UNIVERSITÉ DE GENÈVE

Gravitational Waves from slowly decaying sources in the early Universe

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«Exploring the early Universe with Gravitational Waves and Primordial Magnetic Fields»

Gravitational Wave spectrum from stochastic sources

Caprini, Figueroa [1801.04268]

- Tensor perturbations over FLRW $$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + \ell_{ij})\, dx^i\, dx^j\right]$$

- GW equation (radiation domination) $\xrightarrow[a = \tau/\tau_*]{h_{ij} = a\,\ell_{ij}}$ $$(\partial_\tau^2 + k^2)\, h_{ij}(\tau, \boldsymbol{k}) = 6\,\Pi_{ij}(\tau, \boldsymbol{k})\,\frac{\mathcal{H}_*}{\tau}$$

  $\Pi_{ij} = a^2 \Lambda_{ijkl} T^{kl}/\rho_{crit}$

- Solution $$h_{ij}(\tau, \boldsymbol{k}) = \frac{6\mathcal{H}_*}{k} \int_{\tau_*}^{\min[\tau,\tau_{fin}]} \frac{d\tau_1}{\tau_1}\, \Pi_{ij}(\tau_1, \boldsymbol{k}) \sin k(\tau - \tau_1)$$

  source active for $\tau_* < \tau < \tau_{fin}$

- GW spectrum at present time

$$\Omega_{GW}(t_0) = \frac{\rho_{GW}^0}{\rho_{crit}^0} = \frac{M_{pl}^2}{4\rho_{crit}^0} \langle \left| \partial_t \ell_{ij}(\boldsymbol{x}, t_0) \right|^2 \rangle = \frac{1}{12\, H_0^2} \left(\frac{a_*}{a_0}\right)^4 \langle \left| \partial_\tau h_{ij}(\boldsymbol{x}, \tau_0) - \mathcal{H}_0 h_{ij}(\boldsymbol{x}, \tau_0) \right|^2 \rangle$$

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source active for $\tau_* < \tau < \tau_{fin}$

- GW spectrum at present time

$k \gg \mathcal{H}_0$

$$\Omega_{GW}(t_0) = \frac{\rho_{GW}^0}{\rho_{crit}^0} = \frac{M_{pl}^2}{4\rho_{crit}^0} \langle \left| \partial_t \ell_{ij}(\boldsymbol{x}, t_0) \right|^2 \rangle = \frac{1}{12\, H_0^2} \left(\frac{a_*}{a_0}\right)^4 \langle \left| \partial_\tau h_{ij}(\boldsymbol{x}, \tau_0) - \mathcal{H}_0 h_{ij}(\boldsymbol{x}, \tau_0) \right|^2 \rangle$$

Unequal-time correlator (UETC) of the source

GW spectrum $\int_0^\infty \Omega_{GW}(\tau_0, k)\,\mathrm{d}\ln k \equiv \Omega_{GW}(t_0) \cong \frac{1}{12\,H_0^2}\left(\frac{a_*}{a_0}\right)^4 \langle\, |\partial_\tau h_{ij}(\boldsymbol{x}, \tau_0)|^2 \rangle$

From GW equation's solution $\partial_\tau h_{ij}(\tau, \boldsymbol{k}) = 6\,\mathcal{H}_* \int_{\tau_*}^{\min[\tau,\tau_{fin}]} \frac{d\tau_1}{\tau_1} \Pi_{ij}(\tau_1, \boldsymbol{k}) \cos k(\tau - \tau_1)$

$$\langle\, \Pi_{ij}(\tau_1, \boldsymbol{k}) \Pi^*_{\mathrm{lm}}(\tau_2, \boldsymbol{k}') \rangle = \frac{1}{2}(2\pi)^6 \delta^3(\boldsymbol{k} - \boldsymbol{k}')[\Lambda_{ijlm}(\widehat{\boldsymbol{k}}) \frac{E_\Pi(k, \tau_1, \tau_2)}{4\pi k^3} + i\mathcal{A}_{ijlm}(\widehat{\boldsymbol{k}}) \frac{H_\Pi(k, \tau_1, \tau_2)}{4\pi k^3}]$$

$$\Omega_{GW}(\tau_0, k) = 3\,\mathcal{T}_{GW} \iint_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tau_1}{\tau_1} \frac{d\tau_2}{\tau_2} \cos k(\tau_0 - \tau_1) \cos k(\tau_0 - \tau_2)\, E_\Pi(k, \tau_1, \tau_2)$$

Constant-in-time model for the UETC of the source

- Assuming a constant-in-time UETC for the source for $\tau_* < \tau_{1/2} < \tau_{fin} \longrightarrow E_\Pi(k, \tau_1, \tau_2) = E_\Pi^*(k)$
- GW spectrum

$$\Omega_{GW}(\tau_0, k) = 3\, \mathcal{T}_{GW} E_\Pi^*(k) \int_{\tau_*}^{\min[\tau_0, \tau_{fin}]} \int_{\tau_*}^{\min[\tau_0, \tau_{fin}]} \frac{d\tau_1}{\tau_1} \frac{d\tau_2}{\tau_2} \cos k(\tau_0 - \tau_1) \cos k(\tau_0 - \tau_2)$$

$$\equiv 3\, \mathcal{T}_{GW} E_\Pi^*(k)\, \Delta^2(k, \tau_0)$$

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$$\mathcal{T}_{GW} = \left(\frac{a_*}{a_0}\right)^4 \left(\frac{H_*}{H_0}\right)^2 \approx 1.6 \times 10^{-5} \left(\frac{g_*}{100}\right)^{-\frac{1}{3}}$$

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$$E_\Pi^*(k) \propto \langle \Pi_{ij}(\tau_*, k) \Pi_{ij}^*(\tau_*, k) \rangle$$

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$$\equiv 3\,\mathcal{T}_{GW} E_\Pi^*(k)\,\Delta^2(k,\tau_0)$$

$$\mathcal{T}_{GW} = \left(\frac{a_*}{a_0}\right)^4 \left(\frac{H_*}{H_0}\right)^2 \approx 1.6\times 10^{-5}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}$$

$$E_\Pi^*(k) \propto \langle \Pi_{ij}(\tau_*,k)\Pi_{ij}^*(\tau_*,k)\rangle$$

$$\Delta(k,\tau_0) \equiv \int_{\tau_*}^{\min[\tau_0,\tau_{fin}]} \frac{d\tilde{\tau}}{\tilde{\tau}} \cos k(\tau_0-\tilde{\tau})$$

$$= \cos k\tau_0 \left[\mathrm{Ci}(k,\min[\tau_0,\tau_{fin}]) - \mathrm{Ci}(k,\tau_*)\right] + \sin k\tau_0 [\mathrm{Si}(k,\min[\tau_0,\tau_{fin}]) - \mathrm{Si}(k,\tau_*)]$$

Constant-in-time model for the UETC of the source

Averaging over fast frequency oscillations at present time $k\tau_0 \gg 1$

$$\Omega_{GW}(\tau_0,k) \cong \frac{3}{2}\mathcal{T}_{GW}\,E_\Pi^*(k)\left\{\left[\mathrm{Ci}\left(k,\tau_f\right)-\mathrm{Ci}(k,\tau_*)\right]^2+\left[\mathrm{Si}\left(k,\tau_f\right)-\mathrm{Si}(k,\tau_*)\right]^2\right\}$$

In flat spacetime we have instead $\Omega_{GW}(\tau_0,k) = 3\,\mathcal{T}_{GW}E_\Pi^*(k)\,\Delta_{flat}^2(k,\tau_0)$

$$\Delta_{flat}(k,\tau) = \frac{1}{\tau_*}\int_{\tau_*}^{\min[\tau,\tau_{fin}]} \mathrm{d}\tilde{\tau}\cos k(\tau_0-\tilde{\tau}) = \begin{cases} \dfrac{1}{k\tau_*}\sin k(\tau-\tau_*) & \tau \le \tau_{fin} \\ \dfrac{1}{k\tau_*}\left[\sin k(\tau-\tau_*) - \sin k(\tau-\tau_{fin})\right] & \tau > \tau_{fin} \end{cases}$$

Averaging over fast frequency oscillations at present time $k\tau_0 \gg 1$

$$\Omega_{GW}(\tau_0,k) \cong \frac{3}{2}\mathcal{T}_{GW}\,E_\Pi^*(k)[k\tau_*]^{-2}\left\{\left[\cos k\tau_{fin}-\cos k\tau_*\right]^2+\left[\sin k\tau_{fin}-\sin k\tau_*\right]^2\right\}$$

Constant-in-time model for the UETC of the source

$k(\tau - \tau_*) = k\delta\tau < 1$

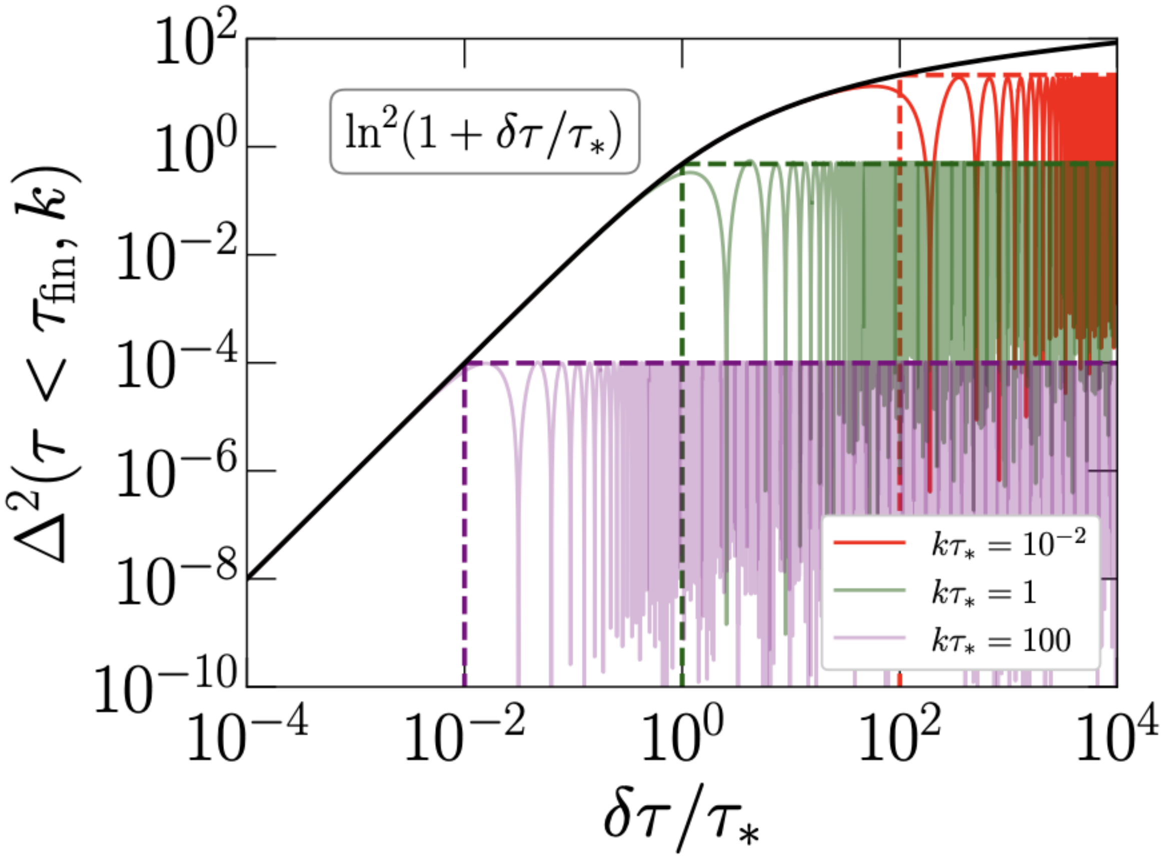
$$\rightarrow \Delta^2(k,\tau) \propto \ln^2\left(1 + \frac{\delta\tau}{\tau_*}\right) \xrightarrow{\delta\tau/\tau_* \ll 1} \approx \left(\frac{\delta\tau}{\tau_*}\right)^2$$

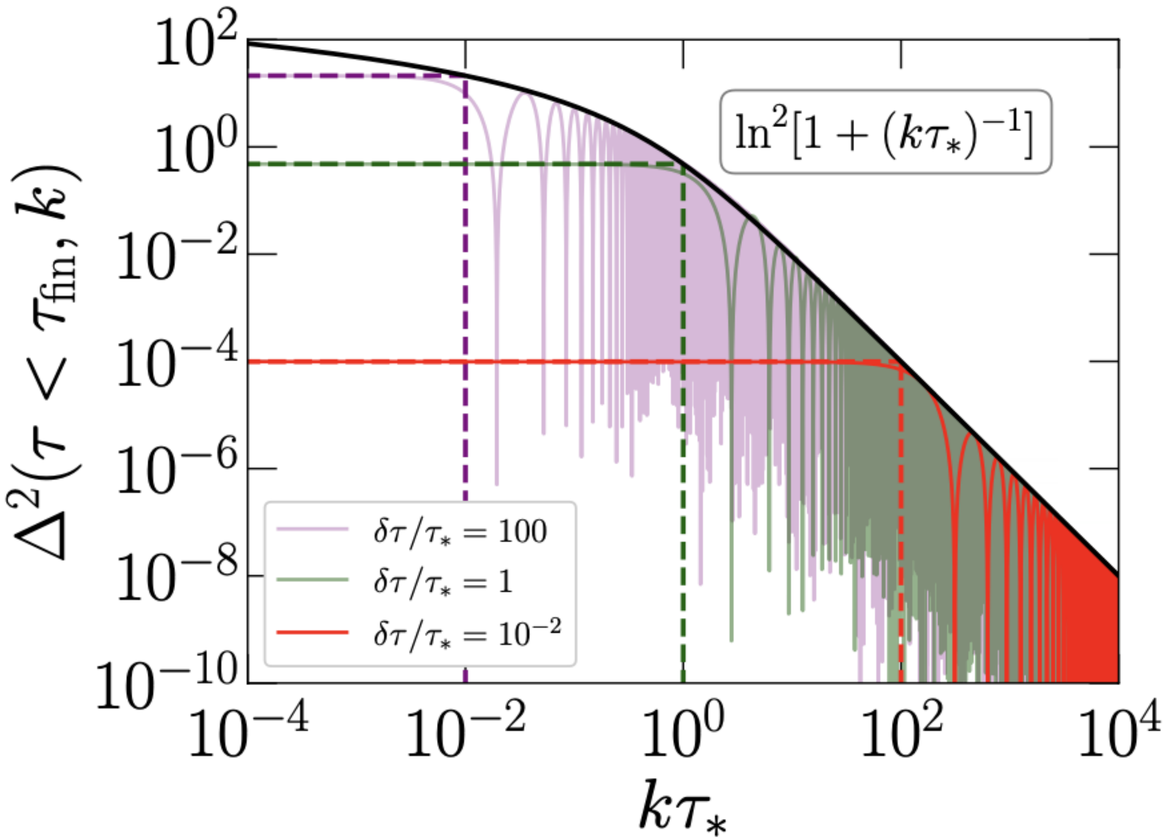
(flat)

$k\delta\tau > 1$

$$\rightarrow \Delta^2(k,\tau) \propto \ln^2\left(1 + \frac{1}{k\tau_*}\right) \xrightarrow{k\tau_* \gg 1} \approx \frac{1}{(k\tau_*)^2}$$

(flat)



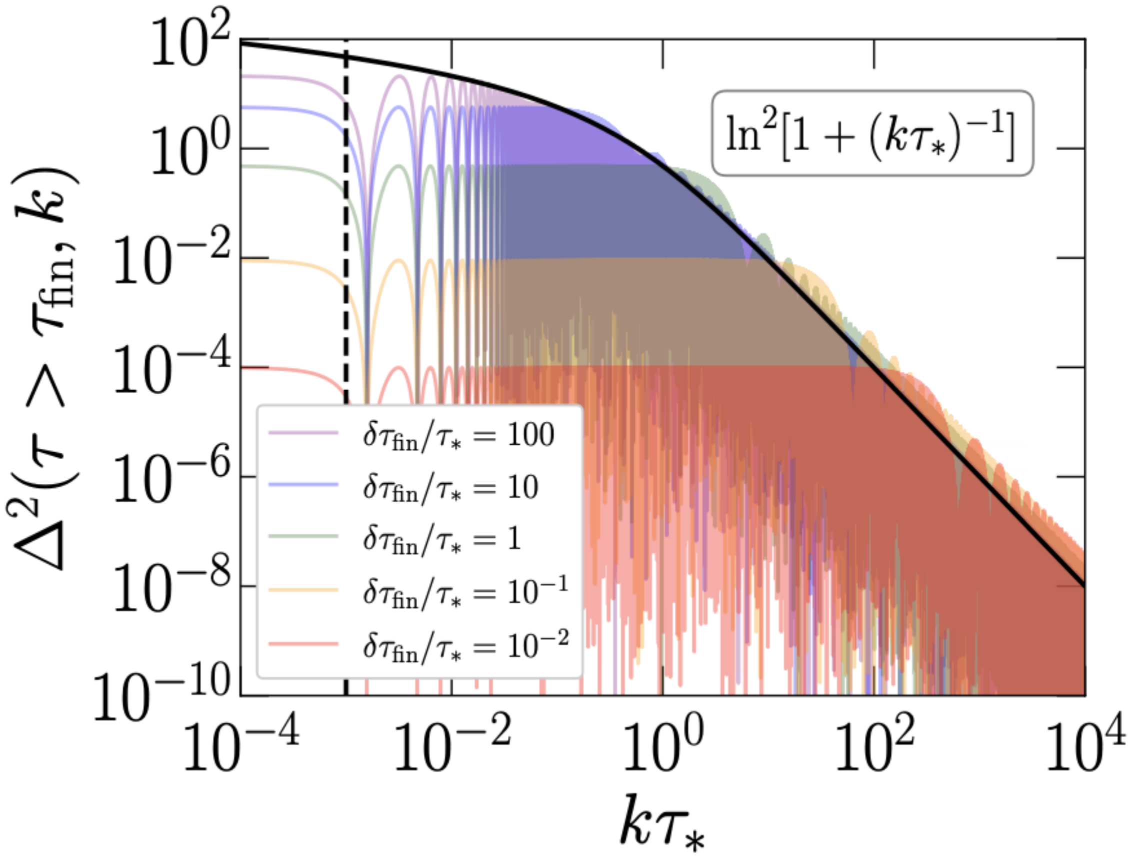


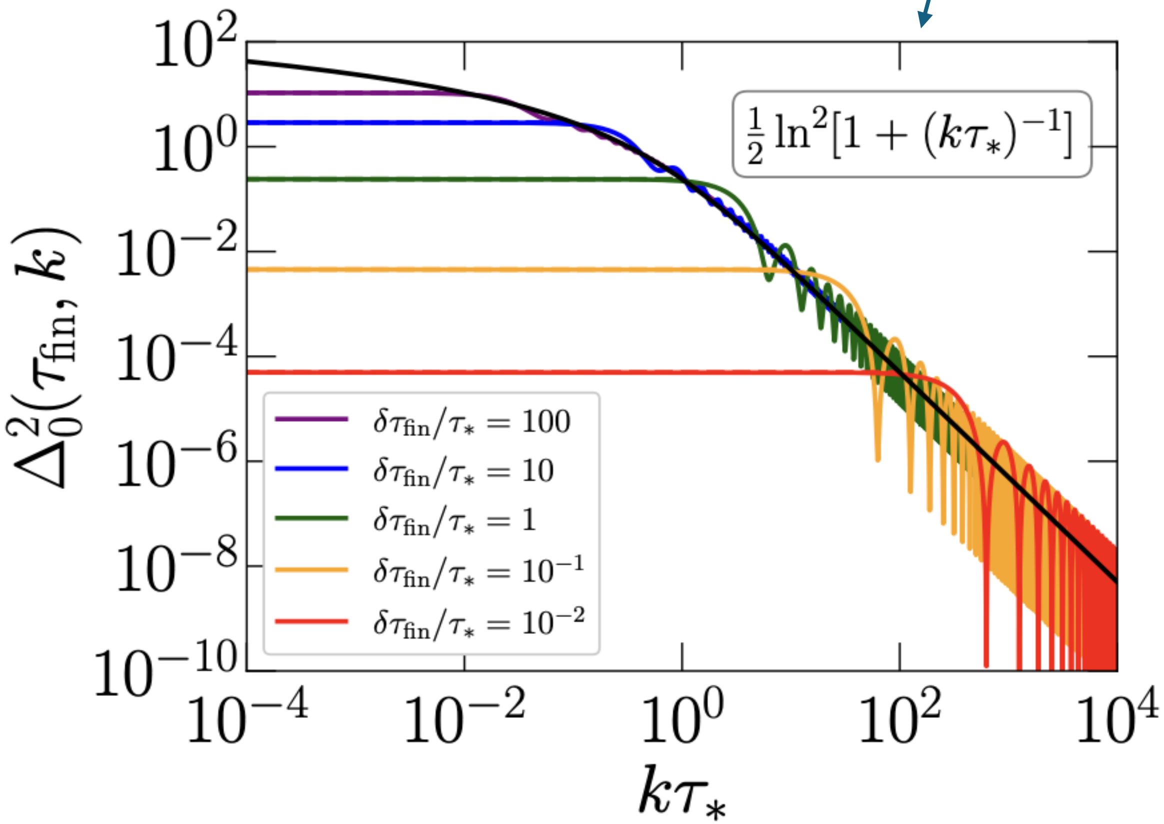
Constant-in-time model for the UETC of the source

Modes $k > 1/\delta\tau_{fin}$ saturated with amplitude $\ln^2[1+(k\tau_*)^{-1}]$

Modes $k < 1/\delta\tau_{fin}$ stop growing at $\tau = \tau_{fin}$ and have amplitude $\ln^2[1+\delta\tau_{fin}/\tau_*]$

averaging over fast frequency oscillations at present time





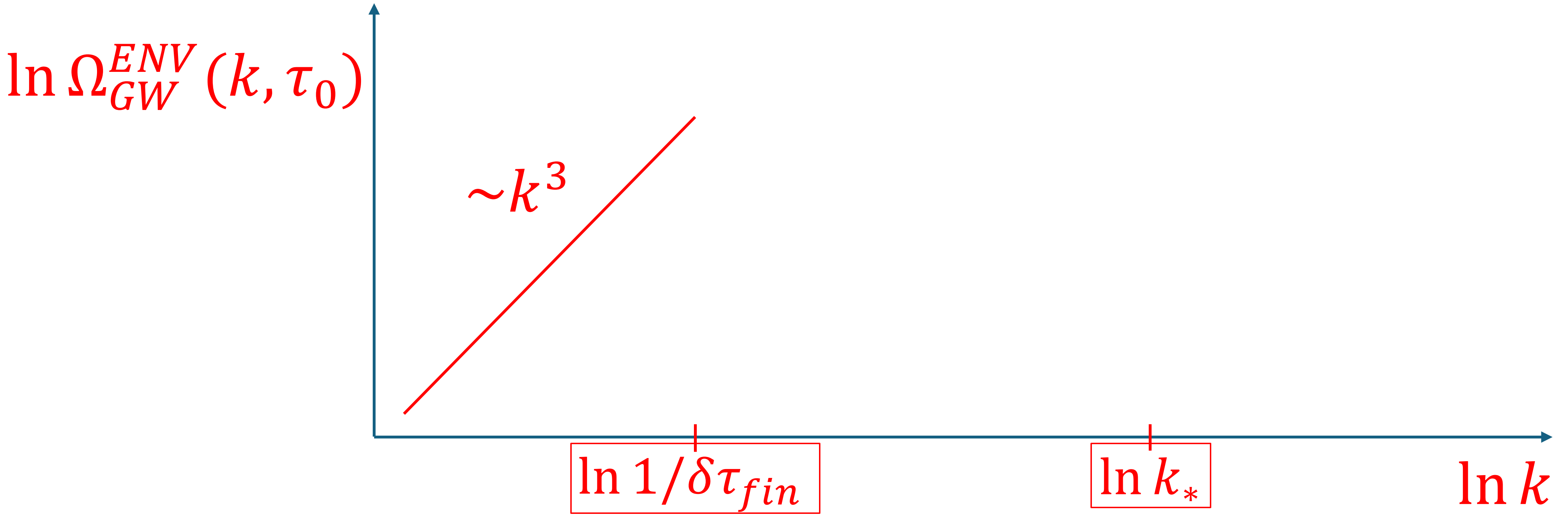
GW spectrum in the constant-in-time model

$$\Omega_{GW}(k,\tau_0) \equiv 3\, \mathcal{T}_{GW} E_\Pi^*(k)\, \Delta_0^2(k,\tau_{fin})$$

$\tau_* = 1/\mathcal{H}_*$

causality

Assuming for the UETC $E_\Pi^*(k) \sim \begin{cases} k^3 & k < k_* \\ k^{-b} & k > k_* \end{cases}$ (e. g. b = 2/3 for Kolmogorov turbulence)



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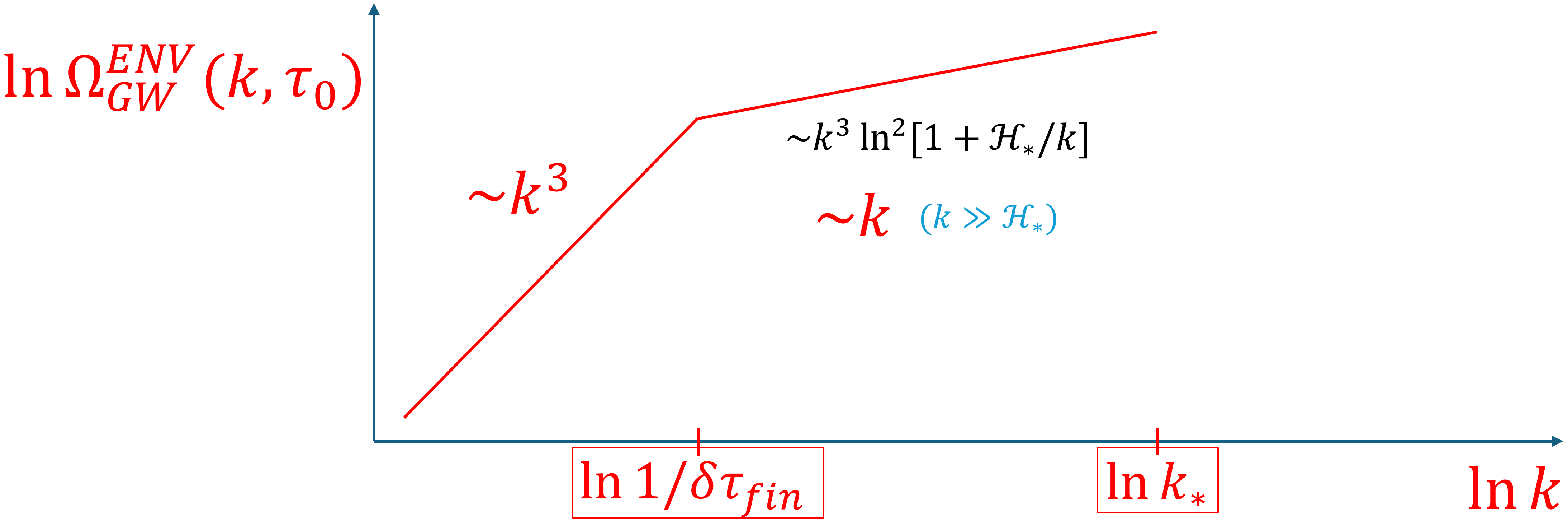

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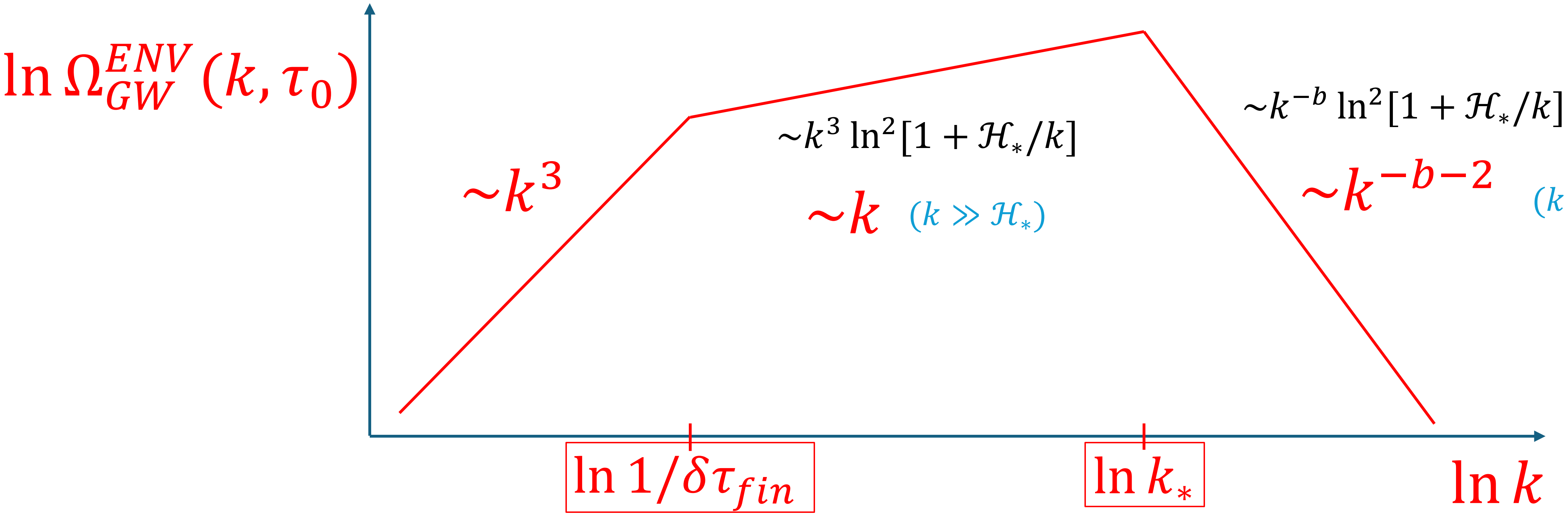
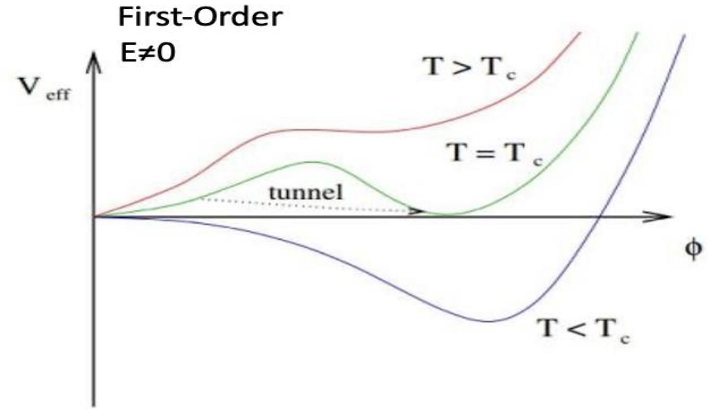
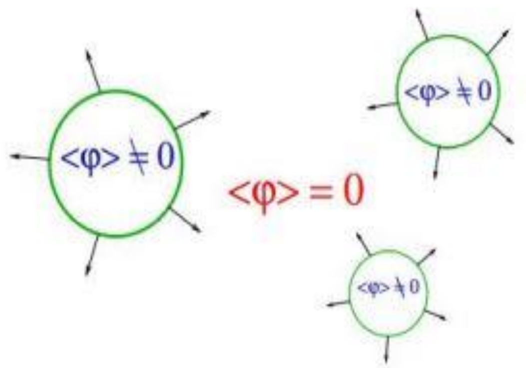


Applications of the constant-in-time model: First-Order Phase Transitions

[Dine et al. hep-ph/9203203]

$$V_{1-Loop}^{eff}(\phi,T) \approx D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4$$

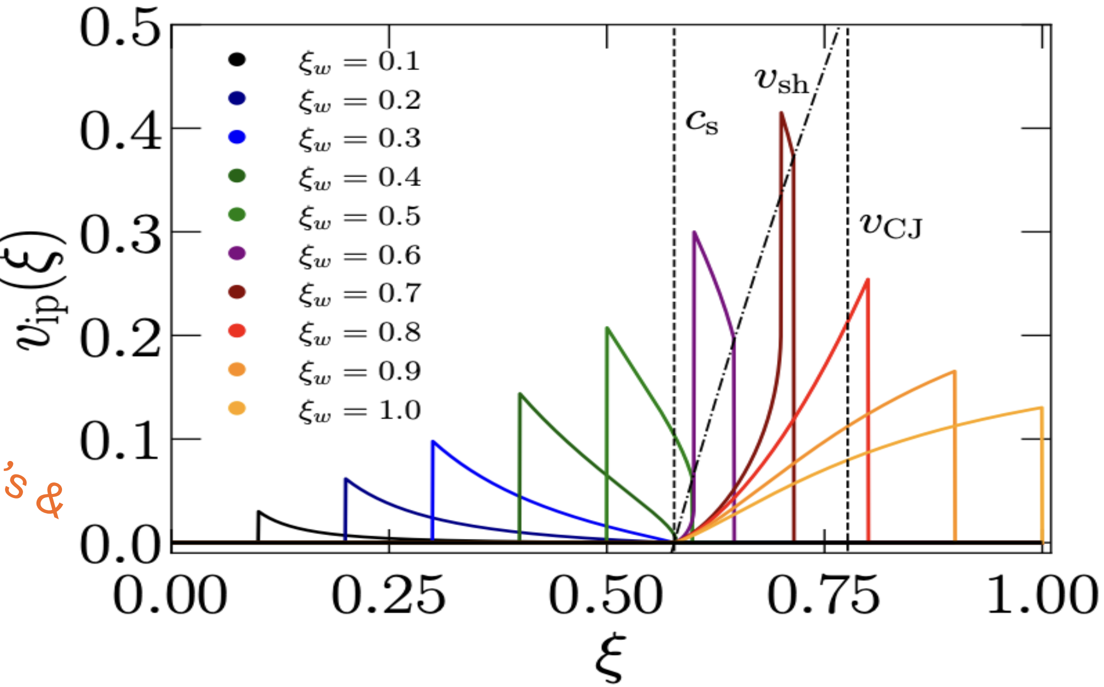




$$T_{ij} \supset \partial_i\phi\, \partial_j\phi + \omega\, \gamma^2 v_i v_j - B_i B_j$$

- BUBBLE COLLISIONS
- SOUND WAVES (fluid compressional modes induced by bubble collisions)
- HD & MHD TURBULENCE (vortical motion produced after the development of nonlinearities)

see Madeline's & Deepen's talks



Velocity Field Contributions

- For a statistically homogeneous and isotropic field

$$\langle v_i(\boldsymbol{k}) v_j^*(\boldsymbol{k}') \rangle = (2\pi)^6 \delta^3(\boldsymbol{k}-\boldsymbol{k}') \left[(\delta_{ij} - \hat{k}_i \hat{k}_j) \frac{E_N^v(k)}{4\pi k^3} + \hat{k}_i \hat{k}_j \frac{E_L^v(k)}{2\pi k^3} + i\epsilon_{ijl}\hat{k}_l \frac{H^v(k)}{8\pi k^3} \right]$$

vortical compressional helical

- Assuming a Gaussian velocity field such that we can use Wick's theorem for the anisotropic stresses UETC

$$E_\Pi(k) = E_\Pi^{vort}(k) + E_\Pi^{comp}(k) + E_\Pi^{mixed}(k) + E_\Pi^{hel}(k)$$

$$E_\Pi^{\rm vort}(k) = \frac{k^3}{2\bar{\rho}^2} \int_0^\infty \frac{E_{\rm N}(p)}{p} \, {\rm d}p \int_{-1}^{1} \frac{E_{\rm N}(\tilde{p})}{\tilde{p}^3} (1+z^2) \left[2 - \frac{p^2}{\tilde{p}^2}(1-z^2) \right] {\rm d}z \,, \quad E_\Pi^{\rm mix}(k) = \frac{2k^3}{\bar{\rho}^2} \int_0^\infty p \, E_{\rm N}(p) \, {\rm d}p \int_{-1}^{1} \frac{E_{\rm L}(\tilde{p})}{\tilde{p}^5} (1-z^4) \, {\rm d}z \,,$$

$$E_\Pi^{\rm comp}(k) = \frac{2k^3}{\bar{\rho}^2} \int_0^\infty p \, E_{\rm L}(p) \, {\rm d}p \int_{-1}^{1} \frac{E_{\rm L}(\tilde{p})}{\tilde{p}^5} (1-z^2)^2 \, {\rm d}z \,, \quad E_\Pi^{\rm hel}(k) = \frac{k^3}{2\bar{\rho}^2} \int_0^\infty \frac{H(p)}{p} \, {\rm d}p \int_{-1}^{1} \frac{H(\tilde{p})}{\tilde{p}^4} z \, (k - pz) \, {\rm d}z \,,$$

see Madeline's talk

Velocity Field Contributions: vortical component

[Monin & Yaglom – Statistical Fluid Mechanics]

For a purely vortical velocity field with a Von Kármán spectrum

$$E_N^v(k) \sim \begin{cases} k^5 & (k/k_{peak} \to 0) \quad Batchelor \\ k^{-2/3} & (k/k_{peak} \to \infty) \quad Kolmogorov \end{cases}$$

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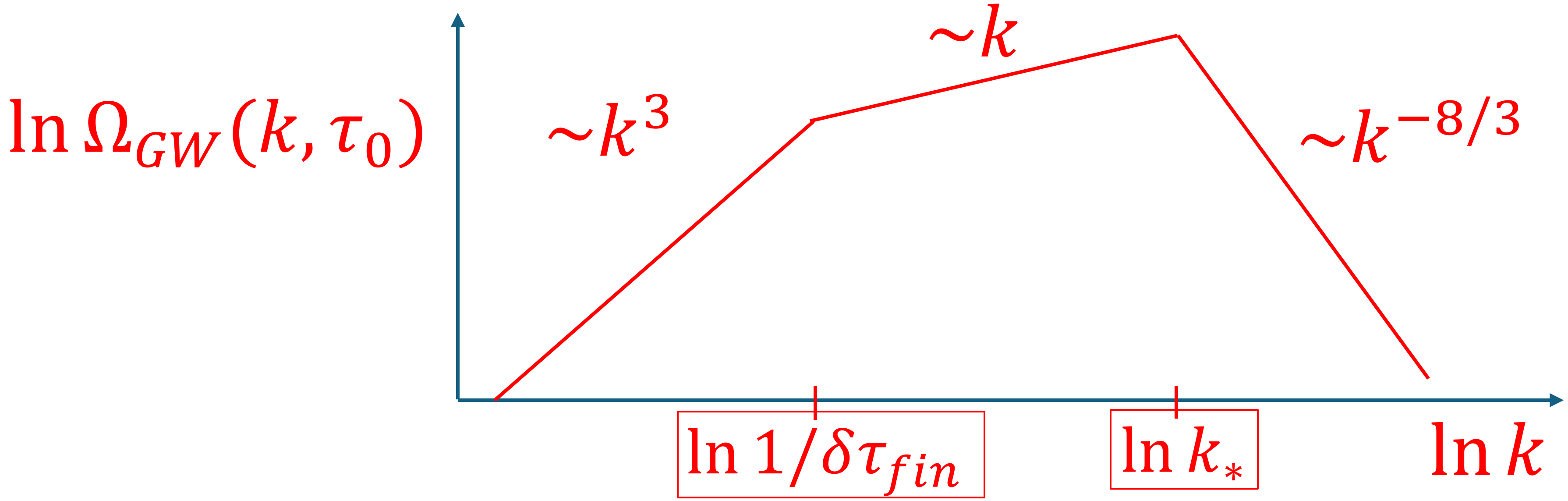
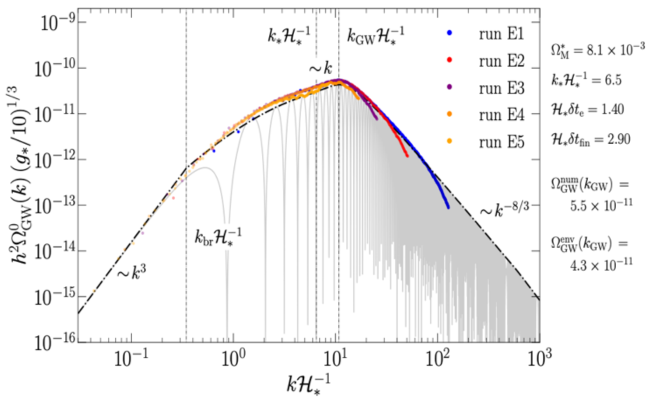
GW spectrum envelope for vortical turbulence in the constant-in-time model (flat spacetime)





Roper Pol et al. [2201.05630]

Relativistic hydrodynamics simulations

Fluid dynamics implementation in *CosmoLattice* ← see Kenneth's talk

Figueroa et al. [2006.15122]

$$T_{\mu\nu}^{fluid} = (\rho + p)\, u_\mu u_\nu + p\, g_{\mu\nu} \longrightarrow \partial_\mu T^{\mu\nu} = 0$$

CONSERVATION FORM (solving for $T^{0\mu}$)

NON-CONSERVATION FORM

Dahl et al. [2407.05826]

$$\partial_\tau \ln\tilde{\rho} = -\frac{1+c_s^2}{1-c_s^2 u^2}\left[\boldsymbol{\nabla}\cdot\boldsymbol{u} + \frac{1-c_s^2}{1+c_s^2}(\boldsymbol{u}\cdot\boldsymbol{\nabla})\ln\tilde{\rho}\right] + \frac{1+u^2}{1-c_s^2u^2}(1-3c_s^2)\,\mathcal{H} + \frac{1}{1-c_s^2u^2}\left[\frac{\tilde{f}^0_{tot}}{\tilde{\rho}}(1+u^2) - 2\,\boldsymbol{u}\cdot\frac{\tilde{\boldsymbol{f}}_{tot}}{\tilde{\rho}}\right]$$

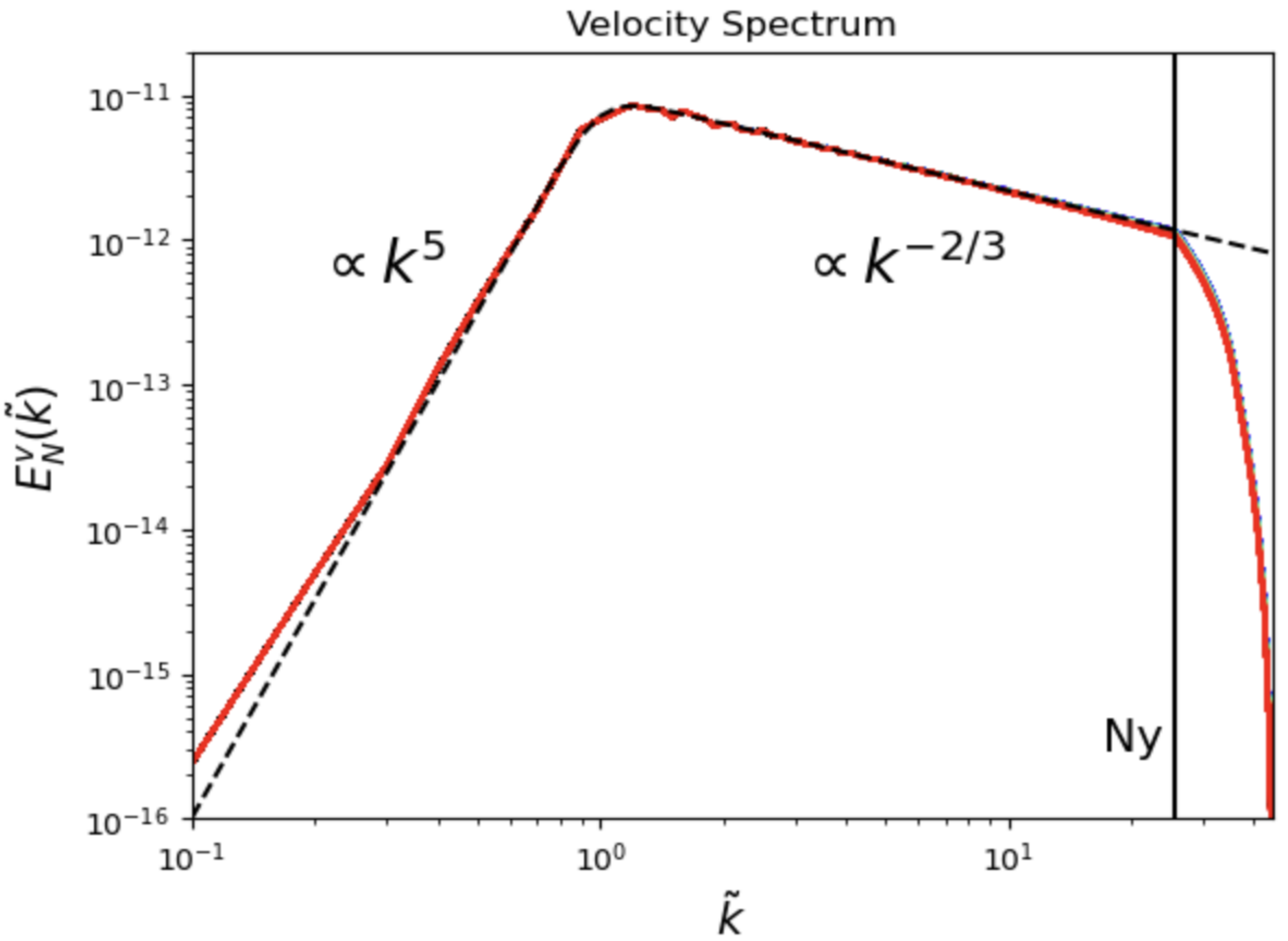
$$\partial_\tau \boldsymbol{u} = -(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} + \frac{c_s^2\,\boldsymbol{u}}{(1-c_s^2u^2)\gamma^2}\left[\boldsymbol{\nabla}\cdot\boldsymbol{u} + \frac{1-c_s^2}{1+c_s^2}(\boldsymbol{u}\cdot\boldsymbol{\nabla})\ln\tilde{\rho} + \frac{3c_s^2-1}{c_s^2}\mathcal{H} - \frac{\tilde{f}^0_{tot}}{c_s^2\tilde{\rho}} + \frac{2}{1+c_s^2}\frac{\boldsymbol{u}\cdot\tilde{\boldsymbol{f}}_H}{\tilde{\rho}}\right] - \frac{c_s^2}{1+c_s^2}\frac{\boldsymbol{\nabla}\ln\tilde{\rho}}{\gamma^2} + \frac{1}{1+c_s^2}\frac{\tilde{\boldsymbol{f}}_{tot}}{\tilde{\rho}\gamma^2}$$

timestepping scheme
Runge Kutta order 3 (Williamson 1980)

finite differences derivative scheme order 6

«Relativistic magnetohydrodynamics in the early Universe» Alberto Roper Pol & ASM [2501.05732]

Simulations of Gravitational Waves from vortical turbulence



<u>Subrelativistic case - numerical setup</u>

$$N = 512, \quad \tilde{k}_{IR} = 0.1, \quad \tilde{k}_{Ny} = \frac{\tilde{k}_{IR} N}{2} = 25.6,$$

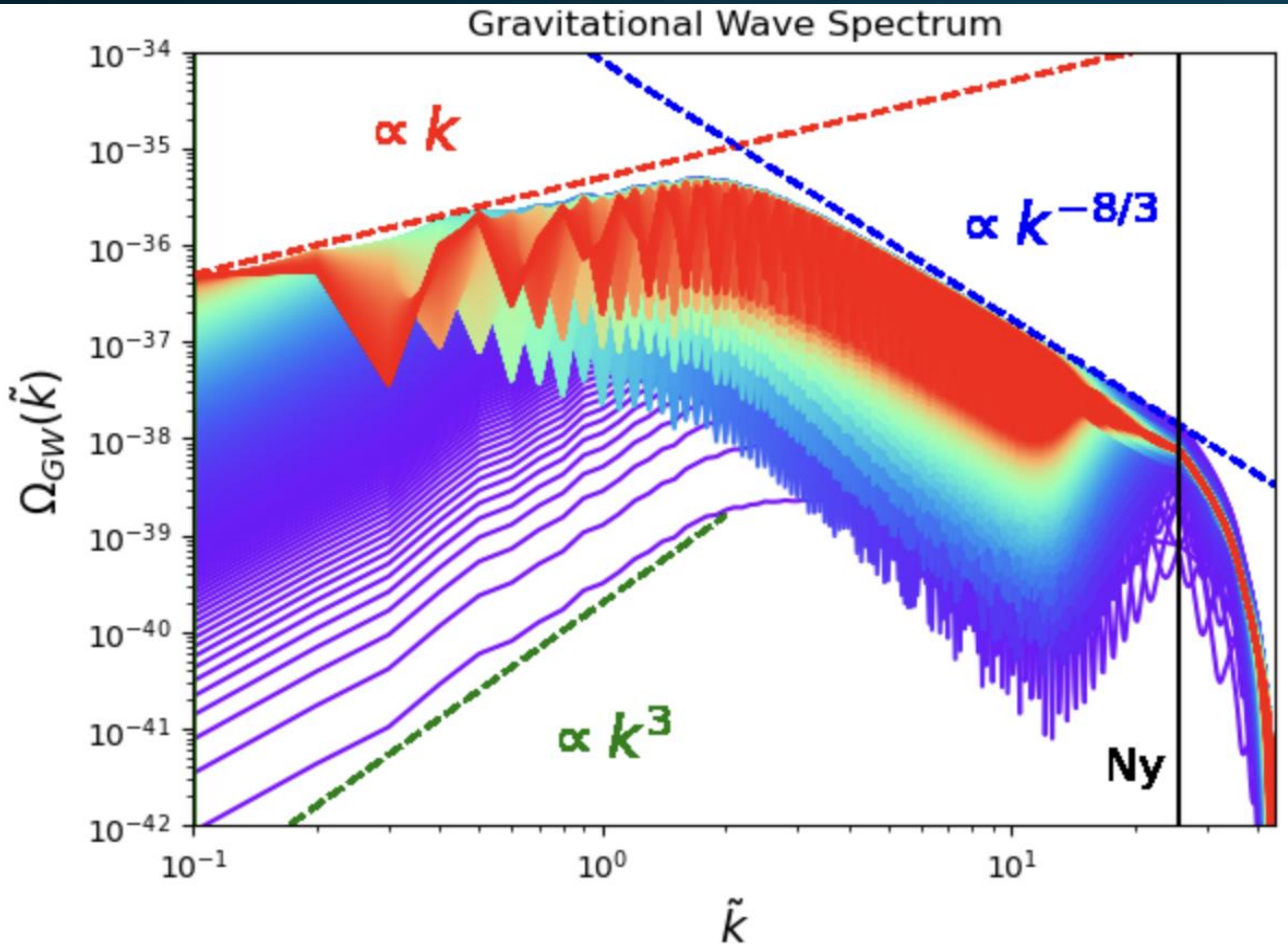
$$\tilde{t}_{fin} - \tilde{t}_* = \frac{1}{\tilde{k}_{IR}} = 10, \quad d\tilde{t} = 10^{-3}, \quad \frac{d\tilde{t}}{d\tilde{x}} < 0.01,$$

$$v_{rms} \approx 4 \times 10^{-6}, \quad \underset{(\eta)}{viscosity} = \frac{v_{rms}}{\tilde{k}_{Ny}} \approx 10^{-7}$$

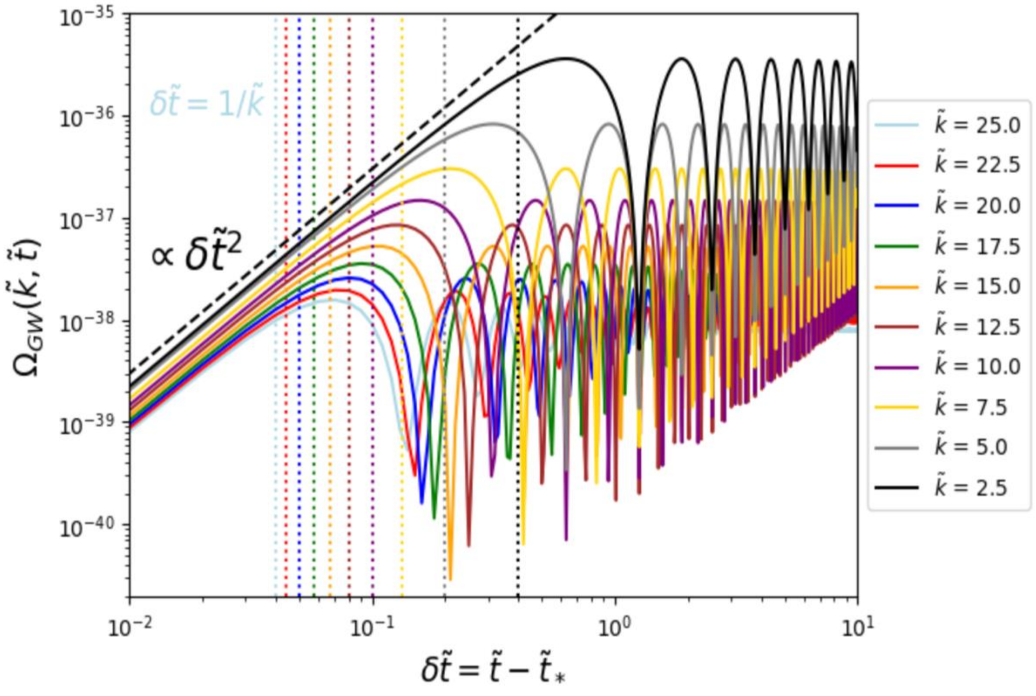
$$\partial_\tau \boldsymbol{u} = \ \dots + \eta \, \boldsymbol{\nabla}^2 \boldsymbol{u}$$

$\delta\tau_{eddy} \sim 10^7$ → almost no decay of the source

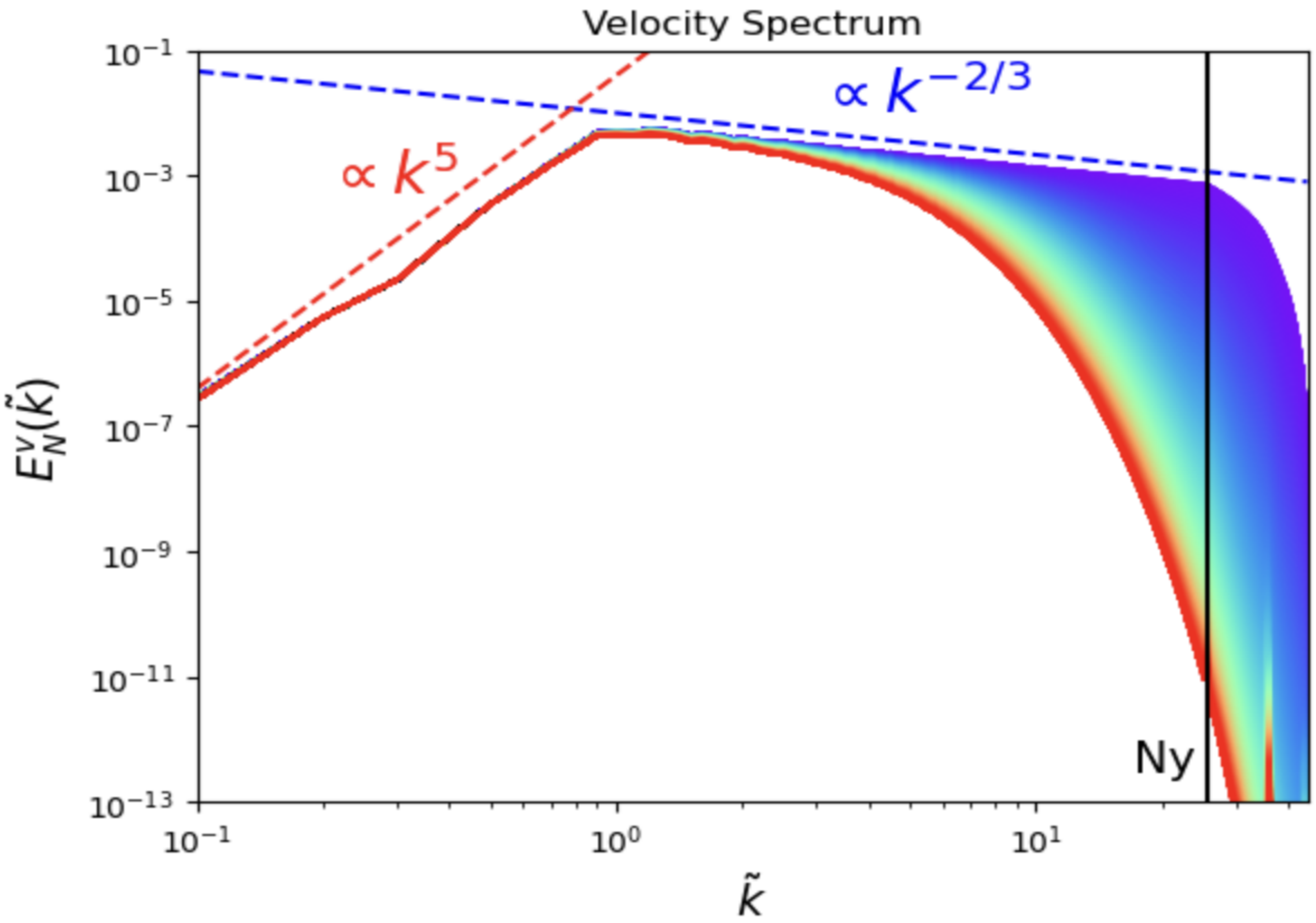
Simulations of Gravitational Waves from vortical turbulence



modes grow as $\delta\tilde{t}^2$
and saturate when $\tilde{k}\delta\tilde{t} \approx 1$



Simulations of Gravitational Waves from vortical turbulence



Relativistic case - numerical setup

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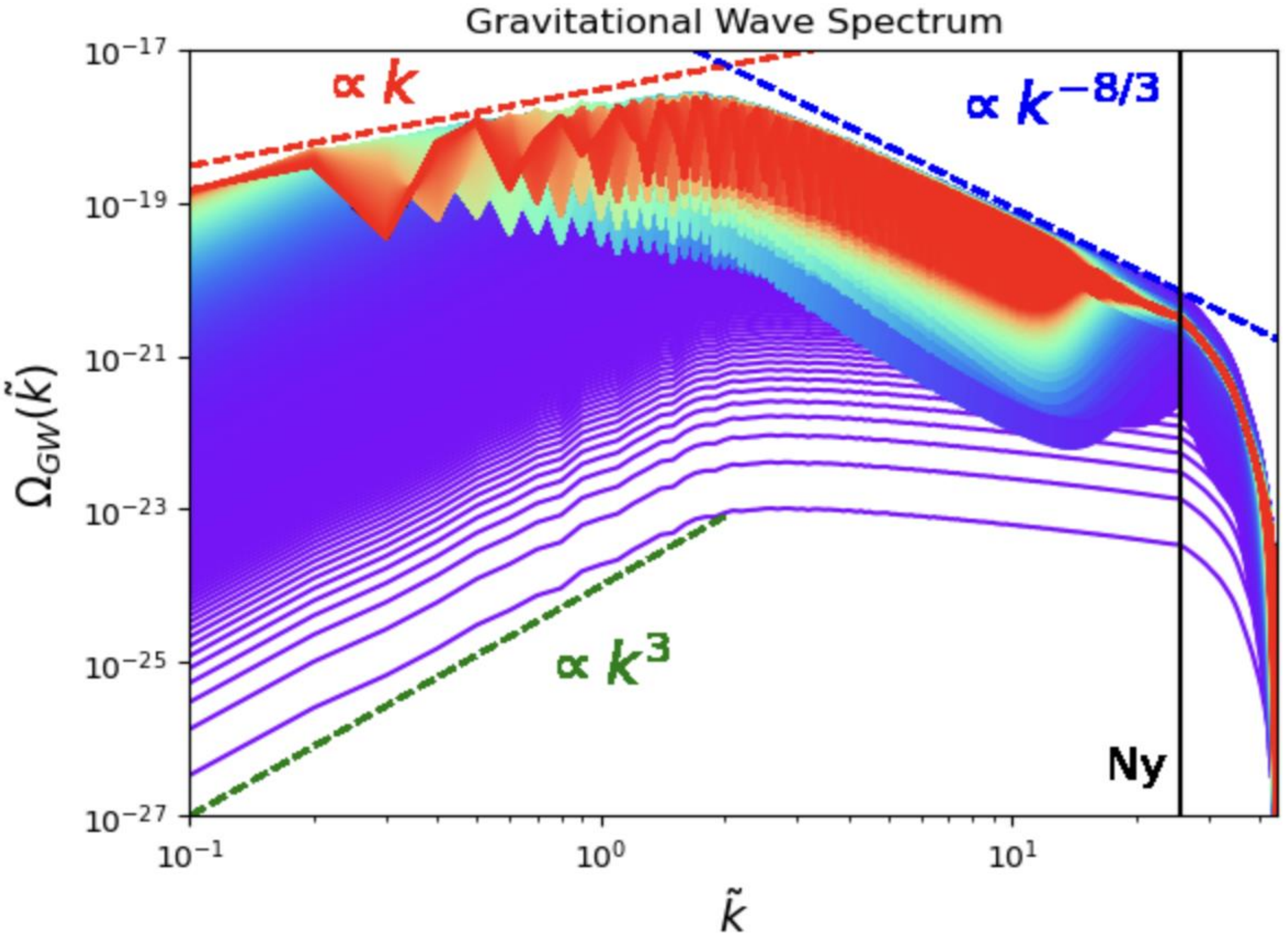
$$\tilde{t}_{fin} - \tilde{t}_* = \frac{1}{\tilde{k}_{IR}} = 10, \quad d\tilde{t} = 10^{-4}, \quad \frac{d\tilde{t}}{d\tilde{x}} < 0.001,$$

$$v_{rms} \approx 0.1, \qquad viscosity = \frac{v_{rms}}{\tilde{k}_{Ny}} \approx 10^{-2}$$

$\delta\tau_{eddy} \sim 10^2$ → visible decay of the source

Simulations of Gravitational Waves from vortical turbulence



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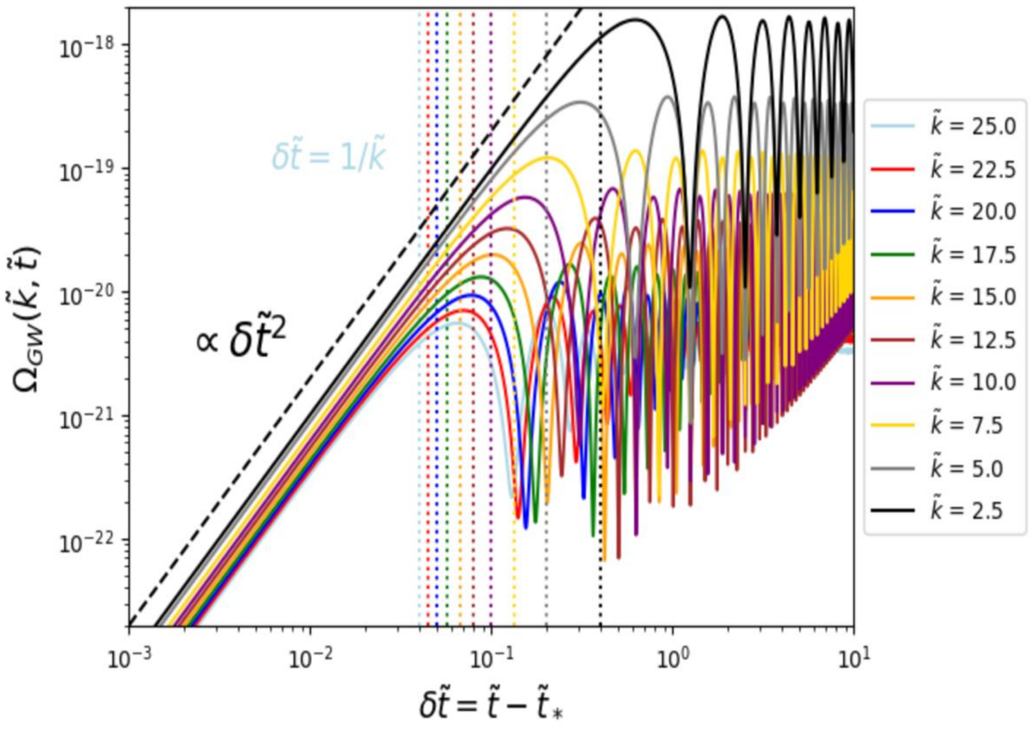

Velocity Field Contributions: compressional component

Hindmarsh et al. [1304.2433]

Weak and moderately strong ($\alpha \lesssim 1$) First-Order Phase Transitions → sound wave phase after collisions

Sound-shell model for GW production from sound waves (see Mark's talk)

$$\Omega_{GW}(k,\tau_0) \equiv 3\,\mathcal{T}_{GW} \int dz\, dP\, f\left(P,\tilde{P}\right) E_L^v(P) E_L^v(\tilde{P})\, \Delta^2_{ssm}\left(k,P,\tilde{P},\tau_{fin}\right) \qquad z = \frac{\boldsymbol{k}\cdot\boldsymbol{P}}{kP}, \quad \tilde{\boldsymbol{P}} = \boldsymbol{k} - \boldsymbol{P}$$

$$\Delta^2_{ssm}\left(k,P,\tilde{P},\tau_{fin}\right) = \frac{1}{4}\sum_{n,m=\pm 1} \Delta^2_{mn} \longrightarrow$$

$$\Delta^2_{mn}(\hat{p}_{mn}) = \left[\mathrm{Ci}\left(\hat{p}_{mn}\tau_{fin}\right) - \mathrm{Ci}(\hat{p}_{mn}\tau_*)\right]^2 + \left[\mathrm{Si}\left(\hat{p}_{mn}\tau_{fin}\right) - \mathrm{Si}(\hat{p}_{mn}\tau_*)\right]^2$$

$$\hat{p}_{mn} = (p + m\tilde{p})c_s + nk$$

Hindmarsh & Hijazi [1909.10040]

Roper Pol, Procacci, Caprini [2308.12943]

$m = 1, n = 1$ ⟶ dominant contribution

Velocity Field Contributions: compressional component

Hindmarsh et al. [1304.2433]

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$$\Omega_{GW}(k,\tau_0) \equiv 3\,\mathcal{T}_{GW}\int dz\, dP\, f(P,\tilde{P}) E_L^v(P) E_L^v(\tilde{P})\, \Delta^2_{ssm}(k,P,\tilde{P},\tau_{fin}) \qquad z = \frac{\boldsymbol{k}\cdot\boldsymbol{P}}{kP},\quad \tilde{\boldsymbol{P}} = \boldsymbol{k} - \boldsymbol{P}$$

$$\Delta^2_{ssm}(k,P,\tilde{P},\tau_{fin}) = \frac{1}{4}\sum_{n,m=\pm1} \Delta^2_{mn} \rightarrow \Delta^2_{mn}(\hat{p}_{mn}) = \left[\mathrm{Ci}(\hat{p}_{mn}\tau_{fin}) - \mathrm{Ci}(\hat{p}_{mn}\tau_*)\right]^2 + \left[\mathrm{Si}(\hat{p}_{mn}\tau_{fin}) - \mathrm{Si}(\hat{p}_{mn}\tau_*)\right]^2$$

$$\hat{p}_{mn} = (p + m\tilde{p})c_s + nk$$

Hindmarsh & Hijazi [1909.10040]

Roper Pol, Procacci, Caprini [2308.12943]

$m = 1, n = 1$ ⟶ dominant contribution

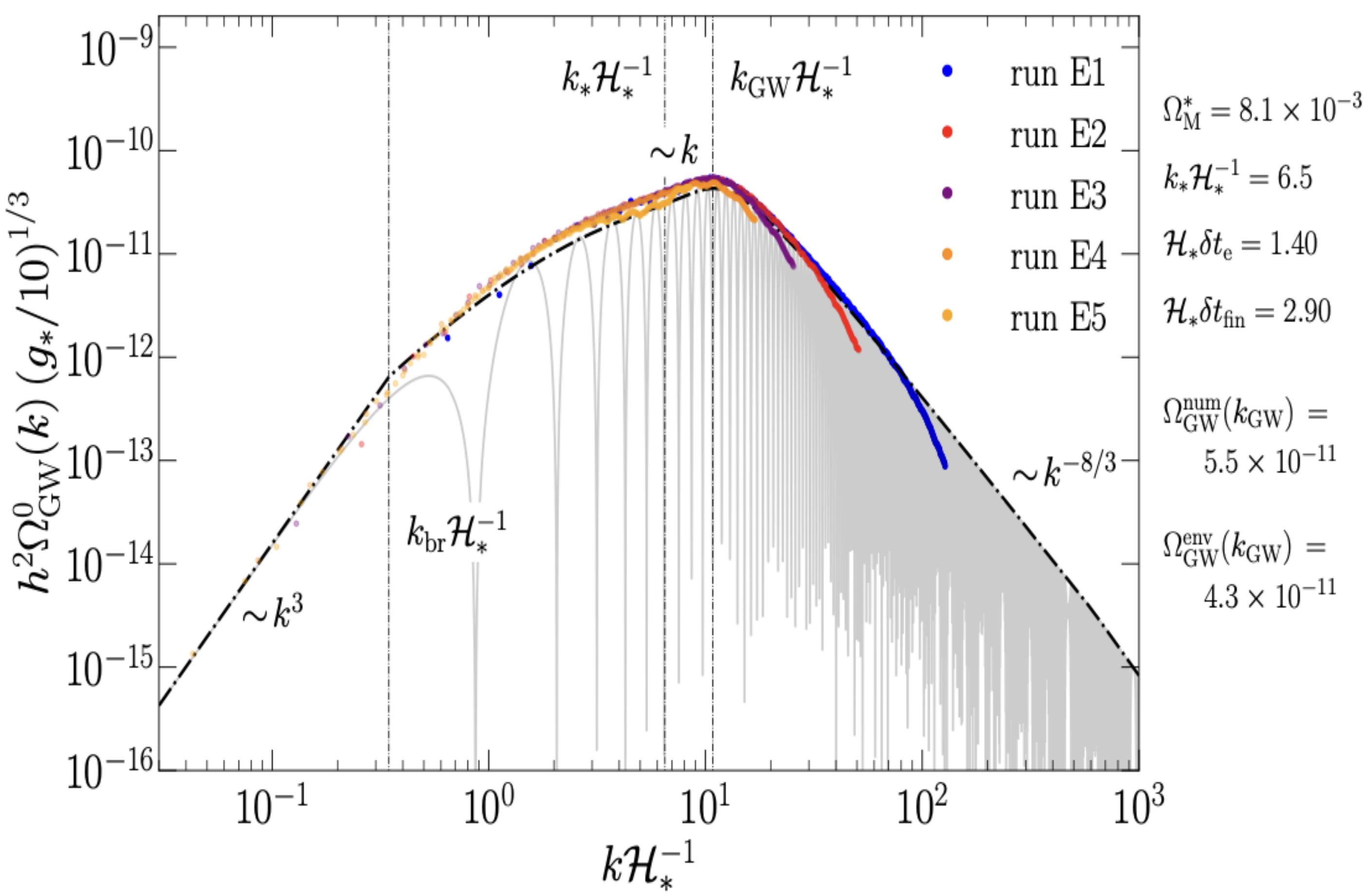
$m = -1, n = \pm 1$ ⟶ if the velocity spectrum is very peaked, i. e. $E_L^v(P) \approx \delta(P - k_*)$, we have approximately

$$p = \tilde{p} = k_* \;\rightarrow\; \tilde{p}_{mn} = \pm k \;\rightarrow\; \Delta^2_{ssm} = \frac{1}{2}\left\{\left[\mathrm{Ci}(k,\tau_f) - \mathrm{Ci}(k,\tau_*)\right]^2 + \left[\mathrm{Si}(k,\tau_f) - \mathrm{Si}(k,\tau_*)\right]^2\right\}$$

⟶ $\Omega_{GW}(k,\tau_0) \equiv 3\,\mathcal{T}_{GW} E_\Pi^*(k)\, \Delta^2_{ssm}$ same result as in the constant-in-time model (see Madeline's talk)

Magnetic Field Contribution

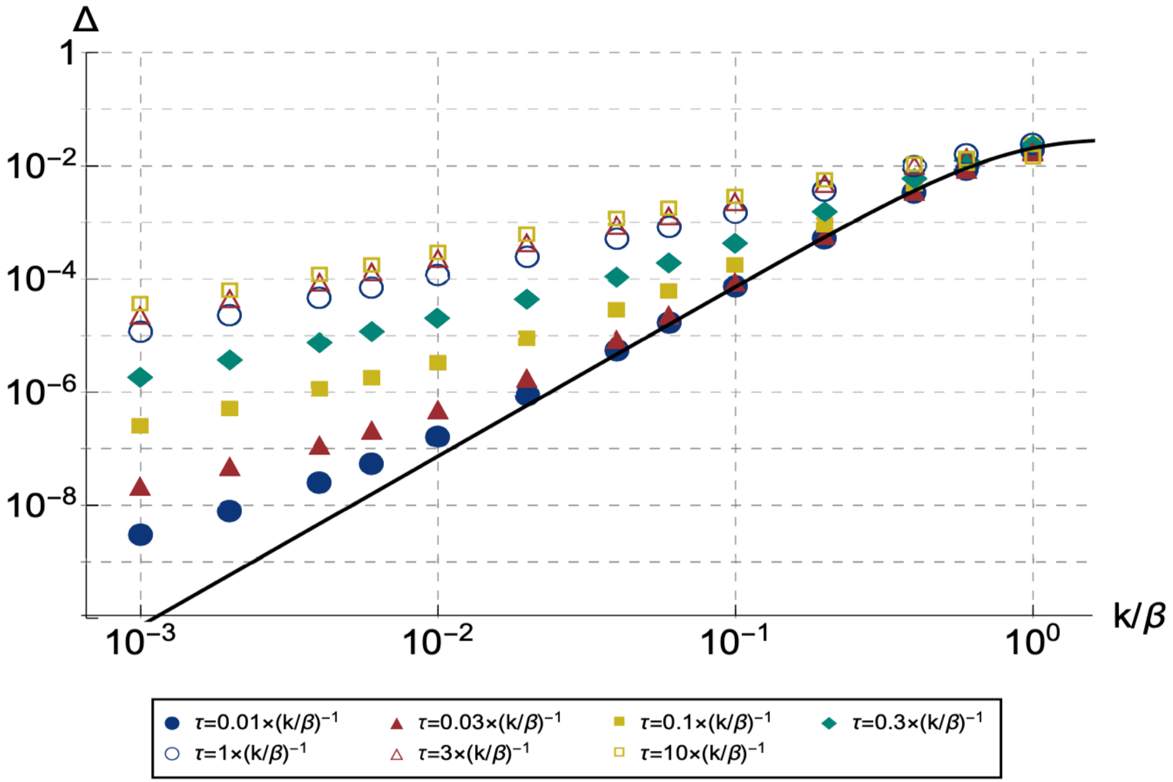
- In a First-Order Phase Transition scalar field gradients can generate magnetic fields (see Patel & Vachaspati [2108.05357]) and/or the pre-existing magnetic fields can be amplified through vortical hydrodynamic turbulence resulting, due to the high conductivity of the primordial plasma (see Arnold, Moore & Yaffe 2003), in MHD turbulence
- Magnetic fields can hence be a source of Gravitational Waves and, being fully vortical, their contribution can be described with the constant-in-time model as shown in numerical simulations (see Axel's talk and Roper Pol et al. [2201.05630])



Which other contributions can be described with the constant-in-time model?

Scalar bubble collisions (see Jinno & Takimoto [1707.03111])

GW spectrum goes from $\propto k^3$ to $\propto k$
in the long-lasting limit of the collided bubbles



Which other contributions can be described with the constant-in-time model?

Scalar bubble collisions (see Jinno & Takimoto [1707.03111])

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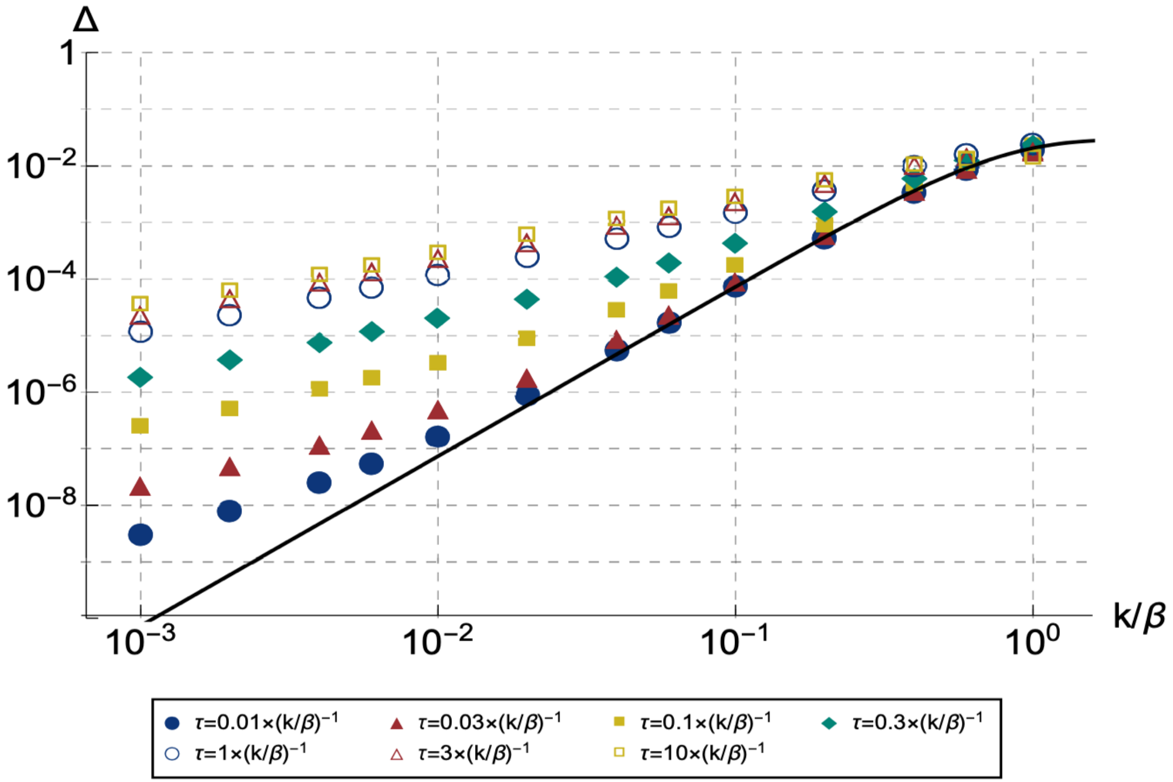
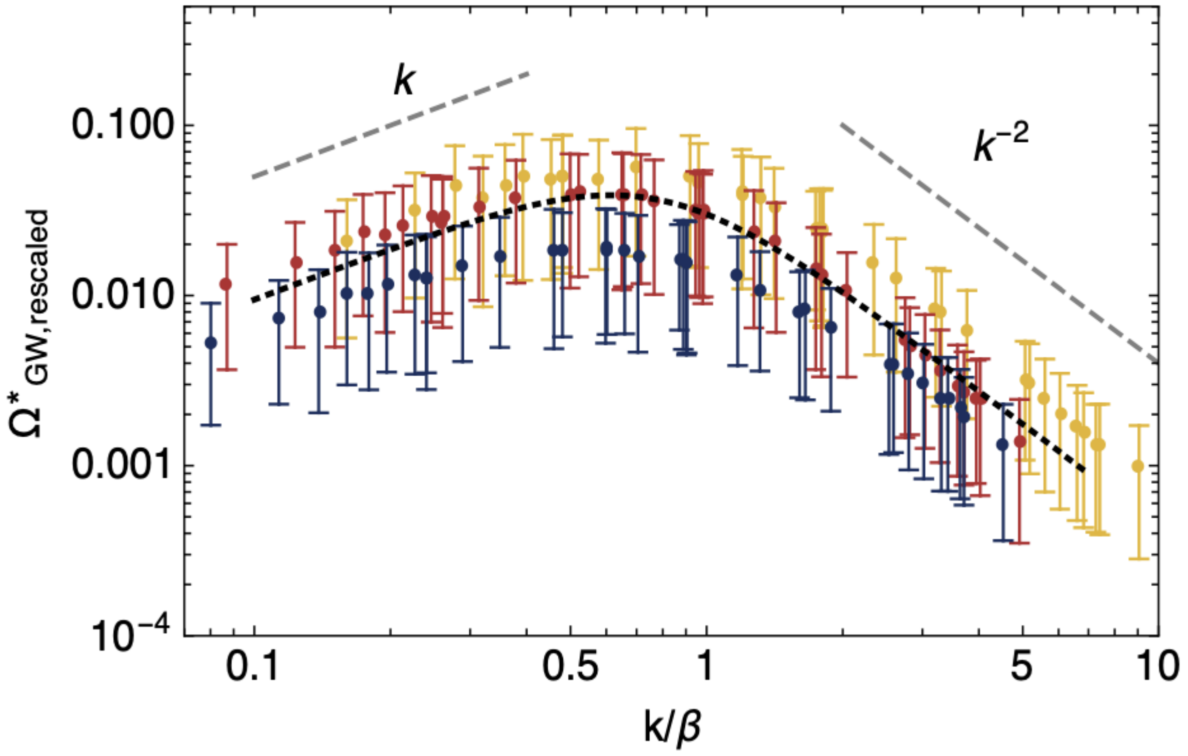


Feebly interacting particles
(see Jinno, Shakya & van de Vis [2211.06405])

GW spectrum with super-Hubble k^3 scaling
and intermediate k scaling



Beyond the constant-in-time model

When the source is decaying faster and the GW spectrum cannot be described with the constant-in-time model there are improved models leading to a different GW spectrum (mostly in the IR)

- Coherent decay model

$$E_\Pi(k,\tau_1,\tau_2) = \sqrt{E_\Pi(k,\tau_1)}\sqrt{E_\Pi(k,\tau_2)}$$

(modeling the decay of the amplitude and peak scale of the source according to its specific evolution)

$$\Omega_{GW}(k,\tau_0) \equiv 3\,\mathcal{T}_{GW}\left[\int_{\tau_*}^{\tau_f} \frac{d\tau_1}{\tau_1}\sqrt{E_\Pi(k,\tau_1,\tau_1)}\cos k(\tau_0-\tau_1)\right]^2$$

$$k_*(t) = k_*\left(1+\frac{\delta t}{\delta t_e}\right)^{-q}, \qquad A(t) = A\left(1+\frac{\delta t}{\delta t_e}\right)^{-p}$$

- Kraichnan decorrelation (see Auclair et al. [2205.02588])

GW spectrum still compatible with the one in the constant-in-time model

$$E_\Pi(k,\tau_1,\tau_2) = E_\Pi(k)\,\exp[-\frac{1}{2}k^2 v_{sw}^2(\tau_1-\tau_2)^2]$$

Conclusions

- Studying the UETC of the anisotropic stresses of the source is important to understand the GW spectrum from stochastic processes in the early Universe
- Various contributions to the GW spectrum produced in a First-Order Phase Transition show a universal k^3 and k behavior which can be described with the constant-in-time model
- The constant-in-time model applies in general when the source is constant or slowly decaying with respect to the time it takes to source GWs (which depends on the specific scale of interest)
- Decaying sources can be described by applying extensions of the constant-in-time model (e. g. coherent decay and Kraichnan decorrelation)
- Fluid compressional modes (like sound waves) require a different description for the UETC

Thanks for your attention!