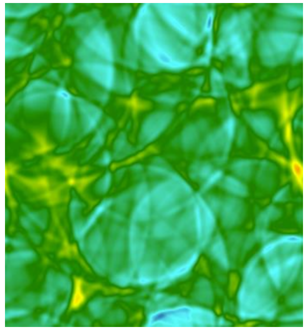

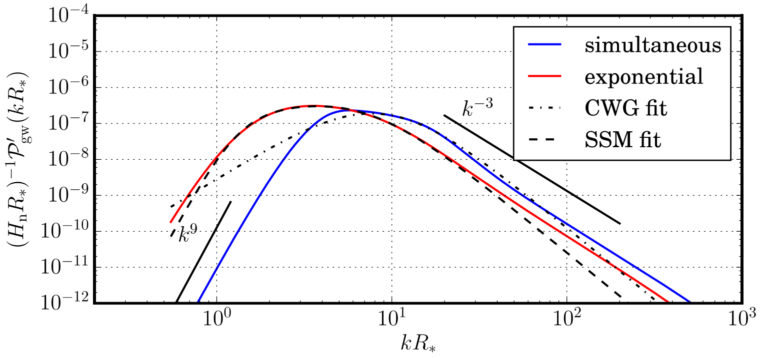

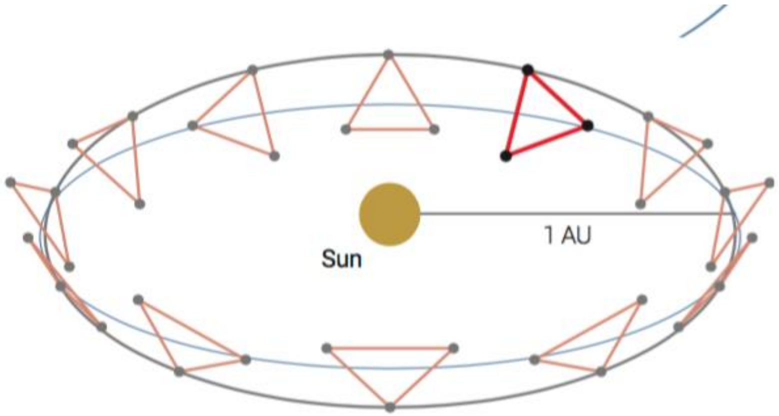

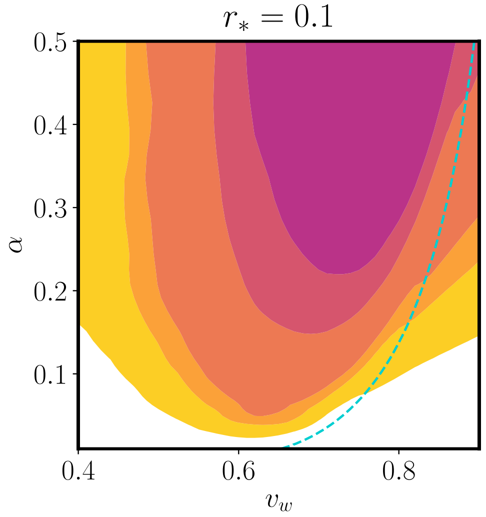

Simulations and modelling of gravitational waves from first order phase transitions

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and

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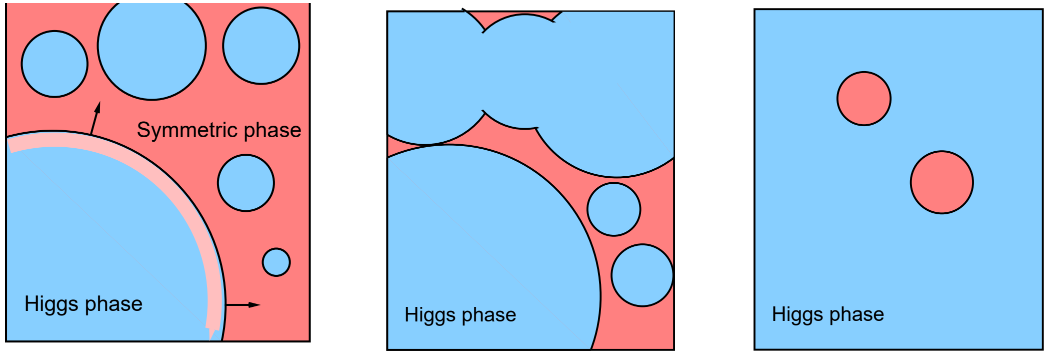
NORDITA

6. elokuuta 2025

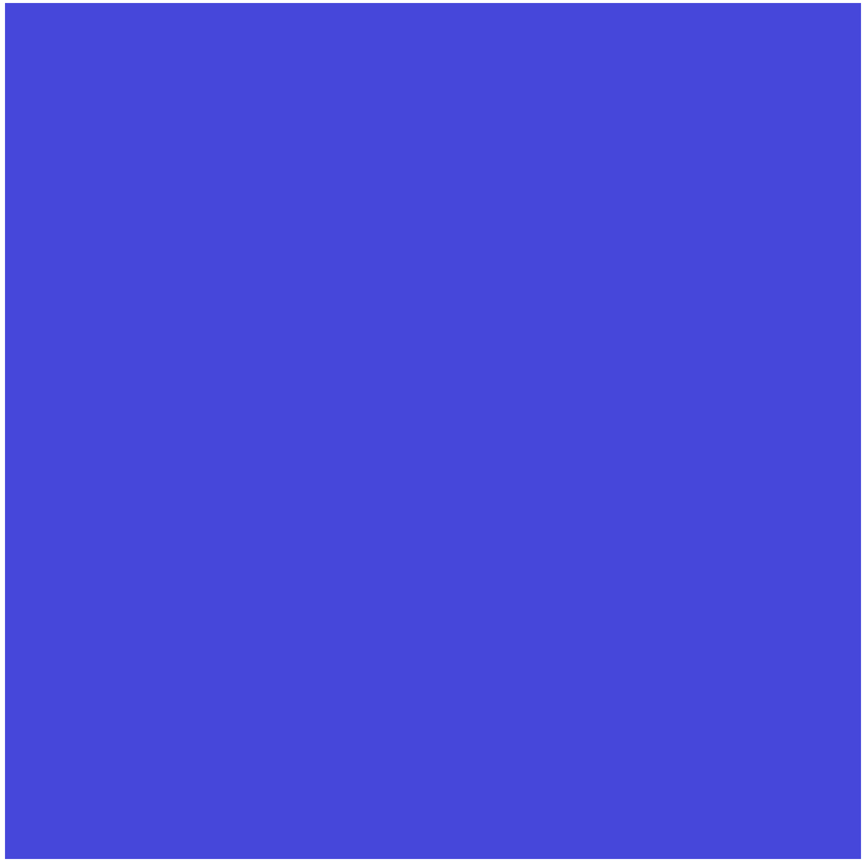
Little bangs in the Big Bang

- 1st order transition by nucleation of bubbles of low-T phase
  Cahn, Hilliard 1959, Langer 1969, Coleman 1974, Linde 1983
- Nucleation rate/volume $p(t)$ rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
- Shear stress, gravitational wave (GW) production
- GW spectrum has information about phase transition
  - Temperature, latent heat, nucleation rate, sound speeds …



Steinhardt (1982); Hogan (1983,86);
Gyulassy et al (1984); Witten (1984)

Fluid kinetic energy

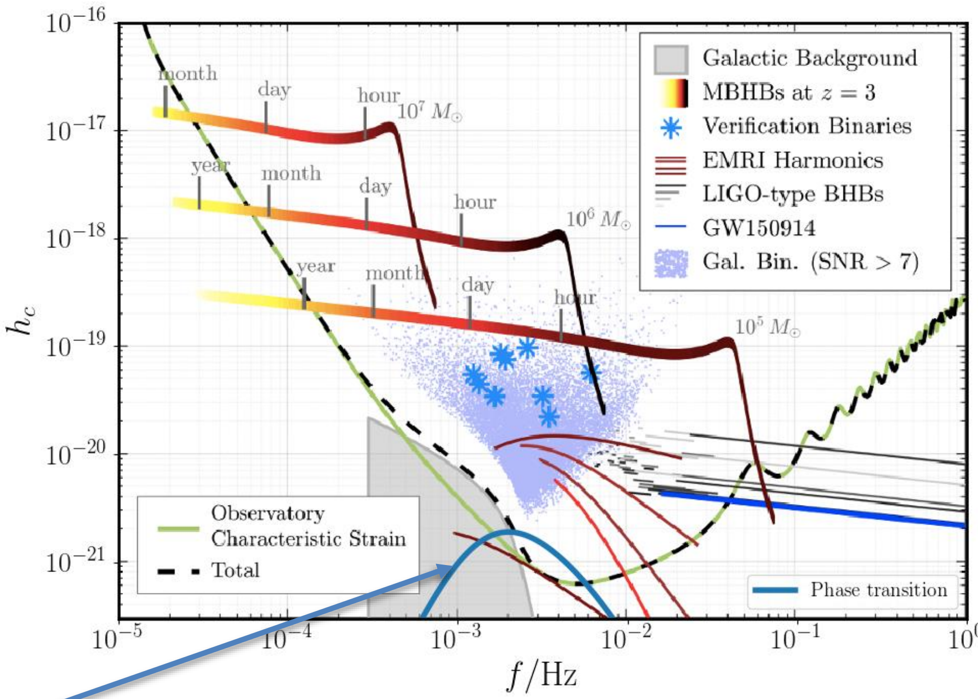


David Weir

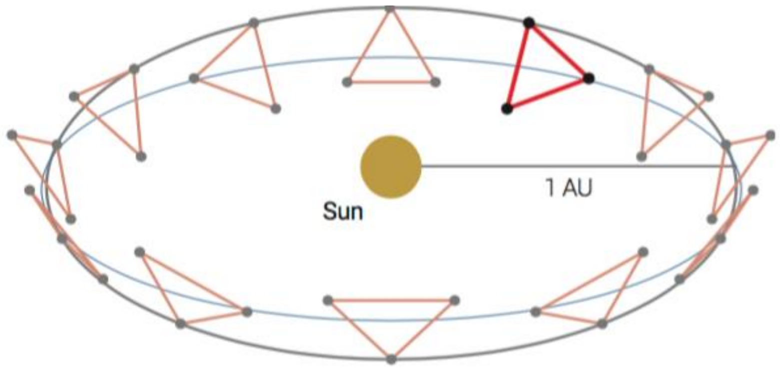
MH, Huber, Rummukainen, Weir (2013,5,7)
Cutting, MH, Weir (2018,9)

Laser Interferometer Space Antenna

- Launch 2035, operations 2038
- 4-year mission (up to 10 years)
- 2.5M km arms, 1 mHz GWs
- Science objectives:
  - White dwarves
  - Black holes
  - Galaxy mergers
  - Extreme gravity
  - TeV-scale early Universe
- Other missions: Taiji, TianQin
- Proposals: DECIGO, BBO



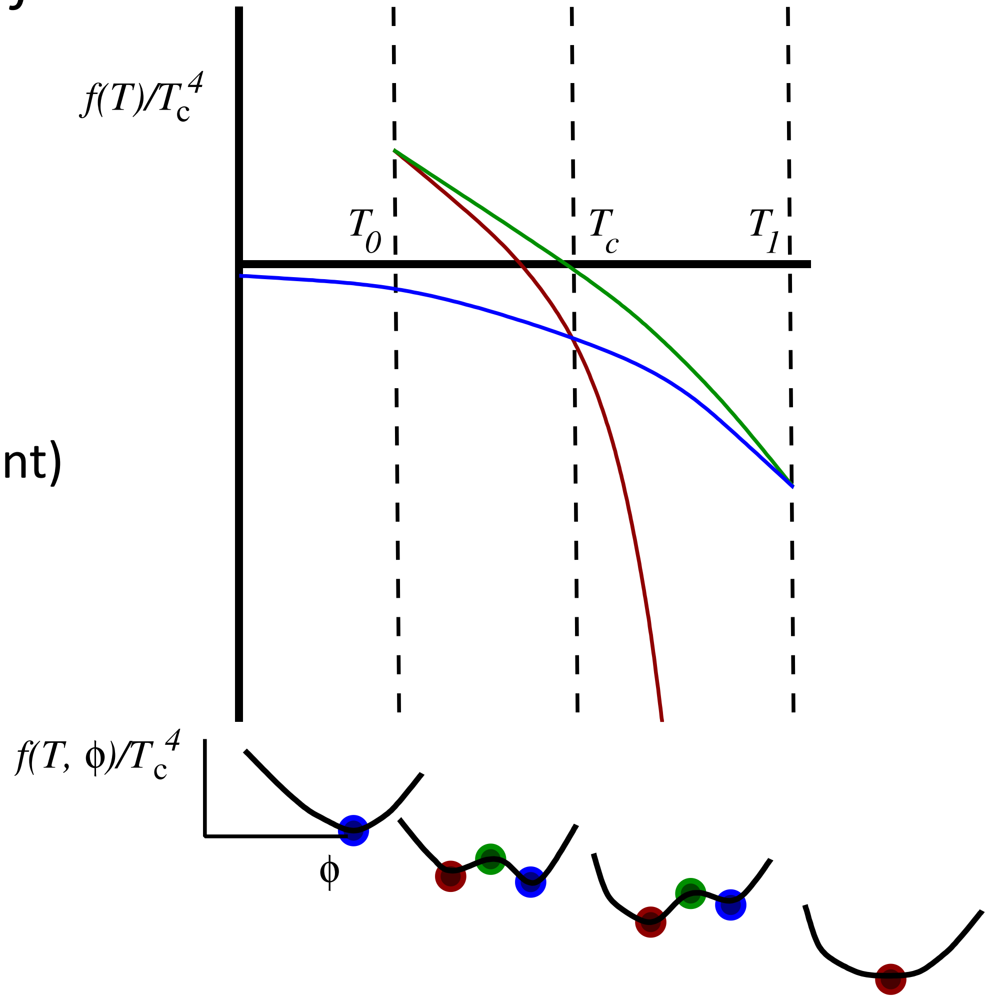
LISA sensitivity



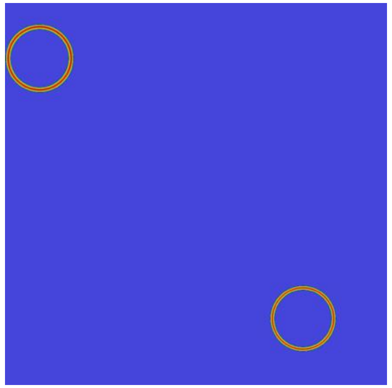
First order phase transitions

e.g. Binder 1987

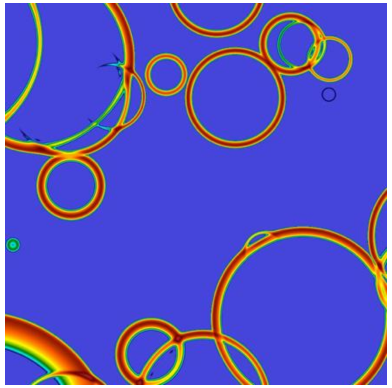
- Phase: a local minimum of the free energy density *f*
- Behaviour of free energy around 1st order PT:
  - $T_2 < T$: one equilibrium phase
  - $T_0 < T < T_1$: two equilibrium phases, one unstable
  - $T = T_c$: equal free energy, critical temperature
  - $T_0 < T < T_c$: high temperature phase is metastable
  - $T = T_0$: high temperature phase is unstable (spinodal point)
- Metastable phase can persist to $T = 0$
  - Example: superfluid ^{3}He, A phase (no spinodal point)
- Keep track of phase with order parameter ϕ
- In equilibrium: $\partial_\phi f(T, \phi_{\text{eq}}) = 0$
- Equilibrium free energy: $f(T) \equiv f(T, \phi_{\text{eq}})$



Phases of a phase transition



1



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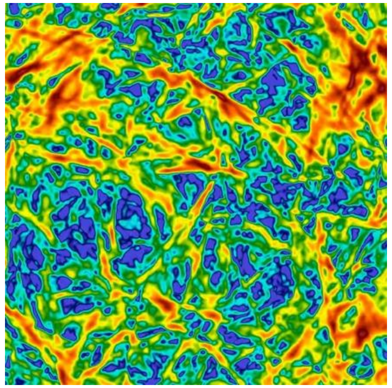


3

4

1. Bubble nucleation and expansion
2. Collision
3. Acoustic waves (vorticity)
4. Non-linear (shocks, turbulence)

$$\tau_{\rm nl} = L/\bar{U}$$

L – fluid flow length scale
$\bar{U}$ – RMS fluid velocity

Thermal transition: approx. exponential time dependence of nucleation rate/volume p(t)

$$p(t) = p_n e^{\beta(t-t_n)}$$

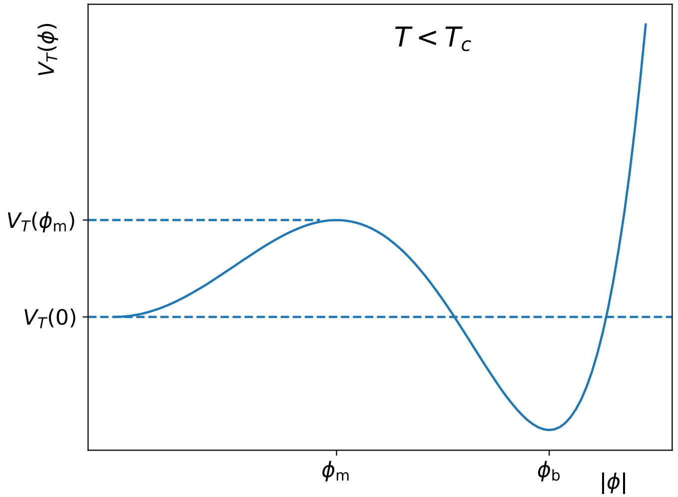
β – transition rate parameter
$\beta > H$ for successful transition

Guth, Weinberg 1983; Enqvist et al 1992;
Turner, Weinberg, Widrow 1992;
Review: MH, Lüben, Lumma, Pauly 2021

Effective theory of early universe phase transitions

Assume: mean free path << order parameter correlation length

Convenient limit: rapid transition, $\beta >> H$, neglect expansion of universe



- **“Higgs” field** $$\Box\phi - V_T'(\phi) = \eta_T(\phi) U \cdot \partial\phi$$
  - $V_T(\phi)$ equation of state — Ignatius et al (1994), Kurki-Suonio, Laine (1996)
  - $\eta_T(\phi)$ field-fluid coupling (models friction)
- **Relativistic fluid (ideal limit)**

$$T_{\mathsf{f}}^{\mu\nu} = (e+P)U^\mu U^\nu + P g^{\mu\nu}$$

$$\partial_\mu T_{\mathsf{f}}^{\mu\nu} + \partial^\nu\phi V_T'(\phi) = \eta_T(\phi)(U\cdot\partial\phi)\partial^\nu\phi$$

- **Metric perturbation** (GW strain)

$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G T_{ij} \quad \Rightarrow \quad \tilde{h}_{ij}(\mathbf{k}) = \Lambda_{ij,kl}^{TT} u_{kl}(\mathbf{k})$$

Garcia-Bellido, Figueroa, Sastre (2008)

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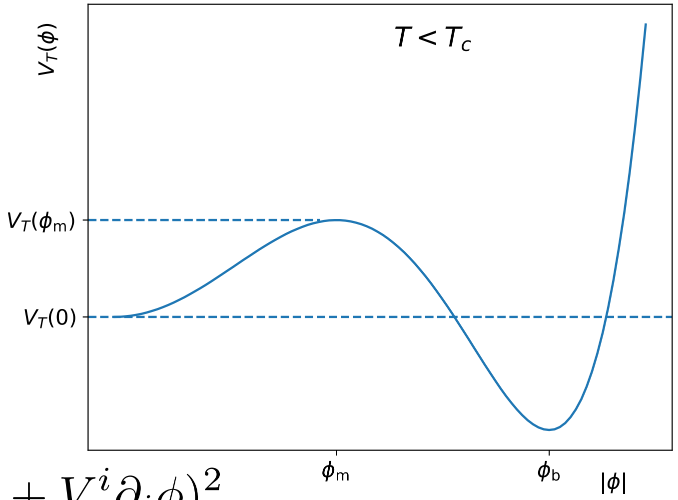


- **"Higgs" field** $$\Box\phi - V_T'(\phi) = \eta_T(\phi) U \cdot \partial\phi$$
  - $V_T(\phi)$ equation of state — Ignatius et al (1994), Kurki-Suonio, Laine (1996)
  - $\eta_T(\phi)$ field-fluid coupling (models friction)
- **Relativistic fluid (ideal limit)**

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial\phi} W(\dot{\phi} + V^i\partial_i\phi) = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2.$$

$$\dot{Z}_i + \partial_j(Z_i V^j) + \partial_i P + \frac{\partial V}{\partial\phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi.$$

  - $E = W*$(energy density), Z_i = momentum density, V_i = 3-velocity, W = Lorentz factor
  - Other variables can be used — Brandenburg, Enqvist, Olesen (1996); Giblin, Mertens (2013); Jinno, Konstandin, Rubira (2020)
- **Metric perturbation** (GW strain)

$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G T_{ij} \quad \Rightarrow \quad \tilde{h}_{ij}(\mathbf{k}) = \Lambda_{ij,kl}^{TT} u_{kl}(\mathbf{k})$$

Garcia-Bellido, Figueroa, Sastre (2008)

Connection to fundamental theory

- Scalar hydrodynamics $-\ddot{\phi} + \nabla^2\phi - V_T'(\phi) = \eta_T(\phi) W(\dot{\phi} + V^i \partial_i \phi)$

- Scalar effective potential $V_T(\phi)$ → equilibrium, quasi-eqm. (T_n, α, β, c_s, $g_{\rm eff}$)
- Scalar-fluid coupling $\eta_T(\phi)$ → non-equilibrium (v_w)

$$\Omega_{\rm gw}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$

Thermodynamic parameters :

T_n = nucleation temperature
$g_{\rm eff}$ = effective d.o.f. in plasma
α ~ (latent heat)/(thermal energy)
c_s = sound speed(s)
β = transition rate
v_w = bubble wall speed

Simulations, Modelling →

$H_n(T_n, g_{\rm eff})$ (Hubble rate)

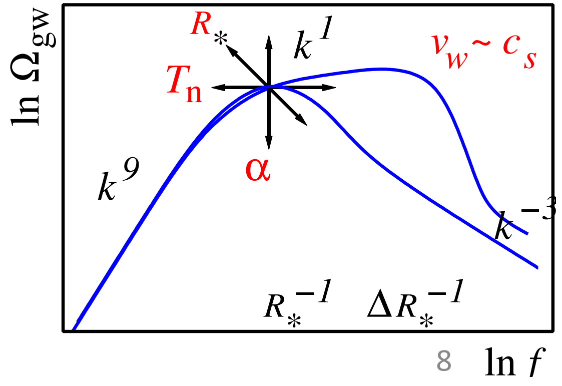
$K(v_w, \alpha, c_s)$ (kinetic energy fraction)

$R_*(\beta, v_w, \alpha)$ (mean bubble separation)

$$f_{\rm p,0} \simeq 26 \left(\frac{1}{H_{\rm n} R_*}\right) \left(\frac{z_{\rm p}}{10}\right) \left(\frac{T_{\rm n}}{10^2\,{\rm GeV}}\right) \left(\frac{h_*}{100}\right)^{\frac{1}{6}} \mu{\rm Hz},$$

GW spectrum

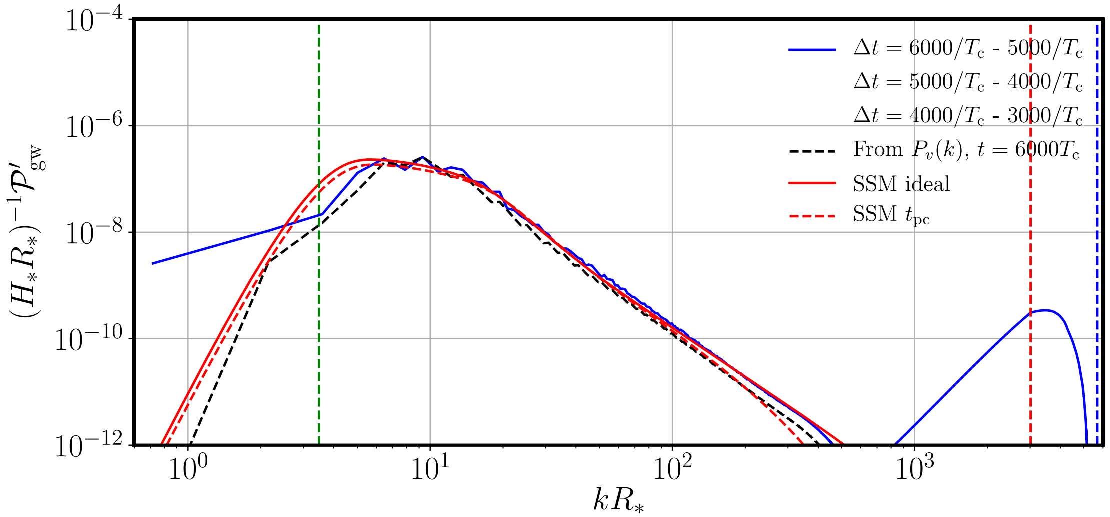
Ω_p = peak amplitude
f_p = peak frequency
σ_i = shape parameters



3D hydrodynamic simulations of phase transitions

MH et al 2013, 2015,2017, 2019; Jinno et al 2023, Caprini et al 2024

- Relativistic fluid + scalar order parameter ("Higgs")
- Linearised gravitational wave production
- Discretise on lattice
  Wilson & Matthews (2003)
- Key output: GW power spectrum
  (fractional GW energy density per log wavenumber)



- While sound waves persist, GW power spectrum grows
- Plot: GW power spectrum **growth rate** (scaled)

$\alpha = 0.5$

$v_w = 0.5$

Cutting, MH, Weir (2019)

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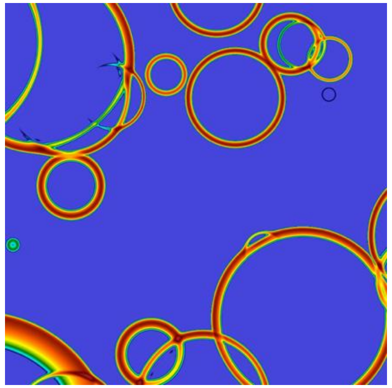
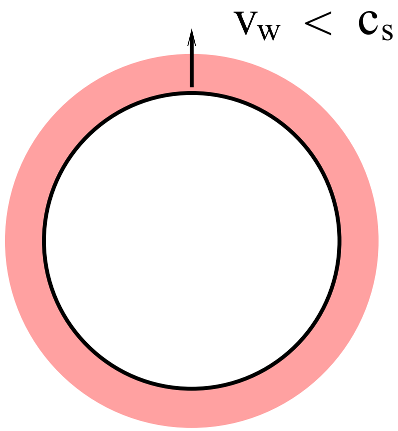
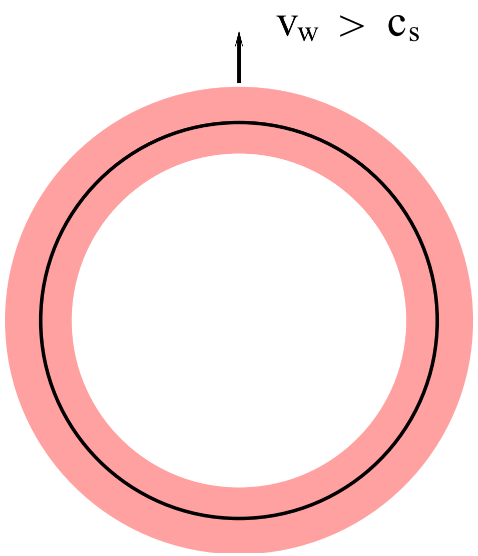
Bubble growth: relativistic combustion



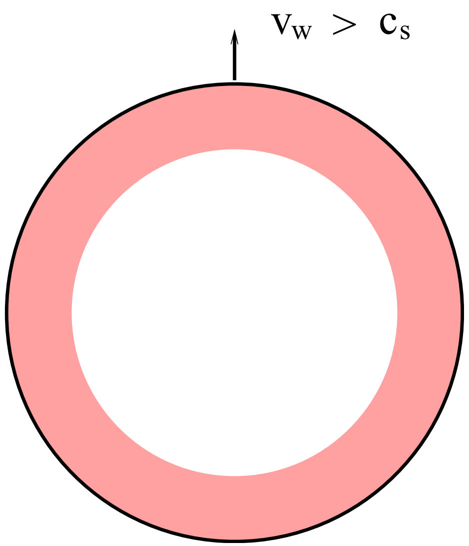


Deflagration

Landau & Lifshitz; Steinhardt (1984)
Kurki-Suonio, Laine (1991), Espinosa et al (2010)

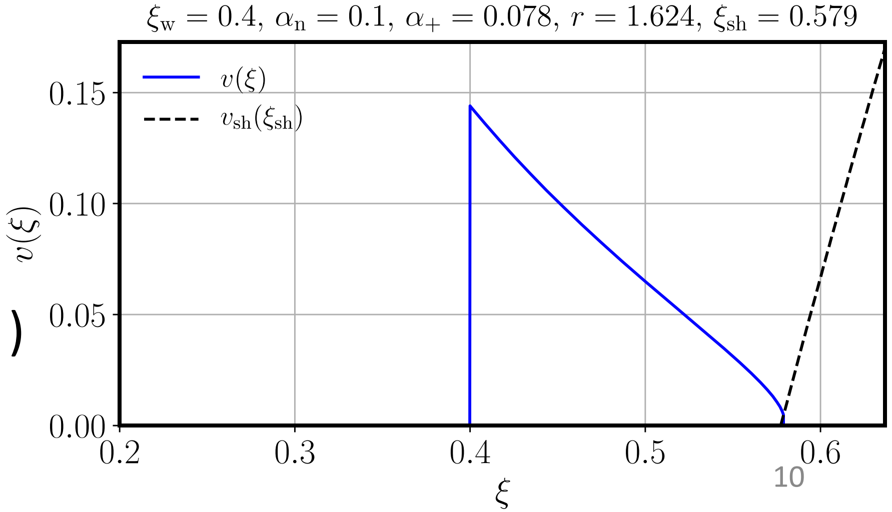


Supersonic deflagration ("hybrid")



Detonation

- Large scales: ideal relativistic hydrodynamics
- Microphysics: $e(T)$, $p(T)$, v_w
- Radial fluid velocity $v(r,t)$ and enthalpy distribution $w(r,t)$
  - **Similarity solution *v*(*x*), *w*(*x*): $\xi = r/t$**
- Low-friction or ultra-strong transition: can be "runaway" ($v_w \rightarrow 1$)
  - (not considered here) Bodeker Moore 2010, 2017



Estimating GW power

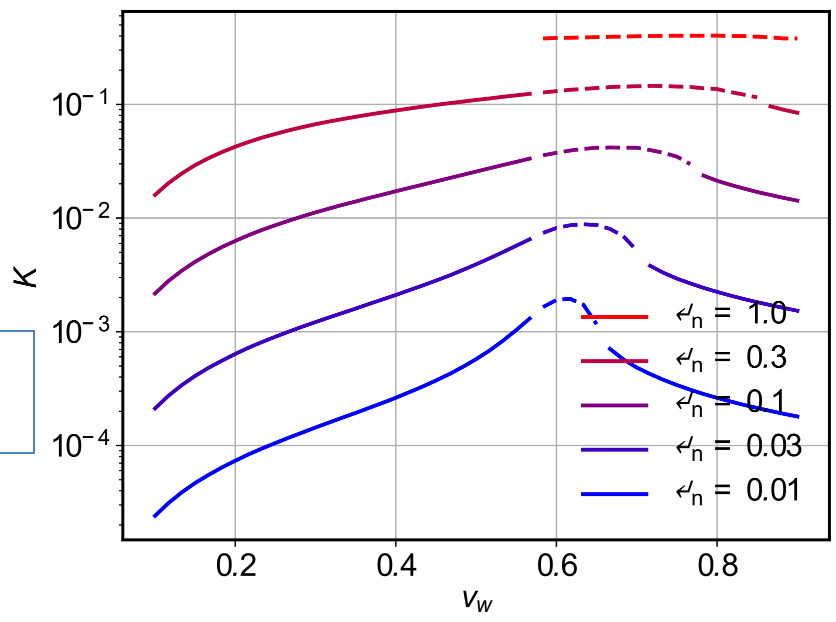
$$\Box h \sim T \longrightarrow P_{\dot{h}}(t,k) \sim \int^t dt_1 \int^t dt_2 \cos[k(t-t_1)]\cos[k(t-t_1)]\langle T_k(t_1) T_k^*(t_2)\rangle$$

- GW energy fraction: $\longrightarrow$ $\Omega_{\rm gw} \sim (H_* \tau_{\rm v})(H_* \tau_{\rm c}) K^2$
  - H_* Hubble rate (max kinetic energy)
  - τ_v duration of stresses
  - τ_c coherence time

$K(v_w, \alpha_n, \ldots)$ (kinetic energy fraction) from self-similar hydro solution

- Kinetic energy fraction $K = \langle w\gamma^2 v^2\rangle / \bar{\epsilon} = \bar{w}\bar{U}^2/\bar{\epsilon}$
  - $\bar{\epsilon}$: mean energy density
  - $\bar{U}$: enthalpy-weighted RMS γv
- Coherence time:
  - $\tau_c \sim R_*$ (mean bubble spacing)
  - Bubble spacing parameter $r_* = R_* H_*$

$R_*(\beta, v_w$, also $\alpha_n)$ from bubble growth dynamics



- Shear stress lifetime (shock lifetime: $t_{\rm sh} = R_*/\bar{U}$)
  - $\tau_v \sim \min(H_*^{-1}, t_{\rm sh}) \sim H_*^{-1}/(1 + \bar{U}/R_* H_*)$

$$\Omega_{\rm gw,0} \simeq F_{\rm gw,0} \frac{r_*}{1+\bar{U}/r_*} K^2 3\tilde{\Omega}_{\rm gw}$$

Numerical simulations:

$$\tilde{\Omega}_{\rm gw} \sim 10^{-2}$$

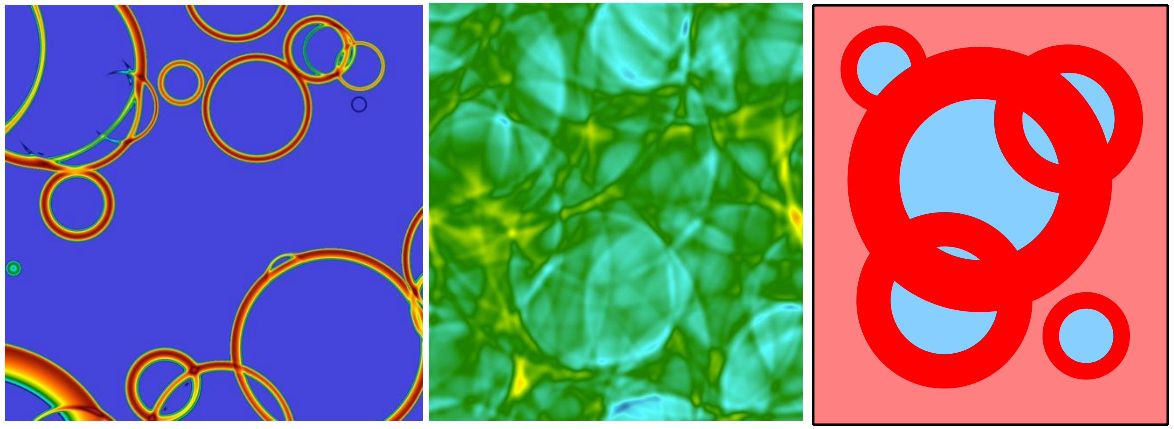
Standard cosmology:

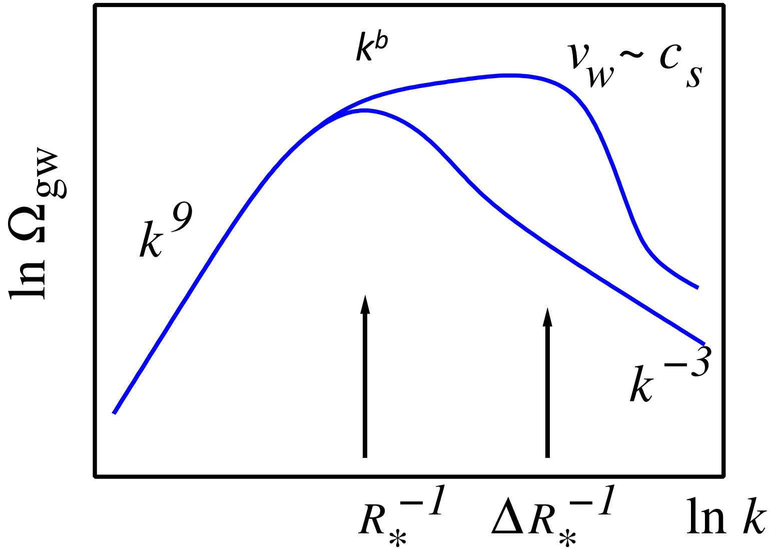
$$F_{\rm gw,0} \simeq 3.6 \times 10^{-5} \left(\frac{100}{g_{\rm eff}}\right)^{1/3}$$

GWs from phase transitions: Sound shell model

- GWs from Gaussian velocity field
  Caprini, Durrer, Servant (2007,2009)
- Velocity field: weighted addition of self-similar sound "shells" $\mathbf{v_q}(t_i)$ from bubbles
  MH 2016, MH, Hijazi (2019)
- Two length scales:
  - Mean bubble spacing R_*
  - Shell width $R_*|v_w - c_s|/c_s$
- Resonant production of GWs
  - $\mathbf{k} = \mathbf{q} + \mathbf{q}'$, $\omega = c_s(q + q')$
- ~ double broken power law ($r_* \equiv H_* R_* \ll 1$)
  - $P_{gw} \sim k^9, k^b, k^{-3}, b \sim 1$
- Amplitude proportional to:
  - Bubble spacing , (Kinetic energy)^2
  - min(flow lifetime, Hubble time)
- Similar: bulk flow model (real space)

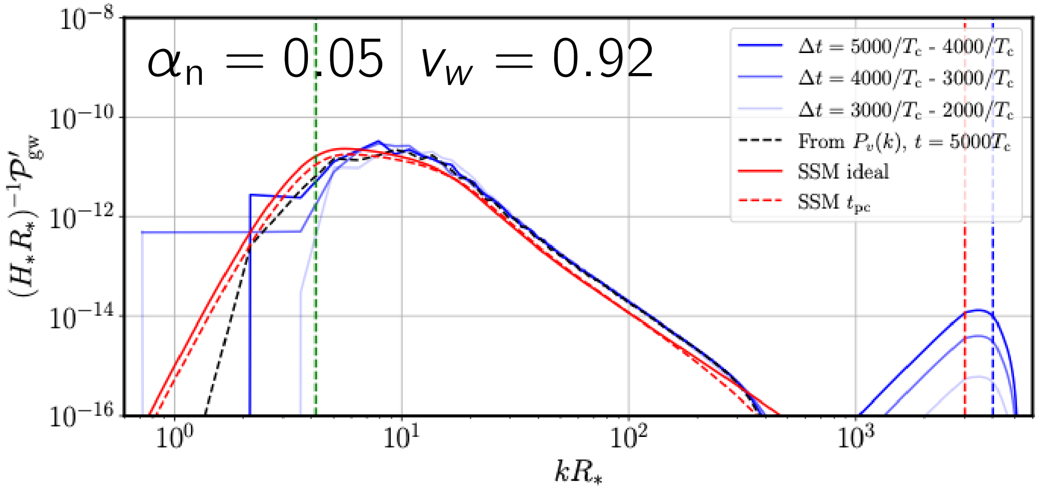
  Jinno, Konstandin, Rubira 2020

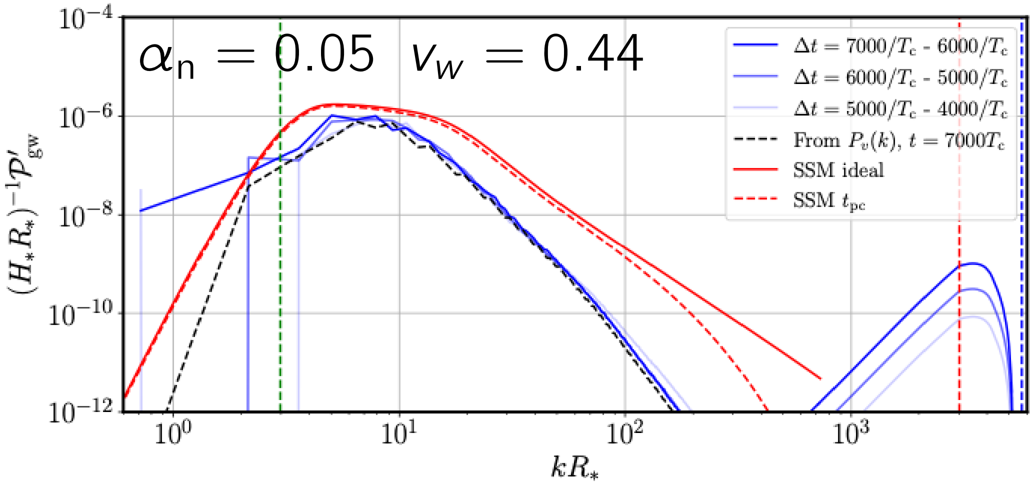


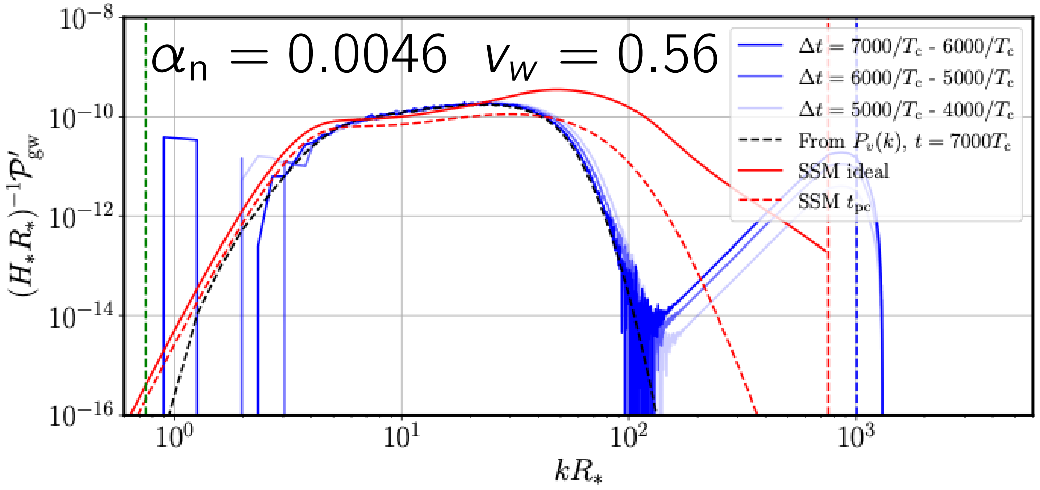


Sound shell model vs. simulations P'_{gw}

- Blue: simulations (**growth rate**): MH et al 2017
  - simultaneous nucleation of bubbles
- Solid: ideal self-similar sound shell
- Dash: evolving sound shell at peak collision time in 1+1D scalar hydro
- SSM good for weak transitions (linear flows)

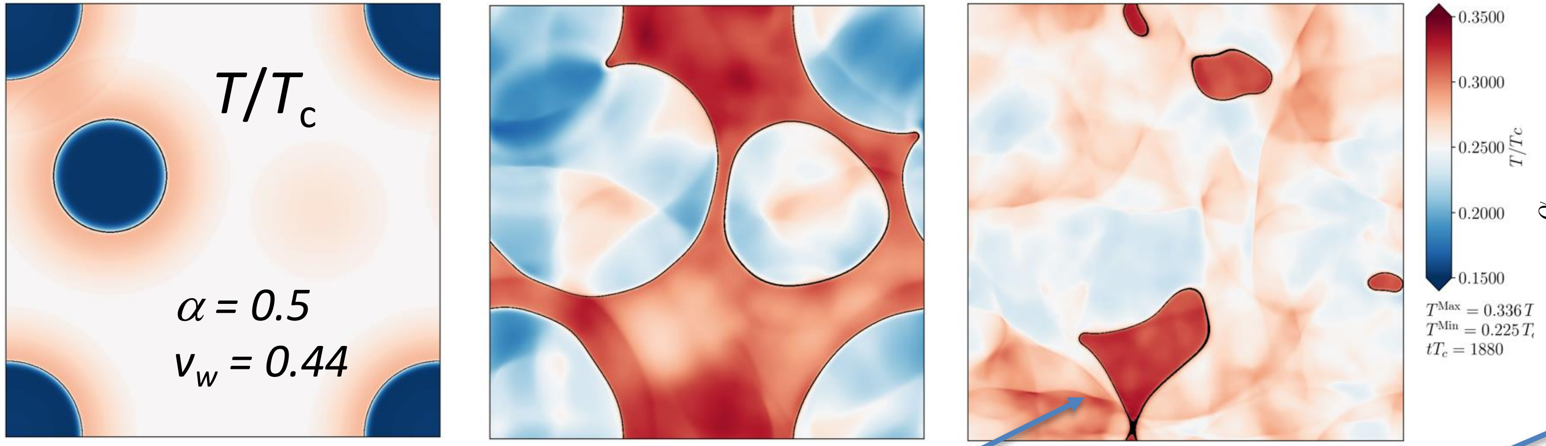


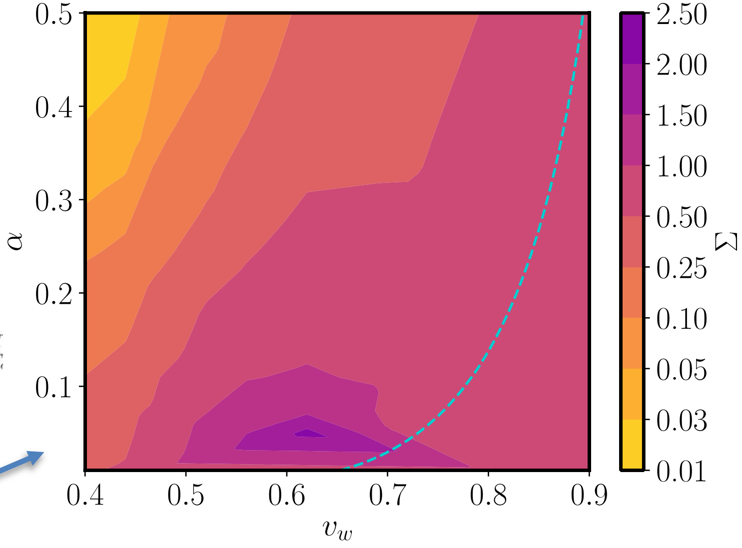




Nonlinearities 1: Kinetic energy & GW suppression

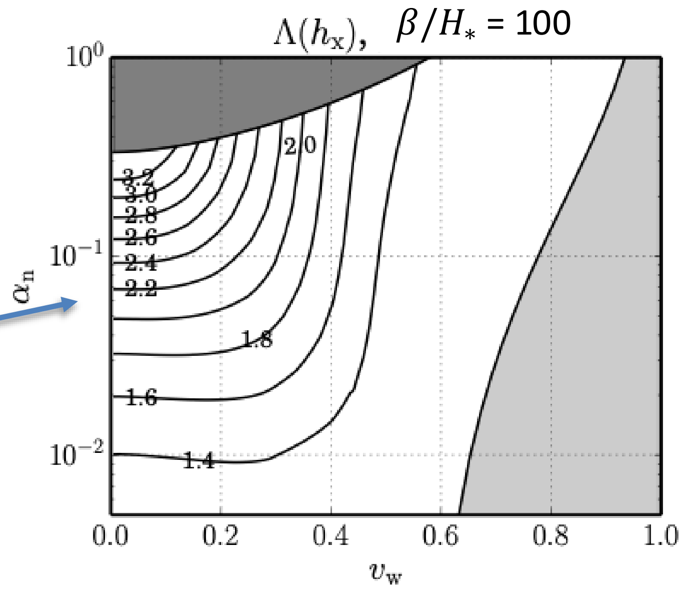
Cutting, MH, Weir, 2020





Gowling, MH (2021)

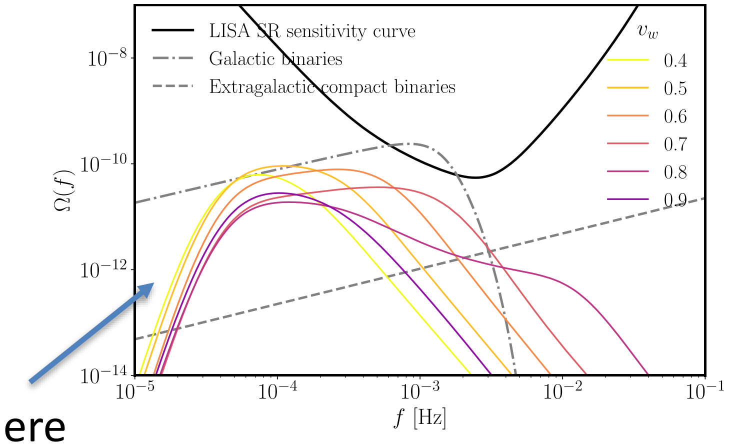
- **Deflagrations**: heat up fluid in front Cutting, MH, Weir, 2020
- Pressure in front of wall increases, walls slow down
- Formation of hot droplets Cutting, Vilhonen, Weir (2022)
- Less transfer into kinetic energy, more into heat.
- Include **GW "suppression factor"** as a numerical parameter (right) – "**pheno-SSM**" Gowling, MH (2021)
- Nucleation suppression in heated regions: Al-Ajmi, MH (2023)
  - Changes bubble spacing – transition rate formula:
    $\Lambda \equiv R_* v_w/(8\pi)^{1/3}\beta > 1$
  - bigger bubbles, boosts signal



Al-Ajmi, MH (2023)

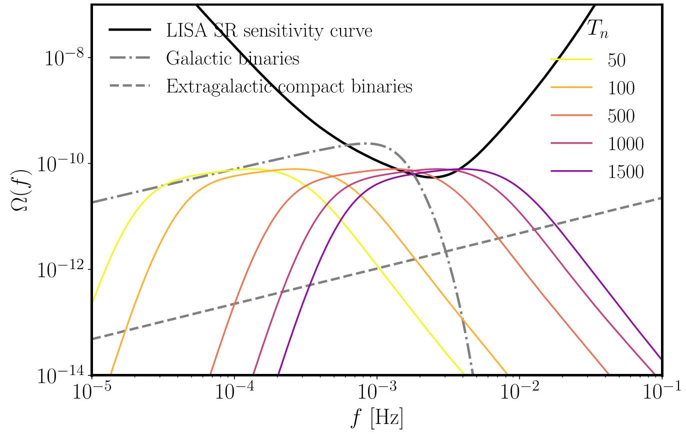
GW power spectra in the SSM

- Pheno-SSM predictions, acceptable accuracy for Gowling, MH (2021)
  - near-linear flows ($\alpha \lesssim 0.1$); sub-Hubble bubble separations ($r_* \ll 1$)
- Encoded in public domain python package **PTtools** MH, Mäki et al (2025)
  - https://github.com/CFT-HY/pttools/

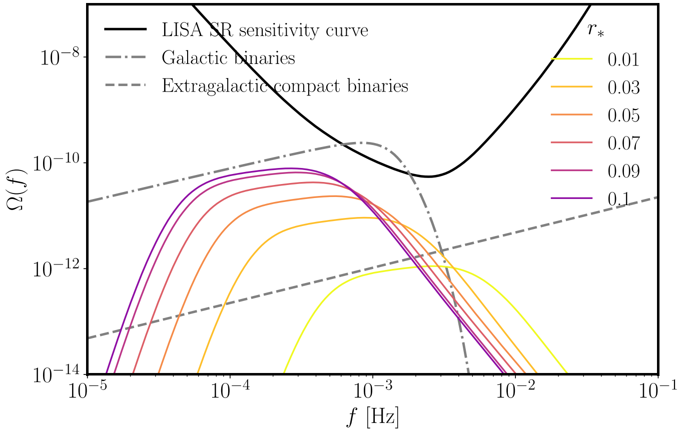


$\sim r_*^2 k^1$ here

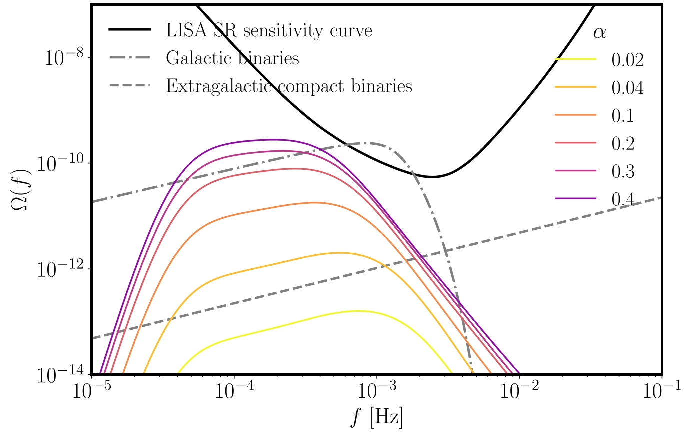
Vary wall speed



Vary transition temp

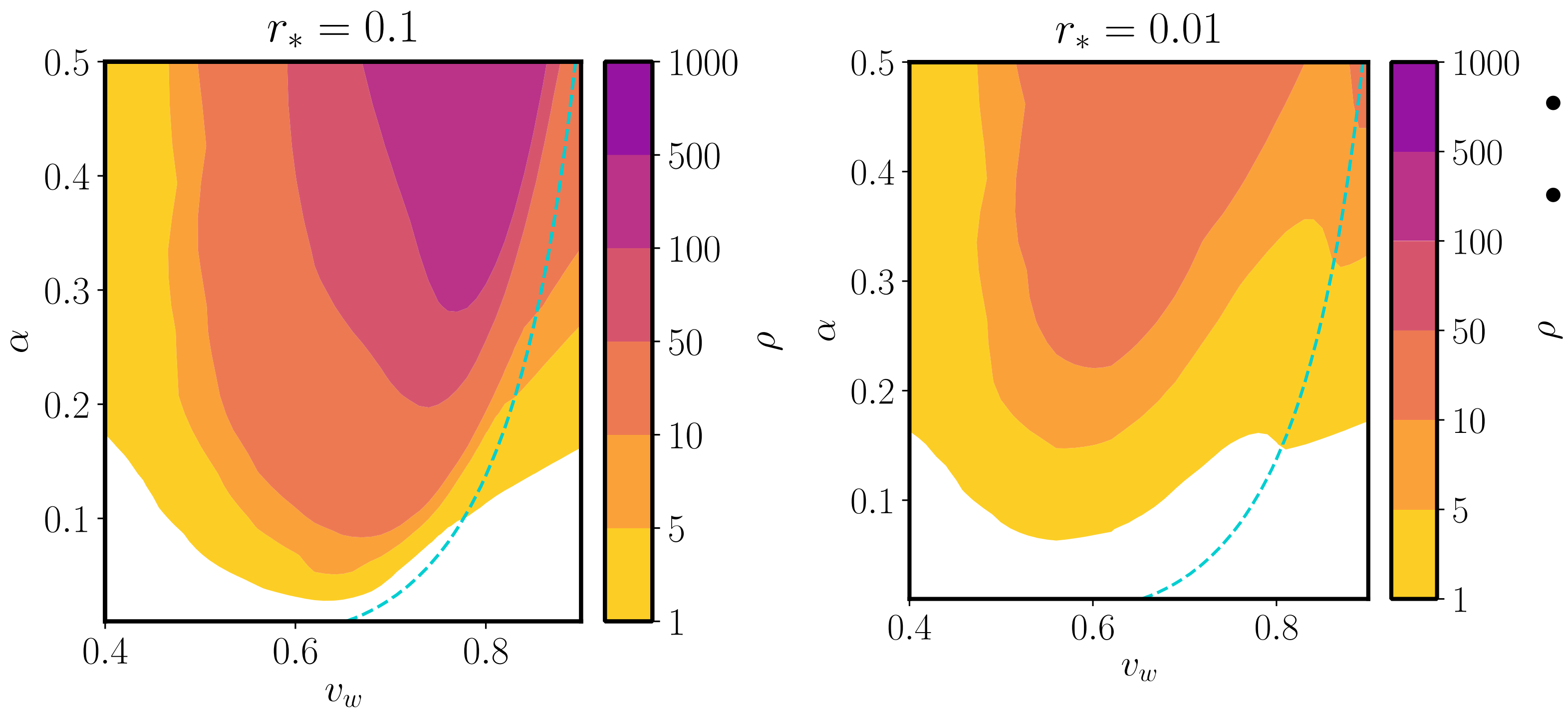


Vary bubble separation



Vary transition strength

Signal-to-noise ratios (LISA)



- T_n = 100 GeV
- $c_s = 1/\sqrt{3}$

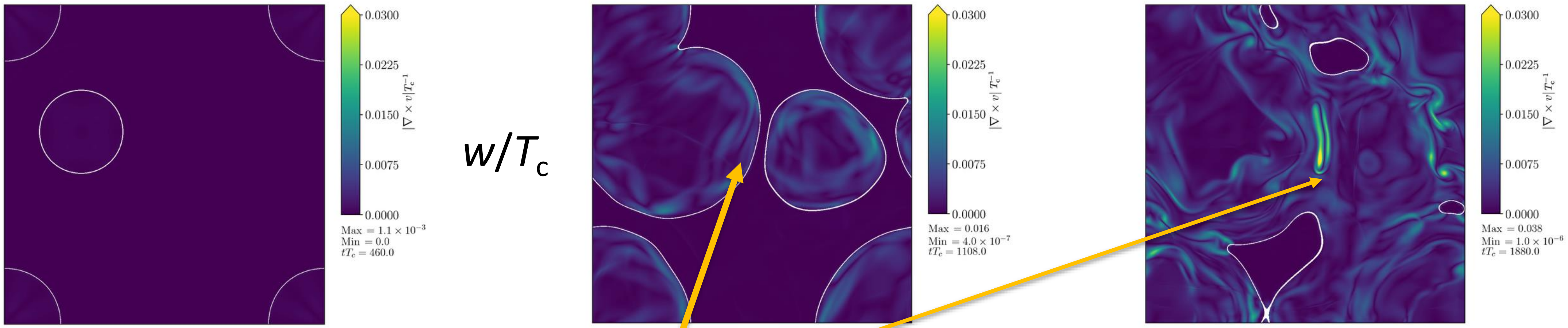
Gowling, MH (2021)

- Signal-to-noise ratio ρ (t_{obs} = 4 years)
- "Worst case" galactic binary foreground
  - (NB annual variation aids removal)
- "LISA science requirements" instrument noise

MH, Hooper, Minkkinen, Weir (2024)

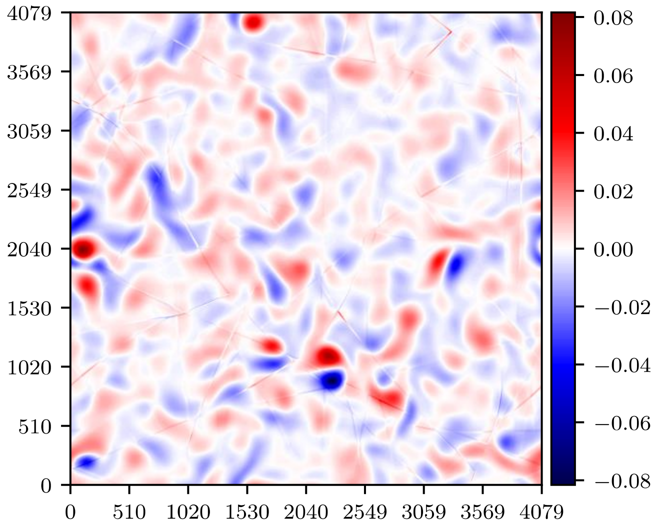
$$\rho^2(\vec{\theta}) = t_{obs} \int df \left(\frac{\Omega_{gw}(f;\vec{\theta})}{\Omega_{noise}(f)} \right)^2$$

Nonlinearities 2: Vorticity and turbulence



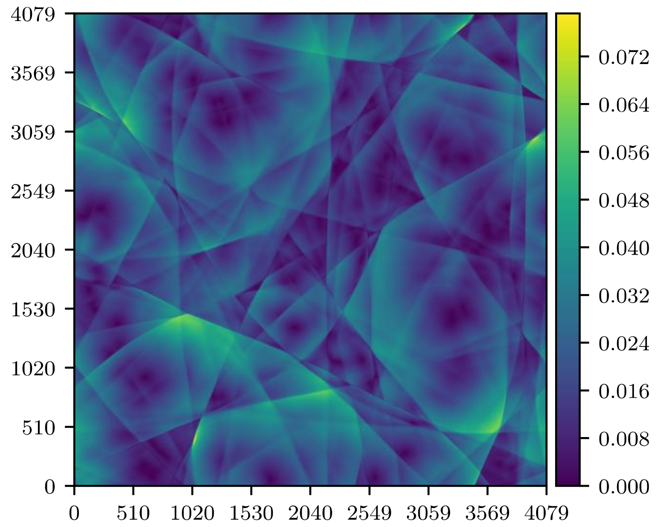
Cutting, MH, Weir, 2020

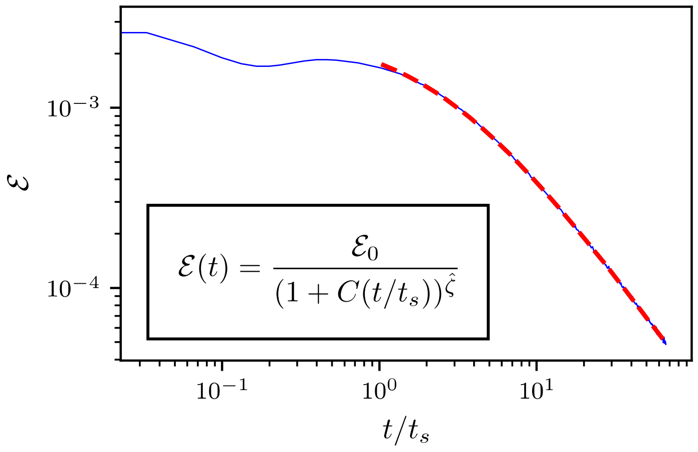
- **Deflagrations**
- Interaction between bubbles/shells generates vorticity
- Vorticity significant for slow walls in strong transitions
- Generation by later shock collision? Pen, Turok 2016
  - Small effect
- Longer simulations needed Auclair et al 2022

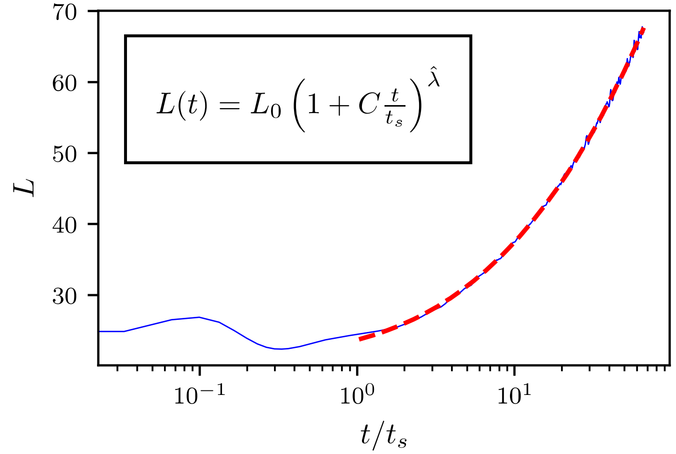


Dahl et al (2021)

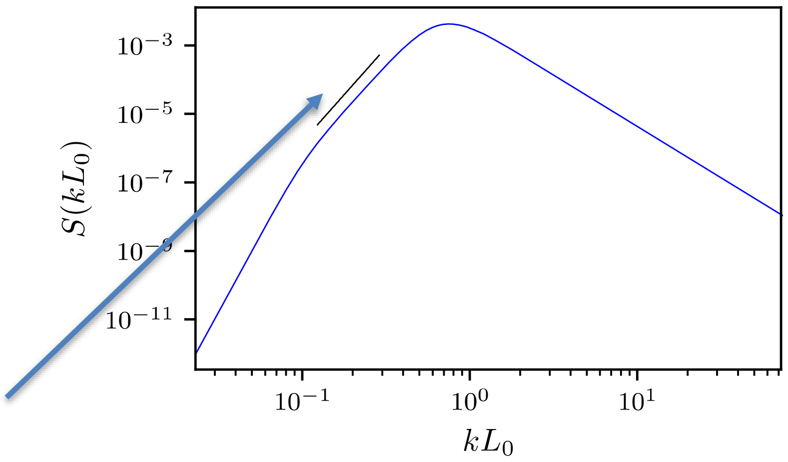
Nonlinearities 3: Shocks and kinetic energy decay







- Shocks develop from any sound wave
- Power spectrum: k^{-3} at high k (any dimension)
- KE decay, length scale growth:
  - Characteristic power laws: $\zeta = 10/7, \lambda = 2/7$
- GW spectrum: intermediate slope change



Dahl, MH, Rukkukainnen, Weir (2021, 2024)

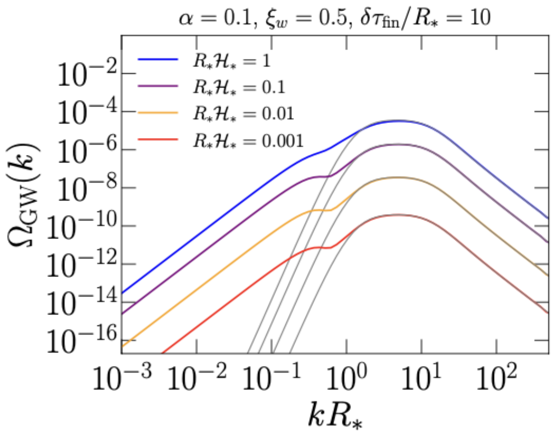
Recent work: hydrodynamics

- Shape of GW spectrum at low *k*

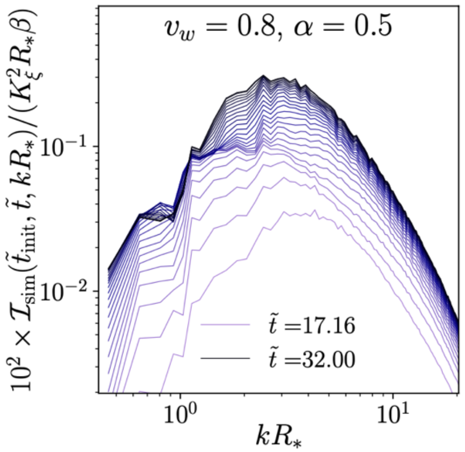
  Sharma, Dahl, Brandenburg, MH 2023; Roper-Pol, Procacci, Caprini 2023
- Effect of sound speed on GW spectrum

  Racco, Poletti 2022, Wang, Huang, Li 2022, Giombi, Dahl, MH 2024
- Effect of self-gravity

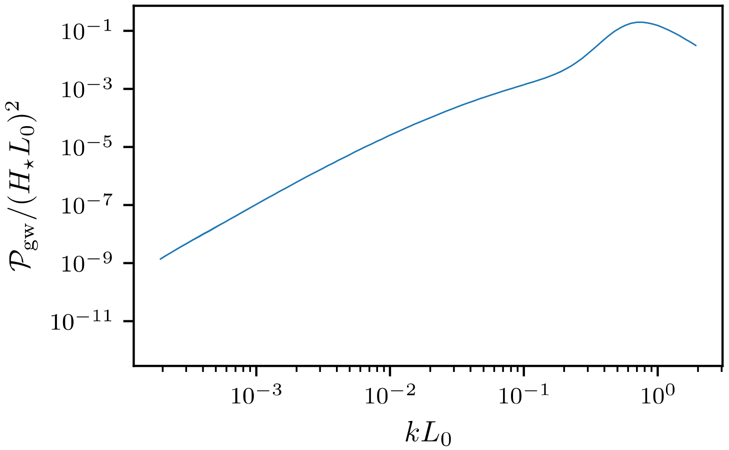
  Giombi, MH 2023; Jinno, Kume 2024; Giombi, Dahl, MH 2025;
- Simulations of GWs from strong transitions
  - v_{rms} ~ 0.1 – 0.3, decaying flow (shock dissipation)
  - convergence of GW spectrum, peak shape change



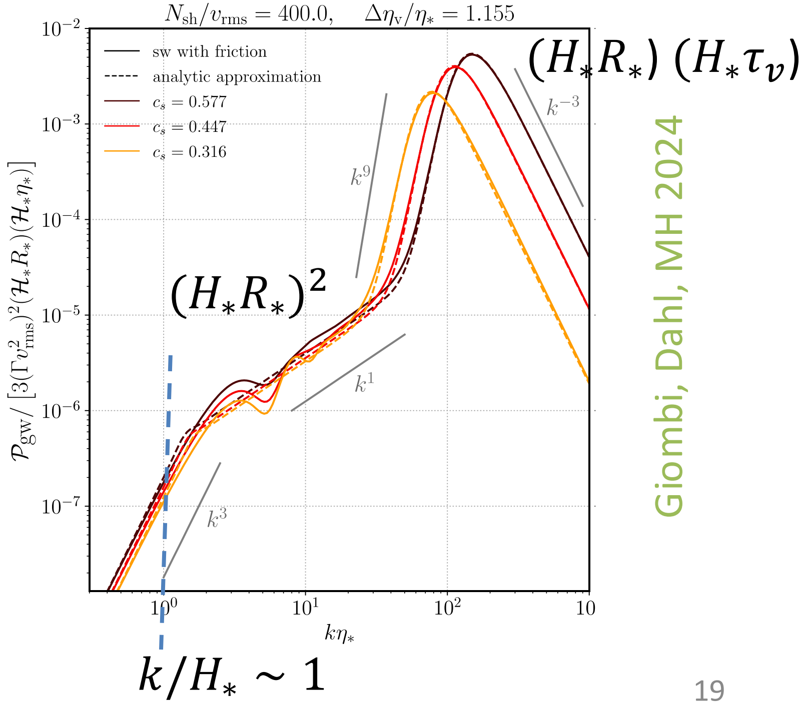
Roper Pol, Procacci, Caprini 2023



Caprini et al 2024



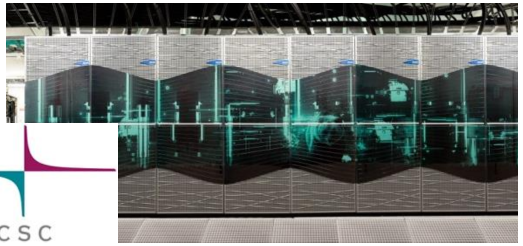
Dahl et al 2024



Giombi, Dahl, MH 2024

3D hydro simulations of strong phase transitions

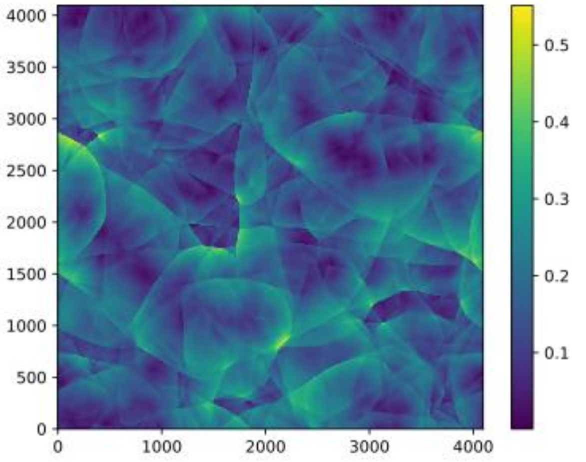
Correia, MH, Rummukainen, Weir (arXiv:2505.17824)



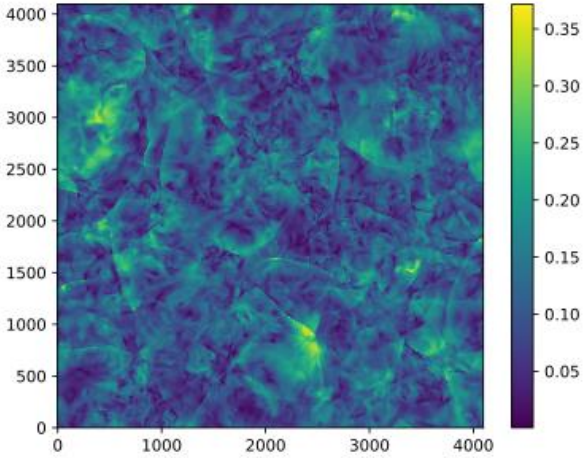
CSC

- Code: SCOTTS, 4096^3 lattice
  - Deflagration: $\alpha = 0.5, v_w = 0.44, \bar{U}_{max} \approx 0.21$
  - Detonation: $\alpha = 0.67, v_w = 0.92, \bar{U}_{max} \approx 0.35$
- Converging GW spectra
- Vorticity 30% in deflagration
  - but GWs nearly all acoustic
- Pheno-SSM
  - good peak amplitude, frequency
  - gets shape wrong for detonation
- Implies:
  - important fluid interactions during collision
  - Non-gaussianity in fluid

Fluid RMS velocity



Detonation



Deflagration

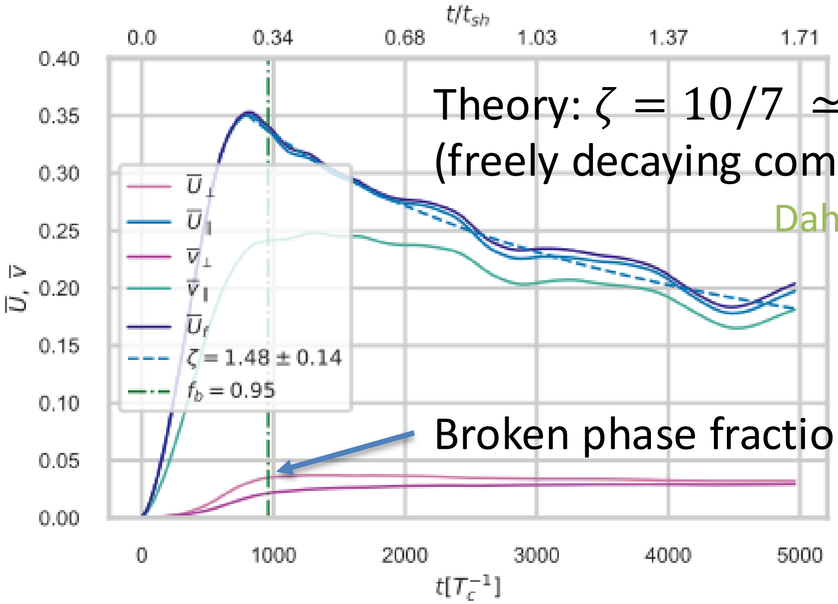
Strong phase transitions: kinetic energy

- Detonation:

$$\alpha = 0.67, v_w = 0.92, \bar{U}_{max} \approx 0.35$$

Enthalpy-weighted spatial part of 4-velocity $U^i = \sqrt{\frac{w}{\bar{w}}}\gamma v^i$

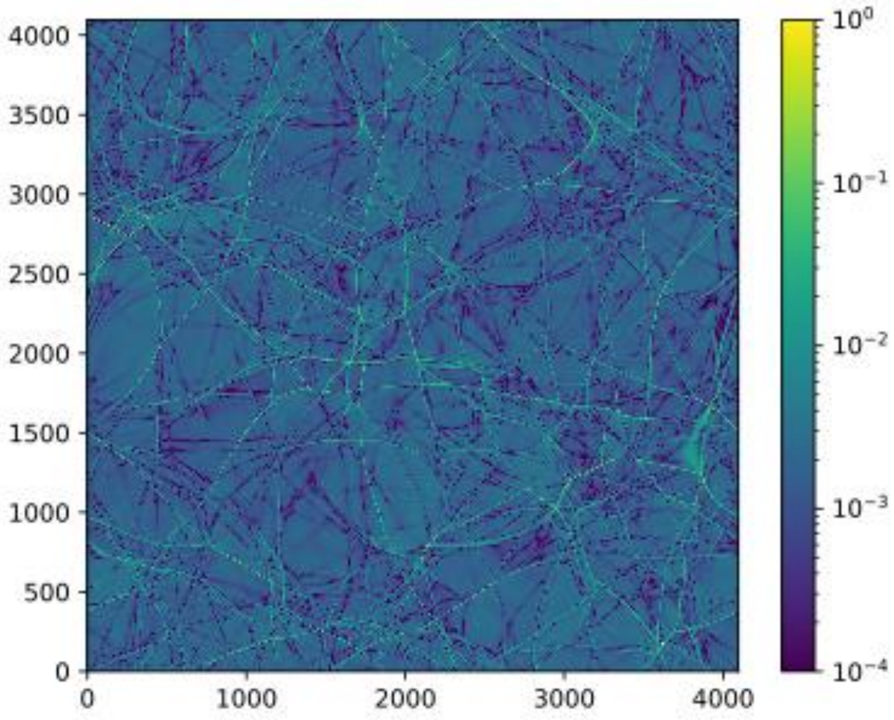
- Vortical kinetic energy < 5%
- Kinetic energy decay: $U_{\parallel} = \max(U_{\parallel}) \left[1 + \frac{\Delta t}{t_*}\right]^{-\zeta/2}$



Theory: $\zeta = 10/7 \simeq 1.43$
(freely decaying compressional modes)
Dahl et al (2021, 2024)

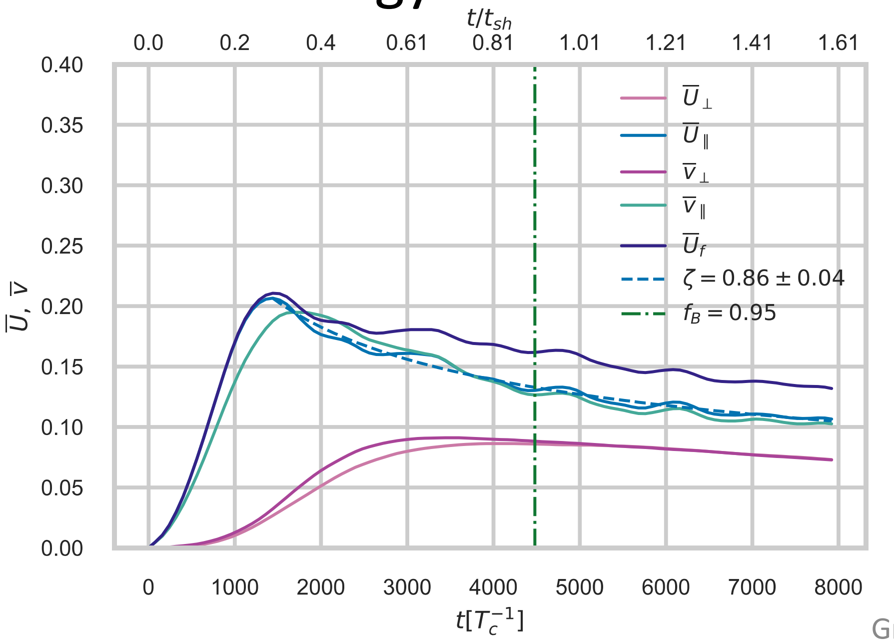
Broken phase fraction 95%

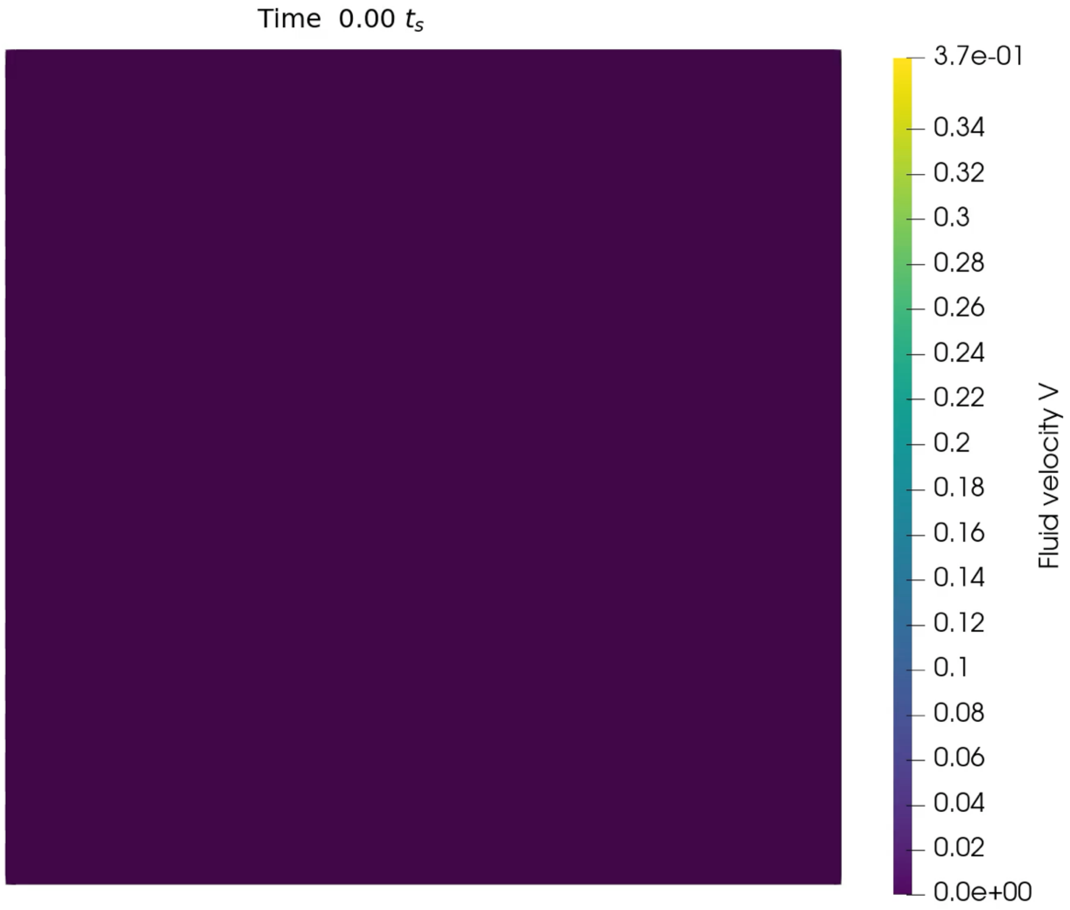
Velocity divergence $t = 5000/T_c$



Strong phase transitions: kinetic energy

- Deflagration:
  $$\alpha = 0.5, v_w = 0.44, \bar{U}_{max} \approx 0.21$$
- Vortical kinetic energy 30%
  – but GWs nearly all acoustic
- Kinetic energy sourced after maximum

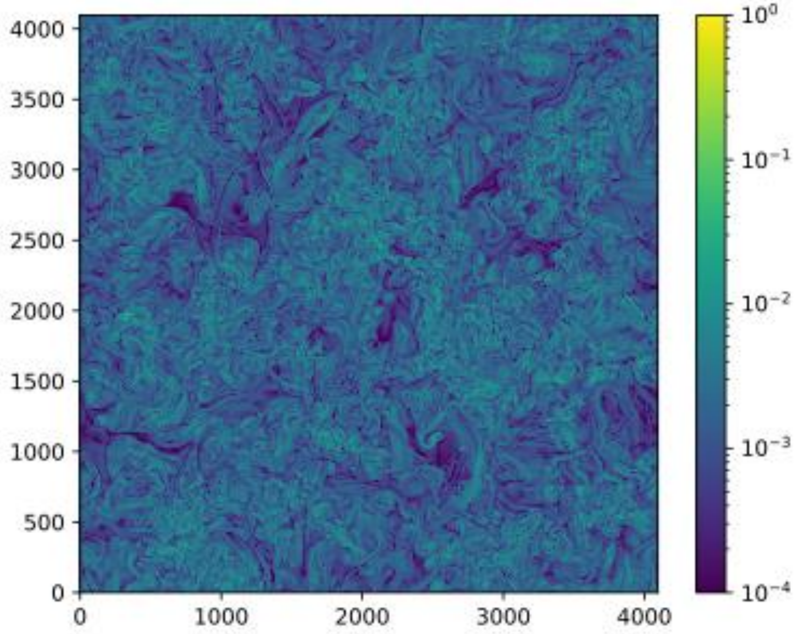


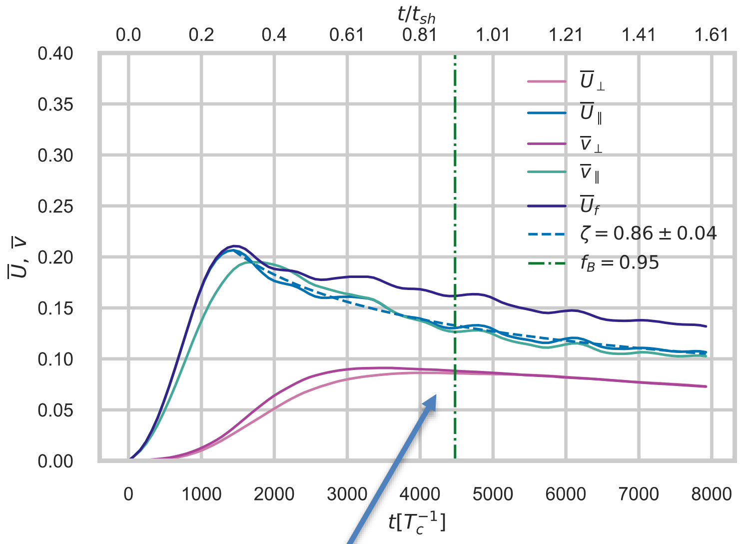


Strong phase transitions: kinetic energy

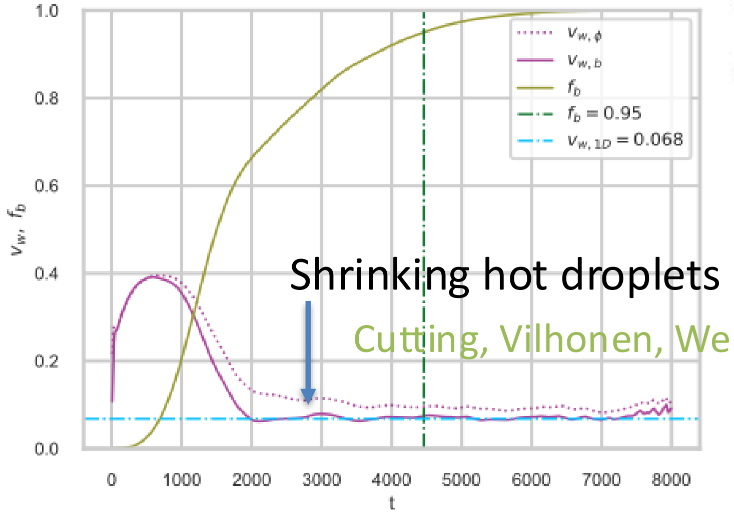
- Deflagration:
  $$\alpha = 0.5, v_w = 0.44, \bar{U}_{max} \approx 0.21$$
- Vortical kinetic energy 30%
- Re-heated symmetric phase, slowing walls

Velocity curl $t = 7700/T_c$





Broken phase fraction 95%



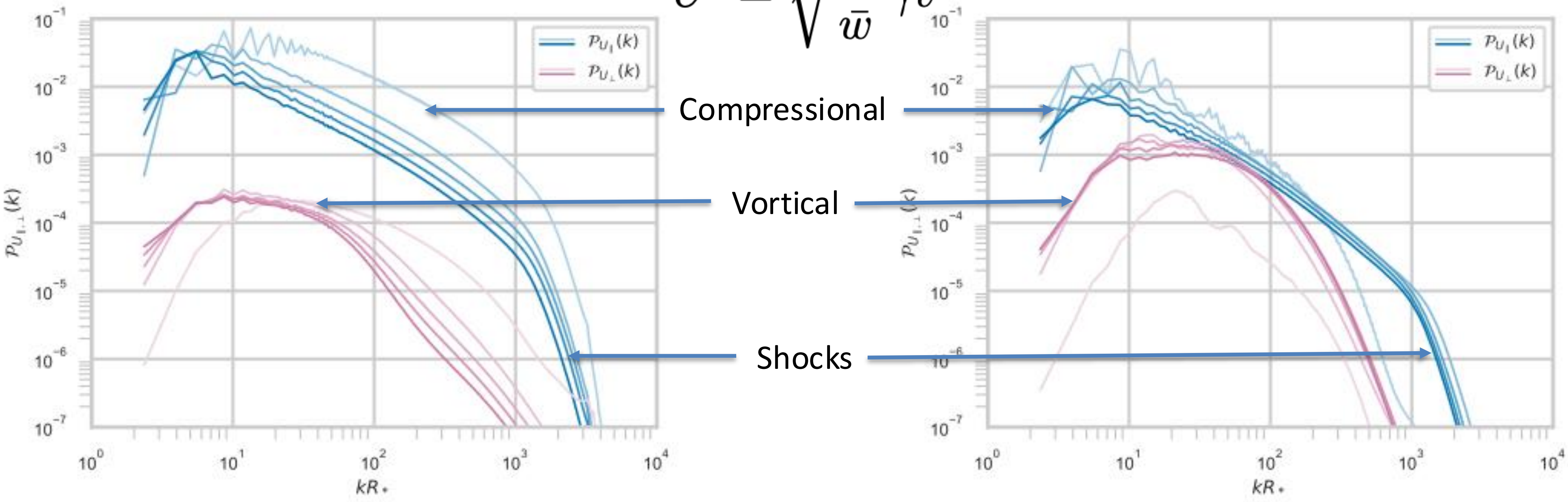
Shrinking hot droplets

Cutting, Vilhonen, Weir (2022)

Weighted 4-velocity power spectra

- Detonation $\alpha = 0.67, v_w = 0.92$
  - $tT_c = 880, 1720, 2560, 3400, 4240$
- Deflagration $\alpha = 0.5, v_w = 0.44$
  - $tT_c = 1440, 2800, 4160, 5520, 6880$

$$U^i = \sqrt{\frac{w}{\bar{w}}}\,\gamma v^i$$

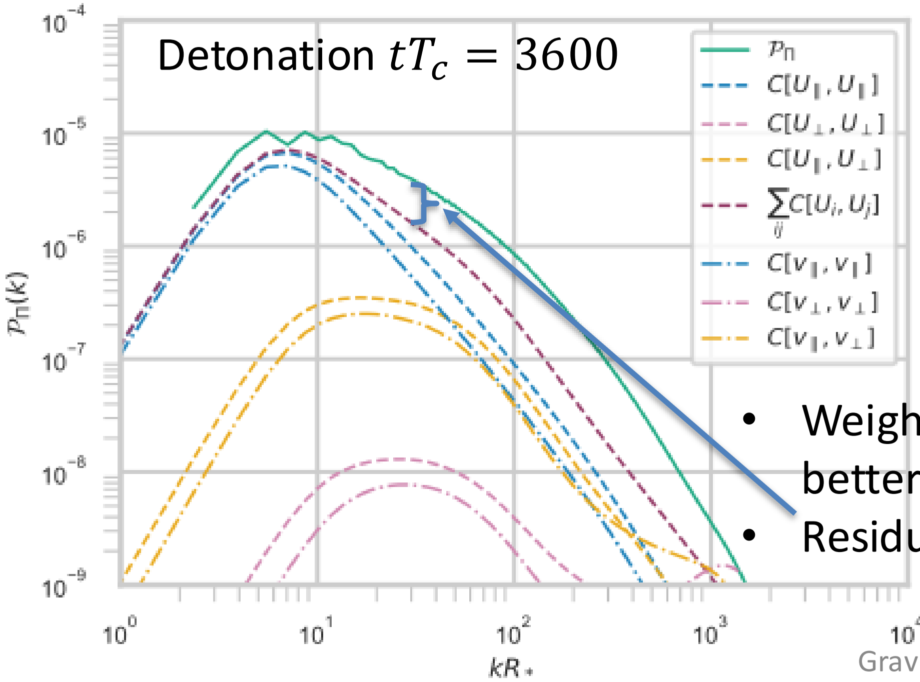


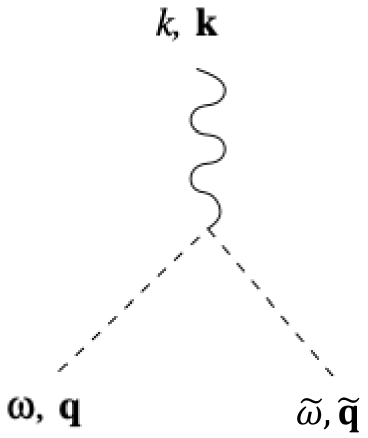
Shear stress power spectra: Gaussian velocity field?

- Shear stress spectral density (if Gaussian velocity field) $P_\Pi(k,t) = R^3(t)\bar{w}^2(t)\sum_{A,B} C[U_A, U_B](k,t) \qquad (A, B \in \{\parallel, \perp\})$

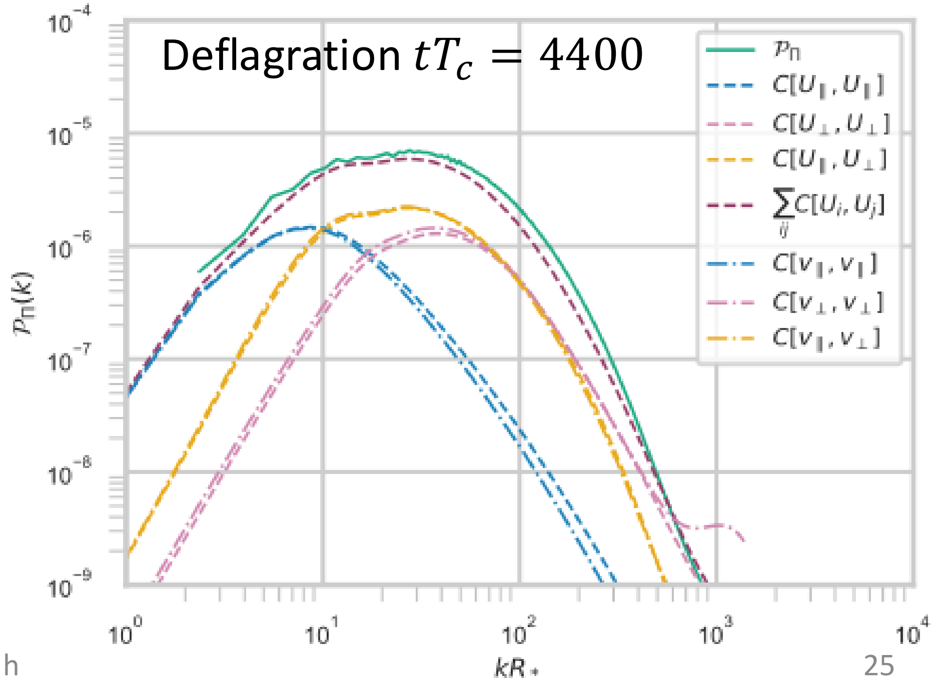
$$C[U_A, U_B](k,t) = R^{-3}\int \frac{d^3q}{(2\pi)^3}\Gamma_{AB}(\mu,\tilde{\mu})P_{U_A}(q,t)P_{U_B}(\tilde{q},t),$$

  - Projection algebra factors $\Gamma_{\parallel\parallel} = (1-\mu^2)(1-\tilde{\mu}^2)$, $\Gamma_{\perp\perp} = (1+\mu^2)(1+\tilde{\mu}^2)$, $\Gamma_{\parallel\perp} = (1-\mu^2)(1+\tilde{\mu}^2)$
  - direction cosines $\mu = \mathbf{q}\cdot\mathbf{k}/qk$, $\tilde{\mu} = \tilde{\mathbf{q}}\cdot\mathbf{k}/\tilde{q}k$
  - Fluid flow length scale R





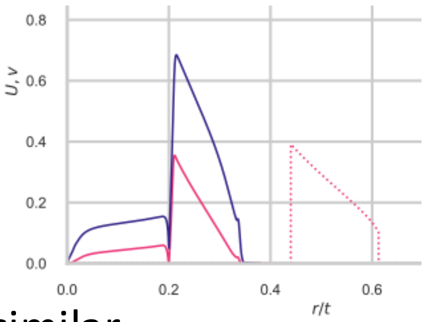
- Weighted 4-velocity ***U*** better than 3-velocity ***v***
- Residual non-Gaussianity

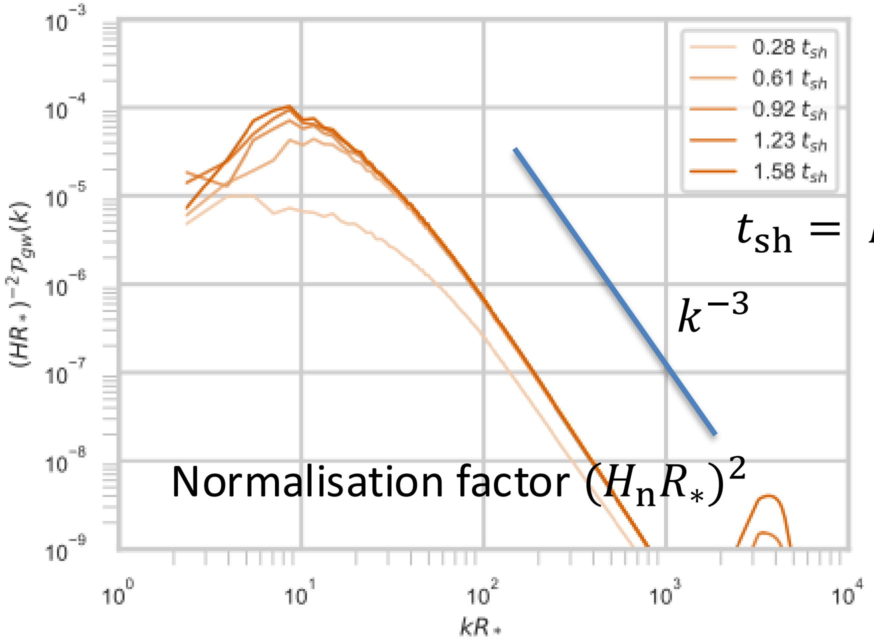


Gravitational wave power spectra

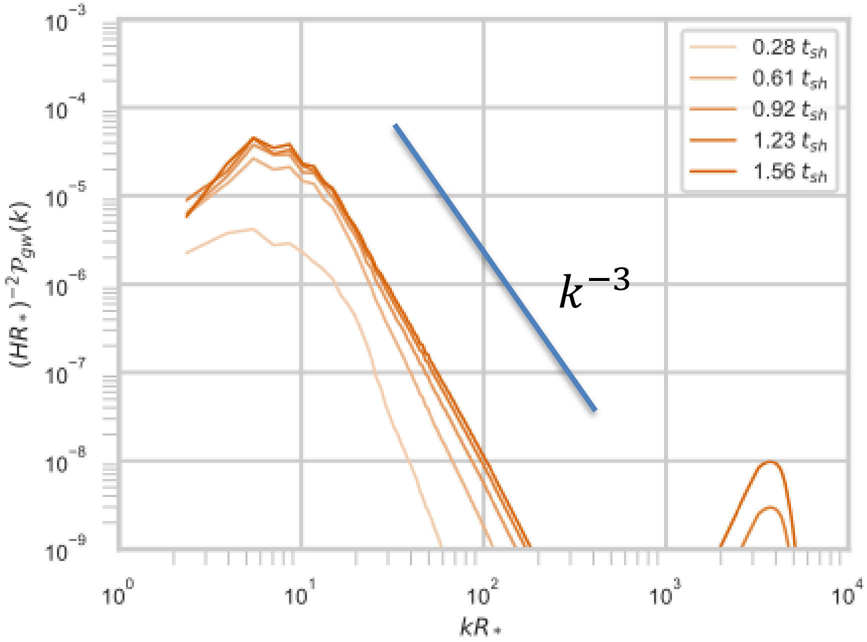
- Detonation $\alpha = 0.67, v_w = 0.92$
  - $tT_c = 880, 1720, 2560, 3400, 4240$
- Convergence at wavenumbers $kR_* \gtrsim 40$
- High wavenumbers k^{-3}, due to shocks

- Deflagration $\alpha = 0.5, v_w = 0.44$
  - $tT_c = 1440, 2800, 4160, 5520, 6880$
- Slower convergence:
  - transition completes at $tT_c = 6880$
- High wavenumbers steeper than k^{-3} colliding fluid profiles not close to self-similar





$t_{\rm sh} = R_* / \max(\bar{U}_\parallel)$



Unequal time correlators & decorrelation functions

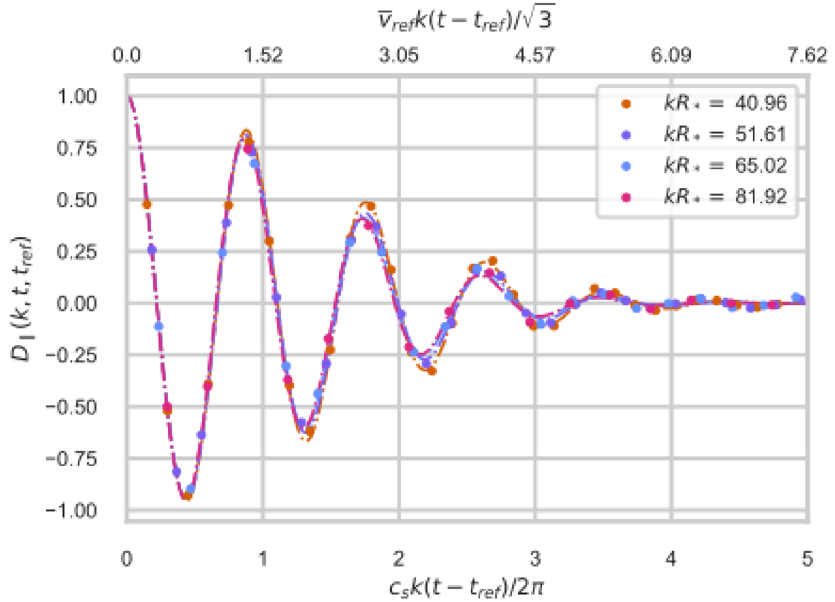
- Unequal time correlator P_Y

$$(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')P_Y(k,t,t_{\rm ref}) = \langle Y(\mathbf{k},t)Y^*(\mathbf{k}',t_{\rm ref})\rangle.$$

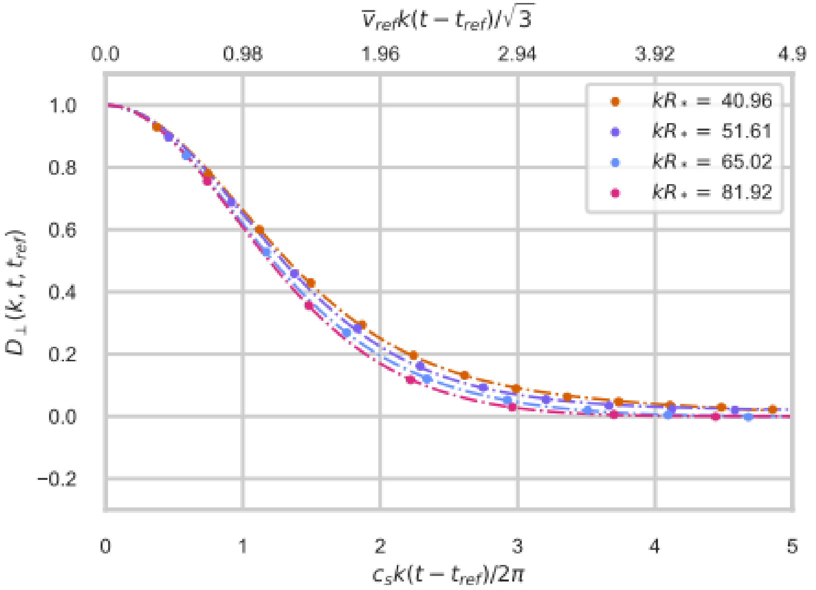
- Decorrelation function

$$D_Y(k,t,t_{\rm ref}) = \frac{P_Y(k,t,t_{\rm ref})}{\sqrt{P_Y(k,t)P_Y(k,t_{\rm ref})}}$$

Detonation, $Y = U_\parallel$, $t_{\rm ref}T_c = 1000$



Deflagration, $Y = U_\perp$, $t_{\rm ref}T_c = 4500$



- Model: $D_\parallel(k,t,t_{\rm ref}) = \cos\left(M_\parallel c_s k(t-t_{\rm ref})\right)\exp\left(-\frac{1}{2}V_\parallel^2k^2(t-t_{\rm ref})^2\right)$
- interacting waves ($M_\parallel \simeq 1.1$, shocks)

- Model: $D_\perp(k,t,t_{\rm ref}) = \exp\left(-\frac{1}{2}V_\perp^2k^2(t-t_{\rm ref})^2\right)$
- small eddies swept by larger ones

Kraichnan (1964)

GW power spectrum growth rate: theory

- GW power spectrum: $$\mathcal{P}_{\rm gw}(k,t) = \frac{(16\pi G)^2}{12H_{\rm n}^2}\int_{t_*}^{t} dt_1 \int_{t_*}^{t} dt_2\, \dot{G}(k,t,t_1)\dot{G}(k,t,t_2)\mathcal{P}_{\Pi}(k,t_1,t_2),$$
  - $H_{\rm n}$ - Hubble rate (assumed constant during GW production)
  - $G(k,t,t_1)$ – metric Green's function
  - $\mathcal{P}_{\Pi}(k,t_1,t_2)$ - shear stress UETC
- Growth of GW PS between $t_{\rm i}$ and t

$$\Delta\mathcal{P}_{\rm gw}(k,t,t_{\rm i}) \simeq 3\int_{t_{\rm i}}^{t} \frac{d\bar{t}}{\bar{R}}(H_{\rm n}\bar{R})^2 \frac{(k\bar{R})^3}{2\pi^2}\left(\frac{\bar{w}}{\bar{\epsilon}_0}\right)^2 \sum_{A,B} C_\Delta[U_A,U_B](k,\bar{t})$$

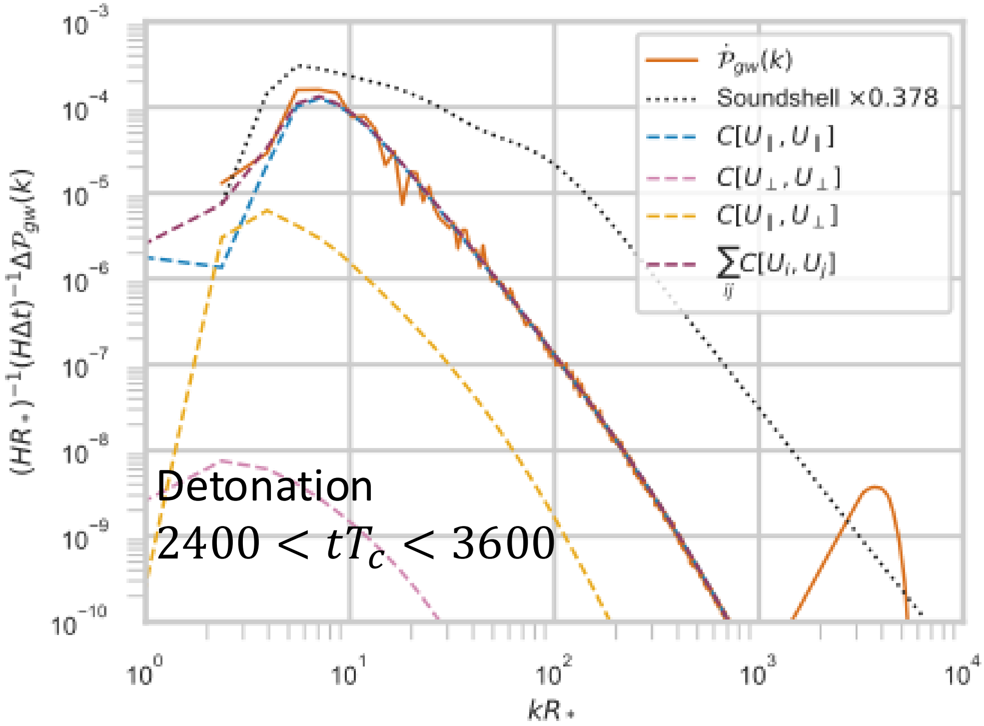
  - $\bar{R} = \sqrt{R(t_1)R(t_2)}$
  - $\bar{t} = \sqrt{t_1 t_2}$, $r = t_1/t_2$
  - $\bar{w} = \sqrt{w(t_1)w(t_2)}$
  - $\bar{\epsilon}_0$ mean energy density

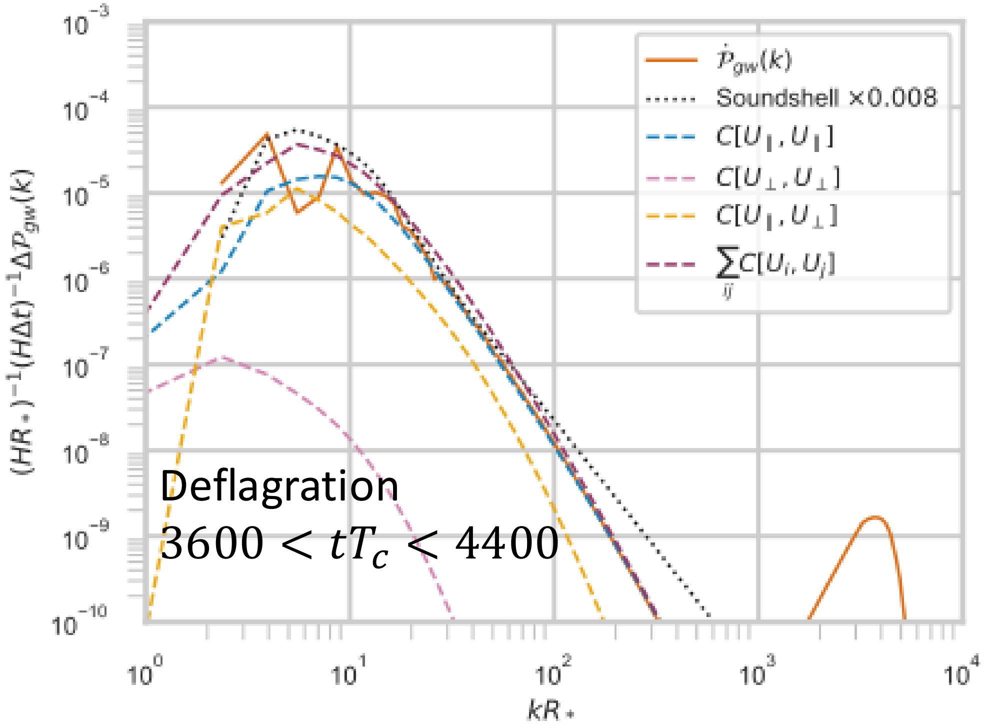
$$C_\Delta[U_A,U_B](k,\bar{t}) = \bar{R}^{-3}\int \frac{d^3q}{(2\pi)^3}\,\Gamma_{AB}(\mu,\tilde{\mu})\Delta_{AB}(q,\tilde{q},k)\bar{P}_{U_A}(q,\bar{t})\bar{P}_{U_B}(\tilde{q},\bar{t}).$$

$$\Delta_{AB}(q,\tilde{q},k) = \frac{\bar{t}}{2\bar{R}}\int_{r_{\rm i}^{-1}}^{r_{\rm i}} \frac{dr}{r}\cos[k(t_1-t_2)]D_A(q,t_1,t_2)D_B(\tilde{q},t_1,t_2),$$

GW power spectra growth rate & pheno-SSM

- Detonation $\alpha = 0.67, v_w = 0.92$
  - $2400 < tT_c < 3600$
- Deflagration $\alpha = 0.5, v_w = 0.44$
  - $3600 < tT_c < 4400$



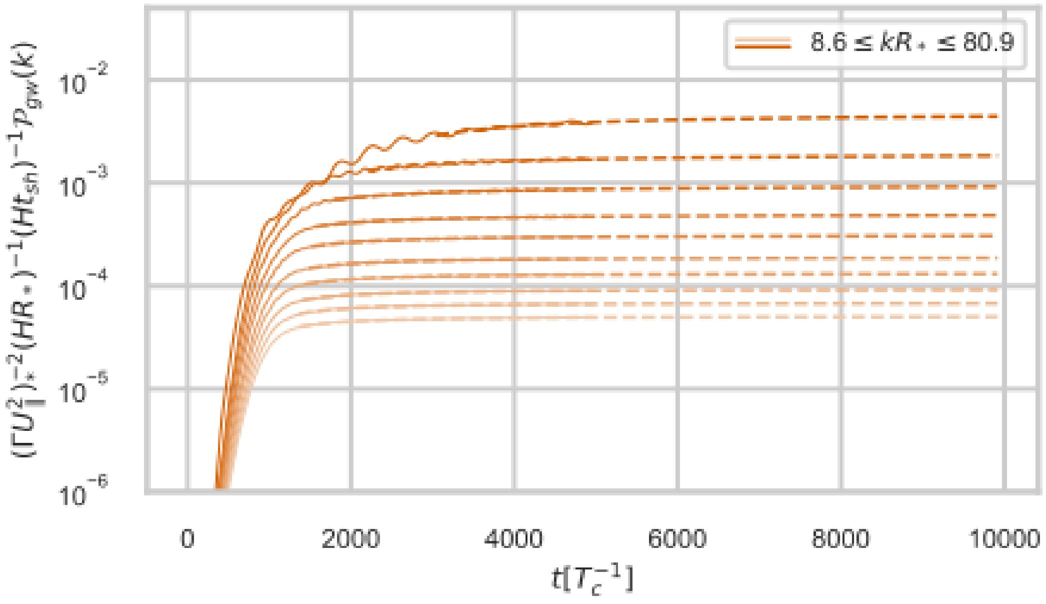


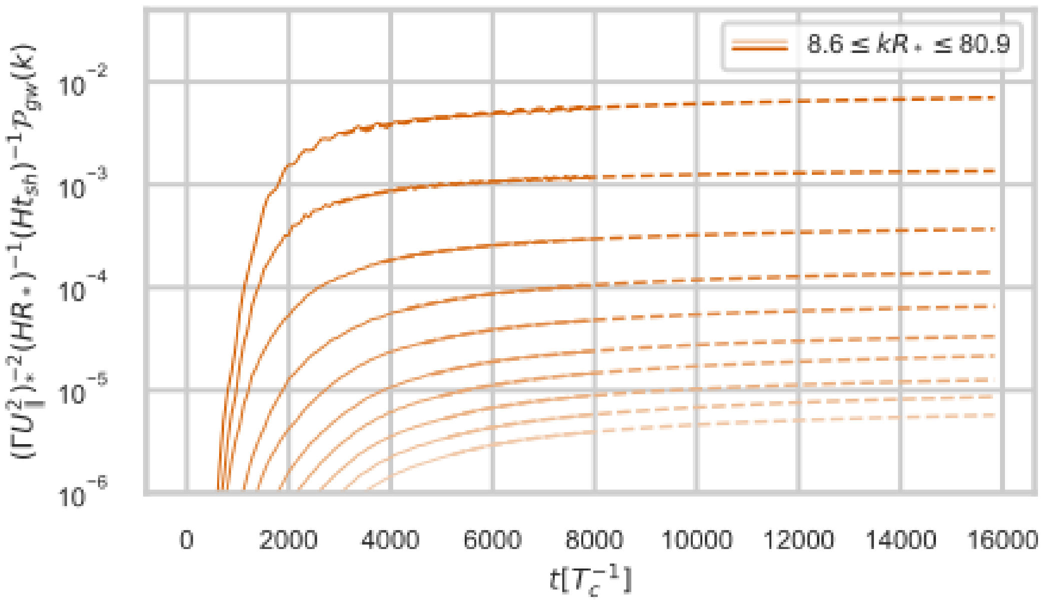
Main contributor to GW power: compressional modes, $\bar{U}_\parallel$

GW power extrapolations by wavenumber

$$\mathcal{P}_{\rm gw}(k,t) = \mathcal{P}_{\rm gw}^{\infty}(k)\left[1-\left(\frac{t}{t_{\rm d}(k)}\right)^{-d(k)}\right],$$

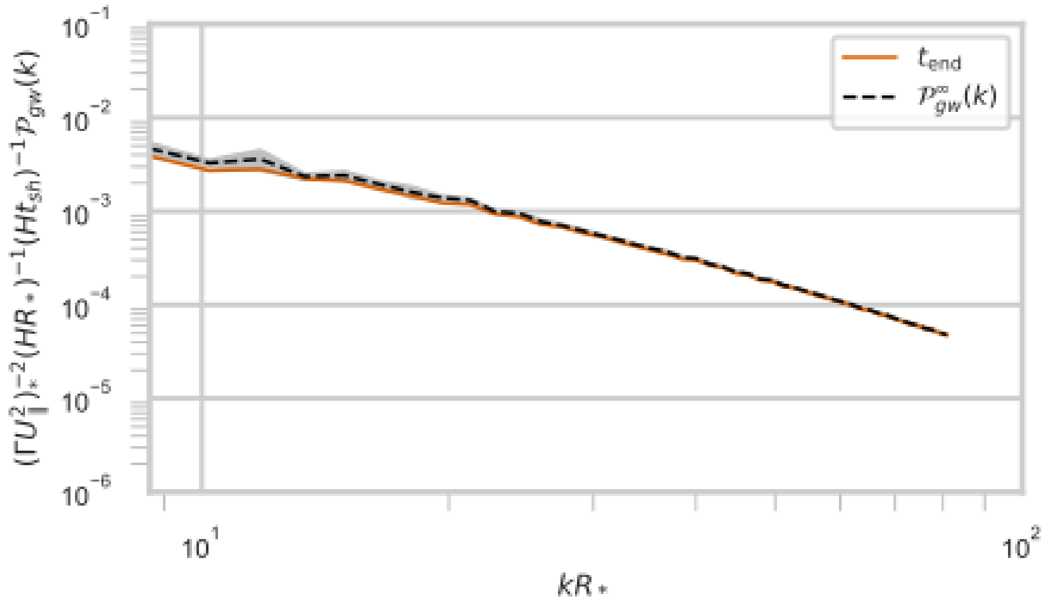
- Detonation $\alpha = 0.67, v_w = 0.92$
- Deflagration $\alpha = 0.5, v_w = 0.44$



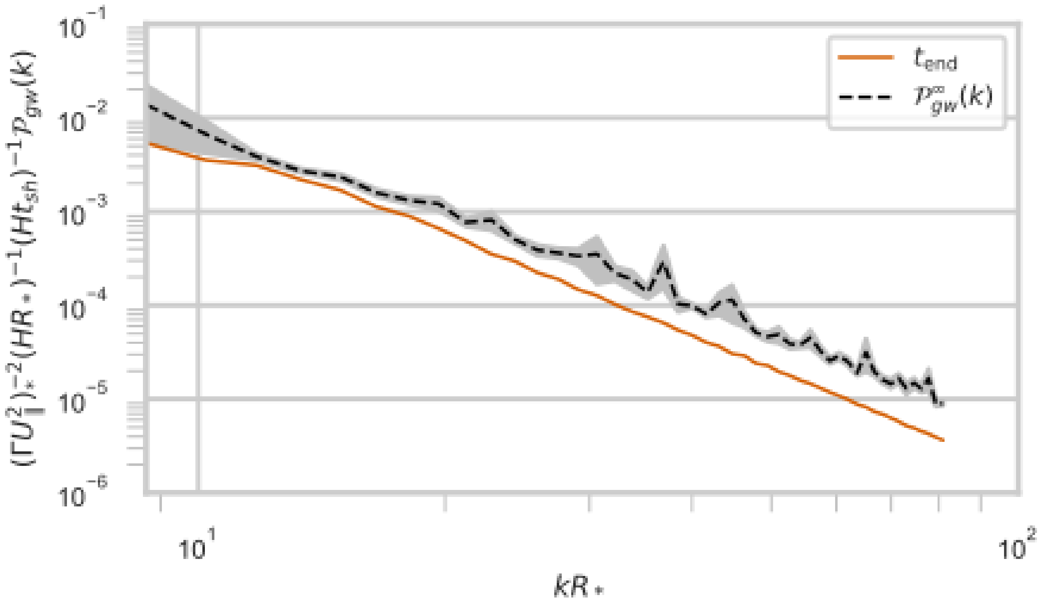


GW power spectra extrapolations

- Detonation $\alpha = 0.67, v_w = 0.92$



- Deflagration $\alpha = 0.5, v_w = 0.44$

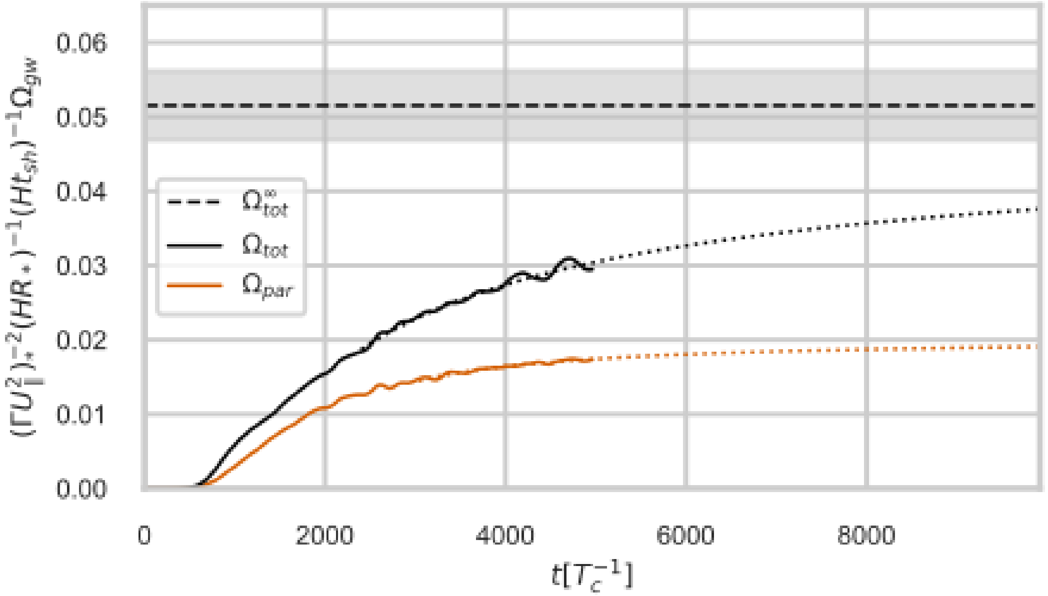


Total GW power extrapolations

$$\Omega_{\rm gw}^{\infty} = (H_{\rm n} t_{\rm sh})(H_{\rm n} R_*)(\Gamma \bar{U}_{\parallel}^2)_*^2 3\tilde{\Omega}_{\rm gw}^{\infty}.$$

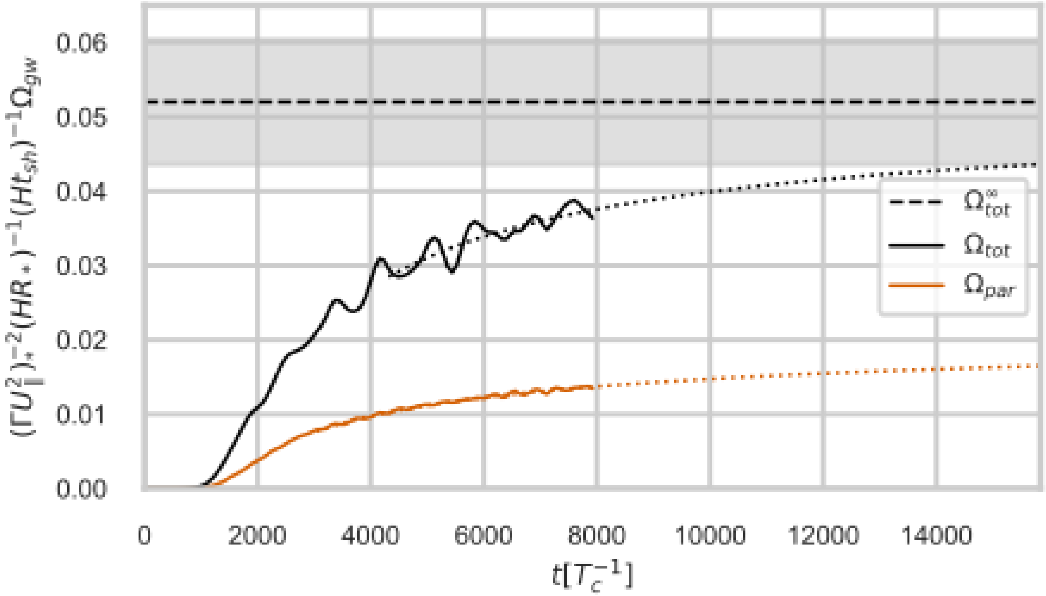
- Detonation $\alpha = 0.67, v_w = 0.92$

$$\tilde{\Omega}_{\rm gw}^{\infty} = 0.017 \pm 0.004$$
$$\tilde{\Omega}_{\rm gw,0}/(H_{\rm n} R_*)^2 = (4.8 \pm 1.1) \times 10^{-8}$$



- Deflagration $\alpha = 0.5, v_w = 0.44$

$$\tilde{\Omega}_{\rm gw}^{\infty} = 0.017 \pm 0.003$$
$$\tilde{\Omega}_{\rm gw,0}/(H_{\rm n} R_*)^2 = (1.3 \pm 0.2) \times 10^{-8}$$



A skeleton in the theory cupboard?

- Is nucleation theory correct?
  - ^{3}He A/B transition rate puzzle

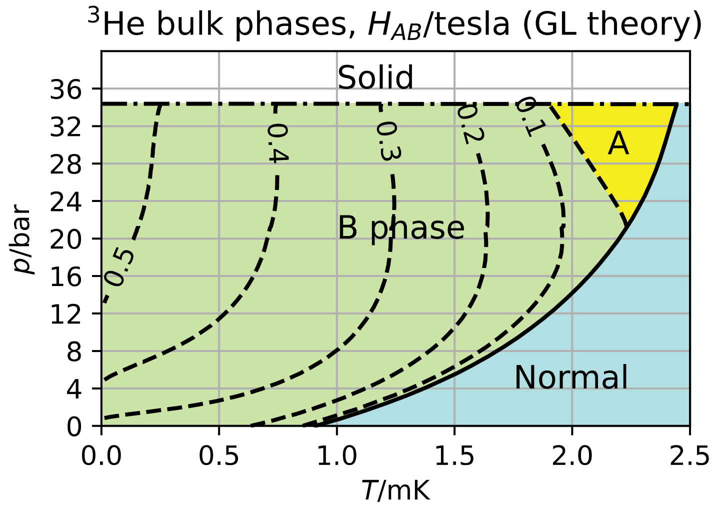
Kaul Kleinert 1980, Bailin, Love 1980, Leggett 1984, Tye Wohns 2011

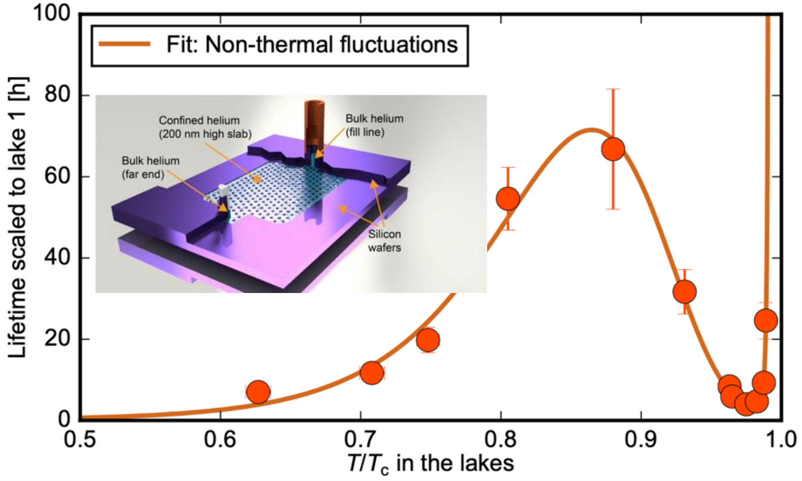
- Classical theory of nucleation rate:
  - Rate density ~ exp(- E_c/T)

    E_c – energy barrier

Cahn-Hilliard 1959, Langer 1969
Coleman 1974, Linde 1983

- ^{3}He A/B theory prediction: E_c/T ~ 10^6
- Lab: metastable ^{3}He A lasts hours/days.
- QUEST-DMC collaboration (UK):
  - resolve the nucleation puzzle
  - study phase boundary propagation



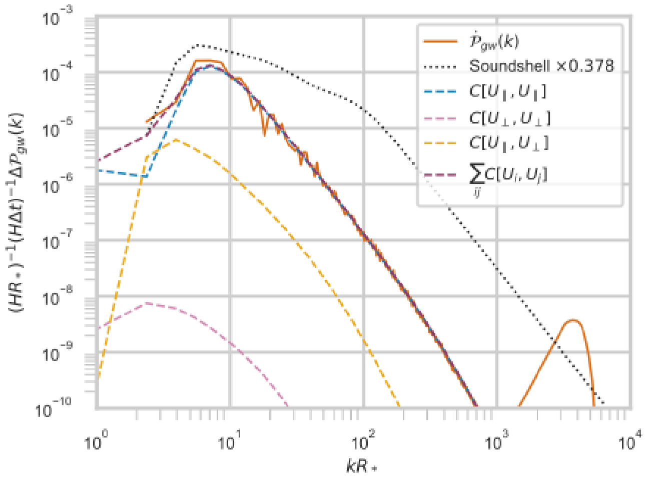


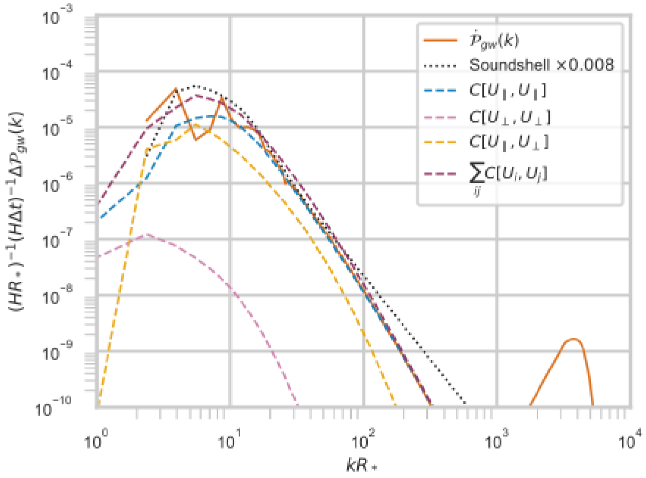
Fit to model of B-phase percolation

Conclusions

- Towards accurate calculations of GW power spectrum from parameters:
  - Sound shell model for GWPS (PTtools/PTPlot) Mäki talk Fri
  - Non-linear evolution becoming clearer Rubira, Stomberg talks Thur
    - Vorticity generated by bubble collisions
    - Compressional kinetic energy decays by shocks
  - Compressional modes make most of the GWs
    - Mixed compressional-vortical contribution Salome talk Thur
  - Pheno-SSM:
    - amplitudes, peak frequencies OK
    - Shape wrong for detonation: non-linearity in collisions
  - GW efficiency parameter $\tilde{\Omega}_{\mathrm{gw}}^{\infty} = 0.017$
    - Consistent order of magnitude with Higgsless simulations

Caprini et al 2024





Can we make GWs as good as the CMB?

- More simulation work needed:
  - Convergence at peak wavenumbers (longer runs)
  - More $(v_w, \alpha_{\rm n})$ - is $\widetilde{\Omega}_{\rm gw}^{\infty} = 0.017$ for both runs a fluke?
  - More realistic equations of state
    - e.g. constant c_s model   Giese et al 2021
  - Temperature-dependent nucleation & friction (deflagrations)
  - Strongly supercooled transitions   Lewicki, Vaskonen 2022
  - Hubble-sized bubbles: gravitational effects
- Models to incorporate simulation developments
  - Reheating effects in deflagrations
  - Kinetic energy decay
  - Large bubble regime