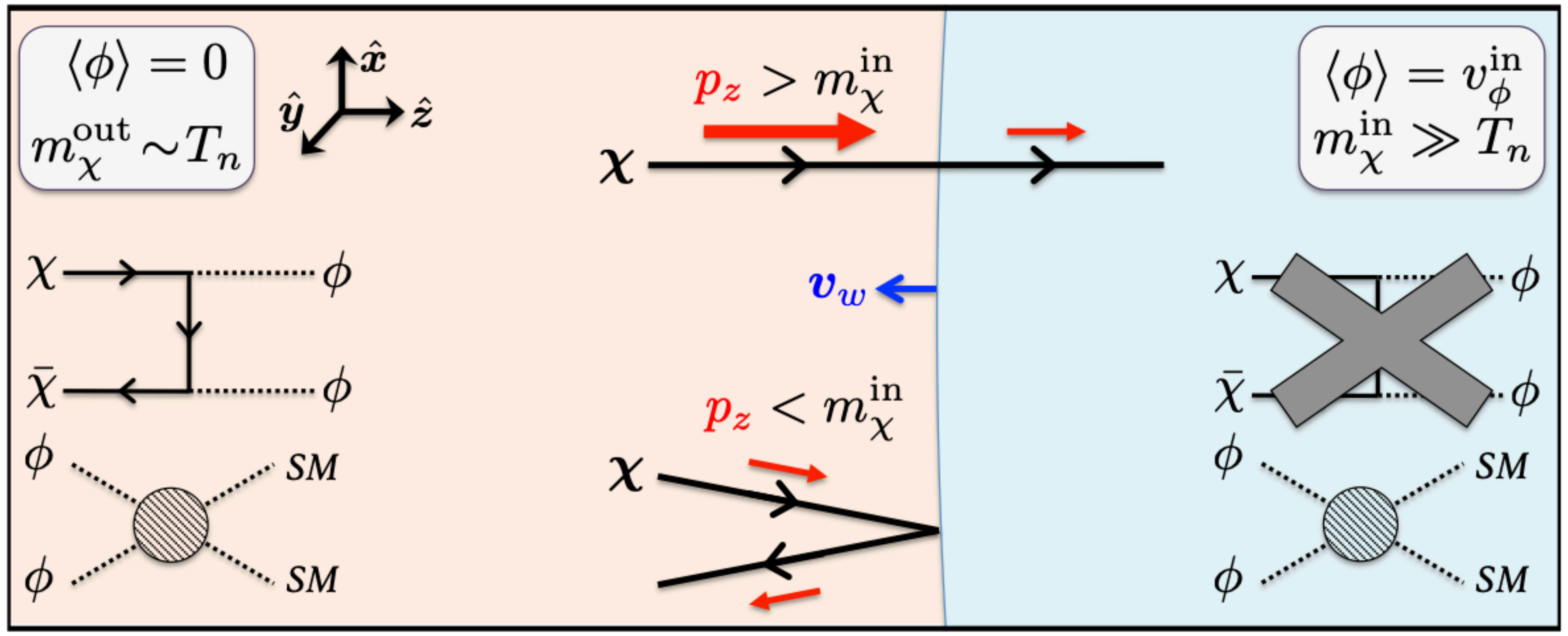


Figures from: Cline, Laurent 2021

Dark matter production

Slow bubbles: Filtered dark matter Baker, Kopp, Long 2019, Chway, Jung, Shin 2019, Marfatia, Tseng 2020

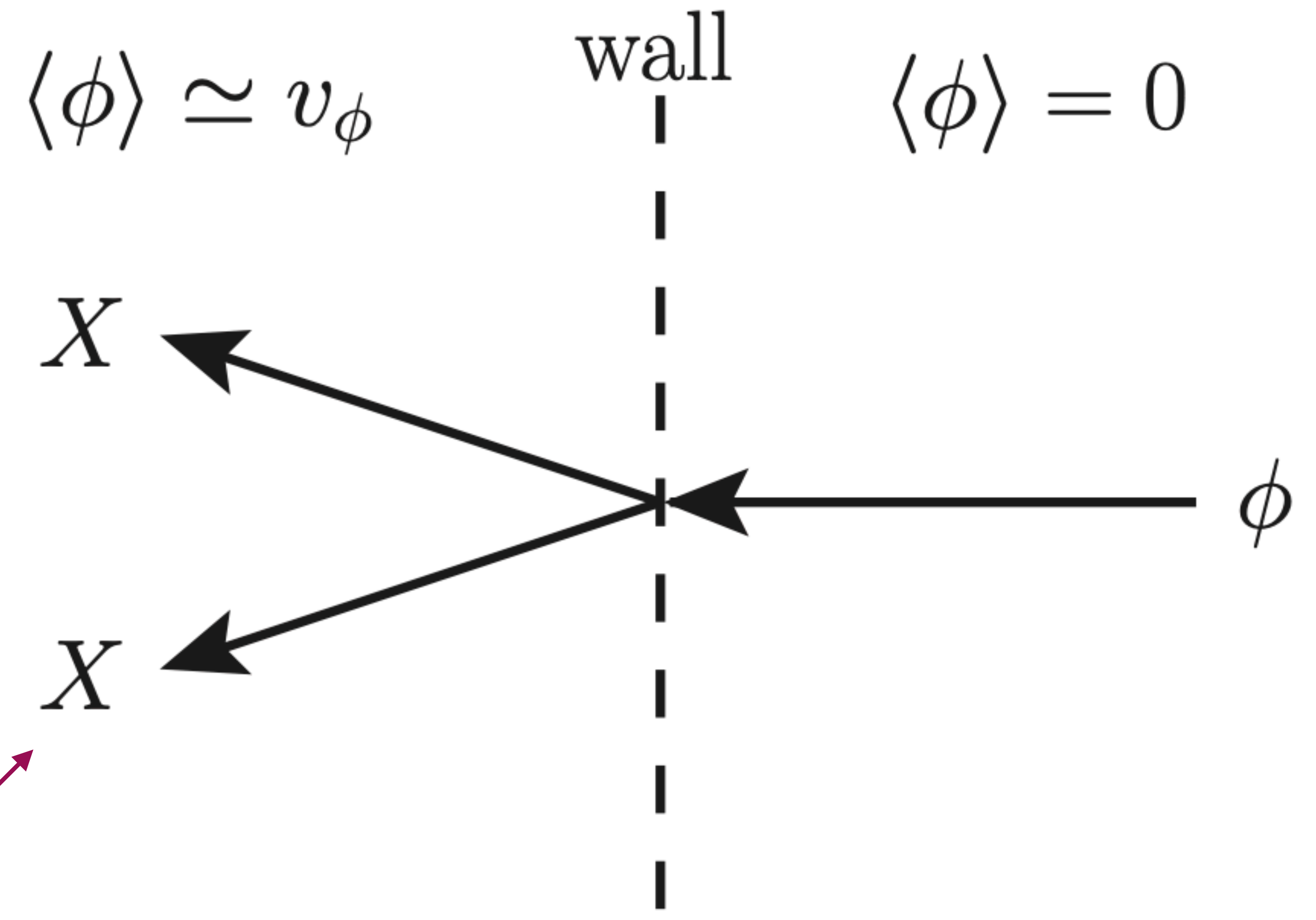




Figures from: Baker, Kopp, Long 2019

Dark matter production

Fast bubbles: Azatov, Vanvlasselaer, Yin 2021; Baldes, Gouttenoire, Sala 2022, ...



Heavy, out-of-equilibrium dark matter

Figure from: Baldes, Gouttenoire, Sala 2022

Gravitational wave spectrum

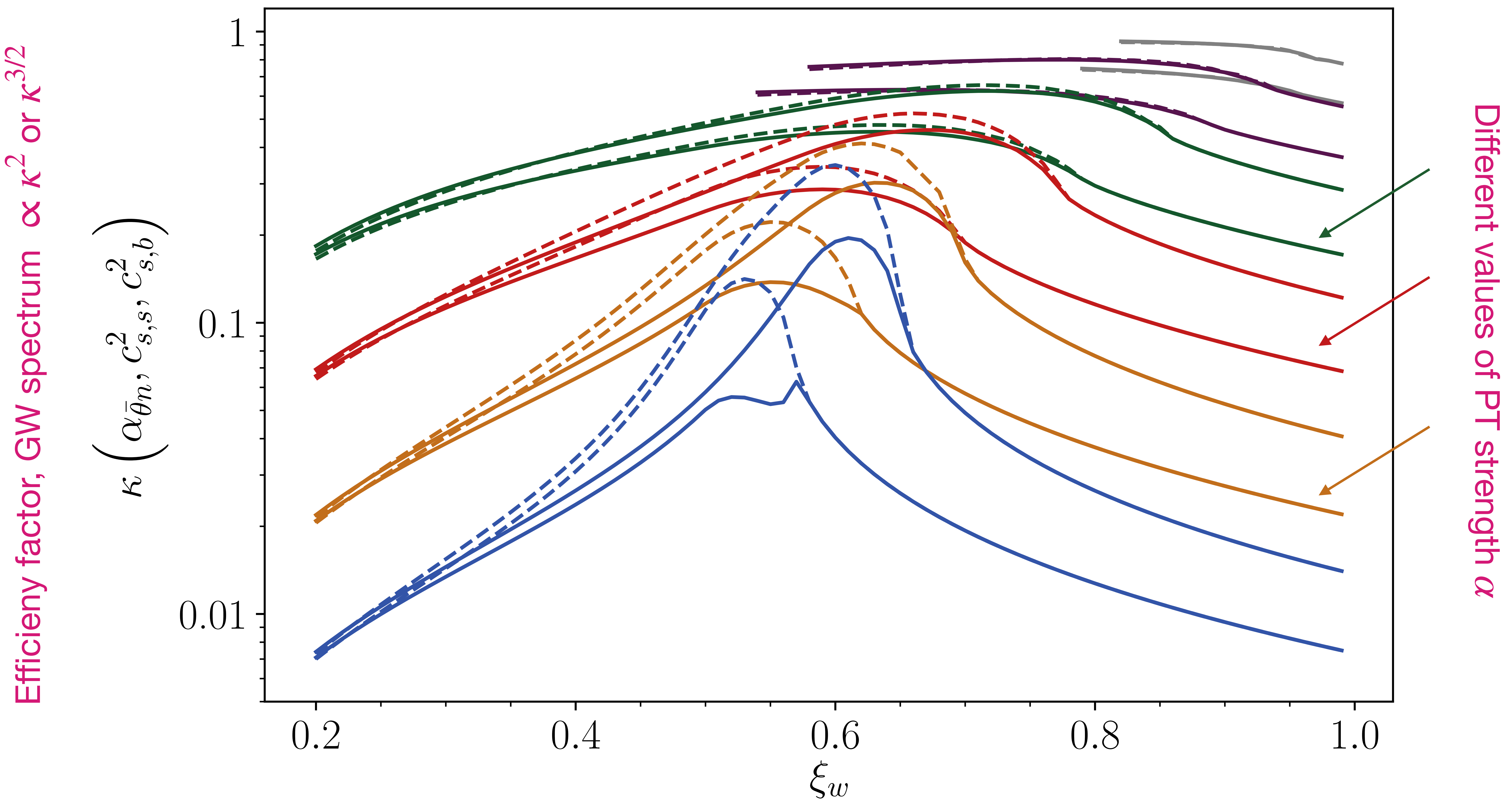
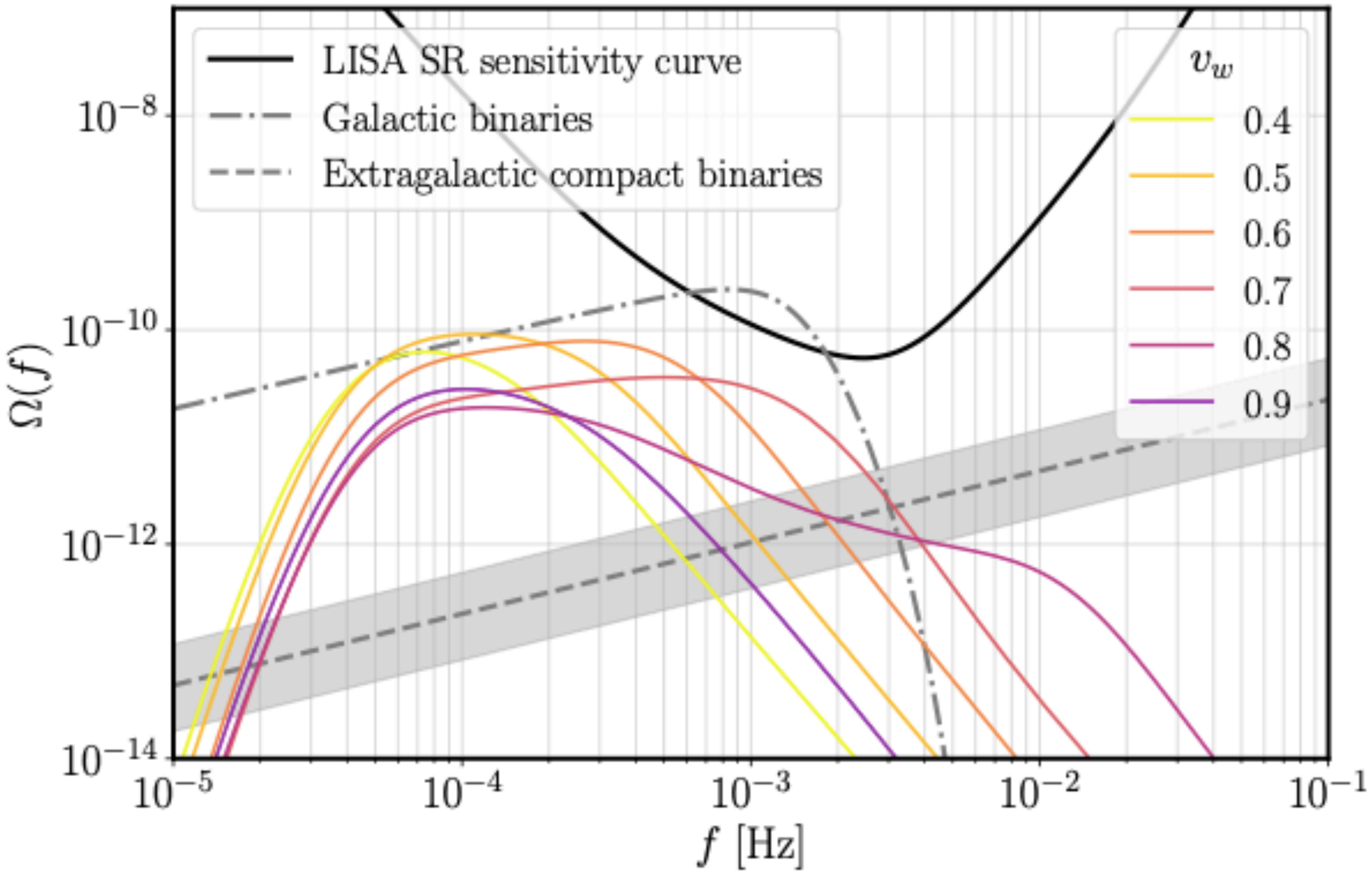


Figure from: Giese, Konstandin, Schmitz, JvdV 2020

Gravitational wave spectrum

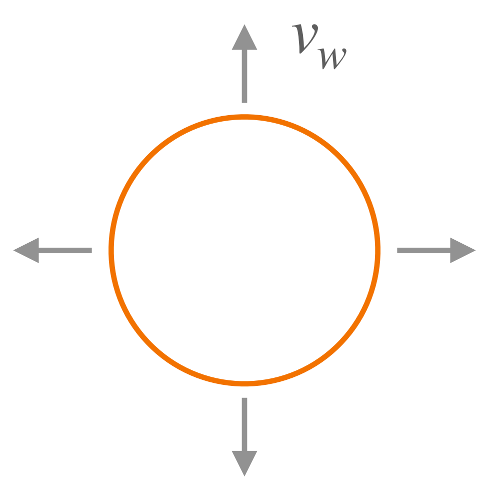


(a) Fixed: $\alpha = 0.2, \quad r_* = 0.1, \quad T_\mathrm{n} = 100$ GeV.

Figure from: [Gowling, Hindmarsh 2021]

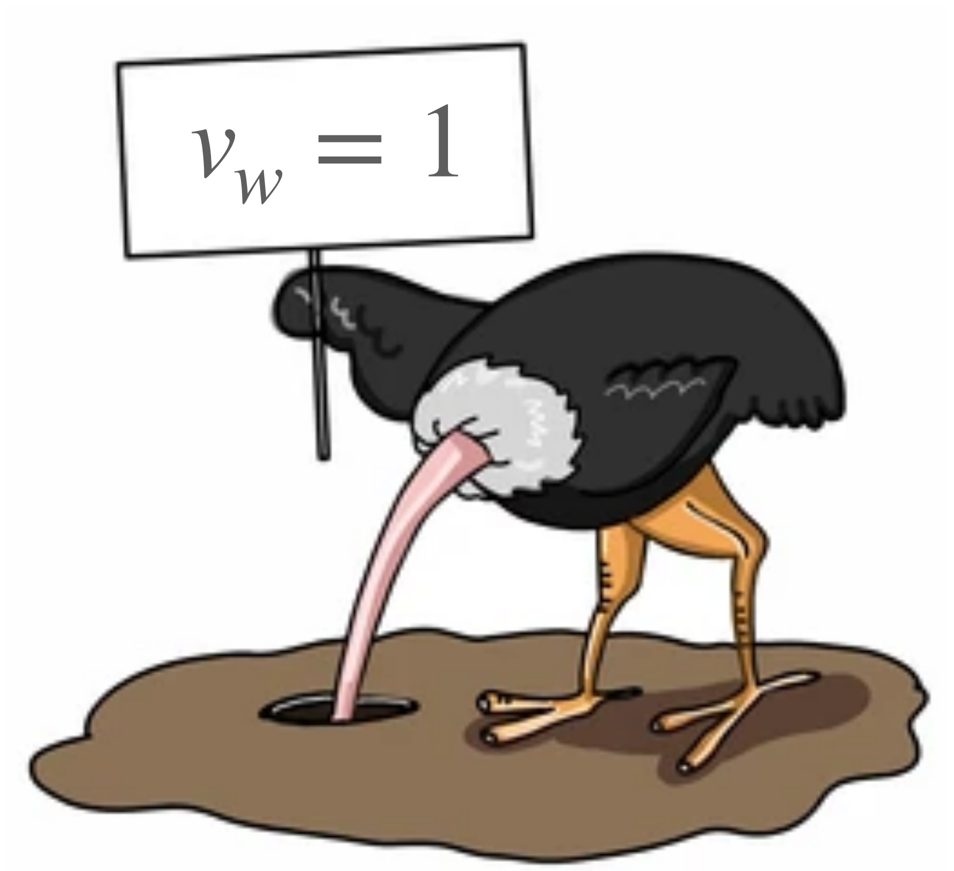
Challenge

- Usually v_w can be determined for one bubble in isolation



Challenge

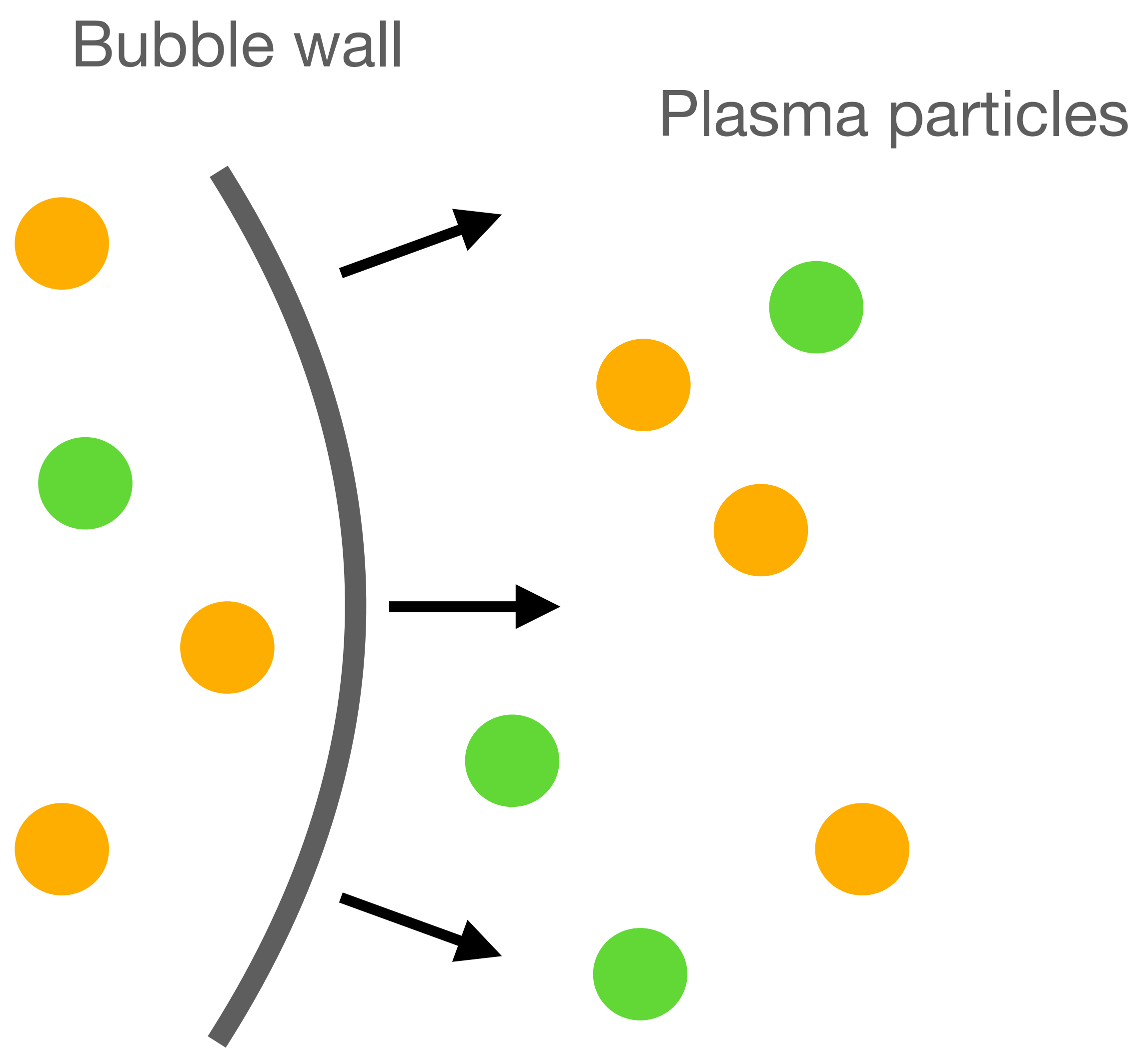
- Usually v_w can be determined for one bubble in isolation
- However…
  - Computation of the wall velocity is numerically challenging
  - Many phenomenological predictions do not include a model-dependent computation of v_w, resulting in significant uncertainties
  - What is the theoretical error in v_w?



Computation of the wall velocity

Weakly coupled bubble wall-plasma system

- Energy release provides outward pressure
- Plasma particles provide friction by reflections and by gaining mass by entering the bubble
- Hydrodynamic backreaction effects
- Wall velocity follows from $|P_{\text{outward}}| = |P_{\text{inward}}|$



Some assumptions

- We can compute the wall velocity when the bubble still expands in isolation $t_{\text{formation}} \ll t_{\text{const}\, v_w} \ll t_{\text{collision}}$
- The bubbles are much smaller than horizon size, $(R_{\text{bubble}} H \ll 1)$, so we assume Minkowski spacetime
- The theory is weakly-coupled
- There is a terminal wall velocity, i.e. the wall does not run away (not always true for very strong phase transitions)

Energy-momentum tensor

Zero-temperature potential

- Scalar field: $T_\phi^{\mu\nu} = (\partial^\mu \phi)(\partial^\nu \phi) - g^{\mu\nu}\left(\frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$
- Fluid: $T_f^{\mu\nu} = (e_f + p_f)u^\mu u^\nu - p_f g^{\mu\nu}$, or $T_f^{\mu\nu} = \sum_i \frac{d^3k}{(2\pi)^3 E_i} k^\mu k^\nu f_i(k,x)$

Fluid velocity $u^\mu u_\mu = 1$

Scalar field equation of motion

See e.g. Prokopec Moore 1995

Effective potential including temperature-corrections

Contribution from out-of-equilibrium particles

$$\partial^2\phi + \frac{\partial V_{\text{eff}}(\vec{\phi}, T)}{\partial \phi} + \sum_a \frac{\partial m_a^2}{\partial \phi} \int_{\vec{p}} \frac{1}{2E} \delta f^a(p^\mu, \xi) = 0$$

Scalar field(s) undergoing the phase transition

Distance from the wall

$$\xi = -\bar{u}_w^\mu x_\mu$$

$$\bar{u}_w^\mu = \gamma_w(v_w, 0, 0, 1)$$

Scalar field equation of motion

Friction force
Dominant contribution from heavy particles (e.g. top)

$$\partial^2\phi + \frac{\partial V_{\rm eff}(\overrightarrow{\phi}, T)}{\partial\phi} + \sum_a \frac{\partial m_a^2}{\partial\phi}\int_{\vec{p}} \frac{1}{2E}\delta f^a(p^\mu, \xi) = 0$$

Driving force and hydrodynamic backreaction

Also for vanishing δf^a, the wall can stop to accelerate

Ignatius, Kajantie, Kurki-Suonio, Laine 1993; Konstandin, No 2011; Barroso Mancha, Prokopec, Swiezewska 2020; Balaji, Spannowski, Tamarit 2020; Ai, Garbrecht, Tamarit 2021; Ai, Laurent, JvdV 2023; Ai, Laurent, JvdV 2024

- Hydrodynamic effects also provide a backreaction force in local thermal equilibrium (LTE) ($\delta f^a = 0$)
- This approximation provides an *upper bound* on the wall velocity*
- Entropy conservation provides a third hydrodynamic matching condition: the wall velocity can be determined from hydrodynamics only

*Fine-print

- Numerical simulations for the xSM show that the LTE solution might not be reached in time-dependent simulations Krajewski, Lewicki, Zych 2024
- Eriksson, Laine 2025 demonstrate that the bound is *unsaturated* due to entropy-generating contributions of the scalar field
- As we will see later, the LTE result can be much larger than the full result

Scalar field equation of motion

$$\partial^2\phi + \frac{\partial V_{\text{eff}}(\overrightarrow{\phi}, T)}{\partial\phi} + \sum_a \frac{\partial m_a^2}{\partial\phi}\int_{\vec{p}} \frac{1}{2E}\delta f^a(p^\mu, \xi) = 0$$

The scalar field equation of motion depends on the temperature profile

Temperature and fluid profile

Energy-momentum tensor of the (perfect) fluid and scalar field

Fluid velocity

$$T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial_\sigma \phi \partial^\sigma \phi \right)$$

Enthalpy

$$w = T \frac{dp}{dT}$$

Pressure

$$p = - V_{\text{eff}}(\vec{\phi}, T)$$

(Including field-independent terms)

$$p = p_f - V(\phi)$$

Temperature and fluid profile

Energy-momentum conservation for a planar wall, moving in the z-direction

$$T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial_\sigma \phi \partial^\sigma \phi \right)$$

$$T^{30} = w \gamma_{\rm pl}^2 v_{\rm pl} + T^{30}_{\rm out} = c_1$$

$$T^{33} = \frac{1}{2}(\partial_z \phi)^2 - V_{\rm eff}(\phi, T) + w \gamma_{\rm pl}^2 v_{\rm pl}^2 + T^{33}_{\rm out} = c_2$$

Contribution from out-of-equilibrium particles

Temperature and fluid profile

Energy-momentum conservation for a planar wall, moving in the z-direction

$$T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial_\sigma \phi \partial^\sigma \phi \right)$$

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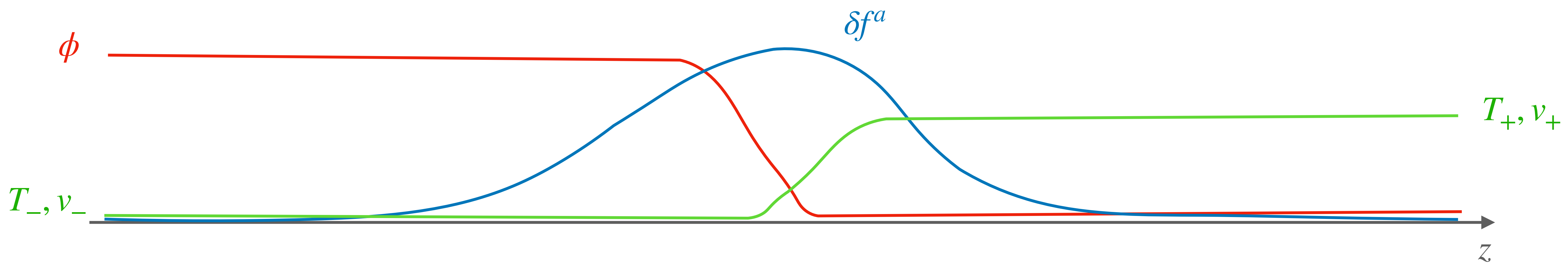
Constants obtained from hydrodynamic solution

Boundary conditions from hydrodynamics

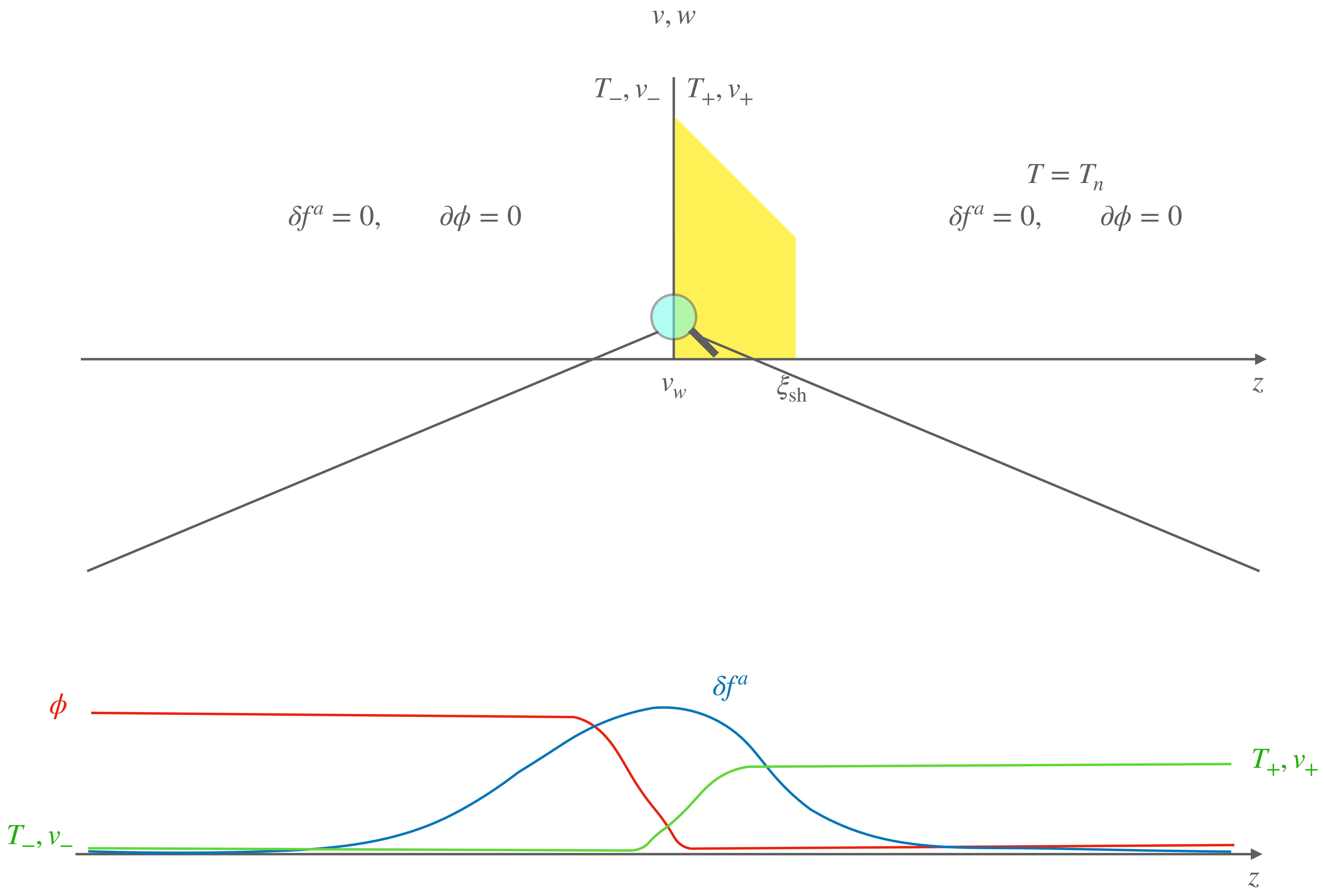
Bubble wall scale: scalar field of motion (and Boltzmann equations)



Units and scale are arbitrary

Boundary conditions from hydrodynamics

On hydrodynamic scales, the wall corresponds to a discontinuity in T, v



Units and scale are arbitrary

The out-of-equilibrium contribution

- Until now we assumed that we already knew the out-of-equilibrium contribution
- In practice one could solve for ϕ, v, T first, and compute δf^a assuming that background solution
- Then plug the solution of δf^a into the equations for ϕ, v, T until it converges

The out-of-equilibrium contribution

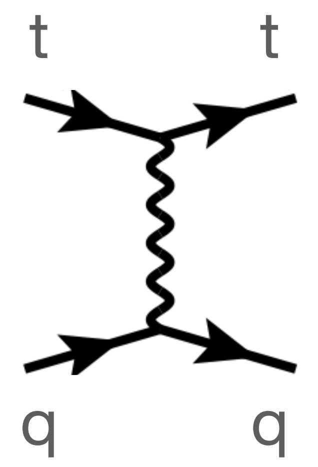
Boltzmann equation

$$\left(p^\mu \partial_\mu + \frac{1}{2} \vec{\nabla} m_a^2 \cdot \nabla_{\vec{p}}\right) f^a(\vec{p}, \xi) = -\,\mathscr{C}_a[\vec{f}]$$

Force term

Collision term: Nine-dimensional integral over distributions of 4 contributing species

E.g.



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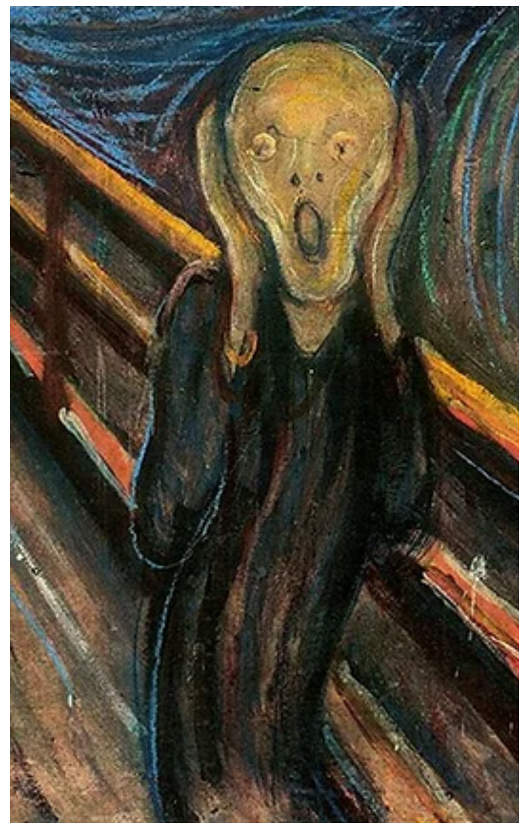

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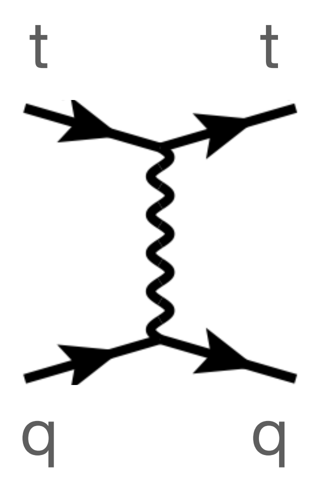

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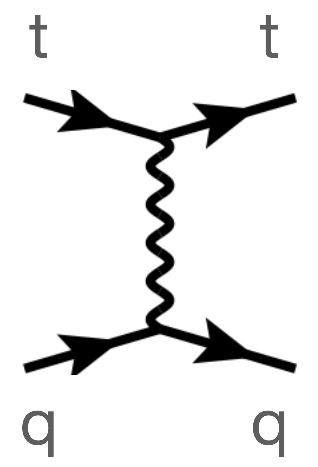

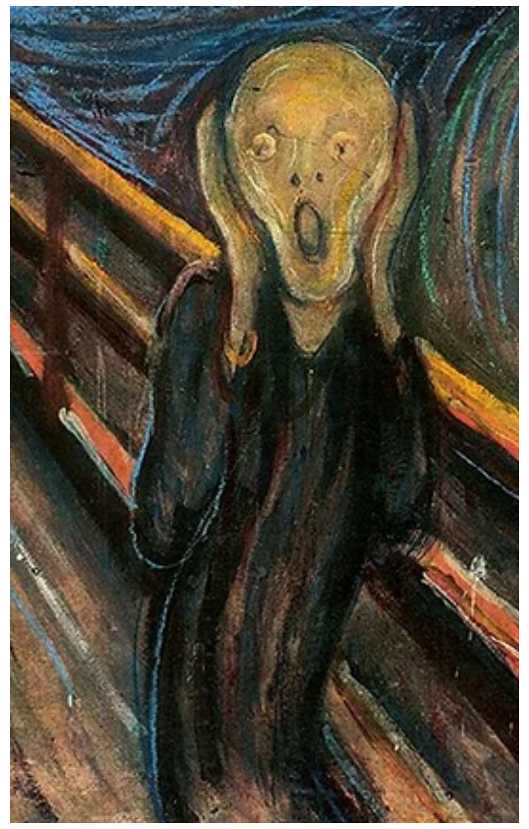

The Boltzmann equations can be simplified by linearizing in the perturbation from equilibrium and using that $\mathscr{C}[f_{\rm eq}] = 0$

Putting the Boltzmann equation in a solvable form

Taking moments Prokopec Moore 1995; Dorsch, Huber, Konstandin 2021; Dorsch, Konstandin, Perboni, Pinto 2024

$$f = f_{\rm eq}(E+\delta), \qquad \delta = -\left[\mu + \mu_{\rm bg} + \frac{E}{T}(\delta T + \delta T_{\rm bg}) + p_z(v + v_{\rm bg})\right]$$

$$-f'_{\rm eq}\left(\frac{p_z}{E}[\partial_z(\mu+\mu_{\rm bg}) + \frac{E}{T}\partial_z(\delta T + \delta T_{\rm bg}) + p_z\partial_z(v+v_{\rm bg})] + \partial_t(\mu+\mu_{\rm bg}) + \frac{E}{T}\partial_t(\delta T + \delta T_{\rm bg}) + p_z\partial_t(v+v_{\rm bg})\right) + C(\mu, \delta T, v) = -f'_{\rm eq}\frac{\partial_t m^2}{2E}$$

Putting the Boltzmann equation in a solvable form

Taking moments Prokopec Moore 1995; Dorsch, Huber, Konstandin 2021; Dorsch, Konstandin, Perboni, Pinto 2024

$$-f'_{\rm eq}\left(\frac{p_z}{E}[\partial_z(\mu+\mu_{\rm bg})+\frac{E}{T}\partial_z(\delta T+\delta T_{\rm bg})+p_z\partial_z(v+v_{\rm bg})]+\partial_t(\mu+\mu_{\rm bg})+\frac{E}{T}\partial_t(\delta T+\delta T_{\rm bg})+p_z\partial_t(v+v_{\rm bg})\right)+C(\mu,\delta T,v)=-f'_{\rm eq}\frac{\partial_t m^2}{2E}$$

Taking moments $\int d^3p/(2\pi)^3, \quad \int E d^3p/(2\pi)^3, \quad \int p_z d^3p/(2\pi)^3$

$$\begin{aligned}
c_2\partial_t(\mu+\mu_{bg})+c_3\partial_t(\delta T+\delta T_{bg})+\frac{c_3T}{3}\partial_z(v+v_{bg})+\int\frac{d^3p}{(2\pi)^3T^2}C[f] &= \frac{c_1}{2T}\partial_t m^2\\
c_3\partial_t(\mu+\mu_{bg})+c_4\partial_t(\delta T+\delta T_{bg})+\frac{c_4T}{3}\partial_z(v+v_{bg})+\int\frac{Ed^3p}{(2\pi)^3T^3}C[f] &= \frac{c_2}{2T}\partial_t m^2\\
\frac{c_3}{3}\partial_z(\mu+\mu_{bg})+\frac{c_4}{3}\partial_t(\delta T+\delta T_{bg})+\frac{c_4T}{3}\partial_t(v+v_{bg})+\int\frac{p_zd^3p}{(2\pi)^3T^3}C[f] &= 0
\end{aligned}$$

$$c_iT^{i+1}\equiv\int E^{i-2}(-f'_{\rm eq})\frac{d^3p}{(2\pi)^3}$$

Putting the Boltzmann equation in a solvable form

Taking moments Prokopec Moore 1995; Dorsch, Huber, Konstandin 2021; Dorsch, Konstandin, Perboni, Pinto 2024

Advantages

- Numerically relatively easily manageable
- Collision terms become very simple;
  E.g. $\int \frac{d^3p}{(2\pi)^3 T^2} C[f] = \mu\Gamma_{\mu 1f} + \delta T \Gamma_{T1f}$
  for top quarks, with
  $\Gamma_{\mu 1f} = 0.00899T, \qquad \Gamma_{T1f} = 0.01752T$
- Different moments correspond to conservation of particle number, energy and momentum

Disadvantages

- Not clear if three moments is sufficient for convergence
- Mixing between different out-of-equilibrium particles is neglected

Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

$$\left(p^\mu \partial_\mu + \frac{1}{2}\vec{\nabla} m_a^2 \cdot \nabla_{\vec{p}}\right) f^a(\vec{p}, x^\mu) = -\mathscr{C}_a[\vec{f}]$$

$$f^a(\vec{p}, \xi) = f^a_{\rm eq}(\vec{p}, \xi) + \delta f^a(\vec{p}, \xi)\,, \qquad f^a_{\rm eq} = \left.\frac{1}{\exp[p_\mu u^\mu_{\rm pl}(\xi)/T(\xi)] \pm 1}\right|_{E_a = \vec{p}^2 + m_a^2}$$

$$\left(-p_\mu \bar{u}^\mu_w \partial_\xi - \frac{1}{2}\partial_\xi(m_a^2)\bar{u}^\mu_w \partial_{p^\mu}\right)\delta f^a = -\mathscr{C}^{\rm lin}_{ab}[\delta f^b] + \mathscr{S}_a, \qquad \mathscr{S}_a = \left(p_\mu \bar{u}^\mu_w \partial_\xi + \frac{1}{2}\partial_\xi(m_a^2)\bar{u}^\mu_w \partial_{p^\mu}\right) f^a_{\rm eq}$$

Mixing between different out-of-eq particles

Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

Rescaled coordinates

Restricted Chebyshev polynomials

$$\delta f^a(\chi, \rho_z, \rho_\parallel) = \sum_{i=2}^{M} \sum_{j=2}^{N} \sum_{k=1}^{N-1} \delta f^a_{ijk} \bar{T}_i(\chi) \bar{T}_j(\rho_z) \tilde{T}_k(\rho_\parallel)$$

Algebraic equation

$$\sum_{i,j,k} \left\{ \partial_\xi \chi \left[\mathscr{P}_w \partial_\chi - \frac{\gamma_w}{2} \partial_\chi(m^2)(\partial_{p_z}\rho_z)\partial_{\rho_z} \right] \bar{T}_i(\chi)\bar{T}_j(\rho_z)\tilde{T}_k(\rho_\parallel)\delta f^a_{ijk} + \bar{T}_i(\chi)\mathscr{C}^{\text{lin}}_{ab}\left[\bar{T}_j(\rho_z)\tilde{T}_k(\rho_\parallel)\right]\delta f^b_{ijk} \right\} = \mathscr{S}_a(\chi, \rho_z, \rho_\parallel)$$

Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

Rescaled coordinates

Restricted Chebyshev polynomials

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Algebraic equation

$$\sum_{i,j,k} \left\{ \partial_\xi \chi \left[\mathscr{P}_w \partial_\chi - \frac{\gamma_w}{2} \partial_\chi(m^2)(\partial_{p_z}\rho_z)\partial_{\rho_z} \right] \bar{T}_i(\chi)\bar{T}_j(\rho_z)\tilde{T}_k(\rho_\parallel)\delta f^a_{ijk} + \bar{T}_i(\chi)\mathscr{C}^{\text{lin}}_{ab}\left[\bar{T}_j(\rho_z)\tilde{T}_k(\rho_\parallel)\right]\delta f^b_{ijk} \right\} = \mathscr{S}_a(\chi, \rho_z, \rho_\parallel)$$

Introduce a grid to convert it to a matrix equation

$$\left(\mathscr{L}[\alpha, \beta, \gamma; i, j, k]\delta_{ab} + \bar{T}_i(\chi^{(\alpha)})\mathscr{C}_{ab}[\beta, \gamma; j, k] \right) \delta f^b_{ijk} = \mathscr{S}_a[\alpha, \beta, \gamma]$$

Grid indices

Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

$$\mathcal{C}^{\text{lin}}_{ab}[\bar{T}_j(\rho_z)\tilde{T}_k(\rho_\parallel)] = \frac{1}{4}\sum_{cde}\int_{\vec{p}_2,\vec{p}_3,\vec{p}_4} \frac{1}{2E_2 2E_3 2E_4}(2\pi)^4\delta^4(P_1+P_2-P_3-P_4)\times$$
$$|M_{ac\to de}(P_1,P_2;P_3,P_4)|^2 f^a f^c f^d f^e\left(\delta_{ab}F^c_a + \delta_{cb}F^a_c - \delta_{db}F^e_d - \delta_{eb}F^d_e\right)\bar{T}_j(\rho_z)\tilde{T}_k(\rho_\parallel)$$

$$F^a_b = \frac{e^{E_a/T}}{(f^b)^2}$$

Nine-dimensional integral for $n_p^2(N-1)^4$ components;

Four integrations are trivial because of the δ-function

Putting the Boltzmann equation in a solvable form

Expand δf^a in polynomials Laurent, Cline 2022; WallGo

Advantages

- Controlled convergence in number of polynomials
- Inclusion of mixing terms in collisions

Disadvantages

- Numerically more intensive than moments
- Individual Chebyshev polynomials have no clear physical interpretation

Obtaining v_w

- Now we have solved ϕ, v, T and δf_a for a given value of v_w
- The pressure on the wall is given by $P = \int dz \frac{d\phi}{dz} \mathrm{EOM}$
- $P(v_w) = 0$ gives us the wall velocity

How fast does the WallGo?

Ekstedt, Gould, Hirvonen, Laurent, Niemi, Schicho, JvdV: 2411.04970

Publicly availabe code for the computation of the wall velocity

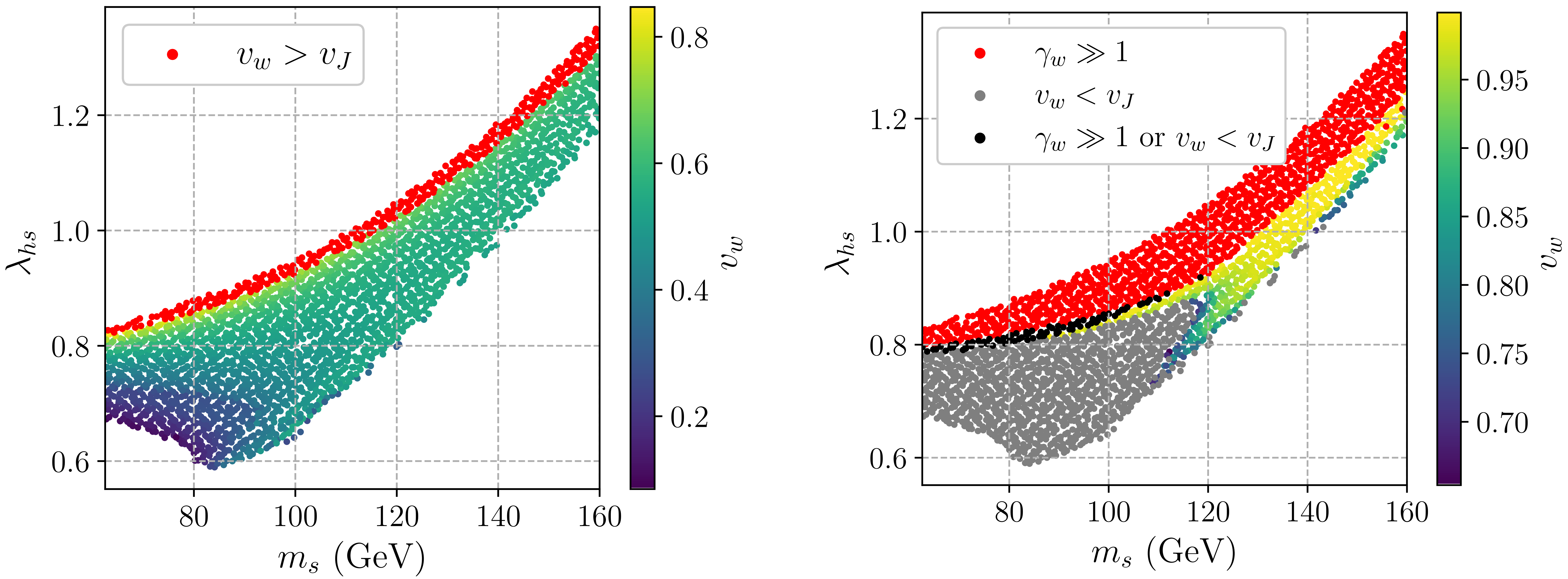
with out-of-equilibrium contributions

What does it do?

- Computes (leading log) matrix elements for out-of-equilibrium particles, based on `DRalgo (Mathematica)` Ekstedt, Schicho, Tenkanen 2022

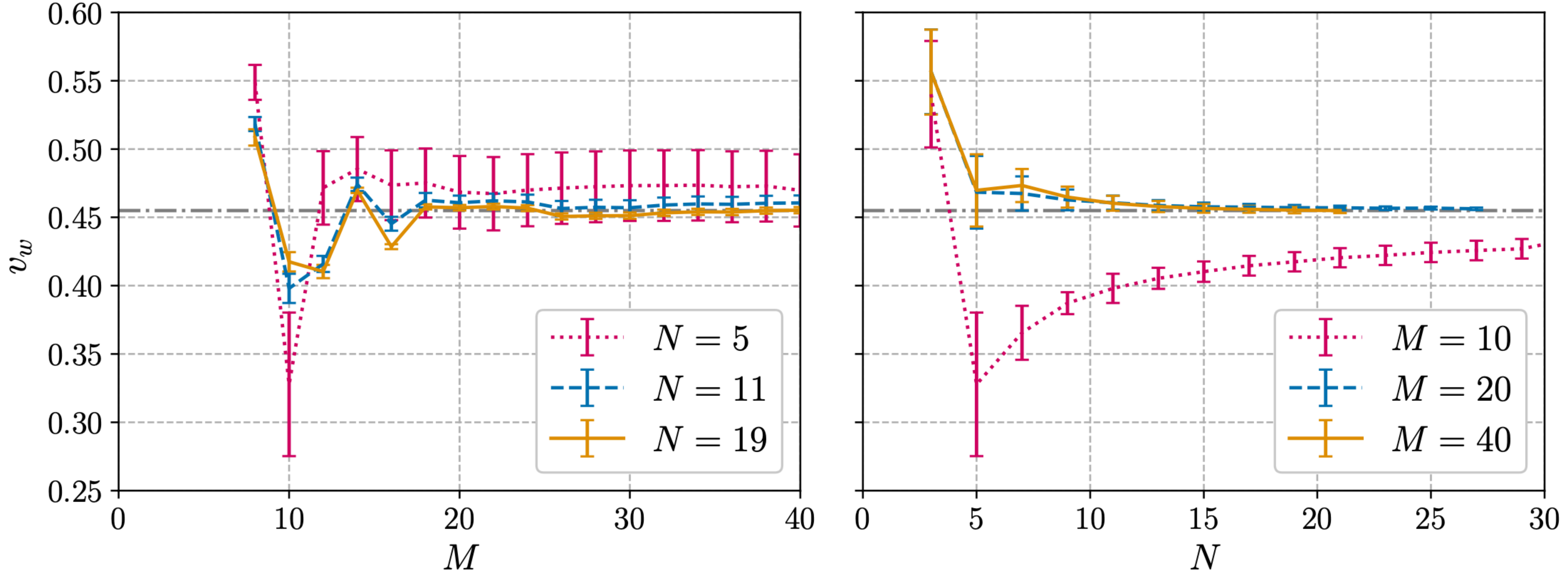
- Computes the corresponding collision terms in `C++`
- Solves the equation of motion for the scalar field(s) with a Tanh-Ansatz, fluid equations and Boltzmann equations in the Chebyshev expansion in `Python`
- The model and the set of out-of-equilibrium particles are user-defined

Result for the xSM



Convergence of the spectral method

- M, N: number of polynomials in position/momentum direction



Towards an estimate (and reduction) of the theoretical error

Two benchmark models

- Standard Model coupled to a gauge singlet (xSM) with Z_2 -symmetry
- We study the parameter space where the phase transition is two-step: first the singlet gets a vev, then the Higgs
- We are interested in the second step of the phase transitions

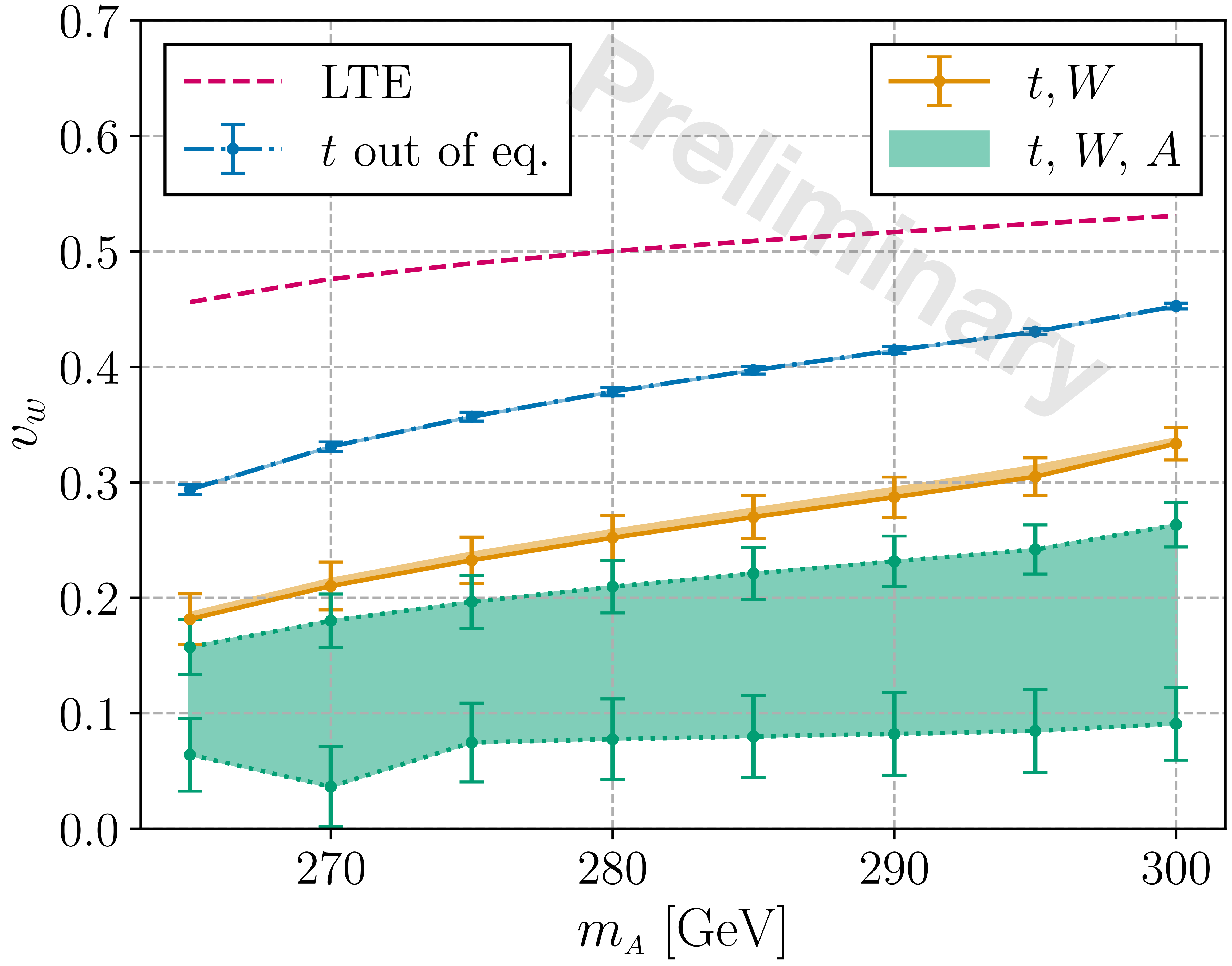
- Inert doublet model (IDM): special case of the two Higgs doublet model where new doublet has no vev at zero-temperature
- The phase transition is radiatively generated; only the Higgs gets a vev
- Four new scalar bosons: $A, H, H^{\pm}$ We take H light, and $m_A = m_{H^{\pm}} \sim \Lambda_{\rm EW}$

Set of out-of-equilibrium particles and interactions

- Particles get pushed out-of-equilibrium by the passing bubble and by the non-constant temperature and fluid profile
- It is numerically very expensive to solve the Boltzmann equation for all particles
- One therefore tracks only the heaviest particles, and focusses on the strongest interactions in the collision term
- Trade-off between numerical cost and accuracy

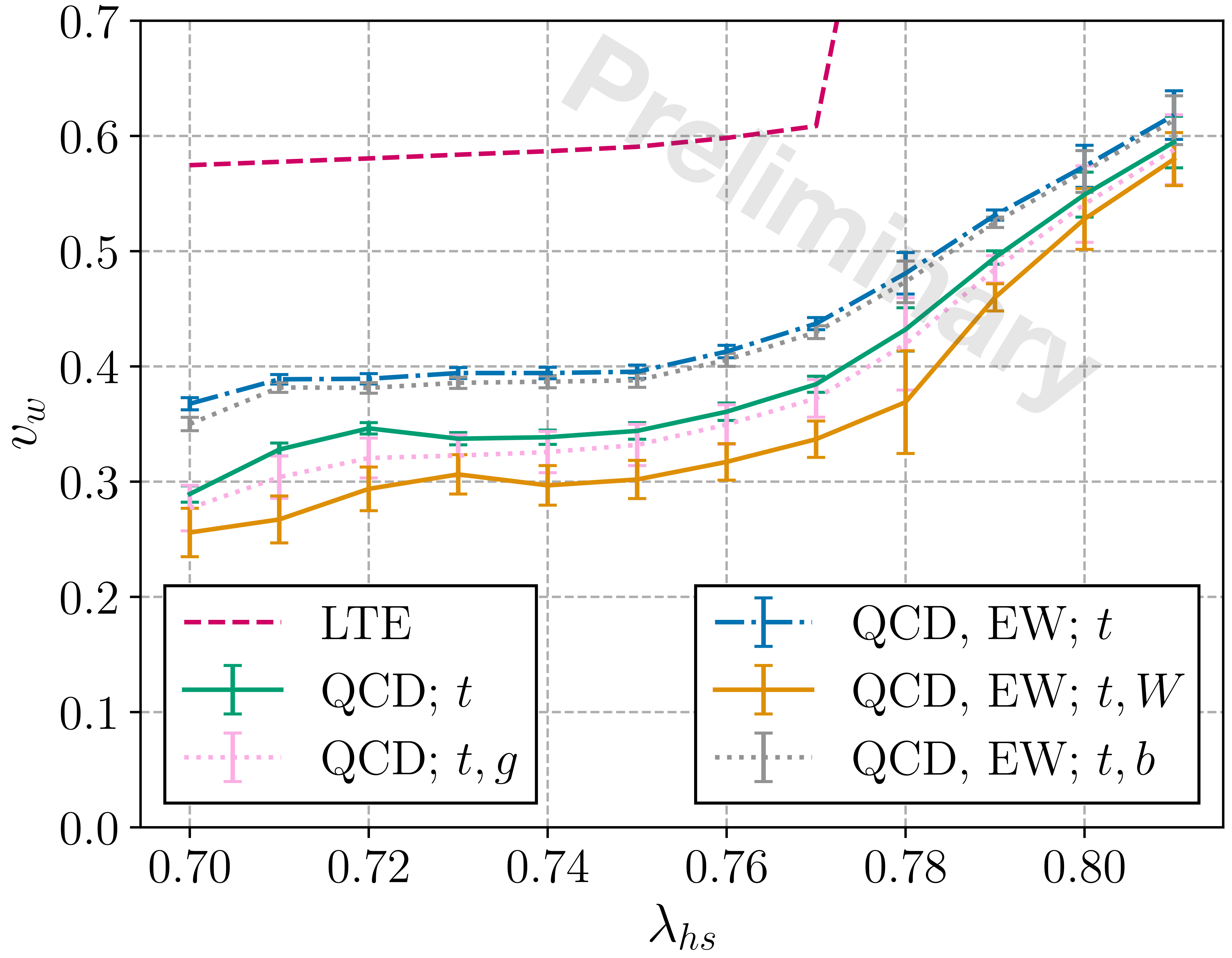
Set of out-of-equilibrium particles and interactions

IDM with strong and weak interactions



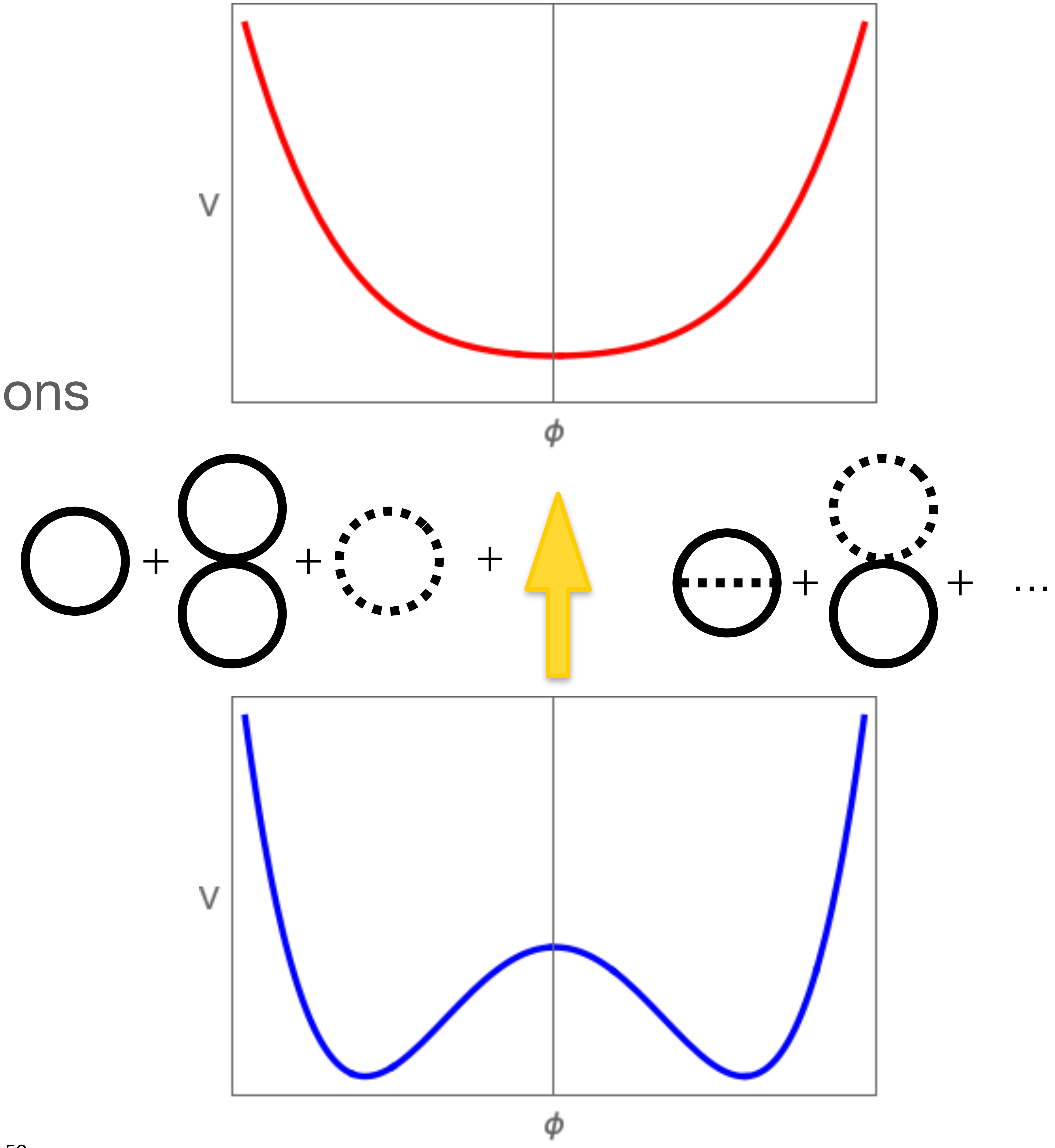
Set of out-of-equilibrium particles and interactions

xSM with strong (and weak) interactions



Effective potential

- Temperature-dependent loop corrections determine $V_{\rm eff}(\phi, T)$



Effective potential

- Temperature-dependent loop correctior determine $V_{\text{eff}}(\phi, T)$
- Accurate results require going beyond one-loop
- The effective potential in the scalar field equations of motion is always evaluated at one-loop only

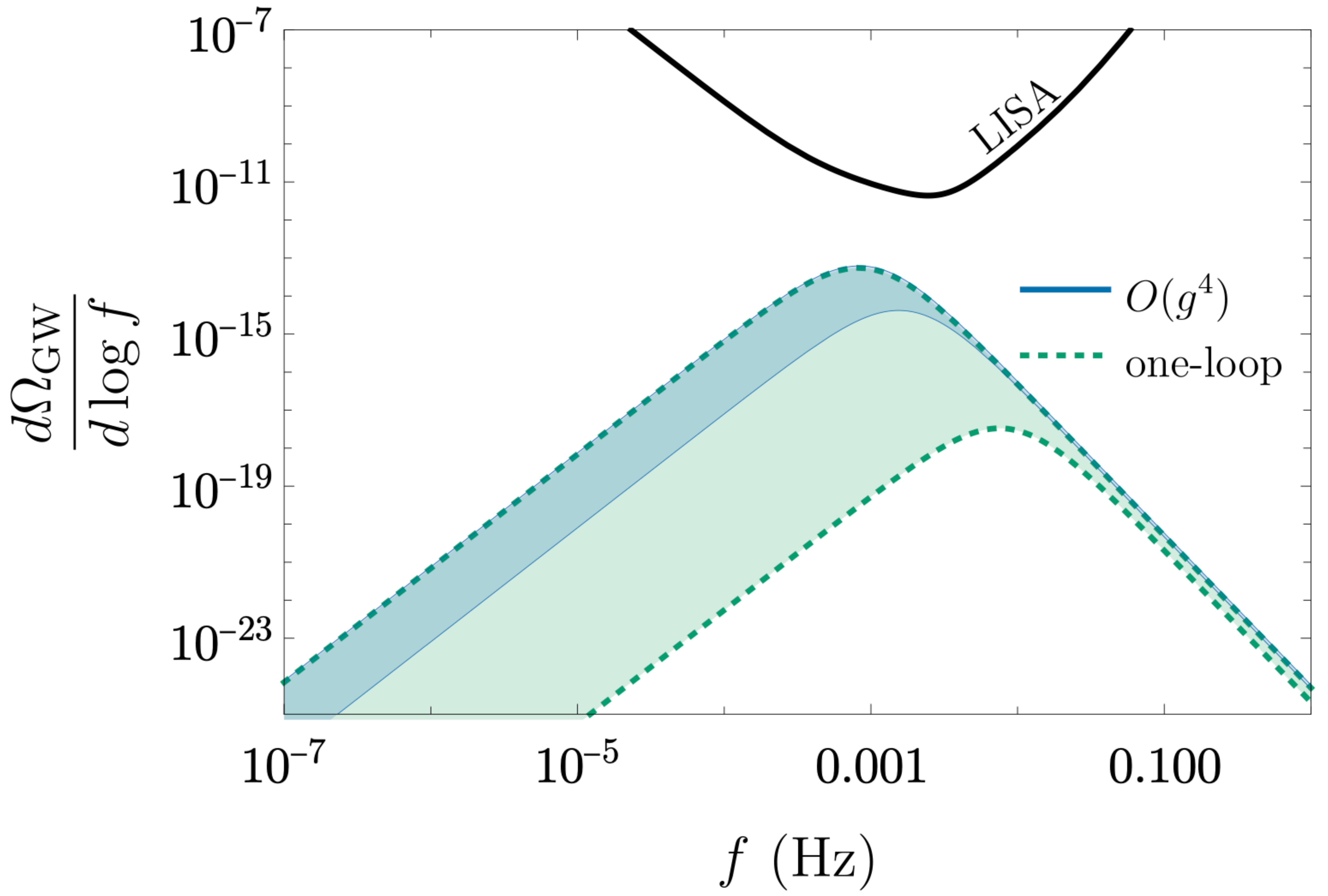
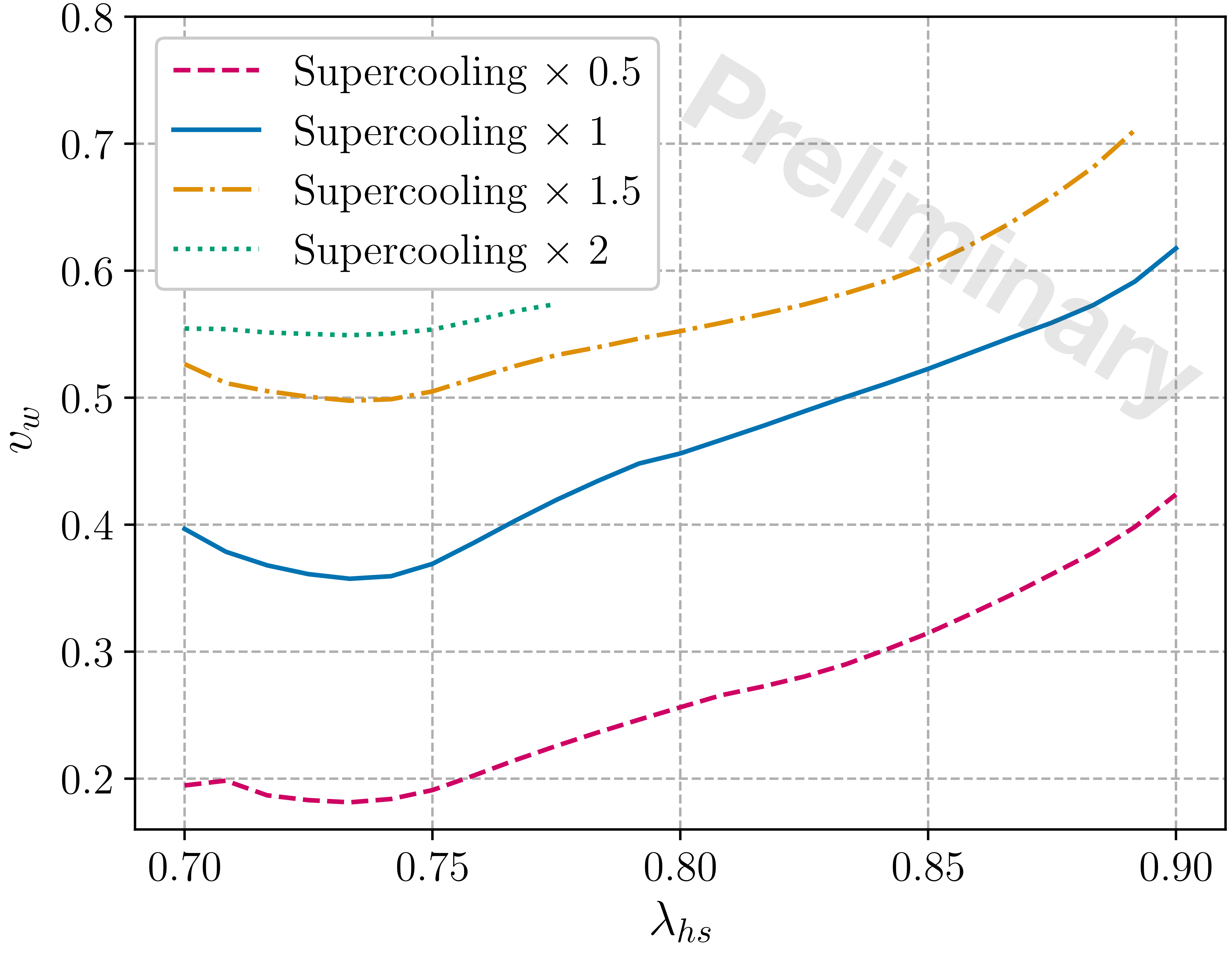


Figure: Gould, Tenkanen 2021

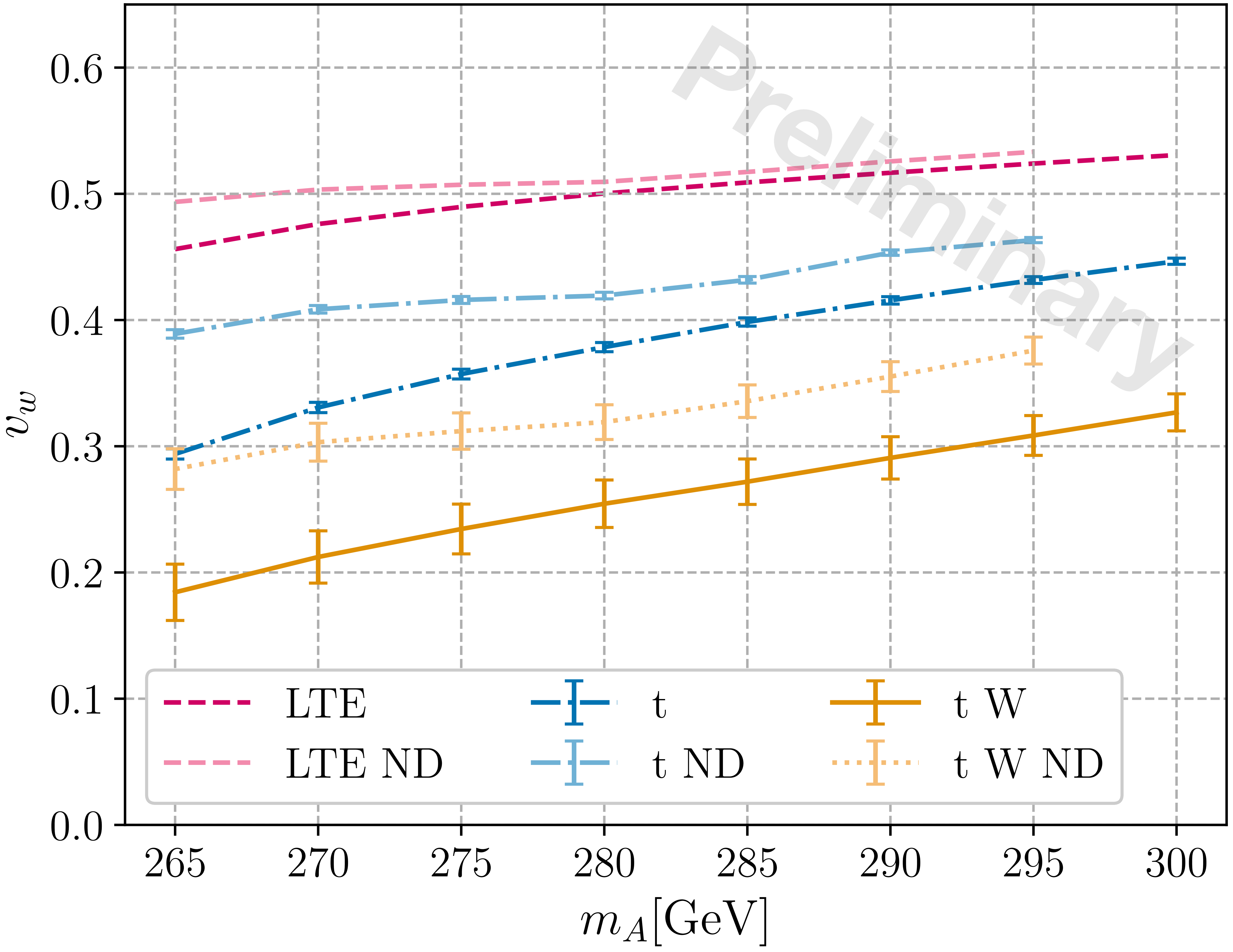
Quanitifying the uncertainty from thermodynamics

Varying the amount of supercooling in the xSM



Quanitifying the uncertainty from thermodynamics

Comparing the potential with and without Daisy resummation in the IDM



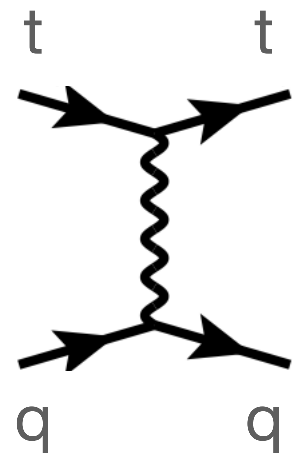
Collision terms at leading logarithmic order

- Linearized collision term

$$\mathscr{C}^{\text{lin}}_{ab}[\delta f] = \frac{1}{4}\sum_{cde}\int_{\vec{p}_2,\vec{p}_3,\vec{p}_4}\frac{1}{2E_2 2E_3 2E_4}(2\pi)^4\delta^4(P_1+P_2-P_3-P_4)\times$$

$$|M_{ac\to de}(P_1,P_2;P_3,P_4)|^2 f^a f^c f^d f^e\left(\delta_{ab}F^c_a+\delta_{cb}F^a_c-\delta_{db}F^e_d-\delta_{eb}F^d_e\right)$$

- Sum over 2 ↔ 2 diagrams, e.g



$$|\mathscr{M}|^2 \propto \frac{s^2+u^2}{t^2}$$

- u- and t-channel diagrams divergent for small momentum transfer!

Collision terms at leading logarithmic order

Arnold, Moore, Yaffe 2000

- u- and t-channel diagrams divergent for small momentum transfer
- Divergence gets regulated by hard-thermal loop self-energy in the propagator
- Leading log approximation:
  - Keep only the u- and t-channels
  - Regularize the propagator by the (momentum-independent) HTL self-energy
  - Remaining collision term is proportional to $1/\log g^{-1}$

Beyond leading log?

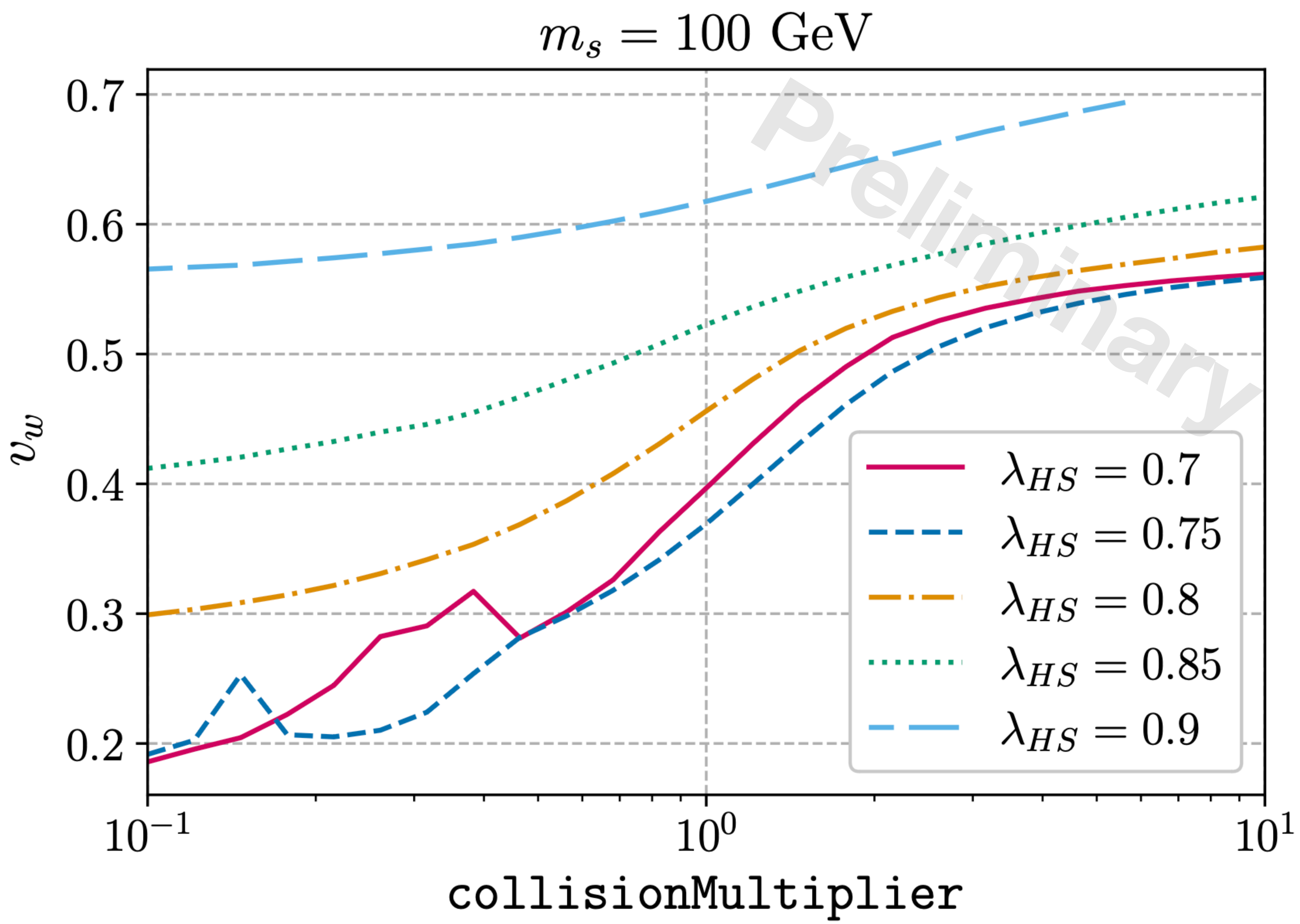
- A full leading order result requires:
  - Inclusion of s-channel diagrams
  - Inclusion of 2↔2 and 1↔2 diagrams
  - Resummation of soft emissions in 1↔2 diagrams (LPM resummation)
  - Including the momentum dependence in the HTL self-energy
- Done for transport coefficients in Arnold, Moore, Yaffe 2003; finding $\mathcal{O}(25\%)$ corrections at next-to-leading-log (NLL)
- No NLL computation of the wall velocity has been done

Estimating the error in v_w from leading-log collision terms

- Multiply *all* collision terms by a factor `collisionMultiplier`, to mimic corrections from NLL results

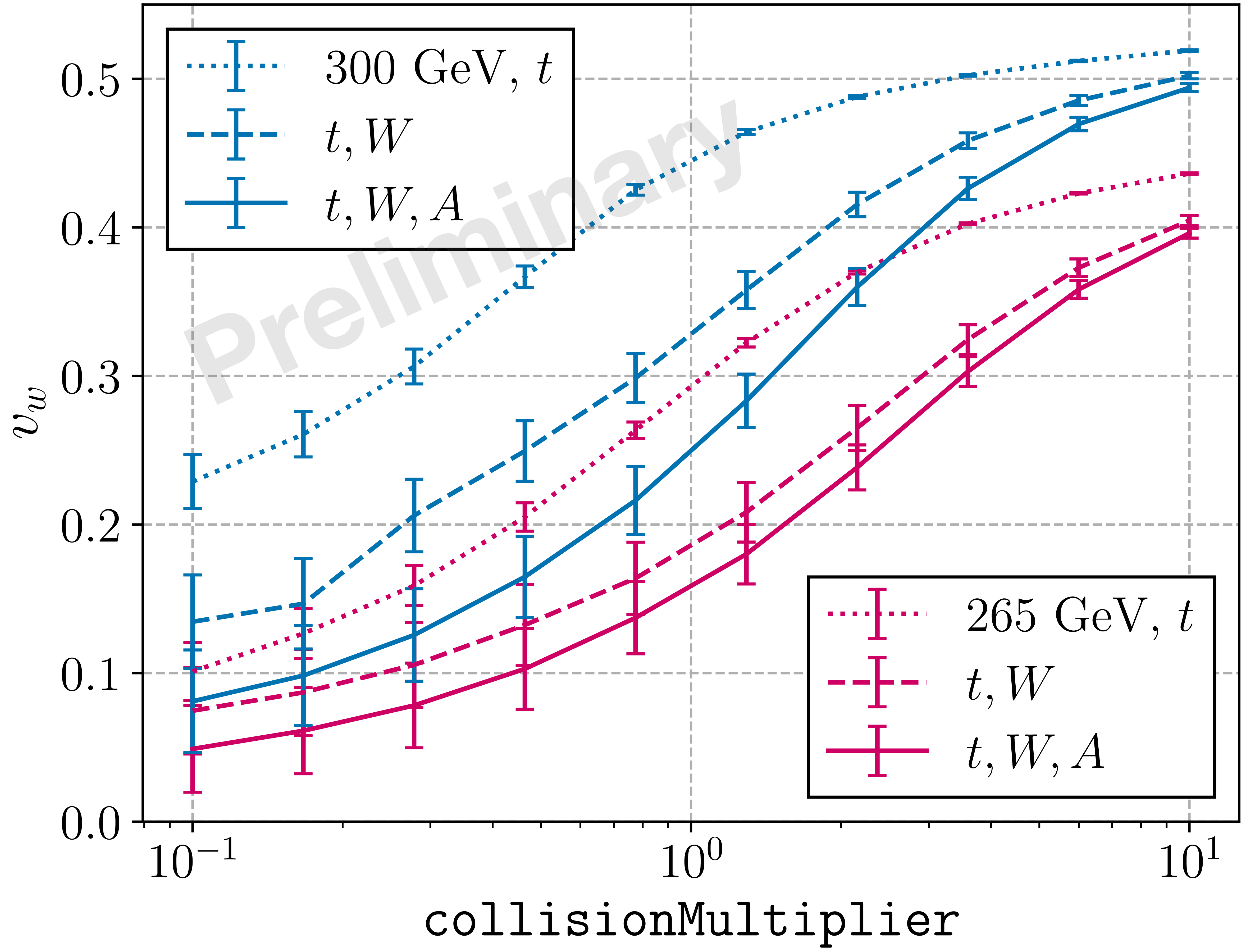
Estimating the error in v_w from leading-log collision terms

xSM benchmark points



Estimating the error in v_w from leading-log collision terms

IDM benchmark points



Summary

- The wall velocity is an important parameter in particle and GW production in first order phase transitions
- WallGo: publicly available code for the computation of v_w with out-of-equilibrium effects
- Study of theoretical uncertainties forthcoming

Back-up

Scalar field equation of motion

Balance of forces Balaji, Spannowski, Tamarit 2020; Ai, Garbrecht, Tamarit 2021

$$\int dz \frac{d\phi}{dz}\left(\partial^2\phi + \frac{\partial V_{\rm eff}(\phi,T)}{\partial\phi} + \sum_a \frac{\partial m_a^2}{\partial\phi}\int_{\vec{p}} \frac{1}{2E}\delta f^a(p^\mu,\xi)\right) = 0$$

$$\int dz \frac{d\phi}{dz}\frac{\partial V_{\rm eff}(\phi,T)}{\partial\phi} = \int dz\left(\frac{dV_{\rm eff}}{dz} - \frac{\partial V_{\rm eff}}{\partial T}\frac{dT}{dz}\right) = \Delta V_{\rm eff} - \int dz \frac{\partial V_{\rm eff}}{\partial T}\frac{dT}{dz}$$

Scalar field equation of motion

Balance of forces Balaji, Spannowski, Tamarit 2020; Ai, Garbrecht, Tamarit 2021

$$\int dz \frac{d\phi}{dz}\left(\partial^2\phi + \frac{\partial V_{\text{eff}}(\phi,T)}{\partial\phi} + \sum_a \frac{\partial m_a^2}{\partial\phi}\int_{\vec{p}} \frac{1}{2E}\delta f^a(p^\mu,\xi)\right) = 0$$

$$\int dz \frac{d\phi}{dz}\frac{\partial V_{\text{eff}}(\phi,T)}{\partial\phi} = \int dz\left(\frac{dV_{\text{eff}}}{dz} - \frac{\partial V_{\text{eff}}}{\partial T}\frac{dT}{dz}\right) = \Delta V_{\text{eff}} - \int dz \frac{\partial V_{\text{eff}}}{\partial T}\frac{dT}{dz}$$

$$\Delta V_{\text{eff}} - \int dz \frac{\partial V_{\text{eff}}}{\partial T}\frac{dT}{dz} + \sum_a \int dz \frac{dm_a^2}{dz}\int_{\vec{p}}\frac{1}{2E}\delta f^a(p^\mu,\xi) = 0$$

- ΔV_{eff}: Driving force
- $\int dz \frac{\partial V_{\text{eff}}}{\partial T}\frac{dT}{dz}$: Hydrodynamic backreaction
- $\sum_a \int dz \frac{dm_a^2}{dz}\int_{\vec{p}}\frac{1}{2E}\delta f^a$: Friction force. Dominant contribution from heavy particles (e.g. top)

Boundary conditions from hydrodynamics

- Spherically symmetric solutions to $\partial_\mu T^{\mu\nu} = 0$ for $\partial_\mu \phi = 0$ and equation of state given by $p_{\rm HT} = -V_{\rm eff}(v_{\rm HT}, T)$ and $p_{\rm LT} = -V_{\rm eff}(v_{\rm LT}, T)$
- Three types of solutions

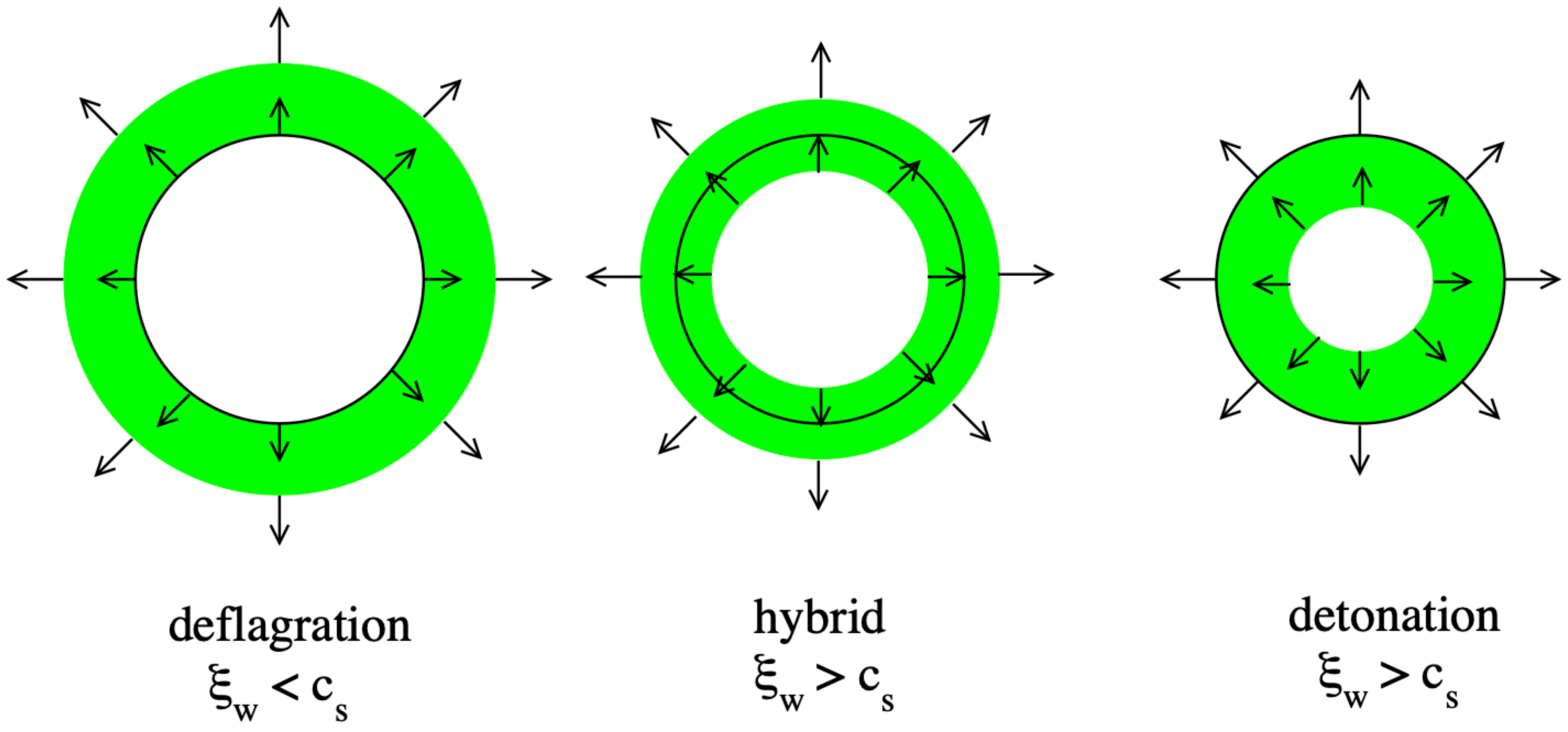
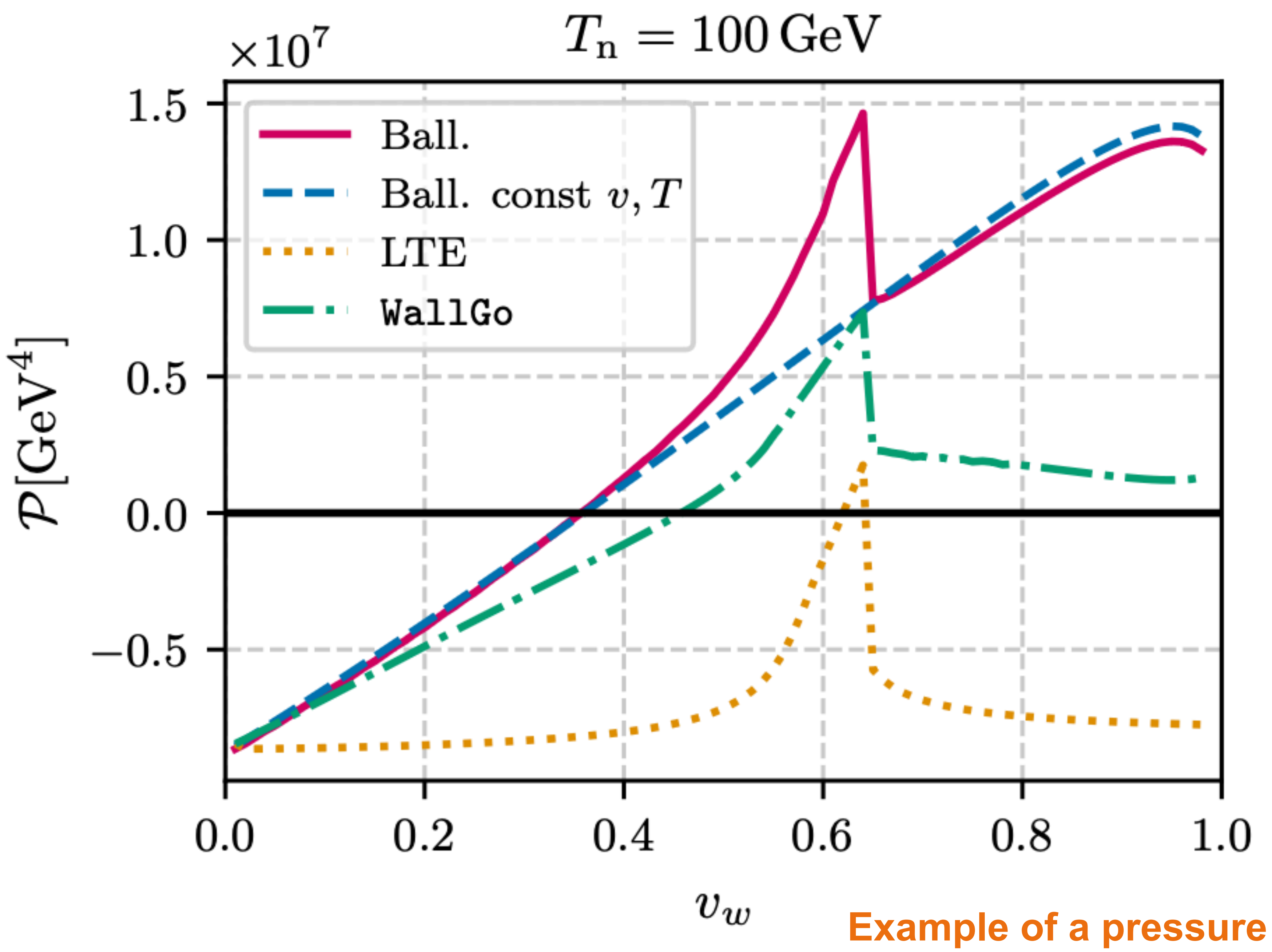


Figure from: Espinosa, Konstandin, No, Servant 2010

Hydrodynamic backreaction

- Due to the hydrodynamic backreaction, the pressure of the deflagration and hybrid solution always increase with v_w
- For detonations the hydrodynamic backreaction *decreases* with v_w



Example of a pressure profile
Fig. Ai, Laurent, JvdV 2024

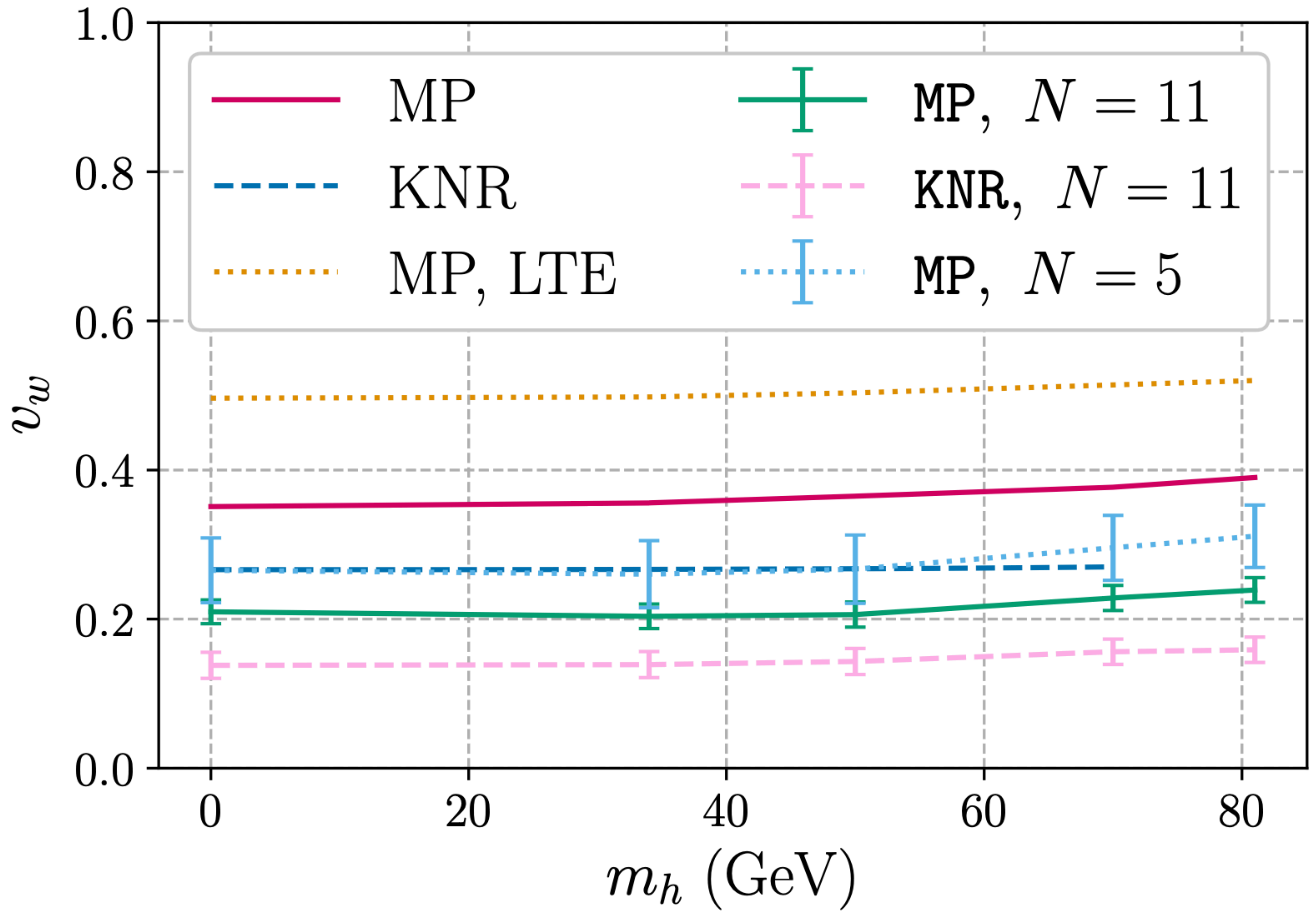
Boundary conditions from hydrodynamics

- Spherically symmetric solutions to $\partial_\mu T^{\mu\nu} = 0$ for $\partial_\mu \phi = 0$ and equation of state given by $p_{\rm HT} = -V_{\rm eff}(v_{\rm HT}, T)$ and $p_{\rm LT} = -V_{\rm eff}(v_{\rm LT}, T)$
- Three types of solutions
- Fixing the nucleation temperature and v_w determines
$T_+, T_-, v_+, v_- \quad \rightarrow \quad c_1, c_2$

Comparison with earlier computation for SM with light Higgs

Moore, Prokopec 1995; Konstandin, Nardini, Rues 2014

- Spectral method ($N = 11$) versus three moments
- Some differences in matrix elements
- Mixing in the Boltzmann equations (e.g. eq. for δf_{top} depends on δf_W)
- Different treatment of hydrodynamics to MP



Comparison for Inert Doublet Model

BM	$\bar{m}_H$ [GeV]	$\bar{m}_A, \bar{m}_{H^\pm}$ [GeV]	λ_L	T_c [GeV]	T_n [GeV]	v_w [49]	v_w [WallGo]
A	62.66	300	0.0015	118.3	117.1	0.165	0.191 ± 0.024
B	65.00	300	0.0015	118.6	117.5	0.164	0.180 ± 0.025
C	63.00	295	0.0015	119.4	118.4	0.164	0.182 ± 0.024

- Comparison with Jiang, Huang, Wang 2022 shows reasonable agreement