

The Higgsless simulations (and PTs to address the H_0 tension)

based on 2010.00971, 2108.11947, 2209.04369, 2302.06952, 2409.0365, 2404.07256, 2508.TODAY

Henrique Rubira
(LMU/Cambridge)



The Higgsless team: Ryusuke Jinno, Thomas Konstandin, Isaac Stromberg

Nordita, Aug 25

+ Chiara Caprini, Alberto Roper Pol, Jorinde v.d. Vis, Simone Blasi, Mathias Garny,
Martin Sloth, Florian Niedermann

Before anything else, cool things always come with videos



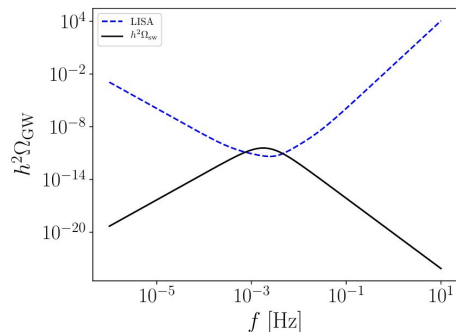
A message to take home...

New simulation scheme for GWs from PTs

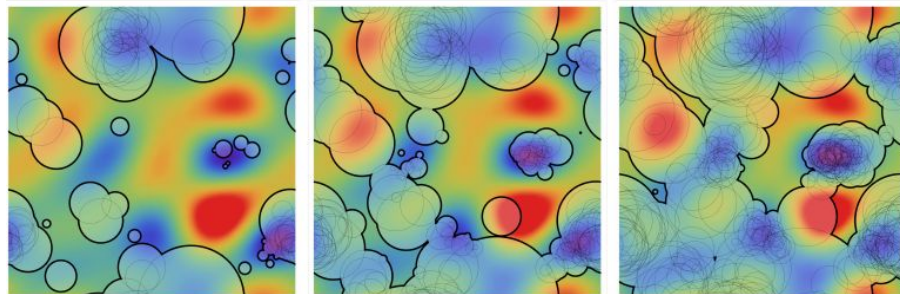
Advantages: fast, shocks, non-linear, Higgsless, different frequencies...

A user-friendly parametrization for ongoing and future data analysis...

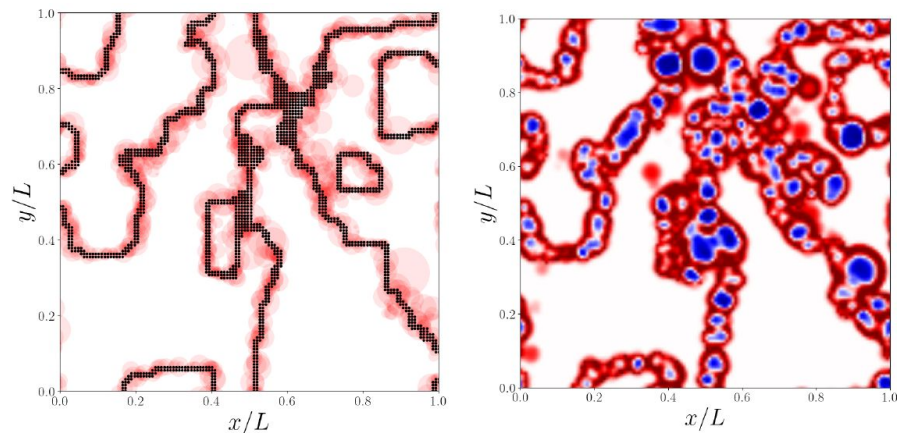
$$\Omega_{\text{GW}} \propto \frac{(q/q_0)^3}{1 + (q/q_0)^2[1 + (q/q_1)^4]}$$



Temperature fluctuations sourcing PT



Domain walls sourcing PT



Outline of this talk

Part I: Higgsless

- **Simulations**
- Results
- Temperature fluctuations and Topological defects

Part II: Phase transitions and H0 tension

Sounds waves are important

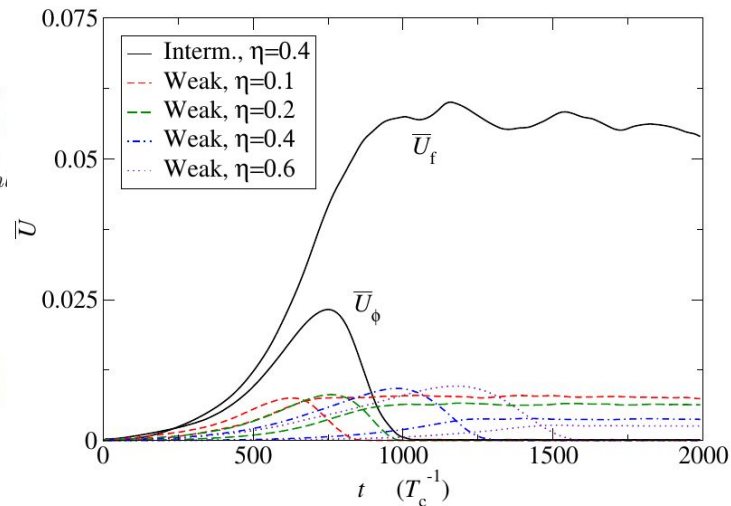
Gravitational waves from the sound of a first order phase transition

Mark Hindmarsh,^{1,2,*} Stephan J. Huber,^{1,†} Kari Rummukainen,^{2,‡} and David J. Weir^{2,§}

¹ *Department of Physics and Astronomy, University of Sussex, Falmer, Brighton BN1 9QH, U.K.*

² *Department of Physics and Helsinki Institute of Physics, PL 64, FI-00014 University of Helsinki, Finland*
(Dated: January 30, 2014)

We report on the first 3-dimensional numerical simulations of first-order phase transitions in the early universe to include the cosmic fluid as well as the scalar field order parameter. We calculate the gravitational wave (GW) spectrum resulting from the nucleation, expansion and collision of bubbles of the low-temperature phase, for phase transition strengths and bubble wall velocities covering many cases of interest. **We find that the compression waves in the fluid continue to be a source of GWs long after the bubbles have merged, a new effect not taken properly into account in previous modelling of the GW source.** For a wide range of models the main source of the GWs produced by a phase transition is therefore the sound the bubbles make.



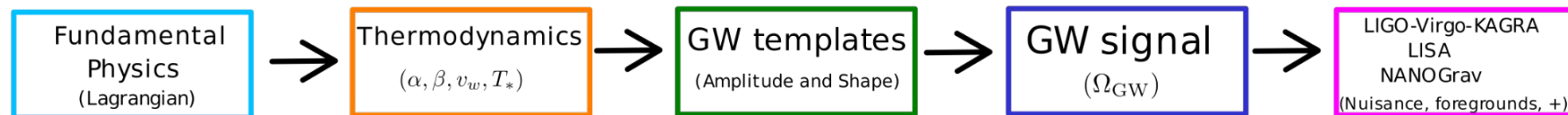
Working flow

Prediction of GW
(analytics, simulations)

Noise

Sensitivity

Foregrounds



How we can connect a Lagrangian to
macroscopic properties of plasma+wall

For how long do sound
waves propagate?

Turbulence

α PT strength

β Bubble nucleation time scale
(and bubble size)

v_w Bubble wall velocity

T_* Temperature of PT

Non-linear dynamics are ultra-relevant!

- **GW Templates**
- **Turbulence**
- **Strong PTs**

Simulations (two schemes)

How can we simulate a scalar field that has TeV mass in the same lattice of a fluid with bubbles of Horizon size (10^{-20} eV)?

Simulations (two schemes)

Scalar field + Fluid, Hindmarsh, Huber, Rummukainen, Weir (13,15,17)

Phenomenological friction term between scalar and fluid

No separation of scales (scalar shell and bubble size)

Wall velocity is determined after stabilization and see plasma

(Hybrid+) Higgsless, Jinno, Konstandin, **HR**, Stromberg

Constant wall velocity (**input**) + Bag equation of state

Model **only the fluid** part

Clear **separation of scales**

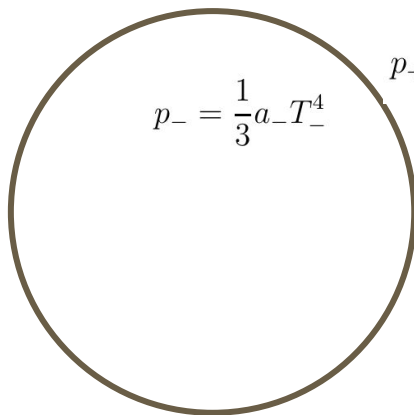
Important for the community:

Two very different schemes, different systematics, in relatively good agreement

Higgsless intuition

Bag Equation of State

Espinosa, Konstandin, No, Servant (10)



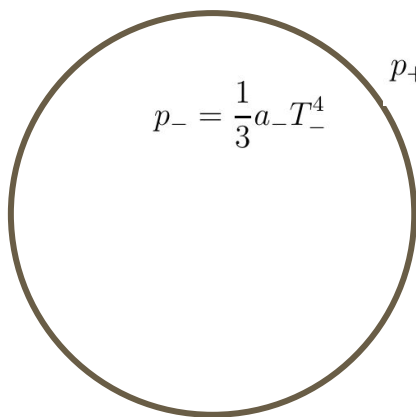
$$p_+ = \frac{1}{3} a_+ T_+^4 - \epsilon$$

Higgs is
incorporated as a
boundary condition

Higgsless intuition

Bag Equation of State

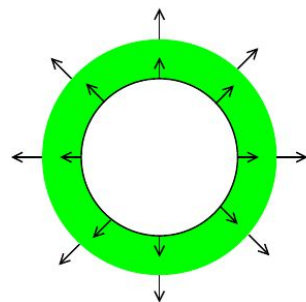
Espinoso, Konstandin, No, Servant (10)



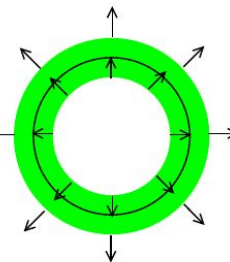
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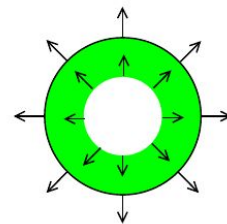
$$\xi = r/t$$



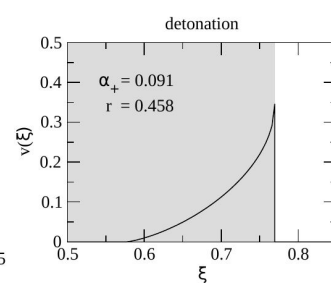
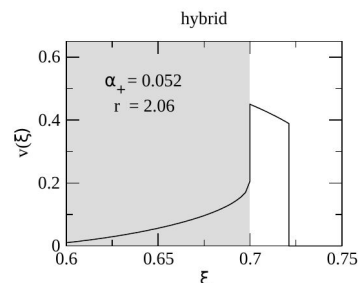
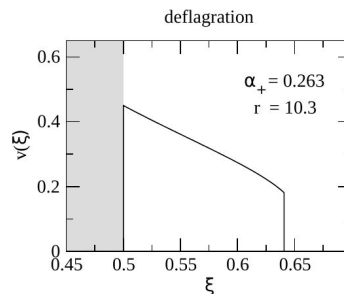
deflagration
 $\xi_w < c_s$



hybrid
 $\xi_w > c_s$



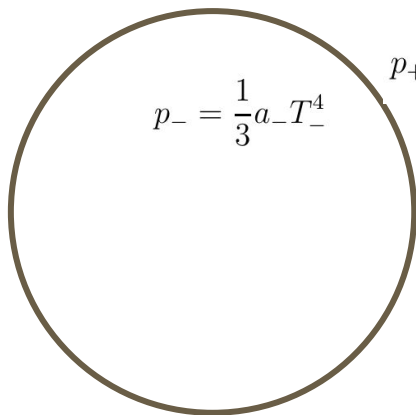
detonation
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Higgsless intuition

Bag Equation of State

Espinosa, Konstandin, No, Servant (10)



$$p_+ = \frac{1}{3} a_+ T_+^4 - \epsilon$$

Higgs is
incorporated as a
boundary condition

How our simulation works:

$$\partial^\mu T_{\mu\nu} = \partial^\mu T_{\mu\nu}^\phi - \partial^\mu T_{\mu\nu}^{\text{plasma}} = 0$$

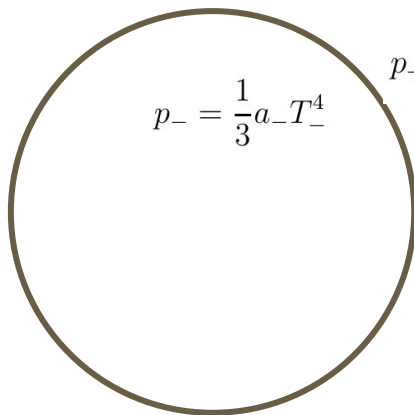
Enters via a time dependent
energy background $\epsilon(\vec{x}, t)$

**You can spend all your
computer energy solving the
fluid part!**

Higgsless intuition

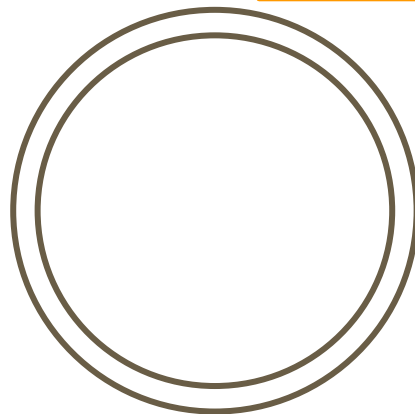
Bag Equation of State

Espinosa, Konstandin, No, Servant (10)



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The scales of the problem:

$$\partial^\mu T_{\mu\nu} = \partial^\mu T_{\mu\nu}^\phi - \partial^\mu T_{\mu\nu}^{\text{plasma}} = 0$$

Enters via a time dependent
energy background

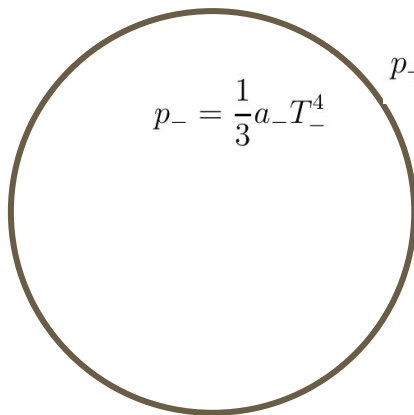
$$\epsilon(\vec{x}, t)$$

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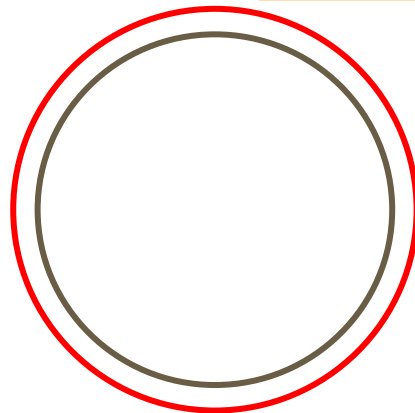
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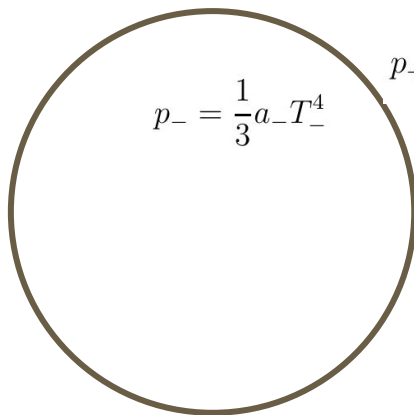
The scales of the problem:

- Bubble thickness: TeV

Higgsless intuition

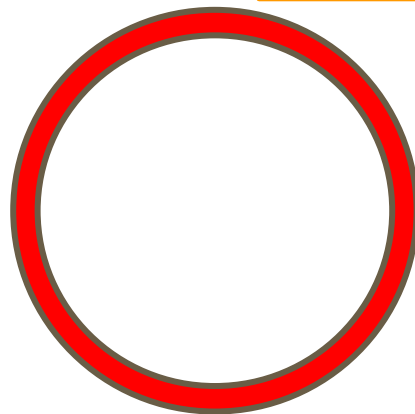
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$$\epsilon(\vec{x}, t)$$

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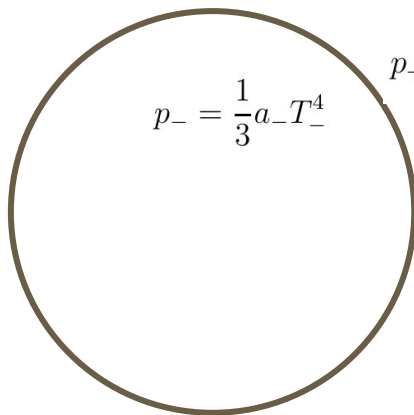
The scales of the problem:

- Bubble thickness: TeV
- Sound Shell

Higgsless intuition

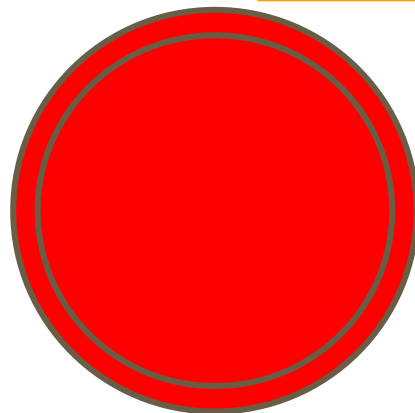
Bag Equation of State

Espinosa, Konstandin, No, Servant (10)



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Enters via a time dependent
energy background $\epsilon(\vec{x}, t)$

**You can spend all your
computer energy solving the
fluid part!**

The scales of the problem:

- Bubble thickness: TeV
- Sound Shell
- Bubble size: Hubble size

What exactly we solve in the lattice

Conservation Eq. $\partial_\mu T^{\mu\nu} = 0$ with $T^{\mu\nu} = u^\mu u^\nu w - \eta^{\mu\nu} p$

Enthalpy and pressure:

$$p = \frac{1}{3}aT^4 - \epsilon, \quad w = T \frac{dp}{dT} = \frac{4}{3}aT^4$$

Modelling of the vacua:

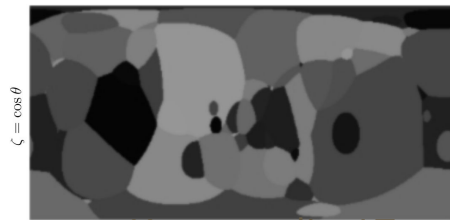
$$\epsilon(t, \vec{x}) = \begin{cases} 0 & \text{inside bubbles,} \\ \epsilon & \text{outside bubbles,} \end{cases}$$

Wall velocity
enters here

PT
strength
enters
here

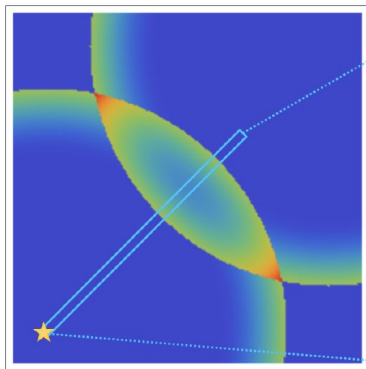
$$\alpha = \frac{4\epsilon}{3w}$$

Hybrid (1d embedded in 3d) Higgsless

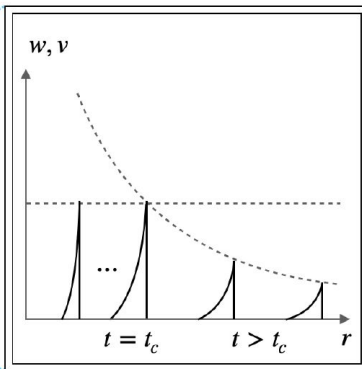


Konstandin 17

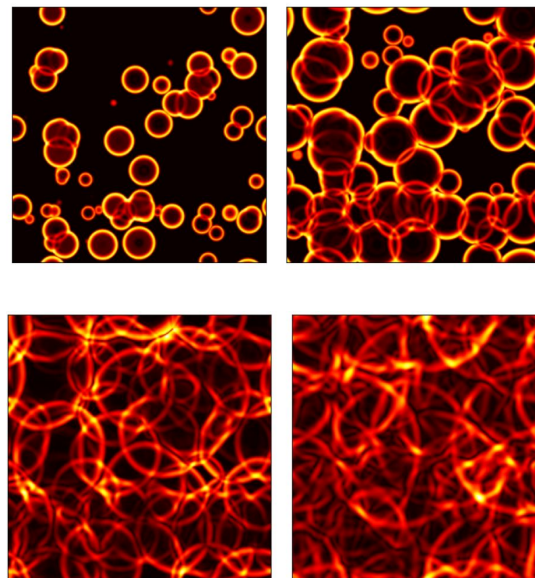
3d simulation



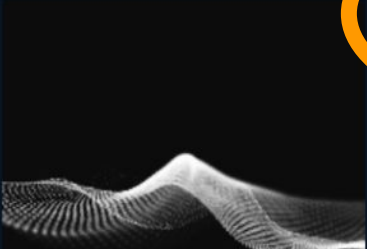
1d simulation

Jinno, Konstandin, **HR**, 2020

The full 3D Higgsless

Jinno, Konstandin, **HR**, Stomberg 2022

This is a part I always skip in my talks but always wanted to focus on




Numerical Simulations of Early Universe Sources of Gravitational Waves

Numerical Simulations of Early Universe Sources of Gravitational Waves

July 28, 2025 to August 15, 2025 — Albano Building 3

Now happening: Curvature perturbation from first-order phase transitions & A positive-definite formulation of tunneling 11:30 AM - 12:30 PM

The numerics

- Higgs->0 leads to fluid discontinuities. How to deal with them?
- Dealing with shocks there is a tradeoff: numerical viscosity vs. unphysical oscil.

1d PDE example: $\partial_t u + \partial_r f + g = 0,$

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} w\gamma^2 - p \\ w\gamma^2 v \end{pmatrix}, \quad f = \begin{pmatrix} w\gamma^2 v \\ w\gamma^2 v^2 + p \end{pmatrix}, \quad g = \frac{d-1}{r} \begin{pmatrix} w\gamma^2 v \\ w\gamma^2 v^2 \end{pmatrix}.$$

Option 0

Lax-Friedrichs (LF)

Linear, first-order

$$u_j^{n+1} = \frac{u_{j+1}^n + u_{j-1}^n}{2} - \frac{\lambda}{2} [f(u_{j+1}^n) - f(u_{j-1}^n)] - g_j^n \Delta t$$

$$\lambda = \Delta t / \Delta x$$

Solving shocks

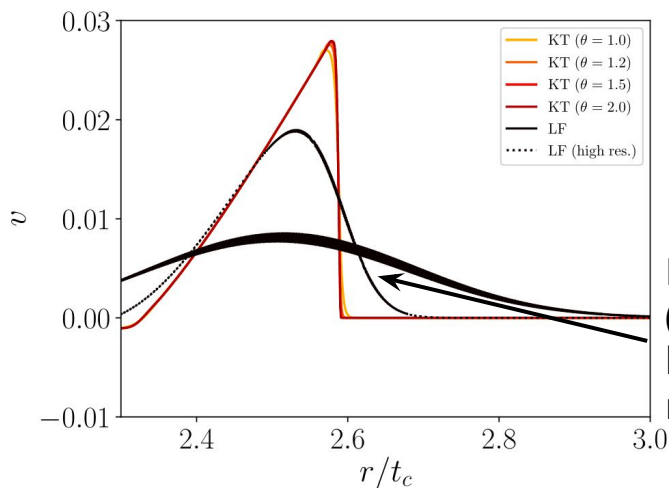
Gudunov's theorem: Linear scheme can only respect positivity at first order

Solution 1: Riemann solvers or Godunov schemes
Solve exact linearized scheme, diagonalize matrix locally

$$\partial_t u + \partial_x f(u) = 0$$

Simple scheme, Lax Friedrichs (LF)

$$u_j^{n+1} = \frac{u_{j+1}^n + u_{j-1}^n}{2} - \frac{\lambda}{2} [f(u_{j+1}^n) - f(u_{j-1}^n)] - g_j^n \Delta t$$

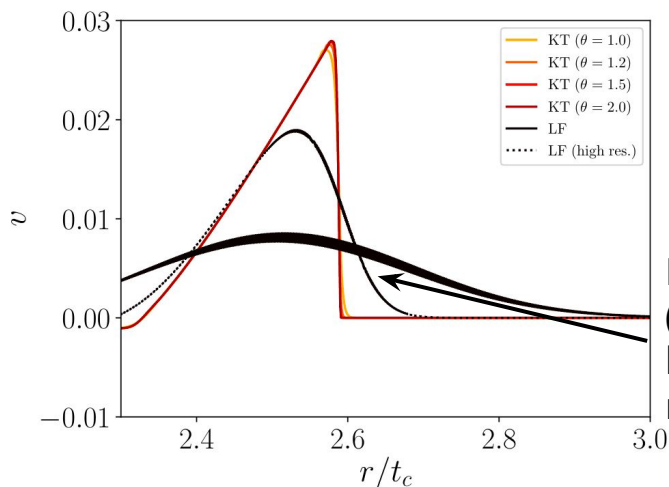


Solving shocks

$$\partial_t u + \partial_x f(u) = 0$$

Simplest scheme, Lax Friedrichs (LF)

$$u_j^{n+1} = \frac{u_{j+1}^n + u_{j-1}^n}{2} - \frac{\lambda}{2} [f(u_{j+1}^n) - f(u_{j-1}^n)] - g_j^n \Delta t$$



Large viscosity
(refined mesh
here would
not help too)

Gudunov's theorem: Linear scheme can only respect positivity at first order

Solution 1: Riemann solvers or Godunov schemes
Solve exact linearized scheme, diagonalize matrix locally

Solution 2: Hybridization
Introduce non-linear terms via 'flux limiters'

Kurganov - Tadmor (KT) for
conservative schemes
Non-linear, second order

$$\partial_\mu T^{\mu\nu} = 0$$

Flux

$$u_j^{n+1} = u_j^n - \frac{\Delta t}{\Delta x} (H_{j+1/2}^n - H_{j-1/2}^n)$$

Viscosity

$$H_{j+1/2} = \frac{f(u_{j+1/2}^+) + f(u_{j+1/2}^-)}{2} - \frac{a_{j+1/2}}{2} [u_{j+1/2}^+ - u_{j+1/2}^-]$$

Local max
velocity

Solving shocks

Gudunov's theorem: Linear scheme can only respect positivity at first order

Solution 1: Riemann solvers or Godunov schemes
diagonalize matrix locally

'flux limiters'

$$\partial_\mu T^{\mu\nu} = 0$$

Flux

$$u_{j+1/2}^n - H_{j-1/2}^n$$

Viscosity

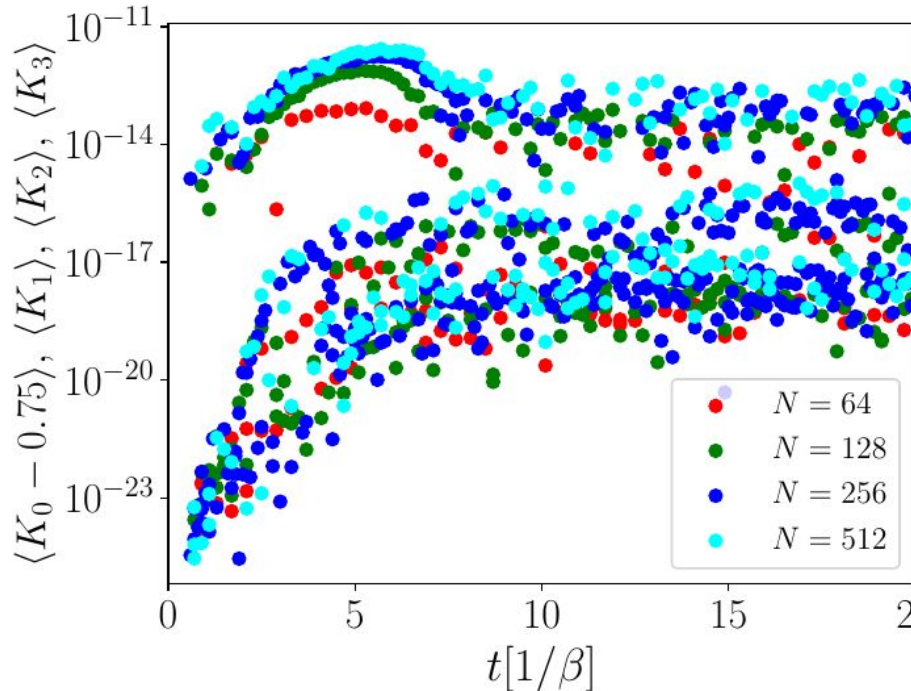
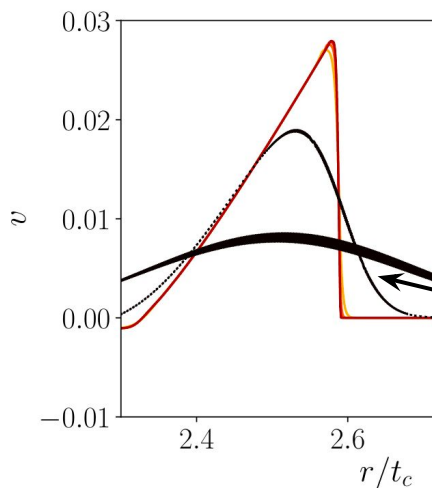
$$\frac{a_{j+1/2}}{2} [u_{j+1/2}^+ - u_{j+1/2}^-]$$

local max
velocity

$$\partial_t u + \partial_x f$$

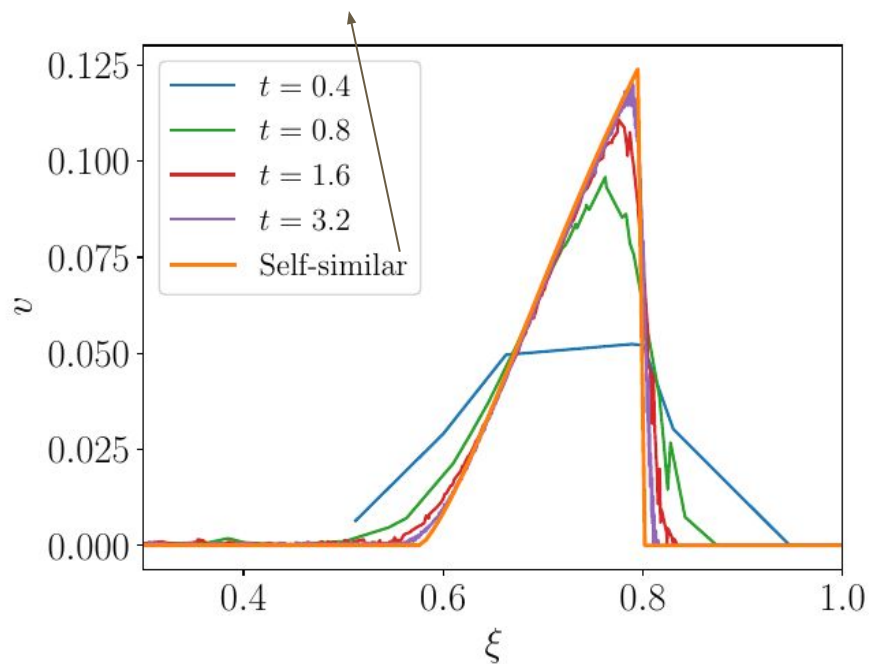
Simple scheme, Lax

$$u_j^{n+1} = \frac{u_{j+1}^n + u_{j-1}^n}{2} - \frac{\lambda}{2} [f_j^n - f_{j-1}^n]$$

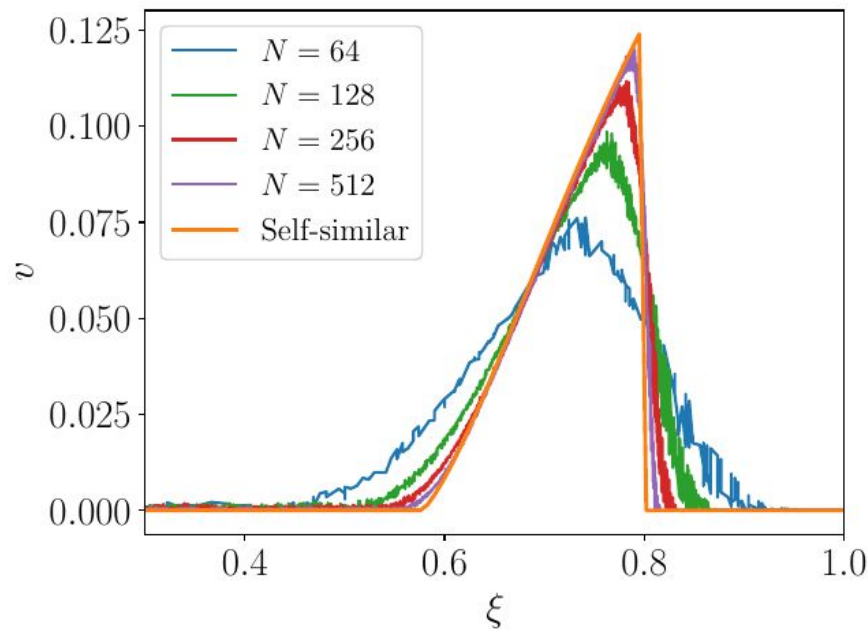


Can we reproduce bag EoS solution?

Comparing to the analytical solution

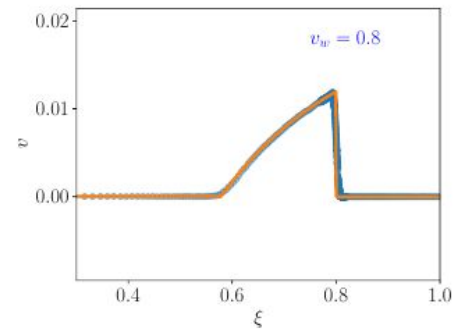
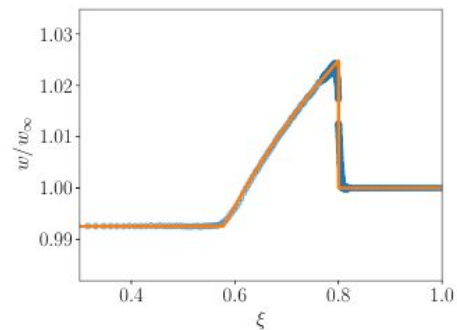
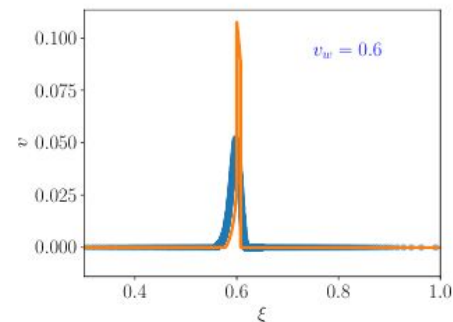
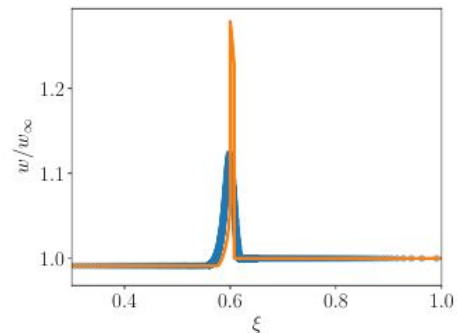
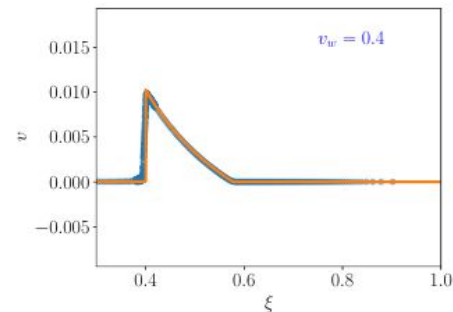
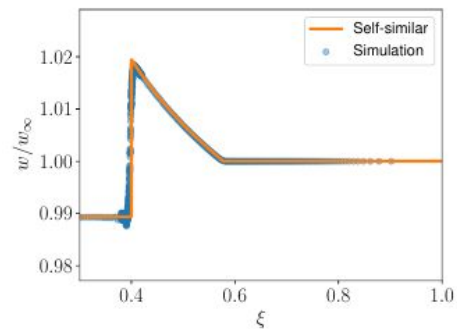


Can we reproduce the correct result with one single bubble?



Can we reproduce bag EoS solution?

Yes!
Struggling more for compact profiles



A short rest for your eyes... questions?



Outline of this talk

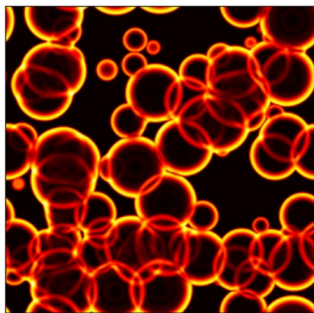
Part I: Higgsless

- Simulations
- **Results**
- Temperature fluctuations and Topological defects

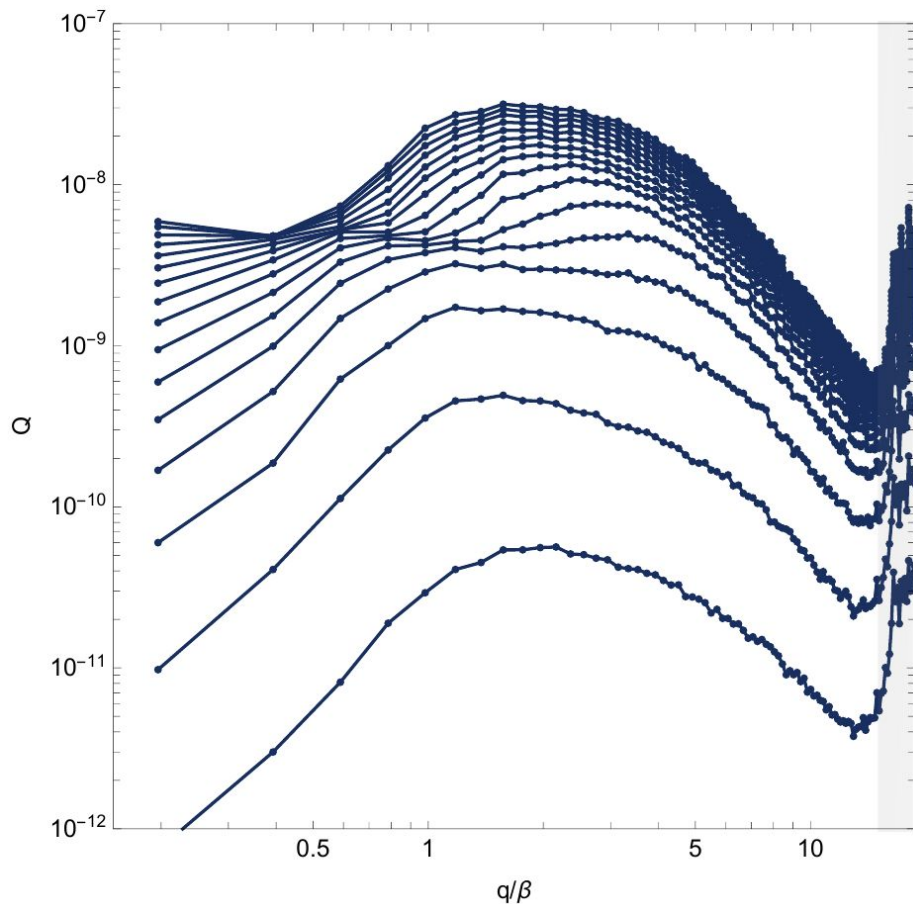
Part II: Phase transitions and H0 tension

Results

Now with many bubbles...



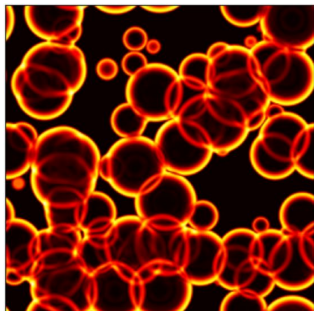
$$\Omega_{\text{GW}} = \frac{w^2 \tau}{4\pi^2 \rho_{\text{tot}} M_P^2 \beta} \times Q',$$



Simulation
time

Results

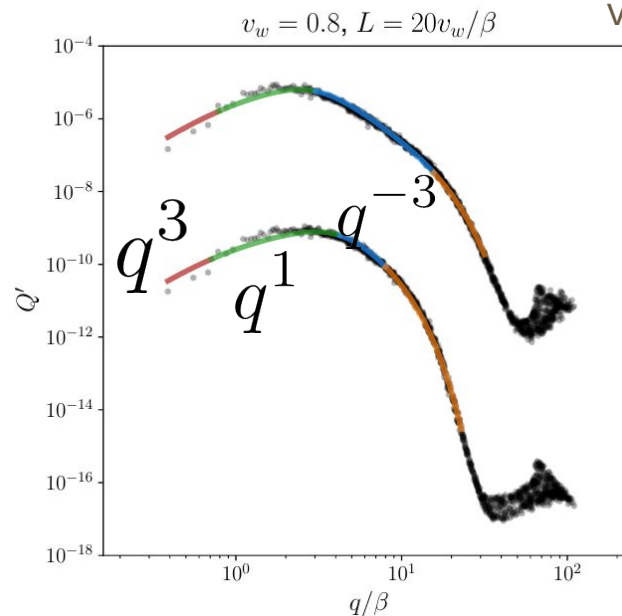
$$Q'(q) = \overset{\text{Amplitude}}{Q'_{\text{int}}} \times \overset{\text{Shape}}{S_f(q)}$$



A double-broken power law

$$S_f(q) = S_0 \times \frac{(q/q_0)^3}{1 + (q/q_0)^2[1 + (q/q_1)^4]} \times e^{-(q/q_e)^2}$$

Exponential
viscosity



Results

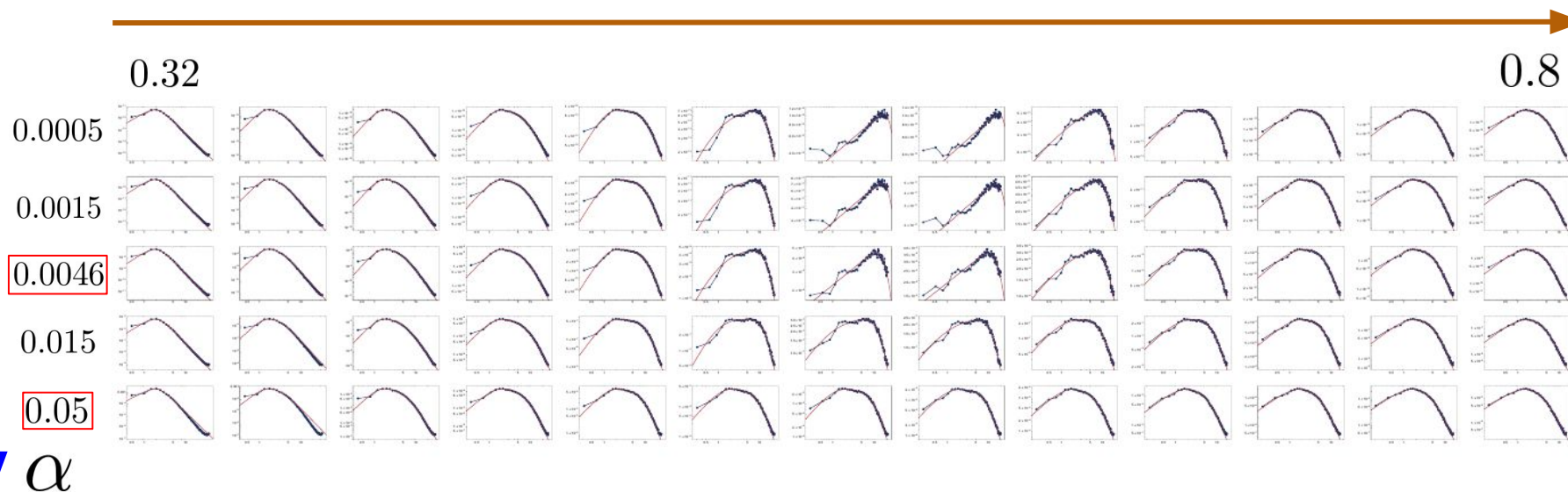
Amplitude Shape

$$Q'(q) = Q'_{\text{int}} \times S_f(q)$$

A double-broken power law

$$S_f(q) = S_0 \times \frac{(q/q_0)^3}{1 + (q/q_0)^2[1 + (q/q_1)^4]} \times e^{-(q/q_e)^2}$$

ξ_w



Results

Amplitude Shape

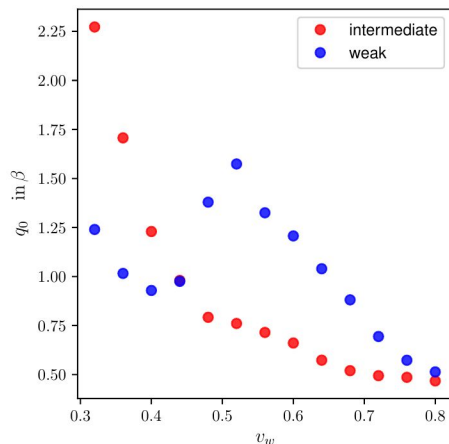
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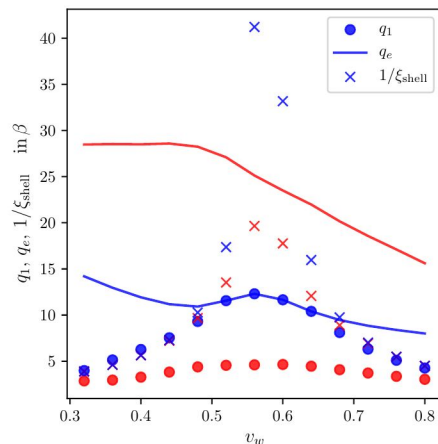
$$S_f(q) = S_0 \times \frac{(q/q_0)^3}{1 + (q/q_0)^2[1 + (q/q_1)^4]} \times e^{-(q/q_e)^2}$$

Position of the peaks

IR peak looks complex

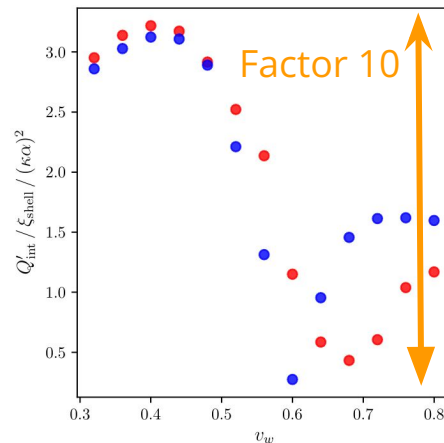


UV peak scales as sound shell thickness

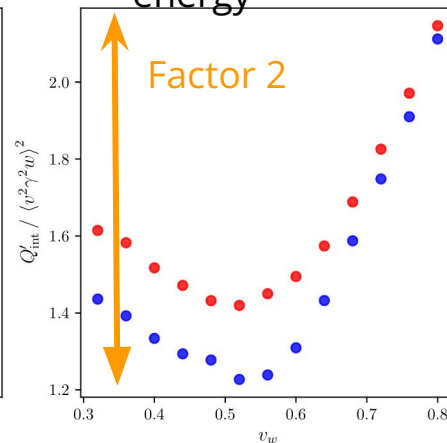


Amplitude of the spectrum

Normalized by kinetic energy



Normalized by 3d simulation kinetic energy



Comparison to other works

We find very **similar GW spectra** when comparing weak and intermediate transitions

1000x faster! (Fluid only)

More bubbles -- $O(2500)$

	IR	Intermediate	UV
Sound shell	9	1	-3
Scalar + fluid lattice	-	1	-3
Hybrid	[2,4]	[-1,0]	[-4,-3]
Higgsless	3	1	-3

Pause for drinking water... questions?



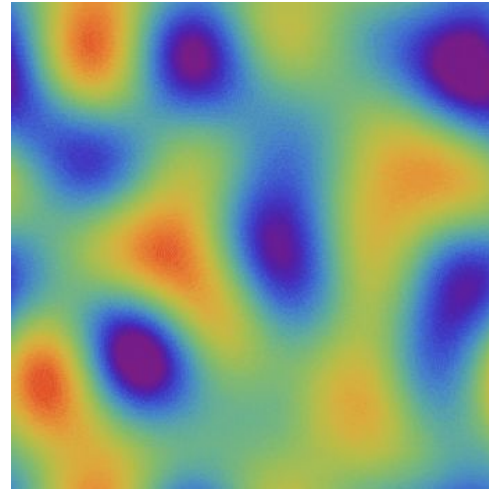
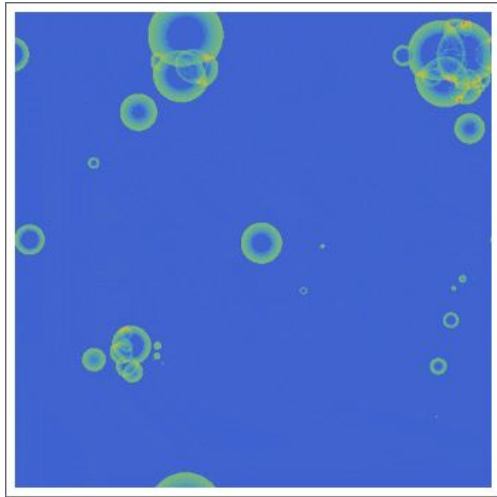
Outline of this talk

Part I: Higgsless

- Simulations
- Results
- **Temperature fluctuations and Topological defects**

Part II: Phase transitions and H0 tension

Can temperature fluctuations also affect the sourcing of GWs from 1st order PTs?



Can temperature fluctuations also affect the sourcing of GWs from 1st order PTs?

Nucleation rate proportional to the 3d bounce action: $\Gamma \propto e^{-S_3/T}$

After expanding it around $t = t_*$

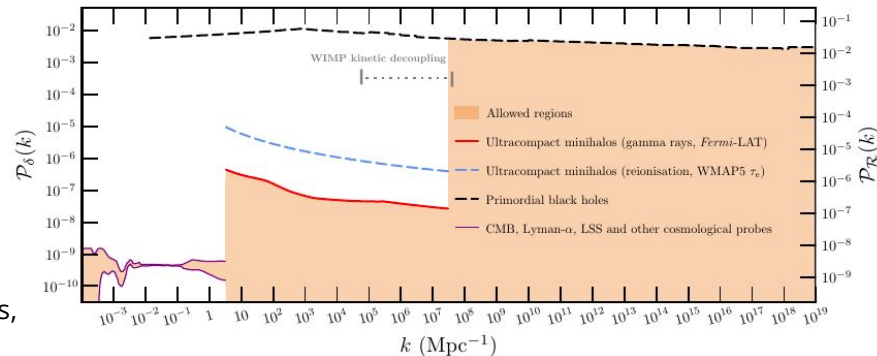
$$\Gamma = \Gamma_* \exp \left[\beta(t - t_*) - \frac{\beta}{H_*} \frac{\delta T}{\bar{T}} \right]$$

This next term in the expansion may be large if

$$\frac{\delta T}{T} \sim \left(\frac{\beta}{H_*} \right)^{-1}$$

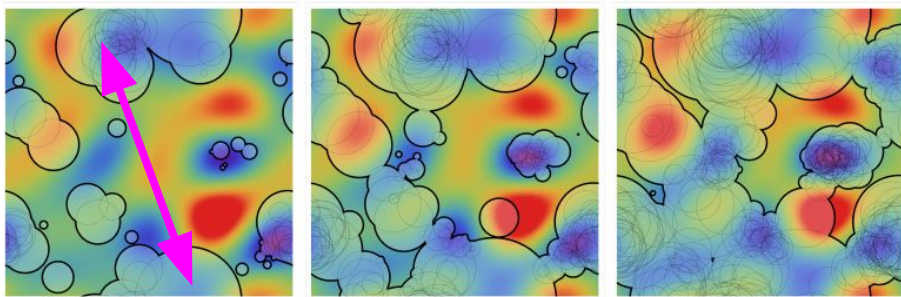
Bringmann, Scott, Akrami, 1110.2484. See also Byrnes, Cole, Patil

Since we are talking about $T \sim \text{TeV}$, it is pretty in the (unconstrained) UV



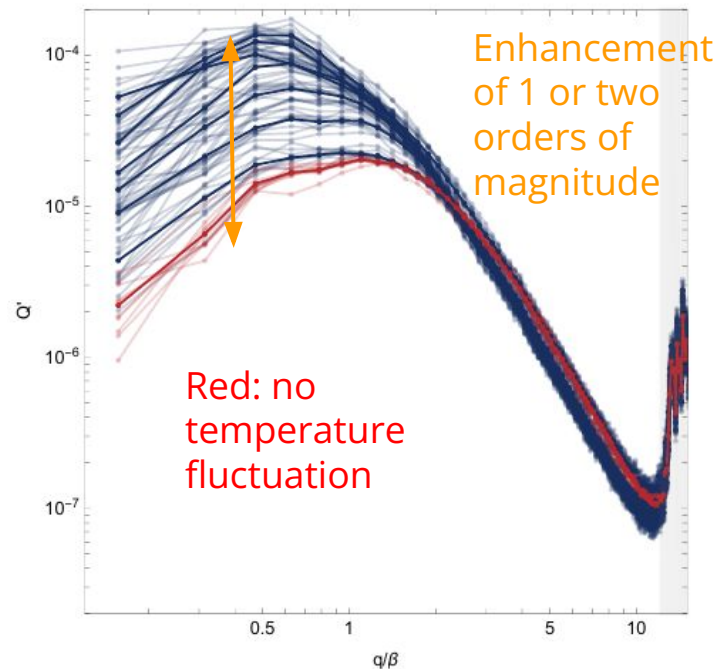
What is the effect in the GW spectra?

We simulate bubble nucleation under temperature fluctuations and plug it into the hybrid simulation



Result: increase the bubbles size and therefore enhance GW spectra

We also parametrized how spectra depends on temperature fluctuations



Motivation

Domain walls appear in many **two-step PTs**
(e.g. singlet extension)

$$V = \frac{\lambda}{4}(\phi^2 - v^2)^2 + \frac{\lambda_m}{4}s^2\phi^2 + \frac{\lambda_m}{4}s^4 - \frac{1}{2}\mu_S^2 s^2$$

s is a singlet with a Z2 sym

This Z2 is spontaneously broken producing DWs

At low Temperatures, this Z2 symmetry is restored when the other scalar ϕ gets a vev

Domain Walls stop to exist!
(they don't overclose the Universe but are relevant at transition time)

COSMIC STRINGS AND DOMAIN WALLS

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Received October 1984

To learn more about the system of walls at formation,
one can use a **Monte Carlo simulation** [53, 54].

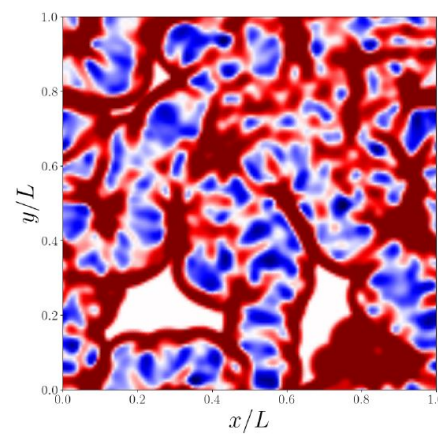
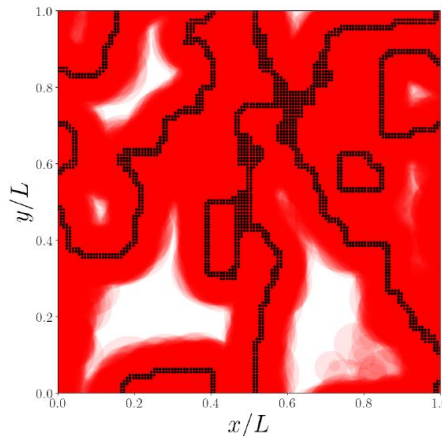
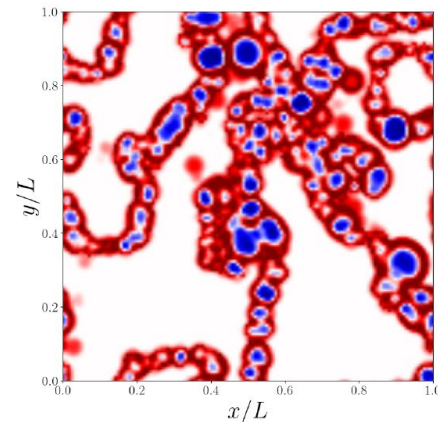
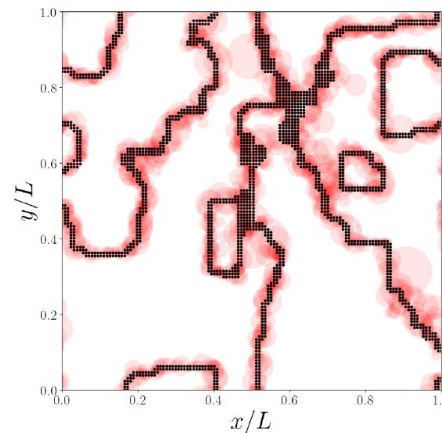
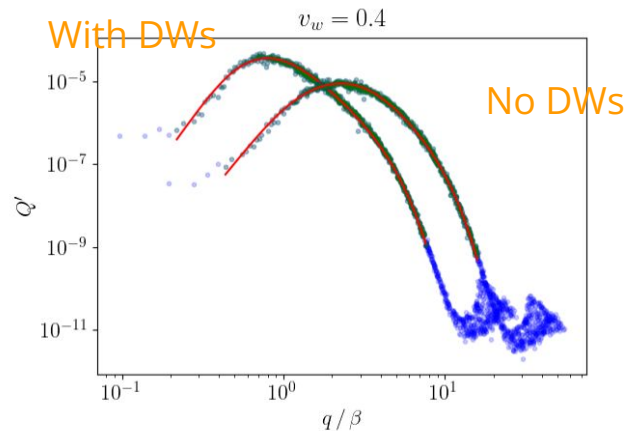
Using the Ising Model to
mimic DWs! Both have a
single scale

The Ising Model and the fluid energy

Henrique Rubira

Relevant whenever $\xi_{\text{DW}} \gg 2\langle r \rangle \simeq 6v_w/\beta$

$$\beta \rightarrow 3/\xi_{\text{DW}}$$



+ Simone Blasi

Conclusions I

We have a fast and precise scheme to calculate GWs from PT that can explore non-linear PTs also alternative scenarios

What comes next? (+ Chiara Caprini, Alberto Roper Pol)

- Deep into non-linear PTs
- Decaying sources



Another short rest for your eyes... questions?



Outline of this talk

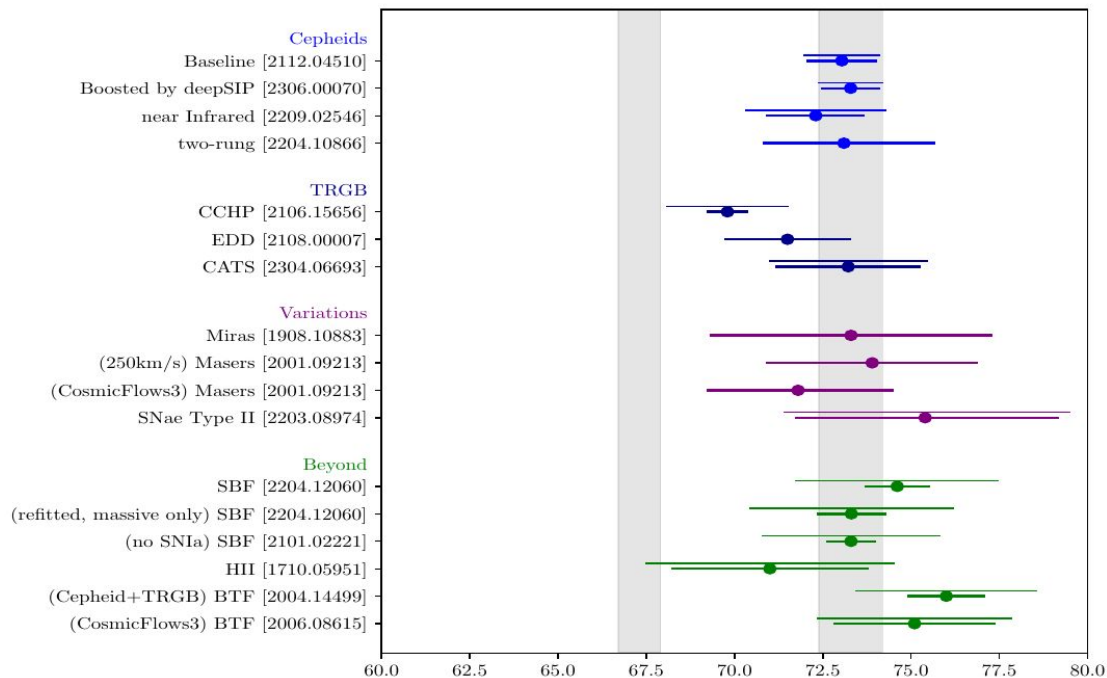
Part I: Higgsless

- Simulations
- Results
- Temperature fluctuations and Topological defects

Part II: Phase transitions and H_0 tension

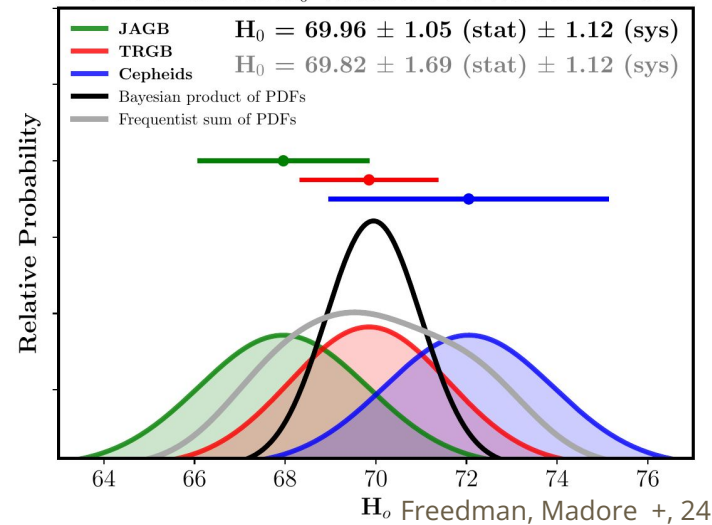
The Hubble tension

Verde, Schoeneberg, Gil-Marín, 23



For now, JWST results are still inconclusive (see Riess+, 24)

Distribution of H_0 Values for 3 JWST Methods



The theorist job

What is a simple model that can address the H_0 tension?

The theorist job

What is a simple model that can address the H0 tension?

$$\mathcal{L} = \bar{\chi}(i\gamma^\mu D_\mu - m_\chi)\chi + |D\Psi|^2 - V_{\text{cl}}(|\Psi|^2) - \frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu}$$

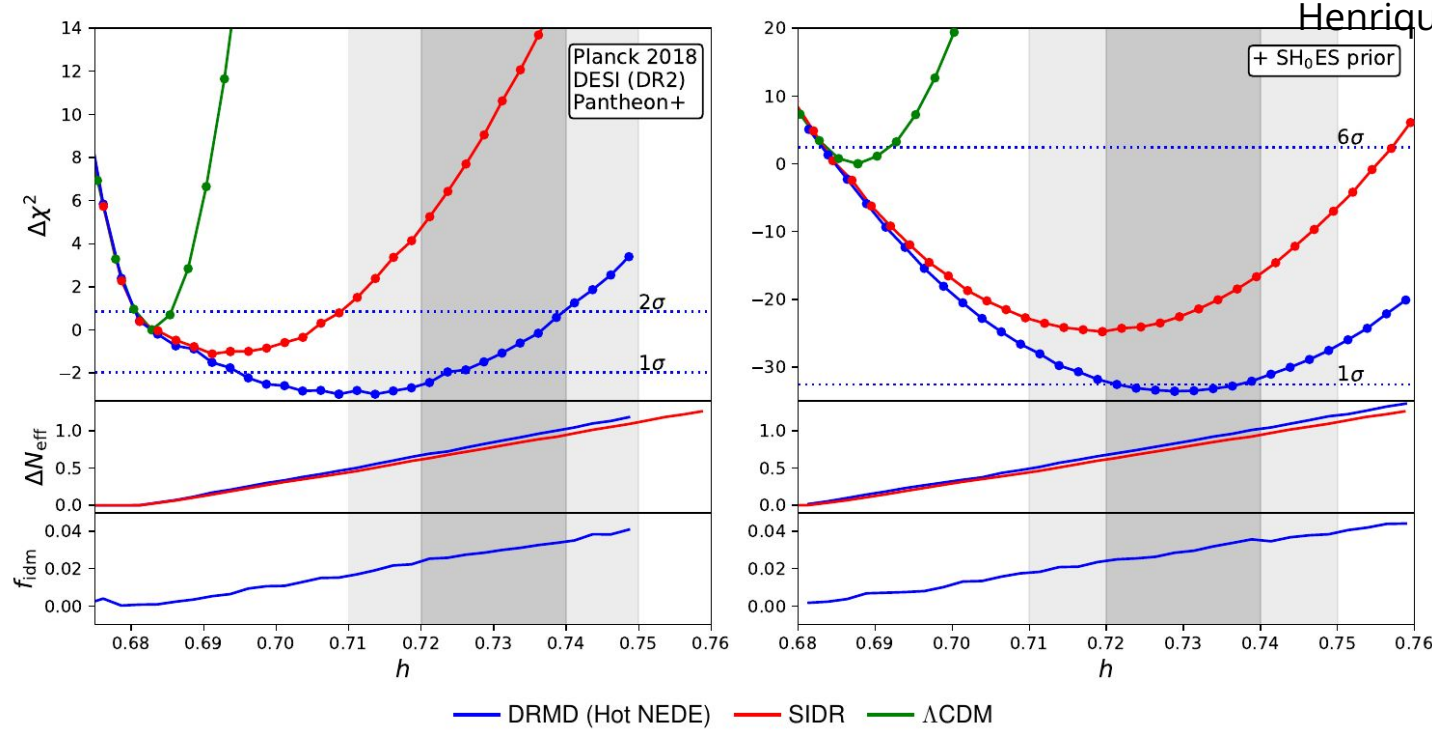
+ Symmetry breaking $SU(N) \rightarrow SU(N-1)$

Produces a fraction of **EDE**
via supercooling
(Niedermann + Sloth)

$$f_{\text{NEDE}} = \Delta V_*/\rho_{\text{tot}}(T_d^*)$$

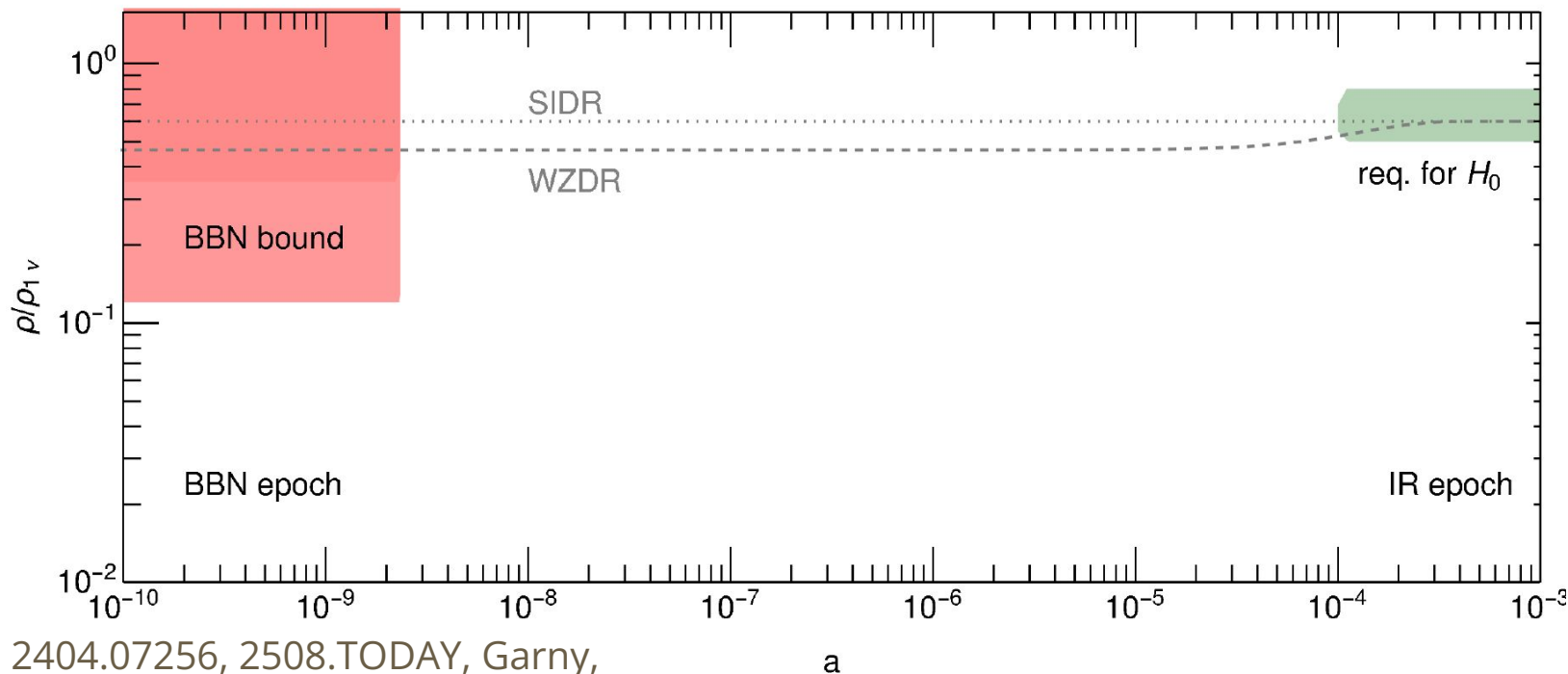
Results

Henrique Rubira

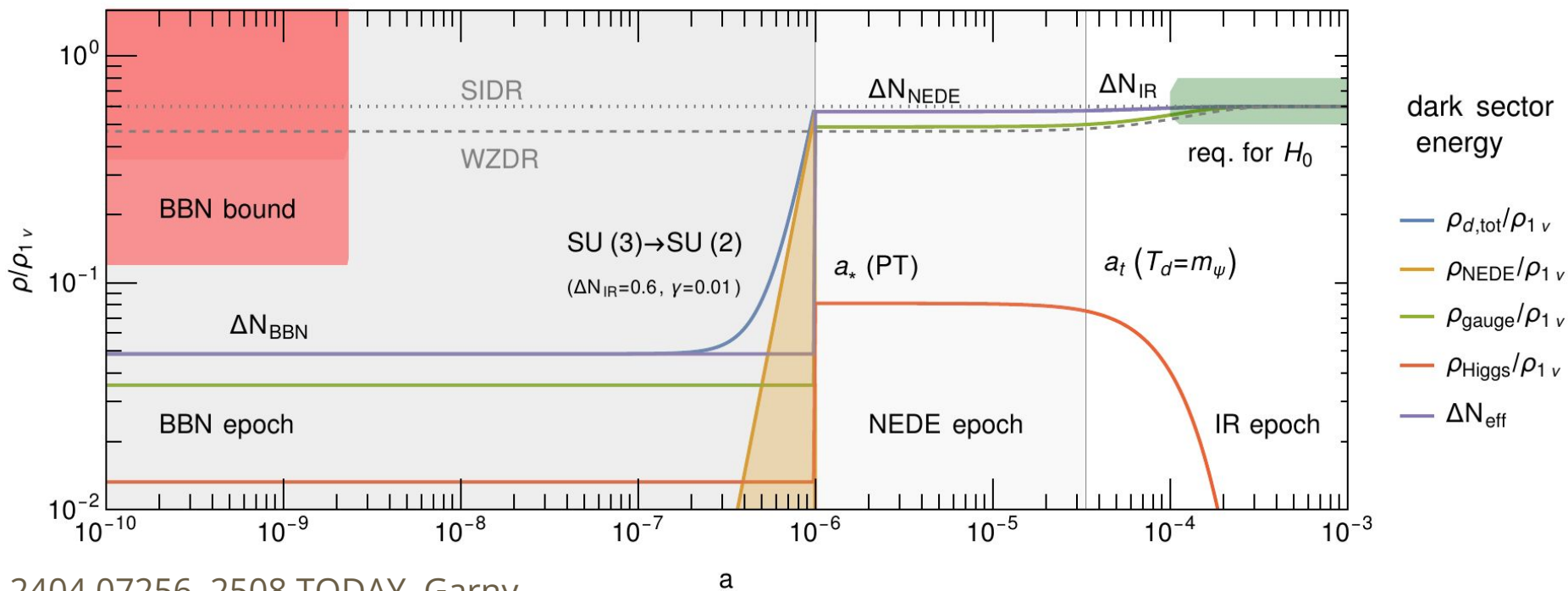


	Base				Base + SH ₀ ES prior				Q_{DMAP}^M	Q_{Gauss}
	H_0	ΔN_{eff}	$\Delta\chi^2$	ΔAIC	H_0	ΔN_{eff}	$\Delta\chi^2$	ΔAIC		
Λ CDM	68.30 ± 0.29	—	—	—	68.70 ± 0.29	—	—	—	5.7σ	4.4σ
SIDR	$69.8^{+0.8}_{-1.0}$ (69.4)	< 0.525 (0.188)	-1.2	0.8	71.9 ± 0.7 (72.0)	0.61 ± 0.13 (0.610)	-24.5	-22.5	3.0σ	2.5σ
DRMD	$70.5^{+1.0}_{-1.6}$ (70.9)	< 0.811 (0.462)	-3.0	5.0	72.8 ± 0.8 (72.9)	0.80 ± 0.16 (0.829)	-33.3	-25.3	1.4σ	1.7σ

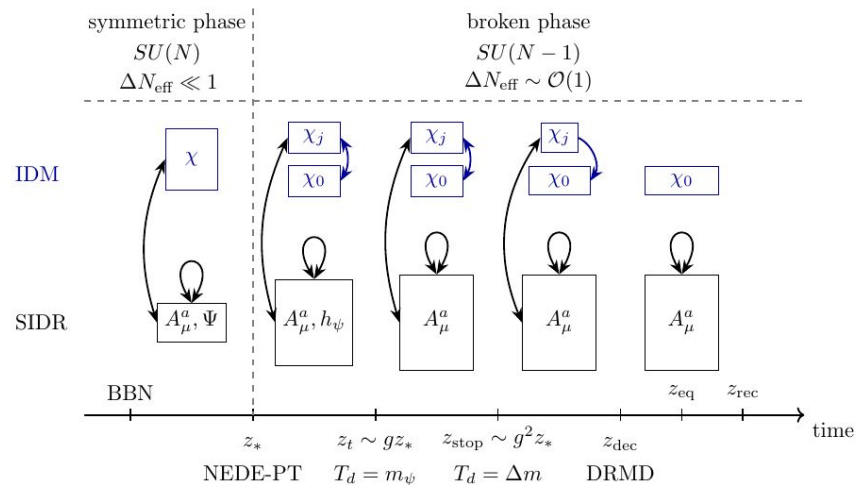
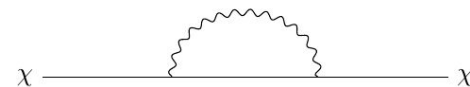
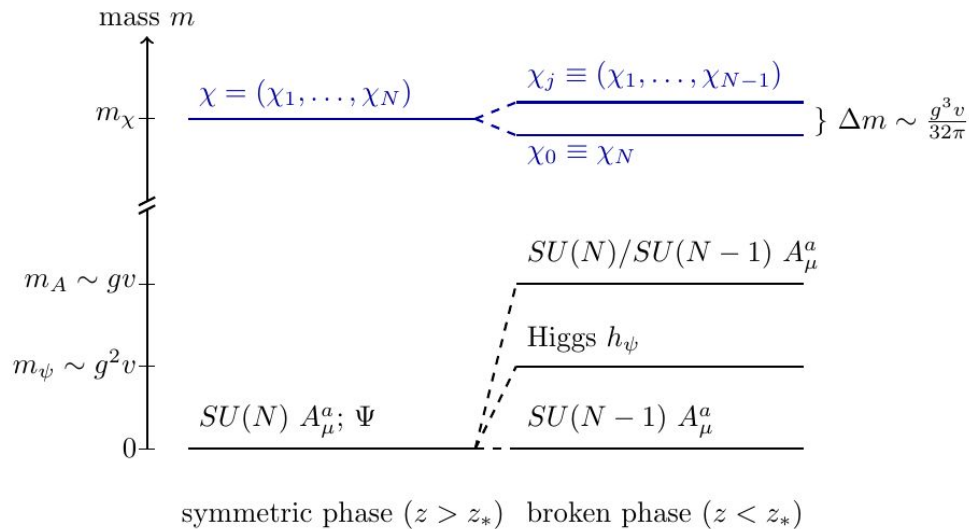
Big picture of the model: evolution of energy density



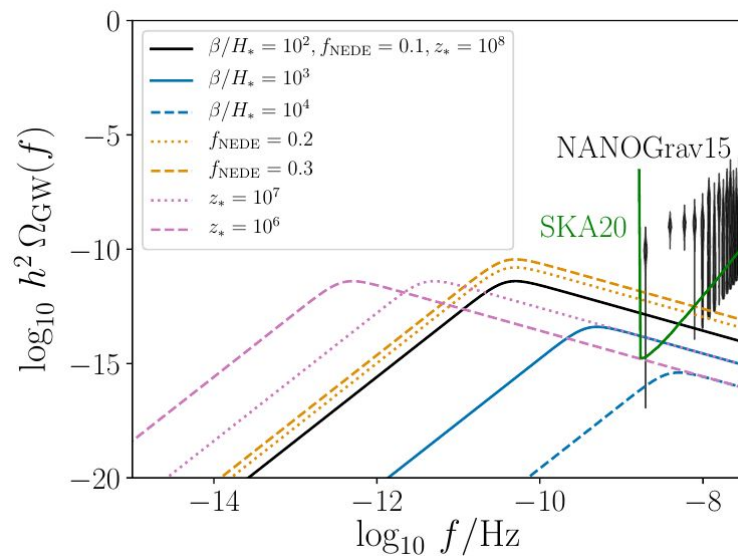
Big picture of the model: evolution of energy density



Phase transitions as a solution for H0

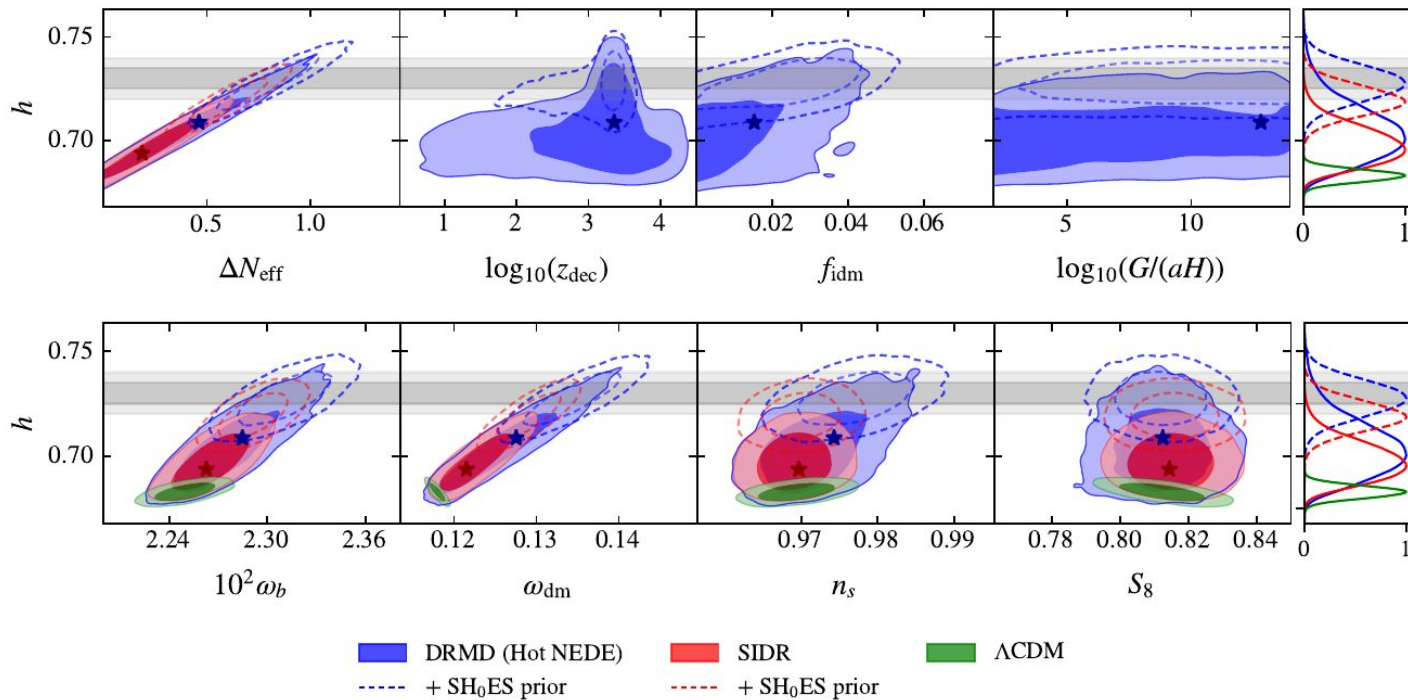


Gravitational waves



2404.07256, 2508.TODAY, Garny,
Niedermann, **HR**, Sloth

The posteriors



Conclusions II

Simple model for a complicated problem.

H0 Tension reduced from 5.7sigma to 1.4sigma

Complex pheno for us to have fun!

Thanks a lot!



Class of solutions to the tension

The three main quantities

- 1) Angular diameter distance

$$D_A(z) = \int_0^z \frac{1}{H(z')} dz'.$$

- 2) The sound horizon

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{c_s(z')}{H(z')} dz'$$

- 3) Angular-scale of the sound horizon

$$\theta_s(z) = \frac{r_s(z_*)}{D_A(z)}$$

Late Universe

- Raise H_0 playing with the DE density;
- Challenge: Myriad of low- z measurements of $H(z')$ make it hard to keep $D_A(z)$ fixed

Early Universe: modifications of the sound horizon, adjusting $D_A(z)$ to keep $\theta_s(z)$ fixed across different scales. In general, perform better.

Supercooling

We work in the light Higgs mass limit (relative to gauge-bosons) $m_A \gg T_d^*, m_\psi$

Potential including loop
gauge-boson loop-corrections
(Coleman-Weinberg)

$$V(\psi; T_d) = V_0 - \frac{\mu_{\text{eff}}^2}{2} \psi^2 \left(1 - \frac{\psi^2}{2v^2} \right) + B\psi^4 \left(\ln \frac{\psi^2}{v^2} - \frac{1}{2} \right) + \Delta V_{\text{thermal}}(\psi; T_d)$$

with: $\Delta V_{\text{thermal}}(\psi; T_d) \rightarrow \frac{1}{24} c_0 n_A g^2 T^2 \psi^2$

The conformal limit is reached at $\mu_{\text{eff}} = 0$ (Witten, 1981)

Defining the
supercooling
parameter

$$\gamma \equiv \frac{\overbrace{12\pi\mu_{\text{eff}}^2}^{\text{Loop corrections}}}{\underbrace{c_0 n_A v^2 g^4}_{\text{Thermal corrections}}}$$

supercooling
happens as
long as

$$\gamma \lesssim \mathcal{O}(1)$$

Reheating the Dark Sector

After the transition:

- kinetic/gradient energy of bubble walls;
- bubble collision leads to bubble fragments and condensate of small-scale anisotropic stress.

Decay of the scalar particles in bubble-wall condensate via $\psi \rightarrow AA$
(1-loop massless gauge bosons, similar to $h \rightarrow \gamma\gamma$)

$$\frac{\Gamma_{\psi \rightarrow AA}^{(\text{cm})}}{H_*} = \mathcal{O}(1) \times \frac{g^9 f_{\text{NEDE}}^{1/4}}{1 + z_*} 10^{24} \quad \text{imposes the condition } g \gtrsim 0.02$$

Thermalization of the gauge bosons via $AA \rightarrow AA$

$$\frac{\Gamma_{\text{therm}}}{H_*} \sim \frac{g^4}{1 + z_*} 10^{31}$$

Coupling with the plasma can also reheat the plasma via other channels