UNIVERSITÉ DE GENÈVE

Gravitational wave production: the interplay between vortical and compressional motions

NORDITA (Stockholm) - August 8th, 2025

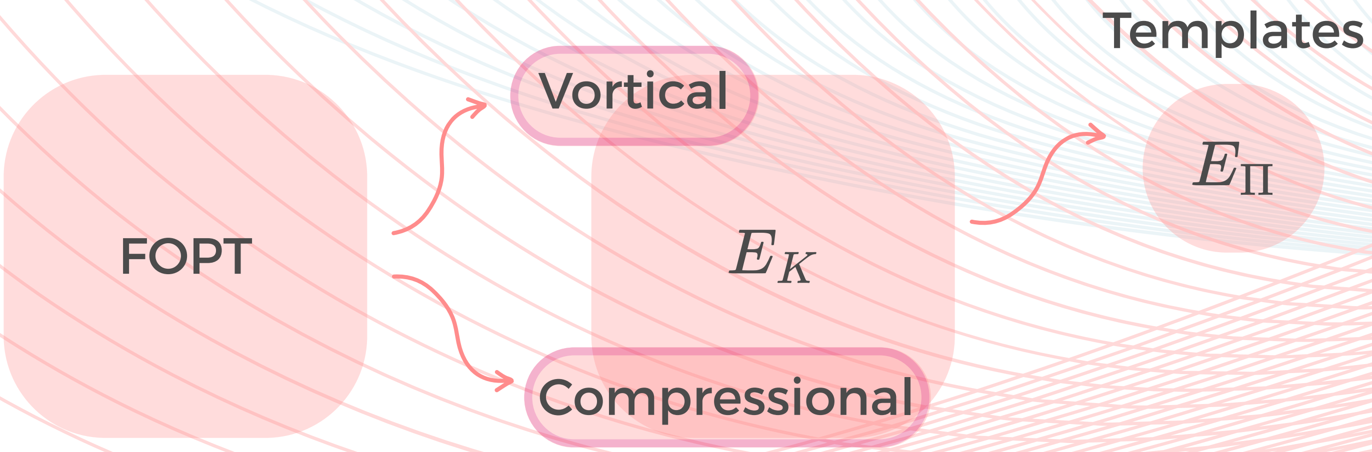
Numerical Simulations of Early Universe Sources of Gravitational Waves

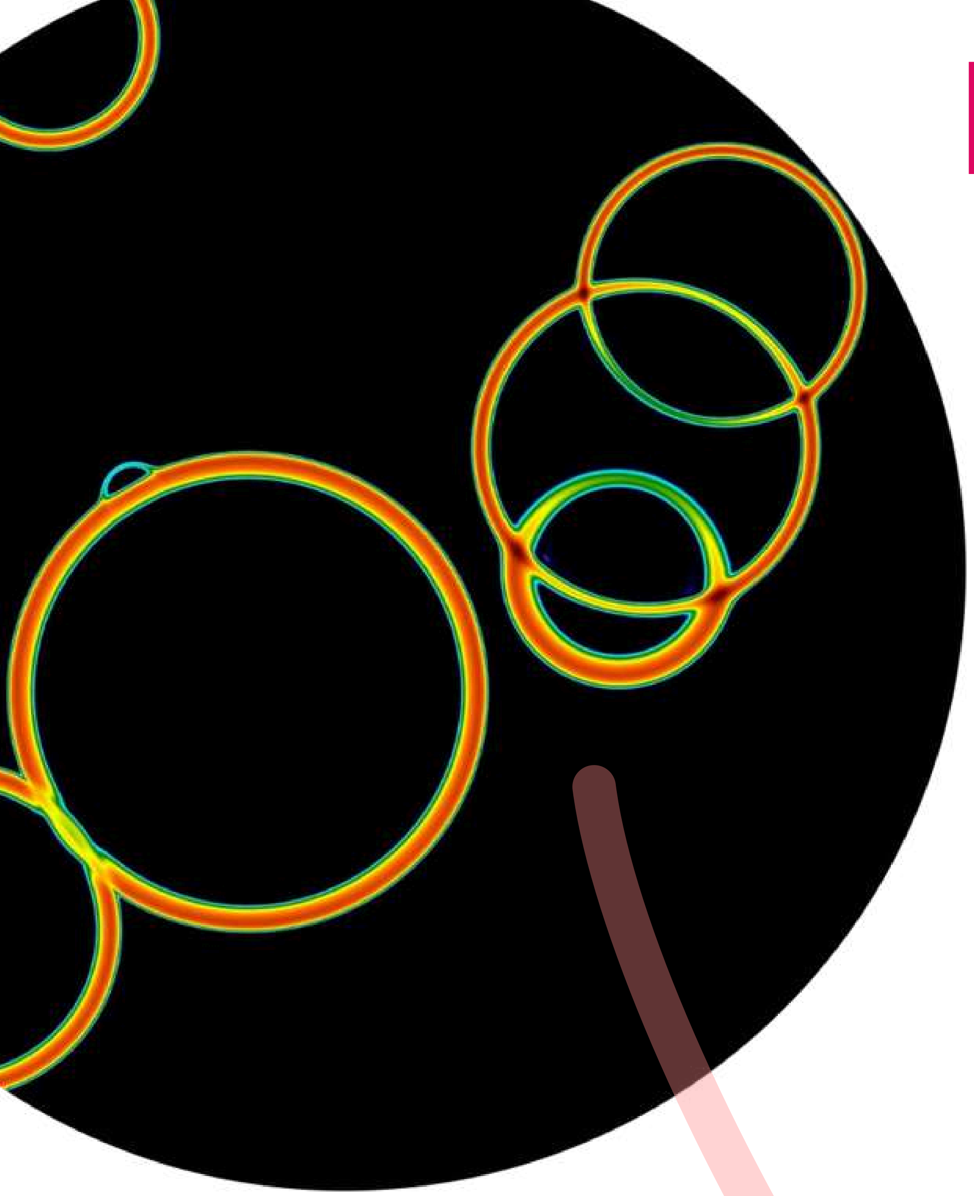
Madeline SALOMÉ

Part I

Introduction

From FOPT to GWs' sourcing





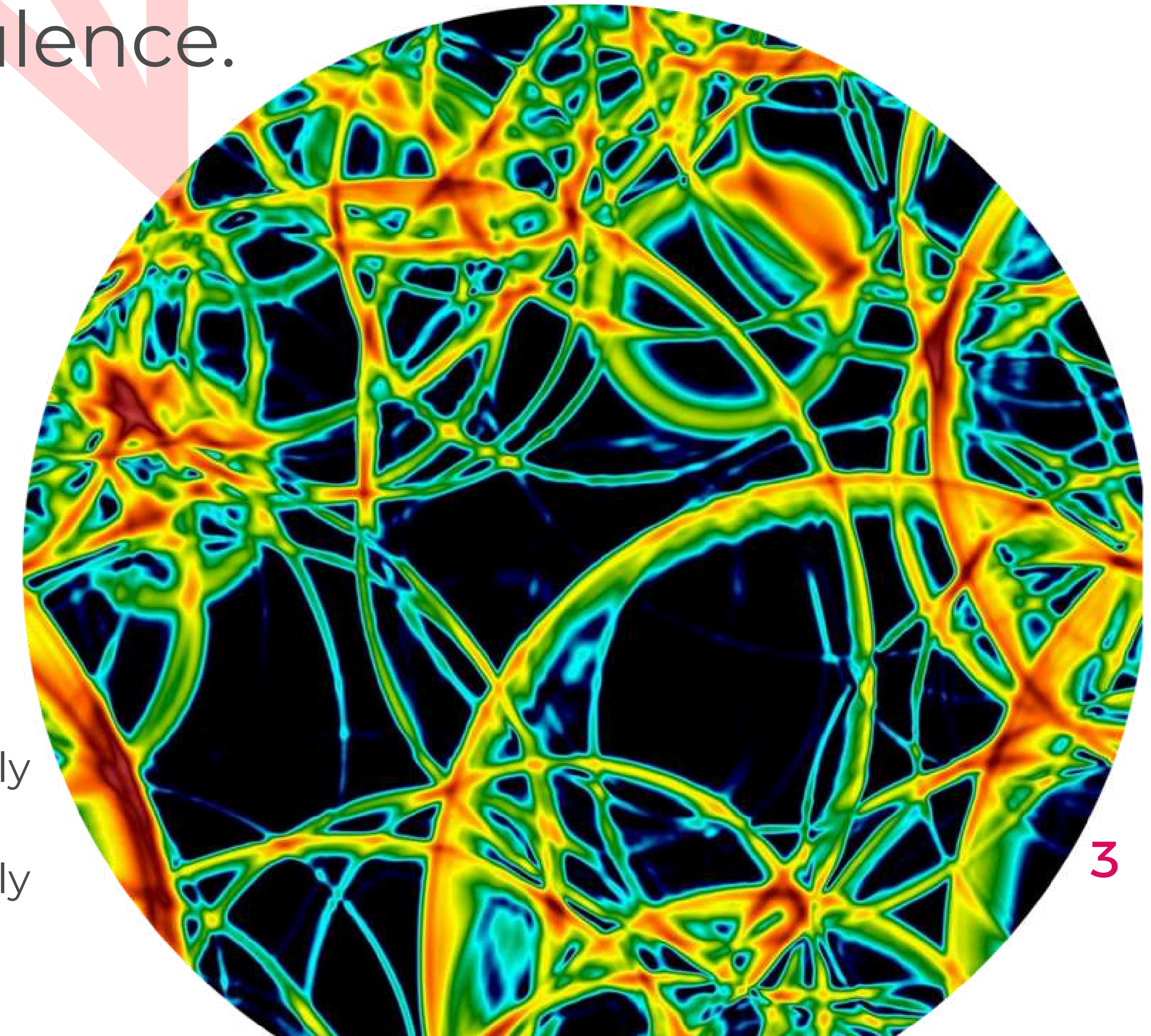
First-order Phase Transition

FOPT Bubble expands and turns the Universe from one state to another. When they collides, sound waves are produced (**compressional** motion in the linear regime).

Turbulence The bubble merging possibly induces also **vortical** motions in the fluid corresponding to magneto-hydrodynamic turbulence.

Stress-Energy Tensor

$$T_{ij} = w\gamma^2 u_i u_j - B_i B_j + \partial_i \phi \, \partial_j \phi$$

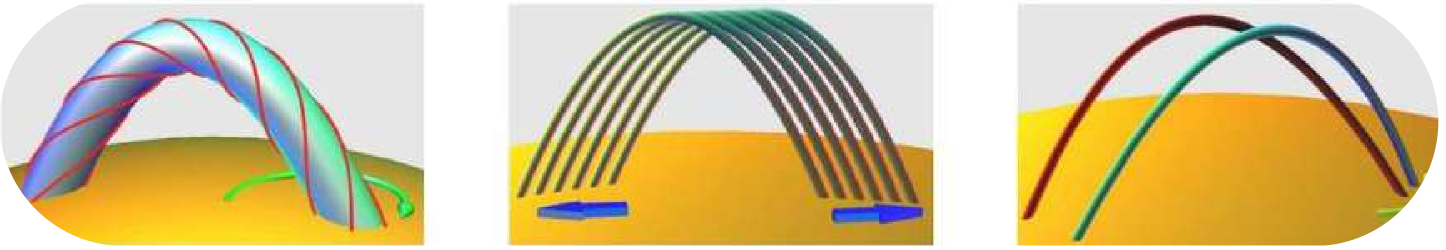


Publications
In preparation.

[1] A. Roper Pol, C. Caprini, A. S. Midiri., M. Salomé "Gravitational wave spectrum from slowly decaying sources in the early Universe: constant-in-time and coherent-decay models."
[2] A. Roper Pol, M. Salomé. "Gravitational wave production from acoustic and fractionally compressible sources."

Properties of the field

https://www.slideserve.com/mikko/injection-photosph-rique-d-h-licit-magn-tique-implications-sur-l-mergence-du-champ-magn-tique#google_vignette

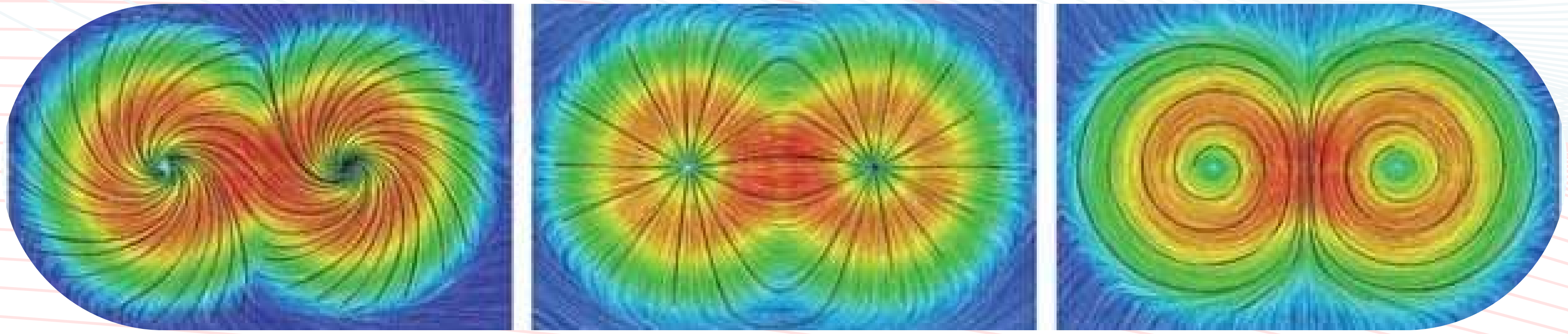


Degree of helicity

Velocity Field Longitudinal & normal components + Helicity.

E_K^{comp} E_K^{vort} E_K^{hel}

Fraction of compressibility



Mix Compressional Vortical

Cuicui Zhang et al. "Characteristic Ocean Flow Visualization Using Helmholtz Decomposition". In: May 2018, pp. 1–4. doi: 10.1109/OCEANSKOBE.2018.8559411.

How to combine them to produce GWs ?

The Unequal-Time Correlator

$$E_\Pi \propto \langle T_{ij}^{TT} T_{ij}^{TT} \rangle \sim \int \int \langle u_i u_j \rangle \langle u_i u_j \rangle \propto \int E_K \int E_K$$

Wick's theorem

Antonino's talk

GW Equation:

$$(\partial_t^2 - c^2 \nabla^2) h_{ij}(\mathbf{x}, t) = \frac{16\pi G}{ac^2} T_{ij}^{TT}(\mathbf{x}, t)$$

Small perturbation in the space-time geometry:

$$ds^2 = a(t)^2 \left[-dt^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \right]$$

GWs

Sources:

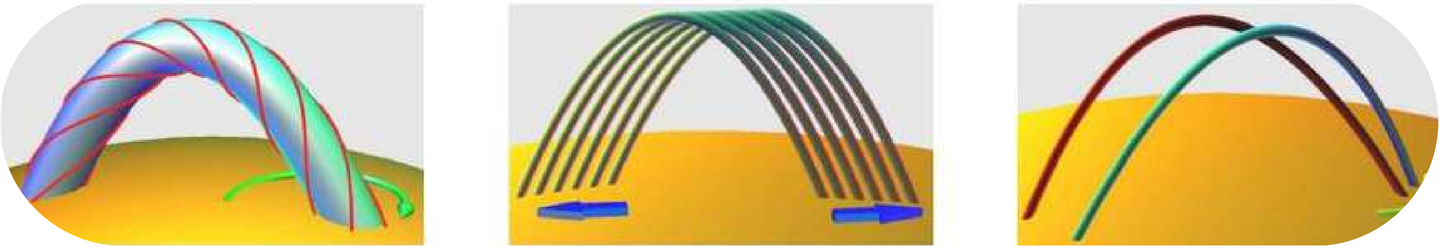
$$T_{ij} = w\gamma^2 u_i u_j$$

Fluid velocity

How to combine them to produce GWs ?

The Unequal-Time Correlator

$$E_\Pi \propto \int E_K \int E_K$$

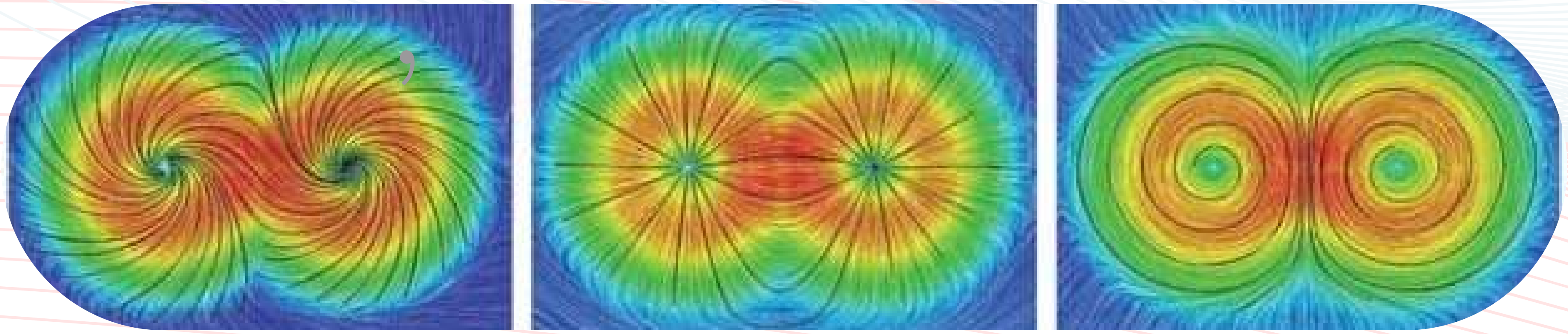


Degree of helicity

$$E_K^{\text{vort}} E_K^{\text{vort}},\ E_K^{\text{comp}} E_K^{\text{comp}},\ E_K^{\text{vort}} E_K^{\text{comp}}$$

$$E_K^{\text{hel}} E_K^{\text{hel}},\ E_K^{\text{hel}} E_K^{\text{comp}},\ E_K^{\text{hel}} E_K^{\text{vort}}$$

Fraction of compressibility

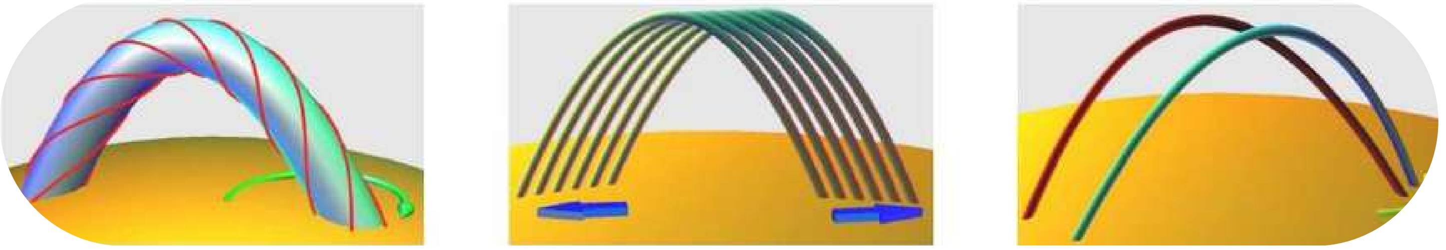


Mix Compressional Vortical

How to combine them to produce GWs ?

The Unequal-Time Correlator

$$E_\Pi \propto \int E_K \int E_K$$



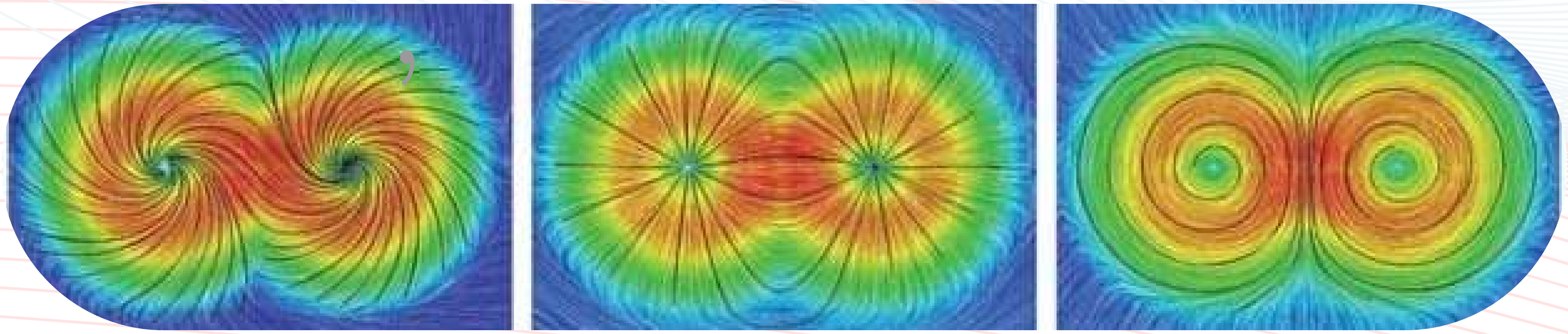
Degree of helicity

$$E_K^{\rm vort} E_K^{\rm vort}, \; E_K^{\rm comp} E_K^{\rm comp}, \; E_K^{\rm vort} E_K^{\rm comp}$$

$$E_K^{\rm hel} E_K^{\rm hel}, \; E_K^{\rm hel} E_K^{\rm comp}, \; E_K^{\rm hel} E_K^{\rm vort}$$

Symmetric part consideration

Fraction of compressibility



Mix Compressional Vortical

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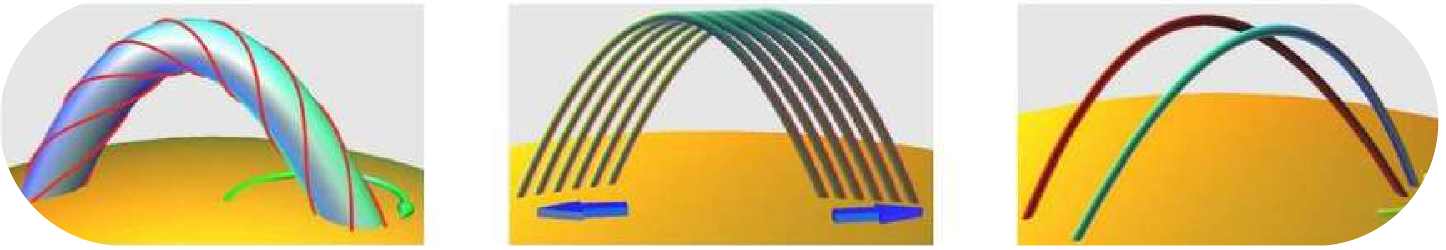
$$E_\Pi \propto \int E_K \int E_K$$

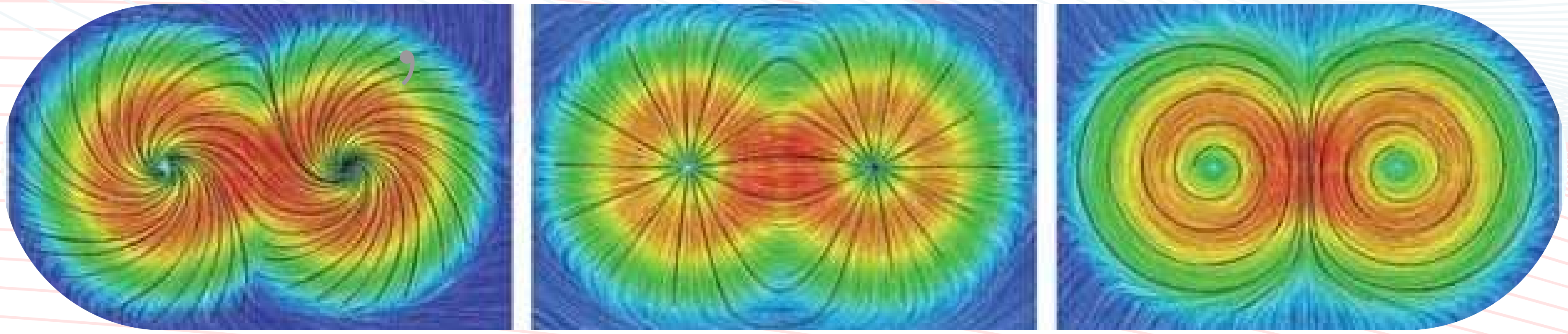


Degree of helicity

$$E_K^{\rm vort} E_K^{\rm vort}\,,\quad E_K^{\rm comp} E_K^{\rm comp}\,,\quad E_K^{\rm vort} E_K^{\rm comp}$$

$$E_K^{\rm hel} E_K^{\rm hel}\,,\quad E_K^{\rm hel} E_K^{\rm comp}\,,\quad E_K^{\rm hel} E_K^{\rm vort}$$

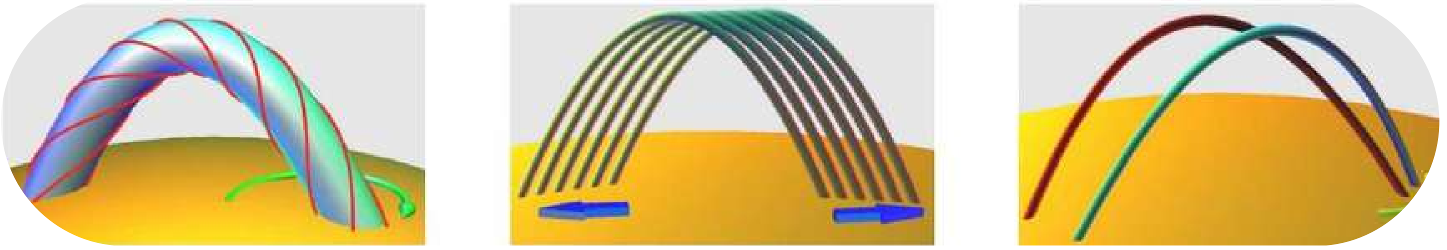
Fraction of compressibility



Mix Compressional Vortical

How to combine them to produce GWs ?

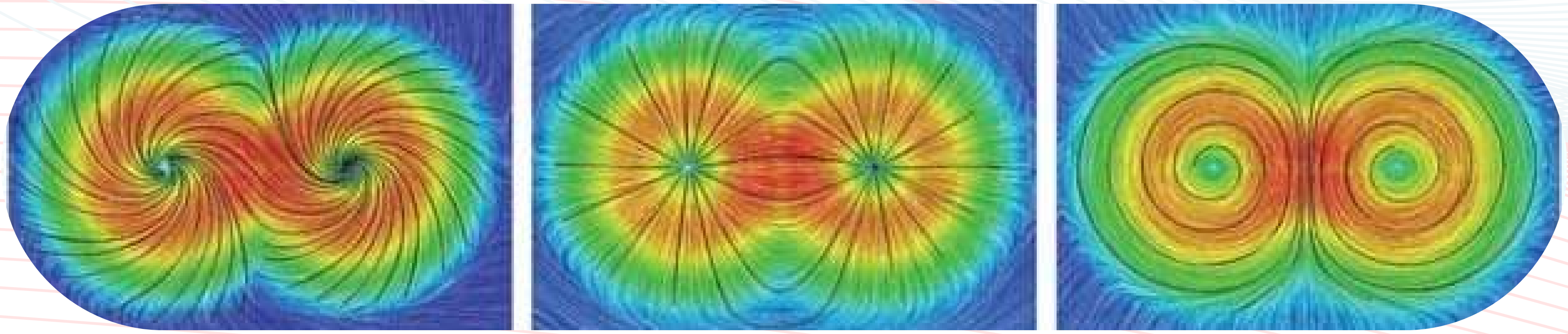
The Unequal-Time Correlator



Degree of helicity

$$(1-q)^2 E_\Pi^{\mathrm{vort}} + q^2 E_\Pi^{\mathrm{comp}} + q(1-q) E_\Pi^{\mathrm{mix}} + \varepsilon^2 E_\Pi^{\mathrm{hel}}$$

Fraction of compressibility



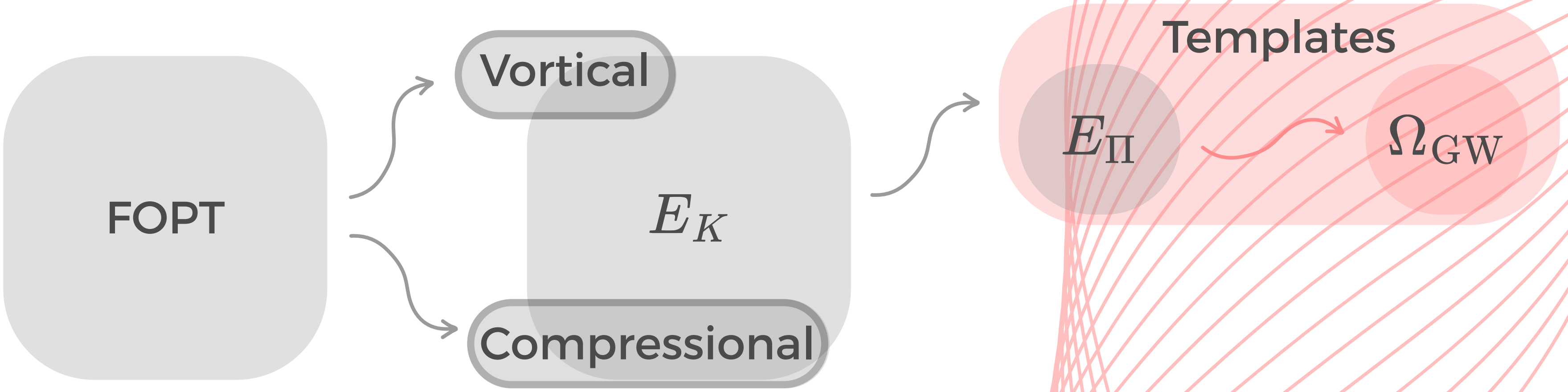
Mix Compressional Vortical

[1] A. Roper Pol, C. Caprini, A. S. Midiri., M. Salomé "Gravitational wave spectrum from slowly decaying sources in the early Universe: constant-in-time and coherent-decay models."

Part II

GW spectrum

GW energy density spectrum



Our observable

$$\Omega_{\rm GW}(k,t) = 3k \int_{t_*}^{\min(t,t_{\rm fin})} dt_1 \int_{t_*}^{\min(t,t_{\rm fin})} dt_2 \frac{E_\Pi(k,t_1,t_2)}{t_1 t_2} \cos\left[k(t-t_1)\right] \cos\left[k(t-t_2)\right]$$

Fully vortical motion

t_* Sourcing

t_e Decay

$t_{\rm fin}$ Dilution

t_0 Today

$$\Omega_{\rm GW}(k, t_0) = F^0_{\rm GW}\,\Omega_{\rm GW}(k, t_{\rm fin})$$

$$F^0_{\rm GW} = \left(\frac{a_*}{a_0}\right)^4 \left(\frac{H_*}{H_0}\right)^2$$

Constant-in-time model

Antonino's talk

$$E_\Pi(k, t_1, t_2) \simeq E_\Pi(k, t_*) = E^*_\Pi(k)$$

$$\Omega_{\rm GW}(k, t_0) = 3k F^0_{\rm GW} E^*_\Pi(k) \left[\int_{t_*}^{t_{\rm fin}} \frac{dt_1}{t_1} \cos[k(t - t_1)]\right]^2$$

[1] A. Roper Pol, C. Caprini, A. S. Midiri., M. Salomé "Gravitational wave spectrum from slowly decaying sources in the early Universe: constant-in-time and coherent-decay models."

Fully compressional motion

Sound-Shell model

$$E_K(k, t_1, t_2) = E_K^*(k) \cos[k c_s (t_2 - t_1)]$$

$$\Omega_{\rm GW}(k, t_0) = \frac{3k}{2} F_{\rm GW}^0 \int_{t_*}^{t_{\rm fin}} \int_{t_*}^{t_{\rm fin}} \frac{dt_1 dt_2}{t_1 t_2} E_\Pi(k, t_1, t_2) \cos[k(t_2 - t_1)]$$

$$= 3 w^2 k^3 F_{\rm GW}^0 \int_0^\infty dp\, p^2 E_K^*(p) \int_{-1}^1 dz\, \frac{(1 - z^2)^2}{\tilde{p}^4} E_K^*(\tilde{p}) \Delta^2(k, p, \tilde{p}, t_{\rm fin})$$

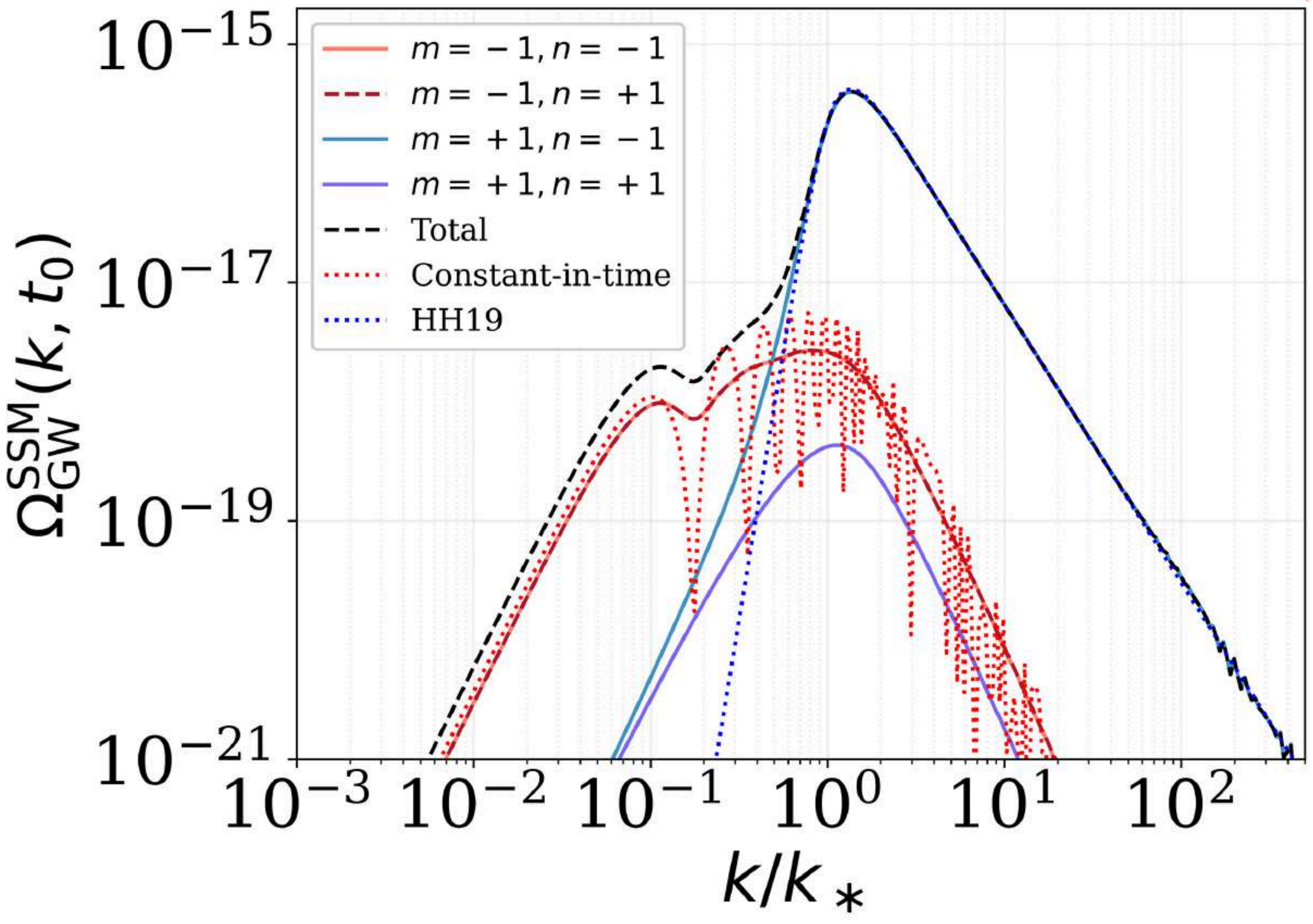
$$\Delta^2(k, p, \tilde{p}, t_{\rm fin}) = \int_{t_*}^{t_{\rm fin}} \int_{t_*}^{t_{\rm fin}} \frac{dt_1 dt_2}{t_1 t_2} \cos[p c_s (t_2 - t_1)] \cos[\tilde{p} c_s (t_2 - t_1)] \cos[k(t_2 - t_1)]$$

The Sound-Shell Model

Component analysis

$$\hat{p}_{mn} = (p + m\tilde{p})c_s + nk$$

$$\Delta^2(k, p, \tilde{p}, t_{\rm fin}) = \frac{1}{4}\sum_{m,n=\pm 1} \Delta^2_{mn}(\hat{p}_{mn})$$



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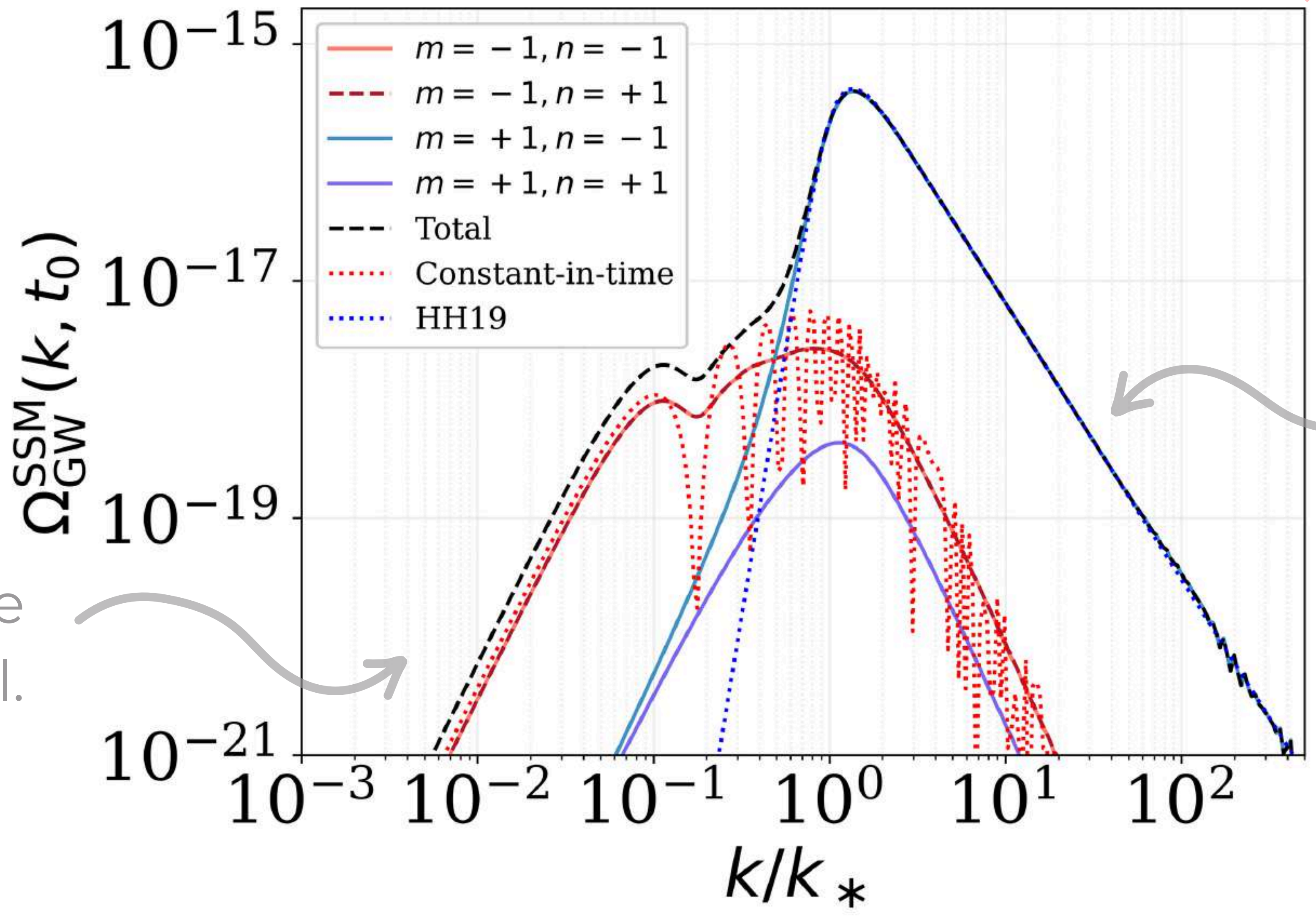

Hindmarsh and Hijazi, 2019

Described by the Constant-in-time model.

The Sound-Shell Model

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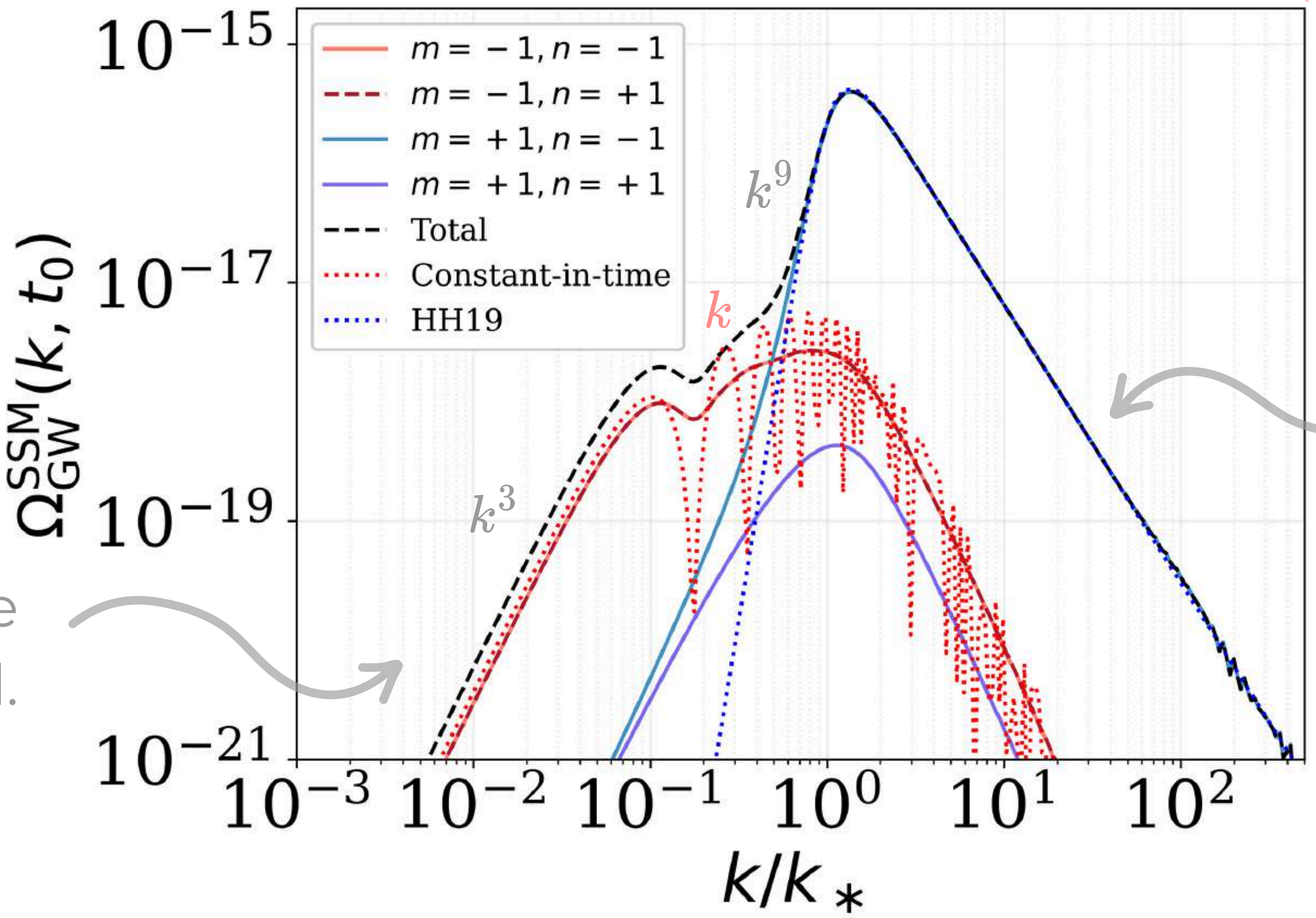

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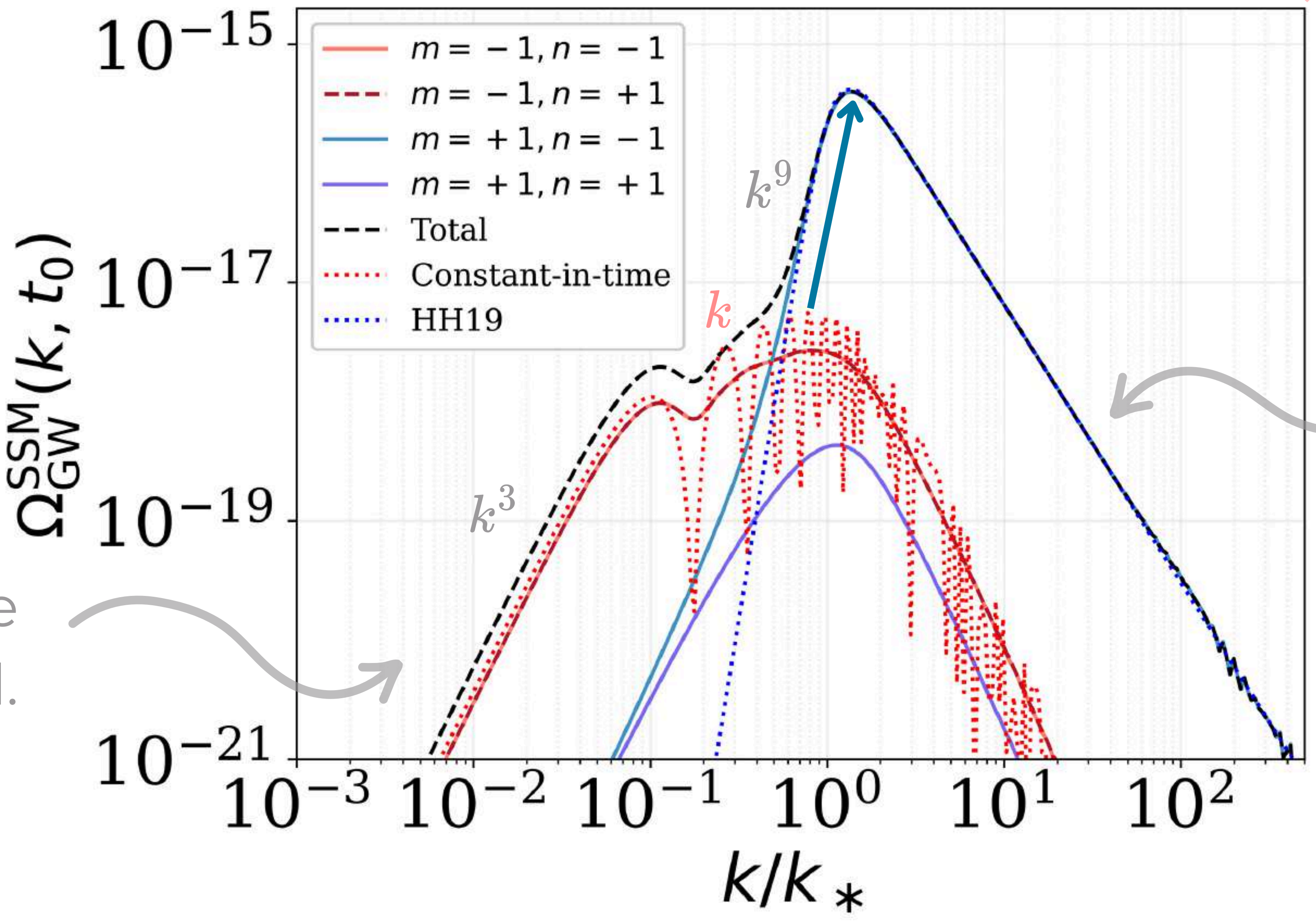
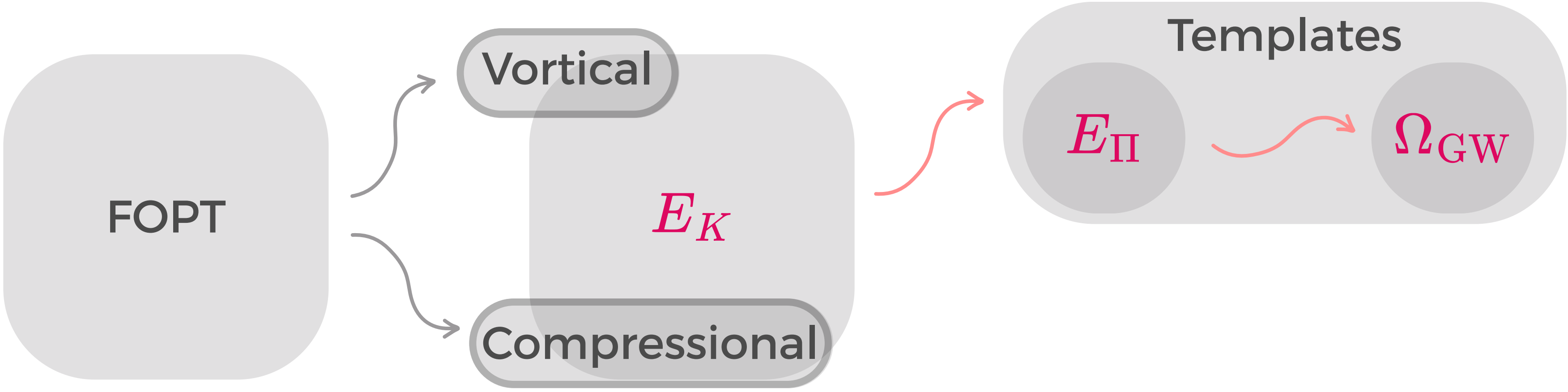


Hindmarsh and Hijazi, 2019

These scales are constructed over time!

CosmoGW

Developed by Alberto Roper Pol



Python library Functions for the study of **cosmological GW backgrounds** from different sources in the early Universe (**phase transitions**).

It includes GW models, postprocessing calculations, numerical computations, plotting routines, and detector sensitivities.

CosmoGW

UETC of the anisotropic stresses

Normal component (Vortical)

$$E_\Pi^{\rm vort}(k) = w^2 \frac{k^2}{2} \int_0^\infty dp\, E_K^{\rm vort}(p) \int_{-1}^1 dz \frac{1}{\tilde{p}^4} E_K^{\rm vort}(\tilde{p})(1+z^2)(2\tilde{p}^2 - p^2(1-z^2))$$

Longitudinal component (Compressional)

$$E_\Pi^{\rm comp}(k) = 2w^2 k^2 \int_0^\infty dp\, p^2 E_K^{\rm comp}(p) \int_{-1}^1 dz \frac{1}{\tilde{p}^4} E_K^{\rm comp}(\tilde{p})(1-z^2)^2$$

Mixed Term

$$E_\Pi^{\rm mix}(k) = 2w^2 k^2 \int_0^\infty dp\, p^2 E_K^{\rm vort}(p) \int_{-1}^1 dz \frac{1}{\tilde{p}^4} E_K^{\rm comp}(\tilde{p})(1-z^4)$$

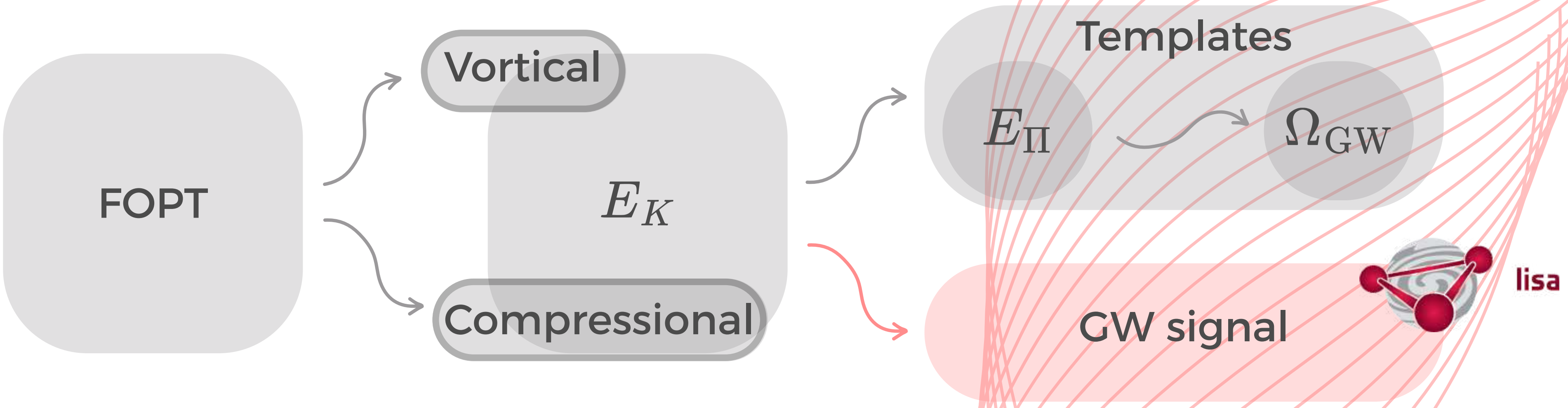
Helicity

$$E_\Pi^{\rm hel}(k) = w^2 \frac{k^2}{2} \int_0^\infty dp\, p E_K^{\rm hel}(p) \int_{-1}^1 dz \frac{1}{\tilde{p}^2} E_K^{\rm hel}(\tilde{p}) z(k - zp)$$

Available in CosmoGW!

[1] A. Roper Pol, C. Caprini, A. S. Midiri., M. Salomé "Gravitational wave spectrum from slowly decaying sources in the early Universe: constant-in-time and coherent-decay models."

Pencil Code



Simulated by solving GW equations in the Pencil Code.

Pencil Code A high-order finite-difference code for compressible hydrodynamic flows with magnetic fields.

Spatial Derivative 6th-order, Finite-difference method.

Time Step 3rd-order Runge-Kutta scheme by Williamson (1980).

CosmoGW Read the output files of the Pencil Code's run.

Pencil Code Collaboration, J. Open Source Software 6, 2807 (2021). The Pencil Code, a modular MPI code for partial differential equations and particles: multi- purpose and multiuser-maintained. arXiv: 2009.08231

Pencil Code

Type of fields Magnetic field $+\sigma$

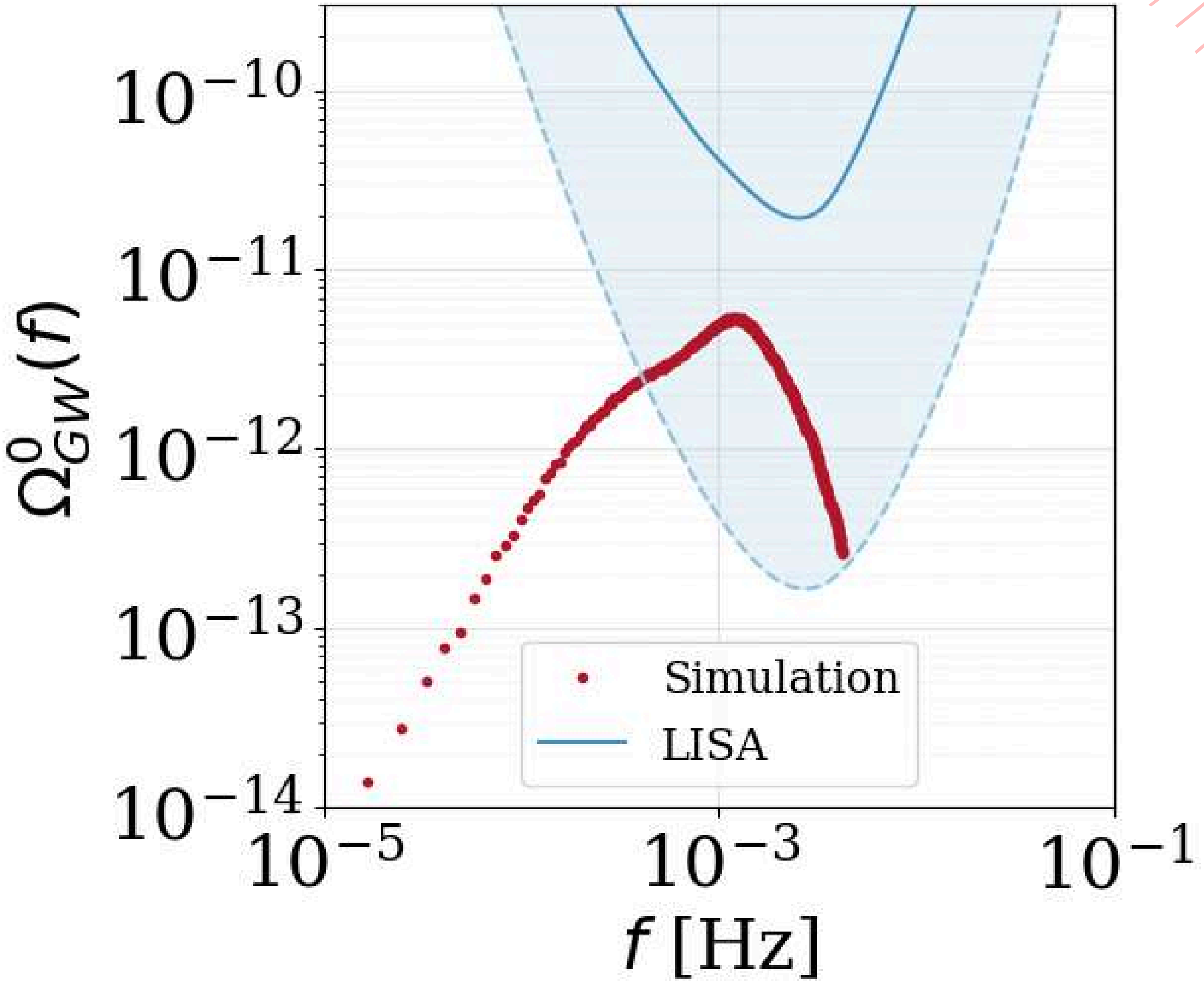
Velocity field $+\sigma + q$

$$(1-q)^2 E_\Pi^{\mathrm{vort}} + q^2 E_\Pi^{\mathrm{comp}} + q(1-q) E_\Pi^{\mathrm{mix}} + \varepsilon^2 E_\Pi^{\mathrm{hel}}$$

Compressional rate

Helicity rate

$$\varepsilon = \frac{2\sigma(1-q)}{1+\sigma^2(1-q)}$$

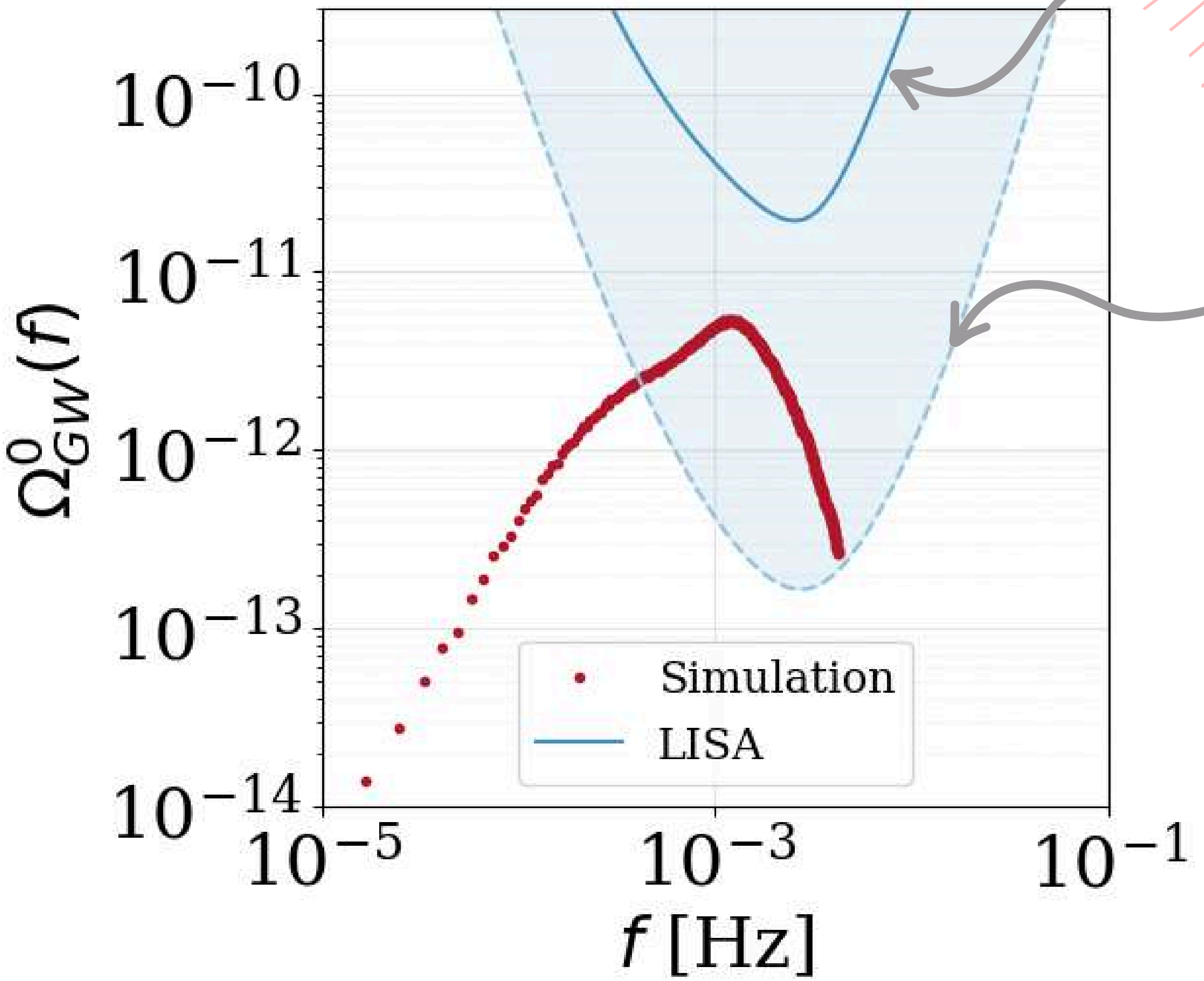


Pencil Code

Type of fields Magnetic field $+\,\sigma$

Velocity field $+\,\sigma\;+\;q$

$$(1-q)^2 E_\Pi^{\rm vort} + q^2 E_\Pi^{\rm comp} + q(1-q) E_\Pi^{\rm mix} + \varepsilon^2 E_\Pi^{\rm hel}$$



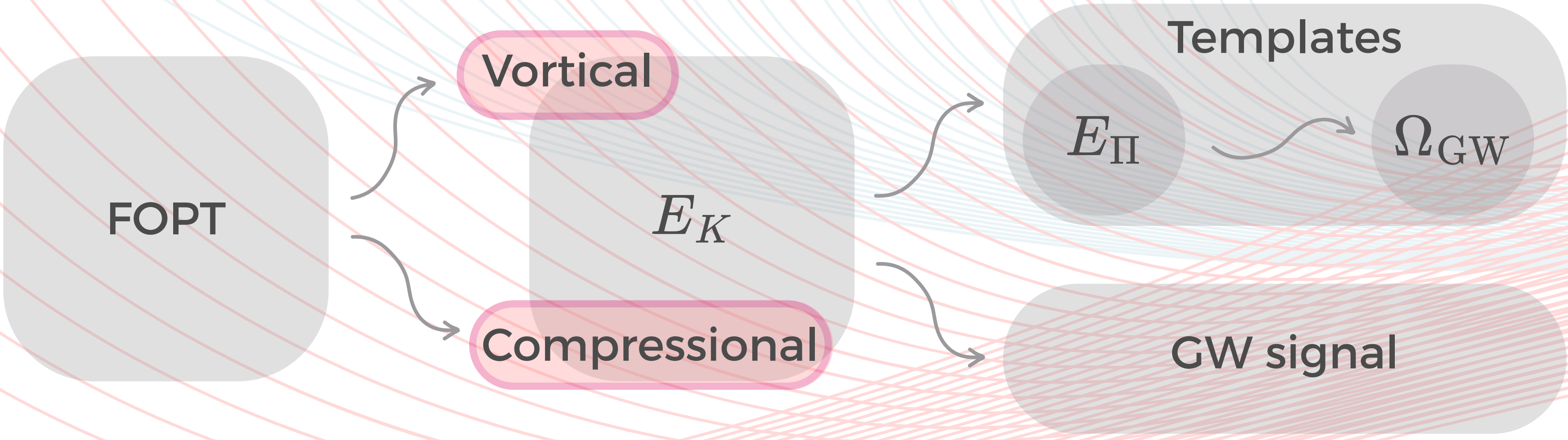
Intrument sensibility of LISA.

Power Law detection sensibility in LISA data. Signal processing.

lisa

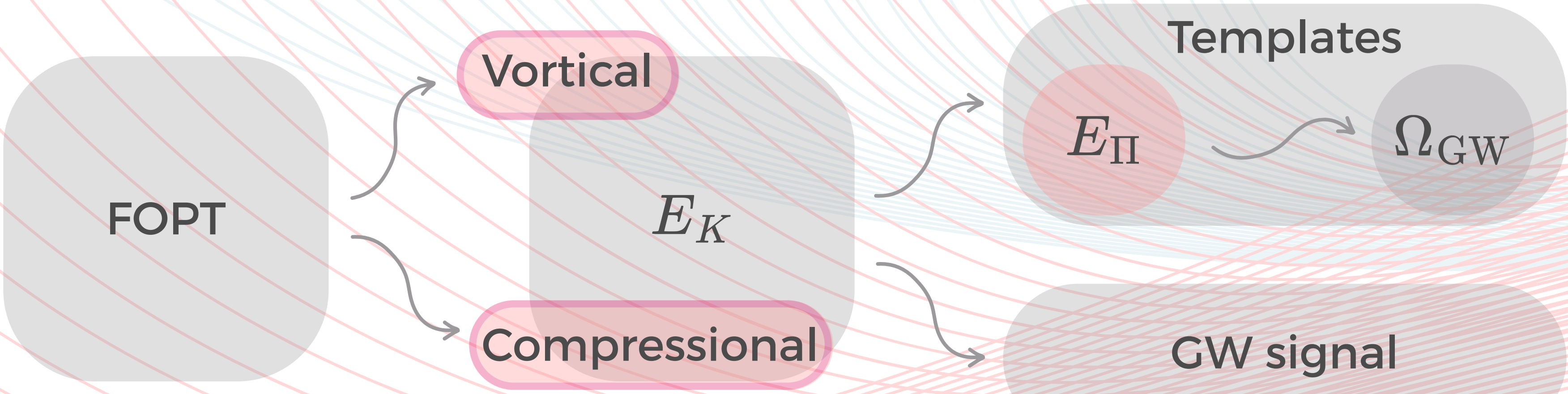
Part III

Mixed Term



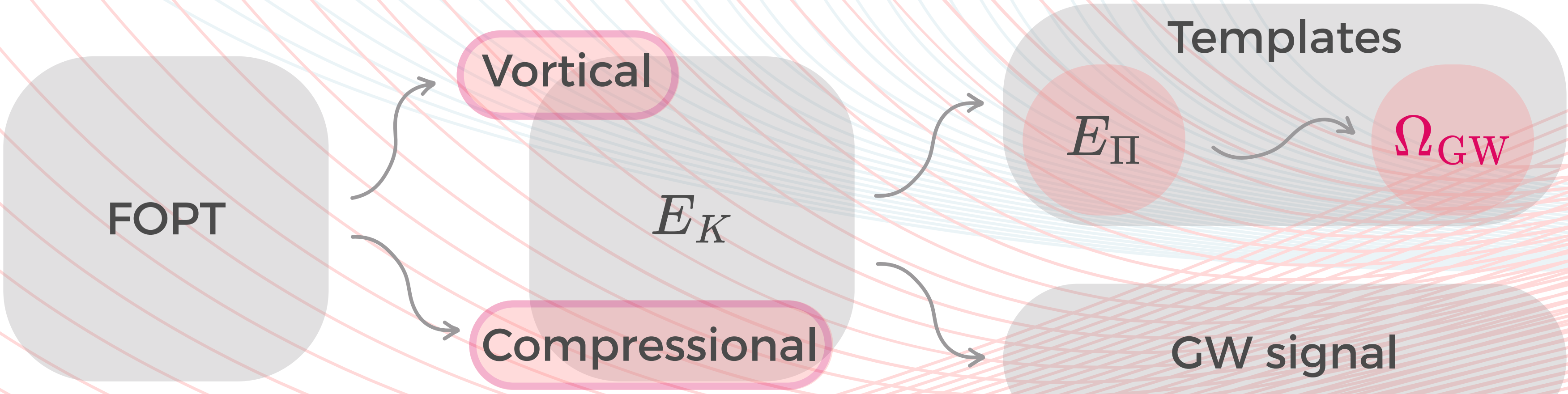
Part III

Mixed Term



Part III

Mixed Term



Simulations with the Pencil Code

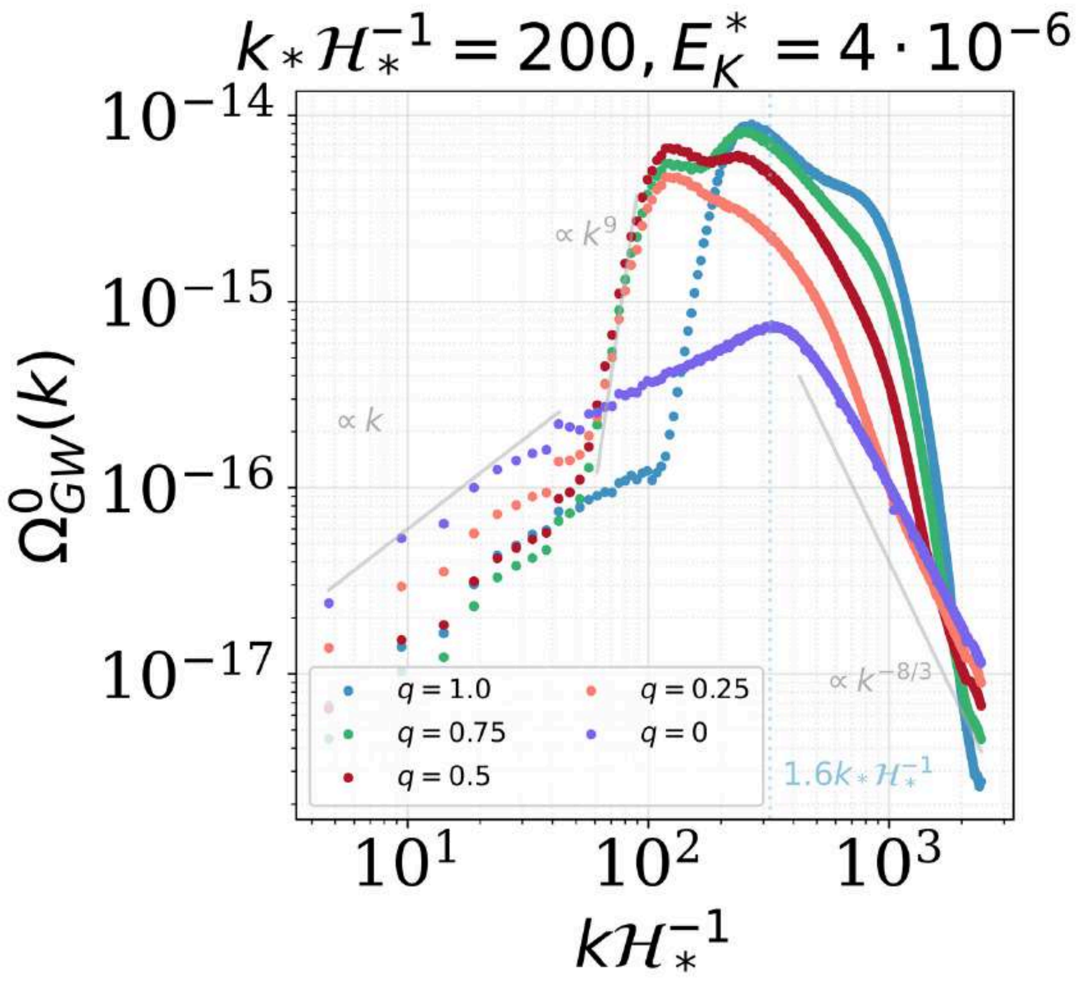
Name	k_*	E_K^*	q	σ	ν	$k_{\rm IR}$	Resolution	nt	δt
q0	200	$4 \cdot 10^{-6}$	0.0	0.0	10^{-7}	$3/2\pi$	1024^3	20000	10^{-4}
q025	200	$4 \cdot 10^{-6}$	0.25	0.0	10^{-7}	$3/2\pi$	1024^3	20000	10^{-4}
q05	200	$4 \cdot 10^{-6}$	0.5	0.0	10^{-7}	$3/2\pi$	1024^3	20000	10^{-4}
q075	200	$4 \cdot 10^{-6}$	0.75	0.0	10^{-7}	$3/2\pi$	1024^3	20000	10^{-4}
q1	200	$4 \cdot 10^{-6}$	1.0	0.0	10^{-7}	$3/2\pi$	1024^3	20000	10^{-4}

Table 3 List of Pencil Code simulations for several fractions of compressibility q.

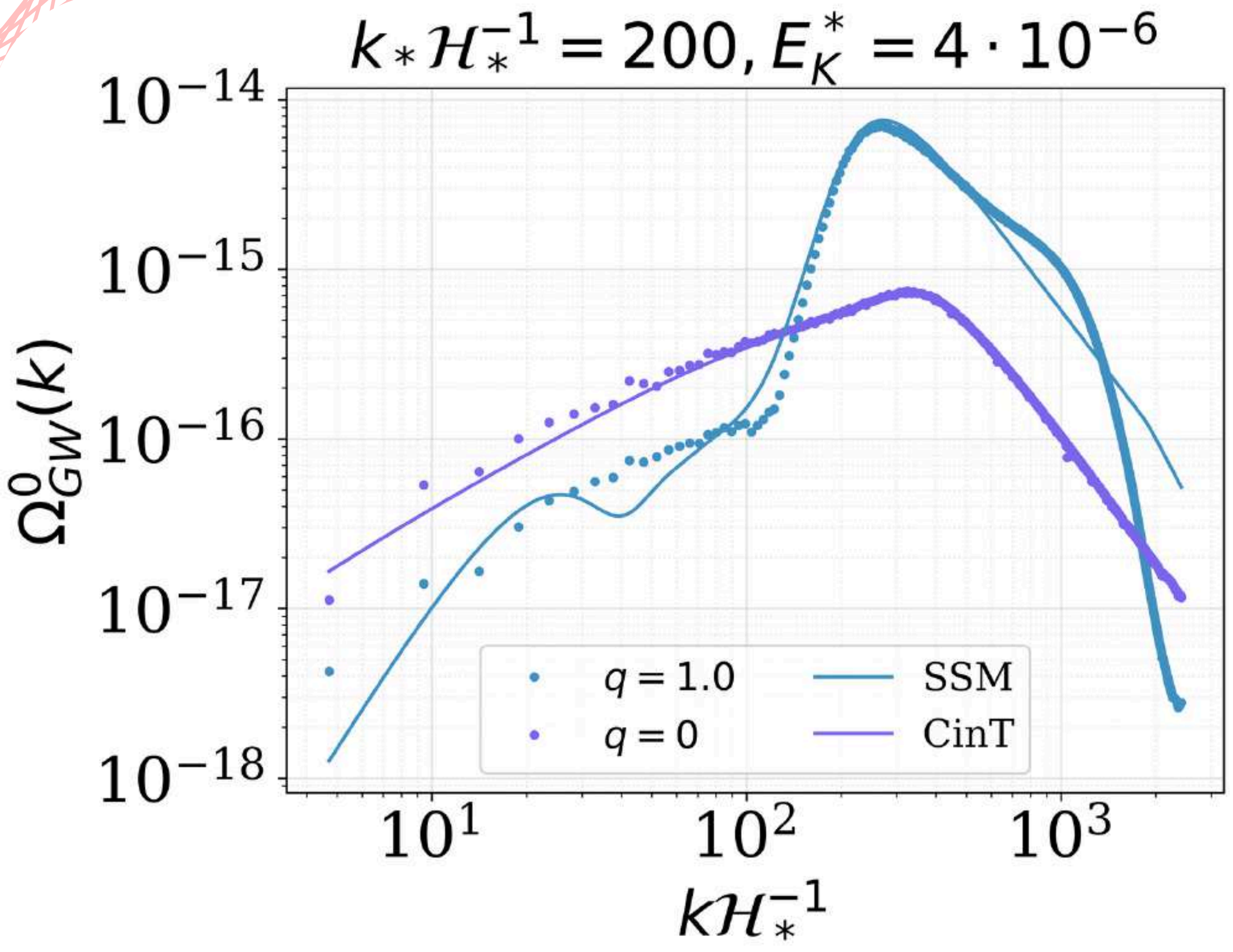
Spectral shape

Isak's Talk

$q \sim 0.7$



Models



[2] A. Roper Pol, M. Salomé. "Gravitational wave production from acoustic and fractionally compressible sources."

Modelizing the mixed motion spectra

$$\Omega_{\rm GW}(k,t) = (1-q)^2\Omega_{\rm GW}^{\rm vort}(k,t) + q^2\Omega_{\rm GW}^{\rm comp}(k,t) + q(1-q)\Omega_{\rm GW}^{\rm mix}(k,t)$$

Constant-in-time Model

$$E_K^{\rm vort}(k,t_1,t_2) = E_K^*(k)$$

$$\Omega_{\rm GW}^{\rm vort}(k,t_0) = \frac{3w^2k^3}{4}F_{\rm GW}^0 \int_0^\infty dp\, E_K^*(p) \int_{-1}^1 dz\, E_K^*(\tilde p)\frac{(1+z^2)}{\tilde p^4}(2\tilde p^2 - p^2(1-z^2))\Delta^2(k,t_{\rm fin})$$

$$\Delta^2(k,t_{\rm fin}) = \int_{t_*}^{t_{\rm fin}}\int_{t_*}^{t_{\rm fin}} \frac{dt_1 dt_2}{t_1 t_2}\cos[k(t_2-t_1)]$$

Sound-Shell Model

$$E_K^{\rm comp}(k,t_1,t_2) = E_K^*(k)\cos[kc_s(t_2-t_1)]$$

$$\Omega_{\rm GW}^{\rm comp}(k,t_0) = 3w^2k^3F_{\rm GW}^0 \int_0^\infty dp\, p^2 E_K^*(p)\int_{-1}^1 dz\, \frac{(1-z^2)^2}{\tilde p^4}E_K^*(\tilde p)\Delta^2(k,p,\tilde p,t_{\rm fin})$$

$$\Delta^2(k,p,\tilde p,t_{\rm fin}) = \int_{t_*}^{t_{\rm fin}}\int_{t_*}^{t_{\rm fin}} \frac{dt_1 dt_2}{t_1 t_2}\cos[pc_s(t_2-t_1)]\cos[\tilde p c_s(t_2-t_1)]\cos[k(t_2-t_1)]$$

Modelizing the mixed motion spectra

$$\Omega_{\rm GW}(k,t) = (1-q)^2\Omega_{\rm GW}^{\rm vort}(k,t) + q^2\Omega_{\rm GW}^{\rm comp}(k,t) + q(1-q)\Omega_{\rm GW}^{\rm mix}(k,t)$$

Constant-in-time Model

$$E_K^{\rm vort}(k,t_1,t_2) = E_K^*(k)$$

Sound-Shell Model

$$E_K^{\rm comp}(k,t_1,t_2) = E_K^*(k)\cos[kc_s(t_2-t_1)]$$

New Model

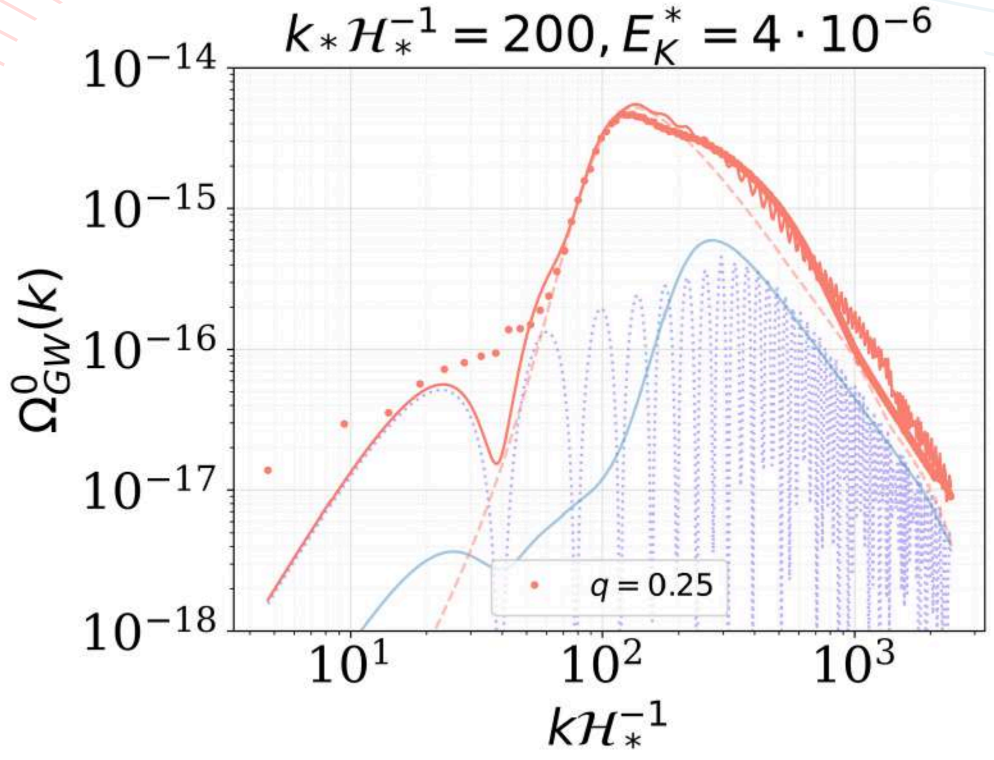
$$\Omega_{\rm GW}^{\rm mix}(k,t_0) = 3w^2k^3F_{\rm GW}^0\int_0^\infty dp\, p^2 E_K^*(p)\int_{-1}^1 dz\,\frac{(1-z^4)}{\tilde{p}^4}E_K^*(\tilde{p})\Delta^2(k,\tilde{p},t_{\rm fin})$$

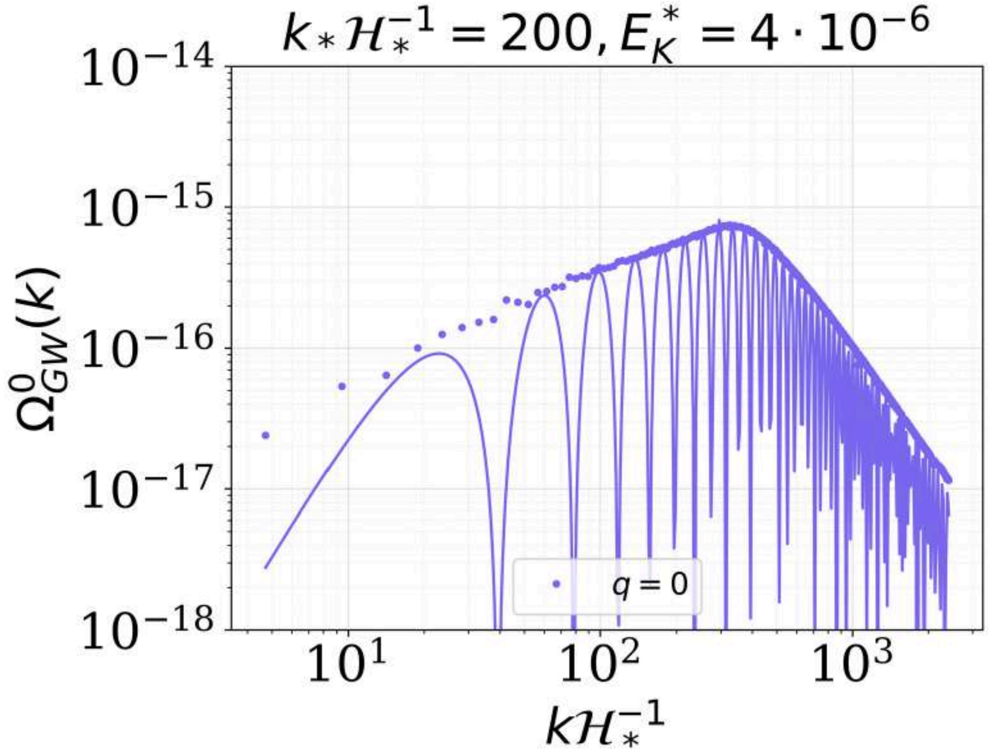
$$\Delta^2(k,\tilde{p},t_{\rm fin}) = \int_{t_*}^{t_{\rm fin}}\int_{t_*}^{t_{\rm fin}}\frac{dt_1dt_2}{t_1t_2}\cos[\tilde{p}c_s(t_2-t_1)]\cos[k(t_2-t_1)]$$

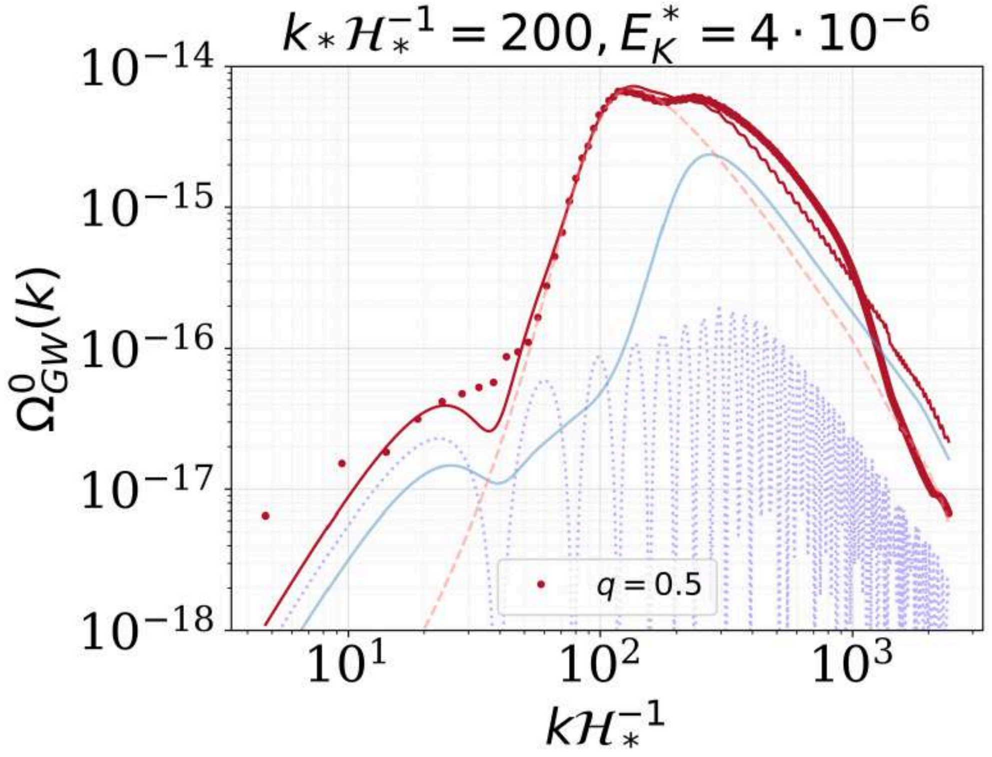
[2] A. Roper Pol, M. Salomé. "Gravitational wave production from acoustic and fractionally compressible sources."
J. Correia & al. - "Gravitational waves from strong first order phase transitions."

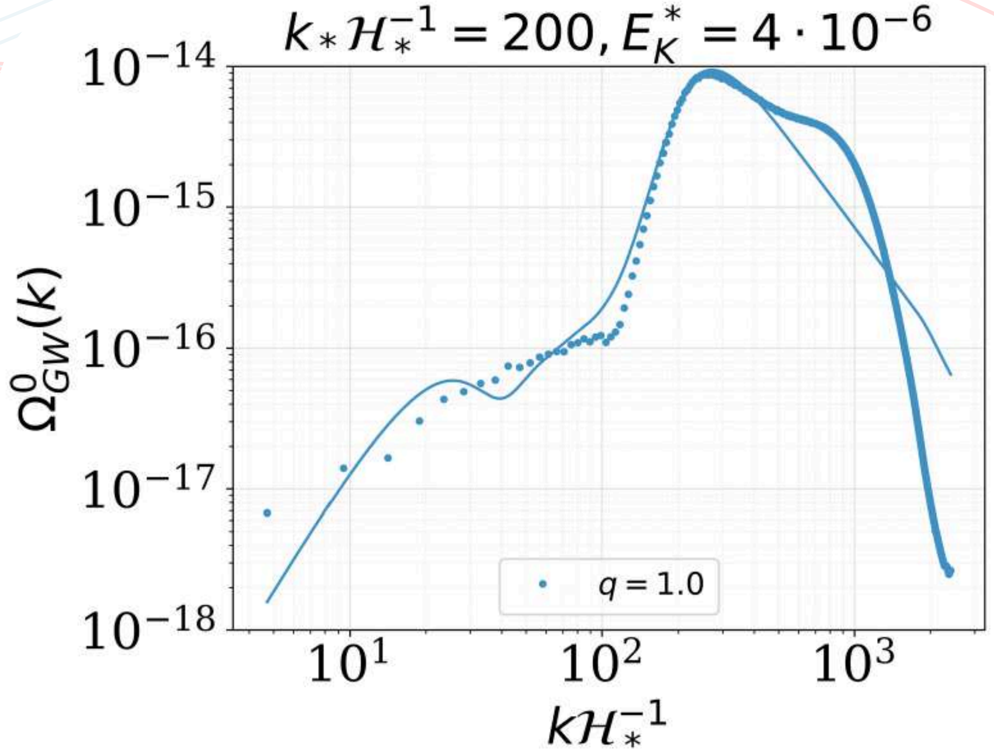
Modelizing the mixed motion spectra

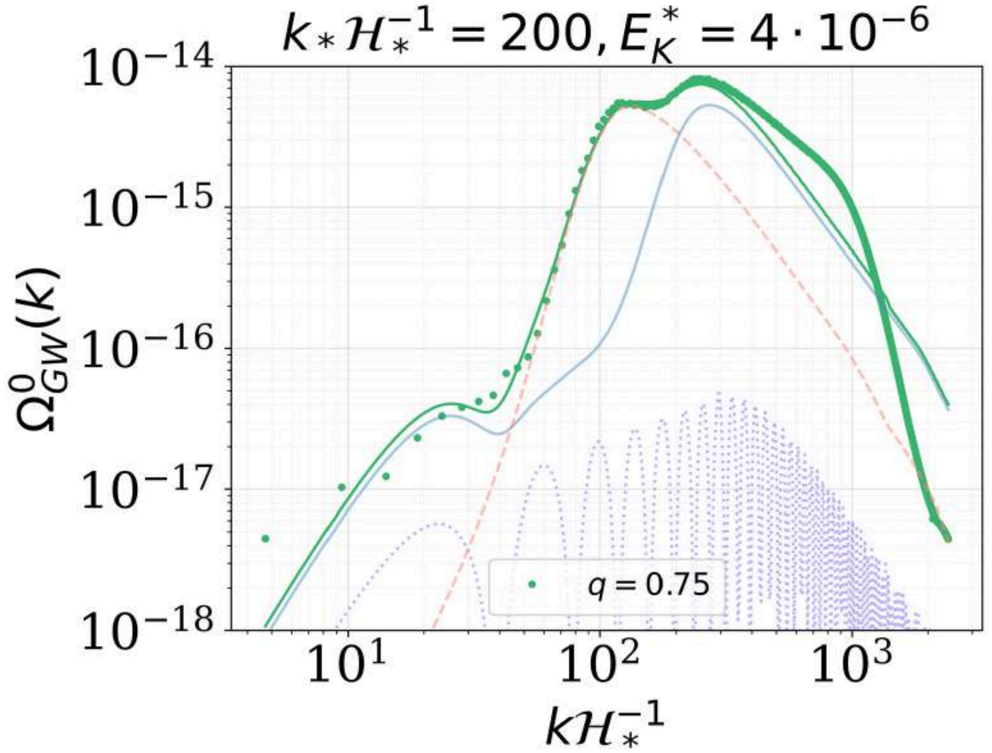
$$\Omega_{\rm GW}(k,t) = (1-q)^2\Omega_{\rm GW}^{\rm vort}(k,t) + q^2\Omega_{\rm GW}^{\rm comp}(k,t) + q(1-q)\Omega_{\rm GW}^{\rm mix}(k,t)$$











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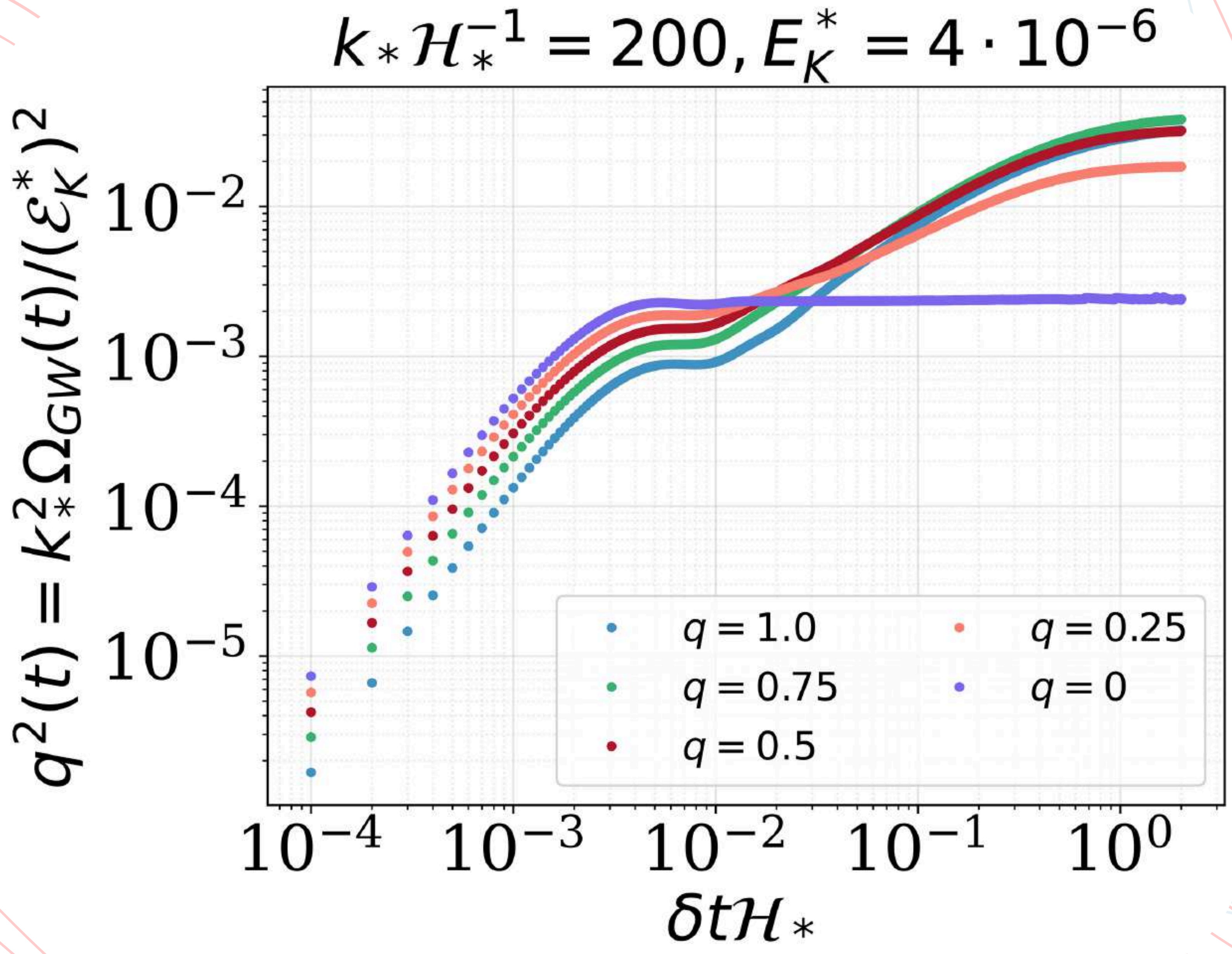
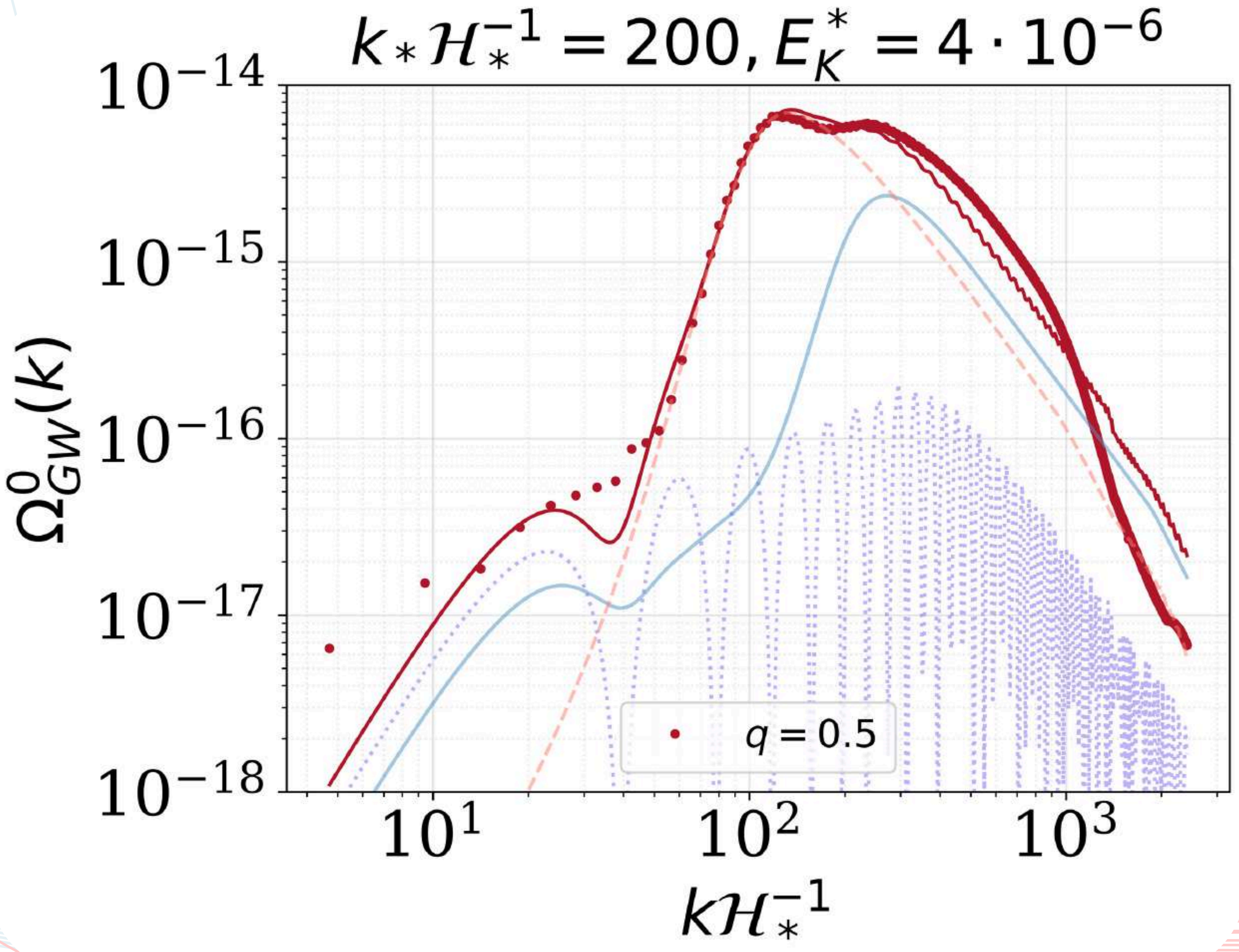
GWs production efficiency

Which mode is the most effective for producing GWs?

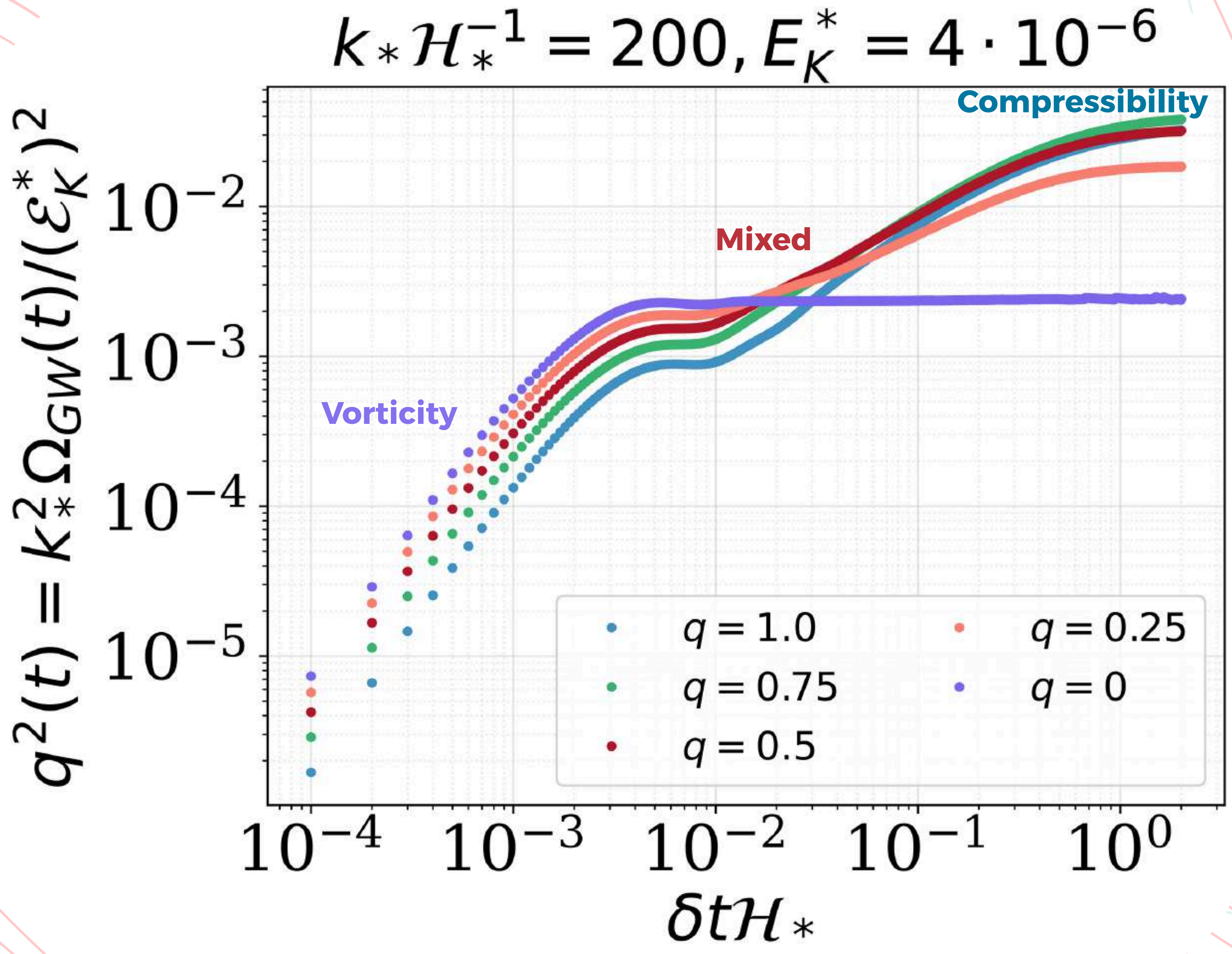


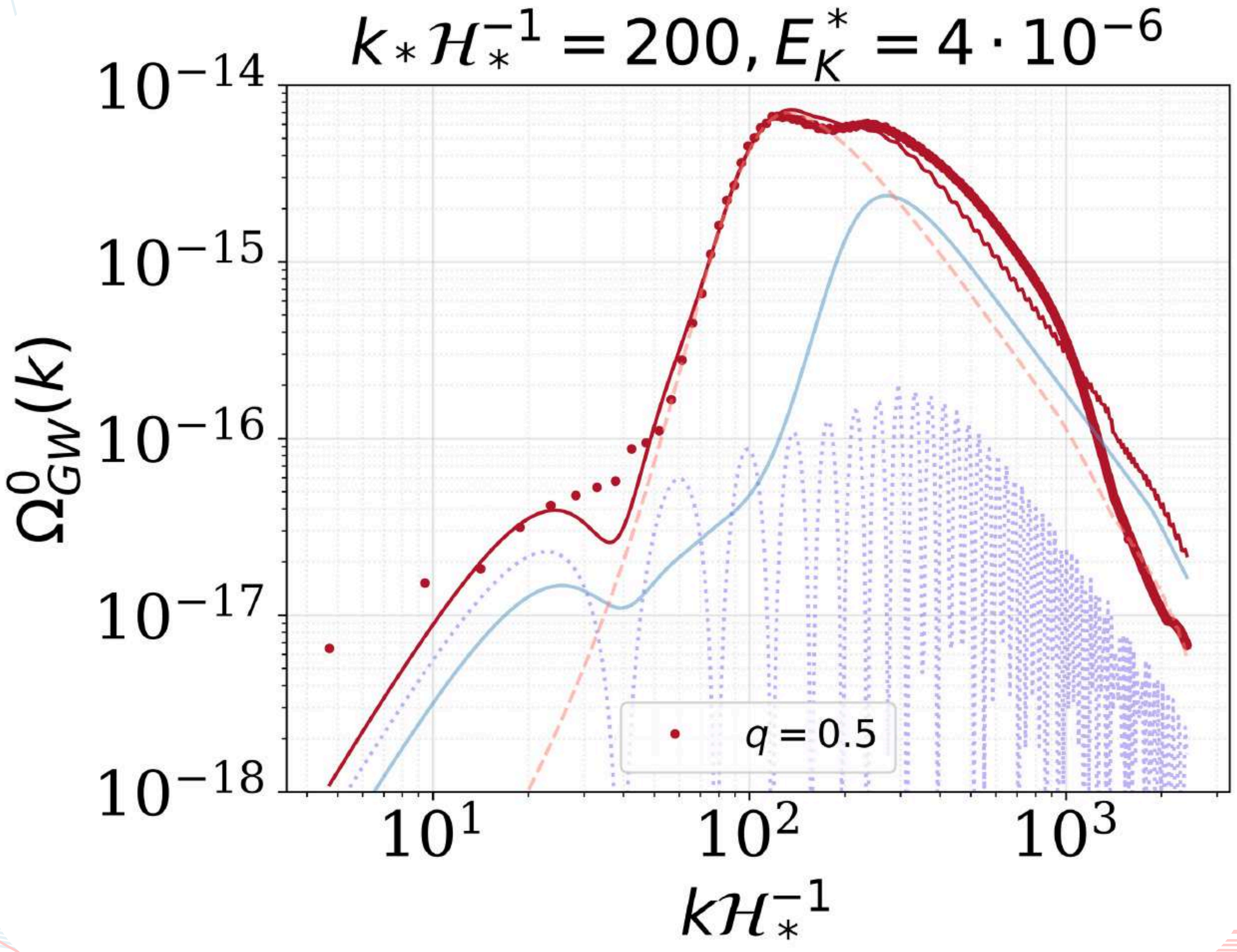


It depends on the sourcing duration and the scale of interest.

GWs production efficiency

Which mode is the most effective for producing GWs?

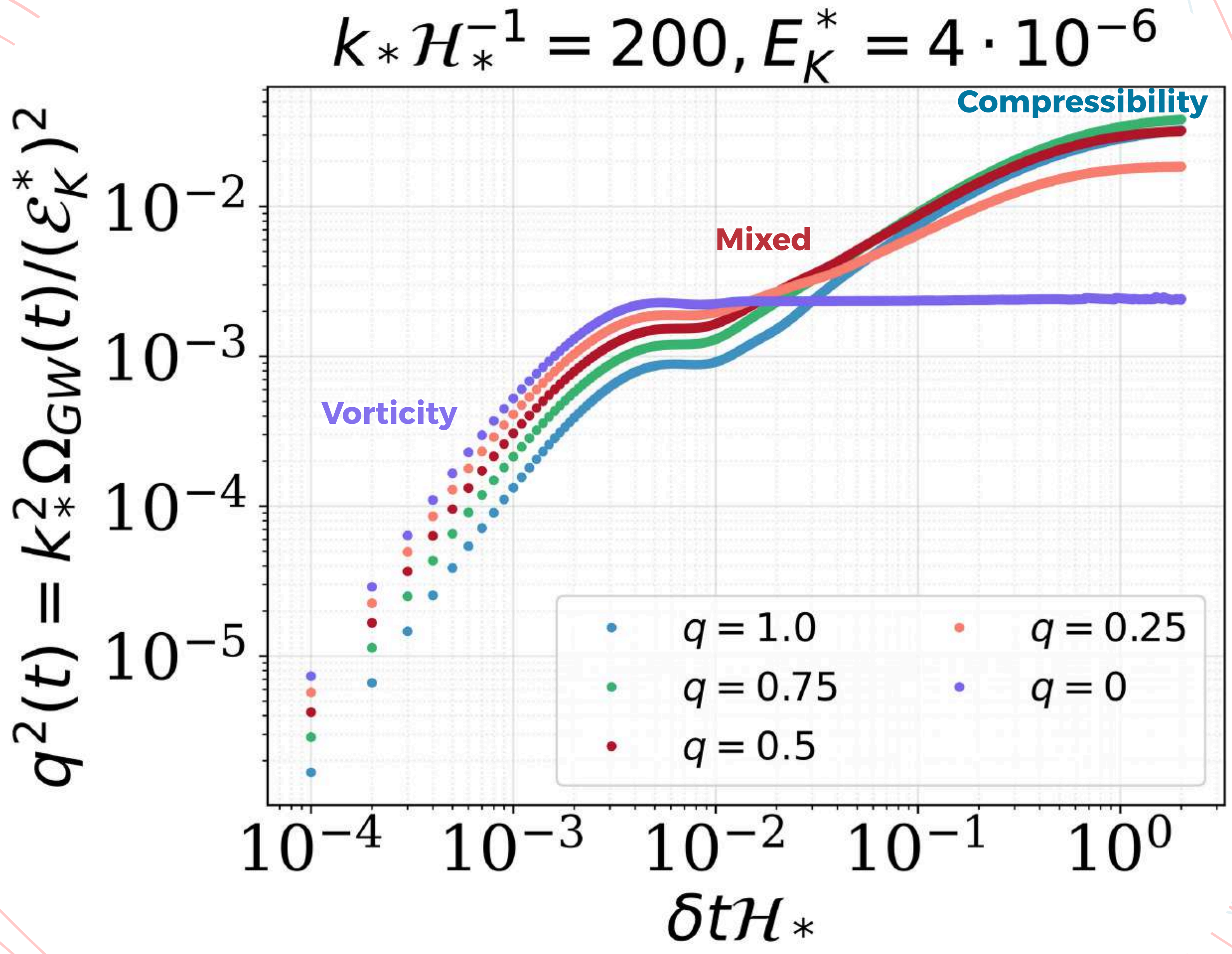


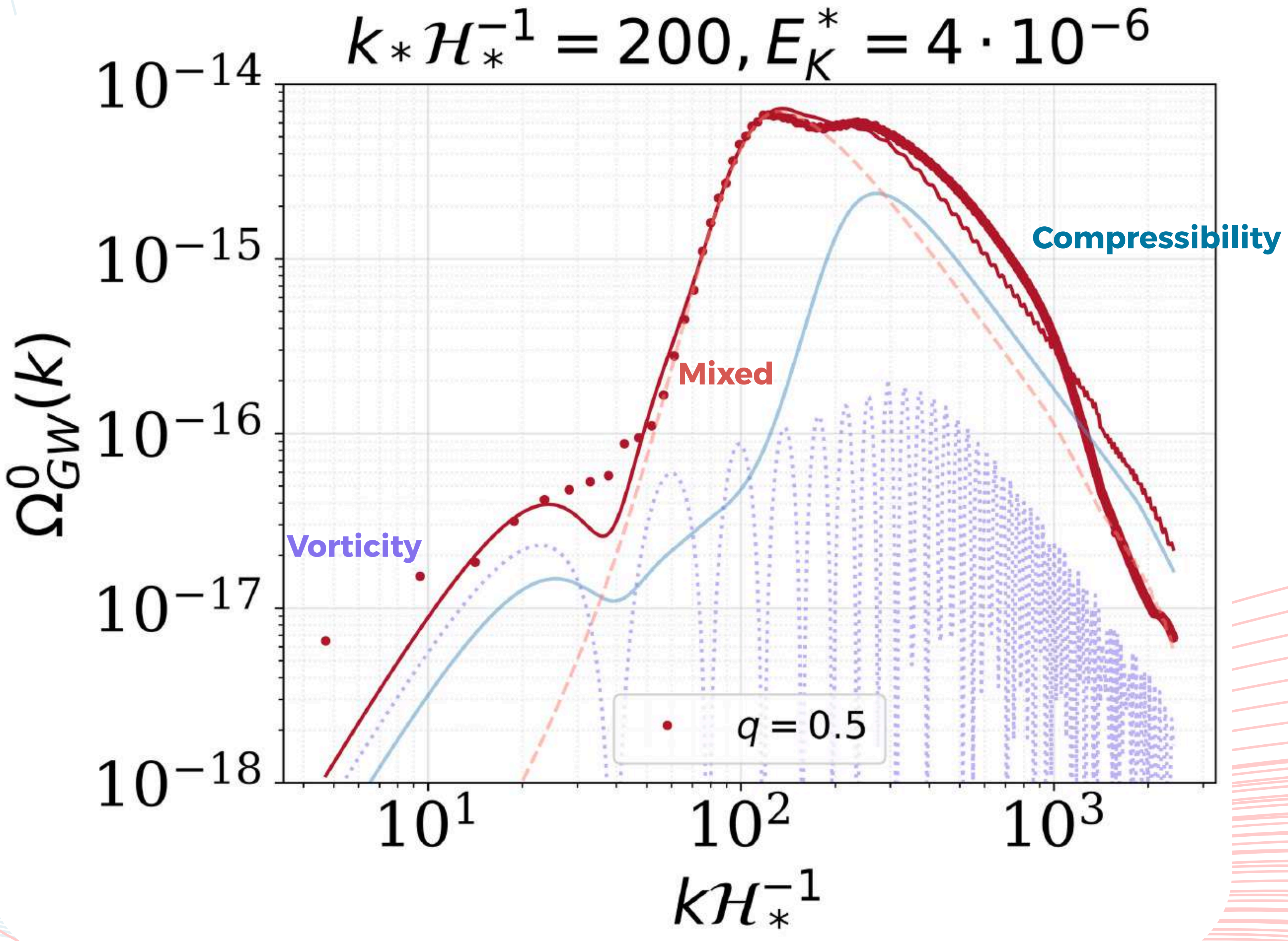


It depends on the sourcing duration and the scale of interest.

GWs production efficiency

Which mode is the most effective for producing GWs?

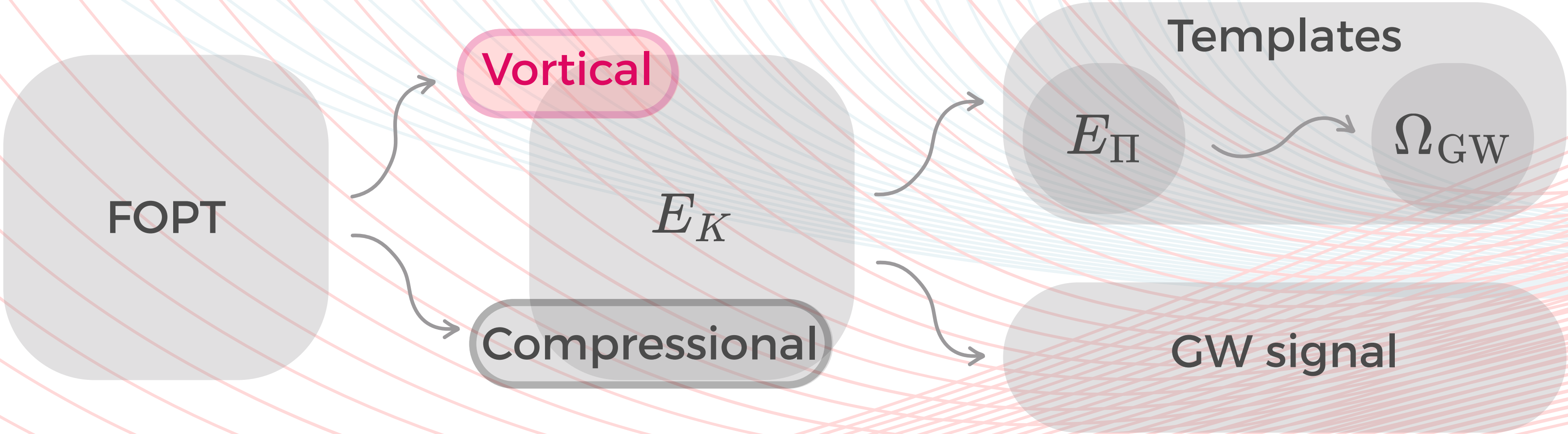




It depends on the sourcing duration and the scale of interest.

Part IV

Vorticity production in FOPT



Vorticity production in FOPT

An overview

$$\omega = \nabla \times \mathbf{u}$$

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} + \frac{\mathbf{u}}{3}\psi - \frac{3}{4}\frac{\nabla p}{\rho} + \frac{3}{4\rho}\mathbf{J} \times \mathbf{B} + \frac{2}{\rho}\nabla \cdot (\rho \nu \mathbf{S})$$

$$\frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla)\omega = (\omega \cdot \nabla)\mathbf{u} - \omega(\nabla \cdot \mathbf{u})$$

$$+ \frac{3}{4}\frac{1}{\rho^2}\nabla \rho \times \nabla p$$

Baroclinic term.

$$+ \frac{1}{3}\omega\psi + \frac{\mathbf{u}}{3}\nabla \times \psi$$

Relativistic term.

$$+ \frac{4}{3}\nabla \times \left(\frac{\mathbf{J} \times \mathbf{B}}{\rho}\right)$$

Magnetic term.

$$+ \nu \nabla^2 \omega + 2\nu \nabla \times (\mathbf{S}\nabla \ln \rho)$$

Viscous term.

Frank Van Den Bosch. “Chapter 10: Vorticity & Circulation”. In: Astrophysical Flows (2025).
A. Brandenburg, E. Scannapieco - “Magnetically assisted vorticity production in decaying acoustic turbulence.”

Vorticity production in FOPT

An overview

$$\frac{\partial \omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = (\omega\cdot\nabla)\mathbf{u} - \omega(\nabla\cdot\mathbf{u})$$
$$+ \frac{3}{4}\frac{1}{\rho^2}\nabla\rho\times\nabla p$$
$$+ \frac{1}{3}\omega\,\psi + \frac{\mathbf{u}}{3}\nabla\times\psi$$
$$+ \frac{4}{3}\nabla\times\left(\frac{\mathbf{J}\times\mathbf{B}}{\rho}\right)$$
$$+ \nu\nabla^2\omega + 2\nu\,\nabla\times(\mathbf{S}\nabla\ln\rho)$$

Baroclinic term. $p = c_s^2\rho$

Relativistic term.

Magnetic term.

Viscous term.

Vorticity production in FOPT

An overview

$$\frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla)\omega = (\omega \cdot \nabla)\mathbf{u} - \omega(\nabla \cdot \mathbf{u})$$
$$+ \frac{3}{4}\frac{1}{\rho^2}\nabla\rho \times \nabla p$$
$$+ \frac{1}{3}\omega\psi + \frac{\mathbf{u}}{3}\nabla \times \psi$$
$$+ \frac{4}{3}\nabla \times \left(\frac{\mathbf{J} \times \mathbf{B}}{\rho}\right)$$
$$+ \nu\nabla^2\omega + 2\nu\nabla \times (\mathbf{S}\nabla \ln \rho)$$

Baroclinic term.

Relativistic term.

Magnetic term.

Viscous term.

Vorticity production in FOPT

An overview

$$\frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla)\omega = \frac{\mathbf{u}}{3} \nabla \times \psi + 2\nu \nabla \times (\mathbf{S} \nabla \ln \rho)$$

Deepen's talk!

Isak's Talk

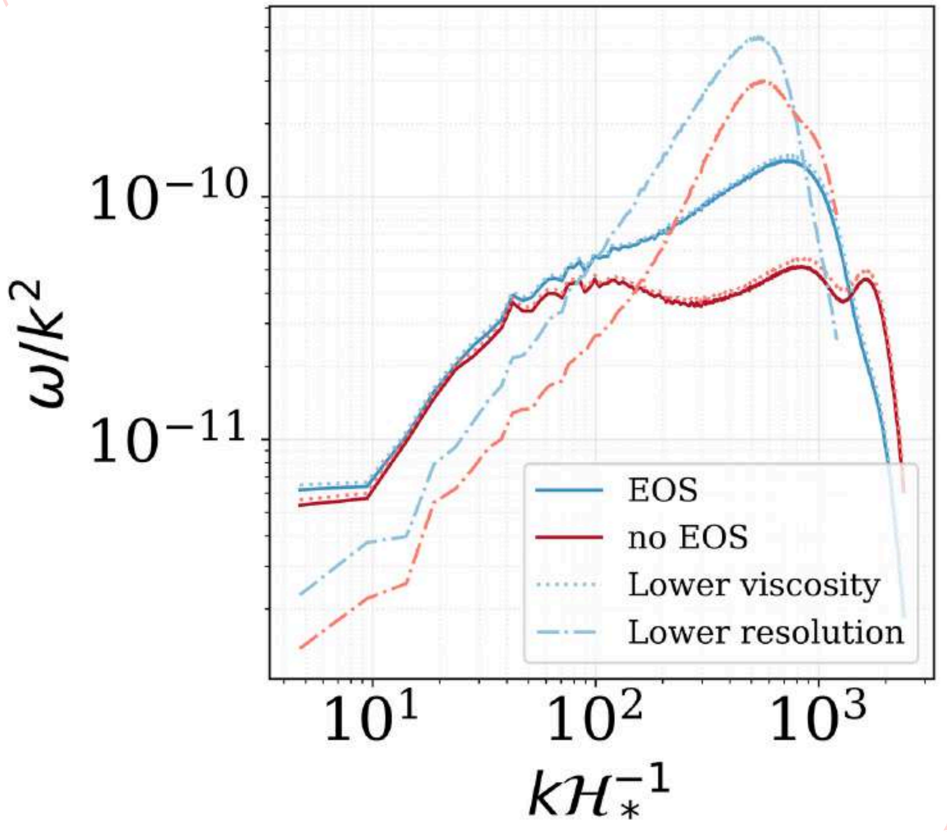
$q \sim 0.7$

Could be even lower with magnetic field !

Relativistic term.

Relativistic EOS = EOS

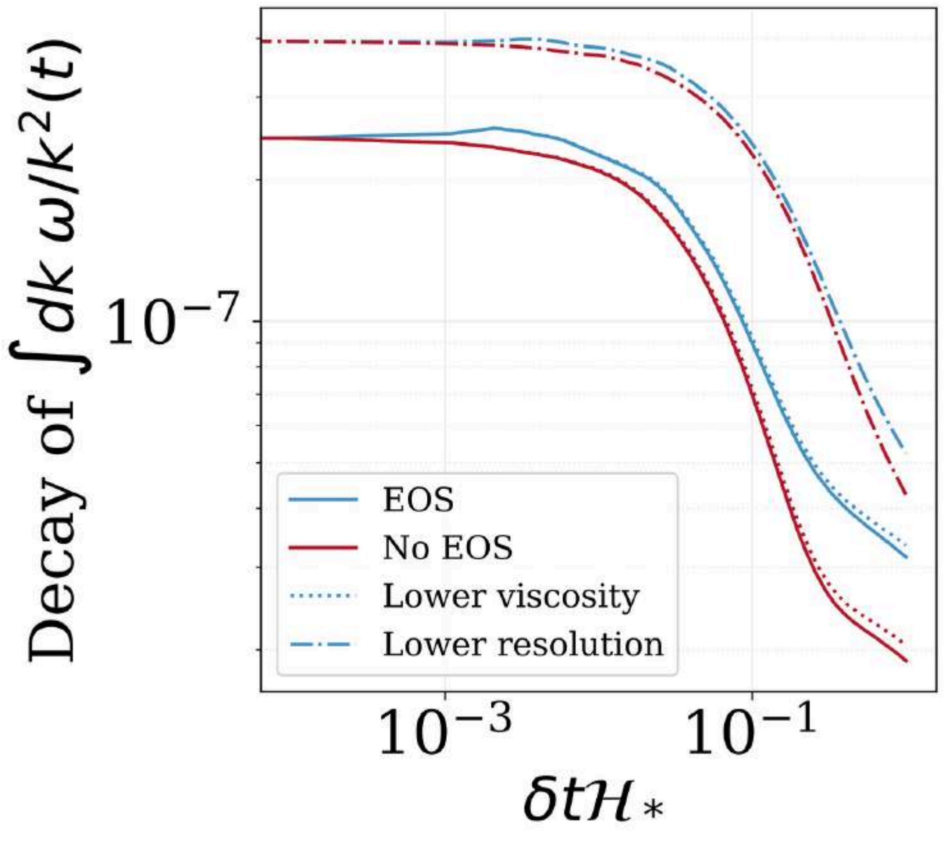
Non-rel. EOS = noEOS



Viscous term.

Simulation stability.

$\implies \nu \neq 0$



Part V

Few more words about the Pencil Code

Pencil Code School

1st Pencil Code school on early Universe physics and gravitational waves (Oct. 20-24)

Learning and developing numerical skills applied to early Universe physics using Pencil Code.

Topics:

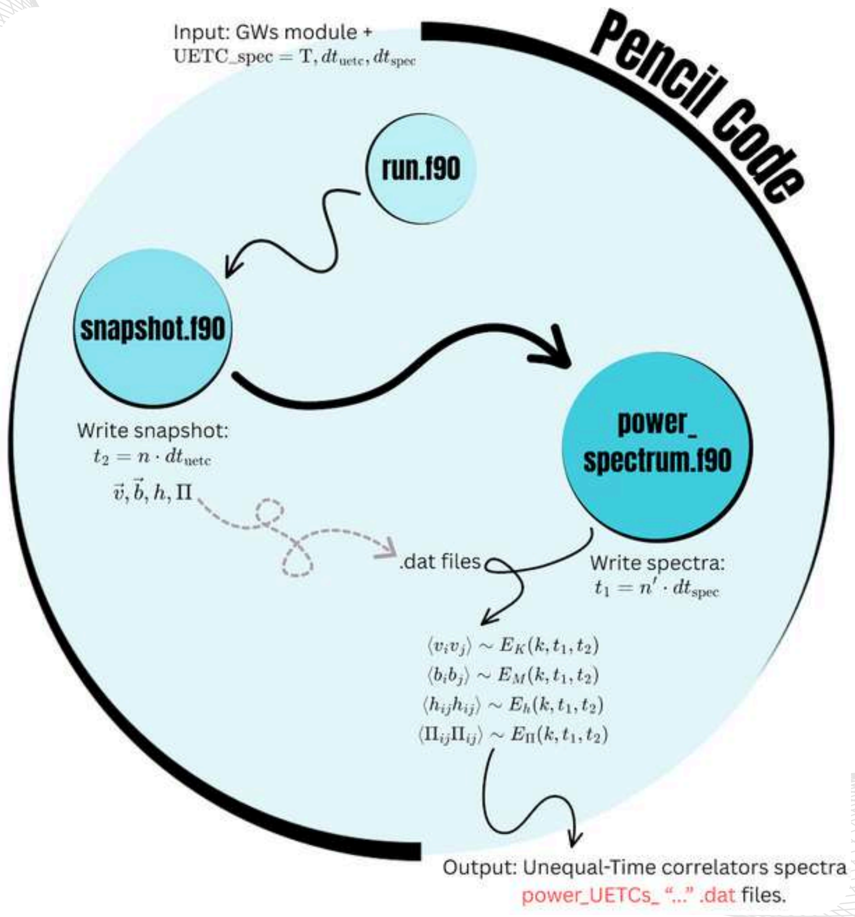
- *Magnetohydrodynamics of the early Universe,*
- *Generation and evolution of primordial magnetic fields,*
- *Chiral magnetohydrodynamics,*
- *First-order phase transitions,*
- *Gravitational wave production,*
- *Axion inflation.*

Pencil Code School

Registration is open and will close on September 10th.

Decorrelation function of the UETCs

UETCs & the Pencil Code



Status To come !

First Results Decorrelation functions.
Kraichnan decorrelation: Gaussian shape is found!

$$E_\Pi(k, t_1, t_2) \sim f(k, t_1) f(k, t_2) \mathcal{D}_{E_\Pi}(k, t_2 - t_1)$$

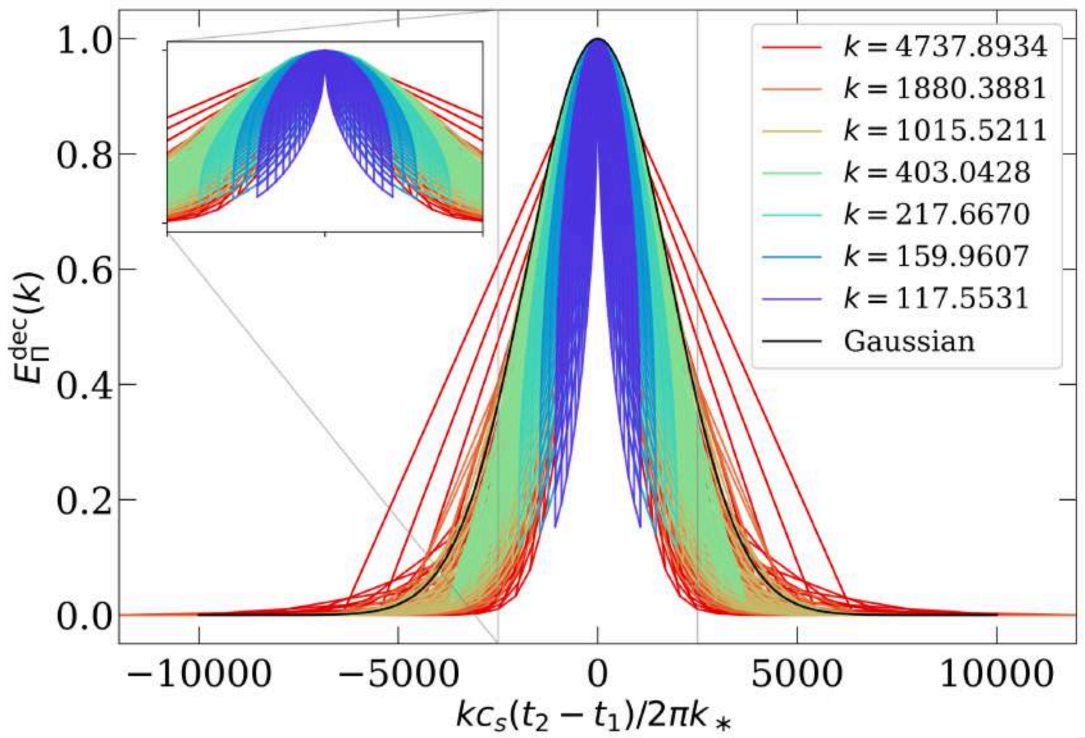
$$f(k, t) = \sqrt{E_\Pi(k, t)}$$

Mark's Talk — J. Correia & M. Hindmarsh & K. Rummukainen & D. J. Weir - "Gravitational waves from strong first order phase transitions."

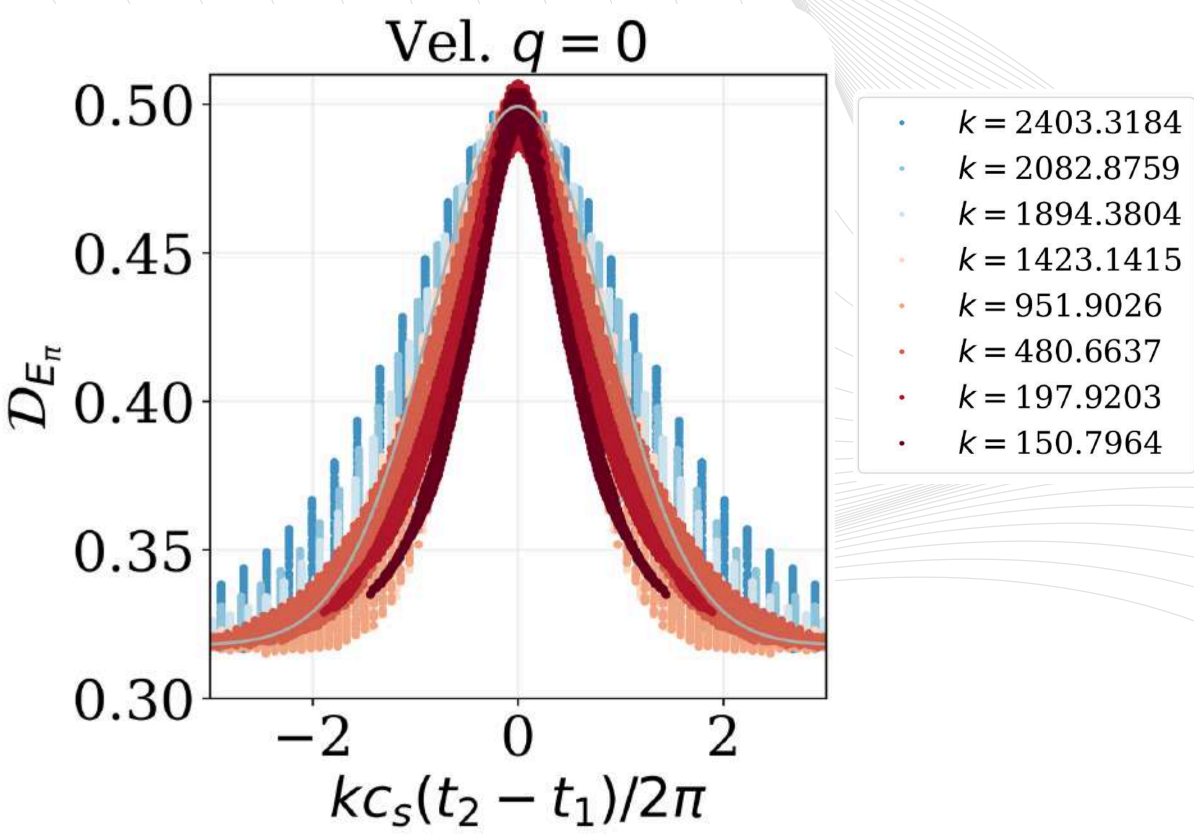
Analytical description

$$\mathcal{D}_{E_K}(k, t_1, t_2) = \frac{v_{\rm dc}(k, t_1, t_2)}{\sqrt{v_{\rm dc}(k, t_1) v_{\rm dc}(k, t_2)}} \exp\left[-\frac{1}{2} k^2 (t_2 - t_1)^2 v_{\rm dc}^2(k, t_1, t_2)\right]$$

$$\mathcal{D}_{E_\Pi}(k, t_1, t_2)$$



Simulation



Conclusion

Gravitational wave production: the interplay between vortical and compressional motions.

Main results

Possibilities to source **vorticity** in relativistic primordial plasma leading to a mixed motion: Vortical & Compressional modes.

An additional peak visible in the GW energy density spectrum from **fractionally compressible motion**. A non-negligeable term in some cases.

New generic **model** to describe the GW energy density spectrum by combining the description of each motion: Constant-in-Time model & Sound Shell model.

Helicity part can be added at this generic template.

Thank you for listening !

References

cosmoGW A. Roper Pol, GitHub project "CosmoGW", https://github.com/AlbertoRoper/CosmoGW.

Pencil Code Pencil Code Collaboration, J. Open Source Software 6, 2807 (2021). *The Pencil Code, a modular MPI code for partial differential equations and particles: multi-purpose and multiuser-maintained*. arXiv: 2009.08231

arXiv:1801.04268 C. Caprini & D.G. Figueroa - *Cosmological* backgrounds of gravitational waves.

arXiv:2308.12916 R. Sharma & J. Dahl & A. Brandenburg & M. Hindmarsh - *Shallow relic gravitational wave spectrum with acoustic peak*.

arXiv:1711.03804 A. Brandenburg & T. Kahniashvili & al. - *Evolution of hydromagnetic turbulence from electroweak phase transition.*

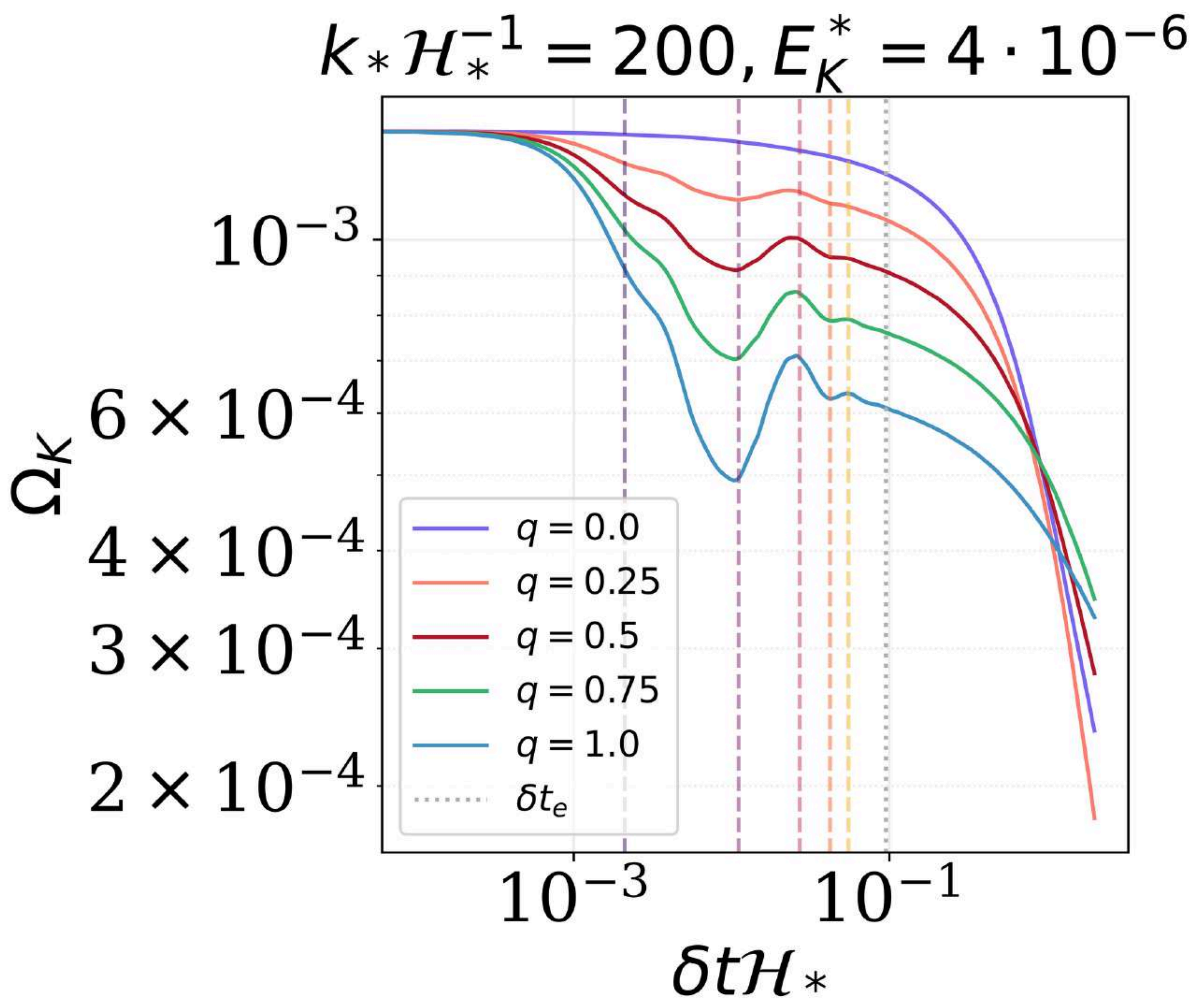
arXiv:2308.12943 A. Roper Pol & S. Procacci & C. Caprini - *Characterization of the gravitational wave spectrum from sound waves within the sound shell model.*

arXiv:2409.01426 L. Giombi & J. Dahl & M. Hindmarsh - *Signatures of the speed of sound on the gravitational wave power spectrum from sound waves.*

arXiv:2505.17824 J. Correia & al. - *Gravitational waves from strong first order phase transitions*.

Slowly decaying sources

Time evolution curve



Vortical description of the decay

$$\Omega_K(t) = \Omega_K^* \left(1 + \frac{\delta t}{\delta t_e}\right)^{-p}$$

Exponentially damped cosinus

$$(e^{-A\delta t} \cos[B \cdot 2c_s k_* \delta t] + C) \cdot D + E$$

J. Correia & al. - “Gravitational waves from strong first order phase transitions.”

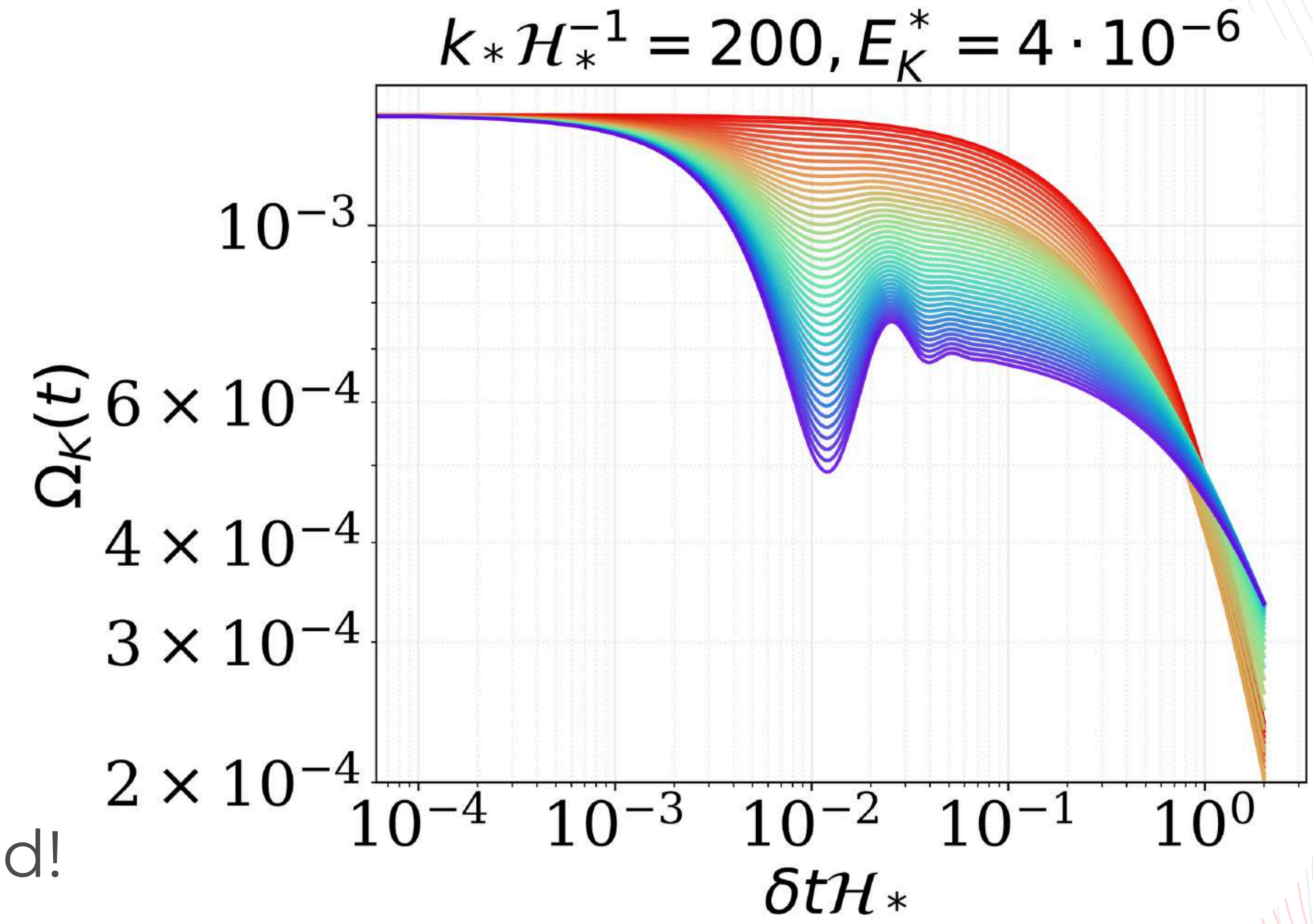
Slowly decaying sources

Parametrization of the decay

$$\Omega_K(t) = \Omega_K^* \times \left(1 + \frac{\delta t}{\delta t_{\mathrm{dec}}}\right)^{-p} \times \left(e^{-A\delta t} \times (\cos\left[B \cdot 2c_s k_* \delta t\right] + C) \cdot D + E\right)$$

Name	δt_{dec}	p	A	B	C	$\log_{10} D$	$\log_{10} E$
q0	1.8	2.35	-	-	-	-	-
q025	1.8	2.35	170	1.0	0.0	−1.4202	−0.6990
q05	1.8	1.70	130	1.0	0.0	−1.4202	−1.0000
q075	1.8	1.10	110	1.0	0.0	−1.4202	−1.2219
q1	1.8	1.00	90	1.0	0.0	−1.4202	−1.3979

$$p(q) = q^{-0.65}$$
$$A(q) = 90 \cdot q^{-0.5}$$
$$E(q) = 0.04 \cdot q^{-1.2}$$



Further studies need to be performed!