Numerical Simulations of Early Universe Sources of Gravitational Waves - August 7, NORDITA, Sweden

VNIVERSITAT DE VALÈNCIA

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Based on 2508.04263 , 2409.03651, 2209.04369

Out today

Gravitational waves

Can we *hear* them?

Yes!

GW151226

Gravitational waves

Can we *hear* them?

Yes!

Can we *hear* GWs from cosmological sources and PTs?

Possibly!

The "*sound*" of the early universe

Phase transitions

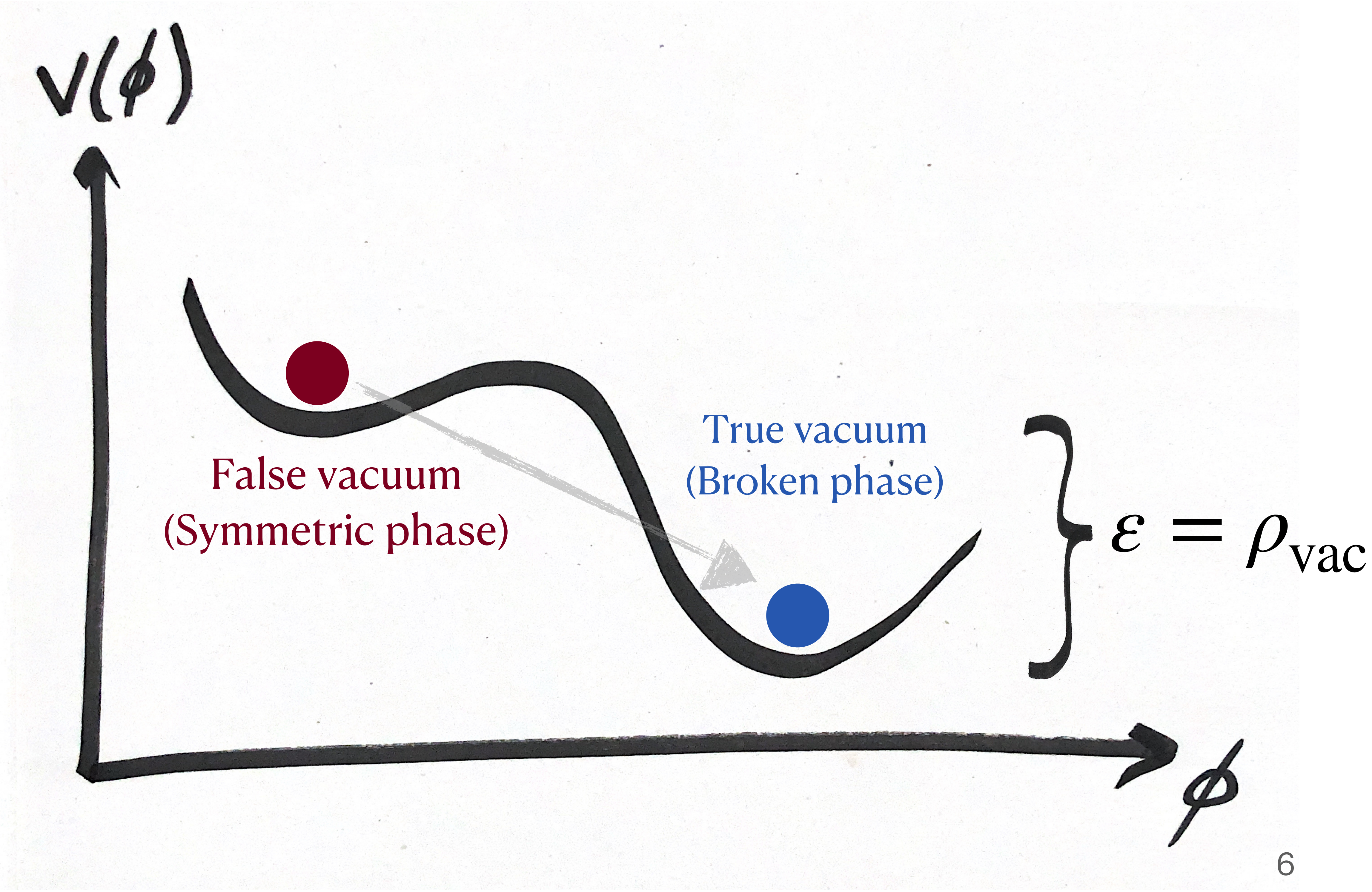
phase transitions

- Phase transitions (PTs) exist in abundance and diversity
- Melting of ice, condensation of snowflakes, **bubbles** in boiling water, ferromagnetism, etc.
- The list is long, but none pertains to a cosmological context.
- Abundance in diverse physical systems ⇒ **cosmological PTs?**

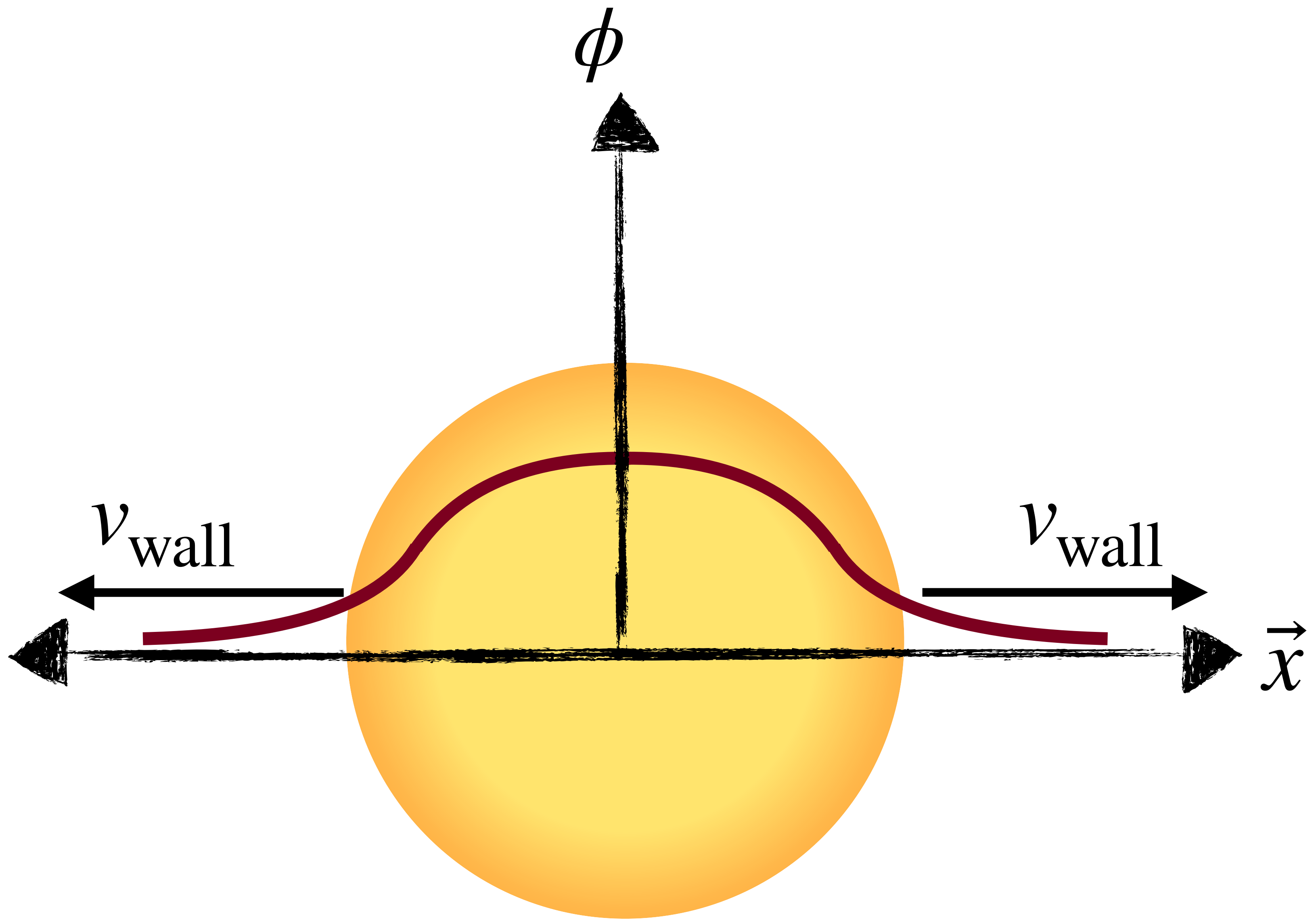
Schematics of a first-order PT

- 1st order PTs proceed through bubble nucleations

Tunneling in field space

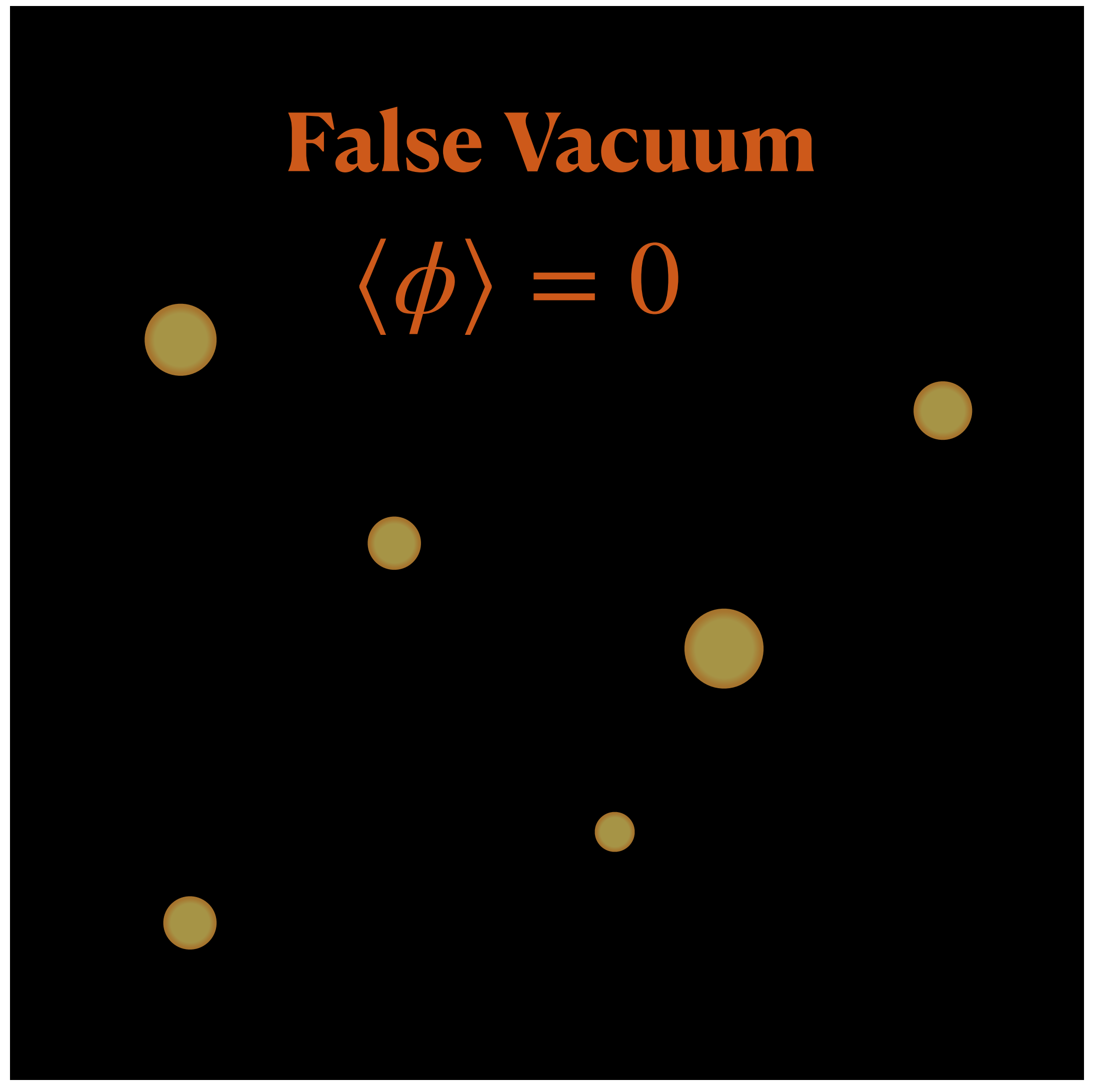


Bubble expansion in physical space



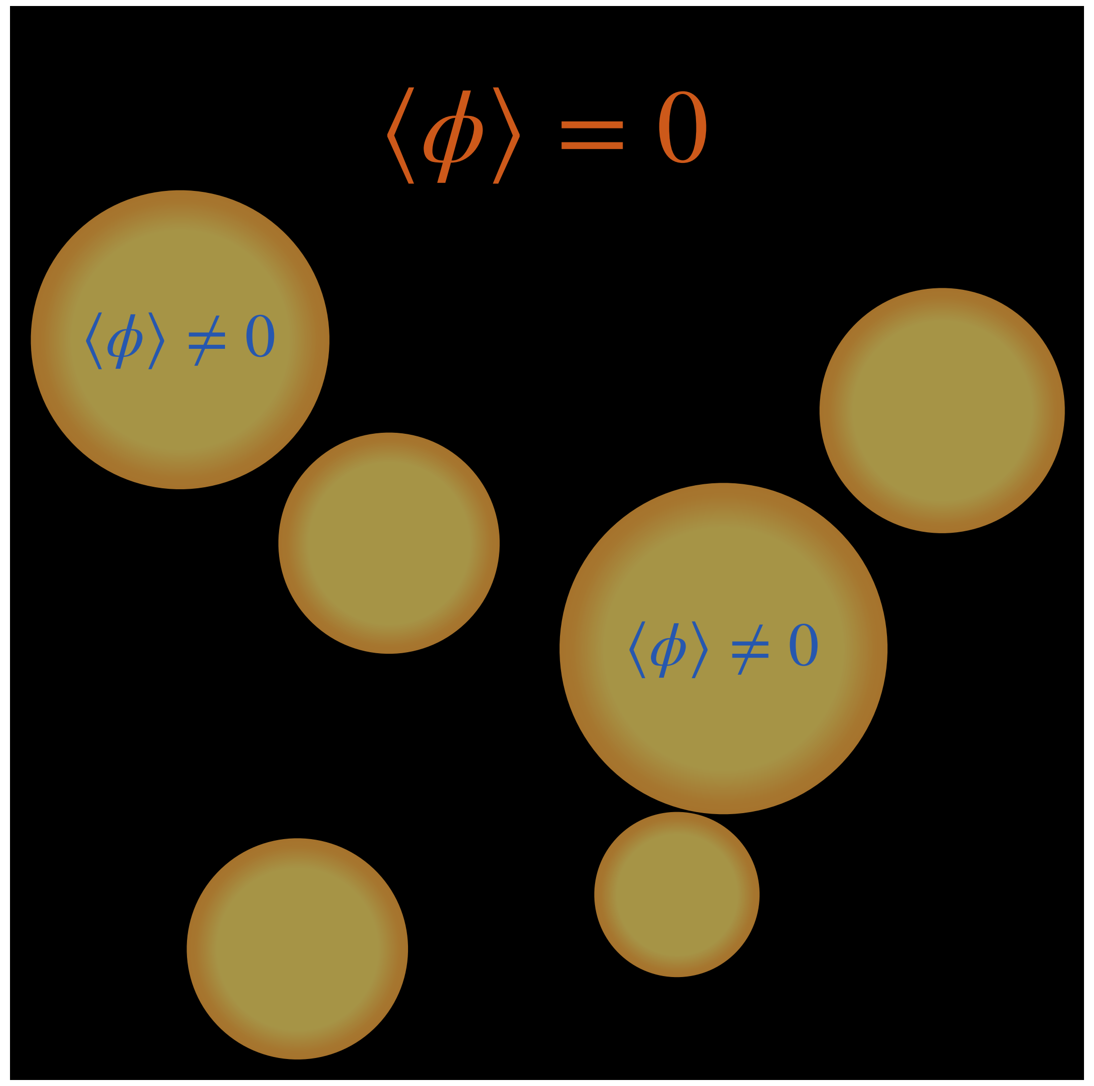
Schematics of a first-order PT

Bubble nucleations



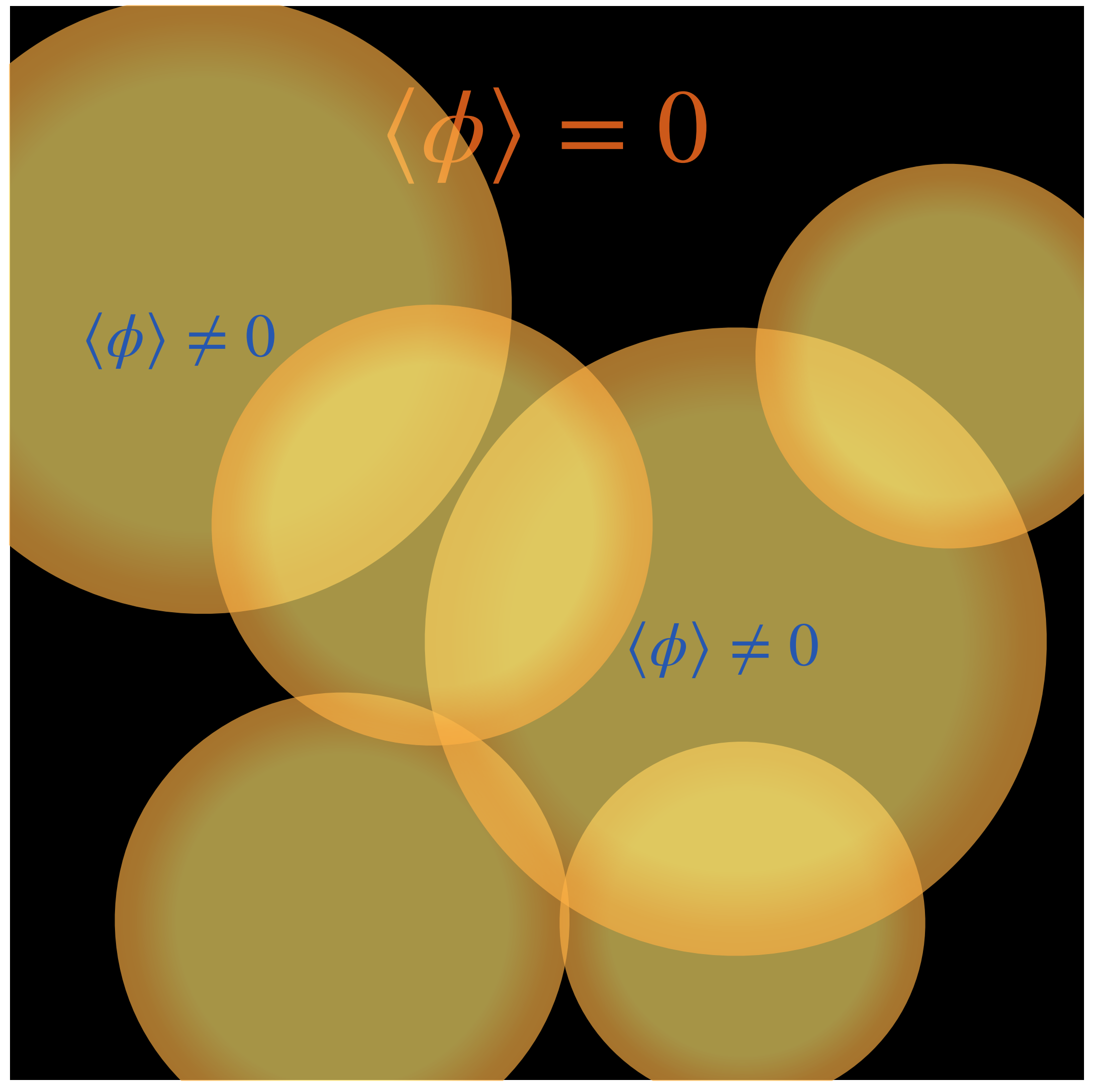
- Universe **initially homogeneous**
- T drops below T_* and ϕ locally **tunnels** to TV
- Bubbles **nucleate** and begin **expanding**

Bubbles expand



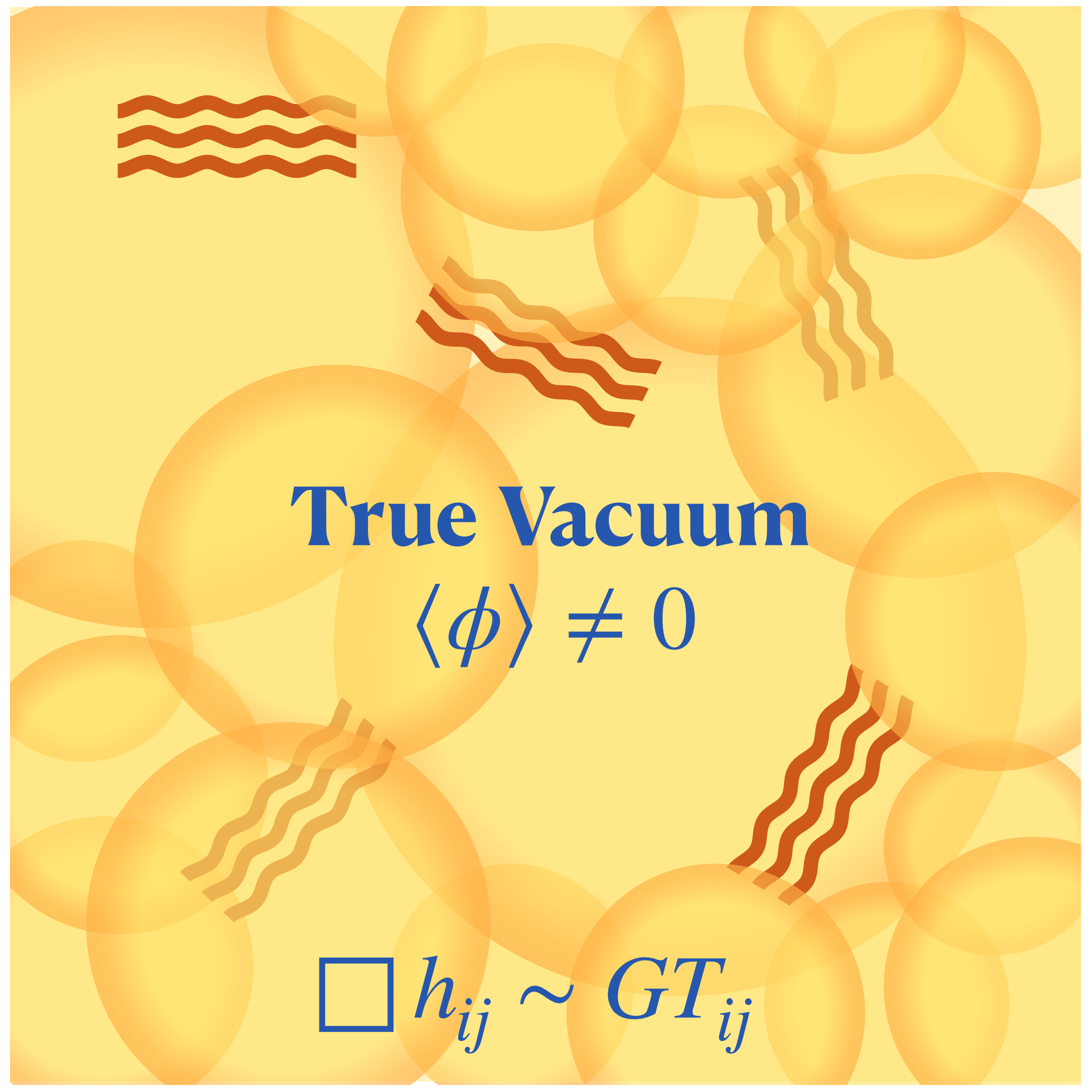
- **Expansion** proceeds
- **Vacuum energy deposits** in fluid
- Fluid **responds**
- **Self-similar** profiles develop

Bubbles collide



- Bubbles **collide**
- **GW production** from **anisotropic** stress T_{ij}^{plasma}

Sound waves



- **PT completion**
- Long-lasting **dynamics (sound waves)** source GWs
- **Non-linear** evolution may produce **shocks** and **turbulence**

PT parameters

Particle physics model
- Lagrangian $\mathscr{L}$
- Thermal field theory
- Effective potential

$\Downarrow$ Theory

PT parameters

- **Temperature** $T_* :\Longrightarrow H_* \to$ Horizon size $\to f_{\text{typcial}}$
- Inverse **duration** β : as in nucleation rate $\Gamma_{\text{nuc}} \propto e^{\beta t} \to R_*(\beta)$,
  $\beta/H \sim \mathcal{O}(100) \Rightarrow$ neglect expansion, $\omega_{\text{peak}} \simeq \frac{\beta}{100H}\frac{T}{100\text{GeV}}\text{mHz}$
- **Strength** $\alpha := \rho_{\text{vac}}/\rho_{\text{rad}}$
- **Wall velocity** $v_w : \longrightarrow$ energy budget and self-similar profiles

$\Rightarrow$ Simulation or theory

Hydrodynamics
- Character of **fluid perturbation**
- **Linear** sound-waves or **non-linear** evolution
- Detailed **evolution** of T_{ij}

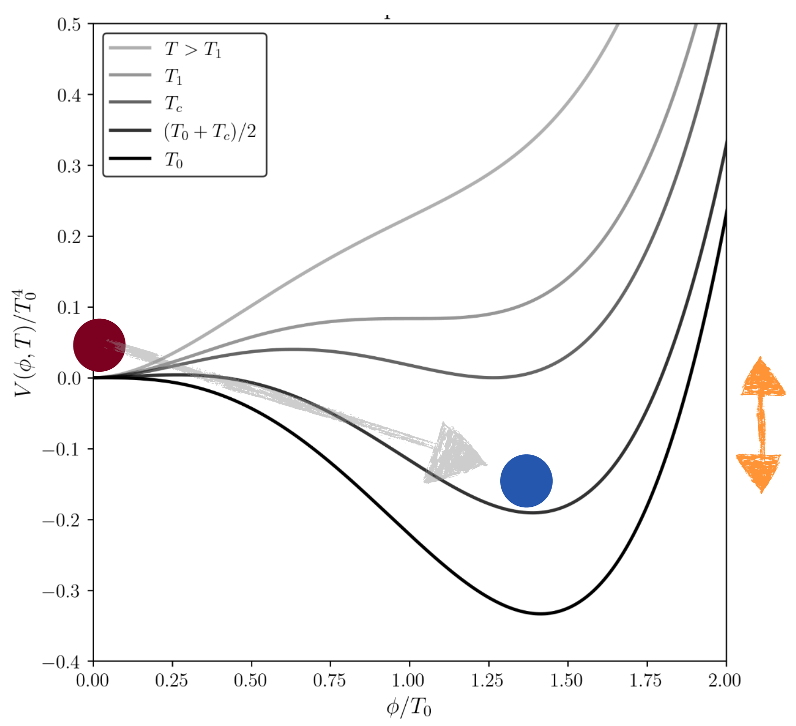
$\Rightarrow$

GW spectrum
- $\Omega_{\text{GW}}(f \mid H_*, \beta, \alpha, v_w)$

Higgsless simulations

Physical setup

Relativistic hydrodynamics



- **Perfect fluid:** $T^{\mu\nu} = (p+\rho)u^{\mu}u^{\nu} - p g^{\mu\nu}$

Thermal

- Bag **E.o.S.**: $\rho = aT^4 + \epsilon$ and. $p = \frac{1}{3}aT^4 - \epsilon$ ← Vacuum

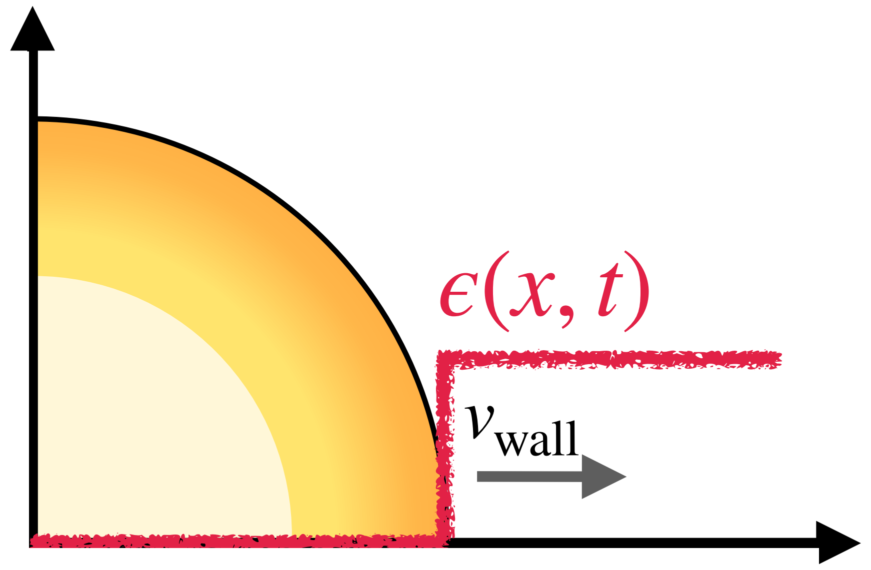
  where $\epsilon = \begin{cases} \varepsilon_+ \equiv \frac{3\alpha}{4} w, & \text{in the symmetric phase,} \\ 0, & \text{in the broken phase} \end{cases}$

is the energy **difference between** the **symmetric** and **broken** phases.

- **E.o.M.** given by **total energy-momentum conservation:** $\partial_\mu T^{\mu\nu} = 0$

- 4 equations: $$\begin{aligned} \partial_t K^0 + \nabla_i K^i &= 0 \\ \partial_t K^j + \nabla_i T^{ij}\left[K^\mu\right] &= 0 \end{aligned}$$ where $$T^{ij}\left[K^\mu\right] = \frac{K^i K^j}{K^0 + p} + p[K^\mu]$$

- 4 dynamical variables: $K^\mu \equiv T^{0\mu}$



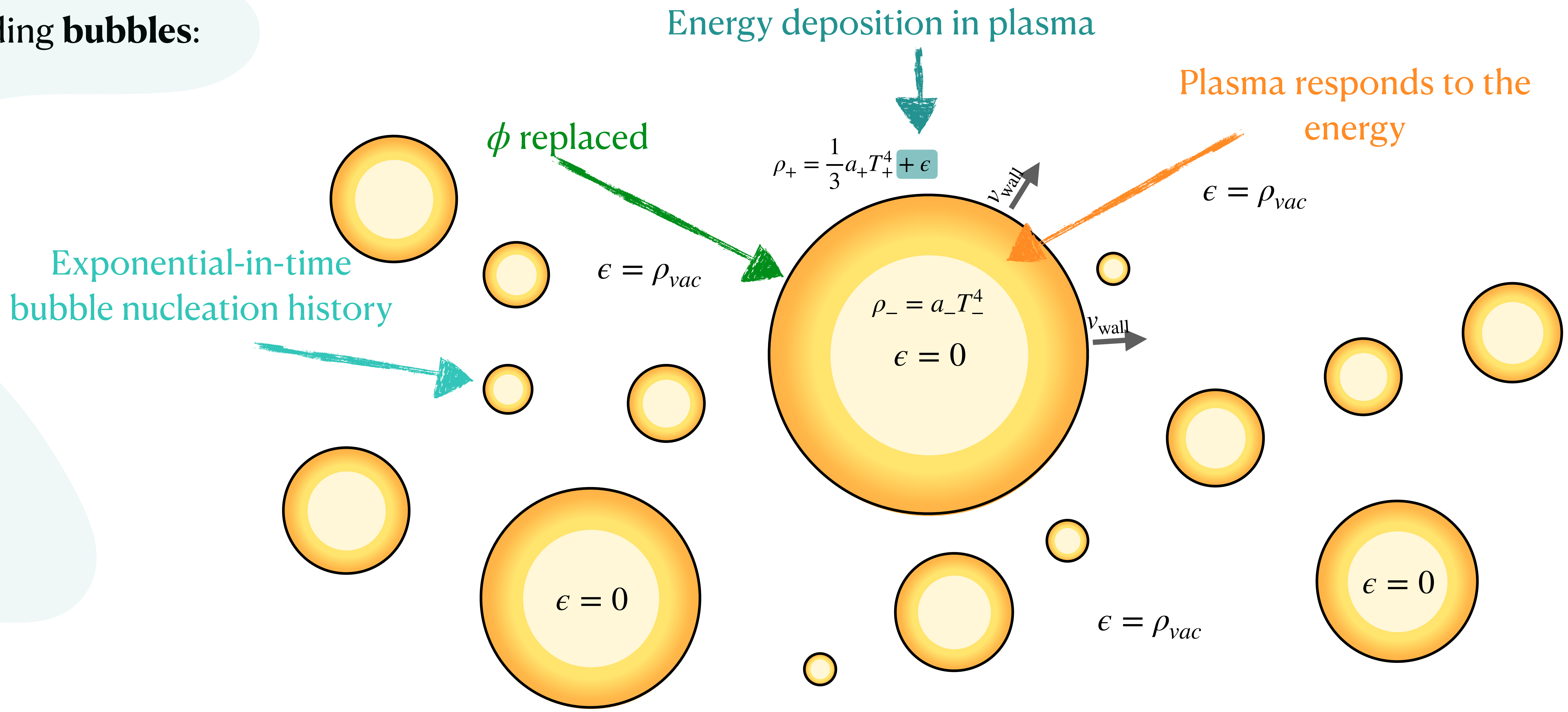
Physical setup

Bubbles and the Equation of State

- **Bag E.o.S.** suggests natural way to **embed** the expanding **bubbles**:
  - Prescribe that $\epsilon = \epsilon(t, \vec{x})$ is a **function** of **space** and **time**
  - **Bubbles** are **radially expanding** regions where $\epsilon = 0$

so that $\epsilon(t, \vec{x}\,|\,\alpha) = \begin{cases} 0, & \textbf{inside bubbles,} \\ \frac{3\alpha}{4} w, & \textbf{outside bubbles} \end{cases}$

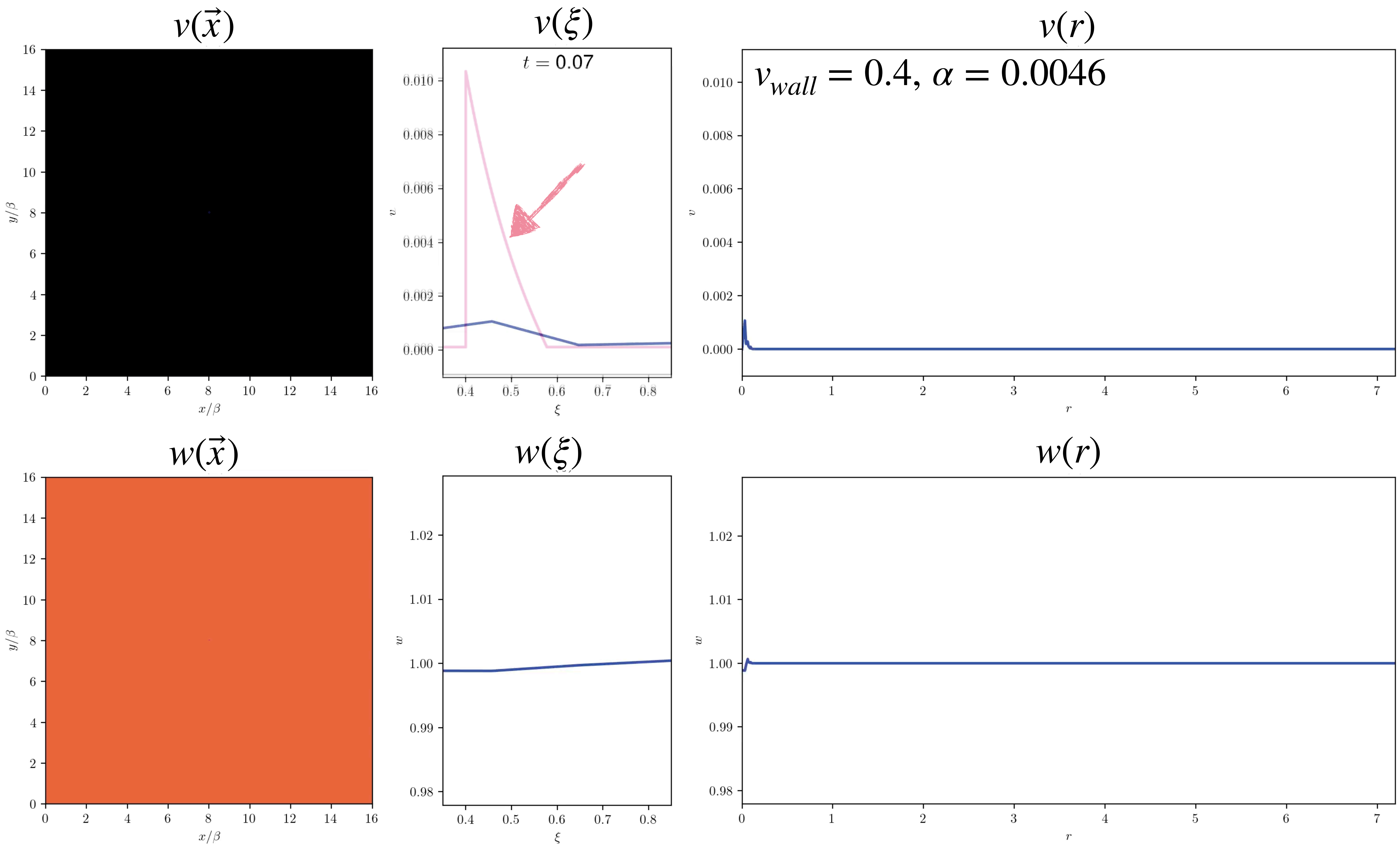
- To determine $\epsilon(t, \vec{x})$, we **need** to specify: v_w, α, and an exponential **bubble nucleation history.**
- **Nucleation rate** $\Gamma(t) \simeq \Gamma_* e^{\beta(t - t_*)}$



- Thus, with these simple 3 **ingredients,** $\epsilon(t, \vec{x}\,|\,\alpha)$ is **predetermined input** to the simulation, effectively **replacing** $\phi \to \epsilon(t, \vec{x}\,|\,\alpha)$
- **Couples to fluid** through $p = \frac{1}{6}\left[-2K^0 - 4\epsilon(t, \vec{x}\,|\,\alpha) + \sqrt{(4K^0 - 4\epsilon(t, \vec{x}\,|\,\alpha))^2 - 12K^iK^i}\right]$

as in $T^{ij}\left[K^\mu\right] = \frac{K^iK^j}{K^0 + p} + p$

Convergence of self-similar solutions



Convergence of self-similar solutions

- **Remarkable agreement** with analytical solutions
- **Resolves** and handles **shocks waves**

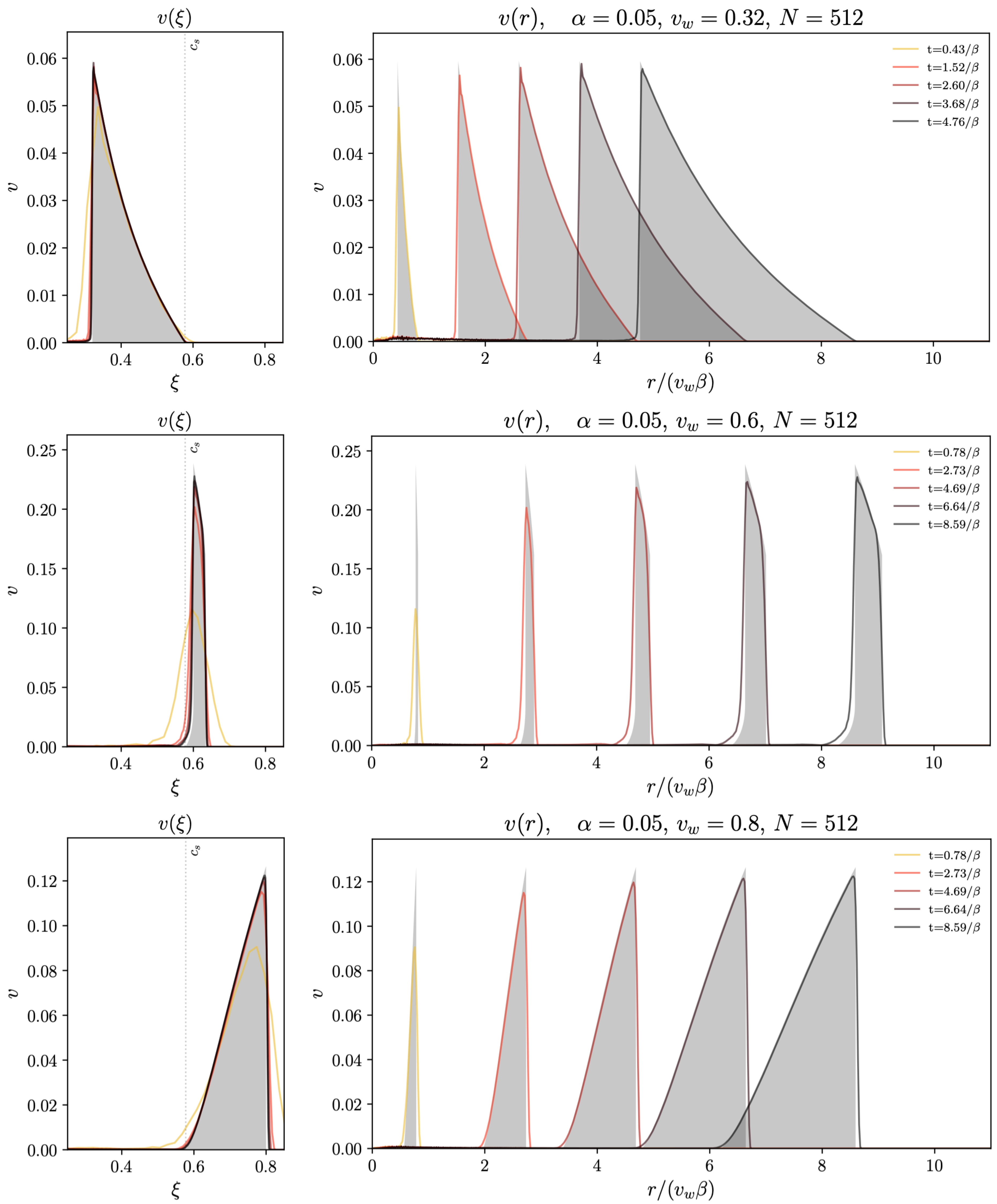
Convergence sufficiently **fast** that modest grid size **ensures** most **bubbles** in **self-similar** regime **before collision**.

Intermediate: $\alpha = 0.05$

Deflagrations

Hybrids

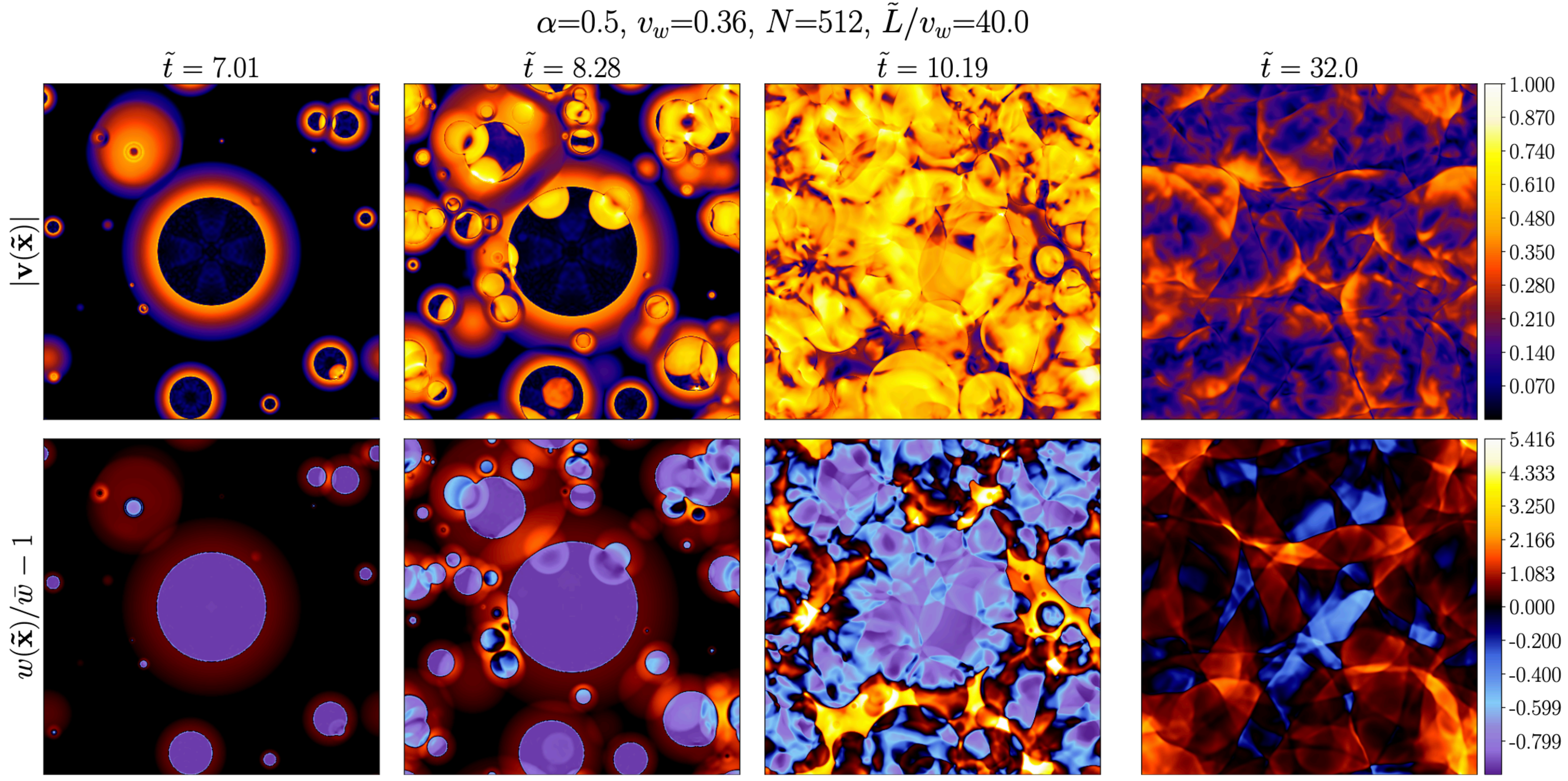
Detonations



We run simulations for...

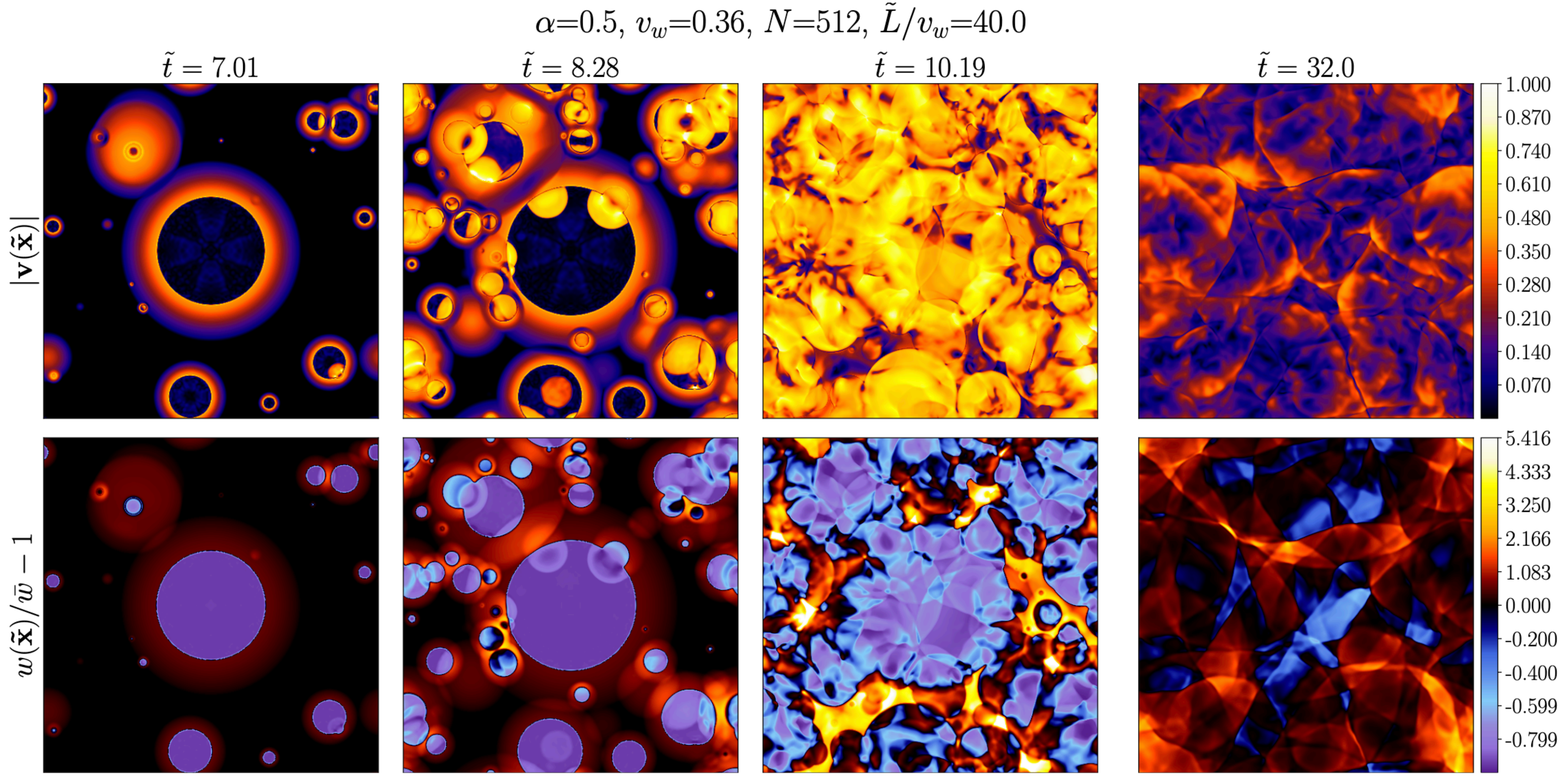
...different wall velocities $v_w \in [0.32, 0.8]$

and strengths $\alpha \in [0.0046, 0.05, 0.5]$



To scan parameter space.

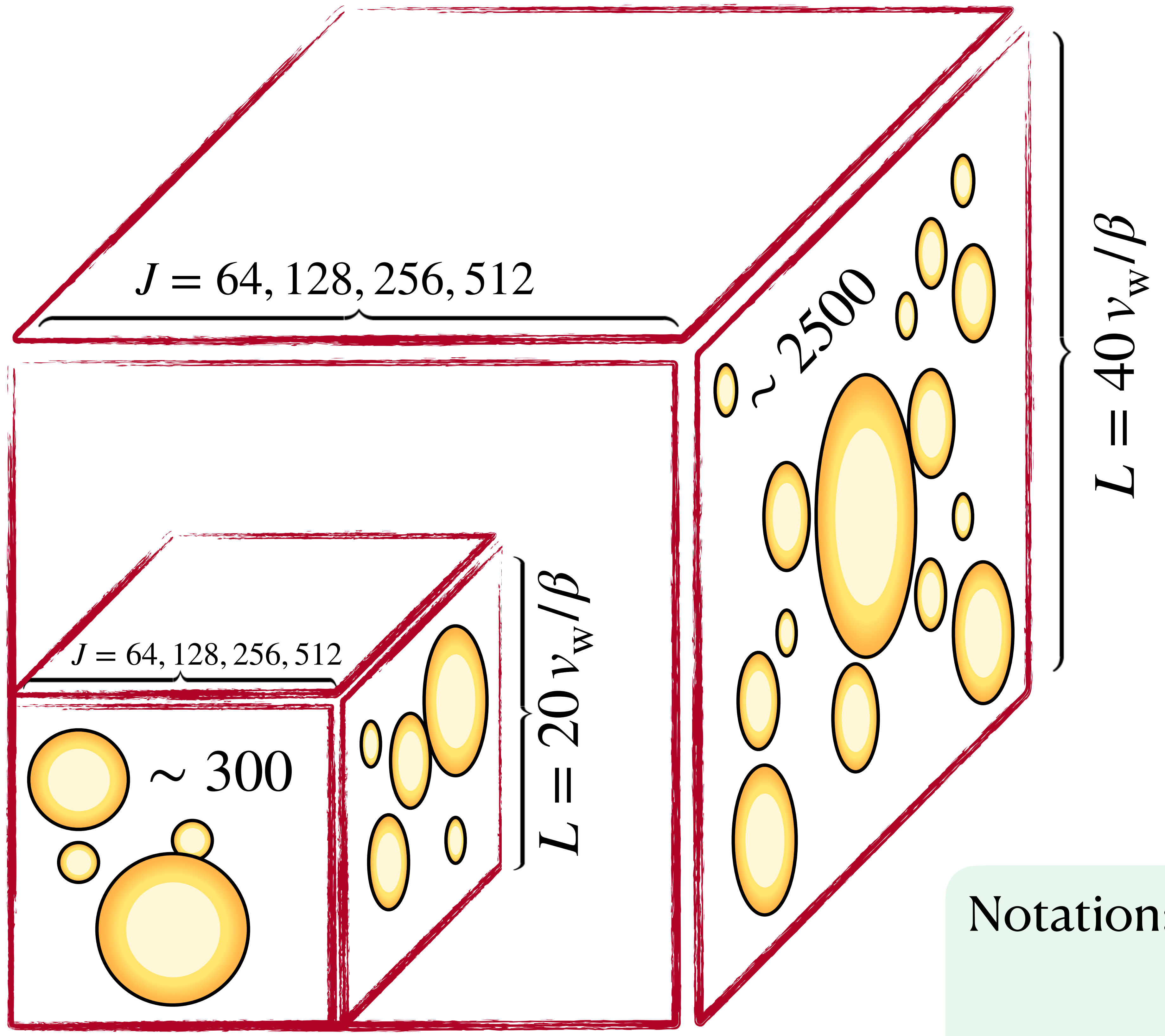
More than 1000 simulations were performed



Simulations

Summary

Grid



Total of
$\sim$ **1000** simulations

Notation: quantities marked by ˜ are normalized to β,

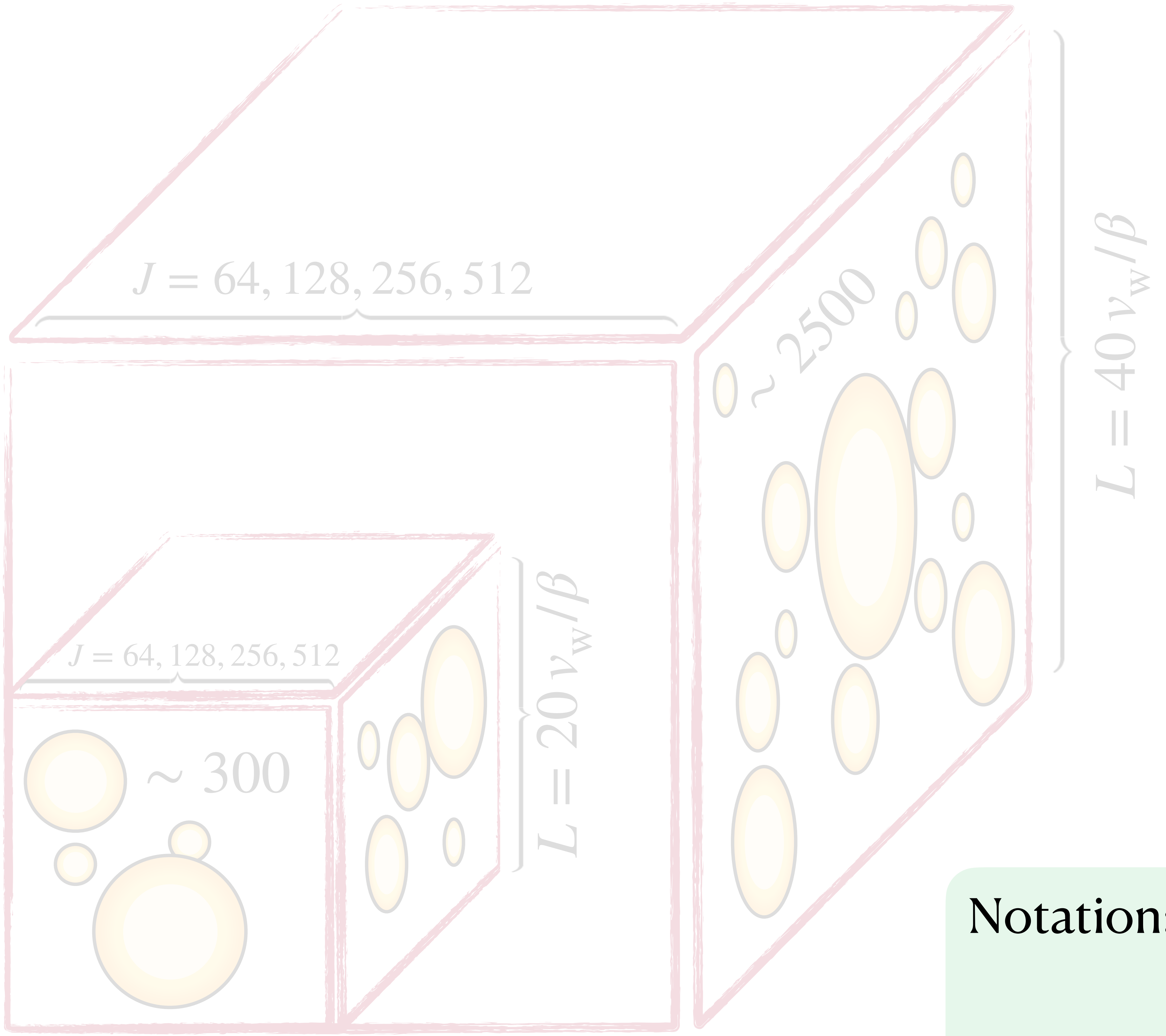
e.g., $\tilde{x} = x\beta$, $\tilde{t} = t\beta$, and $\tilde{k} = k/\beta$

- **Two box sizes** allows to capture both **IR** (more statistics, ~ 2500 bubbles) and **UV** (higher resolution per bubble, ~ 300 bubbles)
- A **series of resolutions** allows to study the **convergence** of energies and to develop an **extrapolation scheme.**

Physics

- Systematic **parameter scan** in v_w, α including **strong PTs**
- Realistic **exponential-in-time bubble nucleation histories**
- 10 different bubble **nucleation histories** for $v_w \in \{0.32, 0.6, 0.8\}$ and all α's to study **statistical variance of extracted parameters**

Grid



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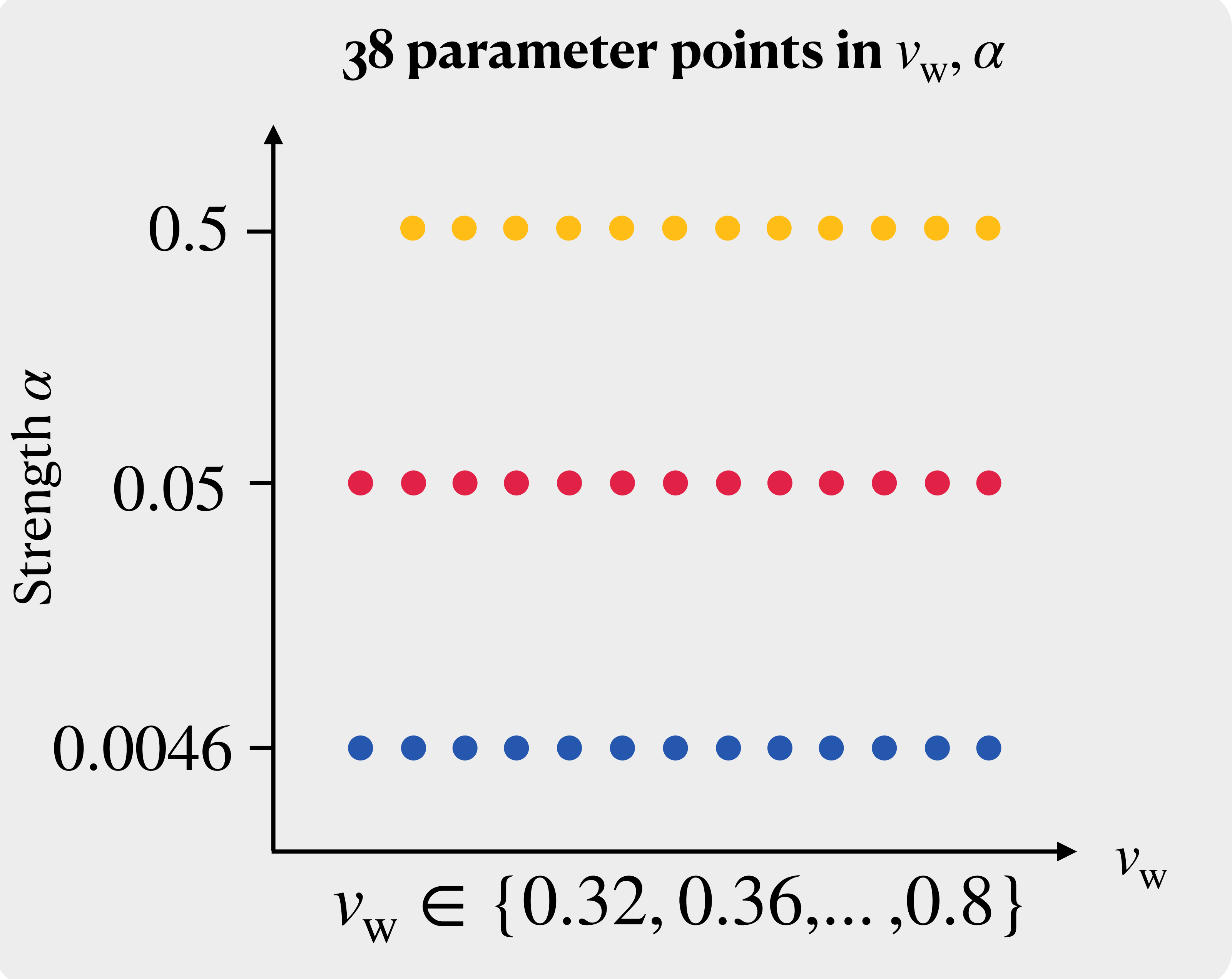

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- Realistic **exponential-in-time bubble nucleation histories**
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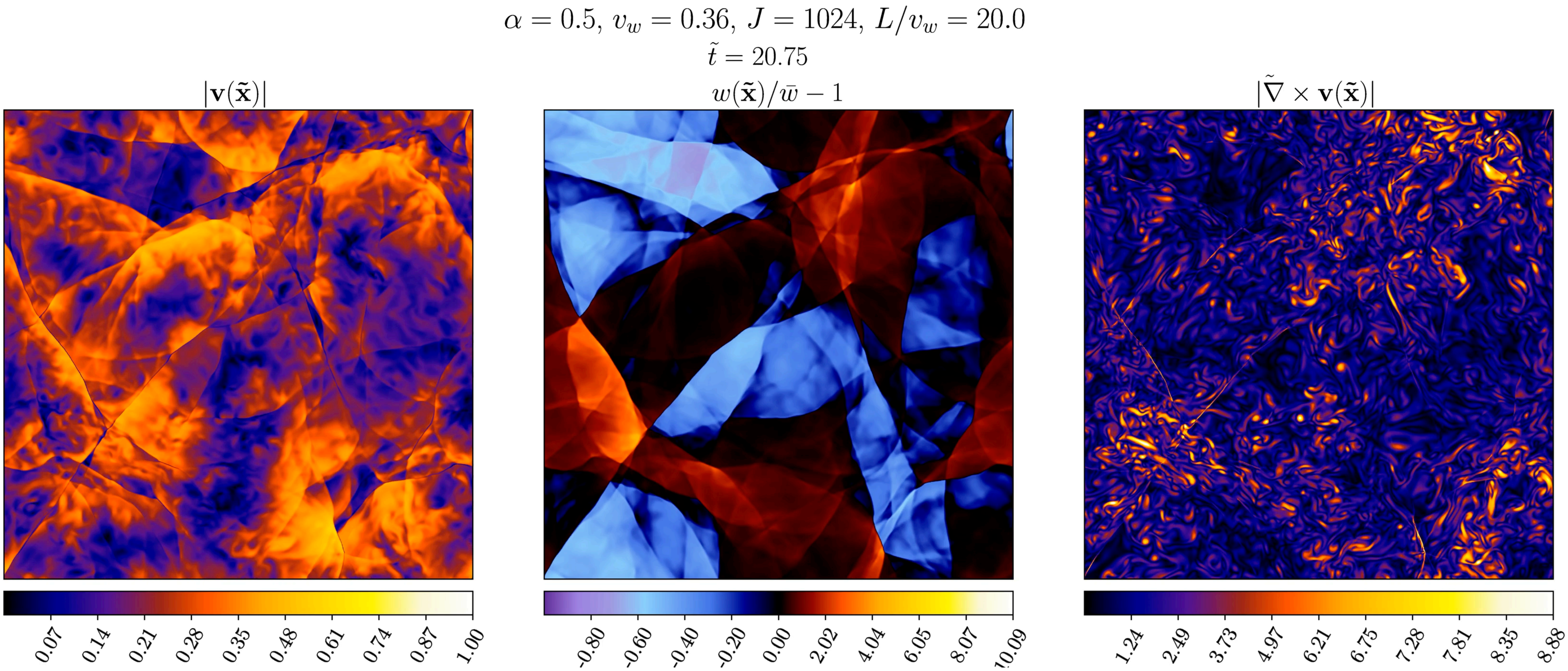
Example

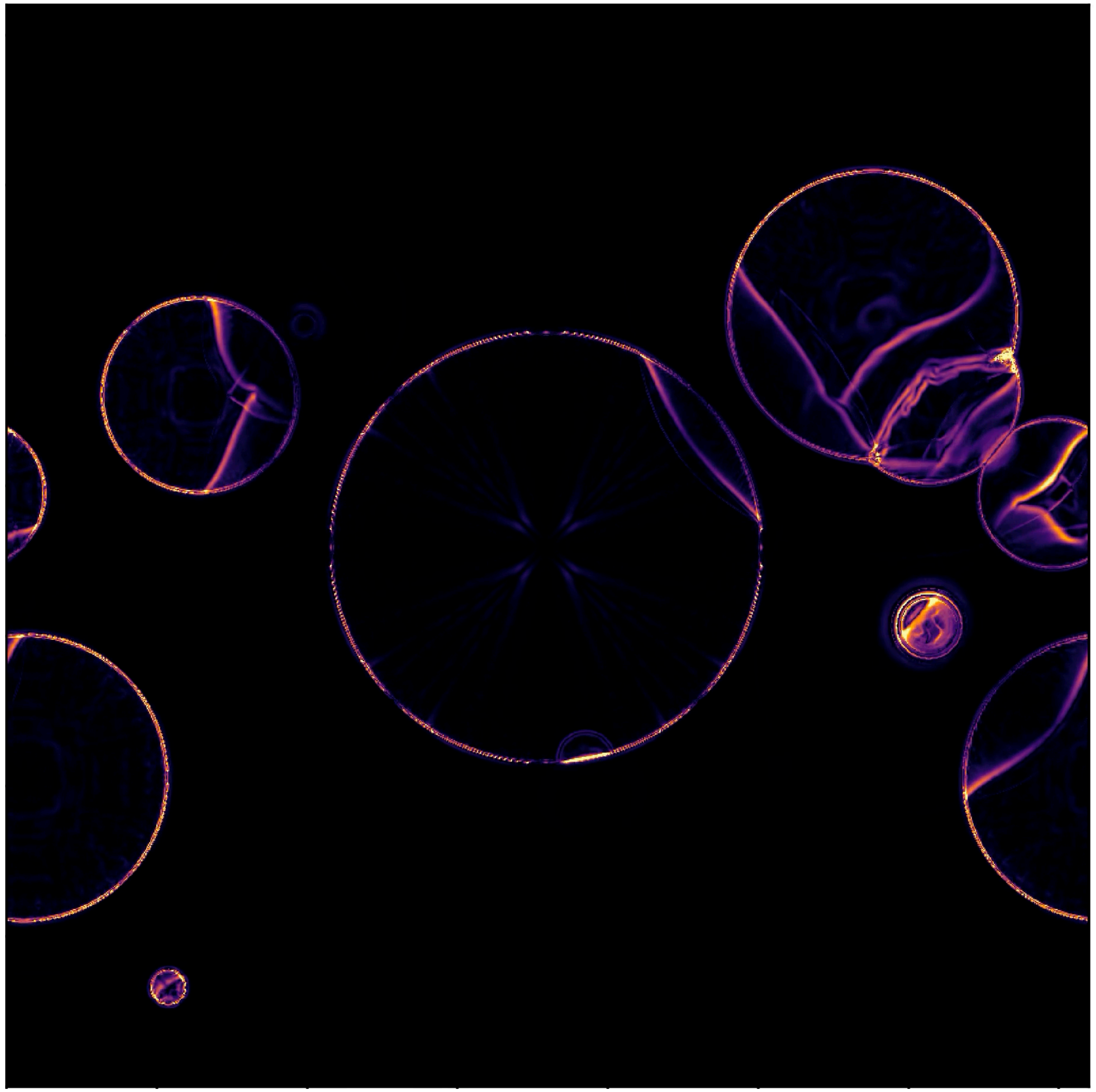


$\alpha = 0.5,\ v_w = 0.36,\ J = 1024,\ L/v_w = 20.0$
$\tilde{t} = 20.75$
$|\mathbf{v}(\tilde{\mathbf{x}})|$
$w(\tilde{\mathbf{x}})/\bar{w} - 1$
$|\tilde{\nabla} \times \mathbf{v}(\tilde{\mathbf{x}})|$
0.07 0.14 0.21 0.28 0.35 0.48 0.61 0.74 0.87 1.00
-0.80 -0.60 -0.40 -0.20 0.00 2.02 4.04 6.05 8.07 10.09
1.24 2.49 3.73 4.97 6.21 6.75 7.28 7.81 8.35 8.88

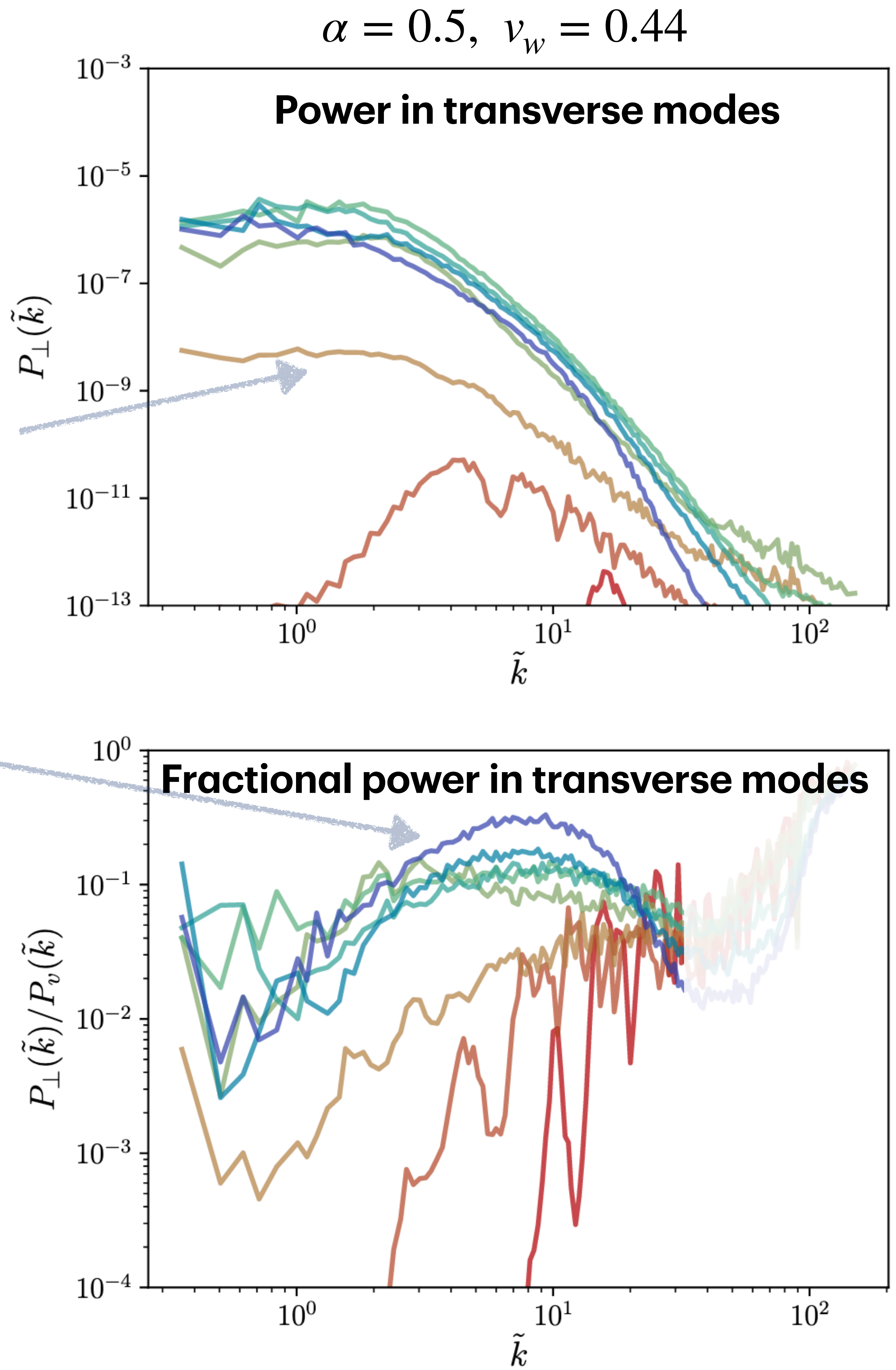
Departure from linearity

Vorticity

$|\nabla \times v|$

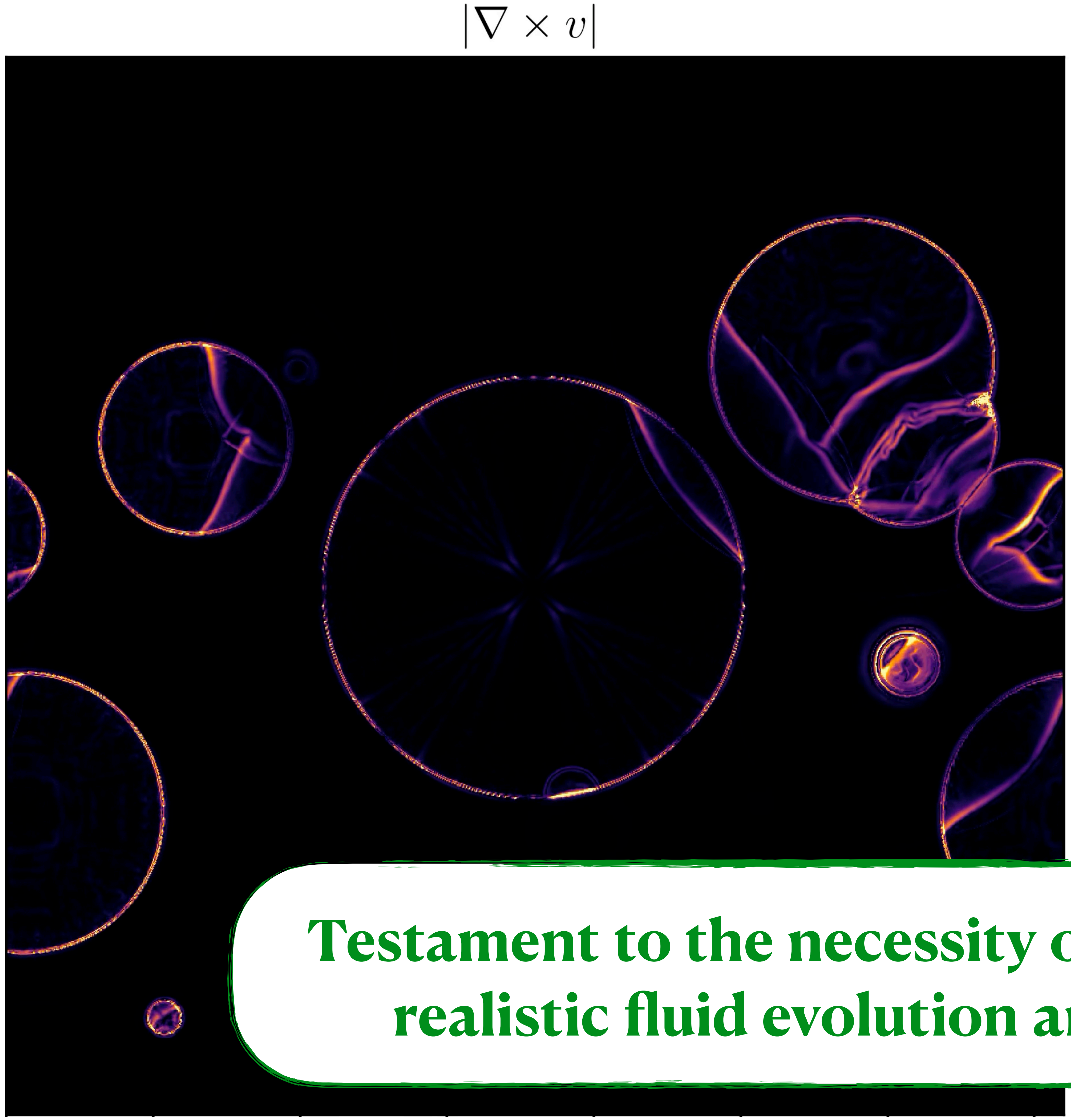


- $P_{\perp}(k)$ **initially vanishing**
- **Transfer** of **energy** from **longitudinal** to **transverse** modes
- **Transverse** velocity spectrum **grows with time**
- **Evolution** is **non-linear**
- **30%** of **power** in **transverse** modes



Departure from linearity

Vorticity



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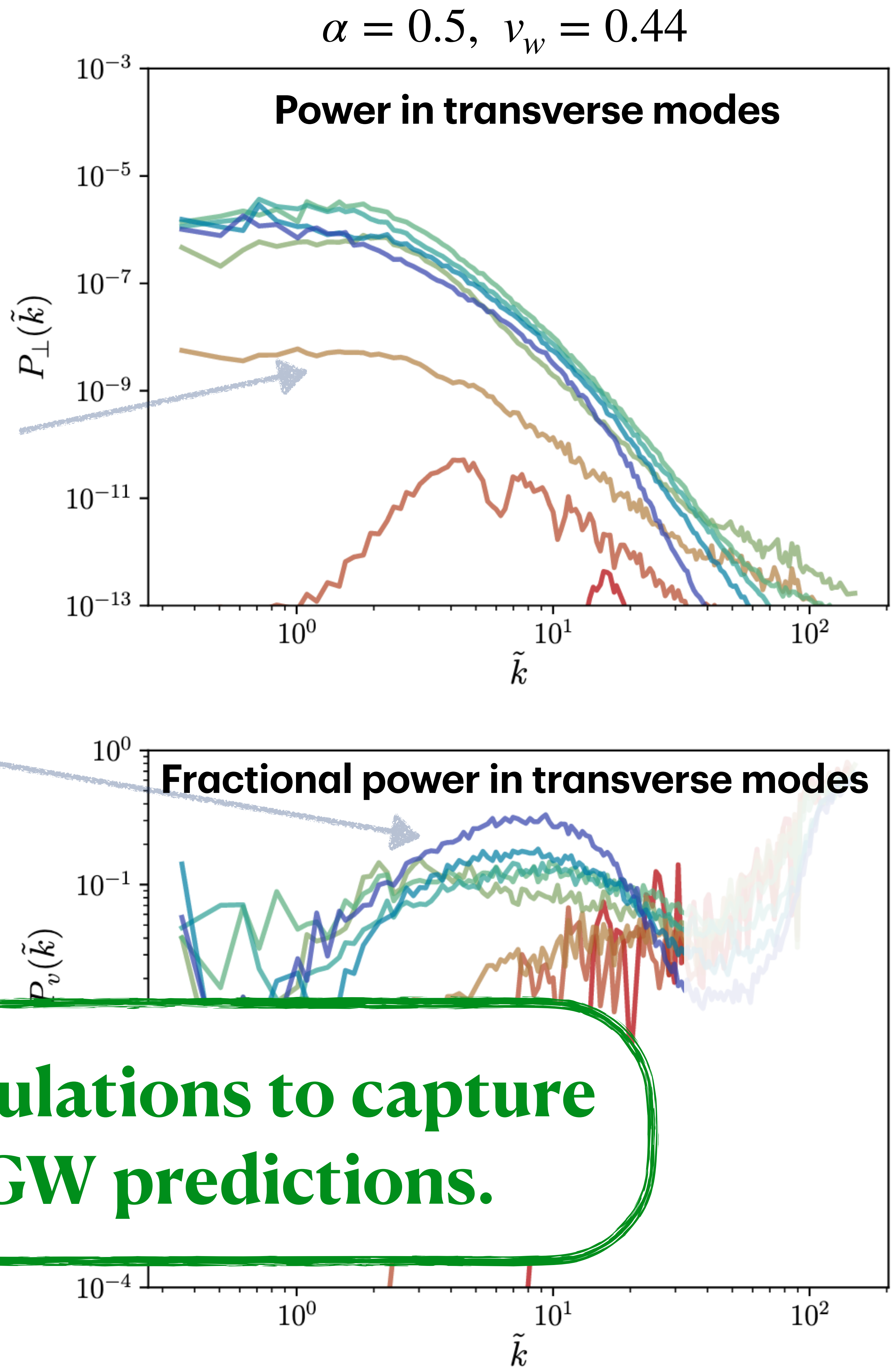

Testament to the necessity of using numerical simulations to capture realistic fluid evolution and to derive accurate GW predictions.

GW production

GW production

GW spectrum

$$\Omega_{\mathrm{GW}}(\mathrm{k}) = \frac{1}{\rho_{\mathrm{c}}}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln \mathrm{k}} = 3\,\mathcal{T}_{\mathrm{GW}}\,\mathcal{J}(k) = 3\,\mathcal{T}_{\mathrm{GW}}\,\frac{k}{2}H_*^2\int_{t_*}^{t_{\mathrm{fin}}}\int_{t_*}^{t_{\mathrm{fin}}} E_\Pi\left(t_1, t_2, k\right)\cos k\left(t_1 - t_2\right)\,dt_1 dt_2 \equiv 3\,\mathcal{T}_{\mathrm{GW}}\left(\frac{H_*}{\beta}\right)^2 \mathcal{I}_{\mathrm{sim}}(\tilde{t}_*, \tilde{t}_{\mathrm{fin}}, \tilde{q})$$

GW production

GW spectrum

Complete knowledge of $T_{\mu\nu}$ from simulations

$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{q}) = \frac{\tilde{q}^3}{4\pi^2 \tilde{V} \tilde{\bar{\rho}}_*^2} \int \frac{d\Omega_k}{4\pi} \left[\Lambda_{ij,kl} \tilde{T}_{ij}(\tilde{q}, \tilde{\mathbf{k}}) \tilde{T}^*_{kl}(\tilde{q}, \tilde{\mathbf{k}})\right]_{\tilde{q}=|\tilde{\mathbf{k}}|}$$

Source GWs from $\tilde{t}_* = 16$ until $\tilde{t}_{\rm fin} = 32$

$$\Omega_{\rm GW}(k) = \frac{1}{\rho_c} \frac{d\rho_{\rm GW}}{d\ln k} = 3\,\mathcal{T}_{\rm GW}\,\mathcal{J}(k) = 3\,\mathcal{T}_{\rm GW} \frac{k}{2} H_*^2 \int_{t_*}^{t_{\rm fin}} \int_{t_*}^{t_{\rm fin}} E_\Pi(t_1, t_2, k) \cos k(t_1 - t_2)\, dt_1 dt_2 \equiv 3\,\mathcal{T}_{\rm GW} \left(\frac{H_*}{\beta}\right)^2 \mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{q})$$

GW production

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UETC

from sound waves

$$E_\Pi (t_1, t_2, k) = 2k^2 K^2 \times f(t_-, k)$$

- $f(t_-, k)$ **parametrizes the strength of the correlations**
- **Stationary source** $\Longrightarrow$
  - **- Kinetic energy $\propto K$ is constant**
  - **- Correlations captured by $f(t_-, k)$ depending only on $t_- = t_2 - t_1$**

E.g. 1304.2433 and 2308.12943

GW production

GW spectrum

Complete knowledge of $T_{\mu\nu}$ from simulations

$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{q}) = \frac{\tilde{q}^3}{4\pi^2 \tilde{V} \tilde{\rho}_*^2} \int \frac{d\Omega_k}{4\pi} \left[\Lambda_{ij,kl} \tilde{T}_{ij}(\tilde{q}, \tilde{\mathbf{k}}) \tilde{T}^*_{kl}(\tilde{q}, \tilde{\mathbf{k}}) \right]_{\tilde{q}=|\tilde{\mathbf{k}}|}$$

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UETC

$$E_\Pi\left(t_1, t_2, k\right) = 2k^2 K^2\left(t_+\right) f\left(t_-, k\right)$$

from ~~sound waves~~ a damped source

- **$f(t_-, k)$ parametrizes the strength of the correlations**
- **Locally Stationary source $\Longrightarrow$**
  - **Kinetic energy $\propto K$ is ~~constant~~ decaying, depending on $t_+ = (t_2 + t_1)/2$**
  - **Correlations captured by $f(t_-, k)$ depending only on $t_- = t_2 - t_1$**

GW production

GW spectrum

Complete knowledge of $T_{\mu\nu}$ from simulations

$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{q}) = \frac{\tilde{q}^3}{4\pi^2 \tilde{V} \tilde{\bar{\rho}}_*^2} \int \frac{d\Omega_k}{4\pi} \left[\Lambda_{ij,kl} \tilde{T}_{ij}(\tilde{q}, \tilde{\mathbf{k}}) \tilde{T}^*_{kl}(\tilde{q}, \tilde{\mathbf{k}}) \right]_{\tilde{q}=|\tilde{\mathbf{k}}|}$$

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$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW}\, K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin})\, (\beta R_*)\, S(k)$$

$\Longrightarrow$ **Generalization of GW spectrum parametrization from sound waves**

GW efficiency

Integrated kinetic energy

Characteristic scale of fluid perturbations

Spectral shape

GW production

GW spectrum from ~~sound waves~~ damped sources

$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW}\, K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin})\, (\beta R_*)\, S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

How large is the GW efficiency?

$$\mathcal{I}_{\text{sim}}(\tilde{t}_*, \tilde{t}_{\text{fin}}, \tilde{k}) = \tilde{\Omega}_{\text{GW}} \; K^2_{\text{int}}(\tilde{t}_*, \tilde{t}_{\text{fin}}) \; (\beta R_*) \; S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

How much of the vacuum energy is transferred to the plasma in the form of kinetic energy and how can the damping be modeled?

How large is the GW efficiency?

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GW production

GW spectrum from ~~sound waves~~ damped sources

How much of the vacuum energy is transferred to the plasma in the form of kinetic energy and how can the damping be modeled?

Which definition of R_* is most meaningful?

How large is the GW efficiency?

$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW}\, K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin})\, (\beta R_*)\, S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

How much of the vacuum energy is transferred to the plasma in the form of kinetic energy and how can the damping be modeled?

Which definition of R_* is most meaningful?

How large is the GW efficiency?

What is the spectral shape? In particular, where is the peak, and what are the IR and UV slopes?

$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW}\, K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin})\, (\beta R_*)\, S(k)$$

Results

$$\mathscr{I}_{\text{sim}}(\tilde{t}_*, \tilde{t}_{\text{fin}}, \tilde{k}) = \tilde{\Omega}_{\text{GW}} \; K_{\text{int}}^2(\tilde{t}_*, \tilde{t}_{\text{fin}}) \; (\beta R_*) \; S(k)$$

$$\mathcal{I}_{\rm sim}(\tilde t_*, \tilde t_{\rm fin}, \tilde k) = \tilde\Omega_{\rm GW}\, K^2_{\rm int}(\tilde t_*, \tilde t_{\rm fin})\,(\beta R_*)\, S(k)$$

Results

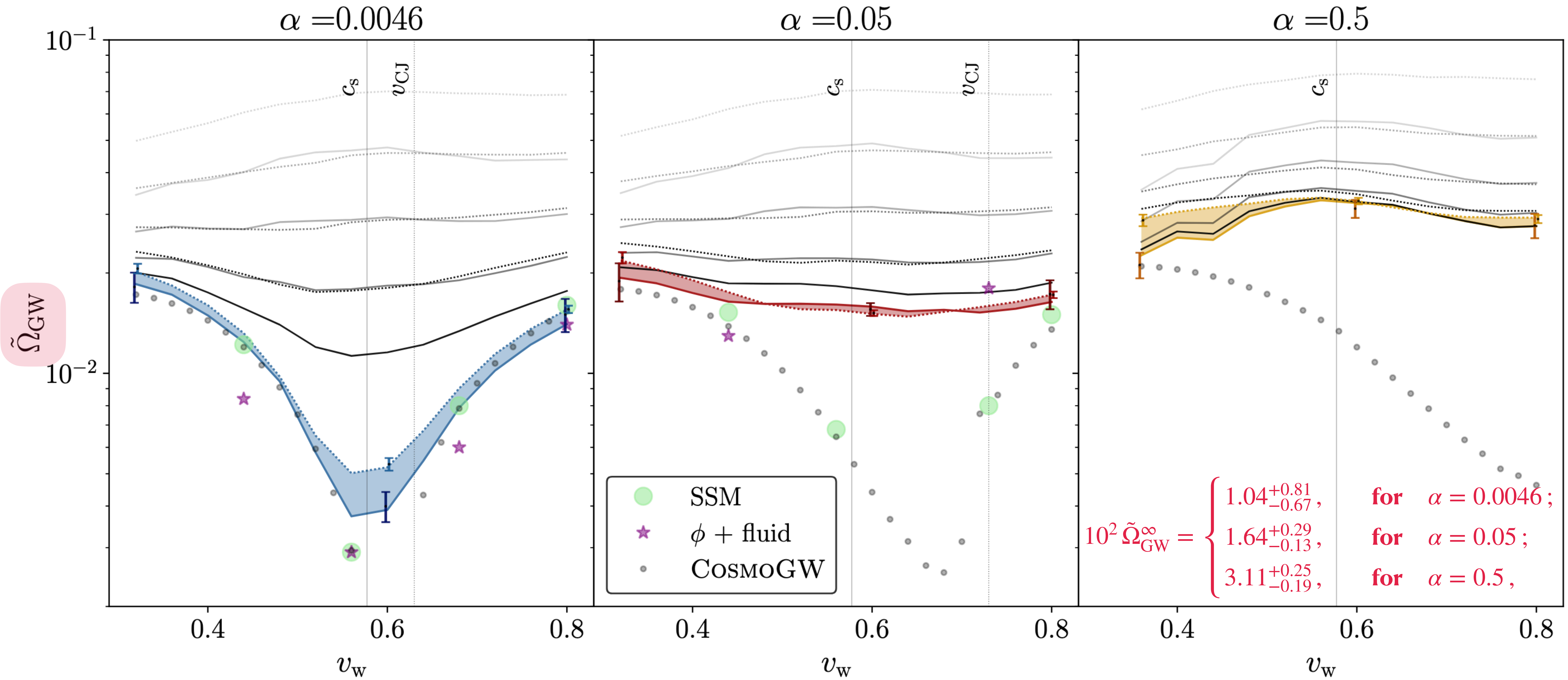
GW efficiency

$$\tilde\Omega_{\rm GW} = \frac{\mathcal{I}^{\rm int}_{\rm sim}(\tilde t_{\rm init}, \tilde t_{\rm end})}{K^2_{\rm int}(\tilde t_{\rm init}, \tilde t_{\rm end})(\beta R_*)}$$

Weak: Agreement with sound shell model and ϕ + fluid simulations

Interm: Agreement with ϕ + fluid simulations but deviations from sound shell model

Strong: new prediction



$$10^2\,\tilde\Omega^\infty_{\rm GW} = \begin{cases} 1.04^{+0.81}_{-0.67}, & \textbf{for} \quad \alpha = 0.0046\,; \\ 1.64^{+0.29}_{-0.13}, & \textbf{for} \quad \alpha = 0.05\,; \\ 3.11^{+0.25}_{-0.19}, & \textbf{for} \quad \alpha = 0.5\,, \end{cases}$$

$$\mathscr{I}_{\text{sim}}(\tilde{t}_*, \tilde{t}_{\text{fin}}, \tilde{k}) = \tilde{\Omega}_{\text{GW}} \; K^2_{\text{int}}(\tilde{t}_*, \tilde{t}_{\text{fin}}) \; (\beta R_*) \; S(k)$$

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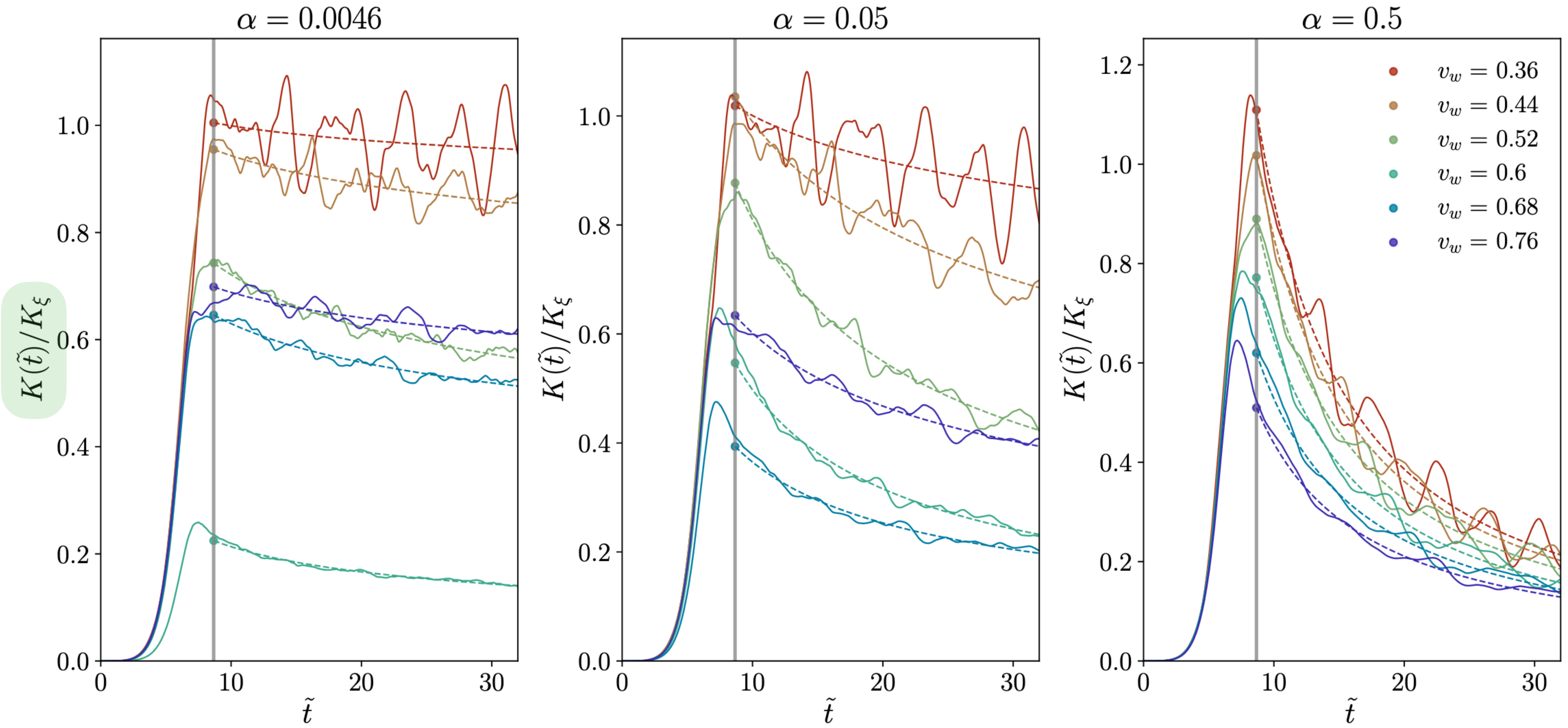
Results

$K^2_{\rm int}$

Weak: Nearly constant kinetic energy or weak damping

Interm: some observed damping

Strong: strong damping



$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW}\, K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin})\, (\beta R_*)\, S(k)$$

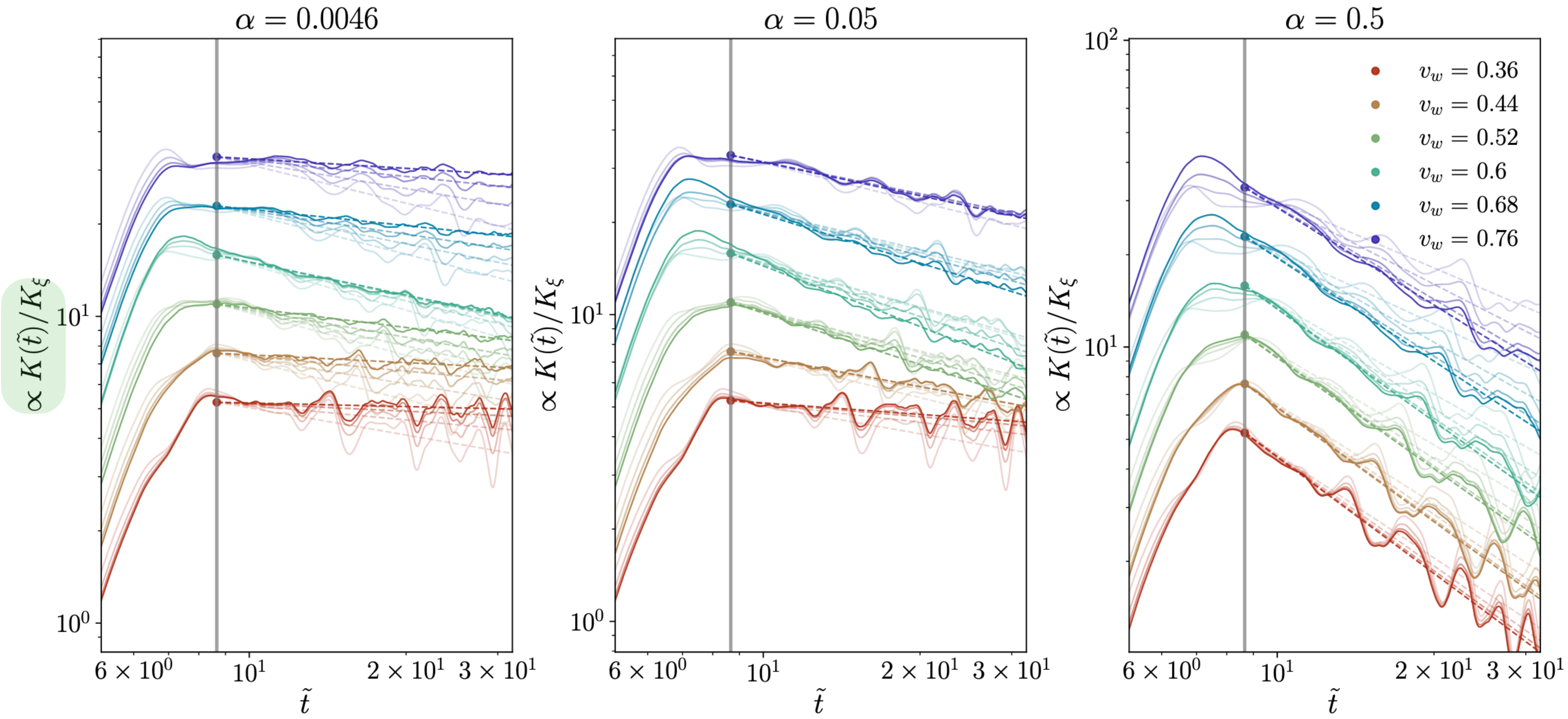
Results

$K^2_{\rm int}$

Weak: Nearly constant kinetic energy or weak damping

Interm: some observed damping

Strong: strong damping

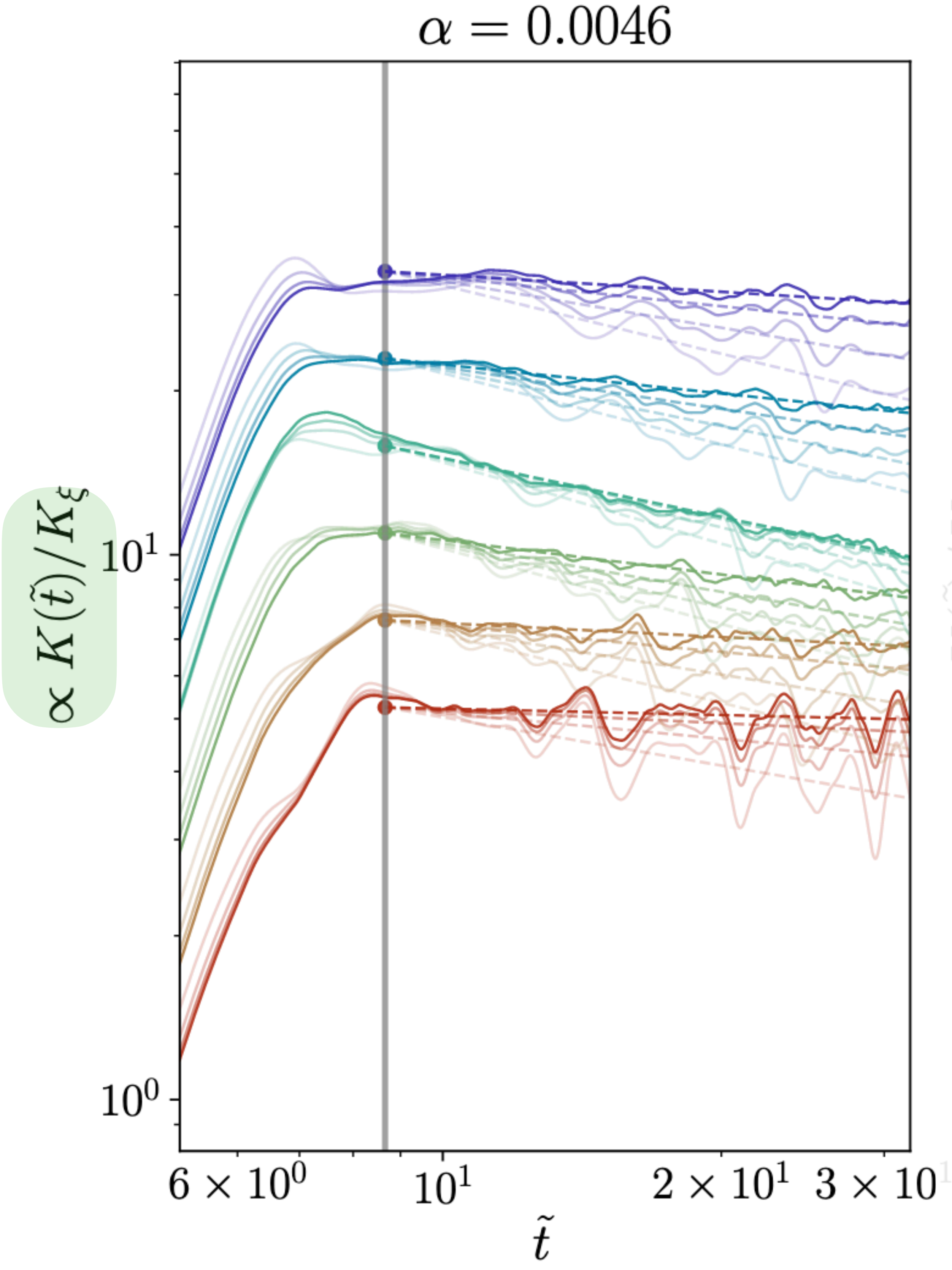


$$\mathcal{I}_{\text{sim}}(\tilde{t}_*, \tilde{t}_{\text{fin}}, \tilde{k}) = \tilde{\Omega}_{\text{GW}} K^2_{\text{int}}(\tilde{t}_*, \tilde{t}_{\text{fin}}) (\beta R_*) S(k)$$

Results

K^2_{int}

Weak: Nearly constant kinetic energy or weak damping



Interm: some observed damping

We fit a **power law**

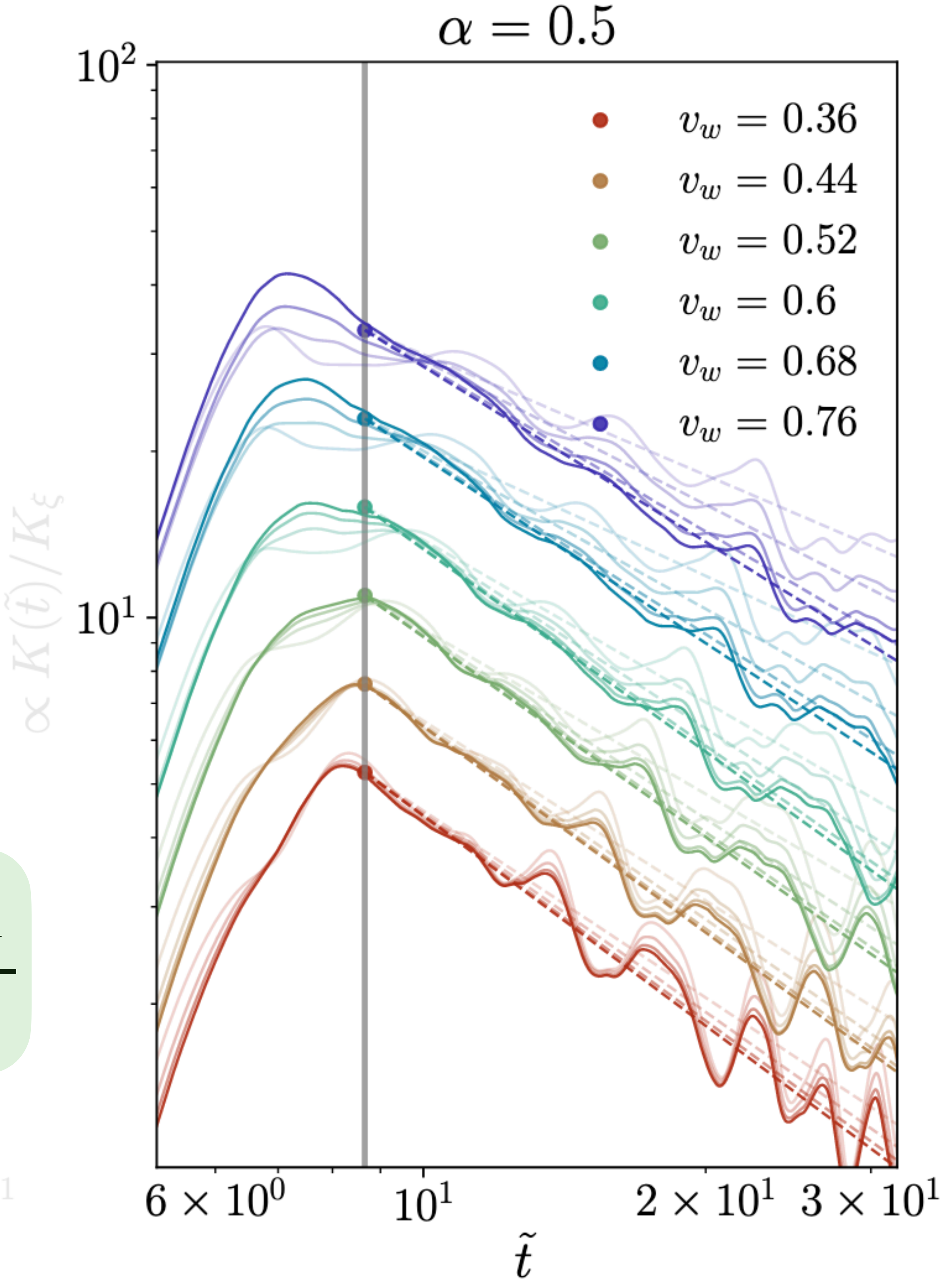
$$K(\tilde{t} > \tilde{t}_0) = K_0 \left(\frac{\tilde{t}}{\tilde{t}_0}\right)^{-b}$$

to the data,

so that

$$K^2_{\text{int}} = K_0^2\, \tilde{t}_0 \frac{(1 + \tilde{t}_{\text{sw}}/\tilde{t}_0)^{1-2b} - 1}{1 - 2b}$$

Strong: strong damping



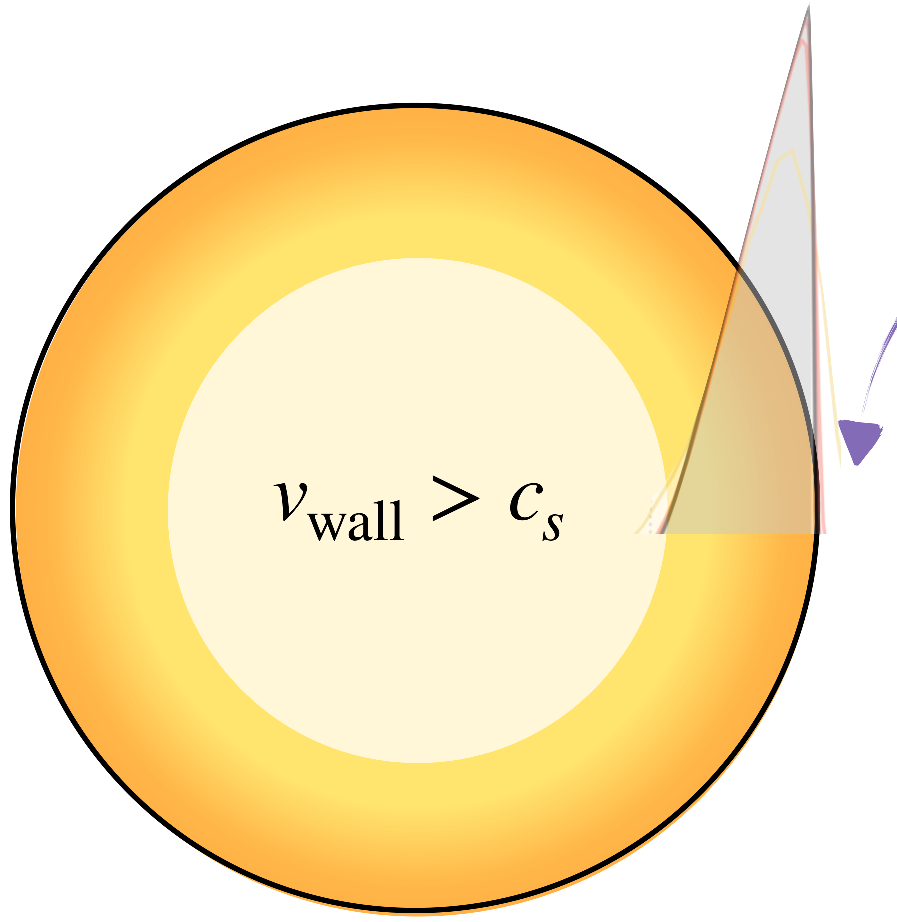
$$\mathscr{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW} \; K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin}) \; (\beta R_*) \; S(k)$$

$\mathcal{I}_{\text{sim}}(\tilde{t}_*, \tilde{t}_{\text{fin}}, \tilde{k}) = \tilde{\Omega}_{\text{GW}}\, K^2_{\text{int}}(\tilde{t}_*, \tilde{t}_{\text{fin}})\, (\beta R_*)\, S(k)$

Results

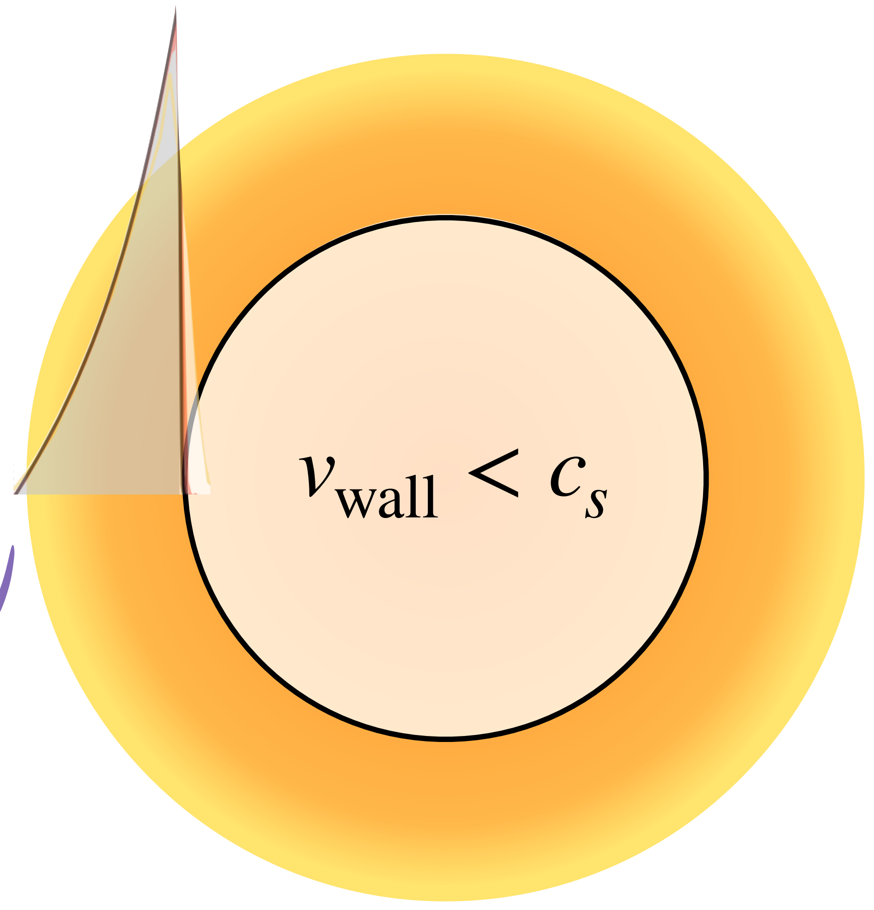
R_*

Detonation



$$R_* = \frac{3 \max\left(v_w, c_s\right)}{\beta}$$

Deflagration



- **Interpret** R_* as the typical **size** of **sound-shells** at collision
- This definition **reduces the dependence** of **other parameters** on v_w

$$\mathscr{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW} \; K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin}) \; (\beta R_*) \; S(k)$$

Results

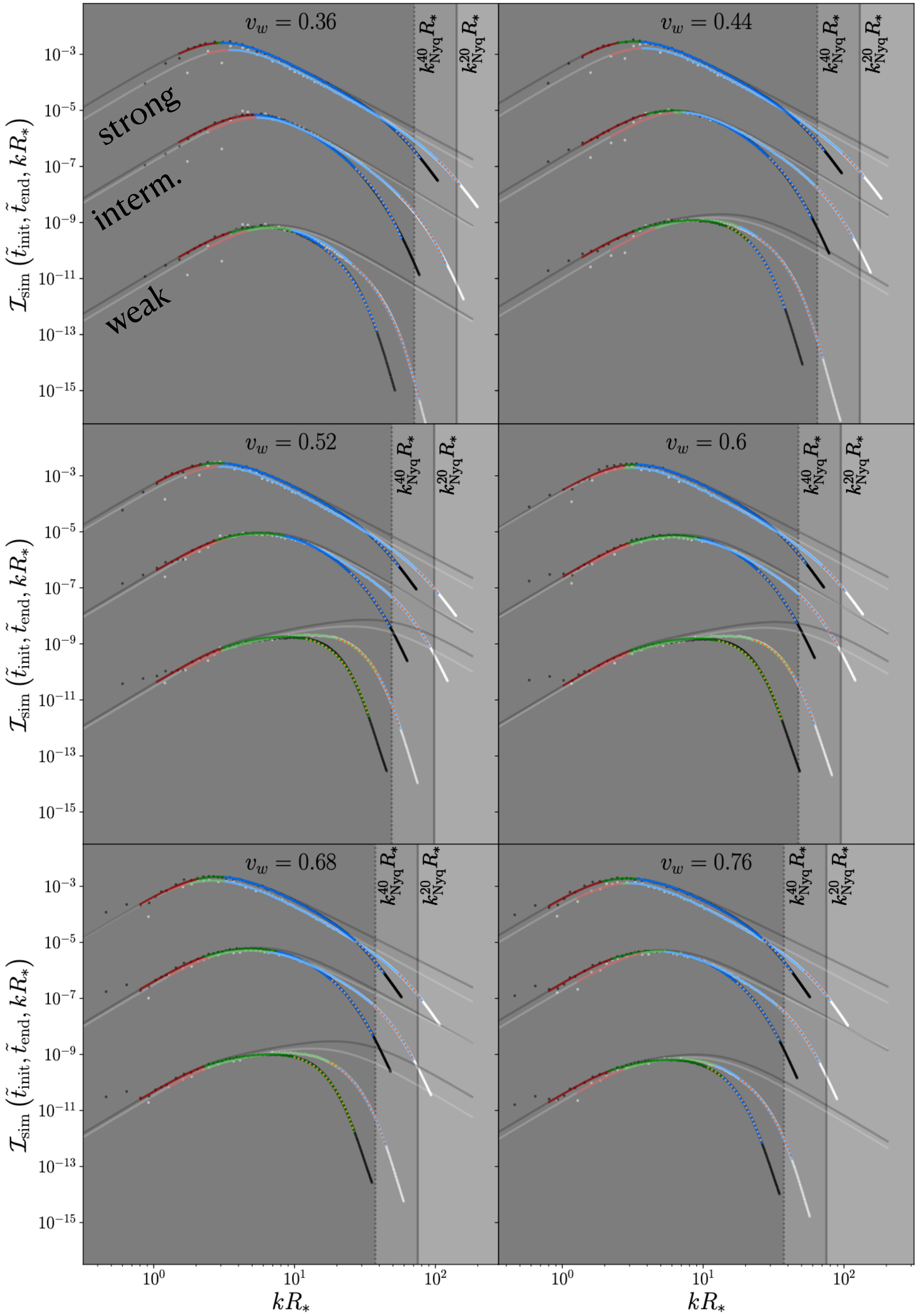
GW spectral shape $S(k)$

Cubic · Linear · ~ -Cubic · Exp. decay

$$S(k) = S_0 \times \left(\frac{k}{k_1}\right)^3 \left[1 + \left(\frac{k}{k_1}\right)^{a_1}\right]^{\frac{-3+1}{a_1}} \left[1 + \left(\frac{k}{k_2}\right)^{a_2}\right]^{\frac{-1+n_3}{a_2}} \times e^{-(k/k_d)^2}$$

- **Double broken power-law** with **exponential damping** in the UV
- Fix **IR** and **Interm**. indices and **fit** $S(k)$ wrt. **free parameters**

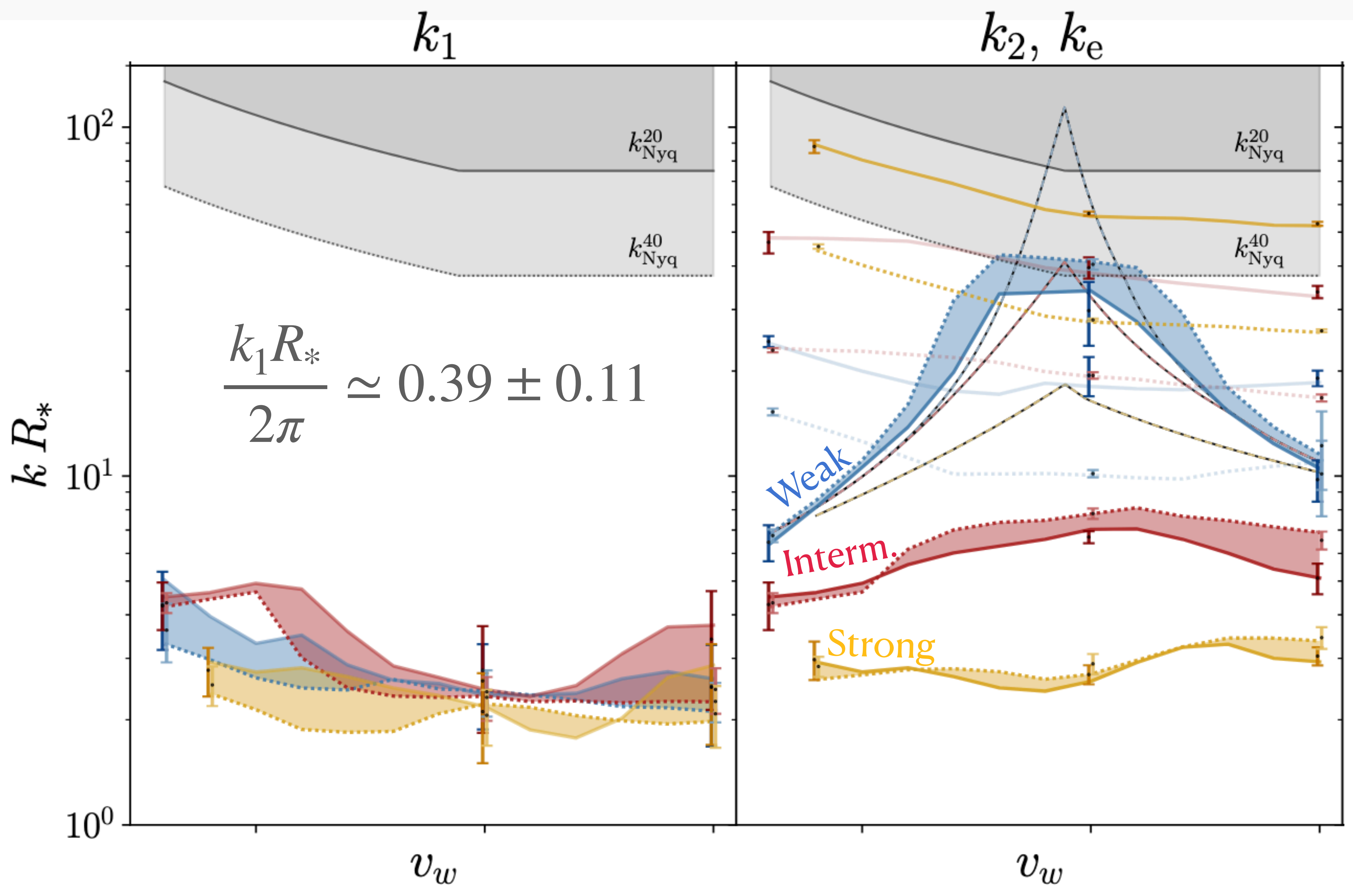
IR: k^3 → k_1 → **Interm:** k^1 → k_2 → **UV:** k^{n_3}, $n_3 \gtrsim -3$ → k_d → **Exp. damping**

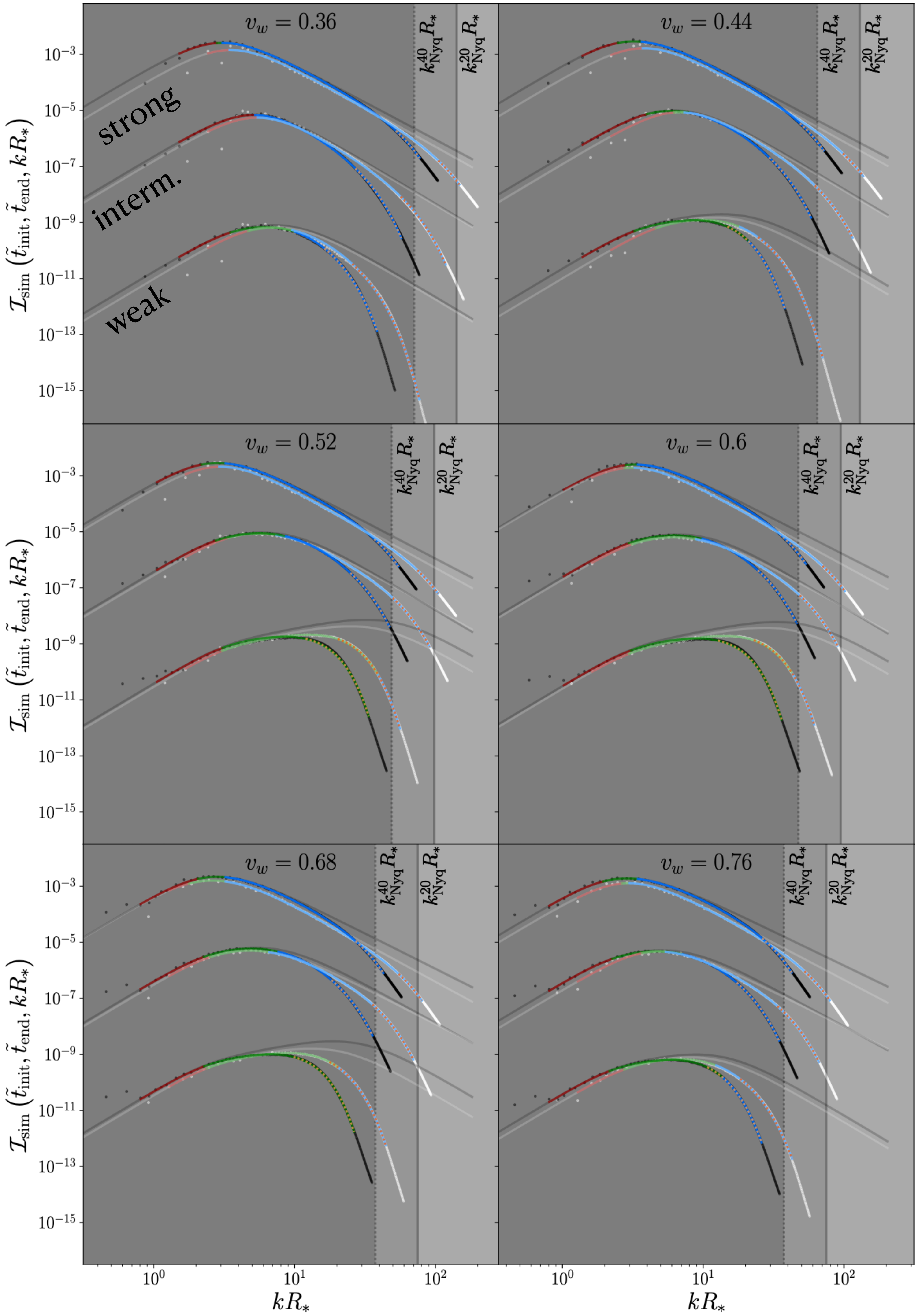


Results

GW spectral shape $S(k)$

$$S(k) = S_0 \times \underbrace{\left(\frac{k}{k_1}\right)^3}_{\text{Cubic}} \underbrace{\left[1+\left(\frac{k}{k_1}\right)^{a_1}\right]^{\frac{-3+1}{a_1}}}_{\text{Linear}} \underbrace{\left[1+\left(\frac{k}{k_2}\right)^{a_2}\right]^{\frac{-1+n_3}{a_2}}}_{\sim\text{-Cubic}} \times \underbrace{e^{-(k/k_\mathrm{d})^2}}_{\text{Exp. decay}}$$





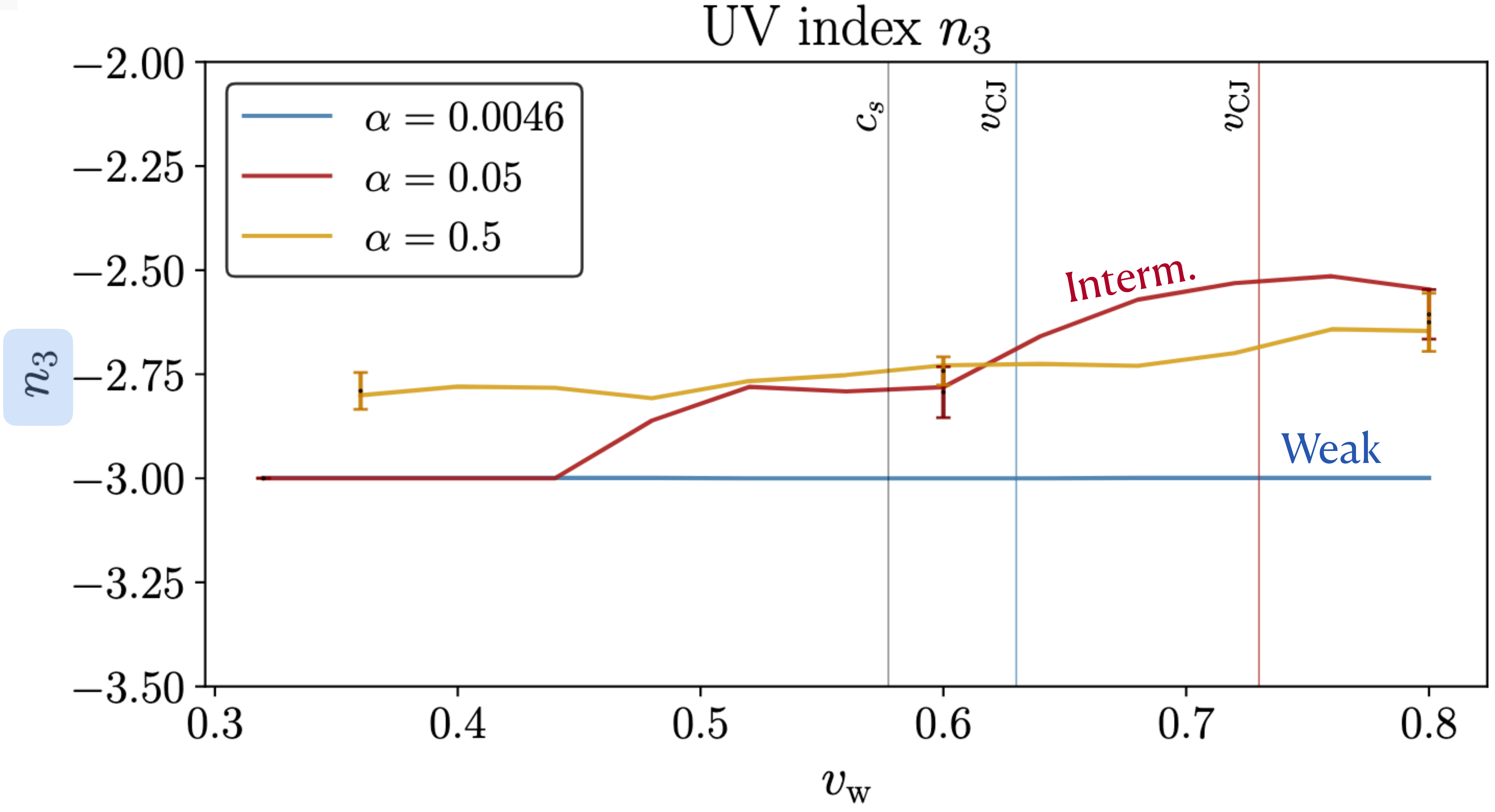
Results

GW spectral shape $S(k)$

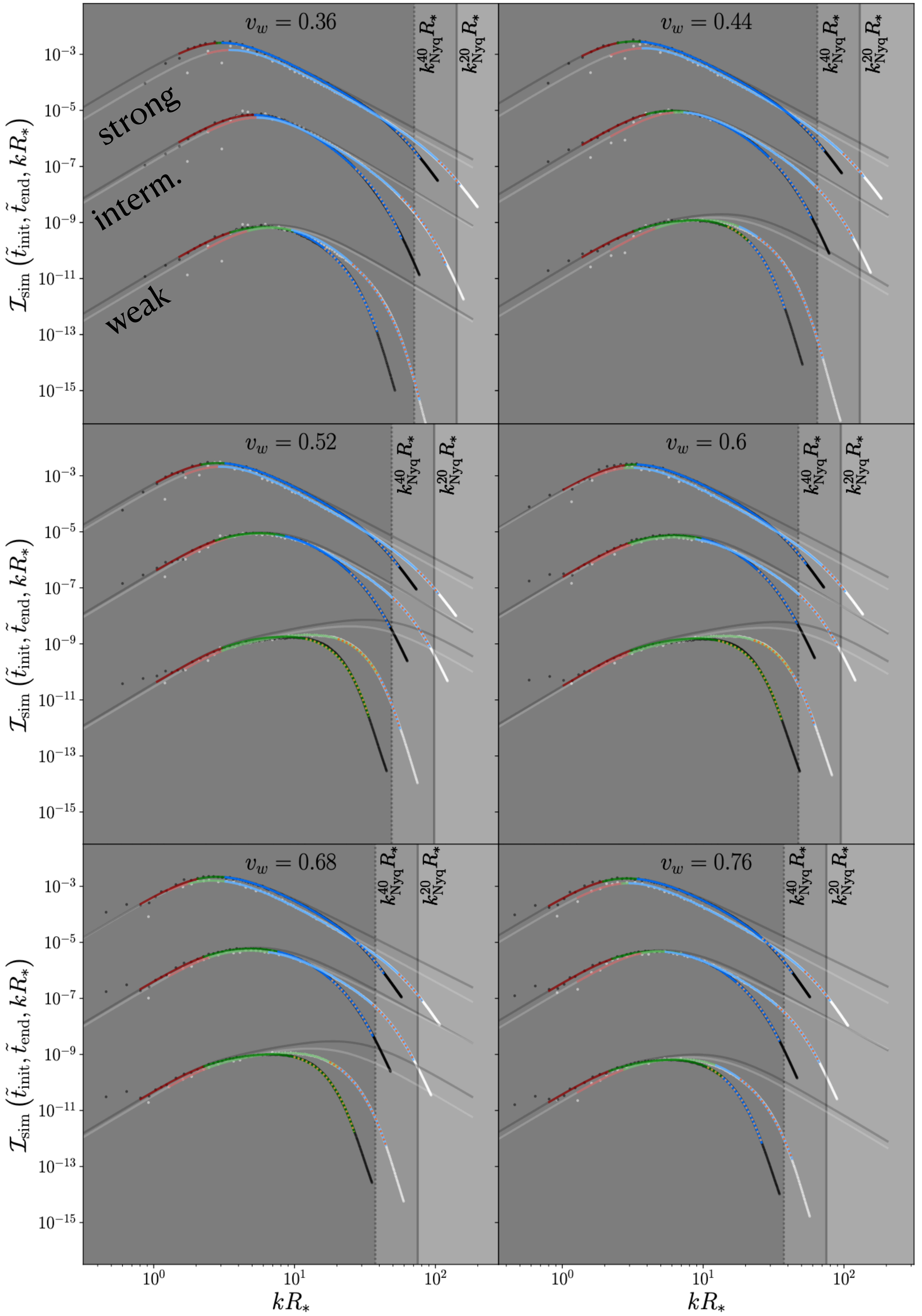
$$S(k) = S_0 \times \left(\frac{k}{k_1}\right)^3 \left[1 + \left(\frac{k}{k_1}\right)^{a_1}\right]^{\frac{-3+1}{a_1}} \left[1 + \left(\frac{k}{k_2}\right)^{a_2}\right]^{\frac{-1+n_3}{a_2}} \times e^{-(k/k_d)^2}$$

Cubic — Linear — ~ -Cubic — Exp. decay

$n_3 \gtrsim -3$



- **Departure** from -3 indicates a **departure from linearity**
- Dynamical depth of **Strong PTs allow accurate estimation**



GW production

GW spectrum from ~~sound waves~~ damped sources

How large is the GW efficiency?

$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW} \, K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin}) \, (\beta R_*) \, S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

$$10^2\,\tilde{\Omega}_{\rm GW}^{\infty} = \begin{cases} 1.04^{+0.81}_{-0.67}\,, & \textbf{for} \quad \alpha = 0.0046\,; \\ 1.64^{+0.29}_{-0.13}\,, & \textbf{for} \quad \alpha = 0.05\,; \\ 3.11^{+0.25}_{-0.19}\,, & \textbf{for} \quad \alpha = 0.5\,, \end{cases}$$

$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW}\; K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin})\; (\beta R_*)\; S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

How much of the vacuum energy is transferred to the plasma in the form of kinetic energy and how can it be modeled?

$$10^2\,\tilde{\Omega}_{\rm GW}^{\infty} = \begin{cases} 1.04^{+0.81}_{-0.67}\,, & \textbf{for} \quad \alpha = 0.0046\,; \\ 1.64^{+0.29}_{-0.13}\,, & \textbf{for} \quad \alpha = 0.05\,; \\ 3.11^{+0.25}_{-0.19}\,, & \textbf{for} \quad \alpha = 0.5\,, \end{cases}$$

$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW}\, K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin})\,(\beta R_*)\, S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

$$K^2_{\rm int}\left(\tilde t_{\rm sw}\right) = \mathscr{K}_0\, \tilde t_0 \frac{(1+\tilde t_{\rm sw}/\tilde t_0)^{1-2b} - 1}{1-2b}$$

$$\mathscr{K}_0 = 0.84^{+0.24}_{-0.29} K_\xi$$

$$10^2\, \tilde\Omega^\infty_{\rm GW} = \begin{cases} 1.04^{+0.81}_{-0.67}, & \textbf{for} \quad \alpha = 0.0046\,; \\ 1.64^{+0.29}_{-0.13}, & \textbf{for} \quad \alpha = 0.05\,; \\ 3.11^{+0.25}_{-0.19}, & \textbf{for} \quad \alpha = 0.5\,, \end{cases}$$

$$\mathscr{I}_{\rm sim}(\tilde t_*, \tilde t_{\rm fin}, \tilde k) = \tilde\Omega_{\rm GW}\, K^2_{\rm int}(\tilde t_*, \tilde t_{\rm fin})\, (\beta R_*)\, S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

$$K_{\rm int}^2\left(\tilde{t}_{\rm sw}\right) = \mathcal{K}_0\,\tilde{t}_0\,\frac{(1+\tilde{t}_{\rm sw}/\tilde{t}_0)^{1-2b}-1}{1-2b}$$

$$\mathcal{K}_0 = 0.84^{+0.24}_{-0.29} K_\xi$$

Which definition of R_* is most meaningful?

$$10^2\,\tilde{\Omega}_{\rm GW}^{\infty} = \begin{cases} 1.04^{+0.81}_{-0.67}\,, & \textbf{for} \quad \alpha = 0.0046\,; \\ 1.64^{+0.29}_{-0.13}\,, & \textbf{for} \quad \alpha = 0.05\,; \\ 3.11^{+0.25}_{-0.19}\,, & \textbf{for} \quad \alpha = 0.5\,, \end{cases}$$

$$\mathcal{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW}\; K_{\rm int}^2(\tilde{t}_*, \tilde{t}_{\rm fin})\; (\beta R_*)\; S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

$$K^2_{\rm int}\left(\tilde{t}_{\rm sw}\right) = \mathscr{K}_0\,\tilde{t}_0\,\frac{(1+\tilde{t}_{\rm sw}/\tilde{t}_0)^{1-2b}-1}{1-2b}$$

$$\mathscr{K}_0 = 0.84^{+0.24}_{-0.29} K_\xi$$

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$$\mathscr{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW}\; K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin})\; (\beta R_*)\; S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

$$K_{\rm int}^2\left(\tilde t_{\rm sw}\right) = \mathscr{K}_0\,\tilde t_0\,\frac{\left(1+\tilde t_{\rm sw}/\tilde t_0\right)^{1-2b}-1}{1-2b}$$

$$\mathscr{K}_0 = 0.84^{+0.24}_{-0.29} K_\xi$$

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What is the spectral shape?
In particular, where is the peak, and what are the IR and UV slopes?

$$\mathscr{I}_{\rm sim}(\tilde t_*, \tilde t_{\rm fin}, \tilde k) = \tilde\Omega_{\rm GW}\; K_{\rm int}^2(\tilde t_*, \tilde t_{\rm fin})\; (\beta R_*)\; S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

$$K^2_{\rm int}\left(\tilde{t}_{\rm sw}\right) = \mathscr{K}_0\,\tilde{t}_0\,\frac{\left(1+\tilde{t}_{\rm sw}/\tilde{t}_0\right)^{1-2b}-1}{1-2b}$$

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$$\frac{k_1 R_*}{2\pi} \simeq 0.39 \pm 0.11$$

$$S(k) = S_0 \times \left(\frac{k}{k_1}\right)^3 \left[1+\left(\frac{k}{k_1}\right)^{a_1}\right]^{\frac{-3+1}{a_1}} \left[1+\left(\frac{k}{k_2}\right)^{a_2}\right]^{\frac{-1+n_3}{a_2}} \times\ e^{-\left(k/k_{\rm d}\right)^2}$$

$$\frac{k_2 R_*}{2\pi} \simeq \begin{cases} 0.49 \pm 0.024/\Delta_w, & \alpha = 0.0046 \\ 0.93 \pm 0.13, & \alpha = 0.05 \\ 0.45 \pm 0.042, & \alpha = 0.5 \end{cases}$$

$$\mathscr{I}_{\rm sim}(\tilde{t}_*, \tilde{t}_{\rm fin}, \tilde{k}) = \tilde{\Omega}_{\rm GW}\ K^2_{\rm int}(\tilde{t}_*, \tilde{t}_{\rm fin})\ (\beta R_*)\ S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

But we can do better!

For, as a matter of fact, in radiation-domination…

GW production

GW spectrum from ~~sound waves~~ damped sources

including cosmic expansion

... $\partial_\mu T^{\mu\nu} = 0$ is conformally invariant

$$\Downarrow$$

Reinterpret measured K as the coming quantity

$$\Downarrow$$

$$K_{\rm int}^2 \longrightarrow K_{\rm int,exp}^2 \equiv \left(\beta/H_*\right)^2 \int_{\tilde\tau_*}^{\tilde\tau_{\rm fin}} \frac{K^2(\tilde\tau)d\tilde\tau}{\tilde\tau^2}$$

Additional damping from cosmic expansion

Now modeling expansion

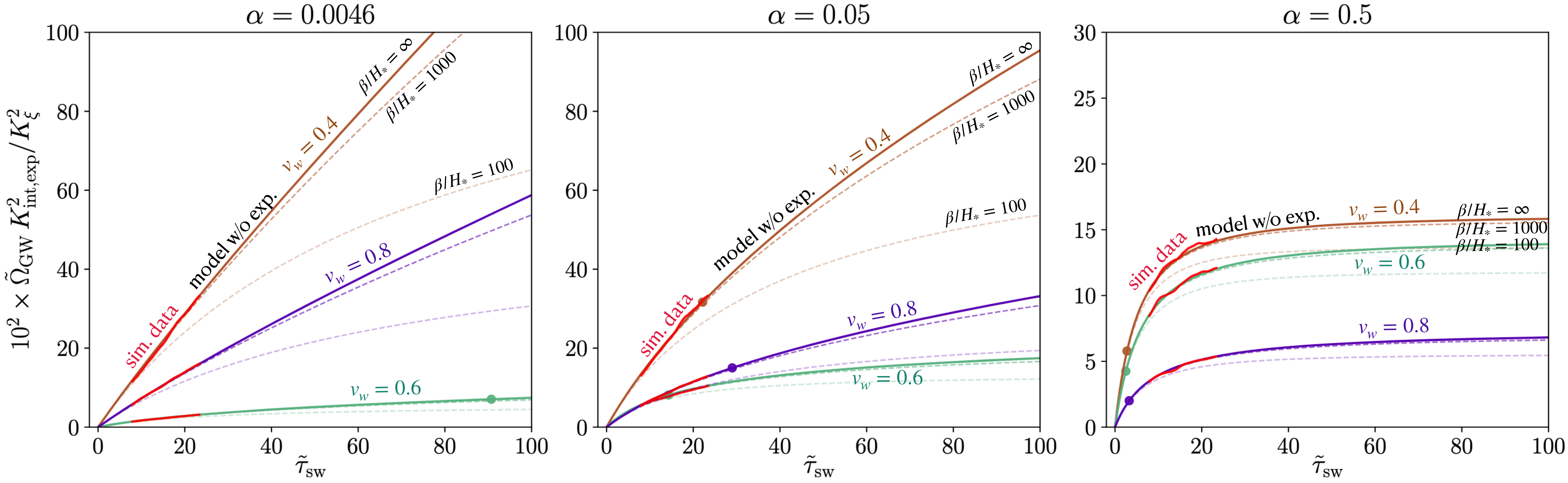
$$\mathcal{I}_{\rm sim}(\tilde\tau_*, \tilde\tau_{\rm fin}, \tilde k) = \tilde\Omega_{\rm GW}\, K_{\rm int,exp}^2(\tilde\tau_*, \tilde\tau_{\rm fin})\,(\beta R_*)\, S(k)$$

GW production

GW spectrum from ~~sound waves~~ damped sources

including cosmic expansion

Growth of the GW amplitude w. and w/o cosmic expansion



Weak: Dominated by Hubble damping

Intermediate: Hybrid damping

Strong: dominated by fluid damping

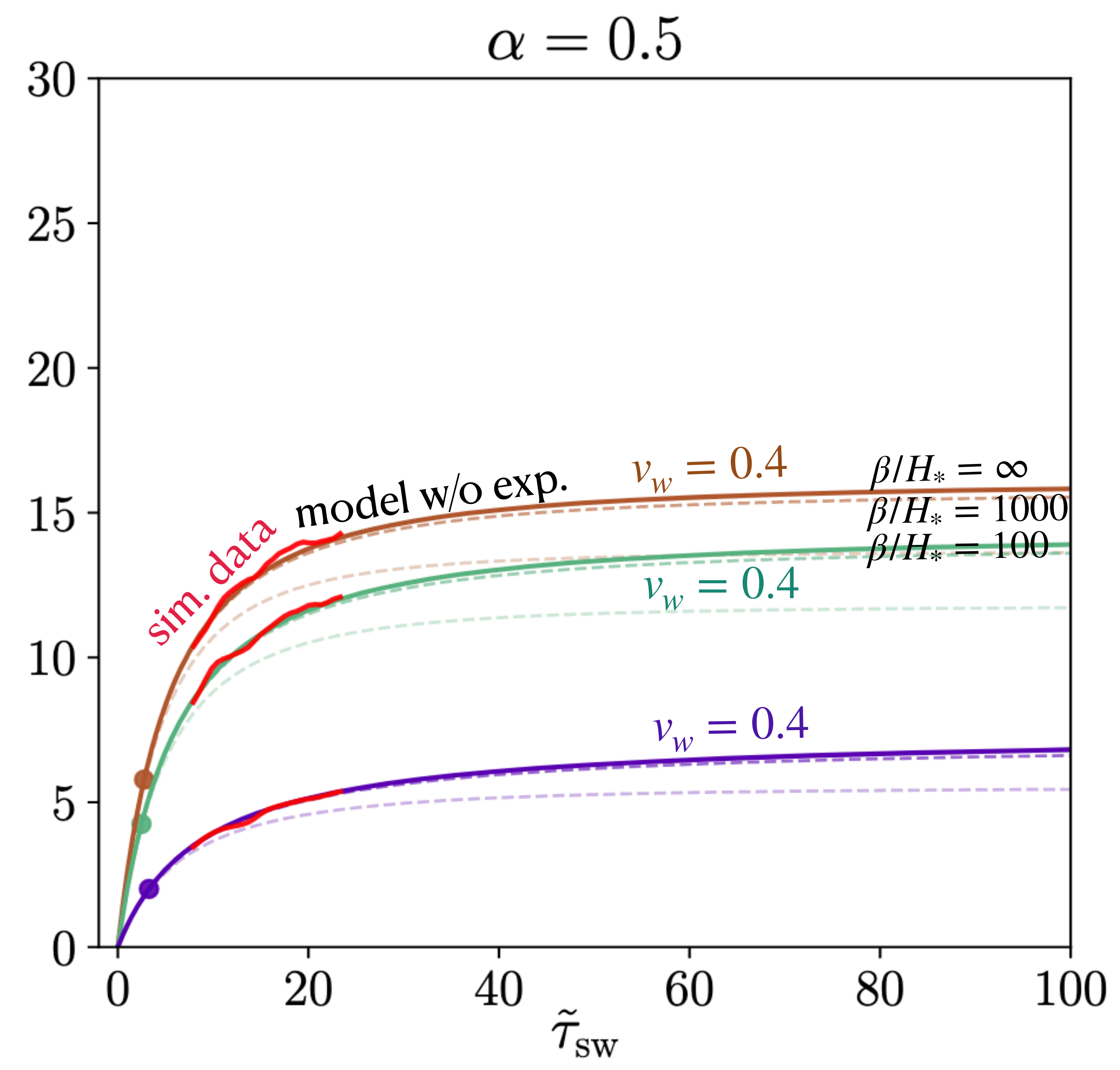
GW production

GW spectrum from ~~sound waves~~ damped sources

including cosmic expansion

Strong PTs:

- Simulating ~ 5 shock formation times
- Within $\sim 10\,\%$ of amplitude saturation
- Impact of Hubble expansion small



Strong: dominated by fluid damping

GW production

GW spectrum from ~~sound waves~~ damped sources

$$\mathscr{K}_0 = 0.84^{+0.24}_{-0.29} K_\xi$$

$$\frac{k_1 R_*}{2\pi} \simeq 0.39 \pm 0.11$$

$$S(k) = S_0 \times \left(\frac{k}{k_1}\right)^3 \left[1 + \left(\frac{k}{k_1}\right)^{a_1}\right]^{\frac{-3+1}{a_1}} \left[1 + \left(\frac{k}{k_2}\right)^{a_2}\right]^{\frac{-1+a_3}{a_2}} \times e^{-(k/k_d)^2}$$

$$\frac{k_2 R_*}{2\pi} \simeq \begin{cases} 0.49 \pm 0.024/\Delta_w, & \alpha = 0.0046 \\ 0.93 \pm 0.13, & \alpha = 0.05 \\ 0.45 \pm 0.042, & \alpha = 0.5 \end{cases}$$

Now modeling expansion

$$\mathscr{I}^{\text{model}}_{\text{sim}}(\tilde{k}) = \tilde{\Omega}^{\infty}_{\text{GW}} K^2_{\text{int,exp}}(\mathscr{K}_0)\,(\beta R_*)\,S(k)$$

Main results:

- **Template** for the **GW spectrum** motivated by **theory** and validated on **simulation data**
- Generalized from **SWs** to **damped sources** and **including** the effect of **cosmic expansion,**
- Potentially captures **GWs** from **full dynamics**, e.g. **compressional motion,** and **turbulence,** simultaneously
- **Requires specification** of the **wall velocity** v_w, the **PT strength** α**, the ratio** β/H_***, and PT temperature** T_*

CosmoGW

CosmoGW

Making the results readily accessible

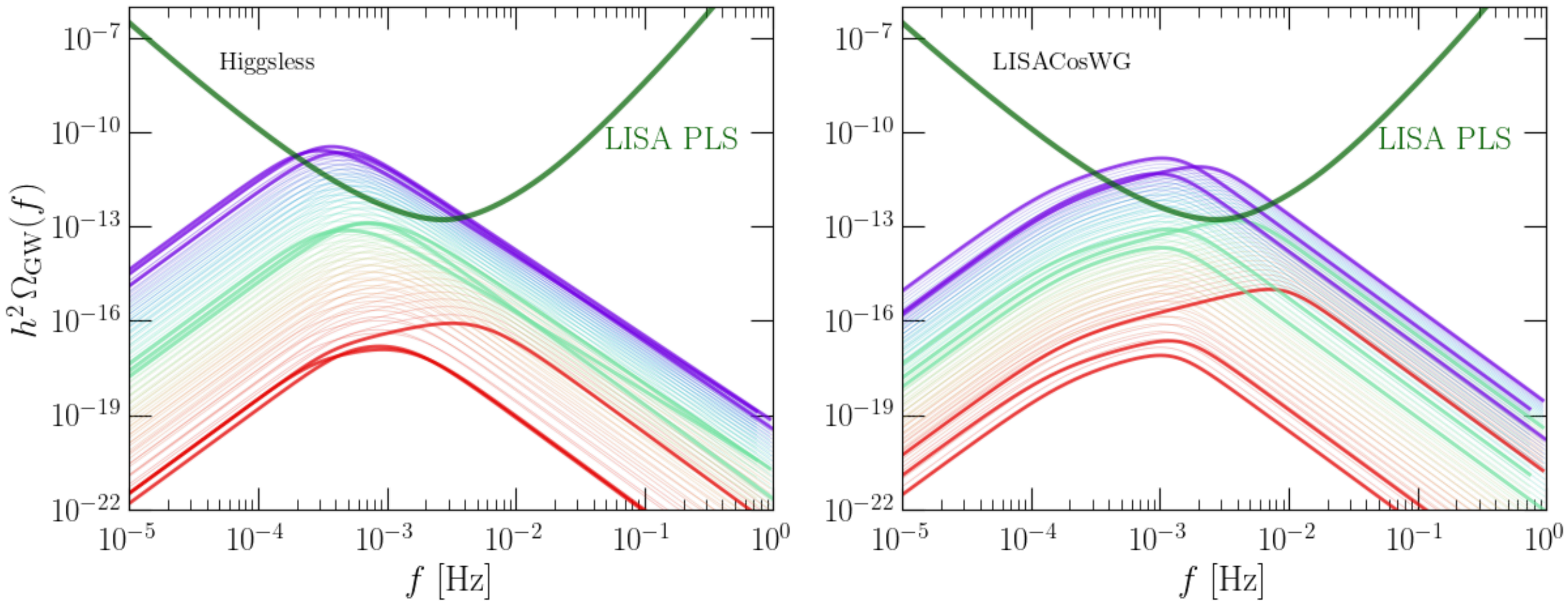
- Python package for cosmological sources of GWs developed by Alberto Roper Pol
- Installation: `pip install cosmoGW`
- Find `CosmoGW` on [Github]
- Implemented the results of 2409.03651 to facilitate community use in e.g. parameter inference studies and detectability forecasts
- New paper summarizing the model 2508.04263 . Out today!
- Allows interpolating between simulations results to obtain GW spectrum predictions for any PT parameter point
- Takes as input $\{v_w, \alpha, \beta/H_*, N_{\rm shock}\}$ for source duration $\delta\eta_{\rm fin} = N_{\rm shock} R_*/v_f$
- Link to [Tutorial]

CosmoGW

Making the results readily accessible

Templates from as outlined in 2508.04263 . Out today!

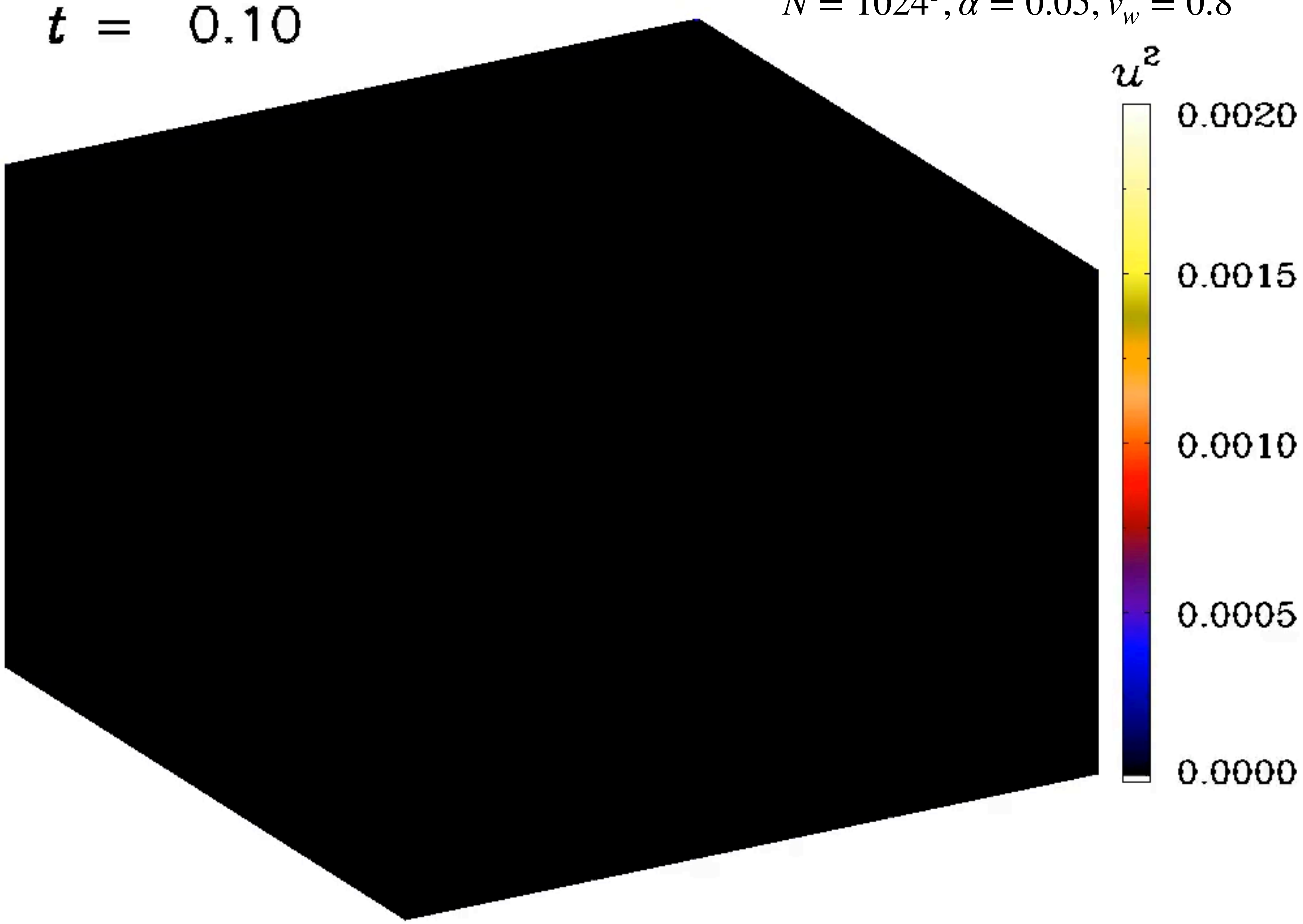
2403.03723



Higgsless in Pencil Code

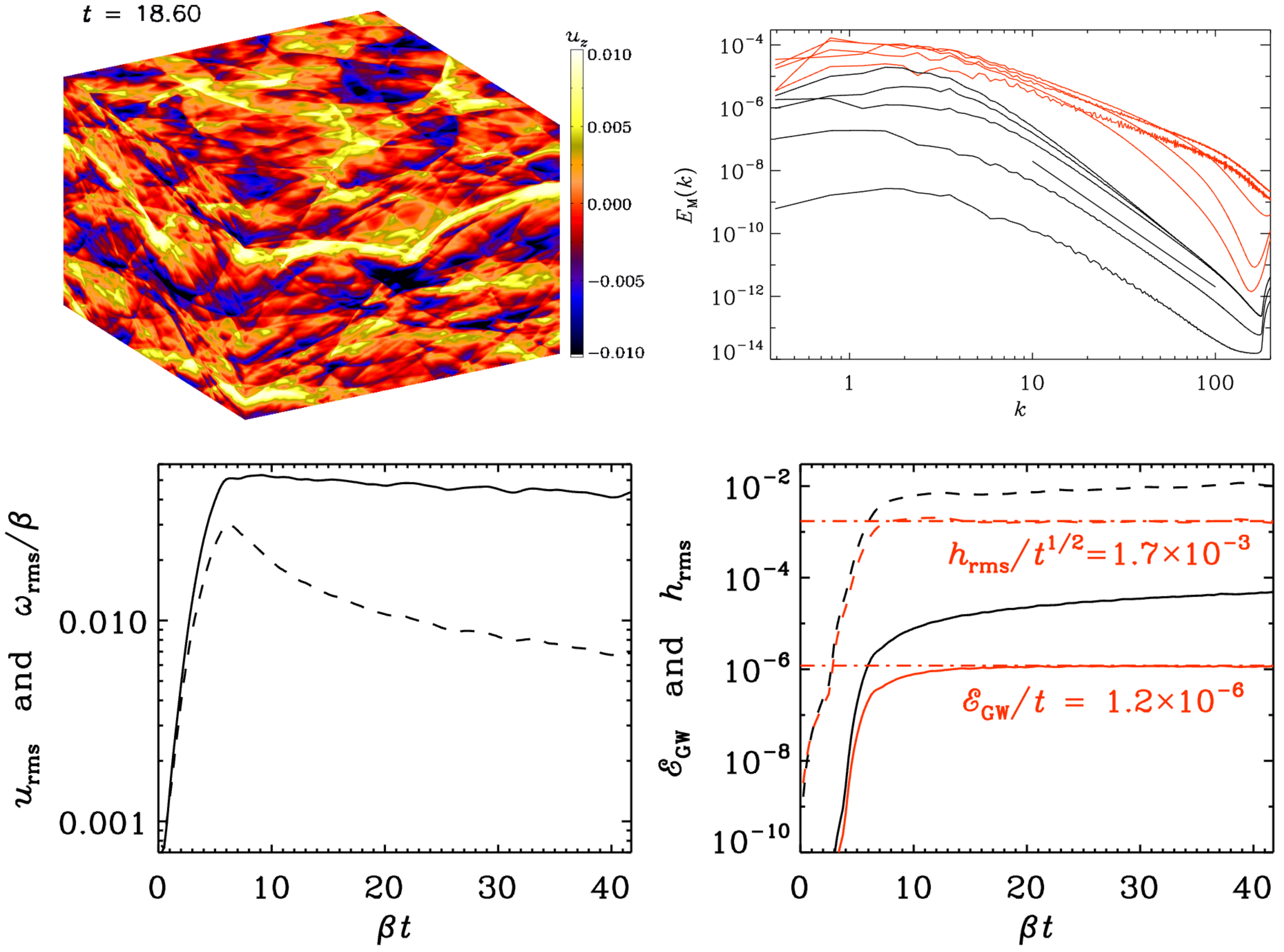
Higgsless in Pencil Code

$N = 1024^3, \alpha = 0.05, v_w = 0.8$



Higgsless in Pencil Code

$N = 1024^3, \alpha = 0.05, v_w = 0.8$



Conclusions and summary

Conclusions and summary

We have...

- Used the novel **Higgsless approach** to simulate the **relativistic hydrodynamics** of 1st-order **cosmological PTs**
- Derived a new **GW-parameterization** generalizing results from sound-waves to **damped sources** accounting also for **cosmic expansion**
- Obtained **GW predictions** based on the **simulation data** for a **large part** of **parameter space** in wall velocity v_w and PT strength α
- **Obtained**, for the **first time**, **GW predictions** for **strong PTs**
- Established that **non-linear evolution dictates** the **shape** and **peak** location of the **GW spectrum**, thus rendering **full simulations necessary** to derive **accurate GW predictions**
- Captured the **saturation** of the **GW amplitude** due to **non-linear dynamics**
- **GW templates** available in the **Python package** `cosmoGW`
- Provided a **tutorial**, allowing the community to **work interactively with the results** and get **GW predictions for any PT parameter choice**

Thank you very much!

Questions?

Isak Stomberg
isak.stomberg@ific.uv.es